

# Homework: Gibbs Sampling from a 3-Dimensional Normal Distribution

Monte Carlo Methods / Bayesian Econometrics

October 19, 2025

## Model

Let

$$\mathbf{X} = (X_1, X_2, X_3)' \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma),$$

where

$$\boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & 0.8 & 0.4 \\ 0.8 & 1 & 0.5 \\ 0.4 & 0.5 & 1 \end{bmatrix}.$$

The goal is to sample from this joint distribution using a **Gibbs sampler**.

## 1. Conditional Distributions

Recall that for a multivariate normal partitioned as

$$\begin{bmatrix} X_i \\ X_{-i} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_i \\ \boldsymbol{\mu}_{-i} \end{bmatrix}, \begin{bmatrix} \sigma_{ii} & \boldsymbol{\sigma}_{i,-i} \\ \boldsymbol{\sigma}_{-i,i} & \boldsymbol{\Sigma}_{-i,-i} \end{bmatrix}\right),$$

the conditional distribution of  $X_i \mid X_{-i}$  is

$$X_i \mid X_{-i} \sim \mathcal{N}\left(\mu_i + \boldsymbol{\sigma}_{i,-i} \boldsymbol{\Sigma}_{-i,-i}^{-1} (X_{-i} - \boldsymbol{\mu}_{-i}), \sigma_{ii} - \boldsymbol{\sigma}_{i,-i} \boldsymbol{\Sigma}_{-i,-i}^{-1} \boldsymbol{\sigma}_{-i,i}\right).$$

## 2. Gibbs Sampler Steps

1. Initialize  $X_1^{(0)}, X_2^{(0)}, X_3^{(0)}$  arbitrarily. 2. At each iteration  $t = 1, \dots, T$ :

$$\begin{aligned} X_1^{(t)} &\sim p(X_1 \mid X_2^{(t-1)}, X_3^{(t-1)}), \\ X_2^{(t)} &\sim p(X_2 \mid X_1^{(t)}, X_3^{(t-1)}), \\ X_3^{(t)} &\sim p(X_3 \mid X_1^{(t)}, X_2^{(t)}). \end{aligned}$$

Repeat for many iterations, discard burn-in, and estimate the sample mean and covariance of the draws. They should converge to  $\boldsymbol{\mu}$  and  $\Sigma$ .

## 3. Tasks

1. Derive analytically the three conditional Normal distributions  $p(X_i \mid X_{-i})$ .
2. Implement the Gibbs sampler in MATLAB using the provided parameters.

3. Simulate 20,000 draws (after 5,000 burn-in). Plot:
  - The trace of each variable.
  - Histograms of the draws.
  - The empirical correlation matrix versus the theoretical  $\Sigma$ .
4. Comment on convergence and whether the Gibbs sampler reproduces the correct mean and covariance.