EX-FINAL_Q2_2017-18_Problema_1.pdf

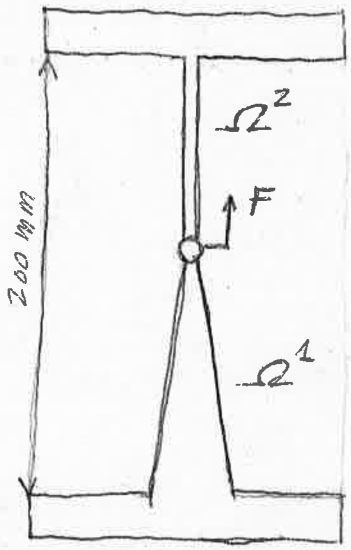
MÈTODES NUMÈRICS: Ex Final

(a) Q2-2017-18

1. We consider the figure where a thin rod made of two parts of the same length is fixed betwen two surfaces at a distance of 200 mm. The upper part of the rod has a constant cross section area of $1\ mm^2$ and Young modulus $100\ N/mm^2$. The lower part has a cross-sectional area that varies in a linear way from $3\ mm^2$ at the bottom to $1\ mm^2$ at the top. The Young modulus of the lower part is $200\ N/mm$

We assume that ~~the~~ at the join point of the two parts we apply an upward load force of 5N.

Hint. Remember that the elastic 1D eq is $-\frac{d}{dx}\left(EA(x)\frac{du}{dx}\right)=0$.



We take a mesh of two 1D linear elements, one for each part, where Ω^1 is the lower one and Ω^2 the upper one. Considering the length units in mm, fill the following table.

$$\psi_2^1(x) = x/100$$

$$K^1 = K^{1,1} = \begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix} \quad (\text{ja que } K^{1,0}=0)$$

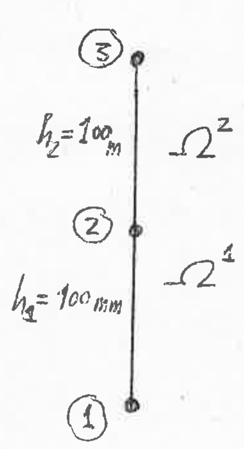
$$K^2 = K^{2,0} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad (\text{" "}\ K^{2,0}=0)$$

Assembled system:
$$\begin{pmatrix} 4 & -4 & \\ -4 & 5 & -1 \\ & -1 & 1 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$$

Boundary conditions: Natural $Q_2 = F = 5$. Essential $U_1 = U_3 = 0$.

Displacement of the central point: $U_2 = F/5 = 5/5 = 1$ mm.

4 Solució:



$h_2 = 100$ mm

$h_1 = 100$ mm

$A_2(x) = 1\ mm^2$

$A_1(x) = 3 - x/50$

$E_2 = 100\ N/mm^2$

$E_1 = 200\ N/mm^2$

Ω^1; $h_1 = 100$ mm, $A_1(x) = 3 - x/50$, $E_1 = 200\ N/mm^2$

$$\psi_1^1(x) = \frac{x-100}{-100} = \frac{1}{100}(100-x) = 1 - \frac{x}{100}; \quad \frac{d\psi_1^1}{dx} = -\frac{1}{100};$$

$$\psi_2^1(x) = \frac{x}{100}; \quad \frac{d\psi_2^1}{dx} = \frac{1}{100}$$

$$K_{11}^{1,1} = \int_{x_1^1}^{x_2^1} E_1 A_1(x) \frac{d\psi_1^1}{dx}\cdot\frac{d\psi_1^1}{dx}\,dx = \frac{200}{10^4}\int_0^{100}\left(3 - \frac{x}{50}\right)dx$$

$$= \frac{200}{10^4}\left(3x - \frac{x^2}{100}\right)\Big|_0^{100} = \frac{200}{10^4}(300-100) = \frac{4\times10^4}{10^4} = 4.$$

Òbviament: $K_{22}^{1,1} = K_{11}^{1,1} = 4, \quad K_{12}^{1,1} = K_{21}^{1,1} = -4$ (vegeu Remarca 1)

Ω^2 : $h_2 = 100$ mm, $A_2(x) = 1\,mm^2$, $E_2 = 100\ N/mm^2$

$$K^2 = K^{2,1} = \frac{100}{100}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} = \begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} \quad \left(K^{2,0} = (0)\right).$$

Assembled system:
$$\begin{pmatrix}4 & -4 & \\ -4 & 5 & -1\\ & -1 & 1\end{pmatrix}\begin{pmatrix}U_1\\ U_2\\ U_3\end{pmatrix} = \begin{pmatrix}Q_1\\ Q_2\\ Q_3\end{pmatrix} + \begin{pmatrix}F_1=0\\ F_2=0\\ F_3=0\end{pmatrix}$$

ja que $f(x) \equiv 0$ a l'equació.

B.C.

- Natural: $Q_2 = F = 5$
- Essential: $U_1 = U_3 = 0$.

Reduced system: $5U_2 = F \Longrightarrow \boxed{U_2 = F/5 = 5/5 = 1 \text{ mm}}$

Remarca 1: podem també aplicar un resultat semblant (de fet, una generalització) del problema 1: Si Ω^e és un element lineal (2 nodes), de longitud h_e, on $a(x)|_{\Omega^e} =$

$$= q_1^e(x) = \alpha x + \beta \longrightarrow K^{e,1} = \frac{1}{h_e}\left(\alpha\,\frac{x_1^e + x_2^e}{2} + \beta\right)\cdot\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} =$$

$$= \frac{1}{100}\left(-4 \times \frac{100}{2} + 600\right)\cdot\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} = \begin{pmatrix}4 & -4\\ -4 & 4\end{pmatrix}$$

$\alpha = 4$

$\beta = 600$

$x_1^e = 0$: posició del 1er node local de Ω^e $(e=1)$

$x_2^e = 100$: " " 2on " " " Ω^e (")