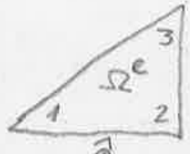
Ex. Final Q2-2020-21_Problema2.pdf

Solució:



$K^e = \frac{k_c}{2ab}\begin{pmatrix} \hat b^2 & -\hat b^2 & 0 \\ -\hat b^2 & \hat a^2+\hat b^2 & -\hat a^2 \\ 0 & -\hat a^2 & \hat a^2 \end{pmatrix}.$

with $k_c = 1/2$, and $\left\{ \begin{array}{l} \hat a = a = 1 \\ \hat b = b = 1.5 \end{array} \right\}$ for $e=1,2$

$\hat a = \hat b = 1$, for $e=3$

Començat el	dimecres, 16 juny 2021, 19:16
Estat	Acabat
Completat el	dimecres, 16 juny 2021, 19:16
Temps emprat	14 segons
Punts	0,13/1,00
Qualificació	**1,25** sobre 10,00 (**13%**)

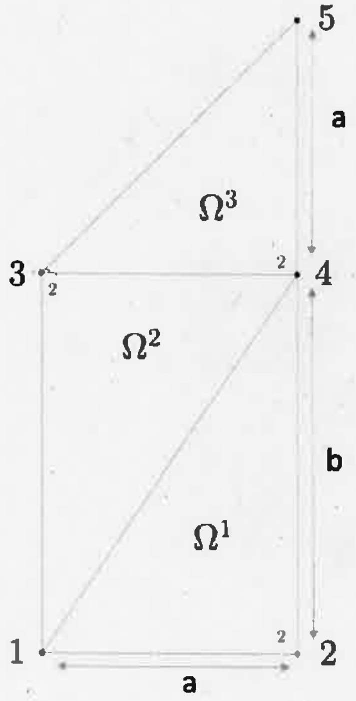
Informació

(a) $K_{43} = K^2_{12} + K^3_{21} = \frac{-1/2}{2\cdot 1\cdot 3/2}\cdot\left(\frac{3}{2}\right)^2 - \frac{1/2}{2\cdot 1^2}1^2 = -\frac{1}{6}\cdot\frac{9}{4} - \frac{1}{4} = -\frac{1}{4}\left(\frac{3}{2}+1\right)$

$= \boxed{-\frac{5}{8}} = -0.625$

Hint. $K_{44} = K^1_{33} + K^2_{11} + K^3_{22} = \frac{1}{6} + \frac{1}{6}\cdot\frac{9}{4} + \frac{1}{2} = \frac{4+9+12}{24}$

$= \boxed{\frac{25}{24}} = 1.0417\ldots$



(b) $K_{33}U_3 = Q_3$ with $Q_3 = \sigma U_3$

$K_{33} = K^2_{22} + K^3_{11} = \frac{1}{6}\left(1+\frac{9}{4}\right) + \frac{1}{4}1 = \frac{13}{24} + \frac{1}{4} = \frac{19}{24}$

$K_{33}U_3 = \sigma U_3 \Leftrightarrow \sigma = K_{33} = \boxed{\frac{19}{24}} = 0.79167\ldots$

(c) $q^3_3(s) = \left\langle \begin{pmatrix} k_c & \\ & k_c \end{pmatrix}\begin{pmatrix} u_x \\ u_y \end{pmatrix}, \frac{1}{|a|\sqrt{2}}\begin{pmatrix} -a \\ a \end{pmatrix} \right\rangle \Big|_{\Gamma^3_3} = \{k_c = \tfrac{1}{2}, a = 1\}$

$= \frac{1}{2\sqrt{2}}\left(\frac{\partial u}{\partial y} - \frac{\partial u}{\partial x}\right)\Big|_{\Gamma^3_3} \equiv \frac{\mu}{2\sqrt{2}}, \quad 0 \le s \le |a|\sqrt{2} \overset{a=1}{=} \sqrt{2}$

$Q_3 = \underbrace{Q^2_2}_{=0} + Q^3_{13} = h^3_3 \frac{q^3_3}{2} = \cancel{\sqrt{2}}\frac{\mu}{4\cancel{\sqrt{2}}} = \frac{\mu}{4} = \frac{2.20}{4} = \boxed{0.55}$

Pregunta **1**

Parcialment correcte

Puntuació 0,13 sobre 1,00

Consider the Poisson equation $-k_c(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) = 0$ on a domain shown below with the three finite elements, nodes and local and global numbering plotted there

(a)(points=4) If $k_c = 0.5$, $a = 1$ and $b = 1.5$ the value of K_{43} of the global stiffness matrix K is

- -2.88e-01 ✖
- Leave it empty (no penalty)
- -3.77e-01
- -5.89e-01
- -6.25e-01

La resposta correcta és: -6,25e-01

Check: the value of K_{44} is equal to $1.04e+00$.

(b)(points=3) Now consider that $u = 0$ at the edges $\Gamma^1_1 \cup \Gamma^1_2 \cup \Gamma^3_2$. Then compute σ such that $Q_3 = \sigma U_3$

- 1.34e+00 ✖
- 7.92e-01

Leave it empty (no penalty)

3.08e-01

3.58e-01

La resposta correcta és: 7.92e-01

(c)(points=3) Now suppose that, besides the essential BC established in the previous part b), we have natural BC: $\frac{\partial u}{\partial y} - \frac{\partial u}{\partial x} = \mu$ in Γ_3^3 and $q_n = 0$ in Γ_2^2. Then, if $\mu = 2.20$, Q_3 is

5.50e-01✔

4.79e-01

1.88e-01

Leave it empty (no penalty)

2.50e-01

La resposta correcta és: 5.50e-01

Check: Remember that the normal vector must be normalized

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Salta a...

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