EX_REAVA_2017_18_Problema_1.pdf

MÈTODES NUMÈRICS

Ex-Reavaluació · 2018 (a)

Problema 1

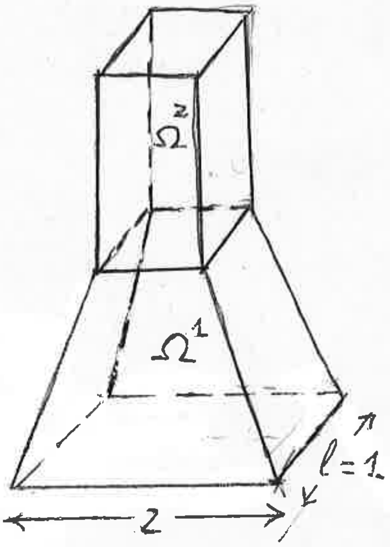
We consider a concrete pillar loaded at the top with a load $-F$, and composed of two 1D elements Ω^1 and Ω^2 with heights a and b respectively, as shown in the figure. The equation that models the displacement $u(x)$ from the equilibrium is

$$-\frac{d}{dx}\left(EA(x)\frac{du}{dx}\right) = -w\,A(x)$$

where w is the specific weight of the concrete, E is the Young modulus, that we suppose constant in all the pillar. As seen in the figure, we suppose the upper part Ω^2 has a constant cross section area $A(x)\equiv 1$; while the lower, Ω^1, $A(x)$ is a rectangle where the smaller side is constantly 1 while the biggest varies linearly between 2 at the bottom to 1 at the top. Now consider $a=3$ and $b=2$.

Fill the following table:

Hint 1. If it is needed, you can use $\int_0^a (2a-x)\,x\,dx = \frac{2}{3}a^3$



$\psi_1^1(x) = 1-x/3$

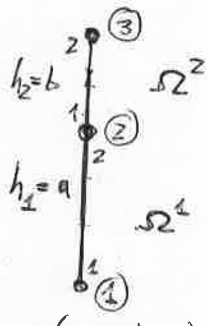
$K^1 = \frac{E}{2}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$

$F^2 = -\frac{W}{2}\begin{pmatrix}2\\2\end{pmatrix}$

Assembled system: $\frac{E}{2}\begin{pmatrix}1 & -1 & \\ -1 & 2 & -1\\ & -1 & 1\end{pmatrix}\begin{pmatrix}U_1\\U_2\\U_3\end{pmatrix} = -\frac{W}{2}\begin{pmatrix}5\\6\\2\end{pmatrix} + \begin{pmatrix}Q_1\\Q_2\\Q_3\end{pmatrix}$

Boundary conditions: $Q_2=0,\ Q_3=-F;\ U_1=0$

Displacement of the central point: $U_2 = -\frac{2}{E}(F+4W)$



Solució: $\psi_1^1(x) = \frac{x-a}{a} = 1-x/a \underset{a=3}{=} 1-x/3$

$$K^1 = K^{1,1} = \frac{1}{a}\left(-\frac{E}{3}\cdot\frac{0+a}{2} + 2E\right)\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} \underset{a=3}{=} \frac{E}{2}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$$

$$q_1(x)\Big|_{\Omega^1} = \alpha x+\beta = EA_1(x) = E\left(2-\frac{x}{a}\right)\cdot l \underset{\substack{a=3\\ l=1}}{=} -\frac{E}{3}x + 2E \quad \text{d'on}:\ \alpha=-\frac{E}{3},\ \beta = 2E$$

i. recordem que (generalització del problema 1), si Ω^e és un element lineal (2 nodes), amb $a_1(x)|_{\Omega^e} = a_1^e(x) = \alpha x + \beta$, llavors:

$$K^{e,1} = \frac{1}{h_e}\left(\alpha \frac{x_1^e + x_2^e}{2} + \beta\right)\cdot\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

on: x_1^e posició del 1er node de l'element Ω^e.

x_2^e " " 2on " " " Ω^e.

$h_e := x_2^e - x_1^e$: longitud de l'element Ω^e.

$$a_1(x)\Big|_{\Omega^2} = a_1^2(x) \equiv EA_2(x) - E \text{ , d'on: } K^2 = K^{3,1} = \frac{EA_2}{b}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = E/2 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$A_2(x) \equiv 1\cdot 1 = 1$$

Calcul dels vectors de carrega elementals F^1 i F^2:

$$\psi_1^1(x) = 1 - \frac{x}{a}, \quad \psi_2^1(x) = \frac{x}{a}; \quad \psi_1^2(x) = \frac{a+b-x}{b}, \quad \psi_2^2(x) = \frac{x-a}{b}$$

$$F_1^1 = -W\int_0^a A_1(x)\psi_1^1(x)\,dx = -W\int_0^a \left(2-\frac{x}{a}\right)\left(1-\frac{x}{a}\right)dx = \left\{\begin{matrix} \text{c.v.} \\ t = 1-\frac{x}{a} \end{matrix}\right\}$$

$$= -Wa\int_0^1 (1+t)\,t\,dt = -aW\left(\frac{1}{2}+\frac{1}{3}\right) = -\frac{5aW}{6} \underset{a=3}{=} -\frac{5W}{2}$$

$$F_2^1 = -W\int_0^a A_1(x)\psi_2^1(x)\,dx = -W\int_0^a\left(2-\frac{x}{a}\right)\frac{x}{a}\,dx = -Wa\int_0^1 (1+t)(1-t)\,dt$$

$$= -Wa\left(1-\frac{1}{3}\right) = -\frac{2aW}{3} = -2W.$$

$$F_1^2 = -W\int_a^{a+b} A_2(x)\psi_1^2(x)\,dx = -\frac{W}{b}\int_a^{a+b}(a+b-x)\,dx = \left\{\begin{matrix} \text{c.v.} \\ t = a+b-x \end{matrix}\right\} = -\frac{W}{b}\int_0^b t\,dt = -\frac{Wb}{2} \underset{b=2}{=} -W$$

$$F_2^2 = -W\int_a^{a+b} A_2(x)\psi_2^2(x)\,dx = -\frac{W}{b}\int_a^{a+b}(x-a)\,dx = \left\{\begin{matrix} \text{c.v} \\ t = x-a \end{matrix}\right\} = -\frac{W}{b}\int_0^b t\,dt = -\frac{Wb}{2} = -W$$

d'on: $F^1 = -\frac{W}{2}\begin{pmatrix} 5 \\ 4 \end{pmatrix}$, $F^2 = -\frac{W}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

- Sistema acoplat: $E/2 \begin{pmatrix} 1 & -1 & \\ -1 & 2 & -1 \\ & -1 & 1 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = -\frac{W}{2}\begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix} + \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$

- B.C. (i) Naturals: $\begin{pmatrix} Q_2 = 0 \\ Q_3 = -P \end{pmatrix}$, (ii) Essencials: $U_1 = 0$.

- Reduced system:

$$+E\,U_2 - E/2\,U_3 = -3W$$
$$-E/2\,U_2 + E/2\,U_3 = -W - P$$

Solució: $U_2 = -2/E\,(P+4W)$

$U_3 = -2/E\,(2P+5W)$