Parcial 10-11-2021

A certain 1D process is simulated by the following coupled problems:

$$\begin{cases} -\dfrac{d}{dx}\left(2e^{x}\dfrac{du}{dx}\right)=10, & x\in(0,0.5) \\ u(0)=0,\ u(0.5)=v(0.2) \end{cases} \qquad \begin{cases} -\dfrac{d}{dy}\left(3\dfrac{dv}{dy}\right)=20, & y\in(0,0.2) \\ v(0)=0,\ v'(0.2)=u'(0.5) \end{cases}$$

which are related by the following compatibility conditions for $u(x)$ and $v(y)$

$$\begin{cases} u(0)=v(0)=0 \\ u(0.5)=v(0.2)=\alpha \\ u'(0.5)=u'(0.2)=\beta \end{cases}$$

with α,β paràmetres to be determined.

We apply the FEM to both equations using <u>only one linear</u> element in each case and let us assume that $[K^1]u=F^1+Q^1$ and $[K^2]v=F^2+Q^2$ are the respective element associated systems.

Questions

(a) The value of K^1_{22} is ...

(b) The value of F^1_2 is ...

(c) Taking into account the compatibility conditions (value and derivative of u and v) at the end nodes of each element, the value α we obtain for the common node is, ...

Solution

$$K^1_{11}=4\int_0^{1/2}2e^{x}dx=8\int_0^{1/2}e^{x}dx=8\cdot(\sqrt{e}-1)$$

$$K^1_{12}=-8(\sqrt{e}-1)=K^1_{21}$$

$$K^1_{22}=K^1_{11}=8(\sqrt{e}-1)\text{, and hence: } \boxed{K^1_{22}\cong 5.1898} \quad (a)$$

$$K^1=8(\sqrt{e}-1)\begin{pmatrix}1&-1\\-1&1\end{pmatrix},\quad F^1=\tfrac{5}{2}\begin{pmatrix}1\\1\end{pmatrix},\quad \text{So: } \boxed{F^1_2=\tfrac{5}{2}=2.5} \quad (b)$$

$$K^2=15\cdot\begin{pmatrix}1&-1\\-1&1\end{pmatrix},\qquad F^2=2\begin{pmatrix}1\\1\end{pmatrix}$$

$$\Omega^1:\quad 8(\sqrt{e}-1)\cdot\begin{pmatrix}1&-1\\-1&1\end{pmatrix}\begin{pmatrix}U^1_1\\U^1_2\end{pmatrix}=\begin{pmatrix}5/2\\5/2\end{pmatrix}+\begin{pmatrix}Q^1_1\\2e^{1/2}\beta\end{pmatrix}:\quad \begin{aligned}&U^1_1=u(0)=0\\&8(\sqrt{e}-1)U^{2\,(*)}_1=8(\sqrt{e}-1)\alpha=\tfrac{5}{2}+2\sqrt{e}\beta\\&{}^{(*)}\,U^2_1=u(0)=\alpha\end{aligned}$$

$$\Omega^2:\quad 15\begin{pmatrix}1&-1\\-1&1\end{pmatrix}\begin{pmatrix}U^2_1\\U^2_2\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}+\begin{pmatrix}Q^2_1\\3\beta\end{pmatrix}:\quad \begin{aligned}&U^2_1=V(0)=0\\&15U^2_2=15V(0.2)=15\alpha=2+3\beta\end{aligned}$$

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$$\begin{cases} 8\left(\sqrt{e}-1\right)\alpha - 2\sqrt{e}\,\beta = 5/2 \\ 15\,\alpha - 3\beta = 2 \end{cases} \Leftrightarrow \begin{cases} \boxed{\alpha \cong -0.0267} \quad (c) \\ \beta \cong -0.8002 \end{cases}$$

$\square$