Sequences

Fibonacci Sequence

$$1, 1, 2, 3, 5, 8, 13, \ldots$$
$$u_n = u_{n-2} + u_{n-1}$$

Series

The sequence of partial sums of an infinite series either converges to a single fixed value or it diverges.

Harmonic Series

Definition 1. *The **harmonic series** is defined as:* $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

Theorem 1. *The harmonic series diverges.*

Proof.

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{1}{n} &= 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \ldots \\ &> 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \ldots \\ &> 1 + \frac{1}{2} + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) + \ldots\end{aligned}$$

RHS diverges $\implies$ LHS diverges. □

Geometric Series

Definition 2. *The **geometric series** is defined as:* $\displaystyle\sum_{n=0}^{\infty} r^n$

Theorem 2. *The geometric series* $\displaystyle\sum_{n=0}^{\infty} r^n$ *converges to* $\dfrac{1}{1-r}$ *when* $|r| < 1$.

Proof.

$$\begin{aligned}\sum_{n=0}^{\infty} r^n &= 1 + r + r^2 + r^3 \ldots \\ &= \lim_{n\to\infty} (1 + r + r^2 + \cdots + r^n) \\ &= \lim_{n\to\infty} \left(\frac{1 - r^{n+1}}{1 - r}\right) \\ &= \frac{1}{1-r} \quad \text{when} \quad |r| < 1\end{aligned}$$

□

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