Programming Assignment 3: Sentiment analysis

In this programming assignment, you will implement logistic regression to predict the sentiment of reviews that come from <code>imdb.com</code>, <code>amazon.com</code>, and <code>yelp.com</code>. Make sure the notebook is in the same folder that contains <code>full set.txt</code>.

1. Load and preprocess data

```
In [1]: %matplotlib inline
    import string
    import numpy as np
    import matplotlib
    import matplotlib.pyplot as plt
    matplotlib.rc('xtick', labelsize=14)
    matplotlib.rc('ytick', labelsize=14)
```

The data set consists of 3000 sentences, each labeled '1' (if it came from a positive review) or '0' (if it came from a negative review). To be consistent with our notation from lecture, we will change the negative review label to '-1'.

```
In [2]: | ## Read in the data set.
        with open("full_set.txt") as f:
            content = f.readlines()
        ## Remove Leading and trailing white space
        content = [x.strip() for x in content]
        ## Separate the sentences from the labels
        sentences = [x.split("\t")[0] for x in content]
        labels = [x.split("\t")[1] for x in content]
        ## Transform the labels from '0 v.s. 1' to '-1 v.s. 1'
        y = np.array(labels, dtype='int8')
        y = 2*y - 1
        for i in range(5):
            print ('Label: ',y[i],'; ', sentences[i])
        Label: -1; So there is no way for me to plug it in here in the US unless I
        go by a converter.
        Label: 1; Good case, Excellent value.
        Label: 1; Great for the jawbone.
        Label: -1; Tied to charger for conversations lasting more than 45 minutes.
```

Label: 1; The mic is great.

MAJOR PROBLEMS!!

Preprocessing the text data

To transform this prediction problem into one amenable to linear classification, we will first need to preprocess the text data. We will do four transformations:

- 1. Remove punctuation and numbers.
- 2. Transform all words to lower-case.
- 3. Remove stop words.
- 4. Convert the sentences into vectors, using a bag-of-words representation.

We begin with first two steps.

```
In [3]: ## full_remove takes a string x and a list of characters removal_list
    ## returns x with all the characters in removal_list replaced by ' '
    def full_remove(x, removal_list):
        for w in removal_list:
            x = x.replace(w, ' ')
        return x

## Remove digits
digits = [str(x) for x in range(10)]
digit_less = [full_remove(x, digits) for x in sentences]

## Remove punctuation
punc_less = [full_remove(x, list(string.punctuation)) for x in digit_less]

## Make everything lower-case
sents_lower = [x.lower() for x in punc_less]
```

Stop words

Stop words are words that are filtered out because they are believed to contain no useful information for the task at hand. These usually include articles such as 'a' and 'the', pronouns such as 'i' and 'they', and prepositions such 'to' and 'from'. We have put together a very small list of stop words, but these are by no means comprehensive. Feel free to use something different; for instance, larger lists can easily be found on the web.

What do the sentences look like so far?

Bag of words

In order to use linear classifiers on our data set, we need to transform our textual data into numeric data. The classical way to do this is known as the *bag of words* representation. In this representation, each word is thought of as corresponding to a number in $\{1, 2, \ldots, d\}$ where d is the size of our vocabulary. And each sentence is represented as a d-dimensional vector x, where x_i is the number of times that word i occurs in the sentence.

To do this transformation, we will make use of the CountVectorizer class in scikit-learn (Note that this is the only time you can call an external function from scikit-learn). We will cap the number of features at 4500, meaning a word will make it into our vocabulary only if it is one of the 4500 most common words in the corpus. This is often a useful step as it can weed out spelling mistakes and words which occur too infrequently to be useful.

Task P1: Once you get the bag-of-words representation, append a '1' to the beginning of each vector to allow our linear classifier to learn a bias term. What is the size of the resulting data_mat matrix?

```
In [6]: from sklearn.feature_extraction.text import CountVectorizer

## Transform to bag of words representation.
vectorizer = CountVectorizer(analyzer = "word", tokenizer = None, preprocessor
= None, stop_words = None, max_features = 4500)
data_features = vectorizer.fit_transform(sents_processed)
print ('The original size: ',data_features.shape)

The original size: (3000, 4500)
```

```
In [7]: | ## STUDENT: YOUR CODE STARTS HERE
        # Task: Append '1' to the beginning of each vector.
        # Hint: You can use data features.toarray() to transform data features into a
         numpy array
        # The output should be a numpy array named data mat
        data mat = data features.toarray()# please assign a numpy array to this
        data mat = np.insert(data mat, 0, 1, axis = 1)
        ## STUDENT: CODE ENDS
        print ('The updated size: ',data_mat.shape)
        print(data mat)
        The updated size: (3000, 4501)
        [[1 0 0 ... 0 0 0]
         [1 0 0 ... 0 0 0]
         [100...000]
         [100...000]
         [1 0 0 ... 0 0 0]
         [1 0 0 ... 0 0 0]]
```

Training / test split

Finally, we split the data into a training set of 2500 sentences and a test set of 500 sentences (of which 250 are positive and 250 negative).

```
In [8]: ## Split the data into testing and training sets
    np.random.seed(0)
    test_inds = np.append(np.random.choice((np.where(y==-1))[0], 250, replace=False),
    e), np.random.choice((np.where(y==1))[0], 250, replace=False))
    train_inds = list(set(range(len(labels))) - set(test_inds))

    train_data = data_mat[train_inds,]
    train_labels = y[train_inds]

    test_data = data_mat[test_inds,]
    test_labels = y[test_inds]

    print("train data: ", train_data.shape)
    print("test data: ", test_data.shape)

    train data: (2500, 4501)
    test data: (500, 4501)
```

2. Fitting a logistic regression model to the training data

In this section, we will implement our own logistic regression solver using gradient descent. As we have seen in the class, to learn the parameters of logistic regression, we need to perform the following optimization:

$$ilde{ ilde{ heta}}_t = \mathop{
m argmin}_{ ilde{ heta}} \; L_{\mathcal{D}}(ilde{ ilde{ heta}}) = \mathop{
m argmin}_{ ilde{ heta}} \; \sum_{i=1}^n \lnigg(1 + e^{y_i \; ilde{ heta}_t^T ilde{x}_i}igg)$$

where $y_i \in \{-1, +1\}$ is the label, $\tilde{ heta}$ is the vector of coefficients:

$$ilde{ heta} = \left[egin{array}{cccc} heta_0 & heta_1 & \dots & heta_d \end{array}
ight]^T,$$

 $ilde{ heta} = \begin{bmatrix} heta_0 & heta_1 & \dots & heta_d \end{bmatrix}^T,$ and $ilde{x}$ is the "augmented" feature vector (of d+1 dimensions), where we stick a 1 in the front of the original features:

$$ilde{x} = egin{bmatrix} 1 & x_1 & \dots & x_d \end{bmatrix}^T.$$

There is no nice, closed-form solution like with least-squares linear regression (http://en.wikipedia.org/wiki/Moore%E2%80%93Penrose pseudoinverse) so we will use gradient descent (http://en.wikipedia.org/wiki/Gradient_descent) instead. Specifically we will use batch gradient descent which calculates the gradient from all data points in the data set. Luckily, the loss function $L_{\mathcal{D}}(\tilde{\theta})$ we want to minimize is convex (http://en.wikipedia.org/wiki/Convex optimization) so there is only one minimum. Thus the minimum we arrive at is the global minimum.

Gradient descent is a general method and requires twice differentiability for smoothness (http://en.wikipedia.org/wiki/Smooth_function). It updates the parameters using a first-order approximation of the error surface.

$$\tilde{ heta}_{t+1} = \tilde{ heta}_t + \nabla L_{\mathcal{D}}(\tilde{ heta}_t)$$

Task P2: Derive the gradient of the loss $L_{\mathcal{D}}(\tilde{\theta})$ with respect to $\tilde{\theta}$, namely $\nabla L_{\mathcal{D}}(\tilde{\theta}_t)$. The answer should depend on data points (x_i,y_i) for $i=1,\dots,n$, and the model parameter $ilde{ heta}$. Make sure you get the sign correct. Also implement the function weight derivative . Print the output of the code.

```
In [21]: def weight derivative(weights, feature matrix, labels):
             # Input:
             # weights: weight vector w, a numpy vector of dimension d
             # feature matrix: numpy array of size n by d, where n is the number of dat
         a points, and d is the feature dimension
             # labels: true labels y, a numpy vector of dimension d, each with value -1
         or +1
             # Output:
             # Derivative of the regression cost function with respect to the weight w,
         a numpy array of dimension d
             ## STUDENT: Start of code ###
             y = labels
             theta = weights
             total = np.zeros(4501)
             for i in range(train labels.size):
                 x = feature matrix[i,]
                 d = np.dot(x, theta)
                 powr = -(v[i]*d)
                 exp = np.exp(powr)
                 du = -(y[i]*x)
                 numerator = du * exp
                 denominator = 1 + exp
                 result = numerator / denominator
                 total = np.add(total, result)
             derivative = total
             return derivative
             # End of code ###
In [22]: | # STUDENT: PRINT THE OUTPUT AND COPY IT TO THE SOLUTION FILE
         my weights = np.ones(data mat.shape[1]) # a weight of all 1s
         derivative = weight_derivative(my_weights,train_data,train_labels)
```

Now, we can just use the same gradient descent algorithm that we wrote in assignment 2 to solve it.

```
In [23]: def gradient descent(feature matrix, labels, initial weights, step size, toler
         ance):
             # Gradient descent algorithm for logistic regression problem
             # Input:
             # feature_matrix: numpy array of size n by d, where n is the number of dat
         a points, and d is the feature dimension
             # labels: true labels y, a numpy vector of dimension d
             # initial weights: initial weight vector to start with, a numpy vector of
          dimension d
             # step size: step size of update
             # tolerance: tolerace epsilon for stopping condition
             # Output:
             # Weights obtained after convergence
             converged = False
             weights = np.array(initial weights) # current iterate
             i = 0
             while not converged:
                 # impelementation of what the aradient descent algorithm does in every
         iteration
                 # Refer back to the update rule listed above: update the weight
                 derivative = weight derivative(weights, feature matrix, labels)
                 weights -= (step_size * derivative)
                 # Compute the gradient magnitude:
                 gradient magnitude = np.sqrt(np.sum(derivative**2))
                 # Check the stopping condition to decide whether you want to stop the
          iterations
                 # print ("grad mag :", gradient_magnitude)
                 #print ("tolerance:", tolerance)
                 if gradient magnitude < tolerance:</pre>
                     converged = True
                  print ("Iteration: ",i,"gradient magnitude: ", gradient magnitude) # f
         or us to check about convergence
             return(weights)
```

Task P3: Specify the initial_weights, step_size, and tolerance for the function <code>gradient_descent</code> . Copy the outputs of the code to the solution file.

```
In [24]: #Initialize the weights, step size and tolerance
    # Start of code
    #STUDENT: Specify the initial_weights, step_size, and tolerance
    initial_weights = np.zeros(data_mat.shape[1])
    step_size = 2e-3
    tolerance = 3
    # end of code

# Use the regression_gradient_descent function to calculate the gradient decen
    t and store it in the variable 'final_weights'
    final_weights = gradient_descent(train_data,train_labels, initial_weights, ste
    p_size, tolerance)

# end of code
    print ("Here are the final weights after convergence:")
    print (final_weights)
```

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            1 gradient magnitude:
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            175 gradient magnitude:
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            178 gradient magnitude:
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            179 gradient magnitude:
                                      27.131333240338744
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            180 gradient magnitude:
                                      27.049396448035022
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Iteration:

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Iteration:

4.534551503741599

2566 gradient magnitude:

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2737 gradient magnitude:

Iteration:

10/27/2020

4.303934248893801

4.30265875288541 Iteration: 2738 gradient magnitude: Iteration: 2739 gradient magnitude: 4.301384060606537 Iteration: 2740 gradient magnitude: 4.30011017129059 Iteration: 2741 gradient magnitude: 4.298837084171944 Iteration: 2742 gradient magnitude: 4.29756479848594 Iteration: 2743 gradient magnitude: 4.296293313468878 Iteration: 2744 gradient magnitude: 4.295022628358024 Iteration: 2745 gradient magnitude: 4.293752742391599 Iteration: 2746 gradient magnitude: 4.292483654808789 2747 gradient magnitude: Iteration: 4.291215364849728 Iteration: 2748 gradient magnitude: 4.289947871755513 Iteration: 2749 gradient magnitude: 4.2886811747681906 Iteration: 2750 gradient magnitude: 4.287415273130762 Iteration: 2751 gradient magnitude: 4.286150166087178 Iteration: 2752 gradient magnitude: 4.28488585288234 Iteration: 2753 gradient magnitude: 4.2836223327620955 Iteration: 2754 gradient magnitude: 4.28235960497324 2755 gradient magnitude: Iteration: 4.281097668763515 Iteration: 2756 gradient magnitude: 4.279836523381602 4.278576168077128 Iteration: 2757 gradient magnitude: Iteration: 2758 gradient magnitude: 4.27731660210066 Iteration: 2759 gradient magnitude: 4.276057824703703 Iteration: 2760 gradient magnitude: 4.274799835138699 Iteration: 2761 gradient_magnitude: 4.27354263265903 Iteration: 2762 gradient magnitude: 4.272286216519008 Iteration: 2763 gradient magnitude: 4.271030585973883 2764 gradient magnitude: Iteration: 4.269775740279833 2765 gradient_magnitude: Iteration: 4.268521678693968 Iteration: 2766 gradient magnitude: 4.267268400474328 Iteration: 2767 gradient_magnitude: 4.266015904879879 Iteration: 2768 gradient magnitude: 4.264764191170515 Iteration: 2769 gradient magnitude: 4.263513258607052 2770 gradient magnitude: Iteration: 4.262263106451232 Iteration: 2771 gradient magnitude: 4.261013733965716 Iteration: 2772 gradient magnitude: 4.25976514041409 2773 gradient_magnitude: 4.258517325060853 Iteration: Iteration: 2774 gradient magnitude: 4.257270287171427 Iteration: 2775 gradient magnitude: 4.256024026012148 Iteration: 2776 gradient magnitude: 4.254778540850266 2777 gradient magnitude: Iteration: 4.253533830953945 Iteration: 2778 gradient magnitude: 4.25228989559226 Iteration: 2779 gradient magnitude: 4.2510467340352 Iteration: 2780 gradient magnitude: 4.24980434555366 Iteration: 2781 gradient magnitude: 4.248562729419442 Iteration: 2782 gradient magnitude: 4.247321884905257 Iteration: 2783 gradient_magnitude: 4.246081811284719 Iteration: 2784 gradient magnitude: 4.244842507832346 Iteration: 2785 gradient magnitude: 4.243603973823561 Iteration: 2786 gradient magnitude: 4.2423662085346825 Iteration: 2787 gradient magnitude: 4.2411292112429315 Iteration: 2788 gradient magnitude: 4.239892981226427 Iteration: 2789 gradient_magnitude: 4.238657517764185 Iteration: 2790 gradient magnitude: 4.237422820136115 Iteration: 2791 gradient magnitude: 4.236188887623022 Iteration: 2792 gradient magnitude: 4.234955719506602 Iteration: 2793 gradient magnitude: 4.233723315069443

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Here are the final weights after convergence:
[-0.26372878 0.1116505 -0.0744277 ... -2.97204879 0.
 -0.21972597]
```

Task P4: Write the code to extract the y-intercept θ_0 and the rest of the parameters $\theta = \begin{bmatrix} \theta_1 & \dots & \theta_d \end{bmatrix}^T$. Copy the code and print the outputs.

```
In [25]: ## STUDENT: CODE STARTS HERE
## Pull out the parameters (theta_0, theta) of the logistic regression model
theta0 = final_weights[0]
theta = final_weights
## STUDENT: CODE ENDS HERE
print ('y intercept: ',theta0)
print ('theta1 and theta2: ',theta[1],theta[2])

y intercept: -0.26372877676156076
theta1 and theta2: 0.11165050235925947 -0.07442770496333988
```

Recall that the logistic/sigmoid function (http://en.wikipedia.org/wiki/Logistic_function) is given by

$$f(z) = \frac{1}{1+e^{-z}}$$

Based on the logistic model, the posterior probability

$$P(y \mid \mathbf{x}) = f(y | ilde{ heta}^T ilde{\mathbf{x}})$$

To make a prediction, we can simply choose the label $y \in \{-1, +1\}$ with the higher posterior probability.

Task P5: Write the code to make the prediction for a given data matrix. Report the training error and test error.

```
def model predict(feature matrix, weights):
# Prediction made by Logistic regression
    # Input:
    # feature matrix: numpy array of size n by d+1, where n is the number of d
ata points, and d+1 is the feature dimension
                      note we have included the dummy feature as the first col
umn of the feature matrix
    # weights: weight vector to start with, a numpy vector of dimension d+1
    # Output:
    # labels: predicted labels, a numpy vector of dimension n
    ## STUDENT: YOUR CODE HERE
    result = np.zeros(feature matrix.shape[0])
    #print(feature matrix.shape[0])
    count = 0
    for i in range(feature matrix.shape[0]):
        positiveY = 1
        positiveY = 1 / (1 + np.exp(-(positiveY * (np.dot(weights, feature mat
rix[i])))))
        negativeY = -1
        negativeY = 1 / (1 + np.exp(-(negativeY * (np.dot(weights, feature ma
trix[i])))))
        if positiveY > negativeY:
            result[i] = 1
            count += 1
        else:
            result[i] = -1
    return result
    ## STUDENT: CODE ENDS
```

```
In [41]: # STUDENT: copy the output of this section to the solution file

## Get predictions on training and test data
preds_train = model_predict(train_data,final_weights)
preds_test = model_predict(test_data,final_weights)

## Compute errors
errs_train = np.sum((preds_train > 0.0) != (train_labels > 0.0))
errs_test = np.sum((preds_test > 0.0) != (test_labels > 0.0))

print ("Training error: ", float(errs_train)/len(train_labels))
print ("Test error: ", float(errs_test)/len(test_labels))
```

Training error: 0.0028 Test error: 0.148

3. Analyzing the margin

As discussed in the lecture, the logistic regression model produces not just classifications but also conditional probability estimates.

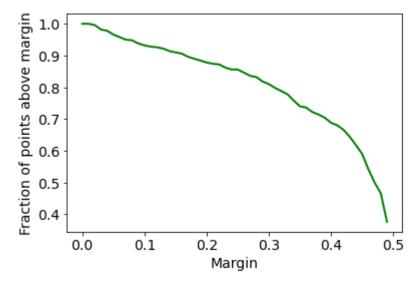
We will say that x has **margin** gamma if (according to the logistic regression model) Pr(y=1|x) > (1/2)+gamma or Pr(y=1|x) < (1/2)-gamma. For example, if Pr(y=1|x) is 0.7 according to the logistic regression model, then the margin is 0.2. If Pr(y=1|x) is 0.15 according to the logistic regression model, then the margin is 0.35.

Task P6: Implement the following function **margin_counts** that takes as input the learned weights $\dot{\theta}$, the feature matrix (feature_matrix), and a value of gamma , and computes how many points in the data have margin at least gamma . Copy the code and the output plot (i.e., visualization of the test set's distribution of margin values) to the solution file.

```
In [42]: def margin counts(feature matrix, weights, gamma):
         ## Return number of points for which Pr(y=1) lies in [0, 0.5 - gamma) or (0.5
          + gamma, 1]
              # Input:
              # feature matrix: numpy array of size n by d+1, where n is the number of d
         ata points, and d+1 is the feature dimension
                                note we have included the dummy feature as the first col
         umn of the feature_matrix
              # weights: weight vector to start with, a numpy vector of dimension d+1
              # gamma: the margin value
              # Output:
              # number of points for which Pr(y=1) lies in [0, 0.5 - gamma) or (0.5 + gamma)
         mma, 1]
              ## STUDENT: YOUR CODE HERE
              counts = 0
              for i in range(feature matrix.shape[0]):
                  positive = 1
                  positive = 1 / (1 + np.exp(-(positive * (np.dot(weights, feature matri
         x[i])))))
                  if 0 <= positive < 0.5 - gamma:</pre>
                      counts+= 1
                  elif 0.5 + gamma < positive <= 1:</pre>
                      counts+= 1
              return counts
              ## STUDENT: CODE ENDS
```

We now visualize the test set's distribution of margin values.

```
In [43]: gammas = np.arange(0,0.5,0.01)
    f = np.vectorize(lambda g: margin_counts(test_data, final_weights,g))
    plt.plot(gammas, f(gammas)/500.0, linewidth=2, color='green')
    plt.xlabel('Margin', fontsize=14)
    plt.ylabel('Fraction of points above margin', fontsize=14)
    plt.show()
```



Next, we investigate a natural question: Are points x with larger margin more likely to be classified correctly?

To address this, we define a function **margin_errors** that computes the fraction of points with margin at least gamma that are misclassified.

Task P7: Implement the function margin_errors that computes the fraction of points with margin at least gamma that are misclassified. Copy the code and the output plot (i.e., visualization of the relationship between margin and error rate) to the solution file. What do you observe from the plot?

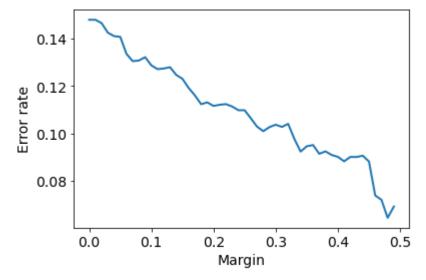
```
def margin errors(feature matrix, labels, weights, gamma):
## Return error of predictions that lie in intervals [0, 0.5 - gamma) and (0.5
+ gamma, 1]
    # Input:
    # feature_matrix: numpy array of size n by d+1, where n is the number of d
ata points, and d+1 is the feature dimension
                       note we have included the dummy feature as the first col
umn of the feature matrix
    # labels: true labels y, a numpy vector of dimension n
    # weights: weight vector to start with, a numpy vector of dimension d+1
    # gamma: the margin value
    # Output:
    # error of predictions that lie in intervals [0, 0.5 - gamma] and (0.5 + gamma)
amma, 1]
    ## STUDENT: YOUR CODE HERE
    withinGamma = 0
    Wrong = 0
    for x in range(feature matrix.shape[0]):
        n = feature_matrix[x]
        probPos = 1 / (1 + np.exp(-(1 * (np.dot(weights, n)))))
        probNeg = 1 / (1 + np.exp(-(-1 * (np.dot(weights, n)))))
        pred = 0
        if probPos > probNeg:
             pred = 1
        else:
             pred = -1
        prob = max(probPos, probNeg)
        if ((1 \ge \text{prob} \ge \text{gamma} + 0.5) \text{ or } (0 \le \text{prob} \le \text{gamma} -0.5)):
             withinGamma += 1
             if pred != labels[x]:
                 Wrong += 1
    return Wrong / withinGamma
    ## STUDENT: YOUR CODE ENDS
```

We now visualize the relationship between margin and error rate.

```
In [47]: ## Create grid of gamma values
    gammas = np.arange(0, 0.5, 0.01)

## Compute margin_errors on test data for each value of g
    f = np.vectorize(lambda g: margin_errors(test_data, test_labels,final_weights,
    g))

## Plot the result
    plt.plot(gammas, f(gammas), linewidth=2)
    plt.ylabel('Error rate', fontsize=14)
    plt.xlabel('Margin', fontsize=14)
    plt.show()
```



4. Words with large influence

Finally, we attempt to partially **interpret** the logistic regression model.

Which words are most important in deciding whether a sentence is positive? As a first approximation to this, we simply take the words whose coefficients in θ have the largest positive values.

Likewise, we look at the words whose coefficients in θ have the most negative values, and we think of these as influential in negative predictions.

Task P8: Report the top 10 positive words (i.e., words with the largest positive coefficients of θ) and the top 10 negative words (i.e., words with the most negative coefficients of θ).

```
In [49]: ## Convert vocabulary into a list:
    ## This is a list where the i-th entry corresponds to the

vocab = np.array([z[0] for z in sorted(vectorizer.vocabulary_.items(), key=lam bda x:x[1])])
    print(vocab)

## STUDENT: YOUR CODE HERE
sortedList = final_weights.argsort()

pos = list()
neg = list()

for i in range(10):
    pos.append(vocab[sortedList[-(i + 1)]])
    neg.append(vocab[sortedList[i]])

print('Positive words are:\t' + str(pos))
print('Negative words are:\t' + str(neg))

## STUDENT: CODE ENDS
```

```
['aailiyah' 'abandoned' 'abhor' ... 'zero' 'zillion' 'zombie']
Positive words are: ['scratch', 'perfected', 'function', 'greatest', 'lov
ed', 'awful', 'likes', 'nicest', 'lovely', 'deliciously']
Negative words are: ['disapppointment', 'poorly', 'wasted', 'sucker', 'ar
epas', 'disappointment', 'worth', 'dog', 'aversion', 'waste']
```

5. (Bonus question) Classifiers that can abstain

Suppose you are building a classifier, and can tolerate an error rate of at most some value e . Unfortunately, every classifier you try has a higher error than this.

Therefore, you decide that the classifier is allowed to occasionally **abstain**: that is, to say "don't know". When it actually makes a prediction, it must have error rate at most e. And subject to this constraint, it should abstain as infrequently as possible.

How would you build an abstaining classifier of this kind, starting from a logistic regression model? To get the bonus score, you need to show the following:

- A general description of the method
- · Your code implementation
- A case study to show how you can use it in practice (including necessary plots)

```
In [ ]: ## STUDENT: YOUR CODE HERE

## STUDENT: CODE ENDS
```