Truncated Bivariate Normal Homework

Model Setup

Consider the bivariate normal vector

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}, \begin{pmatrix} \sigma_X^2 & \rho\,\sigma_X\sigma_Y \\ \rho\,\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix} \right),$$

where $-1 < \rho < 1$ denotes the correlation between X and Y.

For concreteness, take

$$\mu_X = \mu_Y = 0, \qquad \sigma_X = \sigma_Y = 1, \qquad \rho = 0.8.$$

Questions

1. Draw 10,000 samples from the joint distribution (X, Y).
2. Compute and report the sample means, variances, and correlation of X and Y.
3. Generate samples conditional on $X > Y$ (keep only those draws) and compute the same sample moments.
4. Generate samples conditional on $X > 2$.
   - Compute the conditional mean and variance of X and of Y.
   - Compare the marginal distribution of Y under the constraint $X > 2$ to its unconditional distribution.
5. Discuss in a few sentences why truncating X also changes the distribution of Y. (Hint: X and Y are correlated, so a restriction on X implies information about Y.)

MATLAB Exercise

Implement the provided MATLAB script `truncated_bivariate_normal.m`, which:

- Draws samples from the joint $\mathcal{N}_2(0, \Sigma)$ with $\rho = 0.8$,

- Computes empirical means, variances, and correlations for:
  1. the full sample,
  2. the subset with $X > Y$,
  3. the subset with $X > 2$,
- Compares the marginal variance of Y under truncation with its unconditional value.

Deliverables

- A short table summarizing the sample means, variances, and correlations for each case.
- A brief discussion (2–3 sentences) interpreting how the truncation $X > 2$ changes the marginal of Y.