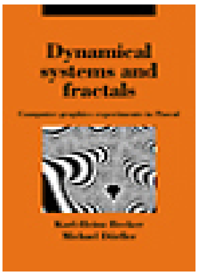

Dynamical Systems and Fractals

Computer Graphics Experiments with Pascal

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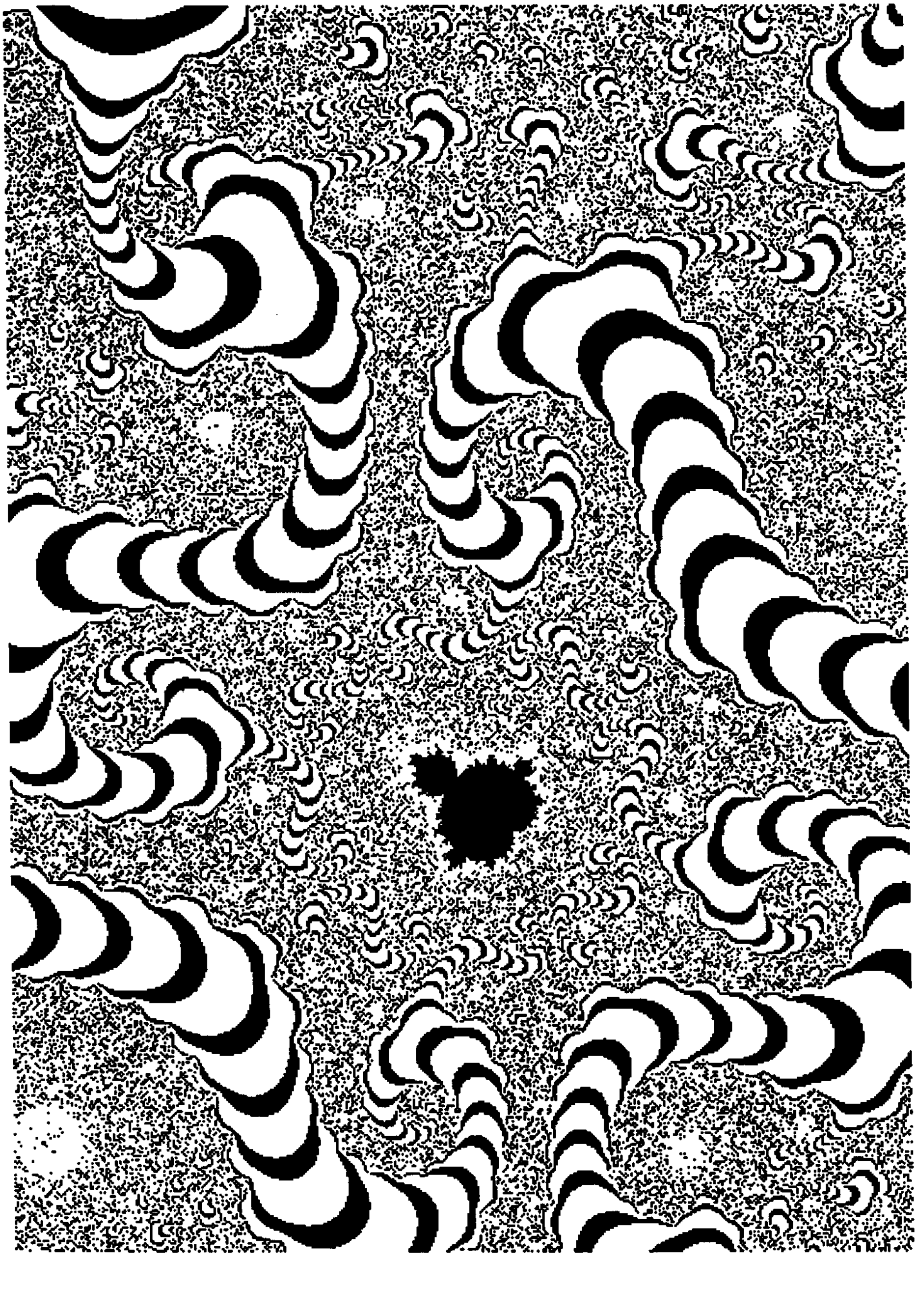
9 Step by Step into Chaos



The people of Bremen are used to all kinds of weather. But what happened in the summer of 1985 in Bremen exceeded the proverbial composure of most inhabitants. On the 24th of July the telephone in the weather office was running hot. Angry callers complained about the weather forecast which they had read in the morning in the *Weserkurier.* There, a day later, you can read a lengthy article on the summer weather:

> 'Lottery players discover twice a week whether they have landed a jackpot or a miserable failure. The game of Bremen weather can now be played every day. For example, whoever took yesterday's forecast from the Bremen weather office as the basis of his bet may as well have thrown it down the drain. Instead of "heavy cloud and intermittent rain with temperatures of 19°" we got a beautiful sunny summer's day with blue skies, real bikini weather.'

What happened to make the meteorologists go so wrong, and what does it have to do with complex systems and the concept of chaos?

The starting point for any weather forecasts is the *synopsis*, that is, a survey of the current weather over a large area. The basis for this is measurements and observations from weather stations on land or at sea. For example, every day radiosondes climb into the atmosphere, and during their climb to almost 40 km in altitude they measure temperature, humidity, and pressure. A whole series of parameters such as pressure, temperature, dewpoint, humidity, cloud cover and windspeed are collected in this manner and input into a forecasting model, which is calculated with the aid of high-speed supercomputers.

Such a forecasting model is a system of differential equations, which describes a closed, complex system. It is derived from particular physical assumptions, which you may perhaps have encountered during your schooldays. Within it are quantities such as:

- impulse
- mass (continuity equation)
- humidity (balance equation for specific heat)
- energy (primary concept in the study of heat)

As already touched upon in the first chapter, the German meteorological department uses a lattice-point model, which lays down a 254 km mesh (at 60° N) over the northern hemisphere. It recognises 9 different heights, so there are about 5000 × 9 = 45 000 lattice points. Influences from ground level (soil, ocean) and topography (mountains, valleys) are taken into consideration at individual lattice points.

For each lattice point the evolution of the atmospheric state is computed by 'mathematical feedback' from the initial state. Every 3.33 minutes the current state of a lattice point is taken as the initial value (input) for a new computation. Such a time-step takes the computer 4 seconds in 'real time'. The preparation of a 24-hour forecast takes about 30 minutes.

The model equations contain the following parameters: temperature, humidity, wind, pressure, cloud cover, radiation balance, temperature and water content of the ground, water content of snow cover, warm currents at ground level, and surface temperature of the ocean.

This is an impressive number of lattice points and parameters, which argues for the complexity, and equally for the realistic nature, of the forecasting model.

But we must be more honest and consider how likely it is, in spite of this, that a false prediction might be made. There is a whole series of evident possibilities for failure in the model:

- Uncertainty and inaccuracy in the analysis, due to poverty of data (e.g. over the ocean) or inadequate description of the topography.
- Space-time solution of the weather parameters in the prediction model. Of course, the finer the lattice and the shorter the time-step, the better the prediction will be. But too small units lead to long computation times!
- Various processes in the atmosphere are only understood empirically: that is, they are not studied through physically grounded equations, but with the help of parameters obtained in experiments. Thus convection, precipitation regions, ground processes, and the interaction of ground and atmosphere are all described empirically.
- Various boundary conditions cannot be represented well. Among these are the influence of 'edges' of the model space (weather in the southern hemisphere, deeper ground and water layers).

One might now imagine that the development of a finer lattice, a still better model, or an increase in the computational capacity of the supercomputer, could lead to an improved success rate of almost 100%.

But in fact this belief in computability, that by setting up all parameters the behaviour of a complex system can be understood, is fallacious - though it is encountered throughout science. This is true of meteorology, physics, and other disciplines. The discovery of chaos theory has cast great doubt upon the scientific principle of the 'computability of the world'. Let us look again at the precise situation on the 23/24 July, to find out what the weather has to do with the concept of chaos.

On 23 July at 11.30 the weather report for 24 July 1985 was dictated by the duty meteorologist on to the teleprinter. According to this, the next day would be sunny and warm. As a rule, the punched tape with the weather report remains untransmitted in the teleprinter, so that later weather changes can be incorporated. The colleague who took over around midday faced a new situation, which led to second thoughts.

A drop in pressure had suddenly appeared to the west of Ireland. Such a tendency often leads to the development of a trough, leading to a worsening of the weather in the direction that the air is moving. In this case the trough was known to be capable of development, and its associated warm/cold front would then move east. This would lead to an air flow in the direction of the North Sea coast over Jutland, passing over the Baltic Sea. The duty meteorologist changed the previously prepared report and put the following message on the teleprinter:

```
23.7.1985
to newspapers
headline: correction

weather conditions.
westerly winds are carrying massive warm and cloudy air-masses
with associated disturbances towards north-west germany.

weather forecast for the weser-ems region wednesday:
heavy cloud, and intermittent rain or showers especially near
the coast.  clouds dispersing in the afternoon leading to
cessation of preciptation.  Warming to around 19 degrees
celsius.  cooling at night to 13 to 9 degrees celsius.
moderate to strong westerly winds.
further outlook:
thursday cloudy, becoming warm, and mainly dry.  temperatures
increasing.

bremen weather bureau.
```

Overnight (24.7.1985, 2.00 MEST[1]), the trough became quite pronounced and lay close to the east of Scotland. According to forecast, it moved further to the east and 12 hours later was off Jutland. The cloud cover associated with the trough then stretched only into the coastal regions and brought a few drops of rain to isolated parts. Intensive and high precipitation passed over Schleswig-Holstein. In contrast, in Bremen almost the entire day was sunny. The weather reports of 24 July then showed how wrong the forecast of 23 July 1985 was.

Weather Report 24 July 1985

	8.00	sun	rain
Bremen	clear	10 h	---
Heligoland	overcast	3 h	0.1 mm (L/m^2)
Schleswig	cloudy	1/2 h	6 mm (L/m^2)

On that day the duty meteorologist[2] could soon recite her explanation of the false prediction by heart:

[1] Middle European Summer Time.

[2] The information was provided by the Bremen weather office on the instructions of qualified meteorologists Sabine Nasdalack and Manfred Klöppel.

'Around 12 midday we prepare the weather forecast for the next day. For Wednesday 24 July we were predicting a sunny day, which it was. Then suddenly a trough appeared on the map and a colleague on a later shift quickly amended the forecast. It was our misfortune that the trough of low pressure with cloud and rain passed to the North of Bremen at a distance of about 100 km. On the North Sea coast it did rain and it was also fairly cool. We did our best to within 100 km.'

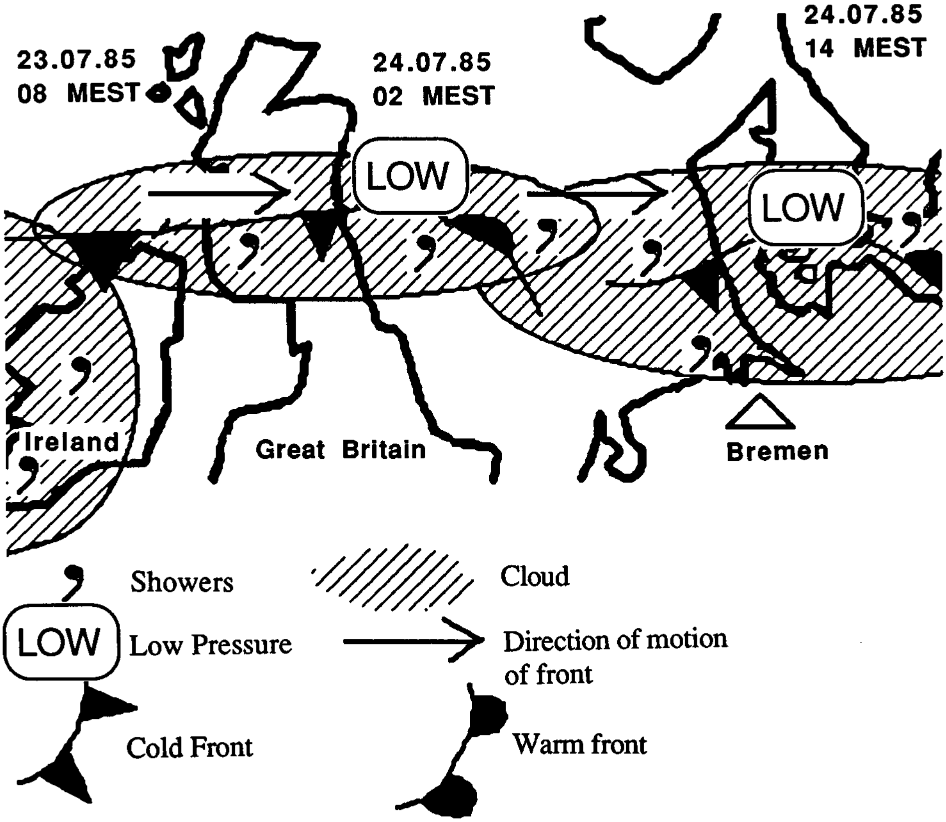


Figure 9-1 Complex weather boundaries around Bremen: 23/24.7.1985.

The weather forecast for 25 July 1985 was 'sunny and warm'. This was doubly unfortunate for the Bremen weathermen, because on that day it then rained. This was another case where a 'weather frontier' 50 or 100 km wide rendered all forecasts useless. Success and failure were only a hair's breadth apart. No wonder. And the Bremen weathermen might have realised this, if they had witnessed a related situation, which we have 'played around with' using computer graphics.

Perhaps the Gingerbread Man had a hand in events?

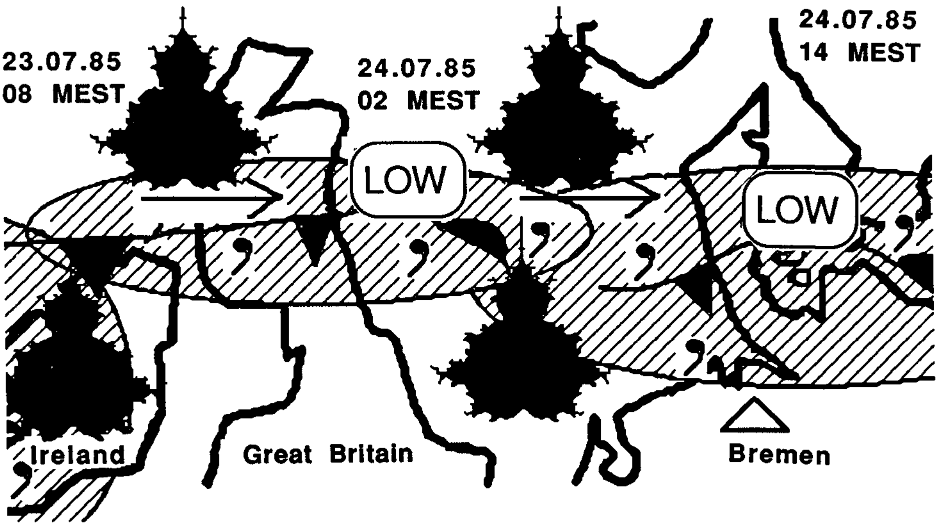


Figure 9.2 Gingerbreadweather?

In the dynamical development of systems like the weather, near complex boundaries all predictions come to nothing. At the frontier between regular and chaotic development of such a system we must make the lattice points infinitely close together to make the model 100 times better, while our computer program becomes larger by a factor of 1000...

Moreover, negligible changes to a single parameter on the borderline of the system can make it go chaotic. In extreme form, in such a situation the 'flap of a butterfly's wing' can change the weather!

In your experiments with Feigenbaum diagrams, Julia sets, and the Gingerbread Man, you have already seen how sensitively systems can react to small changes in parameters. If even simple systems can react so severely, you can see how much more severe it must be for complicated systems like the weather. Astonishingly, however, it seems that even the weather functions according to relatively simple principles, similar to those we have studied. This also applies to other systems whose behaviour depends on many parameters. This explains the interest of many scientists in chaos theory. While there can be no return to the ancient dream of 'computability of the world', at least there is progress in understanding how the change from an ordered computable state into an unpredictable chaotic state can occur.

In particular, chaos theory shows that there is a fundamental limit to the 'computability of the world'. Even with the best supercomputers and rapid technological development, in some cases it may be impossible to predict the behaviour of a system. It might be a political system, economic, physical or otherwise. If we find ourselves in these boundary regions, then any further expenditure is a waste of money.

We nevertheless have a chance. We must learn to understand the 'transition from order to chaos', and to do this we begin with simple systems. At the moment this is done mostly by mathematicians and physicists. You yourself have done it too in your computer graphics experiments - of course without studying intensively the regularities towards which such a process leads.

The final aim of all this investigation is to obtain something like a 'fingerprint' of chaos. Perhaps there is a central mathematical relationship, hidden in a natural constant or a figure like the Gingerbread Man? Perhaps there are 'seismic events' that proclaim chaos? That is what we are seeking. In fact there do seem to be such signs of chaotic phenomena, which signal the transition from order to chaos.

Each of us has a rudimentary understanding of this opposing pair of concepts. We speak of order when 'everything is in its place'. Chaos, according to the dictionary, means among other things 'confusion'. We find another interesting hint there too. By 'chaos', the ancient Greeks meant the primordial substance out of which the world is built. More and more, scientists are coming to the conclusion that chaos is the normal course of events. The much-prized and well-understood order of things is just a special case. This exceptional circumstance has been the centre of scientific interest for centuries, because it is easier to understand and to use. In combination with the indisputable successes of modern science over the past 200 years it has led to a disastrous misconception: that everything is computable. When today a model fails and predicted events do not occur, we simply assume that the model is not good enough, and that that is why the predictions fail. We confidently believe that this can be corrected by more and better measurements and bigger mathematical and computational investment. At first it seems quite a startling idea that there exist problems that simply are not computable, because they lead to 'mathematical chaos'. We have discussed the example of weather at the beginning of this book and in this chapter. When two cars pass each other in the street, a vortex forms in the air between them. As a result, depending on whether we drive on the left or right, whether we live in the northern or southern hemispheres, the global high and low pressure systems are strengthened or weakened. But which weather forecasts take traffic into account? In an extreme case we speak of the 'butterfly effect'. The flap of a butterfly's wing can change our weather! This idea occurred to the American meteorologist Edward N. Lorenz in 1963. He came to the conclusion that long-term weather prediction may not be possible (compare §3.3).

In daily life and science we assume that nothing happens without a reason (the causality principle). Whether we apply this principle to reality or to a mathematical experiment, it makes no difference. This long-standing principle that 'equal causes have equal effects', lies at the heart of our scientific thinking. No experiment would be reproducible if it did not hold. And therein lies the problem. This principle makes no statement about how small changes in causes change their effects. Even the flap of a butterfly's wing can lead to a different outcome. Usually it 'still works', and we obtain similar conclusions from similar assumptions. But often this strengthening of the causality principle fails and the consequences differ considerably. Then we speak of 'chaos'. For example, it shocks a physicist to learn that in fact the strong causality principle need not hold without restriction in classical mechanics: we need only think of a billiard player. The movement of the ball is governed by the classical laws of motion and the reflection principle. But the path of the ball is in no way predictable. After just a few bounces it can no longer be foreseen. Even a manifestly deterministic system can be chaotic. Tiny changes in the forces during the motion, or in initial conditions, lead to unpredictable movements. This can even happen as a result of the gravitational attraction of a spectator standing near the billiard table.

Chaotic systems are the rule in our world, not the exception! This opens up a reversal of our thinking:

- Chaotic systems lie behind significant dependence on initial conditions.
- The strong causality principle does not always hold. Similar causes do not always produce similar effects.
- The long-term behaviour of a system may be uncomputable.

For further examples of chaotic systems we do not need to restrict ourselves to the weather. Simpler systems can also demonstrate the limits to computability.

Time is fundamental to our civilisation. For a long time its measurement depended on the regular swing of a pendulum. Every young physics student learns the ideas that govern the pendulum. Who would imagine that chaos sets in if we add a second pendulum to its end? But that's what happens! Two double pendulums, constructed to be as similar as possible, and started with the same initial conditions, at first display similar behaviour. But quite soon we reach a situation where one of the pendulums is in unstable equilibrium and must decide: do I fall to the left or right? In this situation the system is so sensititve that even the attractive force of a passing bird, the noise of an exhaust, or the cough of an experimentalist can cause the two systems to follow entirely different paths. Many other examples present themselves if we consider flows. The air currents behind buildings or vehicles are chaotic. The eddies in flowing water cannot be pre-computed. Even in the dripping of a tap we find all behaviour from 'purest order' to 'purest chaos'. But there is a further discovery, first made in recent times: whenever a system exhibits both order and chaos, the transition occurs in the same simple manner, the one we were led to in 'step by step into chaos'. Perhaps this simple pattern is one of the first fingerprints of chaos, the first 'seismic event', which signals uncomputability.

What do water drops, heartbeats, and the arms race, have in common? 'Nothing', anyone would probably say, if asked. But all three things are involved in chaos research; all three show the same simple pattern that we saw in 'step by step into chaos'.

The Dripping Tap

Chaos can be found in a dripping tap. Here we can even experiment for ourselves. Normally a tap has two states: open or closed. We are of course interested in the border zone, when the tap is only partially open. The dripping tap then represents an ordered system. The drops are of equal size and fall at regular intervals. If the tap is opened a little more, the drops fall more quickly, until we encounter a phenomenon which we have previously seen only as mathematical feedback: suddenly two drops of different sizes appear, one after the other.

In the same way that a curve in the Feigenbaum diagram can grow two branches, where the sequence alternates between a small and a large value, so also does the water behave. After a large drop there follows a smaller one; after a small one comes a large one.

Unfortunately it is not so easy to observe what happens next. The very rapid events are best seen using photography or with the aid of a stroboscope. Then under some conditions we can see a further period-doubling: a regular sequence of four different drops! Of course, the periods of time between them also differ.

With an accurate timer all this can be made quantitative. The arrangement should be one where the flow rate of the water can be changed reproducibly. Unfortunately a tap is not accurate enough, and it is better to use a water reservoir about the size of an aquarium. A tube is placed across the edge and ends in a spout, made as uniform as possible. It should point downwards. The height of this spout, compared to the water level, controls the quantity of fluid that flows per minute. Because the size of drops is hard to measure, we measure the times at which the drops fall. To do this we let them fall through a suitable light-beam, a few centimetres below the opening. An electronic timer produces a sequence of measurements. We discover:

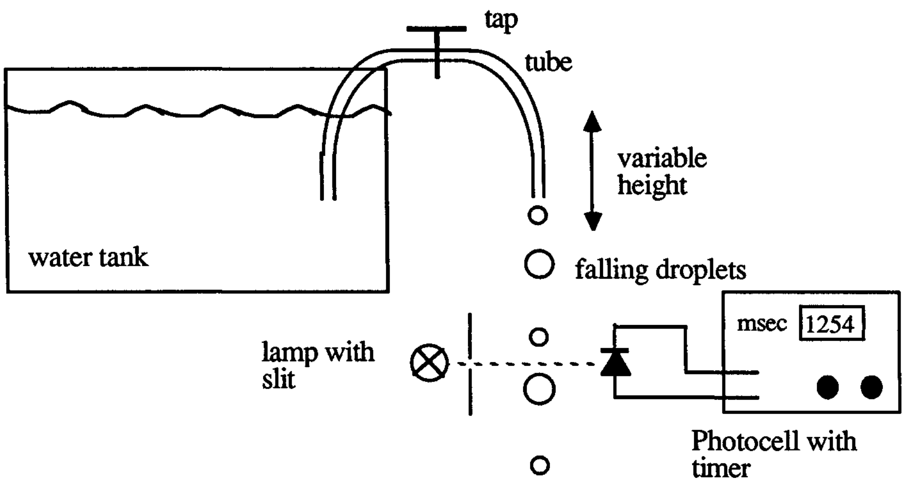


Figure 9–3 Water drop experiment.

- If a little water flows, the drops follow each other at equal intervals.
- By increasing the flow rate, periods 2 and 4 can be clearly detected.
- Finally, shortly before the water flows in a smooth stream, big and small drops follow each other indiscriminately. We have reached chaos!

A possible design of apparatus is shown in Figure 9-3.

The Heartbeat

The heartbeat is another process, in which the complex system 'heart' attains a chaotic state. We consider the process somewhat more precisely.

Normal heartbeats depend upon the synchronised behaviour of many millions of strands of heart muscle. Each of these individual elements of the heart's musculature runs through an electrophysiological cycle lasting about 750 ms. Sodium, potassium and chlorine ions are so arranged, inside and outside the cell wall, that by a combination of chemical events each individiual cell attains a progressively unstable 'explosive' electrophysiological state. Millions of tiny chemical 'catapults' are sprung.

It is now purely a matter of time before this situation leads to a sudden increase in the electrical potential difference across the cell membrane. This is the trigger for the muscular activity of the heart. Normally the initiative for this is controlled by natural pacemaker tissue, which conveys the impulse to the active muscles by a circuit similar to the nervous system.

Once the above chain reaction has begun, it propagates through the musculature of the heart by transmission from each cell to its neighbours. The 'explosive' neighbouring cells are triggered. The process of transmission takes the form of a travelling wave, like a burning fuse, and within 60 - 100 ms this wave has travelled right round the heart.

One consequence, that different parts of the whole heart muscle contract at very similar times, is necessary for the optimal functioning of this organ.

It can sometimes happen that the process of building up the potential can reach a critical level in some places earlier than it does elsewhere in the active musculature. As a result, the stimulus from outside can fail to occur. This is literally fatal, because the outside stimulus provides the starting signal that triggers off the entire heart. Possible causes include a *cardiac infarction*, that is, a localised injury to the heart muscle, which becomes electrophysiologically unstable and capable of providing a chaotic stimulus.

After 'ignition', a buildup of potential and the onset of muscular contraction, there follows a passive condition lasting some 200–300 ms and leading to neither action nor reaction. This is the *refractory phase*. Because of this spatially extended discharging process in the roughly 500 cm^3 muscle mass, this phase is important to synchronise the next work-cycle throughout the entire heart muscle. Ultimately it guarantees the coordinated activity of the millions of muscle strands.

The control system responsible for the regular transmission of the triggering impulses is essential. If a 'firing' impulse arrives at the active muscles too soon, then these will react too soon. The result is an irregular heartbeat: an *extrasystole*.

This is primarily a normal phenomenon. Even in a healthy heart it can be observed, insofar as this impulse may act on a uniformly ready muscle. But if parts of the heart find themselves in an inaccessible stationary state, while other parts are ready to transmit or propagate a spreading impulse, then a fatal condition ensues. It begins with a division of the activity of the heart muscle into regions that are no longer in synchrony. A consequence is the threat that islands of musculature will exhibit cyclic stimulus processes: chaos breaks out, and the result is *fibrillation*. Despite maximal energy use, no output occurs from the biological pump 'heart'. A vicious circle has set in.

Figure 9-4 shows a normal ECG and an ECG showing the typical symptoms of fibrillation. The amplitude is exaggerated compared with the time in relative units.

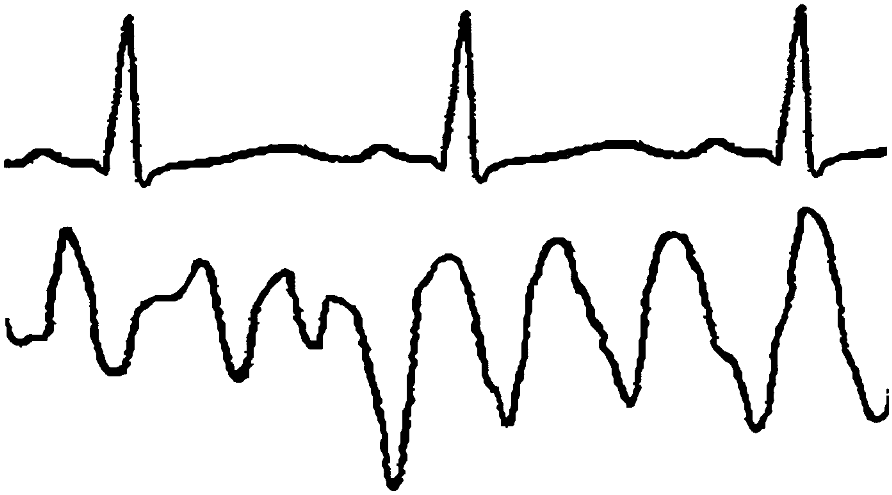


Figure 9-4 ECG curves: normal action (top) and fibrillation.

Astonishingly, this transition from order to chaos, which means fibrillation and death, seems to follow the same simple pattern as water drops and the Feigenbaum diagram. Feigenbaum called the phase shortly before chaos set in *period-doubling*. In this situation complex systems oscillate to and fro between 2, then 4, then 8, then 16, and so on states. Finally nothing regular can be observed.

In the case of the heart, Richard J. Cohen of the Massachusetts Institute of Technology found the same typical pattern in animal experiments. These might imply that an early warning of such extrasystoles in the heart might perhaps be able to save lives.

Outbreaks of War

As a final example we mention that researchers in peace studies have also shown an interest in chaos theory. Again we can see the first tentative beginnings, which as in other examples suggest how effective the applications might become. But it is questionable whether so simplified a formulation of this fundamental problem is really applicable to our times.

Today we live in a world of the balance of terror. Complex systems such as the global environment or national defence switch between phases of order and disorder. Chaotic situations are well known in both systems. Seveso, Bhopal and Chernobyl are environmental catastrophes. World wars and other more local conflicts are peace catastrophes.

History offers many examples of how the precursors of an outbreak of war announce themselves. At the threshold between war and peace political control and predictability are lost. 'I suggest that war be viewed as a breakdown in predictability: a situation in which small perturbations of initial conditions, such as malfunctions of early warning radar systems or irrational acts of individuals disobeying orders, lead to large unforeseen changes in the solutions to the dynamical equations of the model,' says Alvin M. Saperstein (1984), p. 303.

In his article *Chaos - a model for the outbreak of war* he starts out with the same simple mathematical model that we have encountered in the Feigenbaum diagram. The starting point is the following equation, which you should recognise:

$$x_{n+1} = 4*b*x_n*(1-x_n) = F_b(x_n).$$

One can easily show that the attractor for $b < 1/4$ is zero. In the range $1/4 < b < 3/4$ the attractor is $1 - b/4$. For $b > 3/4$ there is no stable state. The critical point is when $b = 0.892$. In certain places there are 2, 4, 8, 16, etc. states. Chaos announces itself through period-doubling. Past the critical value, chaos breaks out. You can easily change your Feigenbaum program to determine the value for period-doubling more accurately.

The Saperstein model applies to a bilateral arms race between two powers X and Y, which proceeds 'step by step'. The symbol n can represent the number of years or the military budget. The model of the arms race consists of the following system:

$$x_{n+1} = 4*a*y_n*(1-y_n) = F_a(y_n)$$
$$y_{n+1} = 4*b*x_n*(1-x_n) = F_b(x_n)$$

with $0 < a,b < 1$. The dependent variables x_n and y_n represent the proportion that the two nations spend on their armaments. Thus the following should hold: $x_n > 0$ and $y_n < 1$.

The armament behaviour of one nation depends of course on that of its opponent up to the current time, and conversely. This is expressed by the dependence between x and y. Much as in the measles example, the factors $(1-y_n)$ and $(1-x_n)$ give the proportion of gross national product that is not invested in armaments. Depending on the values of the parameters a and b, we get stable or unstable behaviour of the system. The two nations can 'calculate' the behaviour of their opponents - or perhaps not.

Saperstein has derived a table from the book *European Historical Statistics 1750-1970*, which shows the military output of several countries between 1934 and 1937 in terms of gross national product (GNP).

	France	Germany	Italy	UK	USSR
1934	0.0276	0.0104	0.0443	0.0202	0.0501
1935	0.0293	0.0125	0.0461	0.0240	0.0552
1936	0.0194	0.0298	0.0296	0.0781	not known
1937	0.0248	0.0359	0.0454	0.0947	not known

Table 9-1 Dependence of military output on GNP, Saperstein (1984) p. 305.

The values in Table 9-1 are now taken in turn for x_0, y_0, respectively x_1, y_1, in the model equations, and *a* and *b* determined from them. Table 9-2 shows the result:

		a	*b*
France-Germany	**(1934-35)**	0.712	0.116
France-Italy	**(1936-37)**	0.214	0.472
UK-Germany	**(1934-35)**	0.582	0.158
UK-Italy	**(1934-35)**	0.142	0.582
USSR-Germany	**(1934-35)**	1.34	0.0657
USSR-Italy	**(1936-37)**	0.819	0.125

Table 9-2 Parameters *a* and *b*, Saperstein (1984), p. 305.

Accordingly, the arms race between USSR and Germany during the chosen period is in the chaotic region. France-Germany and USSR-Italy are, in Saperstein's interpretation, near the critical point.

For such a simple model the results are rather surprising, even if no temporal or historical development is taken into consideration. We suggest that you test the Saperstein model and locate new data. Statistical yearbooks can be found in most libraries. Of course the model is too simple to provide reliable results. We are in the early days of chaos research, and we can only hope that eventually it will be possible to help avoid chaotic situations in real life.

Phase Transitions and Gipsy Moths

These three examples are typical of many other phenomena currently under investigation by scientists in many fields. We briefly describe one of them.

Changes from order to disorder are on the physicist's daily agenda. The transition from water to steam, the change from a conducting to a superconducting state at low temperature, the transition from laminar to turbulent flow or from solid to fluid or gaseous states characterise such *phase transitions*. Phase transitions are extremely complex, because in the system certain elements 'wander' for indeterminate times, oscillating to and fro between two possible states of the system. Mixtures of states can develop, without the system as a whole becoming chaotic.

Of great interest today is the manufacture of new materials. Their magnetic and non-magnetic properties play an important role for different characteristics, such as elasticity or other physical properties. In fact, today, chaos theorists are already 'experimenting' with such materials, albeit hypothetical ones. For this purpose the data of real materials are incorporated in a computer model, which calculates the magnetic and non-magnetic zones. The pictures obtained in this manner are similar to Julia sets. Of course people are particularly interested in the complex boundaries between the two regions.

A surprising result of this research could be the discovery that highly complex phase transitions, as they apply to magnetism, can be understood through simple mechanisms.

Another example from biology shows that even here the Feigenbaum scenario is involved. The biologist Robert M. May investigated the growth of a type of insect, the gipsy moth *Lymantria dispar*, which infests large areas of woodland in the USA. In fact there are all the signs that this sometimes chaotic insect behaviour can be described by the Feigenbaum formula: see May (1976), Breuer (1985).

Chaos theory today concerns itself, as we have seen, with the broad question of how the transition from order to chaos takes place. In particular there are four questions that the researchers pose:

- How can we detect the way in which 'step by step' inevitably leads to chaos?

  Is there a fingerprint of chaos, a characteristic pattern or symptom, which can give advance warning?
- Can this process be formulated and pinned down in simple mathematical terms?

  Are there basic forms, such as the Gingerbread Man, which always occur in different complex systems of an economic, political, or scientific nature?

  Are there basic relations in the form of system constraints or invariants?
- What implications do all these discoveries have for the traditional scientific paradigm?

  What modifications or extensions of existing theoretical constructions are useful or important?
- How do natural systems behave in transition from order to chaos?

Video Feedback

To bring to a close this stepwise excursion into the continuing and unknown development of chaos theory, we will introduce you to a graphical experiment, which will set up the beginnings of a visual journey into the 'Land of Infinite Structures'. This experiment requires apparatus which is already available in many schools and private households. The materials are: a television set, a video camera, and a tripod. Whether you use a black-and-white or a colour TV is unimportant.

The setup is conceptually simple. The video camera is connected to the TV and pointed at the screen. Both machines are switched on. The camera films the picture that it itself creates. In the TV we see a picture of the TV, containing a picture of the TV, containing a picture of the TV, containing...

A recursive, self-similar picture appears. Now bring the camera nearer to the screen. At certain places the screen becomes very bright, at others it stays dark. Feedback leads to a strengthening, in one or the other direction. The interesting thing for us is the border between the light and dark regions. There accidental variations are enhanced, so that scintillations appear, the first symptoms of chaos.

A word about the parameters we can vary in this experiment. On the TV these are brightness and contrast; on the camera they are field of vision (zoom) and sensitivity. If necessary the automatic exposure control must be shut off. Our third piece of apparatus plays an especially important role: the tripod. With this we can tilt the camera at an angle between 0° and 90° above the monitor. A bright speck on the screen can become brighter after being filmed and may strengthen further iterations.

Switch off the light and darken the room. Suddenly the journey begins. Through feedback between the camera and the TV, remarkable dynamically changing structures appear (Figure 9-5), and chaos shows its face. The patterns that arise in this way are so varied that we will make no attempt to describe them.

Nobody can yet compute or predict these structures. Why should it be otherwise?

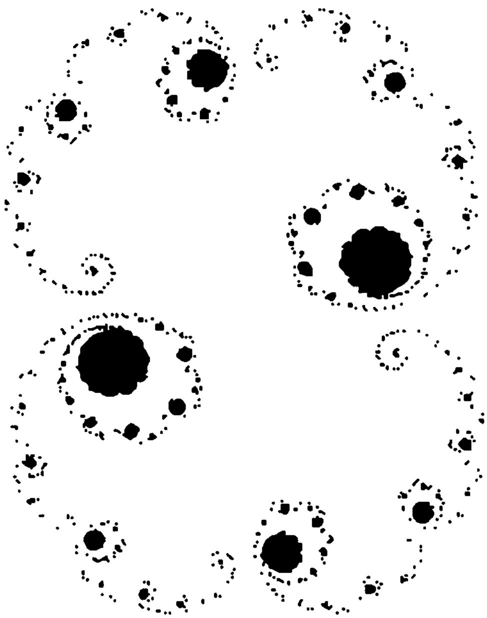


Figure 9-5 Revolving, self-modifying patterns on the screen.[3]

[3]The pictures on the TV screen generally resemble such computer-generated structures.