Electro-Magnetics

Applied Del, Div, Grad, Curl

$\boldsymbol{p}$	position
l	length
$d\boldsymbol{l} \equiv \hat{\mathbf{t}}\, dl$	length segment
A	area
$d\boldsymbol{A} \equiv \hat{\mathbf{n}}\, dA$	surface patch
V	volume

$$\begin{aligned} \operatorname{div} \boldsymbol{F} &\equiv \lim_{A\to 0} \frac{1}{V} \oiint \boldsymbol{F} \cdot d\boldsymbol{A} \equiv \nabla \cdot \boldsymbol{F} \\ \hat{\mathbf{n}} \cdot \operatorname{curl} \boldsymbol{F} &\equiv \lim_{\Delta A_l \to 0} \frac{1}{\Delta A_l} \oint \boldsymbol{F} \cdot d\boldsymbol{l} \equiv \hat{\mathbf{n}} \cdot (\nabla \times \boldsymbol{F}) \\ \operatorname{grad} f &\equiv \nabla f \end{aligned}$$

$$\begin{aligned} \oiint \boldsymbol{F} \cdot d\boldsymbol{A} &= \iiint \nabla \cdot \boldsymbol{F}\, dV \\ \oint \boldsymbol{F} \cdot d\boldsymbol{l} &= \iint \nabla \times \boldsymbol{F} \cdot \hat{\mathbf{n}}\, dA \end{aligned}$$

$$\text{del} \equiv \nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r}\frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r}\frac{\partial}{\partial \theta} \\ \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \end{bmatrix}$$

$$\begin{aligned} \nabla f(x,y,z) &= \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} \\ \nabla \cdot \boldsymbol{F}(x,y,z) &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \\ \nabla \times \boldsymbol{F}(x,y,z) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \\ &= \begin{bmatrix} \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \\ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \\ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{bmatrix} \end{aligned}$$

Physical Constants

c	3.000×10^{8}	speed of light in free space	$\mathrm{m^{1}\,s^{-1}}$
$\varepsilon_0 = 10^7/c^2 4\pi$	8.854×10^{-12}	free space permitivity	$\mathrm{N^{-1}\,C^{2}\,m^{-2}}$
$\mu_0 = 4\pi/10^7$	1.257×10^{-6}	free space permeability	$\mathrm{N^{1}\,C^{-2}\,s^{2}}$

Variable, Quantity, Unit

$\boldsymbol{p}$	position	meter	$\mathrm{m^{1}}$
$\boldsymbol{v} = d\boldsymbol{p}/dt$	velocity	meter/second	$\mathrm{m^{1}\,s^{-1}}$
$\boldsymbol{a} = d\boldsymbol{v}/dt$	acceleration	meter/second^2	$\mathrm{m^{1}\,s^{-2}}$
Q	charge (bulk)	coulomb	$\mathrm{C^{1}}$
$I = dQ/dt$	electric current	ampere	$\mathrm{C^{1}\,s^{-1}}$
$\rho = dQ/dV$	charge density	coulomb/meter^3	$\mathrm{C^{1}\,m^{-3}}$
ε	permittivity	farad/meter	$\mathrm{N^{-1}\,C^{2}\,m^{-2}}$
μ	permeability	henry/meter	$\mathrm{N^{1}\,C^{-2}\,s^{2}}$
$\boldsymbol{E}$	electric field	volt/meter, newton/coulomb	$\mathrm{N^{1}\,C^{-1}}$
$\boldsymbol{B}$	magnetic flux density	weber/meter^2	$\mathrm{N^{1}\,C^{-1}\,m^{-1}\,s^{1}}$
$\boldsymbol{J} = \rho\boldsymbol{v}$	current flux density	ampere/meter^2	$\mathrm{C^{1}\,m^{-2}\,s^{-1}}$
σ	electric conductivity	siemen/meter	$\mathrm{N^{-1}\,C^{2}\,m^{-2}\,s^{-1}}$
$\boldsymbol{D}$	electric flux density	coulomb/meter^2	$\mathrm{C^{1}\,m^{-2}}$
$\boldsymbol{H}$	magnetic field	ampere/meter	$\mathrm{C^{1}\,m^{-1}\,s^{-1}}$
$\boldsymbol{M}$	magnetic		$\mathrm{N^{1}\,C^{-1}\,m^{-1}}$
σ^*	magnetic loss	ohm/meter	$\mathrm{N^{1}\,C^{-2}\,s^{1}}$

Force, Charge, Electric Field

$$\boldsymbol{F}/q = \boldsymbol{E} + \big(\boldsymbol{v} \times \boldsymbol{B}\big)$$

$$\begin{aligned} \boldsymbol{E}(\boldsymbol{p}_0) &= \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i(\boldsymbol{p}_0 - \boldsymbol{p}_i)}{|\boldsymbol{p}_0 - \boldsymbol{p}_i|^3} \\ &= \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho_V(\boldsymbol{p}_0 - \boldsymbol{p}_V)}{|\boldsymbol{p}_0 - \boldsymbol{p}_V|^3} dV \end{aligned}$$

Conservation

check this one

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{J} = 0 \qquad \text{charge}$$

Electrostatic

$$\begin{aligned} \oiint \boldsymbol{E} \cdot d\boldsymbol{A} &= Q/\varepsilon_0 \\ \oint \boldsymbol{E} \cdot d\boldsymbol{l} &= 0 \end{aligned}$$

$$\begin{aligned} \nabla \cdot \boldsymbol{E} &= \rho/\varepsilon_0 \\ \nabla \times \boldsymbol{E} &= 0 \end{aligned}$$

Magnetostatic

$$\begin{aligned} \oiint \boldsymbol{B} \cdot d\boldsymbol{A} &= 0 \\ \oint \boldsymbol{B} \cdot d\boldsymbol{l} &= \mu_0 I \end{aligned}$$

$$\begin{aligned} \nabla \cdot \boldsymbol{B} &= 0 \\ \nabla \times \boldsymbol{B} &= \mu_0 \boldsymbol{J} \end{aligned}$$

Electromagnetic

$$\begin{aligned} \oint \boldsymbol{E} \cdot d\boldsymbol{l} &= -\iint \frac{d\boldsymbol{B}}{dt} \cdot d\boldsymbol{A} \\ \oint \boldsymbol{B} \cdot d\boldsymbol{l} &= \mu_0 I + \mu_0 \varepsilon_0 \iint \frac{d\boldsymbol{E}}{dt} \cdot d\boldsymbol{A} \end{aligned}$$

$$\begin{aligned} \nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \times \boldsymbol{B} &= \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \end{aligned}$$

E/M Equations

$$\nabla \Phi = -\boldsymbol{E}$$

$$\nabla^2 \Phi = \rho/\varepsilon$$

$$c^2 \nabla \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \frac{\boldsymbol{J}}{\varepsilon_0}$$

$$\oiint \boldsymbol{D} \cdot d\boldsymbol{A} = 0$$

$$\oiint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

$$\frac{\partial}{\partial t} \iint \boldsymbol{D} \cdot d\boldsymbol{A} = \oint \boldsymbol{H} \cdot d\boldsymbol{l} - \iint \boldsymbol{J} \cdot d\boldsymbol{A}$$

$$\frac{\partial}{\partial t} \iint \boldsymbol{B} \cdot d\boldsymbol{A} = -\oint \boldsymbol{E} \cdot d\boldsymbol{l} - \iint \boldsymbol{M} \cdot d\boldsymbol{A}$$

$$\nabla \cdot \boldsymbol{D} = 0$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\frac{\partial \boldsymbol{D}}{\partial t} = (\nabla \times \boldsymbol{H}) - \boldsymbol{J}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -(\nabla \times \boldsymbol{E}) - \boldsymbol{M}$$

$$\begin{aligned} \boldsymbol{J} &= \boldsymbol{J}_{source} + \sigma \boldsymbol{E} \\ \boldsymbol{M} &= \boldsymbol{M}_{source} + \sigma^* \boldsymbol{H} \end{aligned}$$

FDTD Model

$$\begin{aligned} \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} &= (\nabla \times \boldsymbol{H}) - (\boldsymbol{J}_{source} + \sigma \boldsymbol{E}) \\ \mu \frac{\partial \boldsymbol{H}}{\partial t} &= -(\nabla \times \boldsymbol{E}) - (\boldsymbol{M}_{source} + \sigma^* \boldsymbol{H}) \end{aligned}$$

$$\begin{aligned} \frac{\partial E_x}{\partial t} &= \frac{1}{\varepsilon}\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - (\boldsymbol{J}_{source_x} + \sigma E_x)\right] \\ \frac{\partial E_y}{\partial t} &= \frac{1}{\varepsilon}\left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - (\boldsymbol{J}_{source_y} + \sigma E_y)\right] \\ \frac{\partial E_z}{\partial t} &= \frac{1}{\varepsilon}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - (\boldsymbol{J}_{source_z} + \sigma E_z)\right] \end{aligned}$$

$$\begin{aligned} \frac{\partial H_x}{\partial t} &= \frac{1}{\mu}\left[\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - (\boldsymbol{M}_{source_x} + \sigma^* H_x)\right] \\ \frac{\partial H_y}{\partial t} &= \frac{1}{\mu}\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - (\boldsymbol{M}_{source_y} + \sigma^* H_y)\right] \\ \frac{\partial H_z}{\partial t} &= \frac{1}{\mu}\left[\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - (\boldsymbol{M}_{source_z} + \sigma^* H_z)\right] \end{aligned}$$

$$x \triangleright y \triangleright z \triangleright$$

$$\begin{aligned} \varepsilon E_t^1 &= H_2^3 - H_3^2 - J_{S1} - \sigma E_1 \\ \mu H_t^1 &= E_3^2 - E_2^3 - M_{S1} - \sigma H_1 \end{aligned}$$

Velocity and Frequency

$$\nu = f\lambda \ \text{ traveling wave speed}$$

$$\nu = \frac{1}{\sqrt{\varepsilon\mu}} \ \text{ em wave speed}$$

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