Spectral hydrology: Resolution and uncertainty in multi-frequency oscillatory hydraulic tomography

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Summary

Imaging the spatial distribution and variability of the physical properties controlling subsurface fluid flow remains a fundamental geophysical challenge. Oscillatory hydraulic tomography is a minimally invasive hydraulic testing approach to image these hydraulic properties; however, the resolution and uncertainty associated with this tomographic method remains an open question. Using linearized and non-linear approaches, we show that multi-frequency oscillatory hydraulic tomography provides additional information content that improves imaging resolution and reduces estimated parameter uncertainty.

Introduction

Understanding the spatial variability in hydraulic properties that control subsurface fluid flow at multiple scales (primarily permeability, hydraulic conductivity, or transmissivity) represents a grand challenge in subsurface imaging and accurate simulation of hydrologic, geothermal, or petroleum reservoirs. Our limited ability to "see" into the subsurface has the effect of limiting our predictive ability in simulating reservoir responses to hydraulic stressors.

The use of near-surface geophysical methods to image spatial variability in hydraulic properties has advanced in recent decades in response to decreasing costs, increased surveying speeds, and commercial "off-the-shelf" geophysical tools and analysis software. Despite these advances in geophysical imaging, the non-uniqueness of geophysical responses, challenging geologic materials (e.g., highly resistive materials), and unreliable petrophysical relationships highlight the need for additional information when characterizing hydraulic properties.

Alternatively, measuring borehole pressure propagation during hydraulic testing and then processing the collected data in a tomographic manner – i.e., hydraulic tomography – provides a direct approach to imaging the structures that control subsurface flow and storage. Like other geophysical imaging methods, hydraulic tomography (HT) parameterizes the spatial variability of hydraulic properties in a flexible manner and quantifies how the properties between sources (pumping locations) and receivers (pressure observation locations) impacts observed data. In contrast to other geophysical methods, hydraulic tomography benefits by directly imaging the hydraulic properties of interest.

Oscillatory hydraulic tomography (OHT) is a recently proposed hydraulic testing method that images hydraulic properties using oscillatory hydraulic pressure signals. While initial studies demonstrate OHT as a promising subsurface imaging method, there is a lack of analysis exploring the resolution and uncertainty associated with this tomographic method. Using numerical tomography experiments, in this presentation we explore OHT resolution and uncertainty under single and multi-frequency conditions using commonly applied geophysical linearized (i.e., singular value decomposition) and non-linear (i.e., checkerboard testing) analysis approaches.

Oscillatory Hydraulic Tomography

In Oscillatory Hydraulic Tomography (OHT), water is alternately injected into and pumped from the subsurface in a periodic manner at a prescribed frequency. The recorded signal is represented by an arriving pressure sinusoid with amplitude and phase delay that can be described by Fourier coefficients. As an example, consider a confined 2D aquifer. In the frequency domain, the groundwater flow equation becomes:

$$i\omega S\Phi = \nabla \cdot (T\nabla\Phi) + q \tag{1}$$

where S is storativity, and T is transmissivity. The source term, q, represents the phase-domain oscillatory input source of the form $q\cos(\omega t)$, and Φ represents the hydraulic head response in terms of Fourier coefficients or "phasor."

In field data, the head phasors are readily extracted through FFT or least squares analysis. These observed phasors provide the necessary inputs for OHT imaging; forward modeling within these inversions numerically solves the frequency-domain governing equation above (Equation 1).

A primary benefit of OHT over traditional hydraulic testing is the ability to extract the pressure signal from instrument drift, signal noise, or other hydrologic noise imprinted upon the recorded pressure time-series (Bakhos et al., 2014). Figure 1 shows a typical observation signal recorded during OHT, with a 4 min wave period and ~ 5 mm amplitude. The signal also contains high frequency Gaussian noise with 0.2 mm amplitude that is easily removed prior to analysis and propagated to parameter uncertainty during inversion using linearized error propagation theory.

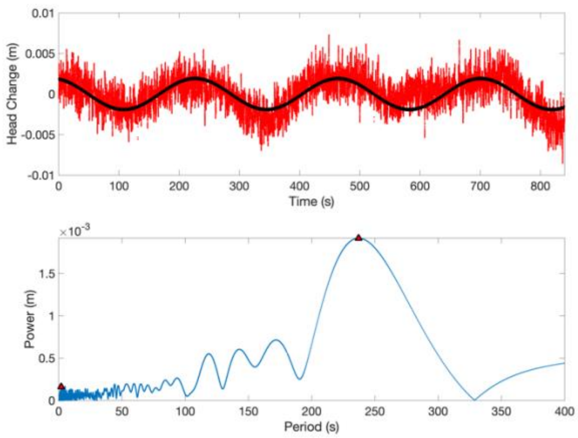

Figure 1: Example OHT data (top) with the red line indicating the recorded signal and the black line representing the de-noised signal. The corresponding Fourier power spectrum (bottom) shows the max power at 240 s (red triangle), corresponding with the observed pressure signal, and a secondary high-frequency power component (red triangle) associated with measurement noise.

Further, OHT samples subsurface heterogeneity across multiple scales by changing the frequency of the pumping signal (Cardiff et al., 2013). Low frequency signals sample far-field regions within the aquifer, smoothing out the heterogeneities to create approximately homogeneous amplitude and phase fields (Figure 2). In contrast, high frequency signals sample near-field areas with significant amplitude attenuation and phase wrapping occurring in the presence of low conductivity regions throughout the subsurface (Figure 2).

Inversion Approach

We solve the inverse problem using the quasi-linear geostatistical approach developed by Kitanidis (1995). The inversion routine performs forward model runs and full model Jacobian updates in an iterative manner to reduce data misfit subject to a geostatistical prior. While the geostatistical approach is not a standard geophysical inversion method, it provides a direct approach to estimate parameter uncertainty. We assume observation signal measurement error of 0.2 mm, consistent with noise amplitude in Figure 1 and prior field data. Following Bakhos et al., (2014) we use linear error propagation theory to translate time-series measurement error into estimated error in phasor observations that populates the data error covariance matrix for inversion.

To construct the geostatistical prior covariance matrix, we assume a stationary, constant-mean random field described by a linear variogram model:

$$\gamma(h) = -\theta h + \max(h) \tag{2}$$

where $-\theta$ is the variogram slope $\left(\frac{\sigma_s^2}{\max(h)}\right)$, and h is the separation distance. We regularize the inversion to determine the minimum parameter variability (σ_s^2) that fits the observed phasor data within the threshold of the estimated phasor error magnitude through the commonly used L-curve approach.

Resolution and Uncertainty Analysis

To understand the information content in multi-frequency OHT data, we implement a linearized approach – singular value analysis – and a non-linear approach – checkerboard testing – to explore OHT resolution and uncertainty.

For these analyses, we conceptualize a synthetic 2-D variable aperture fracture plane with 9 wells arranged in a 3 by 3 regular grid pattern and 20 m spacing between adjacent wells. We specify the variable aperture field in a checkerboard pattern with a 10 m checker size. During OHT, the pumping location is rotated across all wells to generate multiple source-receiver pairs, without considering any reciprocal tests. For each pumping frequency there a total of 36 oscillatory flow tests for a total of 72 data points (i.e., real and imaginary phasor coefficients).

<u>Singular Value Analysis</u>

The singular value decomposition is a method of analyzing and solving ill-conditioned linear inverse problems (Aster et al., 2018). Assuming local linearity, we can apply this linear method to the model Jacobian matrix (Bohling, 2009) and explore how the magnitude of singular values changes as we increase the number of frequencies included in OHT analysis.

Using this linearized approach, we conducted SVD on the full model Jacobian for OHT analysis, from single frequency testing up to seven frequencies. Generally, we see that the magnitude of some singular values increases with the addition of each new pumping frequency, demonstrating increased information content (Figure 3).

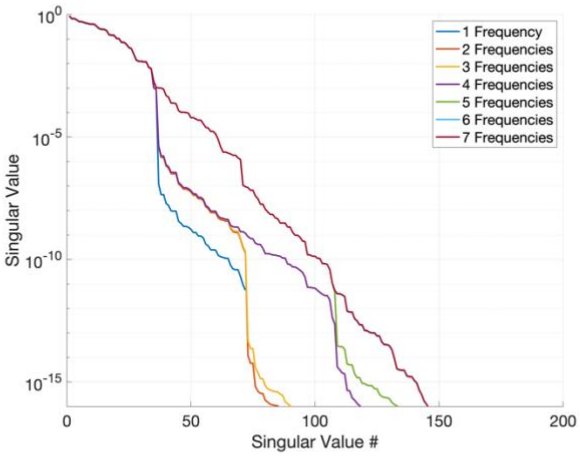

Figure 3: Singular value analysis for single and multi-frequency OHT.

Checkerboard Testing

Building on the linearized singular value analysis, we implement a checkerboard test to further explore subsurface imaging resolution and uncertainty in single and multi-frequency testing. The OHT analysis for these tests uses the modeling domain and inversion approach described above. Following inversion, we estimate parameter uncertainty by calculating and extracting the diagonal elements (i.e., parameter variance) of the posterior covariance matrix following Kitanidis (1995).

With single frequency OHT analysis we find good checkerboard recovery surrounding the central well, but with checkerboard blurring moving towards the edges and beyond the well field (Figure 4). When using four frequencies during OHT analysis we find good checkerboard recovery throughout the well field. Further we see checkerboard recovery beyond the wellfield with blurring noted at the NW and SE corners (Figure 4). Though the differences are subtle, there is an increase in checkerboard recovery in the NW and SE corners of the well field for the 4-frequency test, compared with the 2-frequency OHT analysis (Figure 4).

Similar to the observed improvement in checkerboard recovery, we find decreases in estimated parameter uncertainty - given by the diagonal elements of the posterior covariance matrix - with increasing number of pumping frequencies used during inversion. With single frequency OHT analysis, the area of low parameter uncertainty is confined to the area within the well field (Figure 4). We see parameter uncertainty decreasing within the well field as well as expanding beyond the well field when two pumping frequencies are considered (Figure 4). Finally, there is a drastic decrease in uncertainty throughout the entire domain when using four pumping frequencies during inversion (Figure 4).

Discussion & Conclusions

Overcoming sparse data to geophysically image the spatial distribution and variability of subsurface hydraulic parameters with increasing resolution and decreasing uncertainty remains a fundamental geophysical challenge. Oscillatory hydraulic tomography is a recently developed hydraulic testing method designed to image multi-scale hydraulic parameters using recorded pressure signals.

An open question with OHT is whether including multiple pumping frequencies during inversion provides additional information content. Through singular value analysis (Figure 3) our work shows that using data multi-frequency data provides additional information content to be used during inversion, consistent with previous studies (Cardiff et al., 2013; Patterson & Cardiff, 2022).

Prior to this work, OHT resolution and uncertainty was unexplored. Our idealized synthetic analysis shows that with multi-frequency analysis we resolve subsurface structures that are approximately one-half the size of the well spacing (Figure 4). Further, we see that multi-frequency inversion improves checkerboard recovery, supporting the interpretation that incorporating data from multiple pumping frequencies adds additional information content during the inversion process.

OHT is a minimally invasive hydraulic testing method that shows great promise for imaging subsurface structures that control subsurface fluid flow and storage. This work represents an initial investigation into the resolution and uncertainty associated with this tomographic analysis method and uses common geophysical inversion approaches to explore the information content in single- and multi-frequency OHT data. Further testing can be performed within this framework to assess the limits of OHT resolution in similar 2-D and 3-D synthetic problems, and with different survey arrays.

Acknowledgements

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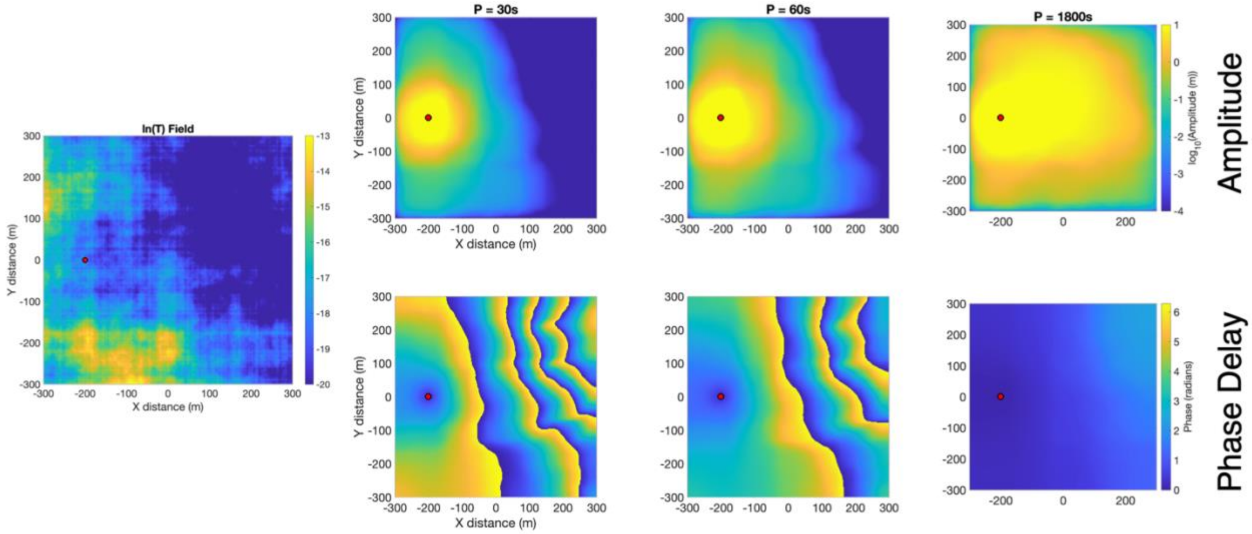

Figure 2: Amplitude and phase field for synthetic aquifer with pumping frequency decreasing to the right. The left panel is a synthetic transmissivity field provided as model inputs. The right panels show amplitude and phase responses across multiple pumping periods with the pumping location indicated by the red dot.

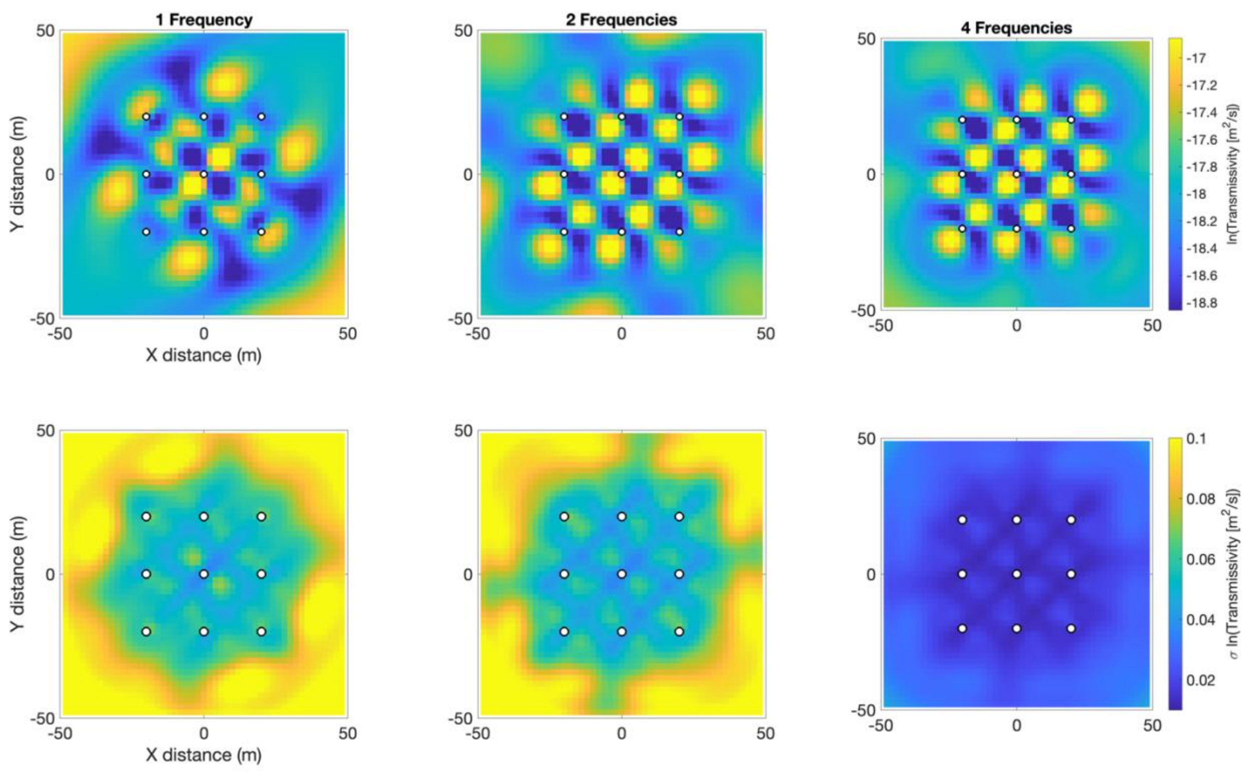

Figure 4: Recovered checkerboard for single and multi-frequency inversions (top) and posterior parameter standard deviation estimates for single and multi-frequency inversions (bottom).