

## Executive Summary

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Our group chose to do the Coin-flipping Game for our third group project. We first approached the problem by clearly identifying the key components and factors. The Coin-flipping Game has 144 turns and requires the player to flip either a dime or a quarter. The dime is worth 2 points, while the quarter is worth three points. If the coin lands on heads then you subtract the value of the coin from your score. If the coin lands on tails then you add the value of the coin to your score. The problem asks us to find an algorithm that maximizes the chances that the user wins. We knew that the algorithm was recursive so we used  $f(s,t)$  to represent the probability that you win the game. The “s” represents the score and the “t” represents the turns that are left. We know that if the score is zero or less and the turns are also zero, then you automatically lose. Additionally, if the score is one or more and the turns are zero, then you automatically win! After thinking about this fact, we created an Excel Sheet and used the rows to represent the turns and the columns to represent the score. We decided to have our scores range from -100 to 100. As a result of our previous thinking, we know that we can hardcode the values for the extremely small scores to “0” and the extremely high scores to “1”. We then deduced the equation to find the different probabilities of winning for dimes and quarters. If a dime is flipped then there is a probability of  $0.5*f(s-1,t-1) + 0.5*f(s+1,t-1)$  that we win. If a quarter is flipped then there is a probability of  $0.5*f(s-2,t-1) + 0.5*f(s+2,t-1)$  that we win. The two probability formulas are extremely similar, but the dime formula only subtracts or adds 1 point from the score while the Quarter formula subtracts or adds 2. We created the two different sheets, “QuartProb” and “DimeProb”, for each formula. We then created a third sheet “Combined Algorithm” that combines the quarter probability and the dime probability. It uses the formula  $f(s,t) = \max\{0.5*f(s-1,t-1) + 0.5*f(s+1,t-1), 0.5*f(s-2,t-1) + 0.5*f(s+2,t-1)\}$ . Finally we created a fourth sheet named “Table” that states which coin you should use to maximize your chances of winning. In conclusion, we have found that if the user’s score is positive and the turn is close to zero, then the best course of action is to flip a dime. If the score is less than or equal to zero and the turn is close to 144, then the user should flip the quarter.