

# Truncated Bivariate Normal Homework

## Model Setup

Consider the bivariate normal vector

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}, \begin{pmatrix} \sigma_X^2 & \rho \sigma_X \sigma_Y \\ \rho \sigma_X \sigma_Y & \sigma_Y^2 \end{pmatrix}\right),$$

where  $-1 < \rho < 1$  denotes the correlation between  $X$  and  $Y$ .

For concreteness, take

$$\mu_X = \mu_Y = 0, \quad \sigma_X = \sigma_Y = 1, \quad \rho = 0.8.$$

## Questions

1. Draw 10,000 samples from the joint distribution  $(X, Y)$ .
2. Compute and report the sample means, variances, and correlation of  $X$  and  $Y$ .
3. Generate samples conditional on  $X > Y$  (keep only those draws) and compute the same sample moments.
4. Generate samples conditional on  $X > 2$ .
  - Compute the conditional mean and variance of  $X$  and of  $Y$ .
  - Compare the marginal distribution of  $Y$  under the constraint  $X > 2$  to its unconditional distribution.
5. Discuss in a few sentences why truncating  $X$  also changes the distribution of  $Y$ . (Hint:  $X$  and  $Y$  are correlated, so a restriction on  $X$  implies information about  $Y$ .)

## MATLAB Exercise

Implement the provided MATLAB script `truncated_bivariate_normal.m`, which:

- Draws samples from the joint  $\mathcal{N}_2(0, \Sigma)$  with  $\rho = 0.8$ ,

- Computes empirical means, variances, and correlations for:
  1. the full sample,
  2. the subset with  $X > Y$ ,
  3. the subset with  $X > 2$ ,
- Compares the marginal variance of  $Y$  under truncation with its unconditional value.

## **Deliverables**

- A short table summarizing the sample means, variances, and correlations for each case.
- A brief discussion (2–3 sentences) interpreting how the truncation  $X > 2$  changes the marginal of  $Y$ .