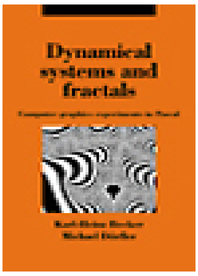

Dynamical Systems and Fractals

Computer Graphics Experiments with Pascal

Karl-Heinz Becker, Michael Dörfler, Translated by I. Stewart

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Chapter

Foreword: New Directions in Computer Graphics: Experimental Mathematic

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Foreword

New Directions in Computer Graphics: Experimental Mathematics

As a mathematician one is accustomed to many things. Hardly any other academics encounter as much prejudice as we do. To most people, mathematics is the most colourless of all school subjects - incomprehensible, boring, or just terribly dry. And presumably, we mathematicians must be the same, or at least somewhat strange. We deal with a subject that (as everyone knows) is actually complete. Can there still be anything left to find out? And if yes, then surely it must be totally uninteresting, or even superfluous.

Thus it is for us quite unaccustomed that our work should so suddenly be confronted with so much public interest. In a way, a star has risen on the horizon of scientific knowledge, that everyone sees in their path.

Experimental mathematics, a child of our 'Computer Age', allows us glimpses into the world of numbers that are breathtaking, not just to mathematicians. Abstract concepts, until recently known only to specialists - for example Feigenbaum diagrams or Julia sets - are becoming vivid objects, which even renew the motivation of students. Beauty and mathematics: they belong together visibly, and not just in the eyes of mathematicians.

Experimental mathematics: that sounds almost like a self-contradiction! Mathematics is supposed to be founded on purely abstract, logically provable relationships. Experiments seem to have no place here. But in reality, mathematicians, by nature, have always experimented: with pencil and paper, or whatever equivalent was available. Even the relationship $a^2+b^2=c^2$, well-known to all school pupils, for the sides of a right-angled triangle, didn't just fall into Pythagoras' lap out of the blue. The proof of this equation came after knowledge of many examples. The working out of examples is a typical part of mathematical work. Intuition develops from examples. Conjectures are formed, and perhaps afterwards a provable relationship is discerned. But it may also demonstrate that a conjecture was wrong: a single counter-example suffices.

Computers and computer graphics have lent a new quality to the working out of examples. The enormous calculating power of modern computers makes it possible to study problems that could never be assaulted with pencil and paper. This results in gigantic data sets, which describe the results of the particular calculation. Computer graphics enable us to handle these data sets: they become visible. And so, we are currently gaining insights into mathematical structures of such infinite complexity that we could not even have dreamed of it until recently.

Some years ago the Institute for Dynamical Systems of the University of Bremen was able to begin the installation of an extensive computer laboratory, enabling its members to carry out far more complicated mathematical experiments. Complex dynamical systems are studied here; in particular mathematical models of changing or self-modifying systems that arise from physics, chemistry, or biology (planetary orbits, chemical reactions, or population development). In 1983 one of the Institute's research groups concerned itself with so-called *Julia sets*. The bizarre beauty of these objects lent wings to fantasy, and suddenly was born the idea of displaying the resulting pictures as a public exhibition.

Such a step down from the 'ivory tower' of science, is of course not easy. Nevertheless, the stone began to roll. The action group 'Bremen and its University', as well as the generous support of Bremen Savings Bank, ultimately made it possible: in January 1984 the exhibition *Harmony in Chaos and Cosmos* opened in the large bank lobby. After the hectic preparation for the exhibition, and the last-minute completion of a programme catalogue, we now thought we could dot the i's and cross the last t's. But something different happened: ever louder became the cry to present the results of our experiments outside Bremen, too. And so, within a few months, the almost completely new exhibition *Morphology of Complex Boundaries* took shape. Its journey through many universities and German institutes began in the Max Planck Institute for Biophysical Chemistry (Göttingen) and the Max Planck Institute for Mathematics (in Bonn Savings Bank).

An avalanche had broken loose. The boundaries within which we were able to present our experiments and the theory of dynamical systems became ever wider. Even in (for us) completely unaccustomed media, such as the magazine *Geo* on ZDF television, word was spread. Finally, even the Goethe Institute opted for a world-wide exhibition of our computer graphics. So we began a third time (which is everyone's right, as they say in Bremen), equipped with fairly extensive experience. Graphics, which had become for us a bit too brightly coloured, were worked over once more. Naturally, the results of our latest experiments were added as well. The première was celebrated in May 1985 in the 'Böttcherstrasse Gallery'. The exhibition *Schönheit im Chaos/Frontiers of Chaos* has been travelling throughout the world ever since, and is constantly booked. Mostly, it is shown in natural science museums.

It's no wonder that every day we receive many enquiries about computer graphics, exhibition catalogues (which by the way were all sold out) and even programming instructions for the experiments. Naturally, one can't answer all enquiries personally. But what are books for? *The Beauty of Fractals*, that is to say the book about the exhibition, became a prizewinner and the greatest success of the scientific publishing company Springer-Verlag. Experts can enlighten themselves over the technical details in *The Science of Fractal Images*; and with *The Game of Fractal Images* lucky Macintosh II owners, even without any further knowledge, can boot up their computers and go on a journey of discovery at once. But what about all the many home computer fans, who themselves like to program, and thus would like simple, but exact. information? The book lying in front of you by Karl-Heinz Becker and Michael Dörfler fills a gap that has too long been open.

The two authors of this book became aware of our experiments in 1984, and through our exhibitions have taken wing with their own experiments. After didactic preparation they now provide, in this book, a quasi-experimental introduction to our field of research. A veritable kaleidoscope is laid out: dynamical systems are introduced, bifurcation diagrams are computed, chaos is produced, Julia sets unfold, and over it all looms the 'Gingerbread Man' (the nickname for the Mandelbrot set). For all of these, there are innumerable experiments, some of which enable us to create fantastic computer graphics for ourselves. Naturally, a lot of mathematical theory lies behind it all, and is needed to understand the problems in full detail. But in order to experiment oneself (even if in perhaps not quite as streetwise a fashion as a mathematician) the theory is luckily not essential. And so every home computer fan can easily enjoy the astonishing results of his or her experiments. But perhaps one or the other of these will let themselves get really curious. Now that person can be helped, for that is why it exists: the study of mathematics.

But next, our research group wishes you lots of fun studying this book, and great success in your own experiments. And please, be patient: a home computer is no 'express train' (or, more accurately, no supercomputer). Consequently some of the experiments may tax the 'little ones' quite nicely. Sometimes, we also have the same problems in our computer laboratory. But we console ourselves: as always, next year there will be a newer, faster, and simultaneously cheaper computer. Maybe even for Christmas... but please with colour graphics, because then the fun *really* starts.

Research Group in Complex Dynamics
University of Bremen *Hartmut Jürgens*