Perturbation Homework

Model Setup

We consider a standard infinite-horizon RBC model with leisure and the following features:

- Infinite horizon representative agent.
- Preferences over consumption and leisure:

$$u(c_t, \ell_t) = \frac{\left(c_t\, \ell_t^{\psi}\right)^{1-\gamma}}{1-\gamma}, \qquad \ell_t = 1 - n_t, \quad \gamma > 0,\ \gamma \neq 1,\ \psi > 0.$$

  In the limit $\gamma \to 1$, $u(c_t, \ell_t) = \log c_t + \psi \log \ell_t$.

- Cobb–Douglas production:

$$Y_t = \exp(z_t)\, K_t^{\alpha} n_t^{1-\alpha}, \qquad 0 < \alpha < 1.$$

- Capital accumulation with partial depreciation:

$$K_{t+1} = (1-\delta)K_t + Y_t - c_t, \qquad \delta < 1.$$

- Stochastic total factor productivity (TFP):

$$z_{t+1} = \rho z_t + \sigma \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \mathcal{N}(0,1).$$

Questions

1. Write the FOCs of the model. (There should be three optimality conditions: the Euler equation, the intratemporal labor–leisure condition, and the law of motion for TFP.)
2. Find the steady state analytically.
3. Write

$$H(c_t, c_{t+1}, n_t, n_{t+1}, k_t, k_{t+1}, z_{t-1}, z_t).$$

4. Write the policy functions we are looking for as functions of $(k_t, z_{t-1}, \varepsilon_t, \sigma)$.

5. Write
$$F(k_t, z_{t-1}, \varepsilon_t, \sigma).$$
6. Write H_i for $i = 1, \ldots, 8$.
7. Write H_i for $i = 1, \ldots, 8$ evaluated at the steady state.
8. Write F_k, F_z, F_ε, and F_σ.
9. Write $\mathbf{A}$ and $\mathbf{B}$.
10. Find $\mathbf{h}_x$, $\mathbf{c}_x$, and $\mathbf{n}_x$.