

# Perturbation Homework

## Model Setup

We consider a standard infinite-horizon RBC model with leisure and the following features:

- Infinite horizon representative agent.
- Preferences over consumption and leisure:

$$u(c_t, \ell_t) = \frac{(c_t \ell_t^\psi)^{1-\gamma}}{1-\gamma}, \quad \ell_t = 1 - n_t, \quad \gamma > 0, \gamma \neq 1, \psi > 0.$$

In the limit  $\gamma \rightarrow 1$ ,  $u(c_t, \ell_t) = \log c_t + \psi \log \ell_t$ .

- Cobb–Douglas production:

$$Y_t = \exp(z_t) K_t^\alpha n_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

- Capital accumulation with partial depreciation:

$$K_{t+1} = (1 - \delta)K_t + Y_t - c_t, \quad \delta < 1.$$

- Stochastic total factor productivity (TFP):

$$z_{t+1} = \rho z_t + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, 1).$$

## Questions

1. Write the FOCs of the model. (There should be three optimality conditions: the Euler equation, the intratemporal labor–leisure condition, and the law of motion for TFP.)
2. Find the steady state analytically.

3. Write

$$H(c_t, c_{t+1}, n_t, n_{t+1}, k_t, k_{t+1}, z_{t-1}, z_t).$$

4. Write the policy functions we are looking for as functions of  $(k_t, z_{t-1}, \varepsilon_t, \sigma)$ .

5. Write

$$F(k_t, z_{t-1}, \varepsilon_t, \sigma).$$

6. Write  $H_i$  for  $i = 1, \dots, 8$ .

7. Write  $H_i$  for  $i = 1, \dots, 8$  evaluated at the steady state.

8. Write  $F_k$ ,  $F_z$ ,  $F_\varepsilon$ , and  $F_\sigma$ .

9. Write **A** and **B**.

10. Find  $\mathbf{h}_x$ ,  $\mathbf{c}_x$ , and  $\mathbf{n}_x$ .