

Homework 1

Model Setup

We consider a standard infinite-horizon RBC model with the following features:

- Infinite horizon representative agent.
- Log utility: $u(c_t) = \log(c_t)$.
- Cobb-Douglas production: $Y_t = A_t K_t^\alpha$, where $0 < \alpha < 1$.
- Full depreciation of capital: $\delta = 1$.
- Stochastic total factor productivity (TFP): $\log A_{t+1} = \varepsilon_{t+1}$ where $\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2)$.

Preferences

The representative agent maximizes expected lifetime utility:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(c_t),$$

subject to the resource constraint:

$$A_t K_t^\alpha = c_t + k_{t+1},$$

where $0 < \beta < 1$ is the discount factor.

Questions

1. Write the FOC of the model, remember there have to be three of as we are looking for policy function for consumption, capital, and TFP shocks.
2. Write them in log terms also.
3. Write

$$F(k_t, A_t, \sigma)$$

for both the level and log version of the FOCs

4. Write the

$$F_k(k_t, A_t, \sigma), F_A(k_t, A_t, \sigma) \text{ and } F_\sigma(k_t, A_t, \sigma)$$

for both the level and log version of the FOCs