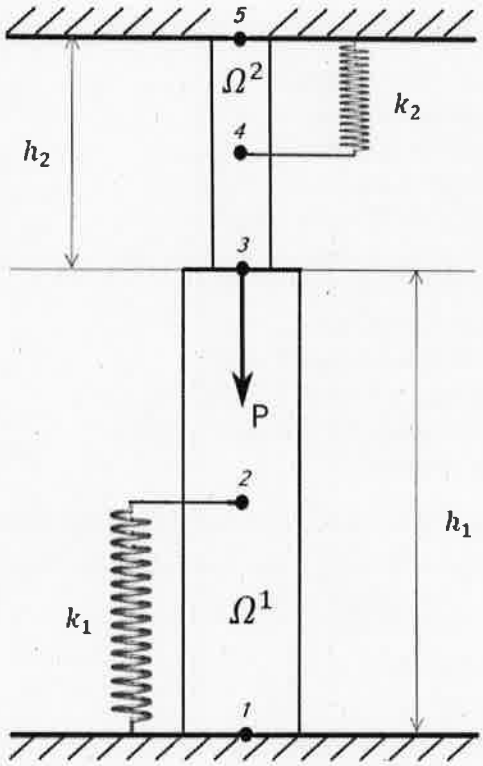
Examen Final 09-07-2021
Problema 1 (T1)

QuizAmbApartats

1. **FEM 1D**

DESCRIPTION



The vertical column in the figure above, embedded between the floor and the ceiling, is made of two block pieces. The lower one with section area $A_1 = 15$ cm^2, length $h_1 = 4$ m, Young modulus $E_1 = 8$ N/m^2 and specific weight $w_1 = 0.001$ N/m^3; the upper piece with section area $A_2 = 10$ cm^2, length $h_2 = 2$ m, Young modulus $E_2 = 6$ N/m^2 and specific weight $w_2 = 0.003$ N/m^3. Attached at the centre of each block, there are two springs, both initially at rest, with constants $k_1 = 0.001$ N/m (for the one fixed at the floor in the figure) and k_2 (for the other one, the fixed at the ceiling). Next, at the point where the two blocks join, we add a load $P = 2 \cdot 10^{-5}$ N acting downwards.

Take two quadratic elements, Ω^1 and Ω^2, to modelise the lower and the upper blocs respectively, and list the global nodes in ascending order (as shown in the figure). This havig said, apply the FEM to compute,

2. Question 1

CLOZE | 1 point | 0.10 penalty

(a) for k_2 = 2.00e-03 N/m, the displacement, U_3, of the joint between the two blocks when the struture is loaded with the weight P

MULTI | 1 point | Single | Shuffle

- U_3 = -3.84149e+00 mm ✓
- U_3 = -3.84020e+00 mm (−25%)
- U_3 = -3.84277e+00 mm (−25%)
- U_3 = -3.83892e+00 mm (−25%)
- U_3 = -3.84406e+00 mm (−25%)
- U_3 = -3.83765e+00 mm (−25%)
- U_3 = -3.84535e+00 mm (−25%)
- U_3 = -3.83637e+00 mm (−25%)
- U_3 = -3.84664e+00 mm (−25%)
- U_3 = -3.83510e+00 mm (−25%)
- None of the given the answers (−25%)
- Blank (no penalty)

Hint: the lower bolck's mid point's diplacement is U_2 = -2.04305e+00

(b) For the same value of k_2, the reaction force, R, made by the floor

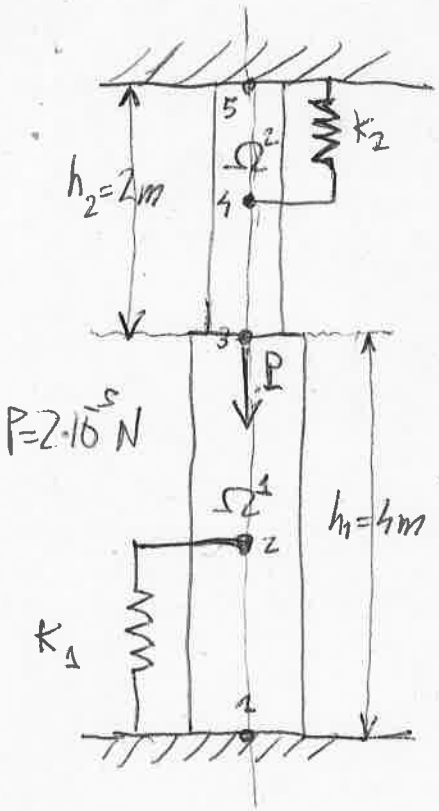
MULTI | 1 point | Single | Shuffle

- R = 1.35029e-05 N ✓
- R = 1.34994e-05 N (−25%)
- R = 1.35065e-05 N (−25%)
- R = 1.34958e-05 N (−25%)
- R = 1.35100e-05 N (−25%)
- R = 1.34923e-05 N (−25%)
- R = 1.35136e-05 N (−25%)
- R = 1.34888e-05 N (−25%)
- R = 1.35172e-05 N (−25%)
- R = 1.34853e-05 N (−25%)
- None of the given the answers (−25%)
- Blank (no penalty)

(c) Still for the same value of k_2, the force acting on the spring fixed to the ceiling

MULTI | 1 point | Single | Shuffle

- F = 3.85910e-06 N ✓



$A_1 = 15\,cm^2 = 15\times10^3\ mm^2$

$E_1 = 8\ N/m^2 = 8\cdot10^{-6}\ N/mm^2$

$h_1 = 4m = 4\cdot10^3$

$W_1 = 0.001\ N/m^3 = 10^{-12}\ N/cm^3$

$A_2 = 10\,cm^2 = 10^3\ mm^2$

$E_2 = 6\ N/m^2 = 6\times10^{-6}\ N/mm^2$

$h_2 = 2m = 2\times10^3\ mm$

$W_2 = 0.003\ N/m^3 = 3\times10^{-12}\ N/mm^3$

$k_1 = 0.001\ N/m = 10^{-3}\ N/m \cdot \frac{1m}{10^3 mm} = \boxed{10^{-6}\ N/mm}$

$k_2 = 0.002\ N/m = 2\times10^{-3}\ N/m \cdot \frac{1m}{10^3 mm} = \boxed{2\times10^{-6}\ N/mm}$

$$\frac{E_1A_1}{3h_1} = \frac{8\cdot10^{-6}\times15\times10^2}{12\cdot10^3} = \frac{120\times10^{-4}}{12\cdot10^3} = 10\times10^{-7} = 10^{-6}$$

$$\frac{W_1A_1h_1}{6} = \frac{10^{-12}\times15\times10^2\times4\times10^3}{6} = \frac{60\times10^{-7}}{6} = 10^{-6}$$

$$K^1 = \frac{E_1A_1}{3h_1}\begin{pmatrix}7&-8&1\\-8&16&-8\\1&-8&7\end{pmatrix} = 10^{-6}\begin{pmatrix}7&-8&1\\-8&16&-8\\1&-8&7\end{pmatrix}$$

$$F^1 = -\frac{W_1A_1h_1}{6}\begin{pmatrix}1\\4\\1\end{pmatrix} = -10^{-6}\begin{pmatrix}1\\4\\1\end{pmatrix}$$

$$\frac{E_2A_2}{3h_2} = \frac{6\times10^{-6}\times10\times10^2}{3\times2\times10^3} = 10^{-6}$$

$$\frac{W_2A_2h_2}{6} = \frac{3\times10^{-12}\times10\times10^2\times2\times10^3}{6} = 10\cdot10^{-7} = 10^{-6}$$

$$K^2 = \frac{E_2A_2}{3h_2}\begin{pmatrix}7&-8&1\\-8&16&-8\\1&-8&7\end{pmatrix} = 10^{-6}\begin{pmatrix}7&-8&1\\-8&16&-8\\1&-8&7\end{pmatrix}$$

$$F^2 = -\frac{W_2A_2h_2}{6}\begin{pmatrix}1\\4\\1\end{pmatrix} = -10^{-6}\begin{pmatrix}1\\4\\1\end{pmatrix}$$

Coupled system:

$$10^{-6}\begin{pmatrix} 7 & -8 & 1 & & \\ -8 & 16 & -8 & & \\ 1 & -8 & 14 & -8 & 1 \\ & & -8 & 16 & -8 \\ & & 1 & -8 & 7 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{pmatrix} = -10^{-6}\begin{pmatrix} 1 \\ 4 \\ 2 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} Q_1 \\ -K_1 U_2 \\ -R \\ -K_2 U_4 \\ Q_5 \end{pmatrix}$$

$P = 2.10^{-5} N$

Reduced System:

$$\begin{pmatrix} 16 & -8 & \\ -8 & 14 & -8 \\ & -8 & 16 \end{pmatrix}\begin{pmatrix} U_2 \\ U_3 \\ U_4 \end{pmatrix} = -\begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} + \begin{pmatrix} -10^6 K_1 U_2 \\ -20 \\ -10^6 K_2 U_4 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 16+10^6 K_1 & -8 & 0 \\ -8 & 14 & -8 \\ & -8 & 16+10^6 K_2 \end{pmatrix}\begin{pmatrix} U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} -4 \\ -22 \\ -4 \end{pmatrix}$$

$(K_1 = 10^{-6},\ K_2 = 2.10^{-6})$

$U_1 = 0$ (from the B.C)

$$\begin{pmatrix} 17 & -8 & 0 \\ -8 & 14 & -8 \\ 0 & -8 & 18 \end{pmatrix}\begin{pmatrix} U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} -4 \\ -22 \\ -4 \end{pmatrix} \Rightarrow$$

$U_2 = -2.04305$ mm.
$U_3 = -3.84149$ mm.
$U_4 = -1.92955$ mm.

$U_5 = 0$ (from the B.C)

$$Q_1 = 10^{-6}\underbrace{(7.U_1}_{=0} - 8U_2 + U_3) - \underbrace{F_1}_{=-10^{-6}} = 1.35029 \times 10^{-5} N$$ (1st component of react. forces).

Force acting on the spring with constant K_2: $R = -K_2 U_4 = 3.85910 \times 10^{-6} N$