Mixed Data-Type Circuit Modelling

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Operator	Operation	Arguments		Result
$+$	addition	numeric	numeric	numeric
-	subtraction	numeric	numeric	numeric
-	negation	numeric	none	numeric
(blank)	multiplication	numeric	numeric	numeric
$/$	division	numeric	numeric	numeric
$<$	less than	numeric	numeric	boolean
$>$	greater than	numeric	numeric	boolean
$=$	equal to	numeric	numeric	boolean
$\wedge$	and	boolean	boolean	boolean
$\vee$	or	boolean	boolean	boolean
$\oplus$	exclusive or	boolean	boolean	boolean
$\Rightarrow$	implication	boolean	boolean	boolean

Operator Summary

When required, convert the Boolean "true" to the real number 1 and the Boolean "false" to the real number 0.

A boolean expression:

$$i = v/R, \text{ where } (i, v, R) \in \mathbb{R}$$

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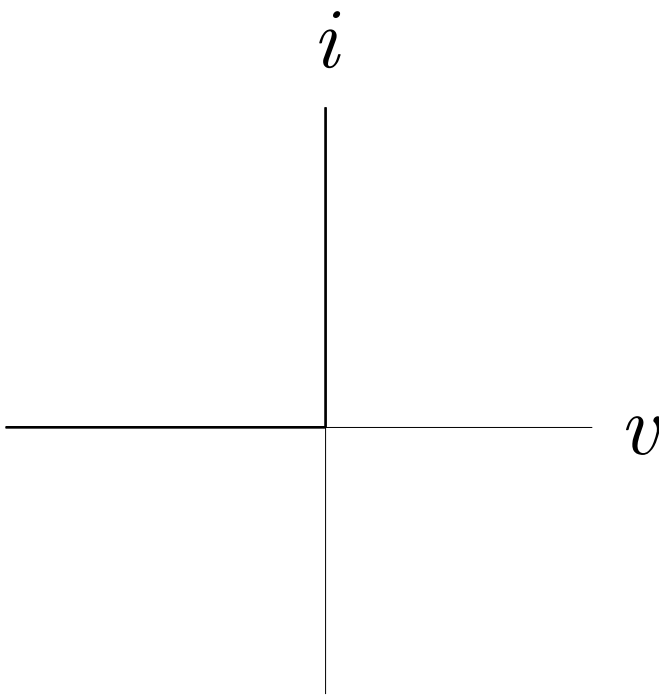
Ohm's Law:

$$\vDash (i = v/R)$$



Diode characteristic vi curve.

1. Either the voltage or the current is zero,
2. if the current is zero then the voltage is less than or equal to zero,
3. and if the voltage is zero then the current is greater than or equal to zero.

$$(v = 0) \vee (i = 0)$$
$$(i = 0) \Rightarrow (v \leq 0)$$
$$(v = 0) \Rightarrow (i \geq 0)$$

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$$\vDash \Big(\big((v = 0) \vee (i = 0) \big) \wedge \big((i = 0) \Rightarrow (v \leq 0) \big) \wedge \big((v = 0) \Rightarrow (i \geq 0) \big) \Big)$$

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{A} \\ -\mathbf{A}^T & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} & \mathbf{N} \end{bmatrix} \begin{bmatrix} \mathbf{e} \\ \mathbf{v} \\ \mathbf{i} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{u}_s \end{bmatrix}$$

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$$\begin{bmatrix} \mathbf{M} & \mathbf{N} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix} = \mathbf{u}_s$$

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$$\begin{bmatrix} \mathbf{M} & \mathbf{N} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix} = \mathbf{u}_s$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} v_b \\ i_b \end{bmatrix} = u_b$$

Selectors:

$$rx + sy = c$$
$$\text{where } (r, s) \in \mathbb{B},\ (x, y, c) \in \mathbb{R}$$
$$\text{and } \vDash (r \oplus s)$$

$$rv + si = 0$$

$$r \oplus s$$

$$(i = 0) \Rightarrow (v \leq 0)$$

$$(v = 0) \Rightarrow (i \geq 0)$$

$$rv + si = 0$$

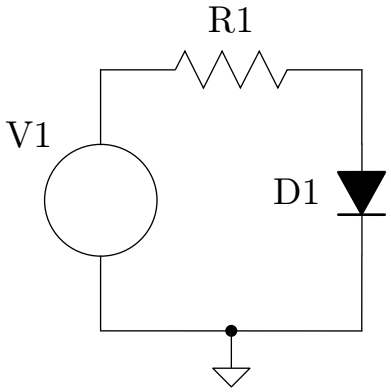
$$r \oplus s$$

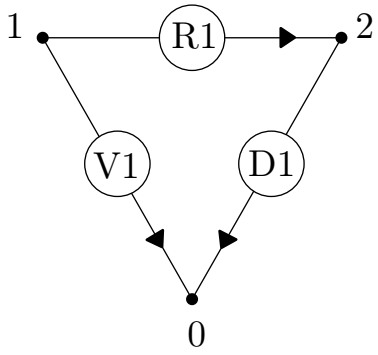
$$(i = 0) \Rightarrow (v \leq 0)$$

$$(v = 0) \Rightarrow (i \geq 0)$$

$$\begin{bmatrix} r & s \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} = 0$$

$$\text{where } \begin{bmatrix} r & s \end{bmatrix} \in \left\{ \begin{bmatrix} 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix} \right\}$$



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V1 1 0
R1 1 2
D1 2 0
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A simple circuit.

$$\left[\begin{array}{cccccccc|c} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & V_{V1} \\ 0 & 0 & 0 & 1 & 0 & 0 & -R_{R1} & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{D1} & 0 & 0 & s_{D1} & 0 \end{array}\right]$$

$$\det \mathbf{T} = s_{D1} - r_{D1} R_{R1}$$

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$\begin{bmatrix} r_{D1} & s_{D1} \end{bmatrix}$	$\det \mathbf{T}$
$\begin{bmatrix} 0 & 1 \end{bmatrix}$	1
$\begin{bmatrix} 1 & 0 \end{bmatrix}$	$-R_{R1}$

$$\det \mathbf{T} = s_{D1} - r_{D1} R_{R1}$$

$[\ r_{D1} \quad s_{D1}\ ]$	$\det \mathbf{T}$
$[\ 0 \quad 1\ ]$	1
$[\ 1 \quad 0\ ]$	$-R_{R1}$

$[\ r_{D1} \quad s_{D1}\ ]$	$\det \mathbf{T}$	constraints
$[\ 0 \quad 1\ ]$	1	none
$[\ 1 \quad 0\ ]$	$-R_{R1}$	$R_{R1} > 0$

$$v_{D1} = \frac{s_{D1} V_{V1}}{\det \mathbf{T}}$$

$$i_{D1} = \frac{-r_{D1} V_{V1}}{\det \mathbf{T}}$$

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$$i_{D1} = \frac{-r_{D1} V_{V1}}{\det \mathbf{T}}$$

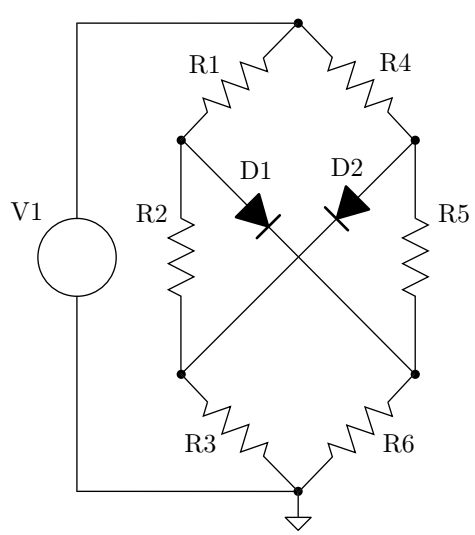
$\begin{bmatrix} r_{D1} & s_{D1} \end{bmatrix}$	constraints	v_{D1}	i_{D1}
$\begin{bmatrix} 0 & 1 \end{bmatrix}$	none	V_{V1}	0
$\begin{bmatrix} 1 & 0 \end{bmatrix}$	$R_{R1} > 0$	0	V_{V1}/R_{R1}

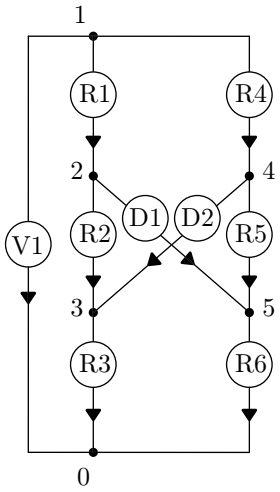
constraints	v_{D1}	i_{D1}
$v_{D1} \leq 0$	V_{V1}	0
$i_{D1} \geq 0, R_{R1} > 0$	0	V_{V1}/R_{R1}

constraints	v_{D1}	i_{D1}
$v_{D1} \leq 0$	V_{V1}	0
$i_{D1} \geq 0, R_{R1} > 0$	0	V_{V1}/R_{R1}

constraints	v_{D1}	i_{D1}
$V_{V1} \leq 0$	V_{V1}	0
$V_{V1} \geq 0, R_{R1} > 0$	0	V_{V1}/R_{R1}

$$\vDash \Big(\big((V_{V1} \leq 0) \implies (v_{D1} = V_{V1}) \wedge (i_{D1} = 0) \big) \wedge$$
$$\big((V_{V1} > 0) \wedge (R_{R1} > 0) \implies (v_{D1} = 0) \wedge (i_{D1} = V_{V1}/R_{R1}) \big) \Big)$$






```
V1 1 0
R1 1 2
R2 2 3
R3 3 0
R4 1 4
R5 4 5
R6 5 0
D1 2 5
D2 4 3
```

A more complex example.