Relations

A relation R is subsets of a cartesian product of 2 sets A and B : $R \subseteq A \times B$

Relations can be defined by a rule or condition that pairs (a,b) must follow : $R = \{(a,b) \in A \times B \mid a < b\}$

A relation could also be represented as a matrix

$R = \{(a,a), (a,b), (b,a)\}$ $\quad R^m = \begin{array}{c c} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} a \\ b \end{array} & \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$ $\quad$ Graph: $\quad a \rightleftarrows b$

Categories of Relations

Reflexive : Every element is paired with itself : $(a,a) \in R$ for all a

Irreflexive : No element is paired with itself : $(a,a) \notin R$ for all a

Symmetric : If elements are paired one way they must be paired the other : $(a,b) \rightarrow (b,a)$

Antisymmetric : If elements are paired both ways they must be equal : $(a,b) \rightarrow (b,a)$ if $a = b$

Asymmetric : Elements must not be paired both ways : $(a,b) \in R \rightarrow (b,a) \notin R$

Transitive : $(a,b) \wedge (b,c) \rightarrow (a,c)$, this is disproven via counter example

Set Proofs

Prove : $(A \cap B) \times C \subseteq A \times (C \cup B)$ $\rightarrow$ Every element of X is in Y. Prove by using arbitrary element x of the first set

Let x be an element of $(A \cap B) \times C$.

This means x is a product of $A \cap B$ and C. $x = yz$ where $y \in A \cap B$ and $z \in C$

Since $y \in A \cap B$, $y \in A$ by definition of intersection

Since $z \in C$, $z \in (C \cup B)$

$y \in A, z \in (C \cup B) \therefore A \times (C \cup B) = yz = x$ $\therefore$ This is proven because the arbitrary element x is in both sets

Proving Relation Properties

R is asymmetric, prove it's antisymmetric and irreflexive

Asymmetric = $(a,b) \in R \rightarrow (b,a) \notin R$ for arbitrary a, b

i) Antisymmetric : $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

However, this premise is false because both cannot be in R by the definition of asymmetry

Since premise is false, this holds and proves antisymmetry

ii) Irreflexive : Assume $b = a$. $(a,a) \in R \rightarrow (a,a) \notin R$ which is a contradiction

Meaning $(a,a) \notin R$ which defines irreflexivity