1. Logic

Use $\wedge$ with $\exists_x$: $\exists_x(W(x) \wedge U(x))$ → "There exist an x that is both"

Use → with $\forall x$: $\forall_x(W(x) \rightarrow U(x))$ → "All western students go to university". If $w(x)$ fails, it is still valid

Disproving Logical Equivalences

Eg. $\forall_x(E(x) \vee O(x)) \equiv (\forall_x E(x) \vee \forall_x O(x))$

Define the predicates to form a contradiction
Domain = $x \in \mathbb{N}$
$E(x)$ = x is even
$O(x)$ = x is odd

For all natural numbers, x is even or x is odd ✓
All natural numbers are odd or all natural numbers are false ✗
lhs is true, rhs is false ∴ not equivalent

Proving Logical Equivalence Formally

$\exists_x(\neg F(x) \vee G(x)) \equiv \forall_x F(x) \rightarrow \exists_x G(x)$
Using transitivity of equivalence, convert lhs to rhs, stating laws used.

$\exists_x(\neg F(x) \vee G(x)) \equiv \exists_x \neg F(x) \vee \exists_x G(x)$ → Existential distribution over disjuction
$\exists_x \neg F(x) \vee \exists_x G(x) \equiv \neg \forall_x F(x) \vee \exists_x G(x)$ → Demorgan's law over existential quantifier
$\neg \forall_x F(x) \vee \exists_x G(x) \equiv \forall_x F(x) \rightarrow \exists_x G(x)$ → Expression of → in terms of $\neg$ and $\vee$

More Laws

$\forall_x(P(x) \wedge G(x)) \equiv \forall_x P(x) \wedge \forall_x G(x)$
$\neg \exists_x P(x) \equiv \forall_x \neg P(x)$

Logical Proofs Using Rules of Inference

$\exists_x(\neg F(x) \vee G(x)) \rightarrow (\forall_x F(x) \rightarrow \exists_x G(x))$

1. $\exists_x(\neg F(x) \vee G(x))$: Imply Assumption
2. $\neg F(c) \vee G(c)$: 1, $\exists$-elim with c
3. $[\forall_x F(x)]$: Imply Assumption
4. $F(c)$: 3, $\forall$-elim with c
5. $[\neg F(c)]$: Neg. assumption → assume $\neg F(c)$ to form contradiction and prove $G(c)$
6. $\perp$: 4,5 $\perp$-intro
7. $\neg\neg F(c)$: 5,6 $\neg$-intro, discharge 5
8. $G(c)$: 2,7, $\vee$-elim → $F(c)$ is true so $\neg F(c) \vee G(c)$ proves $G(c)$, Prove this for
9. $\exists_x G(x)$: 8, $\exists$-intro
10. $\forall_x F(x) \rightarrow \exists_x G(x)$: 3,9, → intro, discharges 3 → We've been working under the assumption that $\forall_x F(x)$, this is now closed
11. $\exists_x(\neg F(x) \vee G(x)) \rightarrow (\forall_x F(x) \rightarrow \exists_x G(x))$: 1,10, → intro, discharges 1

Discharging occurs when an assumption is either proven or disproven