MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY, JAIPUR

Department of Computer Science and Engineering

Machine Learning HW1 2019

Note:

(1) Only Python Lab submission is required for this homework.

(2) Due 01/8/2019 (Thursday) before class

1. (10 points) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, E = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$

   If possible, compute the following

   (a) $(2A)^T$

   (b) $(A - B)^T$

   (c) $(3B^T - A)^T$

   (d) $(-A)^T E$

   (e) $(C + 2D^T + E)^T$

2. (10 points) Let $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

   Is $AB = BA$? Justify your answer.

3. (10 points) Given three vectors $v_1 = (-2, 0, 10), V_2 = (0, 1, 0)$ and $v_3 = (2, 0, 4) \in \mathbb{R}^3$

   - Show that they form an orthogonal set under the standard Euclidean inner product for $\mathbb{R}^3$ but not an orthonormal set.
   - Turn them into a set of vectors that will form an orthonormal set of vectors under the standard Euclidean inner product for $\mathbb{R}^3$.

4. (10 points) Given $x \in \mathbb{R}^m, y \in \mathbb{R}^n$, show that the rank of matrix xy^T is one.

5. (10 points) Given $X = [x_1, x_2, \cdots, x_n] \in \mathbb{R}^{m \times n}$ where $x_i \in \mathbb{R}^m$ for all i, and $Y^T = [y^1, y^2, \cdots, y^n] \in \mathbb{R}^{p \times n}$ where $y^i \in \mathbb{R}^p$ for all i. Show that

$$XY = \sum_{i=1}^{n} x_i (y^i)^T$$

6. (10 points) Given $X \in \mathbb{R}^{m \times n}$, show that the matrix $X^T X$ is symmetric and positive semi-definite. When is it positive definite?

7. (10 points) Given $g(x, y) = \exp^x + \exp^y + \exp^{-2xy} - \ln(-x^2 y)$, compute $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$.

8. (30 points) Consider the matrix $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 5 \end{pmatrix}$

   - Compute the eigenvalues and corresponding eigenvectors of A. You are allowed to use Python to compute the eigenvectors (but not the eigenvalues). Please include the code that you used for computing eigenvectors.
   - What is the eigen-decomposition of A?
   - What is the rank of A?
   - Is A positive definite?
   - Is A positive semi-definite?
   - Is A singular?