Parcial 10-11-2021

A certain 1D process is simulated by the following coupled problems:

$$\begin{cases} -\dfrac{d}{dx}\left(2e^x \dfrac{du}{dx}\right) = 10, & x \in (0, 0.5) \\ u(0) = 0, \; u(0.5) = v(0.2) \end{cases} \qquad \begin{cases} -\dfrac{d}{dy}\left(3 \dfrac{dv}{dy}\right) = 20, & y \in (0, 0.2) \\ v(0) = 0, \; v'(0.2) = u'(0.5) \end{cases}$$

which are related by the following compatibility conditions for $u(x)$ and $v(y)$

$$\begin{cases} u(0) = v(0) = 0 \\ u(0.5) = v(0.2) = \alpha \\ u'(0.5) = u'(0.2) = \beta \end{cases}$$

with α, β paràmetres to be determined.

We apply the FEM to both equations using <u>only one linear</u> element in each case and let us assume that $[K^1]u = F^1 + Q^1$ and $[K^2]v = F^2 + Q^2$ are the respective element associated systems.

Questions

(a) The value of K^1_{22} is ...

(b) The value of F^1_2 is ...

(c) Taking into account the compatibility conditions (value and derivative of u and v) at the end nodes of each element, the value α we obtain for the common node is, ...

Solution

$$K^1_{11} = 4\int_0^{1/2} 2e^x dx = 8\int_0^{1/2} e^x dx = 8\cdot(\sqrt{e}-1)$$

$$K^1_{12} = -8(\sqrt{e}-1) = K^1_{21}$$

$K^1_{22} = K^1_{11} = 8(\sqrt{e}-1)$, and hence: $\boxed{K^1_{22} \cong 5.1898}$ (a)

$K^1 = 8(\sqrt{e}-1)\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, $F^1 = \frac{5}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, So: $\boxed{F^1_2 = \frac{5}{2} = 2.5}$ (b)

$$K^2 = 15\cdot\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \qquad F^2 = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Ω^1: $8(\sqrt{e}-1)\cdot\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} U^1_1 \\ U^1_2 \end{pmatrix} = \begin{pmatrix} 5/2 \\ 5/2 \end{pmatrix} + \begin{pmatrix} Q^1_1 \\ 2e^{1/2}\beta \end{pmatrix}$: $U^1_1 = u(0) = 0$

$8(\sqrt{e}-1)\, U^1_2 \overset{(*)}{=} 8(\sqrt{e}-1)\,\alpha = \frac{5}{2} + 2\sqrt{e}\,\beta$

$(*)\; U^1_2 = u(0) = \alpha$

Ω^2: $15\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} U^2_1 \\ U^2_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} Q^2_1 \\ 3\beta \end{pmatrix}$: $U^2_1 = V(0) = 0$

$15\, U^2_2 = 15\, V(0.2) = 15\alpha = 2 + 3\beta$

pàg següent →

$$\begin{aligned} 8(\sqrt{e}-1)\,\alpha - 2\sqrt{e}\,\beta &= 5/2 \\ 15\,\alpha - 3\beta &= 2 \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} &\boxed{\alpha \cong -0.0267} \quad (c) \\ &\beta \cong -0.8002 \end{aligned}$$

□