

Homework: Gibbs Sampling from a 3-Dimensional Normal Distribution

Monte Carlo Methods / Bayesian Econometrics

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Model

Let

$$\mathbf{X} = (X_1, X_2, X_3)' \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma),$$

where

$$\boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & 0.8 & 0.4 \\ 0.8 & 1 & 0.5 \\ 0.4 & 0.5 & 1 \end{bmatrix}.$$

The goal is to sample from this joint distribution using a **Gibbs sampler**.

1. Conditional Distributions

Recall that for a multivariate normal partitioned as

$$\begin{bmatrix} X_i \\ X_{-i} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_i \\ \boldsymbol{\mu}_{-i} \end{bmatrix}, \begin{bmatrix} \sigma_{ii} & \boldsymbol{\sigma}_{i,-i} \\ \boldsymbol{\sigma}_{-i,i} & \boldsymbol{\Sigma}_{-i,-i} \end{bmatrix}\right),$$

the conditional distribution of $X_i | X_{-i}$ is

$$X_i | X_{-i} \sim \mathcal{N}\left(\mu_i + \boldsymbol{\sigma}_{i,-i}\boldsymbol{\Sigma}_{-i,-i}^{-1}(X_{-i} - \boldsymbol{\mu}_{-i}), \sigma_{ii} - \boldsymbol{\sigma}_{i,-i}\boldsymbol{\Sigma}_{-i,-i}^{-1}\boldsymbol{\sigma}_{-i,i}\right).$$

2. Gibbs Sampler Steps

1. Initialize $X_1^{(0)}, X_2^{(0)}, X_3^{(0)}$ arbitrarily.
2. At each iteration $t = 1, \dots, T$:

$$\begin{aligned} X_1^{(t)} &\sim p(X_1 | X_2^{(t-1)}, X_3^{(t-1)}), \\ X_2^{(t)} &\sim p(X_2 | X_1^{(t)}, X_3^{(t-1)}), \\ X_3^{(t)} &\sim p(X_3 | X_1^{(t)}, X_2^{(t)}). \end{aligned}$$

Repeat for many iterations, discard burn-in, and estimate the sample mean and covariance of the draws. They should converge to $\boldsymbol{\mu}$ and Σ .

3. Tasks

1. Derive analytically the three conditional Normal distributions $p(X_i | X_{-i})$.
2. Implement the Gibbs sampler in MATLAB using the provided parameters.

3. Simulate 20,000 draws (after 5,000 burn-in). Plot:
 - The trace of each variable.
 - Histograms of the draws.
 - The empirical correlation matrix versus the theoretical Σ .
4. Comment on convergence and whether the Gibbs sampler reproduces the correct mean and covariance.