

Abstract Algebra

Group

Definition 1. A set is a **group** iff, given some definition of addition on that set:

1. addition is associative,
2. there is an additive identity,
3. each element has an additive inverse.
4. Additionally, the group is **abelian** iff addition is commutative.

Field

Definition 2. A set is a **field** iff, given some definition of addition and multiplication on that set:

1. addition and multiplication are associative,
2. there is an additive and a multiplicative identity,
3. each element has an additive inverse,
4. each element (other than some “zero”) has a multiplicative inverse,
5. addition and multiplication are commutative,
6. multiplication distributes over addition.