

Zeta Function

Definition 1. The **zeta function** $\zeta(z) \equiv \sum_{n=1}^{\infty} \frac{1}{n^z}$

The zeta function converges when $\operatorname{Re}(z) > 1$.

Zeta Product Formula

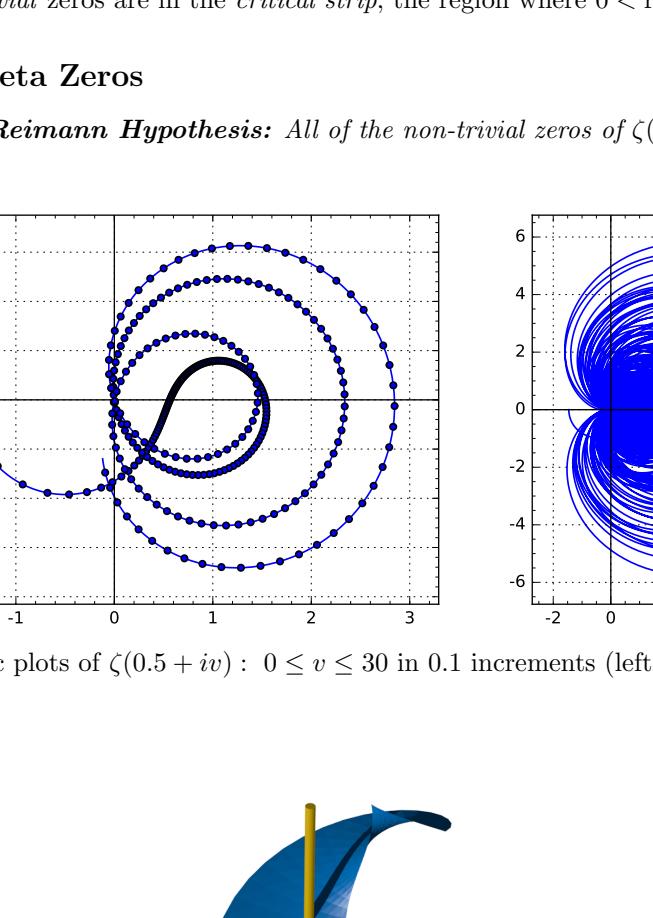
$$\zeta(z) = \prod_{n=1}^{\infty} \frac{1}{1 - p_n^{-z}} \quad \text{converges when } \operatorname{Re}(z) > 1. \quad (1)$$

Reimann Function

Alternative formulation:

$$R(x) = 1 + \sum_{n=1}^{\infty} \frac{(\ln x)^n}{n n! \zeta(n+1)} \quad (2)$$

Analytic Continuation, Zeta Function Computation



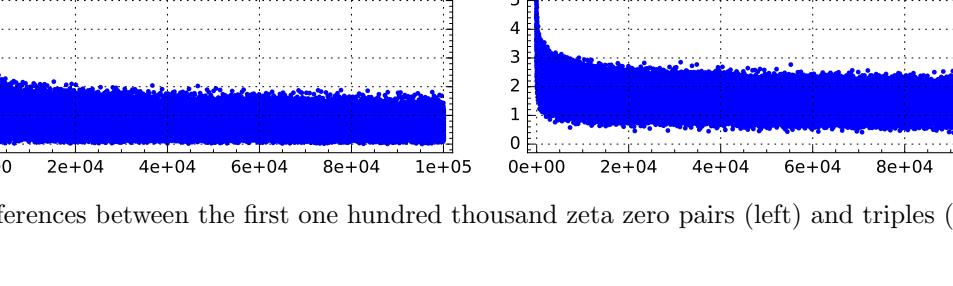
Contour plot: $\arg \zeta(z)$

Notes:

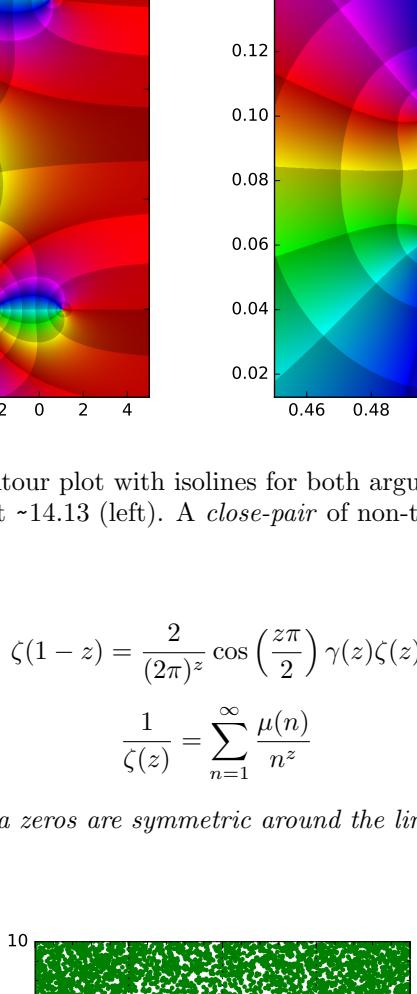
1. The values of $\zeta(z)$ where $\operatorname{re}(z) > 1$ are relatively static.
2. There is a single pole at $z = 1$ and zeros where $z = \{-2, -4, -6, -8, \dots\}$.
3. The *non-trivial* zeros are in the *critical strip*, the region where $0 < \operatorname{re}(z) < 1$.

Non-trivial Zeta Zeros

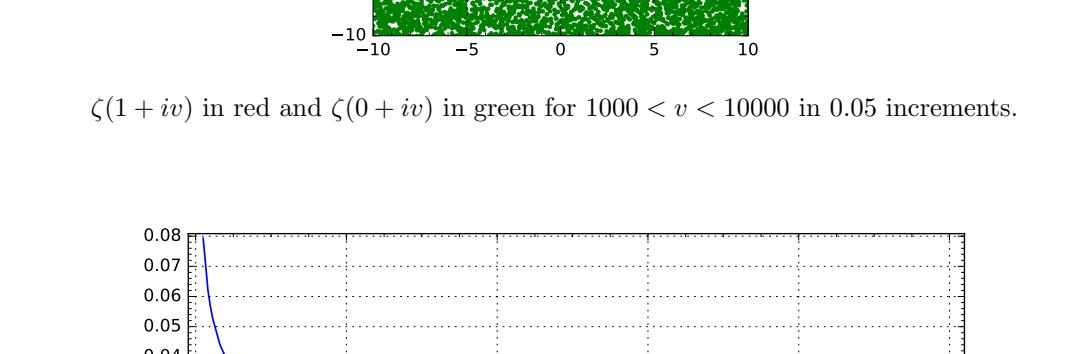
Conjecture 1. Reimann Hypothesis: All of the non-trivial zeros of $\zeta(s)$ are on the line $\operatorname{re}(z) = \frac{1}{2}$.



Parametric plots of $\zeta(0.5 + iv) : 0 \leq v \leq 30$ in 0.1 increments (left), $0 \leq v \leq 500$ (right).



Zeta critical strip spiral and first four zeros.



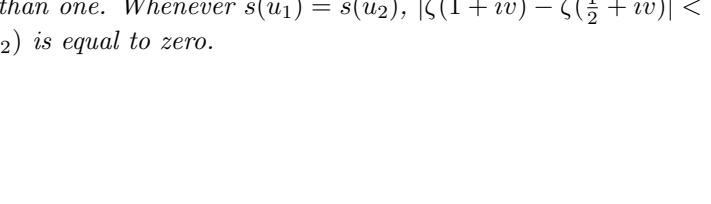
Differences between the first one hundred thousand zeta zero pairs (left) and triples (right).

Some close-pair zeros:

7005.062866175	7005.100564674
71732.901207872	71732.915909348

388858886.0022851203	388858886.0023936899
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777717772.0045702406	777717772.0047873798
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$\zeta(z)$ argument contour plot with isolines for both argument and modulus.

The first non-trivial zero at ~ 14.13 (left). A *close-pair* of non-trivial zeros at ~ 7005 (right).

$$\zeta(1-z) = \frac{2}{(2\pi)^z} \cos\left(\frac{z\pi}{2}\right) \gamma(z) \zeta(z) \quad (3)$$

$$\frac{1}{\zeta(z)} = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^z} \quad (4)$$

Theorem 1. The non-trivial zeta zeros are symmetric around the line where $\operatorname{re}(z) = \frac{1}{2}$.

$\zeta(1+iv)$ in red and $\zeta(0+iv)$ in green for $1000 < v < 10000$ in 0.05 increments.

$|\zeta(1+iv)| / |\zeta(0+iv)|$ for $1000 < v < 10000$

Conjecture 2. Consider the $\mathbb{R} \rightarrow \mathbb{C}$ mapping given by $s(u) = \zeta(u+iv)$ where $0 < u < 1$ and v is any real constant greater than one. If $s(u_1) = s(u_2)$ then $|u_1 - \frac{1}{2}| \neq |u_2 - \frac{1}{2}|$.

Conjecture 3. Consider the $\mathbb{R} \rightarrow \mathbb{C}$ mapping given by $s(u) = \zeta(u+iv)$ where $0 < u < 1$ and v is any real constant greater than one. Whenever $s(u_1) = s(u_2)$, $|\zeta(1+iv) - \zeta(\frac{1}{2}+iv)| < |\zeta(1+iv)|$ and thus neither $s(u_1)$ nor $s(u_2)$ is equal to zero.