

# Multi-Domain Finite Element Meshing for Parotid Acinar Cell Modeling and Simulation

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## Keywords

multi-domain mesh, mesh refinement, surface curvature, finite element modeling, parotid acinar cells

## Abstract

350 words maximum.

Contribution: Multi-domain mesh surface smoothing using iterative coupled refinement.

Termination of refinement guided by reference target surface curvature.

FINISH THIS WHEN EVERYTHING ELSE IS DONE.

## 1 Introduction

The primary role of salivary gland acinar cells is to secrete saliva, the lack of which causes a host of severe medical difficulties [FvdVS<sup>+</sup>85, Mel91]. Thus, an understanding of the mechanisms underlying saliva secretion are vital for the understanding of oral health. The basic mechanism of saliva secretion is well understood [Nau92], and has been previously modeled in detail [STAB<sup>+</sup>03].

However, although much is understood about saliva secretion, important questions, both experimental and theoretical, remain. From the theoretical point of view, one of the most interesting questions is how the structure of the acinar cells affects the properties of the  $\text{Ca}^{2+}$  waves. Along these lines, we have previously reported our first simulation results obtained using an anatomically accurate three-dimensional parotid acinar cell model [SMZ<sup>+</sup>17]. In that initial investigation, we modeled single non-coupled cells. In this paper, we build on that work, extending our mesh model to multiple (seven), coupled cells.

MORE HERE? Why extend to coupled cells?

## 2 Design Considerations

We have been using a straightforward implementation of the Finite Element Method (FEM) [Goc06, Hug, Gos05] for our three-dimentional parotid cell simulations and continue doing so in this latest work. In keeping with relatively common FEM practice, we model the surface of each cell with a triangle mesh, then fill-in the cell interiors with tetrahedrons which results in a complete volumetric mesh for each cell. In the meshes that we developed for our prior work the cells did not physically contact each other i.e. there was a physical gap between the cells.

We have recently extended our mathematical modeling to include dynamic interactions between adjacent cells and this imposed additional demands on our mesh construction. With our new design, each cell now touches one or more other cells, sharing some portion of it's surface. We have chosen to model adjacent cell

surfaces with *conformal faces* in which numerous individual triangle mesh faces are shared in common between adjacent cells. Note that this facilitates a fairly straightforward mathematical implementation of the interactions between cells. The final complete mesh is said to be *multi-domain* (one domain for each cell) in that we still subsequently keep track of what's going on within the physical confines of each cell independently.

As with our prior work, the source data for our acinar cell mesh construction consists of a calibrated confocal microscopy image stack. We have 31 images at 1024 by 1024 pixels each with a resolution of approximately 0.069 micro-meters per pixel and a stack spacing of 0.80 micro-meters.

#### MORE FROM YULE? CITE YULE?

Note that, to maximize the transparency and reproducibility of our computational work, we favor the use of open-source software tools. Our work-flow combines existing tools (building on the work of others whenever possible) along with software tools and utilities that we develop ourselves when existing tools do not suffice.

## 3 Description of Method

We've designed an FEM mesh creation work-flow for our purposes that can overall be described as having three sequential steps:

1. Image segmentation
2. Surface triangulation and refinement
3. Volumetric meshing

### 3.1 Image segmentation

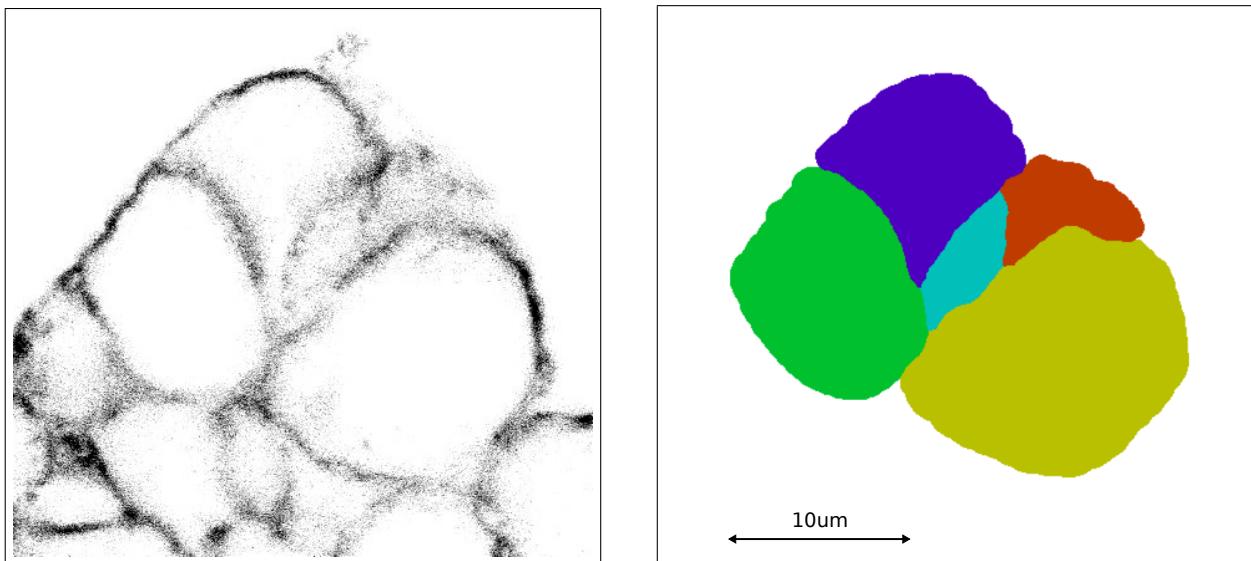


Figure 1: Subsection of a microscopy image slice on the left and its segmentation on the right.

By visual inspection we identified and selected a contiguous clump of seven cells within our source data image stack. This clump spanned thirty of the images and covered a maximum of approximately one-quarter of each image. Segmentation was done manually by tracing the outline of each cell in each image with a distinct color, followed by matched color flood-fill. One-quarter of original image number 16, along with it's segmentation, is shown in Figure 1.

Note that with our image stack the ratio of stack spacing to pixel resolution is 11.6 (being 0.069/0.80). To bring this closer to a ratio of one-to-one, we reduced the X and Y dimensions in the segmented image stack by a factor of four using nearest neighbor interpolation (to retain the distinct coloring of each cell) resulting in a pixel spacing of approximately 0.28 micro-meters. The reduced images were then combined into a single

XYZ TIFF stack for convenience.

### 3.1.1 Curvature

We also wanted to extract some information from the segmented image stack about the smoothness of the surface of each cell. We chose line curvature as a surface smoothness characteristic indicator. Both line and surface curvature have been used successfully by one of us (Rugis) in other work to characterise the surface of objects [Rug05, RK06b, RK06a]. (A description of how we used characteristic line curvature information as a reference for guidance in a cell surface curvature smoothing process in this current work will be given in following sections.)

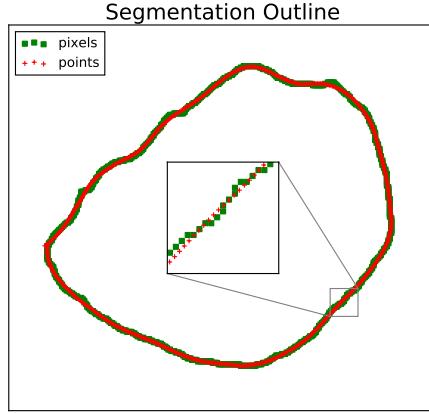


Figure 2: A sample segmentation pixel outline in green with smoothed node points in red.

For curvature calculation purposes, we went back to the 1024 by 1024 segmented images and extracted pixels associated with the closed curve boundary outline for each cell in each image, then selected the outline containing the maximum number of pixels for each cell as being the closest to a “great arc” slice through that cell. (This great arc criterion was based on the fact that only the curvature associated with a great arc of a sphere is equal to the sphere mean surface curvature.)

Next, considering that fact that image pixels are all located on a regular rectangular grid (not very useful for calculating actual local curvature!), a smoothed version of each cell outline was created using a 2D Savitzky-Golay (least-squares) fitting filter [SG64]. See Figure 2 for a sample cell outline showing the original pixel locations as well as the smoothing process result.

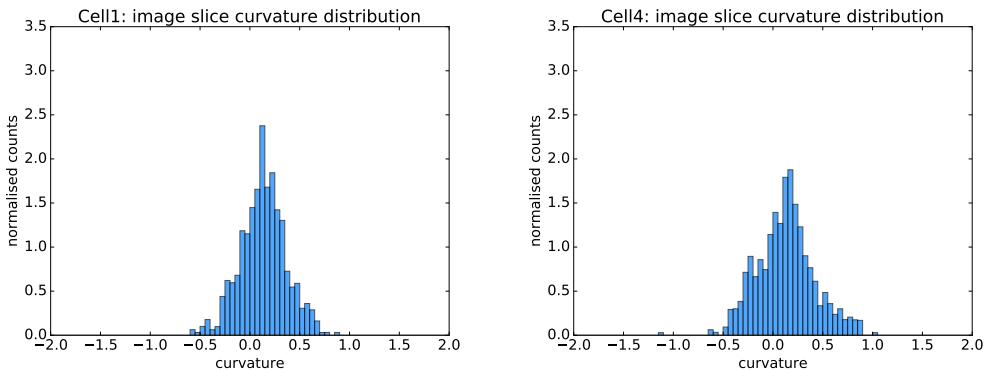


Figure 3: Reference curvature histograms for two of the cells (in  $\mu\text{m}^{-1}$ ). The distributions are “near normal”.

Planar line curvature was calculated at each point on the smoothed outlines using a local estimator as given in [Rug08]. We used weighted histograms to visualise the curvature distribution for each of the seven cells. Two of the histograms are shown in Figure 3. Note that in both cases the curvature distribution is biasing towards the positive, as would be expected with any closed curve from the outside using the convention that positive

curvature is associated with convex line segments.

cell	curvature std ( $\mu\text{m}^{-1}$ )
1	0.2305
2	0.3567
3	0.3864
4	0.2832
5	0.6868
6	0.3627
7	0.4226

Table 1: The standard deviation of curvature values for seven cells extracted from the microscopy image stack.

To simplify our characterisation of cell surface shape we chose the weighted standard deviation of the curvatures for each cell as a single (although imperfect<sup>1</sup>) measure that characterizes the surface smoothness and, in that sense, the shape, of each cell. The weighted curvature standard deviation for each of the seven cells is shown in Table 1.

### 3.2 Surface triangulation and refinement

For the surface triangulation and refinement process we started with the reduced XYZ TIFF image stack described in the previous section. We now thought of this reduced stack as essentially a 256x256x31 solid voxel block within which each voxel is labelled with a color associated with the cell that it belongs to.

#### 3.2.1 Surface triangulation

Significant prior work in extracting triangle surface meshes from labelled voxel blocks has been done by Boltcheva et al.[BYB09] and we used their technique. Sample computer code implementing this technique was found on the Computational Geometry Algorithms Library (CGAL) web site<sup>2</sup>. Note that we needed to convert our voxel block data to Inria (.inr) format<sup>3</sup> before passing it to the CGAL code.

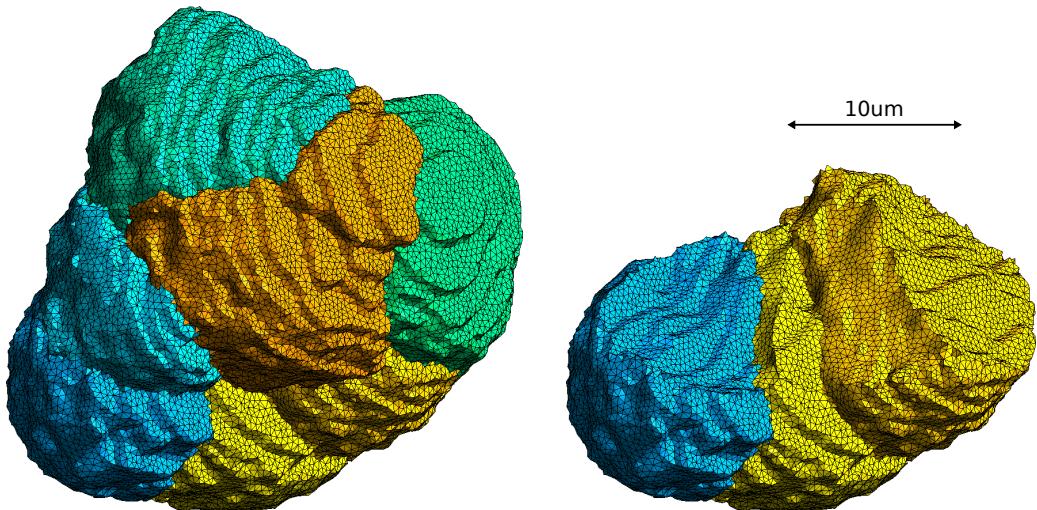


Figure 4: Rough multi-domain surface mesh: all seven cells (left) and three exposed cells (right).

The output from the CGAL code was a multi-domain triangle surface mesh, as shown in Figure 4, where each of the cell faces are labelled in color by cell. However, given our data and subsequent intentions, there are two problems with this mesh: there are ‘‘steps’’ as a result of the relatively limited stack resolution and, on close inspection, the surfaces are rather rough (as can be seen more clearly in the cell labelled ‘‘no smoothing’’ in the top of Figure 5). This lack of surface smoothness has the net effect of increasing the surface area of each cell to a value larger than what it is in the real cells.

<sup>1</sup>Standard deviation is only an unambiguous characterisation given normally distributed data.

<sup>2</sup>[www.cgal.org](http://www.cgal.org)

<sup>3</sup>[www.inria.fr](http://www.inria.fr)

### 3.2.2 Surface smoothing

To reduce the surface area closer to what it should be, we created a constrained surface smoothing process which is the primary contribution of this work. Note that the smoothing process should: 1) minimise the surface area of each cell while 2) maintaining the volume of each cell and 3) keeping the shared conformal faces between cells as being shared.

A psuedo-code outline for our iterative smoothing process is as follows:

```

Main Loop
{
    Cell Loop (for each cell)
    {
        Smooth the cell.
        Restore volume of cell.
        Calculate the cell surface curvature.
    }
    Have all cells reached the target curvature?
    Yes: DONE, exit loop.
    No: Go back to top of Main Loop.
}

```

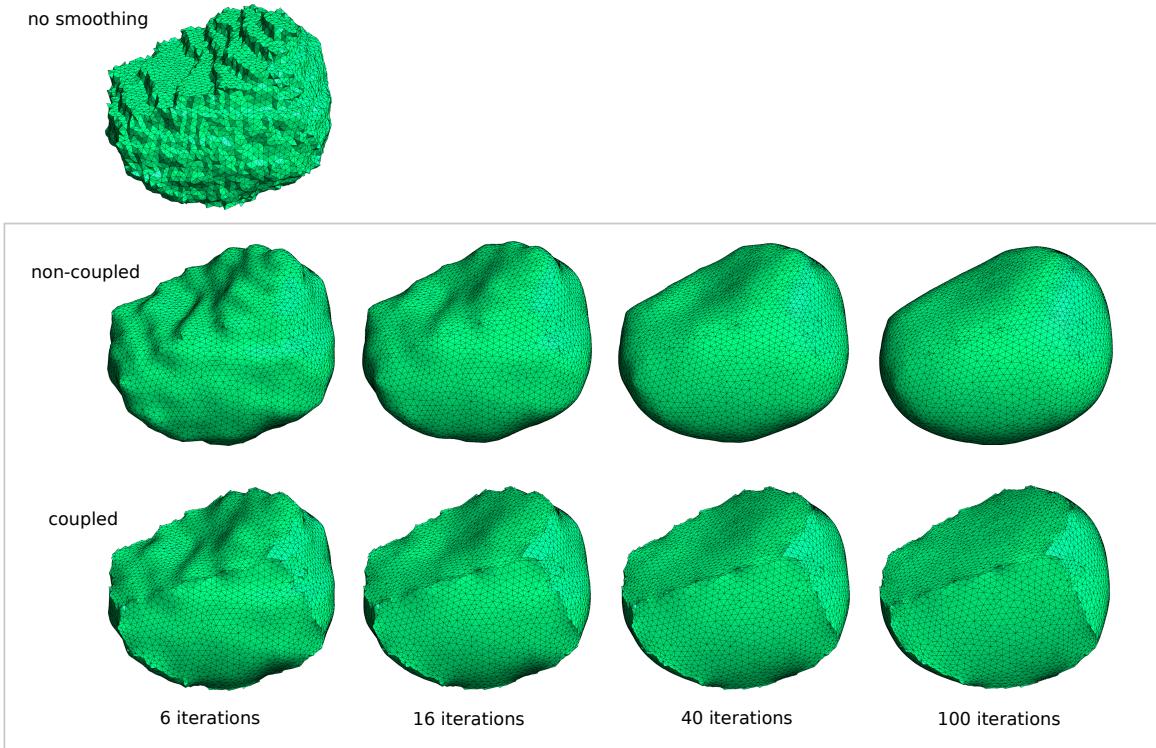


Figure 5: Cell smoothing evolution, with and without inter-cell coupling.

For each cell we used the curvature-flow based smoothing operation described in [DMSB99]. Note that the cell smoothing process works by slightly moving the position of the surface mesh vertices<sup>4</sup> for each cell in turn. Therefore, vertices associated with shared faces get moved twice in each pass through the Main Loop, i.e. once for each cell in every adjacent pair of cells. In this sense, the smoothing is a coupled process. With our approach, if the vertices associated with shared faces were allowed to split from each other, the result would be non-coupled as shown in the middle row of Figure 5, which is not what we want.

Smoothing results after each of each of 6, 16, 40 and 100 iterations are rendered as images in Figure 5. The question remains: After how many iterations should we stop? This is where we found the weighted curvature standard deviation values from Table 1 useful as will be described in the next section.

<sup>4</sup>Also, no new faces are added, nor are any faces removed.

### 3.2.3 Iteration termination

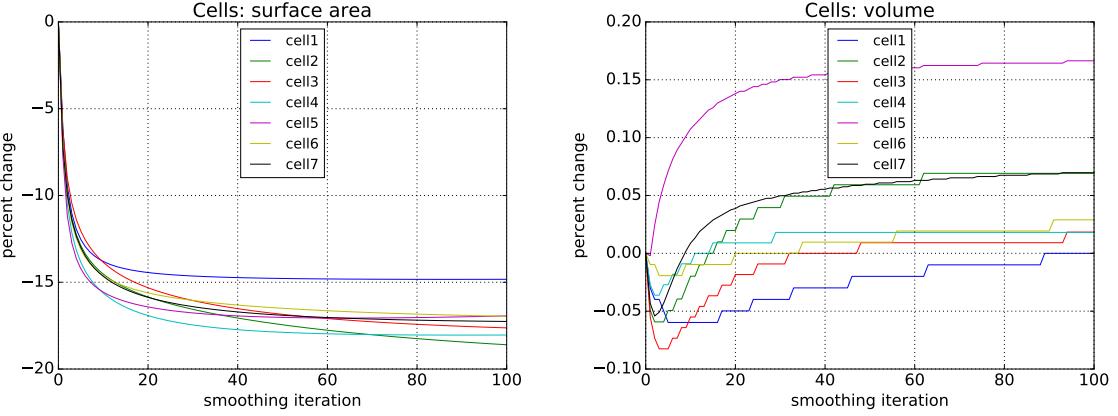


Figure 6: Cell surface area and volume evolution over one hundred coupled smoothing iterations. (Note the much smaller scale range with the plot on the right.)

Recall from the previous section that reducing cell surface area while at the same time maintaining cell volume was an important consideration. The smoothing step in our process has the effect of reducing surface area as shown for one hundred iterations in the left-hand side of Figure 6. Cell volume is explicitly restored in our process, so doesn't vary much as can be seen in the right-hand side of Figure 6. (The raw data for these plots is given in Appendix Tables 3 and 4.) These plots indicate that we've met the surface area and volume consideration after ten or so iterations, but offer no further guidance as to when to terminate the iterative process. For this we looked to surface curvature.

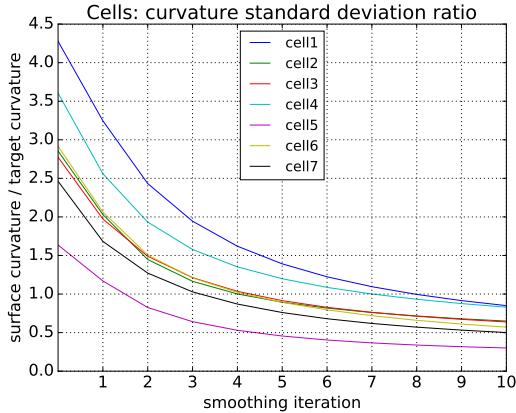


Figure 7: All of the cells hit the target curvature standard deviation ratio of one after nine iterations.

For each cell, in every iteration, we calculated the 2D surface curvature at every mesh vertex using the (local) surface curvature estimator given in [Rug08]. And for each cell, in every iteration, we calculated the weighted standard deviation of those curvatures to get a single smoothness characteristic value. We then compared this value to the associated target standard deviation in Table 1 and expressed it as a ratio as shown in Figure 7. (The raw data for this plot is given in Appendix Table 2.)

With this information in hand, we decided to terminate the iterative process and accept the results after ten iterations by which time all of the cells had more than dropped below their target characteristic curvature.

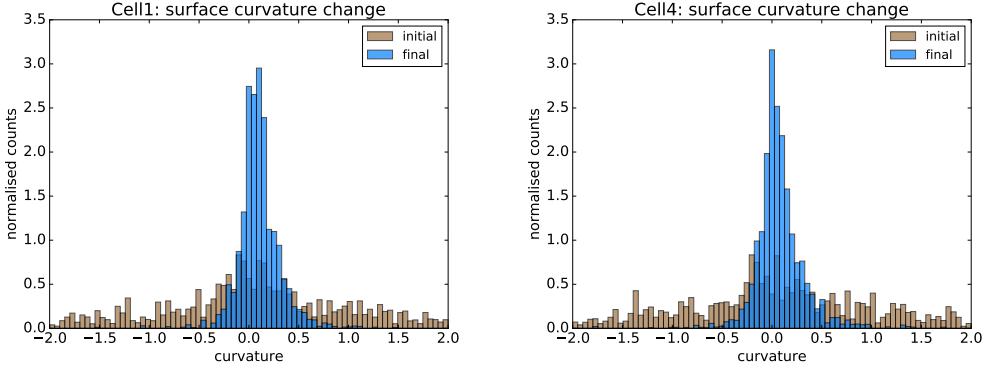


Figure 8: Initial surface curvature distribution in brown and smoothed surface curvature in blue.

For further insight and confirmation of our results we produced initial and final iteration weighted surface curvature histograms for each of the seven cells. Histograms for two of the cells are shown in Figure 8. Note that in all cases, as expected, the initial curvature spread was relatively wide and the final curvature distribution narrower, peaking just to the right (positive) side of zero. The final curvature histograms compared favourably to those associated with the image stacks including those shown earlier in Figure 3.

### 3.3 Volumetric meshing

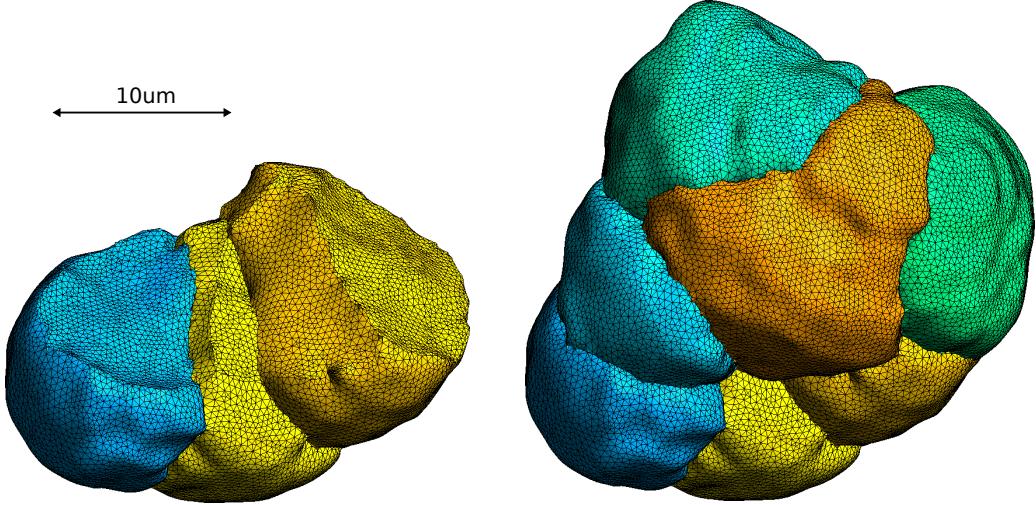


Figure 9: Fully meshed smoothed cells: three exposed cells (left) and all seven cells (right).

GMSH.

## 4 Results

SOME SIMULATION RESULTS FROM NATHAN GO HERE.

## 5 Discussion

Challenge of physically accurate mesh model generation given incomplete data.

Mesh design constrained by both the practical requirements of finite element modeling and the simplification assumptions made in the mathematical model.

In our application, we believe that there is no objective collection of criteria on which to optimally decide precisely when to terminate the iterative smoothing process. However, we do believe that stopping after say  $10 \pm 2$  iterations is well justified as described earlier in this paper.

## 6 Conclusion and Future Plans

Produced multi-domain conformal mesh for FEM simulation. Confirmed by using the meshes in initial simulations with an updated mathematical model that includes interactions between cells.

Future plans include multiple parameter sweep simulation runs to fully explore our new coupled cell model.  
ANY MORE TO GO HERE?

## 7 Acknowledgements

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ANY MORE?

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## A Numerical Data

cell	0	1	2	3	4	5	6	7	8	9	10
1	0.9837	0.7466	0.5595	0.4472	0.3726	0.3201	0.2814	0.2520	0.2289	0.2105	0.1955
2	1.0143	0.7216	0.5146	0.4141	0.3559	0.3177	0.2906	0.2702	0.2543	0.2414	0.2309
3	1.0620	0.7552	0.5703	0.4641	0.3963	0.3501	0.3169	0.2921	0.2727	0.2571	0.2443
4	1.0402	0.7387	0.5579	0.4552	0.3903	0.3459	0.3135	0.2888	0.2691	0.2529	0.2394
5	1.1262	0.8054	0.5683	0.4418	0.3649	0.3139	0.2782	0.2524	0.2331	0.2184	0.2070
6	1.0739	0.7598	0.5541	0.4453	0.3766	0.3286	0.2927	0.2649	0.2428	0.2249	0.2102
7	1.0451	0.7137	0.5396	0.4363	0.3692	0.3226	0.2885	0.2626	0.2422	0.2259	0.2126

Table 2: The evolution of standard deviation of mean surface curvature for the seven cells (in  $\mu\text{m}^{-1}$ ) after each of ten coupled smoothing iterations.

cell	0	1	2	3	4	5	6	7	8	9	10
1	628.64	590.17	569.09	559.23	553.53	549.84	547.26	545.37	543.93	542.80	541.90
2	729.06	680.85	657.39	646.44	639.76	635.12	631.65	628.91	626.66	624.77	623.15
3	681.01	638.45	619.64	609.78	603.46	598.93	595.46	592.68	590.37	588.40	586.70
4	712.88	661.36	637.82	626.19	619.08	614.15	610.47	607.57	605.02	603.22	601.52
5	443.18	408.76	393.58	386.72	382.76	380.16	378.30	376.90	375.81	374.92	374.19
6	684.72	636.81	615.75	605.50	599.26	595.00	591.87	589.46	587.55	585.98	584.67
7	627.42	582.15	564.89	555.74	549.94	545.88	542.82	540.42	538.47	536.84	535.46

Table 3: The evolution of surface area for the seven cells (in  $\mu\text{m}^2$ ) at each coupled smoothing iteration.

cell	0	1	2	3	4	5	6	7	8	9	10
1	1004.3	1004.0	1003.9	1003.9	1003.8	1003.7	1003.7	1003.7	1003.7	1003.7	1003.7
2	1012.4	1011.9	1011.8	1011.8	1011.8	1011.9	1011.9	1012.0	1012.0	1012.1	1012.2
3	1090.2	1089.6	1089.4	1089.3	1089.3	1089.3	1089.4	1089.4	1089.5	1089.5	1089.6
4	1105.7	1105.4	1105.3	1105.3	1105.4	1105.4	1105.5	1105.5	1105.6	1105.6	1105.6
5	492.76	492.75	492.88	492.98	493.05	493.11	493.16	493.20	493.23	493.26	493.29
6	1036.3	1036.2	1036.2	1036.1	1036.1	1036.1	1036.1	1036.1	1036.1	1036.2	1036.2
7	903.99	903.60	903.50	903.53	903.60	903.69	903.78	903.87	903.94	904.01	904.07

Table 4: The evolution of volume for the seven cells (in  $\mu\text{m}^3$ ) at each coupled smoothing iteration.

## B Software tools

CGAL  
GitHub  
GMSH  
ImageMagick  
MatLab  
Photoshop  
Python

cellmesh.git