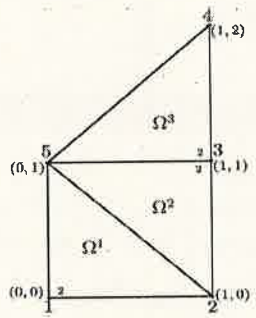
Ex_Final_Q2-2017-18_Problema2.pdf

MÈTODES NUMÈRICS: EX-Final (a) Q2-2017-18

$\psi_2^1(x) =$	$\frac{x}{100}$
$[K^1] =$	$\begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix}$
$[K^2] =$	$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$
Assembled system:	$\begin{pmatrix} 4 & -4 & 0 \\ -4 & 5 & -1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$
Boundary conditions:	$U_1 = 0$, Q_2=5, $U_3 = 0$
Displacement of the central point:	1 mm

(2 points)

2. Consider the domain shown in the figure below, and with the elements,nodes and local and global numbering plotted there.



We want to solve the thermal problem defined by the equation $-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$ with boundary conditions given by:

- $u = 50$ at the edges joining the nodes 1,2,3 and 4.
- $q_n = \frac{2}{\sqrt{2}}$ at the edge joining the nodes 4 and 5
- and at the edge that joints nodes 5 and 1, there is convection with $\beta = 6$ and $T_\infty = 20$,

then:

(a) The coefficient $K(2,2)$ of the global assembled system, is:

K(2,2)=	1

(*Hint:*It must be a value among the following ones: $0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \frac{9}{2}$)

(b) The value of coefficient Q^3_{13} of the local system for Ω^3 is

Q^3_{13}=	1

(*Hint:* It must be a value among the following ones: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$)

(c) When computing the term Q_5 of the global assembled system, it can be expressed as $Q_5 = aU_5 + b$, then:

a=	-2
b=	11

(*Hint*: a must be a value among the following ones: $-4, -2, 0, 2, 4$

and b among: $10, 11, 12, 13, 14, 15, 16, 17, 18, 19$)

(d) The value of U_5 is:

U_5=	$\frac{172}{7}$

(*Hint*: it must be a value among the following ones: $\frac{170}{7}, \frac{172}{7}, \frac{174}{7}, \frac{176}{7}, \frac{178}{7}, \frac{180}{7}, \frac{182}{7}, \frac{184}{7}, \frac{186}{7}, \frac{188}{7}$)

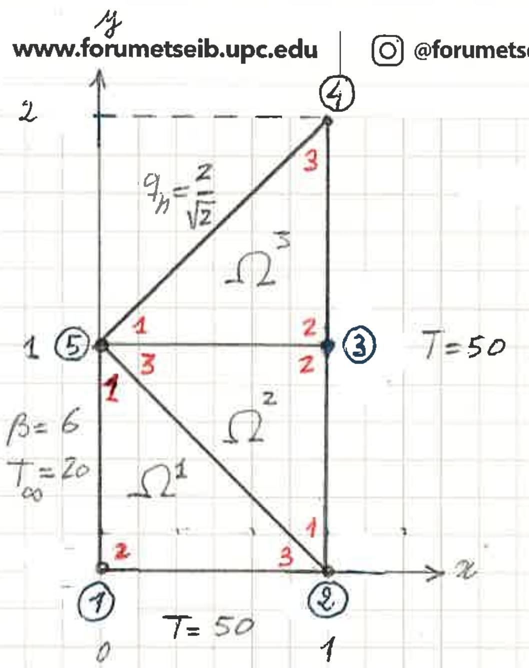
(2 points)

(3) Consider the bar structure shown in the figure below. It is made with bars of $0.005m^2$ section area and Young's module of $0.2GPa$. The length of the horizontal elements is $2.5m$ and for the vertical ones, elements 6 and 10 are also $2.5m$ long and element 8 measures $3m$. Finally, node 5 is fixed in both directions and node 1 is olny fixed in the y direction.

We want to study the displacements when an external load of magnitude $F = 2100N$ is applied on nodes 6 and 8 with the direction from node 7 to these points respectively.

Compute the displacements of node 4 and the node that **moves more** on the x direction (considering the sign).

	dx	dy
node 4	-1.0296e-03m	-1.3399e-03m



$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$

$u \equiv 50$ at the edges joining the nodes 1, 2, 3 and 4

$q_n = \frac{2}{\sqrt{2}}$ at the edge joining the nodes 4 and 5

Convection with $\beta = 6$ and $T_\infty = 20$ at the edges joining nodes 5 and 1.

(a) $K^1 = \frac{c}{2ab}\begin{pmatrix} b^2 & -b^2 & 0 \\ -b^2 & a^2+b^2 & -a^2 \\ 0 & -a^2 & a^2 \end{pmatrix} = \left\{ \begin{matrix} c = k_c = 1 \\ a = b = 1 \end{matrix} \right\} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} = K^2 = K^3$

$K_{22} = K^1_{33} + K^2_{11} = \frac{1}{2}(1+1) = \boxed{1}$

(b) $Q^3_{13} = \frac{1}{2} \cdot \frac{2}{\sqrt{2}} \cdot \sqrt{2} = \boxed{1}$ (constant flow).

(c) $Q^1_{11} = \left(-\frac{\beta T_5}{3} - \frac{\beta T_1}{6} + \frac{\beta T_\infty}{2}\right) h^1_1 = \left\{ \begin{matrix} h^1_1 = 1 \\ \beta = 6,\ T_\infty = 20 \\ T_1 = 50 \text{ (from the BC)} \end{matrix} \right\} = -2T_5 - 50 + 60 = -2T_5 + 10$

$Q_5 = aT_5 + b = Q^1_{11} + Q^3_{13} = -2T_5 + 10 + 1 = -2T_5 + 11 \Rightarrow \boxed{a = -2,\ b = 11}$

(d) Reduced system

$\begin{pmatrix} K^1_{12} & K^1_{13} + K^2_{31} & K^2_{32} + K^3_{12} & K^3_{13} & K^1_{11} + K^2_{33} & K^3_{11} \end{pmatrix} \cdot \begin{pmatrix} T_1 = 50 \\ T_2 = 50 \\ T_3 = 50 \\ T_4 = 50 \\ T_5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 & 0 & -2 & 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 50 \\ 50 \\ 50 \\ 50 \\ T_5 \end{pmatrix}$

$= -2T_5 + 11$

$\Leftrightarrow -75 + \frac{3}{2}T_5 = -2T_5 + 11 \Leftrightarrow \frac{7}{2}T_5 = 86 \Leftrightarrow \boxed{T_5 = \frac{172}{7}}$