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# Zeta Function

**Definition 1.** *The **zeta function*** $\zeta(z) \equiv \sum_{n=1}^{\infty} \frac{1}{n^z}$

The zeta function converges when $\mathrm{Re}(z) > 1$.

## Zeta Product Formula

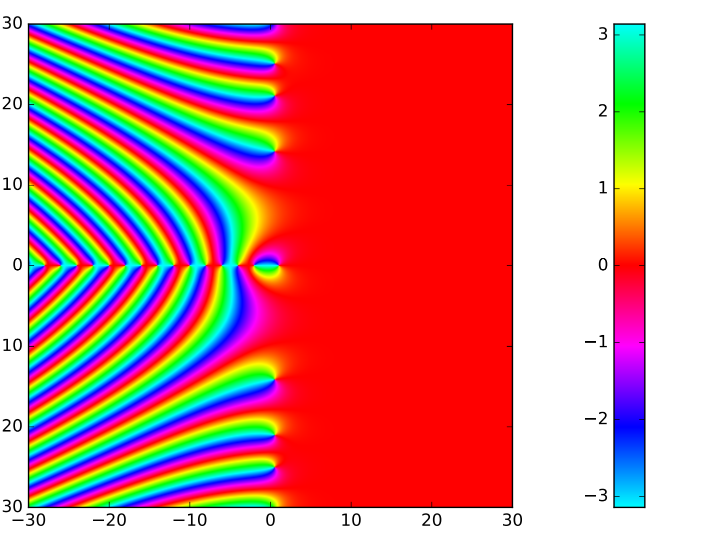
$$\zeta(z) = \prod_{n=1}^{\infty} \frac{1}{1 - p_n^{-z}} \quad \text{converges when } \mathrm{Re}(z) > 1. \tag{1}$$

## Reimann Function

Alternative formulation:

$$\mathrm{R}(x) = 1 + \sum_{n=1}^{\infty} \frac{(\ln x)^n}{n\, n!\, \zeta(n+1)} \tag{2}$$

## Analytic Continuation, Zeta Function Computation



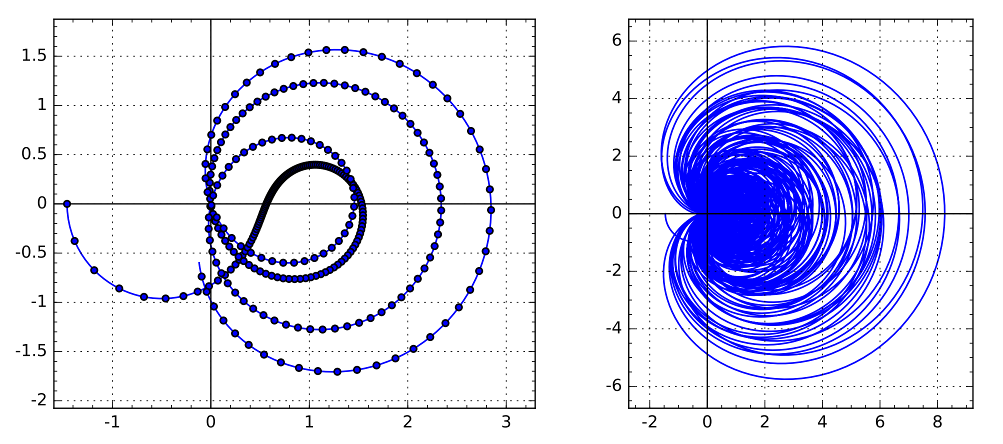
Contour plot: $\arg \zeta(z)$

*Notes:*

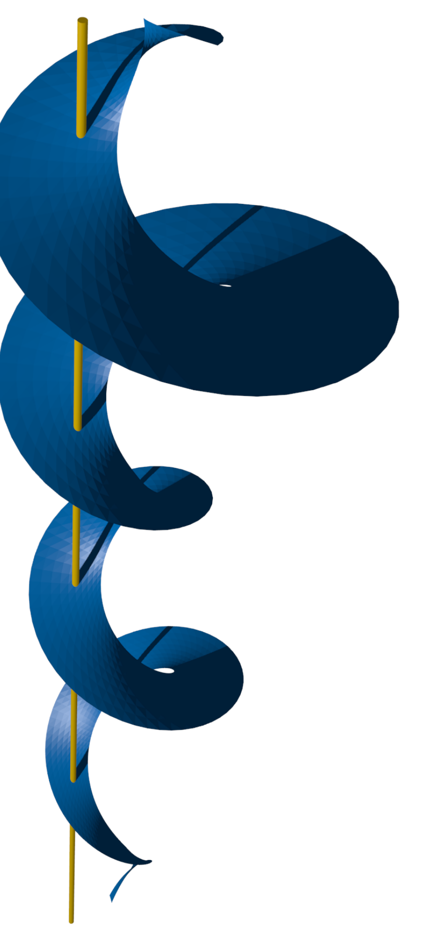
1. The values of $\zeta(z)$ where $\mathrm{re}(z) > 1$ are relatively static.
2. There is a single pole at $z = 1$ and zeros where $z = \{-2, -4, -6, -8, \dots\}$.
3. The *non-trivial* zeros are in the *critical strip*, the region where $0 < \mathrm{re}(z) < 1$.

## Non-trivial Zeta Zeros

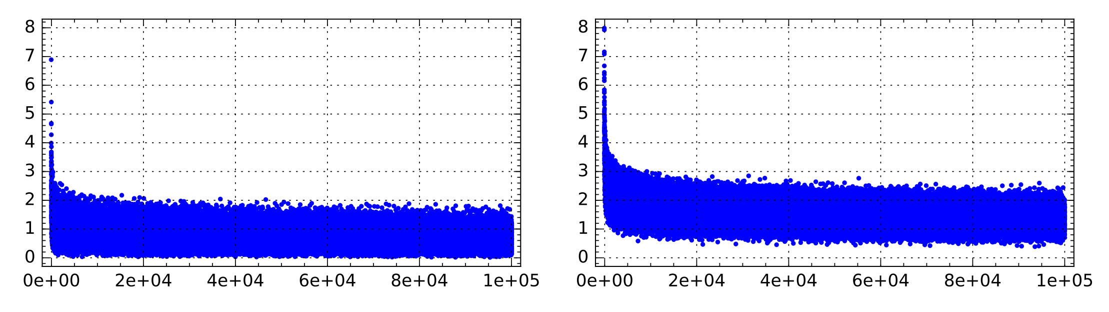
**Conjecture 1.** ***Reimann Hypothesis:*** *All of the non-trivial zeros of* $\zeta(s)$ *are on the line* $re(z) = \frac{1}{2}$.



Parametric plots of $\zeta(0.5 + iv):\ 0 \le v \le 30$ in 0.1 increments (left), $0 \le v \le 500$ (right).



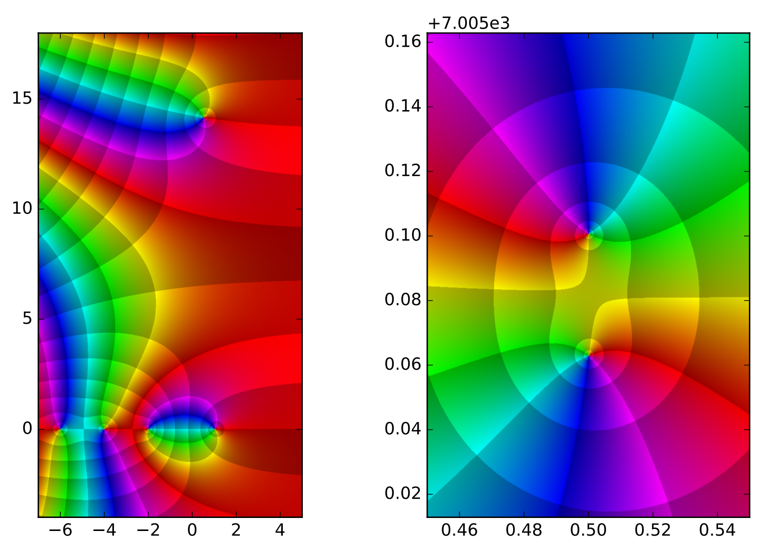
Zeta critical strip spiral and first four zeros.



Differences between the first one hundred thousand zeta zero pairs (left) and triples (right).

**Some *close-pair* zeros:**

| | |
|---|---|
| 7005.062866175 | 7005.100564674 |
| 71732.901207872 | 71732.915909348 |
| 388858886.0022851203 | 388858886.0023936899 |
| 777717772.0045702406 | 777717772.0047873798 |

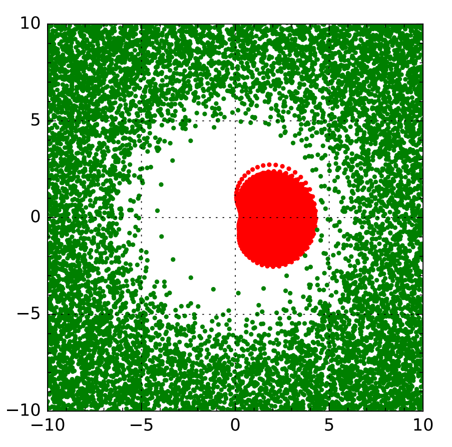


$\zeta(z)$ argument contour plot with isolines for both argument and modulus.
The first non-trivial zero at ~14.13 (left). A *close-pair* of non-trivial zeros at ~7005 (right).

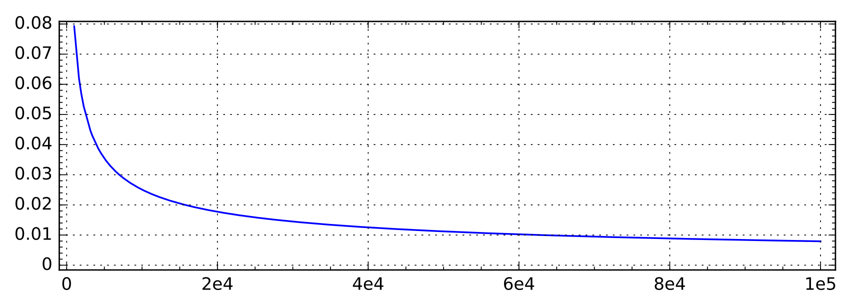
$$\zeta(1-z) = \frac{2}{(2\pi)^z} \cos\left(\frac{z\pi}{2}\right) \gamma(z)\zeta(z) \tag{3}$$

$$\frac{1}{\zeta(z)} = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^z} \tag{4}$$

**Theorem 1.** *The non-trivial zeta zeros are symmetric around the line where* $re(z) = \frac{1}{2}$.



$\zeta(1 + iv)$ in red and $\zeta(0 + iv)$ in green for $1000 < v < 10000$ in 0.05 increments.



$|\zeta(1 + iv)| \ / \ |\zeta(0 + iv)|$ for $1000 < v < 10000$

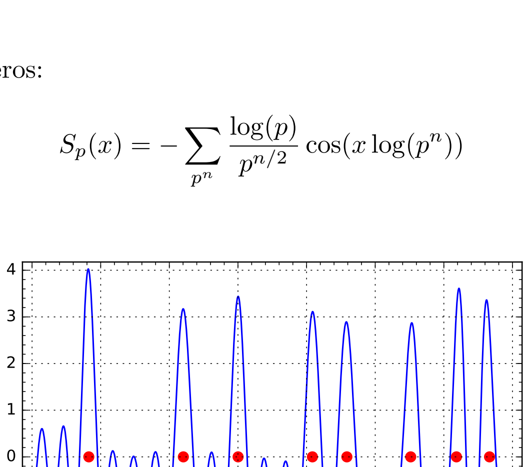
**Conjecture 2.** *Consider the* $\mathbb{R} \to \mathbb{C}$ *mapping given by* $s(u) = \zeta(u + iv)$ *where* $0 < u < 1$ *and* $v$ *is any real constant greater than one. If* $s(u_1) = s(u_2)$ *then* $|u_1 - \frac{1}{2}| \ne |u_2 - \frac{1}{2}|$.

**Conjecture 3.** *Consider the* $\mathbb{R} \to \mathbb{C}$ *mapping given by* $s(u) = \zeta(u + iv)$ *where* $0 < u < 1$ *and* $v$ *is any real constant greater than one. Whenever* $s(u_1) = s(u_2)$, $|\zeta(1 + iv) - \zeta(\frac{1}{2} + iv)| < |\zeta(1 + iv)|$ *and thus neither* $s(u_1)$ *nor* $s(u_2)$ *is equal to zero.*

## Prime Number and Zeta Zero Spectrum

From zeta zeros to prime powers:

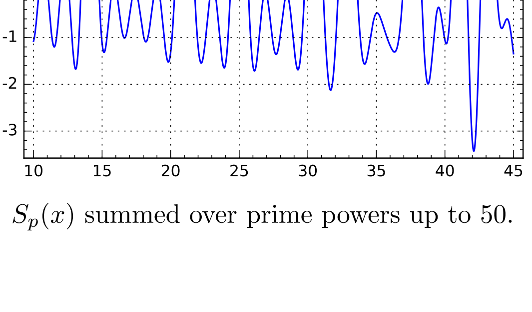
$$S_\rho(x) = -\sum_{\rho} \cos(\log(x)\rho)$$



$S_\rho(x)$ summed over 50 zeta zeros.

From prime powers to zeta zeros:

$$S_p(x) = -\sum_{p^n} \frac{\log(p)}{p^{n/2}} \cos(x \log(p^n))$$



$S_p(x)$ summed over prime powers up to 50.