EX_FINAL_Q1_2017_18_Problema_5.pdf

MÈTODES NUMÈRICS. Ex. Final

(a) Q1-2017-18

(5) Consider a conic column made of a new material with Young's module denoted by E. The column is 3m height and has a circular radius $r=4m$ and $r=1m$ at the bottom and top respectively, and assume it grows linearly. The bottom is fixed to the ground and on the top of the column a weight of value F is applied. We want to study the displacements of the points of heights 2 and 3.

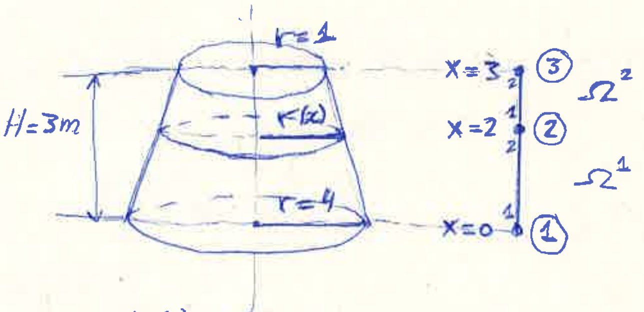
To this end, consider the equation of elasticity

$$-\frac{d}{dx}\left(EA(x)\frac{du}{dx}\right) = -wA(x), \quad x \in (0,3)$$

where $A(x)$ is the section area at height x and w is its specific weight. We will solve this equation using two linear elements $\Omega^1 = [0,2]$ and $\Omega^2 = [2,3]$.

Compute the following values:

(a) The shape functions of both elements: $\psi_1^1(x), \psi_2^1(x), \psi_1^2(x), \psi_2^2(x)$



$r(x) = 4-x$

$A(x) = \pi r^2(x) = \pi(4-x)^2$

$$\psi_1^1(x) = \frac{x-2}{0-2} = \frac{1}{2}(2-x) = 1-\frac{x}{2}, \quad \psi_2^1(x) = \frac{x}{2-0} = \frac{x}{2}$$

$$\psi_1^2(x) = \frac{x-3}{2-3} = 3-x, \quad \psi_2^2(x) = \frac{x-2}{3-2} = x-2$$

(b) For the element stiff matrices K^1 and K^2, the values $K^1_{22}, K^2_{11}, K^2_{12}$ and K^2_{22} (in terms of E and π).

$$K^1_{22} = K^{1,0}_{22} = \frac{E}{4}\int_0^2 \pi(4-x)^2 dx = \frac{\pi E}{4}\left[-\frac{(4-x)^3}{3}\right]_0^2 = \frac{\pi E}{4}\left(-\frac{8}{3}+\frac{64}{3}\right) = \boxed{\frac{14\pi E}{3}}$$

$$K^2_{11} = K^{2,0}_{11} = \pi E\int_2^3 (4-x)^2 dx = -\frac{E\pi}{3}\left[(4-x)^3\right]_2^3 = -\frac{\pi E}{3}(1-8) = \boxed{\frac{7\pi E}{3}} = K^2_{22}$$

$$K_{12}^{2} = +K_{21}^{2} = -K_{22}^{2} = \boxed{-\frac{7\pi E}{3}}$$

(c) For the load vectors F^1, F^2 the values of F_2^1, F_1^2, F_2^2 (as a function of π and W).

Hint: if you need, you can use that $\int_{x_1}^{x_2} (a-x)\cdot(x-4)^2 dx = \frac{(4-x)^3}{12}\cdot\left(4(1-a)+3x\right)\Big|_{x_1}^{x_2}$

$$F_2^1 = -W\int_0^2 A(x)\,\psi_2^1(x)\,dx = -\pi W\int_0^2 (4-x)^2\cdot \frac{x}{2}\,dx = \boxed{-\frac{22\pi W}{3}}$$

$$F_1^2 = -W\int_2^3 \underbrace{A(x)\,\psi_1^2(x)}_{(x-4)^2\cdot(3-x)}\,dx \overset{a=3}{=} -\pi W\left(\frac{1}{12}(-8+9)-\frac{8}{12}(-8+6)\right) = -\pi W\,\frac{1+16}{12} = \boxed{-\frac{17\pi W}{12}}$$

$$F_2^2 = -W\int_2^3 A(x)\,\psi_2^2(x)\,dx = -\pi W\int_2^3 (x-4)^2\cdot(x-2) = W\pi\left(\frac{1}{12}(-4+9)-\frac{8}{12}(-4+6)\right)$$

$$= \pi W\,\frac{5-16}{12} = \boxed{-\frac{11\pi W}{12}}$$

(d) Write the final reduced system to compute U_2 and U_3 (as a function of π, E and W).

$$\frac{7\pi E}{3}\begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} U_2 \\ U_3 \end{pmatrix} = -\frac{\pi W}{12}\begin{pmatrix} 105 \\ 11 \end{pmatrix} - \begin{pmatrix} 0 \\ F \end{pmatrix}$$

Remarca:

$$F_2 = F_2^1 + F_1^2 = -\frac{\pi W}{12}(88+17) = -\frac{105\pi W}{12}$$