

Perturbation Homework

Model Setup

We consider a standard infinite-horizon RBC model with leisure and the following features:

- Infinite horizon representative agent.
- Preferences over consumption and leisure:

$$u(c_t, \ell_t) = \frac{(c_t \ell_t^\psi)^{1-\gamma}}{1-\gamma}, \quad \ell_t = 1 - n_t, \quad \gamma > 0, \gamma \neq 1, \psi > 0.$$

In the limit $\gamma \rightarrow 1$, $u(c_t, \ell_t) = \log c_t + \psi \log \ell_t$.

- Cobb–Douglas production:

$$Y_t = \exp(z_t) K_t^\alpha n_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

- Capital accumulation with partial depreciation:

$$K_{t+1} = (1 - \delta)K_t + Y_t - c_t, \quad \delta < 1.$$

- Stochastic total factor productivity (TFP):

$$z_{t+1} = \rho z_t + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, 1).$$

Questions

1. Write the FOCs of the model. (There should be three optimality conditions: the Euler equation, the intratemporal labor–leisure condition, and the law of motion for TFP.)
2. Find the steady state analytically.

3. Write

$$H(c_t, c_{t+1}, n_t, n_{t+1}, k_t, k_{t+1}, z_{t-1}, z_t).$$

4. Write the policy functions we are looking for as functions of $(k_t, z_{t-1}, \varepsilon_t, \sigma)$.

5. Write

$$F(k_t, z_{t-1}, \varepsilon_t, \sigma).$$

6. Write H_i for $i = 1, \dots, 8$.

7. Write H_i for $i = 1, \dots, 8$ evaluated at the steady state.

8. Write F_k , F_z , F_ε , and F_σ .

9. Write \mathbf{A} and \mathbf{B} .

10. Find \mathbf{h}_x , \mathbf{c}_x , and \mathbf{n}_x .