

## Mathematical Derivation and Explanation for Part 2:

### *Score -*

If there is a customer at location  $x$  then:

$$\text{Score } A(0) = 28 - x - 12 = \mathbf{16 - x}$$

$$\text{Score } B(0) = 28 - (10 - x) - 15 = \mathbf{3 + x}$$

### *Probability -*

$$P(A0) = (16 - x) / ((16 - x) + (3 + x)) = \mathbf{(16 - x) / 19}$$

$$P(B0) = (3 + x) / ((3 + x) + (16 - x)) = \mathbf{(3 + x) / 19}$$

### *Profit -*

If the customer picks A0?  $\$4 - \$2 = \mathbf{\$2}$

If the customer picks B0?  $= \mathbf{\$0}$

### *Customer Location -*

The average location is 5, and since both probability equations are linear, we can assume the average location is also 5, therefore  $\mathbf{x = 5}$ .

### *Putting It All Together -*

$$P(A0) = (16 - 5) / 19 = \mathbf{11 / 19}$$

$$P(B0) = (3 + 5) / 19 = \mathbf{8 / 19}$$

When I multiply, by hand, the probability of A0 by the profit of each cup of coffee sold by cafe A0, I get a value that matches up with a similar value calculated in the simulation.

$$11/19 \times 2 = \mathbf{1.15789474}$$

In the simulation, the expected profit per customer for A0 has a very similar value being:

$$= \mathbf{1.15476}$$

**\*Therefore our expected profit is supported and verified by the simulation\***