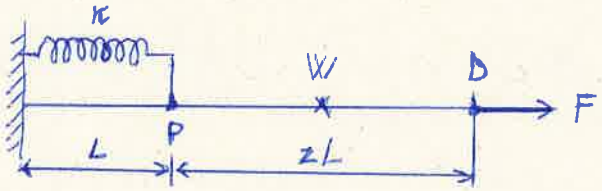
EX_FINAL_Q1_2020_21_Problema_1.pdf

Final 2020-21 Q1

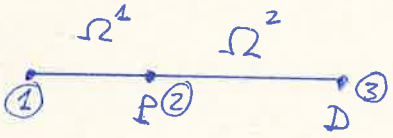
Problema 1

Consider a bar of length $3L$ clamped in a wall at its left end (O) and pulled at the right end (D) by a force F. at a distance L from the wall, the point P of the bar is fixed to a spring of constant K which is clamped to the wall at the other end (see the figure). The point W is located in the middle point between P and D.



Meshing the bar with two linear elements $\Omega^1=[0,L]$ (from O to P) and $\Omega^2=[L,3L]$ (from P to D) and taking the following numerical values and functions (all them assumed in some coherent units), answer the questions that follow.

$$L=4,\quad K=3,\quad a_1(x)=A(x)E(x)=\begin{cases}5, & x\in[0,L],\\ 2x, & x\in[L,3L].\end{cases}$$



◁ Solució (a) The value $K^1_{11}=5/4=1.250$.

$$k^{1,1}=\frac{a_1}{h_1}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}=\frac{5}{4}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}$$

$h_1=L=4$

(b) $K^{2,1}=\frac{2}{2L}\cdot\frac{L+3L}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}=\begin{pmatrix}2&-2\\-2&2\end{pmatrix}$, $K^{2,0}=(0)$; $K^2=K^{2,1}+K^{2,0}=\begin{pmatrix}2&-2\\-2&2\end{pmatrix}$

problema (1) amb $a_1=2$, $h_2=2L$

Assembled matrix $K=\begin{pmatrix}5/4&-5/4&\\-5/4&13/4&-2\\&-2&2\end{pmatrix}$,

The value of K_{22} of the assembled matrix is $K_{22}=13/4=3.25$

(c)
$$\begin{pmatrix} 5/4 & -5/4 & \\ -5/4 & 13/4 & -2 \\ & -2 & 2 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} Q_1 \\ -KU_2 \\ F \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

B.C. Natural $Q_2 = -KU_2$, $Q_3 = F$

B.C. Essential $U_1 = 0$

Reduced system:

$$\left.\begin{aligned} \left(13/4 + K\right) U_2 - 2U_3 &= 0 \\ -2U_2 + 2U_3 &= F \end{aligned}\right\} : \left(\frac{13}{4} + K - 2\right) U_2 = F$$

$$U_3 = \frac{F}{2} + U_2 = \frac{F}{2} + \frac{F}{13/4 + K - 2} \underset{K=3}{=} \frac{F}{2} + \frac{4}{17}F = \frac{25}{34}F = 7.35294 \times 10^{-1} F$$

The displacement of the point D in terms of F is: $U_3 = \frac{25}{34}F = 7.35294 \times 10^{-1} F$

(d) The spring force in terms of F is:

$$F_s = -K U_2 = -3 \cdot \frac{4}{17} F = -\frac{12}{17} F = -7.05882 \times 10^{-1} F$$

$K = 3$:

$$U_2 = \frac{F}{13/4 + K - 2} = \frac{F}{13/4 + 1} = \frac{4}{17} F$$

(e) Compute the displacement of W when the force F is applied:

Interpolant: $U = U_2 \psi_1^2(2L) + U_3 \psi_2^2(2L) = \frac{1}{2}(U_2 + U_3) F = \frac{1}{2}\left(\frac{4}{17} + \frac{25}{34}\right) F = \frac{33}{68} F = 4.8529 \times 10^{-1} F$

$$\psi_1^2(x) = \frac{x - 3L}{-2L}, \quad \psi_1^2(2L) = 1/2$$

$$\psi_2^2(x) = \frac{x - L}{2L}, \quad \psi_2^2(2L) = 1/2$$