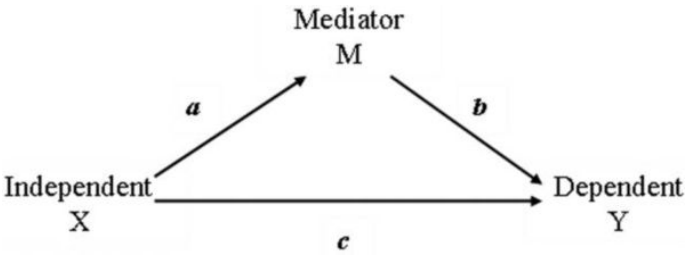
Basic Framework



Parametric Method

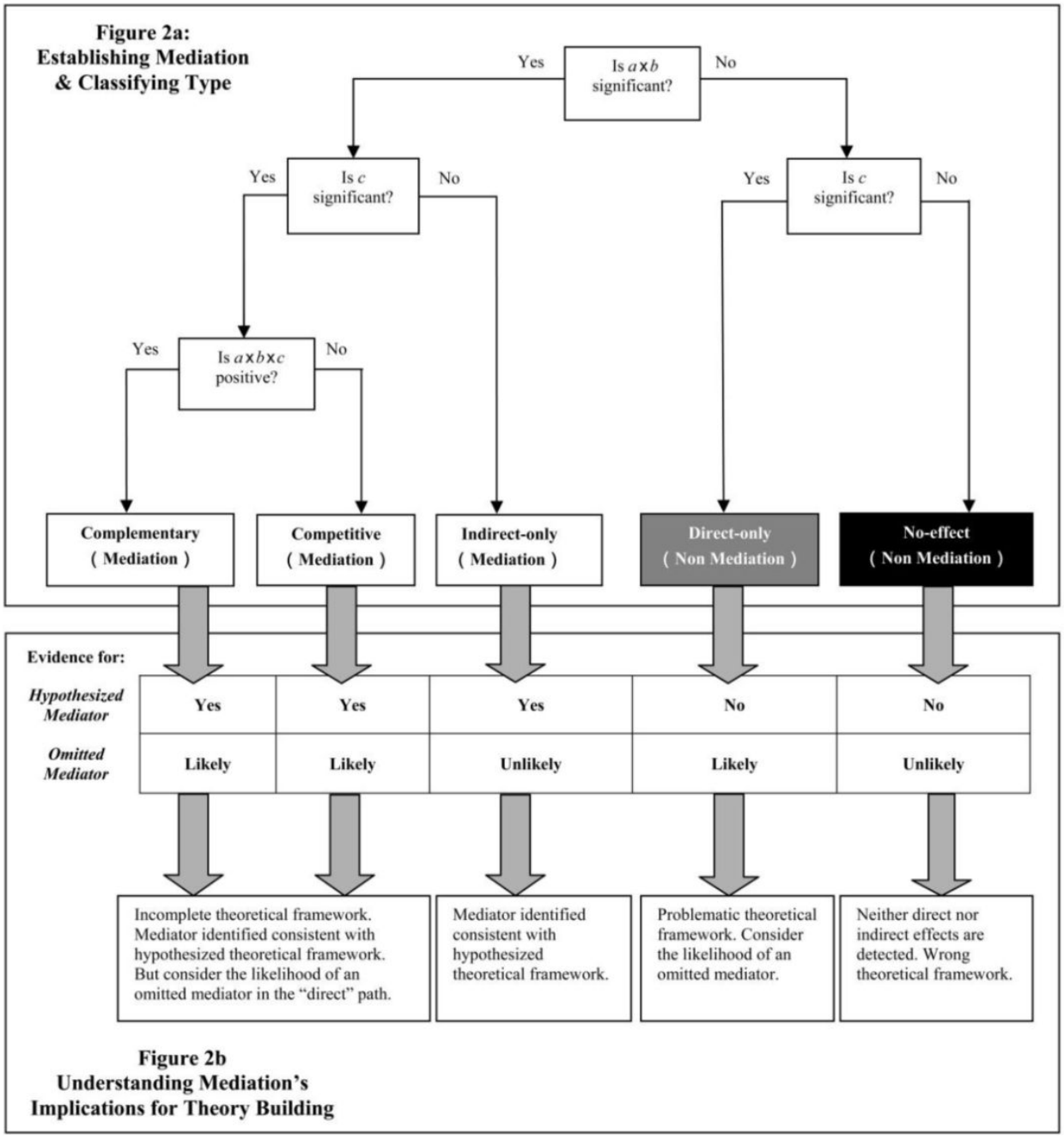
Model Specification

$$M = i_1 + aX + e_1. \quad \textbf{(1)}$$

$$Y = i_2 + c'X + e_2. \quad \textbf{(2)}$$

$$Y = i_3 + cX + bM + e_3. \quad \textbf{(3)}$$

Procedure



How to Check the significance of a*b?

Method 1: Sobel Test

$$se(ab) = \sqrt{\hat{a}^2 se_b^2 + \hat{b}^2 se_a^2}$$

In which the se are the standard errors respectively. The limitation of this method is that it assumes that $\hat{\alpha}\hat{\beta}$ must be normal distributed, which is in general hard to satisfy in reality.

Method 2: Bootstrap

Each time we can take out a subsample with replacement from the original sample. Using the subsample we can estimate the $\hat{\alpha}\hat{\beta}$. repeated this many times, and we can finally get a empirical distribution of $\hat{\alpha}\hat{\beta}$. by checking whether 0 is in the 95 confidence interval of the empirical distribution, we can test the significance of $\hat{\alpha}\hat{\beta}$.

Non-parametric Method:

One important merit of this method is that it seems to need less assumption.
Basically, we can estimate a so-called average causal mediation effect (ACME)
https://imai.fas.harvard.edu/research/files/mediation.pdf

$$E\left\{e_i(1, m_i(1)) - e_i(1, m_i(0))\right\}$$

In which e is the dependent variable, 1 or 0 represents whether got treatment or not. $m(1)$ represents the value of media variable when get treatment, while m(0) is the value of media when not get treatment. Obviously, the second term is unobservable.
Theorem 1 gives the basic results on the identification of ACME.

Calculate the ACME of a single media variable

For time=1, 1000
From the data, take out a sub sample (with replacement)
For this sub sample, Do non-parametric estimation and calculate the ACME. This is a value!
End
Through the process we can get 1000 ACMEs. We can then plot the distribution of ACME and see find the mean of this distribution. You can use this mean as a measurement of the ACME of this media variable

Compare the ACME of two media variables

If we want to compare the ACME of two media variables, then do the following:
For time=1, 1000
From the data, take out a sub sample (with replacement)
For this sub sample, Do non-parametric estimation and calculate the ACME for variable 1. Name this ACME as v1
For this sub sample, Do non-parametric estimation and calculate the ACME for variable 2. Name this ACME as v2
Calculate DIF=v1-v2
End

Reference Link
https://zhuanlan.zhihu.com/p/99435552
https://wemp.app/posts/f14ebc3d-ea5a-4417-b98f-b06b2bcd6204