Let the partial likelihood function for estimating the observed time-to-event Δ_i given covariates $\mathbf{x}$ and under parameters β be $\prod_{i=1}^{n}[f(\Delta_i|\mathbf{x}_i,\beta)]^{\delta_i}[1-F(\Delta_i|\mathbf{x}_i,\beta)]^{1-\delta_i}$ with $\Delta_i = \min(T_i, C_i)$ and $\delta_i = I(T_i \le C_i)$. T_i is the time to failure for individual i, C_i is the time to censoring for individual i, and δ_i is the censoring indicator for individual i. The censoring indicator $\delta_i = 1$ if failure is observed and $\delta_i = 0$ if censoring is observed. Therefore, Δ_i is the time until either the event or censoring occurs, whichever comes first. Then, the log-likelihood function is:

$$
\begin{aligned}
\phi(\Delta_i,\delta_i|\mathbf{x}_i,\beta) &= \log\left[\prod_{i=1}^{n}[f(\Delta_i|\mathbf{x}_i,\beta)]^{\delta_i}[1-F(\Delta_i|\mathbf{x}_i,\beta)]^{1-\delta_i}\right] \\
&= \sum_{i=1}^{n}\log\left[[f(\Delta_i|\mathbf{x}_i,\beta)]^{\delta_i}\right] + \log\left[[1-F(\Delta_i|\mathbf{x}_i,\beta)]^{1-\delta_i}\right] \\
&= \sum_{i=1}^{n}\delta_i\log f(\Delta_i|\mathbf{x}_i,\beta) + (1-\delta_i)\log(1-F(\Delta_i|\mathbf{x}_i,\beta)) \\
&= \sum_{i=1}^{n}\delta_i\log f(\Delta_i|\mathbf{x}_i,\beta) - \delta_i\log(1-F(\Delta_i|\mathbf{x}_i,\beta)) + \log(1-F(\Delta_i|\mathbf{x}_i,\beta)) \\
&= \sum_{i=1}^{n}\delta_i\left[\log f(\Delta_i|\mathbf{x}_i,\beta) - \log(1-F(\Delta_i|\mathbf{x}_i,\beta))\right] + \log(1-F(\Delta_i|\mathbf{x}_i,\beta)) \\
&= \sum_{i=1}^{n}\delta_i\left[\log\frac{f(\Delta_i|\mathbf{x}_i,\beta)}{(1-F(\Delta_i|\mathbf{x}_i,\beta))}\right] + \log(1-F(\Delta_i|\mathbf{x}_i,\beta)) \\
&= \sum_{i=1}^{n}\delta_i\log\lambda(\Delta_i|\mathbf{x}_i,\beta) + \log(1-F(\Delta_i|\mathbf{x}_i,\beta)) \\
&= \sum_{i=1}^{n}\delta_i\alpha(\Delta_i|\mathbf{x}_i,\beta) + \log\exp(-\int_0^{\Delta_i}(\exp(\alpha(u_i|\mathbf{x}_i,\beta))))du \\
&= \sum_{i=1}^{n}\delta_i\alpha(\Delta_i|\mathbf{x}_i,\beta) - \int_0^{\Delta_i}\exp(\alpha(u_i|\mathbf{x}_i,\beta))du, \Delta_i \ge 0, \delta_i \in \{0,1\}.
\end{aligned}
$$

The first several steps simply use rules of logarithms and the distributive property. Then using $\log A - \log B = \log(\frac{A}{B})$ and the definition of the hazard function as the conditional probability of failure at Δ given survival up to Δ, $\lambda(\Delta) = \frac{f(\Delta)}{1-F(\Delta)}$, the partial log-likelihood function can be simplified to $\sum_{i=1}^{n}\delta_i\alpha(\Delta_i|\mathbf{x}_i,\beta) - \int_0^{\Delta_i}\exp(\alpha(u_i|\mathbf{x}_i,\beta))du, \Delta_i \ge 0, \delta_i \in \{0,1\}$.

Therefore, the time-to-event model is determined entirely by its hazard function and censoring indicator.