(3) Since $\sigma(z) = \frac{1}{1+e^{-z}}$, then $\sigma'(z) = \frac{1}{(1+e^{-z})^2} \cdot e^{-z} = \frac{e^{-z}}{(1+e^{-z})^2} = (1-\sigma(z))\sigma(z)$

Also,

$$\log L(\beta|y,x) = \Sigma(y_i \log(\sigma(x_i\beta)) + (1-y_i)\log(1-\sigma(x_i\beta)))$$

Then we have the F.O.C of $\max \frac{1}{N}\log L(\beta|y,x)$ is the scores,

which is $S_N(\beta) = \frac{\partial}{\partial\beta'}\frac{1}{N}\log L(\beta|y,x))$

Since we know that $\frac{\partial}{\partial\beta}\log(\sigma(x\beta)) = \frac{1}{\sigma(x\beta)}\sigma(x\beta)(1-\sigma(x\beta))x = (1-\sigma(x\beta))x.$

$$\frac{\partial}{\partial\beta}\log(1-\sigma(x\beta)) = \frac{1}{1-\sigma(x\beta)}\cdot(-1)\cdot(1-\sigma(x\beta))\sigma(x\beta)x = -\sigma(x\beta)\cdot x$$

Thus, $S_N(\beta) = \frac{\partial}{\partial\beta}L(\beta|y,x) = \Sigma y_i(1-\sigma(x_i\beta))x_i + (1-y_i)(-\sigma(x_i\beta))x_i$

$$= \frac{1}{N}\Sigma y_i x_i - \sigma(x_i\beta)x_i$$

Then $E[S_N(\beta)] = E[\frac{1}{N}\Sigma y_i x_i - \sigma(x_i\beta)x_i]$

$$= \frac{1}{N}\Sigma E[y_i x_i - \sigma(x_i\beta)x_i]$$

$$= \frac{1}{N}\Sigma E[E[y_i x_i - \sigma(x_i\beta)x_i|x]]$$

$$= \frac{1}{N}\Sigma E[E[y_i|x_i]x_i - \sigma(x_i\beta)x_i]$$

$$= \frac{1}{N}\Sigma E[0] = 0$$

Since $E[S_N(\beta)] = 0$, it could serve as the estimating equation of a GMM estimator.

where $g_N(\beta) = \frac{1}{N}\sum_{i=1}^{N} [y - 6(x_i\beta)]x_i$