

Zeta Function

Definition 1. The zeta function $\zeta(z) \equiv \sum_{n=1}^{\infty} \frac{1}{n^z}$

The zeta function converges when $\operatorname{Re}(z) > 1$.

Zeta Product Formula

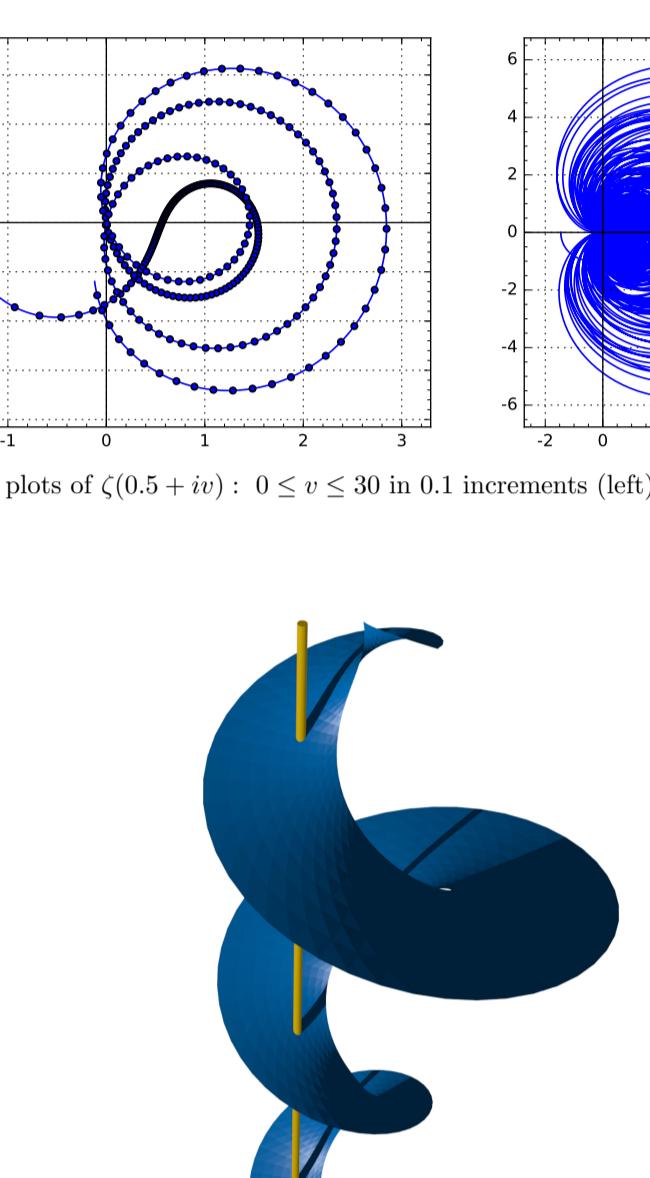
$$\zeta(z) = \prod_{n=1}^{\infty} \frac{1}{1 - p_n^{-z}} \quad \text{converges when } \operatorname{Re}(z) > 1. \quad (1)$$

Reimann Function

Alternative formulation:

$$R(x) = 1 + \sum_{n=1}^{\infty} \frac{(\ln x)^n}{n n! \zeta(n+1)} \quad (2)$$

Analytic Continuation, Zeta Function Computation

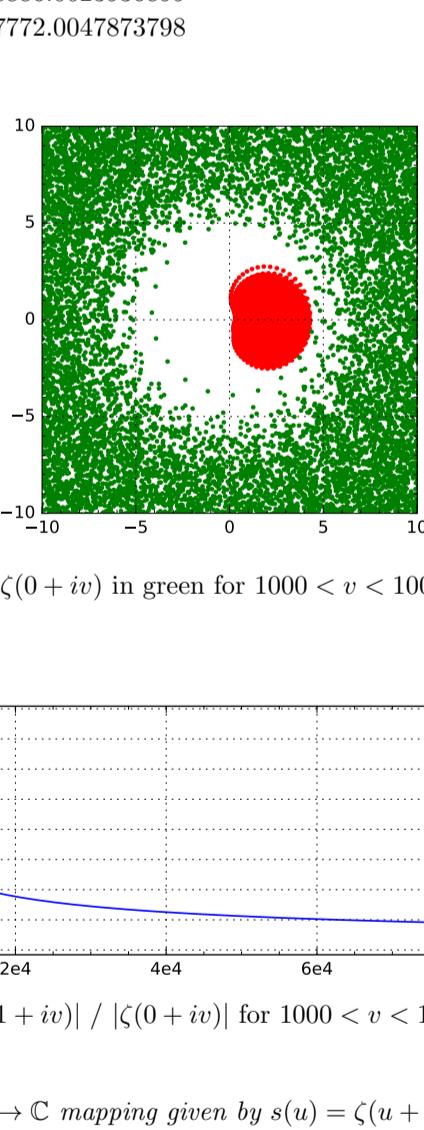
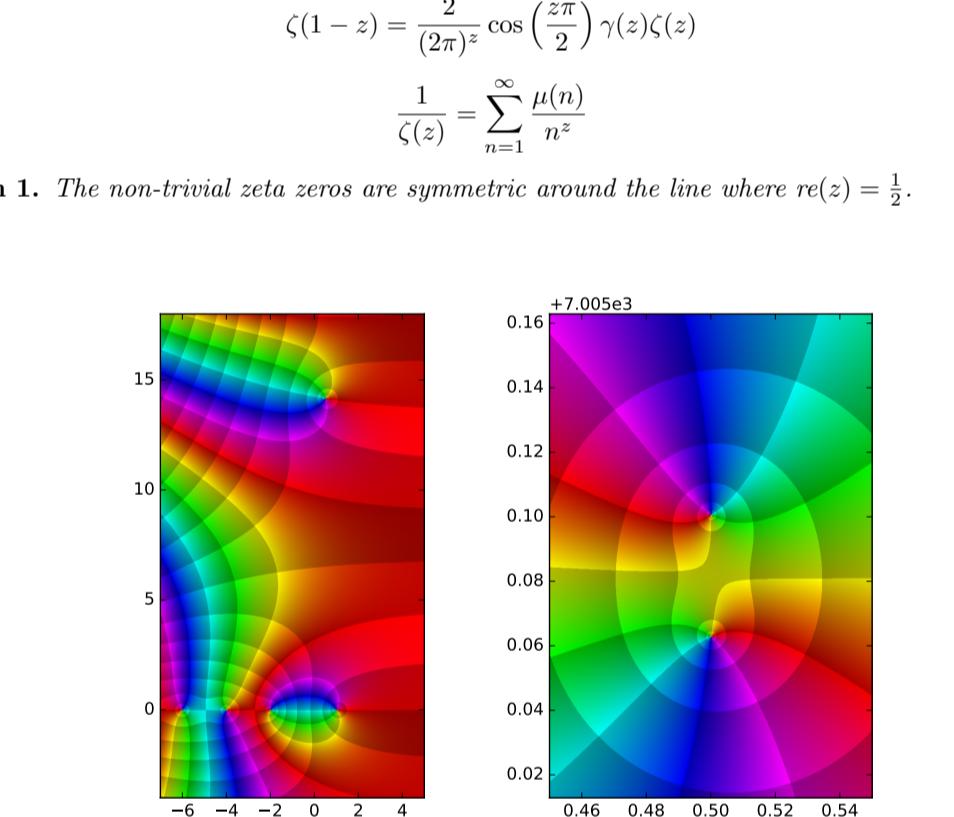


Notes:

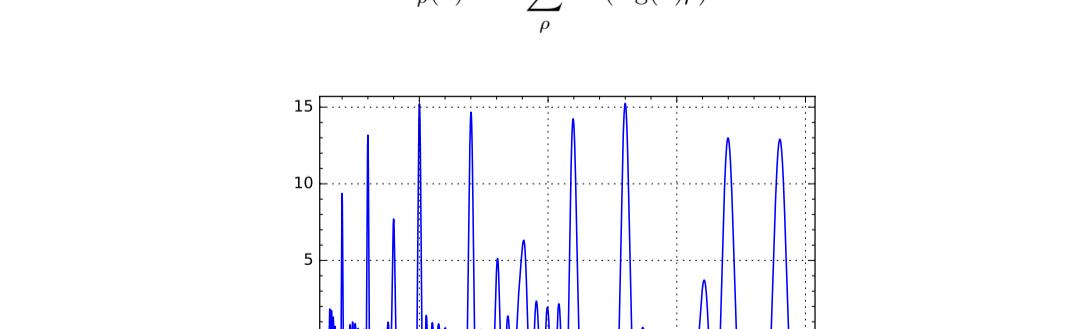
1. The values of $\zeta(z)$ where $\operatorname{re}(z) > 1$ are relatively static.
2. There is a single pole at $z = 1$ and zeros where $z = \{-2, -4, -6, -8, \dots\}$.
3. The *non-trivial* zeros are in the *critical strip*, the region where $0 < \operatorname{re}(z) < 1$.

Non-trivial Zeta Zeros

Conjecture 1. Reimann Hypothesis: All of the non-trivial zeros of $\zeta(s)$ are on the line $\operatorname{re}(z) = \frac{1}{2}$.



Zeta critical strip spiral and first four zeros.

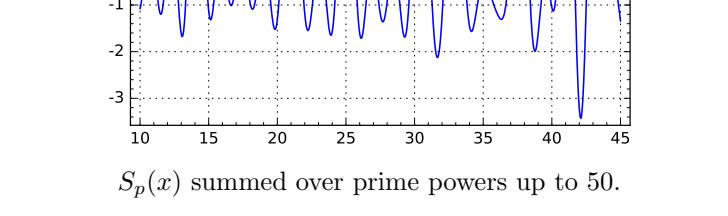


Differences between the first one hundred thousand zeta zero pairs (left) and triples (right).

$$\zeta(1-z) = \frac{2}{(2\pi)^z} \cos\left(\frac{z\pi}{2}\right) \gamma(z) \zeta(z) \quad (3)$$

$$\frac{1}{\zeta(z)} = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^z} \quad (4)$$

Theorem 1. The non-trivial zeta zeros are symmetric around the line where $\operatorname{re}(z) = \frac{1}{2}$.



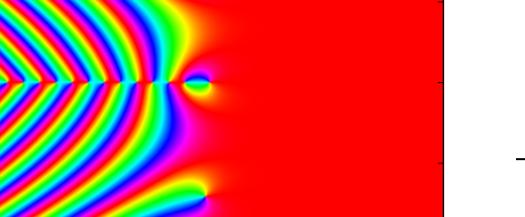
$\zeta(z)$ argument contour plot with isolines for both argument and modulus.

The first non-trivial zero at ~ 14.13 (left). A close-pair of non-trivial zeros at ~ 7005 (right).

Some close-pair zeros:
 7005.062866175 7005.100564674
 71732.901207872 71732.915909348
 388858886.0022851203 388858886.0023936899
 777717772.0045702406 777717772.0047873798

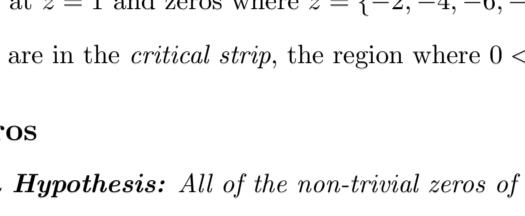
From zeta zeros to prime powers:
 $S_p(x) = - \sum_{\rho} \cos(\log(x)\rho)$

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$S_p(x)$ summed over 50 zeta zeros.

$$S_p(x) = - \sum_{p^n} \frac{\log(p)}{p^{n/2}} \cos(x \log(p^n))$$



$S_p(x)$ summed over prime powers up to 50.