EX_PARCIAL_Q2_2016-17_Problema_3.pdf

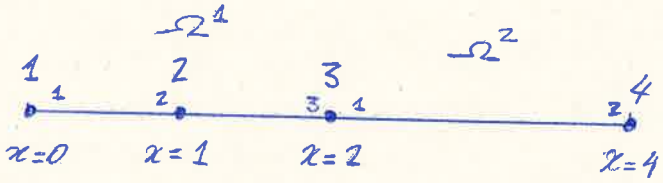
MÈTODES NUMÈRICS : Ex Parcial (a) Q2-2016-17

Problema 3.

We want to study the equation:

$$-\frac{d}{dx}\left((x+4)\frac{du}{dx}\right) = f(x), \quad x \in (0,4)$$

using FEM with a quadratic element $\Omega^1=[0,2]$ and a linear element $\Omega^2=[2,4]$ with the numbering shown in the figure:



(a) The element K^1_{33} of the local stiff matrix K^1 of the element Ω^1 is (Hint: you can use the formula $\int_0^2 (ax+b)(cx^2+dx+e)dx = a(4c+\frac{8}{3}d+2e)+b(\frac{8}{3}c+2d+2e)$)

$$\psi_3^1(x) = \frac{x(x-1)}{2\cdot(2-1)} = \frac{1}{2}(x^2-x), \quad \frac{d\psi_3^1}{dx} = x-\frac{1}{2}$$

$$K^1_{33} = K^{1,1}_{33} = \int_{x_1^1}^{x_3^1} q_1(x)\,\frac{d\psi_3^1}{dx}\cdot\frac{d\psi_3^1}{dx}dx = \int_0^1 (x+4)\left(x-\frac{1}{2}\right)^2 dx$$

$$= \int_0^2 (x+4)\left(x^2-x+\frac{1}{4}\right)dx = 4-\frac{8}{3}+\frac{1}{2}+4\left(\frac{8}{3}-2+\frac{1}{2}\right)$$

$a=1$, $b=4$, $c=1$, $d=-1$, $e=1/4$

$$= \frac{24-16+3}{6} + 4\,\frac{16-12+3}{16} = \frac{11}{6}+\frac{28}{6}$$

$$= \frac{39}{6} = \frac{13}{2} = \boxed{6.5}$$

(b) The element K_{33} of the global stiff matrix K

- Using prob. 1 $K^2 = K^{2,1} = \frac{1}{2}\left(1\cdot\frac{2+4}{2}+4\right)\cdot\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} = \frac{7}{2}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$

Per tant: $K^2_{11} = \frac{7}{2} = 3.5$. Aleshores $K_{33} = K^1_{33} + K^2_{11} = 6.5 + 3.5 = \boxed{10}$