Programming Assignment 2: Linear regression

As we know, the COVID-19 pandemic has been disrupting people's life around the world. To get it under control, a crucial aspect is to be able to accurate forecast the spread of the disease, which can be helpful as a planning tool for policymakers, clinicians, and public health officers to deal with this crisis. In this notebook,we will try to do some forecasting on the covid-19 epidemic progression using machine learning. We will use a dataset based on the COVID-19 Data Repository at John Hopkins university.

In [99]:
```
import pandas as pd
from datetime import datetime, timedelta
import matplotlib.pyplot as plt
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import numpy as np
from math import sqrt
import sys
```

In [100]:
```
print (sys.version)
```

```
3.8.1 (tags/v3.8.1:1b293b6, Dec 18 2019, 23:11:46) [MSC v.1916 64 bit (AMD6
4)]
```

First, let us load the data.

In [101]:
```
data_orig = pd.read_csv("us_covid_data.csv")
data = data_orig.copy()
print (data_orig.columns)
data_orig
```

```
Index(['date', 'Country', 'hospitalized_with_symptom', 'Intensive_care',
       'Total_hospitalized', 'Home_Isolation', 'Total_positive',
       'Daily_change_in_positive_cases', 'New_positive_cases', 'Recovered',
       'Deaths', 'Total_cases', 'People_tested'],
      dtype='object')
```

Out[101]:

	date	Country	hospitalized_with_symptom	Intensive_care	Total_hospitalized	Home_I
0	2020-02-24T18:00:00	ITA	101	26	127	
1	2020-02-25T18:00:00	ITA	114	35	150	
2	2020-02-26T18:00:00	ITA	128	36	164	
3	2020-02-27T18:00:00	ITA	248	56	304	
4	2020-02-28T18:00:00	ITA	345	64	409	
...	...	...	...	...	...	
201	2020-09-12T17:00:00	ITA	1951	182	2133	
202	2020-09-13T17:00:00	ITA	2042	187	2229	
203	2020-09-14T17:00:00	ITA	2122	197	2319	
204	2020-09-15T17:00:00	ITA	2222	201	2423	
205	2020-09-16T17:00:00	ITA	2285	207	2492	

206 rows × 13 columns

In [102]:
```
# change the date format
dates = data['date']
date_format = [pd.to_datetime(d) for d in dates]
```

Data Visualization

Task P1: complete the following **three** visualization graphs that show the trend of the epidemic progression. Copy them to the solution file.

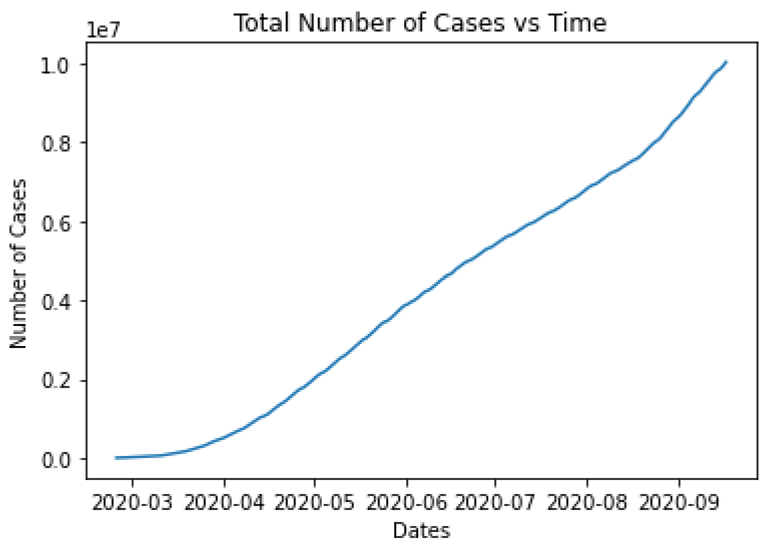
Graph 1: plot the total number of people tested for the entire period of the dataset. Your X axis will be the dates ("Dates") and Y-axis will be the total number of cases ("People_tested") over the period of time.

In [103]:
```
# Add code to plot the trend of the total number of people being tested as day
s progressed.
# X axis -> dates('Dates')
# Y axis -> number of people tested.('People_tested')

### STUDENT: Start of Code ###
plt.title("Total Number of Cases vs Time")
plt.xlabel("Dates")
plt.ylabel("Number of Cases")
People_tested = data['People_tested']

plt.plot(date_format, People_tested)

plt.show()
### End of code ####
```



Graph 2: plot the total number of deaths for the entire period. Your X axis will be the dates ("Dates") and Y-axis will be the total number of death cases("Deaths") over the period of time.

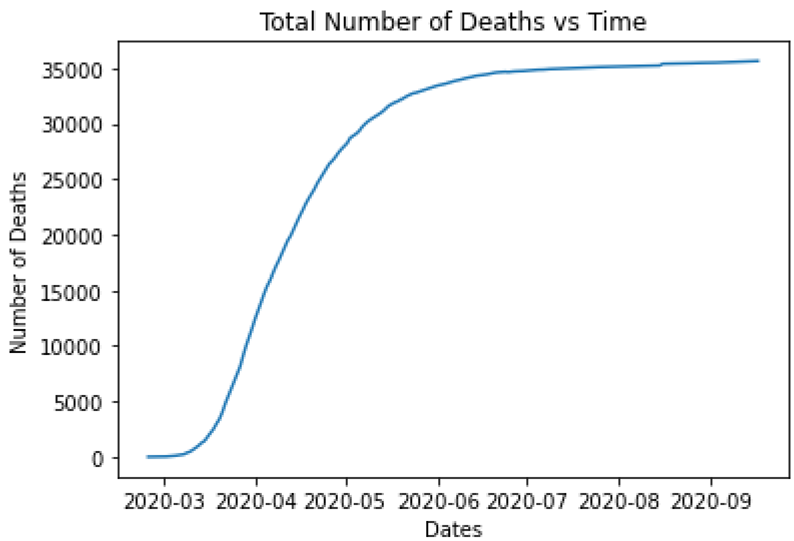
In [104]:
```
# Add code to plot the trend of total deaths as days progressed.
# X axis -> dates ('Dates')
# Y axis -> number of deaths ('Deaths')


### STUDENT: Start of Code ###
plt.title("Total Number of Deaths vs Time")
plt.xlabel("Dates")
plt.ylabel("Number of Deaths")
Deaths = data['Deaths']

plt.plot(date_format, Deaths)

plt.show()

### End of code ####
```



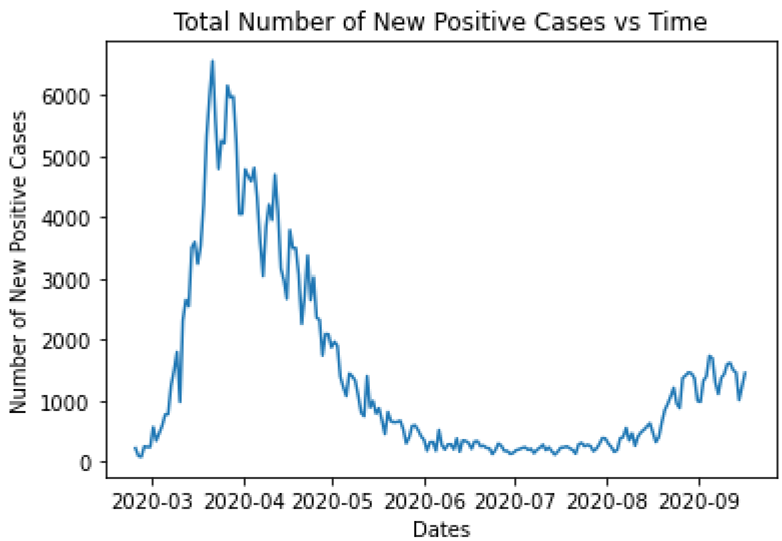
Graph 3: plot the total number of infected cases for the entire period. Your X axis will be the dates ("Dates") and Y-axis will be the total number of infected cases ('New_positive_cases') over the period of time.

In [105]:
```
### STUDENT: Start of Code ###
plt.title("Total Number of New Positive Cases vs Time")
plt.xlabel("Dates")
plt.ylabel("Number of New Positive Cases")
New_positive_cases = data['New_positive_cases']

plt.plot(date_format, New_positive_cases)

plt.show()


### End of code ####
```



As we can see that our data has different ranges of values for every feature and this can cause problems in our model, so here we will normalize our data (ignoring the categorical variables) so that our data is scaled between 0 and 1. The downside, however, is that the numbers are no longer interpretable. To interpret it, you need to multiply back by the scaling factor.

IMPORTANT: From now on, we will work with the normalized features to build the regression model. However, in **Task P8**, you need to convert the number back to the actual units.

In [106]:
```
data_list = data_orig.columns.values.tolist()

for i in data_list[-11:]:
    data[[i]]=(data_orig[i]-data_orig[i].min())/(data_orig[i].max()-data_orig[
i].min())

data
```

Out[106]:

	date	Country	hospitalized_with_symptom	Intensive_care	Total_hospitalized	Home_I
0	2020-02-24T18:00:00	ITA	0.000000	0.000000	0.000000	(
1	2020-02-25T18:00:00	ITA	0.000450	0.002227	0.000700	(
2	2020-02-26T18:00:00	ITA	0.000934	0.002474	0.001125	(
3	2020-02-27T18:00:00	ITA	0.005085	0.007422	0.005384	(
4	2020-02-28T18:00:00	ITA	0.008440	0.009401	0.008577	(
...	...	...	...	...	...	
201	2020-09-12T17:00:00	ITA	0.063994	0.038595	0.061015	(
202	2020-09-13T17:00:00	ITA	0.067142	0.039832	0.063935	(
203	2020-09-14T17:00:00	ITA	0.069909	0.042306	0.066673	(
204	2020-09-15T17:00:00	ITA	0.073368	0.043295	0.069836	(
205	2020-09-16T17:00:00	ITA	0.075547	0.044780	0.071935	(

206 rows × 13 columns

Calculate the feature matrix

The following is a function that accepts a list of feature names (e.g. ['Total_Hospitalized', 'People_tested']) and an target feature e.g. ('Deaths') and returns two things:

1. A numpy matrix whose columns are the desired features plus a column with a constant value 1, which is also known as the 'intercept'.
2. A numpy array that contains the values of the target output.

In [107]:
```
def get_numpy_data(data_frame, features, output):
    # Steps
    # select the columns of data_Frame given by the features list into the var
iable features_sframe which will include the constant
    # Convert the features_frame into a numpy matrix
    # assign the column of data_frame associated with the output to the array
 output_array
    # convert the array into a numpy array by first converting it to a list
    # return feature_matrix,output_array

    data_frame['constant'] = 1 # here we are adding a constant column
    # add the column 'constant' to the front of the features list.
    features = ['constant'] + features

    # select the columns of data_Frame given by the features list into the var
iable features_sframe which will include the constant)
    features_frame = data_frame[features]

    # Convert the features_frame into a numpy matrix
    feature_matrix = features_frame.to_numpy()
    # print ("feature_matrix:", feature_matrix)

    # assign the column of data_frame associated with the output to the array
 output_array
    output_array = data_frame[output]

    # convert the array into a numpy array by first converting it to a list
    output_array = output_array.to_numpy()

    return(feature_matrix, output_array)
```

For dates, we need to convert them into a sequence of numbers. We now add a new column to our dataframe corresponding to the number of days since the start of the dataset.

In [108]:
```
X = date_format
day_numbers = []
for i in range(1, len(X) + 1):
    day_numbers.append([i])

data['Days'] = pd.DataFrame(day_numbers,columns = ['Days'])
data["Days"] = data["Days"].astype(int)
```

Test the above function for a particular input and output feature.

In [109]:
```
(example_features, example_output) = get_numpy_data(data, ['Days'], 'New_posit
ive_cases')
print (example_features[0,:])
print (example_output[0])
```

```
[1 1]
0.022071307300509338
```

Predict the outputs with given regression weights

Suppose we had the weights $[1, 1]$ corresponding to the features $[1, 100]$, to compute the predicted output, we can simply take the dot product between them, so the output is $1 * 1 + 1 * 100 = 101$. Now, let's create the data with

In [110]:
```
(test_features, output) = get_numpy_data(data, ['Days'], 'People_tested')
```

Task P2: Complete the following function 'predict_output'. Copy the the outputs of the code to the solution file.

In [111]:
```
def predict_output(feature_matrix, weights):
    # Inputs:
    # feature_matrix: a numpy matrix containing the features as columns (inclu
ding the intercept),
    #                 and each row corresponds to a data point
    # weights: a numpy array for the corresponding regression weights (includi
ng the intercept)
    # Output:
    # a numpy array that contains the predicted outputs (according to the prov
ided weights)
    # for all the data points in the feature_matrix

    # STUDENT: Start of code ####
    features = np.array(feature_matrix)
    regress_weights = np.array(weights)

    return np.dot(features, regress_weights)
    ## end of code
```

In [112]:
```
# Copy the outputs of this code to the solution file
my_weights = np.array([1., 1.])
test_predictions = predict_output(example_features, my_weights)
print ("(normalized) prediction at day 5: ", test_predictions[5])
print ("(normalized) prediction at day 20 ", test_predictions[20])
```

```
(normalized) prediction at day 5:  7.0
(normalized) prediction at day 20  22.0
```

Compute the derivative

We will now compute the derivative of the regression cost function:

$$L_D(w) = \frac{1}{n} \sum_{i=1}^{n} (y_i - w \cdot x_i)^2,$$

where $x_i \in \mathrm{R}^d$ is the input feature of dimension d, $y_i \in \mathrm{R}$ is the output response, and $w \in \mathrm{R}^d$ is the regression weights.

Task P3: Complete the function 'weight_derivative' to calculate the derivative of the cost function with respect to regression weights w, i.e., $\frac{\partial}{\partial w} L_D(w)$. Note that this should be a d dimensional vector. Also copy the output of the code for the test example to the solution file.

In [113]:
```
def weight_derivative(weights, feature_matrix, labels):
    # Input:
    # weights: weight vector w, a numpy vector of dimension d
    # feature_matrix: numpy array of size n by d, where n is the number of dat
a points, and d is the feature dimension
    # labels: true labels y, a numpy vector of dimension d
    # Output:
    # Derivative of the regression cost function with respect to the weight w,
a numpy array of dimension d

    ## STUDENT: Start of code ###

    error = np.subtract( labels, np.dot(feature_matrix, weights ) )
    derivative = -2/len(labels)*np.dot(np.transpose(feature_matrix), error)

#     print("Size of Weights:", weights.shape )
#     print("Size of features:", feature_matrix.shape )
#     print("Size of Labels:", labels.shape )

    return derivative
    # End of code ###
```

In [114]:
```
# NOTE: copy the output to the solution file.

(example_features, example_output) = get_numpy_data(data, ['Days'], 'People_te
sted')
my_weights = np.array([0., 0.]) # this makes all the predictions 0
derivative = weight_derivative(my_weights, example_features,example_output)

print (derivative)
```

```
[  -0.82103242 -120.60087518]
```

Gradient descent algorithm

Here, we will write a function to perform gradient descent algorithm on the lineare regression cost. Given an initial point, we will update the current weights by moving in the negative gradient direction to minimize the cost function. Thus, in each iteration we obtain the updated weight w_{t+1} from the current iterate w_t as follows:

$$w_{t+1} = w_t - h\frac{\partial}{\partial w}L_D(w_t),$$

where h is the 'step_size' that is the amount by which we move in the negative gradient direction.

We stop when we are sufficiently close to the optimum (where gradient is the zero vector) by checking the condition with respect to the magnitude (length) of the gradient vector:

$$\|\frac{\partial}{\partial w}L_D(w_t)\|_2 \leq \epsilon,$$

where ϵ is the 'tolerance' parameter.

Task P4: Complete the code section to perform the gradient decent in the function `regression_gradient_descent`. Copy the code to the solution file.

In [115]:

```
def regression_gradient_descent(feature_matrix, labels, initial_weights, step_
size, tolerance):
    # Gradient descent algorithm for linear regression problem

    # Input:
    # feature_matrix: numpy array of size n by d, where n is the number of dat
a points, and d is the feature dimension
    # labels: true labels y, a numpy vector of dimension d
    # initial_weights: initial weight vector to start with, a numpy vector of
 dimension d
    # step_size: step size of update
    # tolerance: tolerace epsilon for stopping condition
    # Output:
    # Weights obtained after convergence

    converged = False
    weights = np.array(initial_weights) # current iterate
    i = 0
    while not converged:
        i += 1
        # STUDENT: Start of code: your impelementation of what the gradient de
scent algorithm does in every iteration
        # Refer back to the update rule listed above: update the weight
        derivative = weight_derivative(weights, feature_matrix, labels);
        weights = weights - step_size*derivative

        # Compute the gradient magnitude:

        gradient_magnitude = np.linalg.norm( weight_derivative(weights, featur
e_matrix, labels) )

        # Check the stopping condition to decide whether you want to stop the
 iterations
        if  gradient_magnitude <= tolerance:   # STUDENT: check the stopping c
ondition here
            converged = True

        # End of code

        print ("Iteration: ",i,"gradient_magnitude: ", gradient_magnitude) # f
or us to check about convergence

    return(weights)
```

Use gradient descent for linear regression

Let's test the gradient descent algorithm for linear regression with a single feature ('Day'). Here we are using first 180 days' data as our training data.

In [116]:

```
#train_data
train_data = data[:180]
```

Task P5: Specify the initial_weights, step_size, and tolerance for the function `regression_gradient_descent` . Copy the outputs of the code to the solution file.

In [117]:

```
simple_features = ['Days']
my_output = 'People_tested'

# Use get_numpy_data method to calculate the feature matrix and output.
(simple_feature_matrix, output) = get_numpy_data(train_data, simple_features,
my_output)

#Initialize the weights, step size and tolerance
# Start of code
#STUDENT: Specify the initial_weights, step_size, and tolerance
initial_weights = [1.0, 1.0]
step_size = 7e-8
tolerance = 1.2e4
# end of code

# Use the regression_gradient_descent function to calculate the gradient decen
t and store it in the variable 'final_weights'
final_weights = regression_gradient_descent(simple_feature_matrix, output, ini
tial_weights, step_size, tolerance)

# end of code
print ("Here are the final weights after convergence:")
print (final_weights)
```

```
<ipython-input-107-8b2916f5b454>:9: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/s
table/user_guide/indexing.html#returning-a-view-versus-a-copy
  data_frame['constant'] = 1 # here we are adding a constant column
```

```
Iteration:  1 gradient_magnitude:  21840.81707866705
Iteration:  2 gradient_magnitude:  21807.51575965956
Iteration:  3 gradient_magnitude:  21774.265216126485
Iteration:  4 gradient_magnitude:  21741.06537064902
Iteration:  5 gradient_magnitude:  21707.91614592638
Iteration:  6 gradient_magnitude:  21674.81746477566
Iteration:  7 gradient_magnitude:  21641.769250131623
Iteration:  8 gradient_magnitude:  21608.77142504654
Iteration:  9 gradient_magnitude:  21575.82391269002
Iteration:  10 gradient_magnitude:  21542.92663634879
Iteration:  11 gradient_magnitude:  21510.079519426563
Iteration:  12 gradient_magnitude:  21477.282485443844
Iteration:  13 gradient_magnitude:  21444.53545803773
Iteration:  14 gradient_magnitude:  21411.838360961778
Iteration:  15 gradient_magnitude:  21379.191118085775
Iteration:  16 gradient_magnitude:  21346.593653395597
Iteration:  17 gradient_magnitude:  21314.045890993017
Iteration:  18 gradient_magnitude:  21281.547755095533
Iteration:  19 gradient_magnitude:  21249.099170036196
Iteration:  20 gradient_magnitude:  21216.70006026343
Iteration:  21 gradient_magnitude:  21184.350350340832
Iteration:  22 gradient_magnitude:  21152.04996494705
Iteration:  23 gradient_magnitude:  21119.79882887556
Iteration:  24 gradient_magnitude:  21087.59686703451
Iteration:  25 gradient_magnitude:  21055.444004446545
Iteration:  26 gradient_magnitude:  21023.340166248632
Iteration:  27 gradient_magnitude:  20991.28527769187
Iteration:  28 gradient_magnitude:  20959.279264141343
Iteration:  29 gradient_magnitude:  20927.32205107593
Iteration:  30 gradient_magnitude:  20895.41356408814
Iteration:  31 gradient_magnitude:  20863.55372888393
Iteration:  32 gradient_magnitude:  20831.74247128252
Iteration:  33 gradient_magnitude:  20799.979717216258
Iteration:  34 gradient_magnitude:  20768.26539273041
Iteration:  35 gradient_magnitude:  20736.599423983014
Iteration:  36 gradient_magnitude:  20704.98173724469
Iteration:  37 gradient_magnitude:  20673.412258898476
Iteration:  38 gradient_magnitude:  20641.89091543965
Iteration:  39 gradient_magnitude:  20610.417633475583
Iteration:  40 gradient_magnitude:  20578.99233972552
Iteration:  41 gradient_magnitude:  20547.614961020485
Iteration:  42 gradient_magnitude:  20516.28542430301
Iteration:  43 gradient_magnitude:  20485.003656627076
Iteration:  44 gradient_magnitude:  20453.769585157825
Iteration:  45 gradient_magnitude:  20422.583137171507
Iteration:  46 gradient_magnitude:  20391.44424005523
Iteration:  47 gradient_magnitude:  20360.352821306817
Iteration:  48 gradient_magnitude:  20329.308808534643
Iteration:  49 gradient_magnitude:  20298.312129457467
Iteration:  50 gradient_magnitude:  20267.36271190423
Iteration:  51 gradient_magnitude:  20236.46048381396
Iteration:  52 gradient_magnitude:  20205.605373235514
Iteration:  53 gradient_magnitude:  20174.797308327477
Iteration:  54 gradient_magnitude:  20144.036217357967
Iteration:  55 gradient_magnitude:  20113.322028704482
Iteration:  56 gradient_magnitude:  20082.654670853706
Iteration:  57 gradient_magnitude:  20052.03407240138
```

```
Iteration:  58 gradient_magnitude:  20021.46016205212
Iteration:  59 gradient_magnitude:  19990.932868619224
Iteration:  60 gradient_magnitude:  19960.45212102455
Iteration:  61 gradient_magnitude:  19930.017848298332
Iteration:  62 gradient_magnitude:  19899.629979579015
Iteration:  63 gradient_magnitude:  19869.28844411305
Iteration:  64 gradient_magnitude:  19838.993171254828
Iteration:  65 gradient_magnitude:  19808.744090466414
Iteration:  66 gradient_magnitude:  19778.54113131744
Iteration:  67 gradient_magnitude:  19748.38422348492
Iteration:  68 gradient_magnitude:  19718.273296753105
Iteration:  69 gradient_magnitude:  19688.208281013285
Iteration:  70 gradient_magnitude:  19658.189106263653
Iteration:  71 gradient_magnitude:  19628.215702609137
Iteration:  72 gradient_magnitude:  19598.28800026125
Iteration:  73 gradient_magnitude:  19568.405929537894
Iteration:  74 gradient_magnitude:  19538.56942086323
Iteration:  75 gradient_magnitude:  19508.7784047675
Iteration:  76 gradient_magnitude:  19479.03281188685
Iteration:  77 gradient_magnitude:  19449.33257296321
Iteration:  78 gradient_magnitude:  19419.677618844125
Iteration:  79 gradient_magnitude:  19390.06788048253
Iteration:  80 gradient_magnitude:  19360.503288936685
Iteration:  81 gradient_magnitude:  19330.98377536993
Iteration:  82 gradient_magnitude:  19301.50927105059
Iteration:  83 gradient_magnitude:  19272.07970735178
Iteration:  84 gradient_magnitude:  19242.69501575124
Iteration:  85 gradient_magnitude:  19213.355127831197
Iteration:  86 gradient_magnitude:  19184.059975278215
Iteration:  87 gradient_magnitude:  19154.809489882977
Iteration:  88 gradient_magnitude:  19125.603603540203
Iteration:  89 gradient_magnitude:  19096.442248248433
Iteration:  90 gradient_magnitude:  19067.325356109905
Iteration:  91 gradient_magnitude:  19038.25285933037
Iteration:  92 gradient_magnitude:  19009.224690218958
Iteration:  93 gradient_magnitude:  18980.240781187997
Iteration:  94 gradient_magnitude:  18951.301064752897
Iteration:  95 gradient_magnitude:  18922.405473531922
Iteration:  96 gradient_magnitude:  18893.55394024609
Iteration:  97 gradient_magnitude:  18864.74639771902
Iteration:  98 gradient_magnitude:  18835.982778876747
Iteration:  99 gradient_magnitude:  18807.26301674756
Iteration:  100 gradient_magnitude:  18778.58704446187
Iteration:  101 gradient_magnitude:  18749.954795252062
Iteration:  102 gradient_magnitude:  18721.366202452296
Iteration:  103 gradient_magnitude:  18692.821199498412
Iteration:  104 gradient_magnitude:  18664.319719927713
Iteration:  105 gradient_magnitude:  18635.861697378845
Iteration:  106 gradient_magnitude:  18607.447065591656
Iteration:  107 gradient_magnitude:  18579.075758407
Iteration:  108 gradient_magnitude:  18550.747709766623
Iteration:  109 gradient_magnitude:  18522.462853712976
Iteration:  110 gradient_magnitude:  18494.221124389092
Iteration:  111 gradient_magnitude:  18466.022456038405
Iteration:  112 gradient_magnitude:  18437.866783004625
Iteration:  113 gradient_magnitude:  18409.754039731557
Iteration:  114 gradient_magnitude:  18381.684160762976
```

```
Iteration:  115 gradient_magnitude:  18353.657080742418
Iteration:  116 gradient_magnitude:  18325.672734413143
Iteration:  117 gradient_magnitude:  18297.731056617842
Iteration:  118 gradient_magnitude:  18269.83198229859
Iteration:  119 gradient_magnitude:  18241.975446496646
Iteration:  120 gradient_magnitude:  18214.161384352312
Iteration:  121 gradient_magnitude:  18186.389731104784
Iteration:  122 gradient_magnitude:  18158.660422092016
Iteration:  123 gradient_magnitude:  18130.97339275053
Iteration:  124 gradient_magnitude:  18103.328578615303
Iteration:  125 gradient_magnitude:  18075.7259153196
Iteration:  126 gradient_magnitude:  18048.165338594838
Iteration:  127 gradient_magnitude:  18020.646784270404
Iteration:  128 gradient_magnitude:  17993.17018827355
Iteration:  129 gradient_magnitude:  17965.735486629208
Iteration:  130 gradient_magnitude:  17938.34261545987
Iteration:  131 gradient_magnitude:  17910.991510985405
Iteration:  132 gradient_magnitude:  17883.68210952294
Iteration:  133 gradient_magnitude:  17856.414347486694
Iteration:  134 gradient_magnitude:  17829.18816138785
Iteration:  135 gradient_magnitude:  17802.00348783438
Iteration:  136 gradient_magnitude:  17774.86026353092
Iteration:  137 gradient_magnitude:  17747.75842527861
Iteration:  138 gradient_magnitude:  17720.69790997496
Iteration:  139 gradient_magnitude:  17693.678654613675
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Here are the final weights after convergence:
[0.99621425 0.54625306]
```

Task P6: Use the learned weights to predict 'People*tested' in the last three weeks in the dataset. Copy the predictions to the solution file, and calculate the test error $$\frac{1}{n*{\mathrm{tst}}}\sum*{i=1}^{n*{\mathrm{tst}}}(y_i^{\mathrm{tst}}-\hat{y}*i^{\mathrm{tst}})^2,$$ where $n*{\mathrm{tst}} $is the number of test data,$ y_i^{\mathrm{tst}} $is the true label,$ \hat{y}_i^{\mathrm{tst}}$ is the predicted label.

In [118]:
```
# Create the test data
test_data = data.iloc[-21:]
(test_simple_feature_matrix, test_output) = get_numpy_data(test_data, simple_features, my_output)
test_predictions = predict_output(test_simple_feature_matrix, final_weights)
print (test_predictions)
```

```
[102.59928352 103.14553658 103.69178964 104.2380427  104.78429576
 105.33054882 105.87680188 106.42305494 106.969308   107.51556106
 108.06181412 108.60806718 109.15432024 109.7005733  110.24682637
 110.79307943 111.33933249 111.88558555 112.43183861 112.97809167
 113.52434473]
```

```
<ipython-input-107-8b2916f5b454>:9: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  data_frame['constant'] = 1 # here we are adding a constant column
```

In [121]:
```
# Calculate the test error
# STUDENT: Start of code

test_error = (1/len(test_predictions))* np.sum(np.square( np.subtract(test_data.People_tested, test_predictions)))
print(test_error)

#end of code
```

```
11490.99138344532
```

Linear regression using multiple features

Here, we will be considering multiple input features (`Intensive_care` , `New_positive_cases` , `Days`) to predict the `People_tested` in the future.

Task P7: Specify the initial_weights, step_size, and tolerance for the function `regression_gradient_descent` . Print the outputs of the code.

In [127]:
```
model_features = ['Intensive_care','New_positive_cases','Days']
my_output = 'People_tested'

#call the get_nupy_data method to calculate the feature matrix and output. Sto
re them in the variables "multi_feature_matrix" & "output"

(multi_feature_matrix, output) = get_numpy_data(data, model_features, my_outpu
t)

# Initialize the weights, step size and tolerance
# STUDENT: Start of code
# STUDENT: Specify the initial_weights, step_size, and tolerance
initial_weights = [ 1.0, 1.0, 1.0, 1.0 ]
step_size = 7e-8
tolerance = 1.2e4
# end of code
weight_2 = regression_gradient_descent(multi_feature_matrix, output, initial_w
eights, step_size, tolerance)
print ("Here are the final weights after convergence:")
print (weight_2)
```

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Iteration:  1 gradient_magnitude:  28577.826184864996
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Here are the final weights after convergence:
[0.99572553 0.99950436 0.99941219 0.41591547]
```

Task P8: Use the learned weights to predict 'People_tested' in the last three weeks in the dataset.Find the value of the model predictions on the 10th day of the forecasting period.Also print the actual number of people tested on that particular day. Copy the predictions to the solution file, and calculate the test error. Note: here we are asking you to report the number before normalization. So you need to convert the prediction back to the unit of people.

In [128]:
```
(test_feature_matrix, test_output) = get_numpy_data(test_data, model_features, my_output)

test_predictions_2 = predict_output(test_feature_matrix, weight_2)

#Prediction for the 10th day of the forecasting period.
print (test_predictions_2[10])

#Convert the normalized data back to original figures using the same min-max normalization
prediction_10th_day = test_predictions_2[10] * (data_orig['People_tested'].max() - data_orig['People_tested'].min()) + data_orig['People_tested'].min()

print ("Model prediction of the 10th day:",int(prediction_10th_day))

# Get the actual number of people tested from our test data on 10 th day of forecasting period.
actual_people_tested = data_orig["People_tested"].iloc[190]

print ("Actual number of people tested on the 10th day:",actual_people_tested)
```

```
82.72965202021162
Model prediction of the 10th day: 830628809
Actual number of people tested on the 10th day: 8725909

<ipython-input-107-8b2916f5b454>:9: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  data_frame['constant'] = 1 # here we are adding a constant column
```

In [129]:
```
# Calculate the test error
# STUDENT: Start of code
test_error = (1/len(test_predictions_2))* np.sum(np.square( np.subtract(test_data.People_tested, test_predictions_2)))
print(test_error)

# end of code
```

```
6701.844086335183
```

Explore on your own

Now that you have tried two models for predictions, in this section, you can explore on your own an aspect of the problem that interests you. Here are some examples:

- What features or what combination of features are most predictive for 'People_tested'?
- How does tolerance for convergence affect prediction errors?
- How does step size affect prediction errors?
- How can we use validation to select the set of features to improve prediction?

Report your question of investigation, as well as your results/interpretation in the solution file.

In [138]:
```
def regression_gradient_descent_2(feature_matrix, labels, initial_weights, step_size, tolerance):
    # Gradient descent algorithm for linear regression problem

    # Input:
    # feature_matrix: numpy array of size n by d, where n is the number of data points, and d is the feature dimension
    # labels: true labels y, a numpy vector of dimension d
    # initial_weights: initial weight vector to start with, a numpy vector of dimension d
    # step_size: step size of update
    # tolerance: tolerace epsilon for stopping condition
    # Output:
    # Weights obtained after convergence

    converged = False
    weights = np.array(initial_weights) # current iterate
    i = 0
    while not converged:
        i += 1
        # STUDENT: Start of code: your impelementation of what the gradient descent algorithm does in every iteration
        # Refer back to the update rule listed above: update the weight
        derivative = weight_derivative(weights, feature_matrix, labels);
        weights = weights - step_size*derivative

        # Compute the gradient magnitude:

        gradient_magnitude = np.linalg.norm( weight_derivative(weights, feature_matrix, labels) )

        # Check the stopping condition to decide whether you want to stop the iterations
        if  gradient_magnitude <= tolerance:   # STUDENT: check the stopping condition here
            converged = True

        # End of code

        #print ("Iteration: ",i,"gradient_magnitude: ", gradient_magnitude) # for us to check about convergence

    return(weights)
```

In [141]:
```
# Explore an aspect of the model that interests you
### STUDENT: Start of code

simple_features = ['Days']
my_output = 'People_tested'

# Use get_numpy_data method to calculate the feature matrix and output.
(simple_feature_matrix, output) = get_numpy_data(train_data, simple_features,
my_output)

#Initialize the weights, step size and tolerance
# Start of code
#STUDENT: Specify the initial_weights, step_size, and tolerance
initial_weights = [1.0, 1.0]
step_size = 7e-8

tolerance_1 = 0.8e4
tolerance_2 = 1.0e4
tolerance_3 = 1.2e4
tolerance_4 = 1.4e4
# end of code

# Use the regression_gradient_descent function to calculate the gradient decen
t and store it in the variable 'final_weights'
final_weights_1 = regression_gradient_descent_2(simple_feature_matrix, output,
initial_weights, step_size, tolerance_1)
final_weights_2 = regression_gradient_descent_2(simple_feature_matrix, output,
initial_weights, step_size, tolerance_2)
final_weights_3 = regression_gradient_descent_2(simple_feature_matrix, output,
initial_weights, step_size, tolerance_3)
final_weights_4 = regression_gradient_descent_2(simple_feature_matrix, output,
initial_weights, step_size, tolerance_4)
# end of code

#print ("Here are the final weights after convergence with tolerance of 0.8e
4:")
#print (final_weights_1)
#print ("Here are the final weights after convergence with tolerance of 1.0e
4:")
#print (final_weights_2)
#print ("Here are the final weights after convergence with tolerance of 1.2e
4:")
#print (final_weights_3)
#print ("Here are the final weights after convergence with tolerance of 1.4e
4:")
#print (final_weights_4)

test_data = data.iloc[-21:]
(test_simple_feature_matrix, test_output) = get_numpy_data(test_data, simple_f
eatures, my_output)
test_predictions_1 = predict_output(test_simple_feature_matrix, final_weights_
1)
test_predictions_2 = predict_output(test_simple_feature_matrix, final_weights_
2)
test_predictions_3 = predict_output(test_simple_feature_matrix, final_weights_
3)
```

```
test_predictions_4 = predict_output(test_simple_feature_matrix, final_weights_
4)

test_error = (1/len(test_predictions_1))* np.sum(np.square( np.subtract(test_d
ata.People_tested, test_predictions_1)))
print("Error for tolerance of 0.8e4:", test_error)
test_error = (1/len(test_predictions_2))* np.sum(np.square( np.subtract(test_d
ata.People_tested, test_predictions_2)))
print("Error for tolerance of 1.0e4:", test_error)
test_error = (1/len(test_predictions_3))* np.sum(np.square( np.subtract(test_d
ata.People_tested, test_predictions_3)))
print( "Error for tolerance of 1.2e4:", test_error)
test_error = (1/len(test_predictions_4))* np.sum(np.square( np.subtract(test_d
ata.People_tested, test_predictions_4)))
print("Error for tolerance of 1.4e4:", test_error)
#end of code

### End of code
```

```
Error for tolerance of 0.8e4: 5067.1321292306275
Error for tolerance of 1.0e4: 7969.441808885362
Error for tolerance of 1.2e4: 11490.99138344532
Error for tolerance of 1.4e4: 15670.65863747879

<ipython-input-107-8b2916f5b454>:9: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/s
table/user_guide/indexing.html#returning-a-view-versus-a-copy
  data_frame['constant'] = 1 # here we are adding a constant column
```

My question of investigation was how does tolerance for convergence affect the test error. I used 4 different tolerance values ranging from 0.8e4 to 1.4e4. As I increased the tolerance value, the error also increased as well. It seems like the smaller the tolerance I give it, the less room for error there is for the model to make.