

# Problem Set #3

ECE 4424 / CS 4824 - Machine learning  
Virginia Tech

October 11, 2020

- Feel free to collaborate with other classmates in doing the homework. Please indicate your collaborators with their student ID. You should, however, write down your solution yourself. Please try to keep the answers brief and clear.
- Whenever you need clarification, please post the related questions on Piazza under the corresponding homework folder.
- Total: 100 points + 10 bonus points
- **Due date: 10/24/2020, 11:59PM ET**
- Late submission: each student will have a total of **four** free late (calendar) days to use for homeworks. **Note: this is the total number of late days accumulated through all the homeworks so far. It's NOT reset after each homework!** Once these late days are exhausted, any assignments turned in late will be penalized 20% per late day. However, no assignment will be accepted more than three days after its due date. Each 24 hours or part thereof that a homework is late uses up one full late day.

## 1 Convex optimization basics

**Question 1.1 (10 points):** Is the matrix  $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$  positive semidefinite? If yes, prove it; if no, provide a counter example.

1. To be positive semi-definite,  $x^T M_n \geq 0 \quad \forall n \in \mathbb{R}^n$

$$M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} x^T M_n &= \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+2y & 2x+y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= x^2 + 2xy + 2xy + y^2 \\ &= x^2 + y^2 + 4xy \end{aligned}$$

Now take  $\begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}$

$$x^T M_n = (-1)^2 + (1)^2 + 4(1)(-1) = -2 < 0$$

Thus this matrix is not positive semi-definite

**Question 1.2 (20 points):** Recall that in the example of spam, to calculate the maximum likelihood estimation for Naive Bayes, we first derive the log-likelihood function:

$$f(\theta) = n_1 \log \theta_{y=1} + (n - n_1) \log(1 - \theta_{y=1})$$

$$+ \sum_{j=1}^d \left( r_0^{(j)} \log \theta_{j|0} + (n - n_1 - r_0^{(j)}) \log(1 - \theta_{j|0}) + r_1^{(j)} \log \theta_{j|1} + (n_1 - r_1^{(j)}) \log(1 - \theta_{j|1}) \right)$$

where  $n_1 = \sum_{i=1}^n y_i$  is the number of spam emails (label 1),  $r_0^{(j)} = \sum_{i=1}^n x_i^{(j)} \mathbb{1}(y_i = 0)$  (for every word  $j \in \{1, \dots, d\}$ ) is number of non-spam emails that contain word  $j$ ,  $r_1^{(j)} = \sum_{i=1}^n x_i^{(j)} \mathbb{1}(y_i = 1)$  (for every word  $j \in \{1, \dots, d\}$ ) is number of spam emails that contain word  $j$ .

The parameters of Naive Bayes  $\theta$  include  $\theta_{j|1}$  for every  $j \in \{1, \dots, d\}$  (i.e., probability that word  $j$  is present given a spam email),  $\theta_{j|0}$  for every  $j \in \{1, \dots, d\}$  (i.e., probability that word  $j$  is present given a non-spam email), and  $\theta_{y=1}$  (i.e., the prior probability of a spam email).

- a) (10 pts) Explain why the Hessian of the function is a diagonal matrix, i.e.,  $\frac{\partial^2}{\partial \theta_p \partial \theta_q} f(\theta) = 0$  for any two different parameters  $\theta_p$  and  $\theta_q$ . For example,  $\frac{\partial^2}{\partial \theta_{j|1} \partial \theta_{y=1}} f(\theta) = 0$  for any  $j \in \{1, \dots, d\}$ , and  $\frac{\partial^2}{\partial \theta_{j|0} \partial \theta_{j|1}} f(\theta) = 0$  for any  $j \in \{1, \dots, d\}$ . Hint: write out the partial derivative of the function with respect to each parameter, and observe if it depends on the other parameters.
- b) (10 pts) Show that the **negative log-likelihood function**,  $-f(\theta)$  is convex. Hint: Calculate the second-order derivative of the negative log-likelihood for each parameter and show that it is non-negative. Use the fact that the Hessian is diagonal to make the case that it is positive semidefinite.

$$a.) f(\theta) = n_1 \log \theta_{y=1} + (n-n_1) \log (1-\theta_{y=1}) + \sum_{j=1}^d (r_0^{(j)}) \log \theta_{j|0} + (n-n_1 - r_0^{(j)}) \log (1-\theta_{j|0}) + r_1^{(j)} \log \theta_{j|1} + (n_1 - r_1^{(j)}) \log (1-\theta_{j|1})$$

$$\frac{\partial f(\theta)}{\partial \theta_{y=1}} = \frac{n_1}{\theta_{y=1}} - \frac{n-n_1}{(1-\theta_{y=1})} + 0$$

$$\frac{\partial f(\theta)}{\partial \theta_{j|1}} = 0 + \frac{r_1^{(j)}}{\theta_{j|1}} - \frac{n_1 - r_1^{(j)}}{1-\theta_{j|1}}$$

$$\frac{\partial f(\theta)}{\partial \theta_{j|0}} = 0 + \frac{r_0^{(j)}}{\theta_{j|0}} - \frac{n-n_1 - r_0^{(j)}}{1-\theta_{j|0}} + 0$$

In every case parameters do not depend on each other since when you take partial derivatives of each parameter you will get zero for those other parameter

$$b.) -f(\theta) = -n_1 \log \theta_{y=1} - (n-n_1) \log (1-\theta_{y=1}) - \sum_{i=1}^n (r_0^{(i)}) \log \theta_{j|0} + (n-n_1 - r_0^{(i)}) \log (1-\theta_{j|0}) + r_1^{(i)} \log \theta_{j|1} + (n_i - r_1^{(i)}) \log (1-\theta_{j|1})$$

$$\frac{\partial -f(\theta)}{\partial \theta_{j|0}} = \frac{r_0^{(j)}}{\theta_{j|0}} - \frac{(n-n_1 - r_0^{(j)})}{1-\theta_{j|0}}$$

$$\frac{\partial -f(\theta)}{\partial \theta_{j|1}} = \frac{r_1^{(j)}}{\theta_{j|1}} - \frac{n_1 - r_1^{(j)}}{1-\theta_{j|1}}$$

$$\frac{\partial^2 -f(\theta)}{\partial \theta_{j|0}^2} = -\frac{r_0^{(j)*}}{\theta_{j|0}^2} - \frac{n-n_1 - r_0^{(j)*}}{(1-\theta_{j|0})^2}$$

$$\frac{\partial^2 -f(\theta)}{\partial \theta_{j|1}^2} = -\frac{r_1^{(j)*}}{\theta_{j|1}^2} - \frac{(n_i - r_1^{(j)*})}{(1-\theta_{j|1})^2}$$

$$\frac{\partial f}{\partial \theta_{y=1}} = \frac{-n_1}{\theta_{y=1}} - \frac{(n-n_1)}{(1-\theta_{y=1})}$$

$$\frac{\partial^2 f(\theta)}{\partial \theta_{y=1}^2} = \frac{n_1**}{\theta_{y=1}^2} - \frac{(n-n_1)**}{(1-\theta_{y=1})^2}$$

\* squared makes value always  $\geq 0^4$

\* numerators are  $\geq 0$  by definitions of  $n_i$  and  $r_0^{(i)}$

Based on the observations above we can also now say the following regarding the results of each partial derivative

$$\frac{\partial^2}{\partial \theta_j \partial \theta_0} g(\theta) \geq 0, \quad \frac{\partial^2}{\partial \theta_j \partial \theta_1} g(\theta) \geq 0, \text{ and } \frac{\partial^2}{\partial \theta_j \partial A_{y=1}} g(\theta) \geq 0$$

thus this means every parameter of the Hessian will be non-negative, which also means it's positive semi definite.

Being Positive Semi-definite also implies that  $g\theta$  will be convex.

**Question 1.3 (20 points):** Given a dataset  $D = \{(x_1, y_1), \dots, (x_n, y_n)\}$ , where  $x_i \in \mathbb{R}^d$  and  $y_i \in \mathbb{R}$ , the goal of linear least squares is to learn the parameters  $\tilde{\theta} = [\theta_0 \quad \theta^\top]^\top$  of a linear function  $h(x) = \theta_0 + \theta \cdot x = \tilde{\theta}^\top \tilde{x}$ , where  $\theta_0 \in \mathbb{R}$ ,  $\theta \in \mathbb{R}^d$ ,  $\tilde{\theta} \in \mathbb{R}^{d+1}$ ,  $\tilde{x} = [1 \quad x^\top]^\top \in \mathbb{R}^{d+1}$ .

The method aims to minimize the mean squared error:

$$L_D(\tilde{\theta}) = \frac{1}{n} \sum_{i=1}^n (y_i - \tilde{\theta}^\top \tilde{x}_i)^2$$

- a) (10 pts) Derive the derivative  $\frac{\partial}{\partial \tilde{\theta}} L_D(\tilde{\theta})$ . Note: this should be a  $d+1$ -dimensional vector.
- b) (10 pts) Show that the loss function  $L_D(\tilde{\theta})$  is convex. Hint: Derive the Hessian and show that it is positive semidefinite.

a.)

$$L_D(\tilde{\theta}) = \frac{1}{n} \sum_i^n (y_i - \tilde{\theta}^\top \tilde{x}_i)^2$$

$$\frac{\partial}{\partial \tilde{\theta}} L_D(\tilde{\theta}) = \begin{bmatrix} \frac{\partial}{\partial \theta_0} L_D(\tilde{\theta}) \\ \frac{\partial}{\partial \theta_1} L_D(\tilde{\theta}) \\ \vdots \\ \frac{\partial}{\partial \theta_d} L_D(\tilde{\theta}) \end{bmatrix}$$

$$\begin{aligned} \frac{\partial}{\partial \theta_j} L_D(\tilde{\theta}) &= \frac{\partial}{\partial \theta_j} \frac{1}{n} \sum_{i=1}^n (y_i - \tilde{\theta}^\top \tilde{x}_i)^2 \\ &= \frac{1}{n} \sum_{i=1}^n 2(y_i - \tilde{\theta}^\top \tilde{x}_i) \frac{\partial}{\partial \theta_j} (y_i - \tilde{\theta}^\top \tilde{x}_i) \quad (\text{Chain Rule}) \\ &= -\frac{1}{n} \sum_{i=1}^n 2(y_i - \tilde{\theta}^\top \tilde{x}_i) x_i^{(j)} \end{aligned}$$

So,

$$\frac{\partial}{\partial \tilde{\theta}} L_D(\tilde{\theta}) = \begin{bmatrix} -\frac{1}{n} \sum_{i=1}^n (y_i - \tilde{\theta}^\top \tilde{x}_i) x_i^{(0)} \\ -\frac{1}{n} \sum_{i=1}^n (y_i - \tilde{\theta}^\top \tilde{x}_i) x_i^{(1)} \\ \vdots \\ -\frac{1}{n} \sum_{i=1}^n (y_i - \tilde{\theta}^\top \tilde{x}_i) x_i^{(d)} \end{bmatrix}$$

b.) Using part a.)

$$\frac{\partial}{\partial \theta_j} L_D(\tilde{\theta}) = -\frac{1}{n} \sum_{i=1}^n 2(y_i - \tilde{\theta}^T \tilde{x}_i) x_i^{(j)}$$

$$\begin{aligned} \frac{\partial^2}{\partial \theta_k \partial \theta_j} L_D(\tilde{\theta}) &= -\frac{2}{n} \sum_{i=1}^n x_i^{(j)} \frac{\partial}{\partial \theta_k} [y_i - \tilde{\theta}^T \tilde{x}_i] \\ &= -\frac{2}{n} \sum_{i=1}^n x_i^{(j)} (-x_i^{(k)}) \\ &\doteq \frac{2}{n} \sum_{i=1}^n x_i^{(j)} x_i^{(k)} \end{aligned}$$

Hessian matrix is given by,

$$H = \left[ a_{jk} \right]_{(d+1) \times (d+1)}$$

$$\text{where } a_{jk} = \frac{\partial^2}{\partial \theta_k \partial \theta_j} L_D(\tilde{\theta})$$

Now, a function convex if it's Hessian is positive semi definite

$$\begin{aligned} a_{jk} &= \frac{\partial^2}{\partial \theta_k \partial \theta_j} L_D(\tilde{\theta}) \\ &\doteq \frac{2}{n} \sum_{i=1}^n x_i^{(j)} x_i^{(k)} \\ &= \frac{2}{n} \sum_{i=1}^n x_i^{(k)} x_i^{(j)} \\ &= \frac{\partial^2}{\partial \theta_j \partial \theta_k} L_D(\tilde{\theta}) \\ &= a_{kj} \end{aligned}$$

Hence,  $H$  is symmetric. By the property of symmetric matrix  
 $H \geq D$   $H$  is positive semi definite

Hence the loss function is convex

Programming Assignment Included on Next Page

\*My P3 takes about 4000 iterations to converge but  
the resulting error is pretty good.

## 2 Programming assignment (50 + 10 pts)

For the following programming assignment, please download the datasets and iPython notebooks from Canvas and submit the following:

- Completed and ready-to-run iPython notebooks. Note: we will inspect the code and run your notebook if needed. If we cannot run any section of your notebook, you will not receive any points for the task related to that section.
- Responses (texts, codes, and/or figures) to the following problems/tasks

In this programming exercise, you will build a logistic regression model for sentiment analysis.

**Task P1 (4 pts):** Once you get the bag-of-words representation, append a '1' to the beginning of each vector to allow our linear classifier to learn a bias term. What is the size of the resulting data matrix?

**Task P2 (10 pts):** As we have seen in the class, to learn the parameters of logistic regression, we need to perform the following optimization:

$$\tilde{\theta}_t = \operatorname{argmin}_{\tilde{\theta}} L_{\mathcal{D}}(\tilde{\theta}) = \operatorname{argmin}_{\tilde{\theta}} \sum_{i=1}^n \ln \left( 1 + e^{y_i \tilde{\theta}_t^T \tilde{x}_i} \right)$$

where  $y_i \in \{-1, +1\}$  is the label,  $\tilde{\theta}$  is the vector of coefficients:

$$\tilde{\theta} = [\theta_0 \ \theta_1 \ \dots \ \theta_d]^T,$$

and  $\tilde{x}$  is the "augmented" feature vector (of  $d + 1$  dimensions), where we stick a 1 in the front of the original features:

$$\tilde{x} = [1 \ x_1 \ \dots \ x_d]^T.$$

Derive the gradient of the loss  $L_{\mathcal{D}}(\theta)$  with respect to  $\theta$ , namely  $\nabla L_{\mathcal{D}}(\theta_t)$ . The answer should depend on data points  $(x_i, y_i)$  for  $i = 1, \dots, n$ , and the model parameter  $\theta$ . Make sure you get the sign correct. Also implement the function 'weight\_derivative'. Print the output of the code.

**Task P3 (5 pts):** Specify the initial weights, step size, and tolerance for the function 'gradient descent'. Copy the outputs of the code to the solution file.

**Task P4 (4 pts):** Write the code to extract the y-intercept  $\theta_0$  and the rest of the parameters  $\theta = [\theta_1 \ \dots \ \theta_d]^T$ . Copy the code and print the outputs.

**Task P5 (6 pts):** Write the code to make the prediction for a given data matrix. Report the training error and test error.

**Task P6 (8 pts):** Implement the function ‘margin counts’ that takes as input the learned weights  $\theta^*$ , the feature matrix ('feature\_matrix'), and a value of ‘gamma’, and computes how many points in the data have margin at least ‘gamma’. Copy the code and the output plot (i.e., visualization of the test set’s distribution of margin values) to the solution file.

**Task P7 (7 pts):** Implement the function ‘margin errors’ that computes the fraction of points with margin at least ‘gamma’ that are misclassified. Copy the code and the output plot (i.e., visualization of the relationship between margin and error rate) to the solution file. What do you observe from the plot?

**Task P8 (6 pts):** Report the top 10 positive words (i.e., words with the largest positive coefficients of  $\theta_j$ ) and the top 10 negative words (i.e., words with the most negative coefficients of  $\theta_j$ ).

**Bonus questions (10 pts):** Suppose you are building a classifier, and can tolerate an error rate of at most some value  $\epsilon \in (0,1)$ . Unfortunately, every classifier you try has a higher error than this.

Therefore, you decide that the classifier is allowed to occasionally “abstain”: that is, to say “don’t know”. When it actually makes a prediction, it is expected to have error rate at most  $\epsilon \in (0,1)$ . And subject to this constraint, it should abstain as infrequently as possible.

How would you build an abstaining classifier of this kind, starting from a logistic regression model? To get the bonus score, you need to show the following:

- A general description of the method
- Your code implementation
- A case study to show how you can use it in practice (including necessary plots)

# Programming Assignment 3: Sentiment analysis

In this programming assignment, you will implement logistic regression to predict the sentiment of reviews that come from `imdb.com`, `amazon.com`, and `yelp.com`. Make sure the notebook is in the same folder that contains `full_set.txt`.

## 1. Load and preprocess data

```
In [1]: %matplotlib inline
import string
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
matplotlib.rc('xtick', labelsize=14)
matplotlib.rc('ytick', labelsize=14)
```

The data set consists of 3000 sentences, each labeled '1' (if it came from a positive review) or '0' (if it came from a negative review). To be consistent with our notation from lecture, we will change the negative review label to '-1'.

```
In [2]: ## Read in the data set.
with open("full_set.txt") as f:
    content = f.readlines()

## Remove Leading and trailing white space
content = [x.strip() for x in content]

## Separate the sentences from the Labels
sentences = [x.split("\t")[0] for x in content]
labels = [x.split("\t")[1] for x in content]

## Transform the Labels from '0 v.s. 1' to '-1 v.s. 1'
y = np.array(labels, dtype='int8')
y = 2*y - 1

for i in range(5):
    print ('Label: ',y[i],'; ', sentences[i])
```

Label: -1 ; So there is no way for me to plug it in here in the US unless I go by a converter.  
 Label: 1 ; Good case, Excellent value.  
 Label: 1 ; Great for the jawbone.  
 Label: -1 ; Tied to charger for conversations lasting more than 45 minutes.  
 MAJOR PROBLEMS!!  
 Label: 1 ; The mic is great.

## Preprocessing the text data

To transform this prediction problem into one amenable to linear classification, we will first need to preprocess the text data. We will do four transformations:

1. Remove punctuation and numbers.
2. Transform all words to lower-case.
3. Remove *stop words*.
4. Convert the sentences into vectors, using a bag-of-words representation.

We begin with first two steps.

```
In [3]: ## full_remove takes a string x and a list of characters removal_list
## returns x with all the characters in removal_list replaced by ' '
def full_remove(x, removal_list):
    for w in removal_list:
        x = x.replace(w, ' ')
    return x

## Remove digits
digits = [str(x) for x in range(10)]
digit_less = [full_remove(x, digits) for x in sentences]

## Remove punctuation
punc_less = [full_remove(x, list(string.punctuation)) for x in digit_less]

## Make everything Lower-case
sents_lower = [x.lower() for x in punc_less]
```

## Stop words

Stop words are words that are filtered out because they are believed to contain no useful information for the task at hand. These usually include articles such as 'a' and 'the', pronouns such as 'i' and 'they', and prepositions such 'to' and 'from'. We have put together a very small list of stop words, but these are by no means comprehensive. Feel free to use something different; for instance, larger lists can easily be found on the web.

```
In [4]: ## Define our stop words
stop_set = set(['the', 'a', 'an', 'i', 'he', 'she', 'they', 'to', 'of', 'it',
'from'])

## Remove stop words
sents_split = [x.split() for x in sents_lower]
sents_processed = [" ".join(list(filter(lambda a: a not in stop_set, x))) for
x in sents_split]
```

What do the sentences look like so far?

```
In [5]: sents_processed[0:10]
```

```
Out[5]: ['so there is no way for me plug in here in us unless go by converter',
'good case excellent value',
'great for jawbone',
'tied charger for conversations lasting more than minutes major problems',
'mic is great',
'have jiggle plug get line up right get decent volume',
'if you have several dozen or several hundred contacts then imagine fun send
ing each them one by one',
'if you are razr owner you must have this',
'needless say wasted my money',
'what waste money and time']
```

## Bag of words

In order to use linear classifiers on our data set, we need to transform our textual data into numeric data. The classical way to do this is known as the *bag of words* representation. In this representation, each word is thought of as corresponding to a number in  $\{1, 2, \dots, d\}$  where  $d$  is the size of our vocabulary. And each sentence is represented as a  $d$ -dimensional vector  $x$ , where  $x_i$  is the number of times that word  $i$  occurs in the sentence.

To do this transformation, we will make use of the `CountVectorizer` class in `scikit-learn` (Note that this is the only time you can call an external function from `scikit-learn`). We will cap the number of features at 4500, meaning a word will make it into our vocabulary only if it is one of the 4500 most common words in the corpus. This is often a useful step as it can weed out spelling mistakes and words which occur too infrequently to be useful.

**Task P1:** Once you get the bag-of-words representation, append a '1' to the beginning of each vector to allow our linear classifier to learn a bias term. What is the size of the resulting `data_mat` matrix?

```
In [6]: from sklearn.feature_extraction.text import CountVectorizer

## Transform to bag of words representation.
vectorizer = CountVectorizer(analyzer = "word", tokenizer = None, preprocessor = None, stop_words = None, max_features = 4500)
data_features = vectorizer.fit_transform(sents_processed)
print ('The original size: ',data_features.shape)
```

The original size: (3000, 4500)

```
In [7]: ## STUDENT: YOUR CODE STARTS HERE
# Task: Append '1' to the beginning of each vector.
# Hint: You can use data_features.toarray() to transform data_features into a
#       numpy array
# The output should be a numpy array named data_mat

data_mat = data_features.toarray()# please assign a numpy array to this
data_mat = np.insert(data_mat, 0, 1, axis = 1)

## STUDENT: CODE ENDS
print ('The updated size: ',data_mat.shape)
print(data_mat)
```

```
The updated size: (3000, 4501)
[[1 0 0 ... 0 0 0]
 [1 0 0 ... 0 0 0]
 [1 0 0 ... 0 0 0]
 ...
 [1 0 0 ... 0 0 0]
 [1 0 0 ... 0 0 0]
 [1 0 0 ... 0 0 0]]
```

## Training / test split

Finally, we split the data into a training set of 2500 sentences and a test set of 500 sentences (of which 250 are positive and 250 negative).

```
In [8]: ## Split the data into testing and training sets
np.random.seed(0)
test_inds = np.append(np.random.choice((np.where(y==1))[0], 250, replace=False),
                     np.random.choice((np.where(y==0))[0], 250, replace=False))
train_inds = list(set(range(len(labels))) - set(test_inds))

train_data = data_mat[train_inds,:]
train_labels = y[train_inds]

test_data = data_mat[test_inds,:]
test_labels = y[test_inds]

print("train data: ", train_data.shape)
print("test data: ", test_data.shape)

train data: (2500, 4501)
test data: (500, 4501)
```

## 2. Fitting a logistic regression model to the training data

In this section, we will implement our own logistic regression solver using gradient descent. As we have seen in the class, to learn the parameters of logistic regression, we need to perform the following optimization:

$$\tilde{\theta}_t = \operatorname{argmin}_{\tilde{\theta}} L_{\mathcal{D}}(\tilde{\theta}) = \operatorname{argmin}_{\tilde{\theta}} \sum_{i=1}^n \ln \left( 1 + e^{y_i \tilde{\theta}_t^T \tilde{x}_i} \right)$$

where  $y_i \in \{-1, +1\}$  is the label,  $\tilde{\theta}$  is the vector of coefficients:

$$\tilde{\theta} = [\theta_0 \quad \theta_1 \quad \dots \quad \theta_d]^T,$$

and  $\tilde{x}$  is the "augmented" feature vector (of  $d + 1$  dimensions), where we stick a 1 in the front of the original features:

$$\tilde{x} = [1 \quad x_1 \quad \dots \quad x_d]^T.$$

There is no nice, closed-form solution like with [least-squares linear regression](#)

([http://en.wikipedia.org/wiki/Moore%20-%20Penrose\\_pseudoinverse](http://en.wikipedia.org/wiki/Moore%20-%20Penrose_pseudoinverse)) so we will use [gradient descent](#)

([http://en.wikipedia.org/wiki/Gradient\\_descent](http://en.wikipedia.org/wiki/Gradient_descent)) instead. Specifically we will use batch gradient descent which

calculates the gradient from all data points in the data set. Luckily, the loss function  $L_{\mathcal{D}}(\tilde{\theta})$  we want to minimize is [convex](#) ([http://en.wikipedia.org/wiki/Convex\\_optimization](http://en.wikipedia.org/wiki/Convex_optimization)) so there is only one minimum. Thus the minimum we arrive at is the global minimum.

Gradient descent is a general method and requires twice differentiability for [smoothness](#) ([http://en.wikipedia.org/wiki/Smooth\\_function](http://en.wikipedia.org/wiki/Smooth_function)). It updates the parameters using a first-order approximation of the error surface.

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t + \nabla L_{\mathcal{D}}(\tilde{\theta}_t)$$

**Task P2:** Derive the gradient of the loss  $L_{\mathcal{D}}(\tilde{\theta})$  with respect to  $\tilde{\theta}$ , namely  $\nabla L_{\mathcal{D}}(\tilde{\theta}_t)$ . The answer should depend on data points  $(x_i, y_i)$  for  $i = 1, \dots, n$ , and the model parameter  $\tilde{\theta}$ . Make sure you get the sign correct. Also implement the function `weight_derivative`. Print the output of the code.

```
In [21]: def weight_derivative(weights, feature_matrix, labels):
    # Input:
    # weights: weight vector w, a numpy vector of dimension d
    # feature_matrix: numpy array of size n by d, where n is the number of dat
    a points, and d is the feature dimension
    # Labels: true Labels y, a numpy vector of dimension d, each with value -1
    or +1
    # Output:
    # Derivative of the regression cost function with respect to the weight w,
    a numpy array of dimension d

    ## STUDENT: Start of code ##
    y = labels
    theta = weights
    total = np.zeros(4501)

    for i in range(train_labels.size):
        x = feature_matrix[i,:]
        d = np.dot(x, theta)
        powr = -(y[i]*d)
        exp = np.exp(powr)
        du = -(y[i]*x)
        numerator = du * exp
        denominator = 1 + exp
        result = numerator / denominator
        total = np.add(total, result)

    derivative = total

    return derivative
    # End of code ###
```

```
In [22]: # STUDENT: PRINT THE OUTPUT AND COPY IT TO THE SOLUTION FILE
my_weights = np.ones(data_mat.shape[1]) # a weight of all 1s
derivative = weight_derivative(my_weights,train_data,train_labels)

print (derivative[:10])
```

```
[ 1.23415330e+03 -4.13993755e-08  1.00000000e+00  9.99993856e-01
 1.99987630e+00  9.99859072e-01  9.52574127e-01  3.59772270e+01
 2.99996572e+00 -1.38879439e-11]
```

Now, we can just use the same gradient descent algorithm that we wrote in assignment 2 to solve it.

```
In [23]: def gradient_descent(feature_matrix, labels, initial_weights, step_size, tolerance):
    # Gradient descent algorithm for logistic regression problem

    # Input:
    # feature_matrix: numpy array of size n by d, where n is the number of data points, and d is the feature dimension
    # labels: true labels y, a numpy vector of dimension d
    # initial_weights: initial weight vector to start with, a numpy vector of dimension d
    # step_size: step size of update
    # tolerance: tolerance epsilon for stopping condition
    # Output:
    # Weights obtained after convergence

    converged = False
    weights = np.array(initial_weights) # current iterate
    i = 0
    while not converged:
        # implementation of what the gradient descent algorithm does in every iteration
        # Refer back to the update rule listed above: update the weight
        i += 1
        derivative = weight_derivative(weights, feature_matrix, labels)

        weights -= (step_size * derivative)

        # Compute the gradient magnitude:
        gradient_magnitude = np.sqrt(np.sum(derivative**2))

        # Check the stopping condition to decide whether you want to stop the iterations
        # print ("grad mag : ", gradient_magnitude)
        #print ("tolerance: ", tolerance)

        if gradient_magnitude < tolerance:
            converged = True

        print ("Iteration: ", i, "gradient_magnitude: ", gradient_magnitude) # for us to check about convergence

    return(weights)
```

**Task P3:** Specify the initial\_weights, step\_size, and tolerance for the function `gradient_descent`. Copy the outputs of the code to the solution file.

```
In [24]: #Initialize the weights, step size and tolerance
# Start of code
#STUDENT: Specify the initial_weights, step_size, and tolerance
initial_weights = np.zeros(data_mat.shape[1])
step_size = 2e-3
tolerance = 3
# end of code

# Use the regression_gradient_descent function to calculate the gradient decent
# and store it in the variable 'final_weights'
final_weights = gradient_descent(train_data,train_labels, initial_weights, step_size, tolerance)

# end of code
print ("Here are the final weights after convergence:")
print (final_weights)
```

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Iteration: 1 gradient_magnitude: 209.21400526733385
Iteration: 2 gradient_magnitude: 178.58496774522143
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Iteration: 4 gradient_magnitude: 146.5002270747251
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Here are the final weights after convergence:
[-0.26372878  0.1116505 -0.0744277 ... -2.97204879  0.
 -0.21972597]

```

**Task P4:** Write the code to extract the y-intercept  $\theta_0$  and the rest of the parameters  $\theta = [\theta_1 \dots \theta_d]^T$ . Copy the code and print the outputs.

```
In [25]: ## STUDENT: CODE STARTS HERE
## Pull out the parameters (theta_0, theta) of the Logistic regression model
theta0 = final_weights[0]
theta = final_weights
## STUDENT: CODE ENDS HERE
print ('y intercept: ',theta0)
print ('theta1 and theta2: ',theta[1],theta[2])
```

```
y intercept: -0.26372877676156076
theta1 and theta2: 0.11165050235925947 -0.07442770496333988
```

Recall that the [logistic/sigmoid function](http://en.wikipedia.org/wiki/Logistic_function) ([http://en.wikipedia.org/wiki/Logistic\\_function](http://en.wikipedia.org/wiki/Logistic_function)) is given by

$$f(z) = \frac{1}{1 + e^{-z}}$$

Based on the logistic model, the posterior probability

$$P(y | \mathbf{x}) = f(y \tilde{\theta}^T \tilde{\mathbf{x}})$$

To make a prediction, we can simply choose the label  $y \in \{-1, +1\}$  with the higher posterior probability.

**Task P5:** Write the code to make the prediction for a given data matrix. Report the training error and test error.

```
In [40]: def model_predict(feature_matrix,weights):
    # Prediction made by logistic regression

        # Input:
        # feature_matrix: numpy array of size n by d+1, where n is the number of data points, and d+1 is the feature dimension
        # note we have included the dummy feature as the first column of the feature_matrix
        # weights: weight vector to start with, a numpy vector of dimension d+1
        # Output:
        # Labels: predicted labels, a numpy vector of dimension n

    ## STUDENT: YOUR CODE HERE
    result = np.zeros(feature_matrix.shape[0])
    #print(feature_matrix.shape[0])
    count = 0
    for i in range(feature_matrix.shape[0]):

        positiveY = 1
        positiveY = 1 / (1 + np.exp(-(positiveY * (np.dot(weights, feature_matrix[i])))))
        negativeY = -1
        negativeY = 1 / (1 + np.exp(-(negativeY * (np.dot(weights, feature_matrix[i])))))

        if positiveY > negativeY:
            result[i] = 1
            count += 1
        else:
            result[i] = -1

    return result
## STUDENT: CODE ENDS
```

```
In [41]: # STUDENT: copy the output of this section to the solution file

## Get predictions on training and test data
preds_train = model_predict(train_data,final_weights)
preds_test = model_predict(test_data,final_weights)

## Compute errors
errs_train = np.sum((preds_train > 0.0) != (train_labels > 0.0))
errs_test = np.sum((preds_test > 0.0) != (test_labels > 0.0))

print ("Training error: ", float(errs_train)/len(train_labels))
print ("Test error: ", float(errs_test)/len(test_labels))
```

Training error: 0.0028  
Test error: 0.148

### 3. Analyzing the margin

As discussed in the lecture, the logistic regression model produces not just classifications but also conditional probability estimates.

We will say that  $x$  has **margin**  $\gamma$  if (according to the logistic regression model)  $\Pr(y=1|x) > (1/2)+\gamma$  or  $\Pr(y=1|x) < (1/2)-\gamma$ . For example, if  $\Pr(y=1|x)$  is 0.7 according to the logistic regression model, then the margin is 0.2. If  $\Pr(y=1|x)$  is 0.15 according to the logistic regression model, then the margin is 0.35.

**Task P6:** Implement the following function **margin\_counts** that takes as input the learned weights  $\tilde{\theta}$ , the feature matrix (`feature_matrix`), and a value of `gamma`, and computes how many points in the data have margin at least `gamma`. Copy the code and the output plot (i.e., visualization of the test set's distribution of margin values) to the solution file.

```
In [42]: def margin_counts(feature_matrix, weights, gamma):
    """ Return number of points for which Pr(y=1) lies in [0, 0.5 - gamma) or (0.5 + gamma, 1]

    # Input:
    # feature_matrix: numpy array of size n by d+1, where n is the number of data points, and d+1 is the feature dimension
    # note we have included the dummy feature as the first column of the feature_matrix
    # weights: weight vector to start with, a numpy vector of dimension d+1
    # gamma: the margin value
    # Output:
    # number of points for which Pr(y=1) lies in [0, 0.5 - gamma) or (0.5 + gamma, 1]

    ## STUDENT: YOUR CODE HERE
    counts = 0
    for i in range(feature_matrix.shape[0]):

        positive = 1
        positive = 1 / (1 + np.exp(-(positive * (np.dot(weights, feature_matrix[i])))))

        if 0 <= positive < 0.5 - gamma:
            counts+= 1

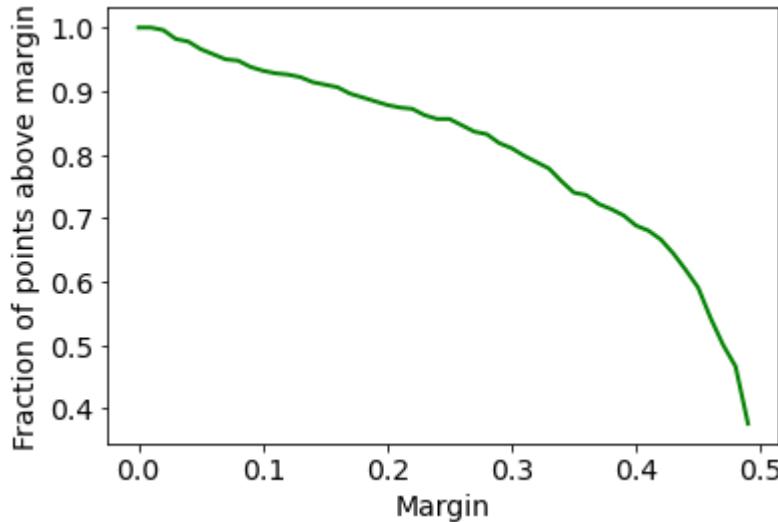
        elif 0.5 + gamma < positive <= 1:
            counts+= 1

    return counts

    ## STUDENT: CODE ENDS
```

We now visualize the test set's distribution of margin values.

```
In [43]: gammas = np.arange(0,0.5,0.01)
f = np.vectorize(lambda g: margin_counts(test_data, final_weights,g))
plt.plot(gammas, f(gammas)/500.0, linewidth=2, color='green')
plt.xlabel('Margin', fontsize=14)
plt.ylabel('Fraction of points above margin', fontsize=14)
plt.show()
```



Next, we investigate a natural question: Are points  $x$  with larger margin more likely to be classified correctly?

To address this, we define a function `margin_errors` that computes the fraction of points with margin at least `gamma` that are misclassified.

**Task P7:** Implement the function `margin_errors` that computes the fraction of points with margin at least `gamma` that are misclassified. Copy the code and the output plot (i.e., visualization of the relationship between margin and error rate) to the solution file. What do you observe from the plot?

```
In [46]: def margin_errors(feature_matrix, labels, weights, gamma):
    ## Return error of predictions that lie in intervals [0, 0.5 - gamma) and (0.5 + gamma, 1]

    # Input:
    # feature_matrix: numpy array of size n by d+1, where n is the number of data points, and d+1 is the feature dimension
    # note we have included the dummy feature as the first column of the feature_matrix
    # Labels: true labels y, a numpy vector of dimension n
    # weights: weight vector to start with, a numpy vector of dimension d+1
    # gamma: the margin value
    # Output:
    # error of predictions that lie in intervals [0, 0.5 - gamma) and (0.5 + gamma, 1]

    ## STUDENT: YOUR CODE HERE
    withinGamma = 0
    Wrong = 0

    for x in range(feature_matrix.shape[0]):
        n = feature_matrix[x]

        probPos = 1 / (1 + np.exp(-(1 * (np.dot(weights, n))))) )
        probNeg = 1 / (1 + np.exp(-(-1 * (np.dot(weights, n))))) )

        pred = 0
        if probPos > probNeg:
            pred = 1
        else:
            pred = -1

        prob = max(probPos, probNeg)

        if ((1 >= prob > gamma + 0.5) or (0 <= prob < gamma - 0.5)):
            withinGamma += 1

        if pred != labels[x]:
            Wrong += 1
    return Wrong / withinGamma

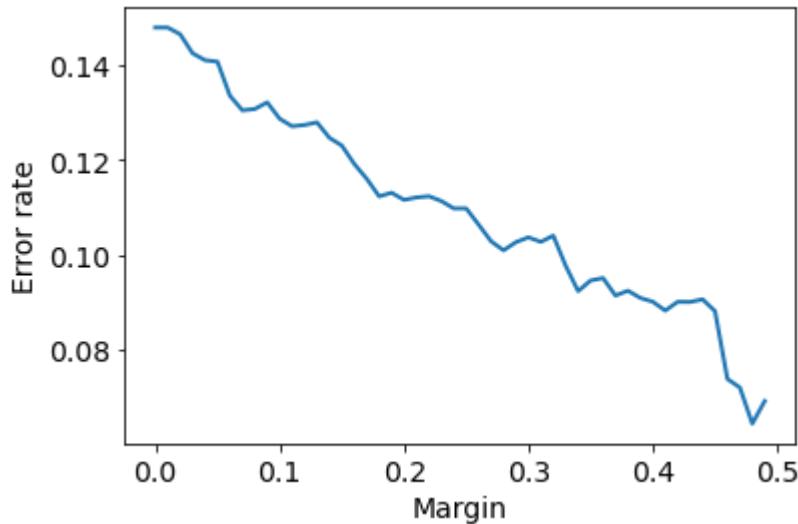
    ## STUDENT: YOUR CODE ENDS
```

We now visualize the relationship between margin and error rate.

```
In [47]: ## Create grid of gamma values
gammas = np.arange(0, 0.5, 0.01)

## Compute margin_errors on test data for each value of g
f = np.vectorize(lambda g: margin_errors(test_data, test_labels, final_weights,
g))

## Plot the result
plt.plot(gammas, f(gammas), linewidth=2)
plt.ylabel('Error rate', fontsize=14)
plt.xlabel('Margin', fontsize=14)
plt.show()
```



## 4. Words with large influence

Finally, we attempt to partially **interpret** the logistic regression model.

Which words are most important in deciding whether a sentence is positive? As a first approximation to this, we simply take the words whose coefficients in  $\theta$  have the largest positive values.

Likewise, we look at the words whose coefficients in  $\theta$  have the most negative values, and we think of these as influential in negative predictions.

**Task P8:** Report the top 10 positive words (i.e., words with the largest positive coefficients of  $\theta$ ) and the top 10 negative words (i.e., words with the most negative coefficients of  $\theta$ ).

```
In [49]: ## Convert vocabulary into a list:
## This is a list where the i-th entry corresponds to the

vocab = np.array([z[0] for z in sorted(vectorizer.vocabulary_.items(), key=lambda x:x[1])])
print(vocab)

## STUDENT: YOUR CODE HERE
sortedList = final_weights.argsort()

pos = list()
neg = list()

for i in range(10):
    pos.append(vocab[sortedList[-(i + 1)]])
    neg.append(vocab[sortedList[i]])

print('Positive words are:\t' + str(pos))
print('Negative words are:\t' + str(neg))

## STUDENT: CODE ENDS

['aailiyah' 'abandoned' 'abhor' ... 'zero' 'zillion' 'zombie']
Positive words are:      ['scratch', 'perfected', 'function', 'greatest', 'loved', 'awful', 'likes', 'nicest', 'lovely', 'deliciously']
Negative words are:      ['disappointment', 'poorly', 'wasted', 'sucker', 'arepas', 'disappointment', 'worth', 'dog', 'aversion', 'waste']
```

## 5. (Bonus question) Classifiers that can abstain

Suppose you are building a classifier, and can tolerate an error rate of at most some value  $\epsilon$ . Unfortunately, every classifier you try has a higher error than this.

Therefore, you decide that the classifier is allowed to occasionally **abstain**: that is, to say "*don't know*". When it actually makes a prediction, it must have error rate at most  $\epsilon$ . And subject to this constraint, it should abstain as infrequently as possible.

How would you build an abstaining classifier of this kind, starting from a logistic regression model? To get the bonus score, you need to show the following:

- A general description of the method
- Your code implementation
- A case study to show how you can use it in practice (including necessary plots)

```
In [ ]: ## STUDENT: YOUR CODE HERE
```

```
## STUDENT: CODE ENDS
```