Electro-Magnetic FDTD Model

Variable - Quantity - Unit

ε	permittivity	farad/meter	$\mathrm{N}^{-1}\,\mathrm{C}^{2}\,\mathrm{m}^{-2}$
$\boldsymbol{E}$	electric field	volt/meter	$\mathrm{N}^{1}\,\mathrm{C}^{-1}$
$\boldsymbol{J}$	current flux density	ampere/meter^2	$\mathrm{C}^{1}\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}$
σ	electric conductivity	siemen/meter	$\mathrm{N}^{-1}\,\mathrm{C}^{2}\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}$

μ	permeability	henry/meter	$\mathrm{N}^{1}\,\mathrm{C}^{-2}\,\mathrm{s}^{2}$
$\boldsymbol{H}$	magnetic field	ampere/meter	$\mathrm{C}^{1}\,\mathrm{m}^{-1}\,\mathrm{s}^{-1}$
$\boldsymbol{M}$	magnetization	volt/meter^2	$\mathrm{N}^{1}\,\mathrm{C}^{-1}\,\mathrm{m}^{-1}$
σ_m	magnetic loss	ohm/meter	$\mathrm{N}^{1}\,\mathrm{C}^{-2}\,\mathrm{s}^{1}$

Electro-Magnetic Differential Equations

Electro-Magnetic Vectors

$$
\begin{aligned}
\varepsilon\frac{\partial \boldsymbol{E}}{\partial t} &= (\nabla\times\boldsymbol{H}) - \sigma\boldsymbol{E}\\
\mu\frac{\partial \boldsymbol{H}}{\partial t} &= -(\nabla\times\boldsymbol{E}) - \sigma_m\boldsymbol{H}
\end{aligned}
$$

Electro-Magnetic Components

$$
\begin{aligned}
\varepsilon\frac{\partial E_x}{\partial t} &= \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\\
\varepsilon\frac{\partial E_y}{\partial t} &= \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\\
\varepsilon\frac{\partial E_z}{\partial t} &= \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\partial H_x}{\partial t} &= \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \sigma_m H_x\\
\mu\frac{\partial H_y}{\partial t} &= \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \sigma_m H_y\\
\mu\frac{\partial H_z}{\partial t} &= \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \sigma_m H_z
\end{aligned}
$$

Finite Difference Equations

$$
\begin{aligned}
E_x\begin{bmatrix}t+1\\ i+\frac{1}{2}\\ j\\ k\end{bmatrix} &= \left(\frac{1-\frac{\sigma\Delta t}{2\epsilon}}{1+\frac{\sigma\Delta t}{2\epsilon}}\right)E_x\begin{bmatrix}t\\ i+\frac{1}{2}\\ j\\ k\end{bmatrix}\\
&+\left(\frac{1}{1+\frac{\sigma\Delta t}{2\epsilon}}\right)\left(\frac{\Delta t}{\epsilon\Delta y}\left(H_z\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j+\frac{1}{2}\\ k\end{bmatrix}-H_z\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j-\frac{1}{2}\\ k\end{bmatrix}\right)-\frac{\Delta t}{\epsilon\Delta z}\left(H_y\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j\\ k+\frac{1}{2}\end{bmatrix}-H_y\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j\\ k-\frac{1}{2}\end{bmatrix}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
E_y\begin{bmatrix}t+1\\ i\\ j+\frac{1}{2}\\ k\end{bmatrix} &= \left(\frac{1-\frac{\sigma\Delta t}{2\epsilon}}{1+\frac{\sigma\Delta t}{2\epsilon}}\right)E_y\begin{bmatrix}t\\ i\\ j+\frac{1}{2}\\ k\end{bmatrix}\\
&+\left(\frac{1}{1+\frac{\sigma\Delta t}{2\epsilon}}\right)\left(\frac{\Delta t}{\epsilon\Delta y}\left(H_x\begin{bmatrix}t+\frac{1}{2}\\ i\\ j+\frac{1}{2}\\ k+\frac{1}{2}\end{bmatrix}-H_x\begin{bmatrix}t+\frac{1}{2}\\ i\\ j+\frac{1}{2}\\ k-\frac{1}{2}\end{bmatrix}\right)-\frac{\Delta t}{\epsilon\Delta z}\left(H_z\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j+\frac{1}{2}\\ k\end{bmatrix}-H_z\begin{bmatrix}t+\frac{1}{2}\\ i-\frac{1}{2}\\ j+\frac{1}{2}\\ k\end{bmatrix}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
E_z\begin{bmatrix}t+1\\ i\\ j\\ k+\frac{1}{2}\end{bmatrix} &= \left(\frac{1-\frac{\sigma\Delta t}{2\epsilon}}{1+\frac{\sigma\Delta t}{2\epsilon}}\right)E_z\begin{bmatrix}t\\ i\\ j\\ k+\frac{1}{2}\end{bmatrix}\\
&+\left(\frac{1}{1+\frac{\sigma\Delta t}{2\epsilon}}\right)\left(\frac{\Delta t}{\epsilon\Delta x}\left(H_y\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j\\ k+\frac{1}{2}\end{bmatrix}-H_y\begin{bmatrix}t+\frac{1}{2}\\ i-\frac{1}{2}\\ j\\ k+\frac{1}{2}\end{bmatrix}\right)-\frac{\Delta t}{\epsilon\Delta y}\left(H_x\begin{bmatrix}t+\frac{1}{2}\\ i\\ j+\frac{1}{2}\\ k+\frac{1}{2}\end{bmatrix}-H_x\begin{bmatrix}t+\frac{1}{2}\\ i\\ j-\frac{1}{2}\\ k+\frac{1}{2}\end{bmatrix}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
H_x\begin{bmatrix}t+1\\ i+\frac{1}{2}\\ j\\ k\end{bmatrix} &= \left(\frac{1-\frac{\sigma\Delta t}{2\epsilon}}{1+\frac{\sigma\Delta t}{2\epsilon}}\right)H_x\begin{bmatrix}t\\ i+\frac{1}{2}\\ j\\ k\end{bmatrix}\\
&+\left(\frac{1}{1+\frac{\sigma\Delta t}{2\epsilon}}\right)\left(\frac{\Delta t}{\epsilon\Delta y}\left(E_z\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j+\frac{1}{2}\\ k\end{bmatrix}-E_z\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j-\frac{1}{2}\\ k\end{bmatrix}\right)-\frac{\Delta t}{\epsilon\Delta z}\left(E_y\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j\\ k+\frac{1}{2}\end{bmatrix}-E_y\begin{bmatrix}t+\frac{1}{2}\\ i+\frac{1}{2}\\ j\\ k-\frac{1}{2}\end{bmatrix}\right)\right)
\end{aligned}
$$

Field Update Code

```
n = i + j * sX + k * sXY

c[n].ex = c[n].cexe * c[n].ex
        + c[n].cexh * ((c[n].hz - c[n - sX].hz) - (c[n].hy - c[n - sXY].hy))
c[n].ey = c[n].ceye * c[n].ey
        + c[n].ceyh * ((c[n].hx - c[n - sXY].hx) - (c[n].hz - c[n - 1].hz))
c[n].ez = c[n].ceze * c[n].ez
        + c[n].cezh * ((c[n].hy - c[n - 1].hy) - (c[n].hx - c[n - sX].hx))

c[n].hx = c[n].chxh * c[n].hx
        + c[n].chxe * ((c[n + sXY].ey - c[n].ey) - (c[n + sX].ez - c[n].ez))
c[n].hy = c[n].chyh * c[n].hy
        + c[n].chye * ((c[n + 1].ez - c[n].ez) - (c[n + sXY].ex - c[n].ex))
c[n].hz = c[n].chzh * c[n].hz
        + c[n].chze * ((c[n + sX].ex - c[n].ex) - (c[n + 1].ey - c[n].ey))
```

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