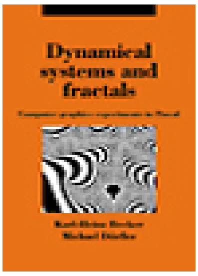

Dynamical Systems and Fractals

Computer Graphics Experiments with Pascal

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Chapter

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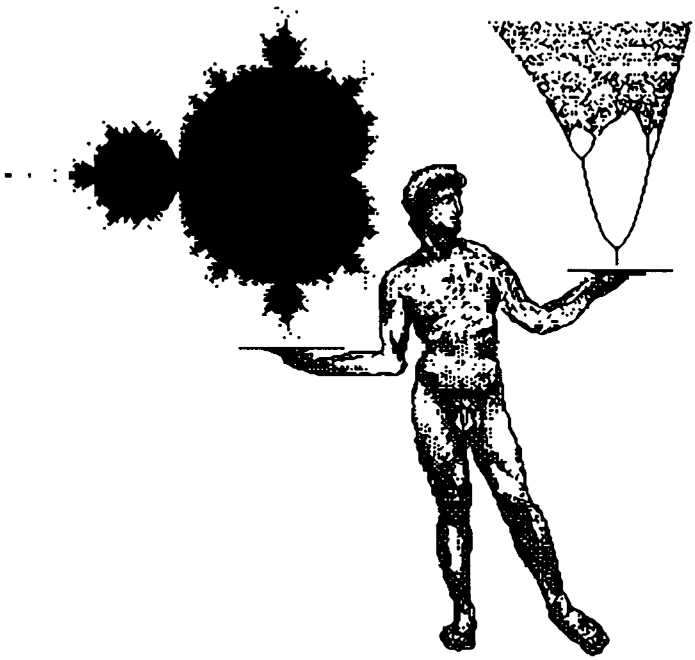
1 Researchers Discover Chaos



The story which today so fascinates researchers, and which is associated with chaos theory and experimental mathematics, came to our attention around 1983 in Bremen. At that time a research group in dynamical systems under the leadership of Professors Peitgen and Richter was founded at Bremen University. This starting-point led to a collaboration lasting many years with members of the Computer Graphics Laboratory at the University of Utah in the USA.

Equipped with a variety of research expertise, the research group began to install its own computer graphics laboratory. In January and February of 1984 they made their results public. These results were startling and caused a great sensation. For what they exhibited was beautiful, coloured computer graphics reminiscent of artistic paintings. The first exhibition, *Harmony in Chaos and Cosmos,* was followed by the exhibition *Morphology of Complex Frontiers.* With the next exhibition the results became internationally known. In 1985 and 1986, under the title *Frontiers of Chaos* and with assistance from the Goethe Institute, this third exhibition was shown in the UK and the USA. Since then the computer graphics have appeared in many magazines and on television, a witches' brew of computer-graphic simulations of dynamical systems.

What is so stimulating about it?

Why did these pictures cause so great a sensation?

We think that these new directions in research are fascinating on several grounds. It seems that we are observing a 'celestial conjunction' - a conjunction as brilliant as that which occurs when Jupiter and Saturn pass close together in the sky, something that happens only once a century. Similar events have happened from time to time in the history of science. When new theories overturn or change previous knowledge, we speak of a *paradigm change.* [1]

The implications of such a paradigm change are influenced by science and society. We think that may also be the case here. At any rate, from the scientific viewpoint, this much is clear:

- A new theory, the so-called *chaos theory*, has shattered the scientific world-view. We will discuss it shortly.
- New techniques are changing the traditional methods of work of mathematics and lead to the concept of *experimental mathematics.*

For centuries mathematicians have stuck to their traditional tools and methods such as paper, pen, and simple calculating machines, so that the typical means of progress in mathematics have been proofs and logical deductions. Now for the first time some mathematicians are working like engineers and physicists. The mathematical problem under investigation is planned and carried out like an experiment. The experimental apparatus for this investigatory mathematics is the computer. Without it, research in this field would be impossible. The mathematical processes that we wish to understand are

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[1] Paradigm = 'example'. By a paradigm we mean a basic point of view, a fundamental unstated assumption, a dogma, through which scientists direct their investigations.

visualised in the form of computer graphics. From the graphics we draw conclusions about the mathematics. The outcome is changed and improved, the experiment carried out with the new data. And the cycle starts anew.

- Two previously separate disciplines, mathematics and computer graphics, are growing together to create something qualitatively new.

Even here a further connection with the experimental method of the physicist can be seen. In physics, bubble-chambers and semiconductor detectors are instruments for visualising the microscopically small processes of nuclear physics. Thus these processes become representable and accessible to experience. Computer graphics, in the area of dynamical systems, are similar to bubble-chamber photographs, making dynamical processes visible.

Above all, this direction of research seems to us to have social significance:

- The 'ivory tower' of science is becoming transparent.

In this connection you must realise that the research group is interdisciplinary. Mathematicians and physicists work together, to uncover the mysteries of this new discipline. In our experience it has seldom previously been the case that scientists have emerged from their own 'closed' realm of thought, and made their research results known to a broad lay public. That occurs typically here.

- These computer graphics, the results of mathematical research, are very surprising and have once more raised the question of what 'art' really is.
  Are these computer graphics to become a symbol of our 'hi-tech' age?
- For the first time in the history of science the distance between the utmost frontiers of research, and what can be understood by the 'man in the street', has become vanishingly small.

Normally the distance between mathematical research, and what is taught in schools, is almost infinitely large. But here the concerns of a part of today's mathematical research can be made transparent. That has not been possible for a long time.

Anyone can join in the main events of this new research area, and come to a basic understanding of mathematics. The central figure in the theory of dynamical systems, the *Mandelbrot set* - the so-called 'Gingerbread Man' - was discovered only in 1980. Today, virtually anyone who owns a computer can generate this computer graphic for themselves, and investigate how its hidden structures unravel.

1.1 Chaos and Dynamical Systems - What Are They?

An old farmer's saying runs like this: 'When the cock crows on the dungheap, the weather will either change, or stay as it is.' Everyone can be 100 per cent correct with this weather forecast. We obtain a success rate of 60 per cent if we use the rule that tomorrow's weather will be the same as today's. Despite satellite photos, worldwide measuring networks for weather data, and supercomputers, the success rate of computer-generated predictions stands no higher than 80 per cent.

Why is it not better?

Why does the computer - the very incarnation of exactitude - find its limitations here?

Let us take a look at how meteorologists, with the aid of computers, make their predictions. The assumptions of the meteorologist are based on the *causality principle*. This states that equal causes produce equal effects - which nobody would seriously doubt. Therefore the knowledge of all weather data must make an exact prediction possible. Of course this cannot be achieved in practice, because we cannot set up measuring stations for collecting weather data in an arbitrarily large number of places. For this reason the meteorologists appeal to the *strong causality principle*, which holds that similar causes produce similar effects. In recent decades theoretical models for the changes in weather have been derived from this assumption.

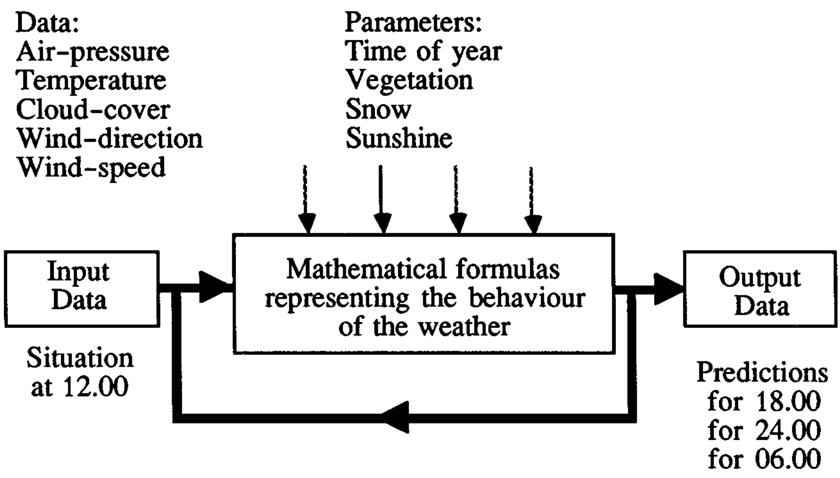

Figure 1.1-1 Feedback cycle of weather research.

Such models, in the form of complicated mathematical equations, are calculated with the aid of the computer and used for weather prediction. In practice weather data from the worldwide network of measuring stations, such as pressure, temperature, wind direction, and many other quantities, are entered into the computer system, which calculates the resulting weather with the aid of the underlying model. For example, in principle the method for predicting weather 6 hours ahead is illustrated in Figure 1.1-1. The 24-hour forecast can easily be obtained, by feeding the data for the 18-hour computation back into the model. In other words, the computer system generates output data with the aid of the weather forecasting program. The data thus obtained are fed back in again as input data. They produce new output data, which can again be treated as input data. The data are thus repeatedly fed back into the program.

One might imagine that the results thus obtained become ever more accurate. The opposite can often be the case. The computed weather forecast, which for several days has matched the weather very well, can on the following day lead to a catastrophically false prognosis. Even if the 'model system weather' gets into a 'harmonious' relation to the predictions, it can sometimes appear to behave 'chaotically'. The stability of the computed weather forecast is severely over-estimated, if the weather can change in unpredictable ways. For meteorologists, no more stability or order is detectable in such behaviour. The model system 'weather' breaks down in apparent disorder, in 'chaos'. This phenomenon of unpredictablity is characteristic of complex systems. In the transition from 'harmony' (predictability) into 'chaos' (unpredictability) is concealed the secret for understanding both concepts.

The concepts 'chaos' and 'chaos theory' are ambiguous. At the moment we agree to speak of chaos only when 'predictability breaks down'. As with the weather (whose correct prediction we classify as an 'ordered' result), we describe the meteorologists - often unfairly - as 'chaotic', when yet again they get it wrong.

Such concepts as 'order' and 'chaos' must remain unclear at the start of our investigation. To understand them we will soon carry out our own experiments. For this purpose we must clarify the many-sided concept of a *dynamical system*.

In general by a *system* we understand a collection of elements and their effects on each other. That seems rather abstract. But in fact we are surrounded by systems.

The weather, a wood, the global economy, a crowd of people in a football stadium, biological populations such as the totality of all fish in a pond, a nuclear power station: these are all systems, whose 'behaviour' can change very rapidly. The elements of the dynamical system 'football stadium', for example, are people: their relations with each other can be very different and of a multifaceted kind.

Real systems signal their presence through three factors:

- They are *dynamic*, that is, subject to lasting changes.
- They are *complex*, that is, depend on many parameters.
- They are *iterative*, that is, the laws that govern their behaviour can be described by feedback.

Today nobody can completely describe the interactions of such a system through mathematical formulas, nor predict the behaviour of people in a football stadium.

Despite this, scientists try to investigate the regularities that form the basis of such dynamical systems. In particular one exercise is to find simple mathematical models, with whose help one can simulate the behaviour of such a system.

We can represent this in schematic form as in Figure 1.1-2.

Of course in a system such as the weather, the transition from order to chaos is hard to predict. The cause of 'chaotic' behaviour is based on the fact that negligible changes to quantities that are coupled by feedback can produce unexpected chaotic effects. This is an apparently astonishing phenomenon, which scientists of many disciplines have studied with great excitement. It applies in particular to a range of problems that might bring into question recognised theories or stimulate new formulations, in biology, physics, chemistry and mathematics, and also in economic areas.

The research area of dynamical systems theory is manifestly interdisciplinary. The theory that causes this excitement is still quite young and - initially - so simple mathematically that anyone who has a computer system and can carry out elementary programming tasks can appreciate its startling results.

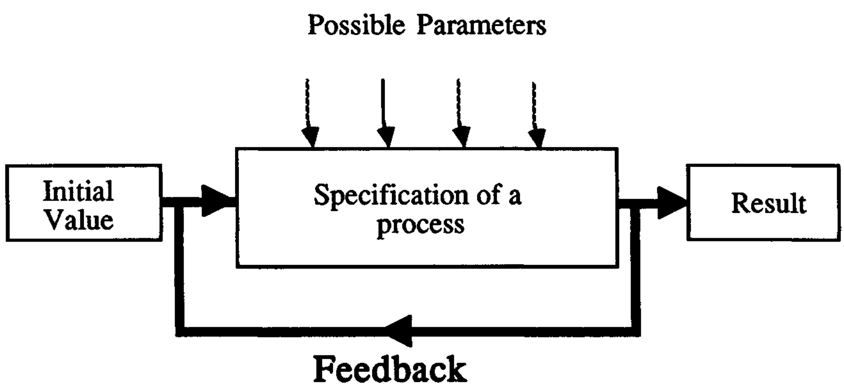

Figure 1.1-2 General feedback scheme.

The aim of chaos research is to understand in general how the transition from order to chaos takes place.

An important possibility for investigating the sensitivity of chaotic systems is to represent their behaviour by computer graphics. Above all, graphical representation of the results and independent experimentation has considerable aesthetic appeal, and is exciting.

In the following chapters we will introduce you to such experiments with different dynamical systems and their graphical representation. At the same time we will give you - a bit at a time - a vivid introduction to the conceptual world of this new research area.

1.2 Computer Graphics Experiments and Art

In their work, scientists distinguish two important phases. In the ideal case they alternate between experimental and theoretical phases. When scientists carry out an experiment, they pose a particular question to Nature. As a rule they offer a definite point of departure: this might be a chemical substance or a piece of technical apparatus, with which the experiment should be performed. They look for theoretical interpretations of the answers, which they mostly obtain by making measurements with their instruments.

For mathematicians, this procedure is relatively new. In their case the apparatus or measuring instrument is a computer. The questions are presented as formulas, representing a series of steps in an investigation. The results of measurement are numbers, which must be interpreted. To be able to grasp this multitude of numbers, they must be represented clearly. Often graphical methods are used to achieve this. Bar-charts and pie-charts, as well as coordinate systems with curves, are widespread examples. In most cases not only is a picture 'worth a thousand words': the picture is perhaps the only way to show the precise state of affairs.

Over the last few years experimental mathematics has become an exciting area, not just for professional researchers, but for the interested layman. With the availability of efficient personal computers, anyone can explore the new territory for himself.

The results of such computer graphics experiments are not just very attractive visually - in general they have never been produced by anyone else before.

In this book we will provide programs to make the different questions from this area of mathematics accessible. At first we will give the programs at full length; but later - following the building-block principle - we shall give only the new parts that have not occurred repeatedly.

Before we clarify the connection between experimental mathematics and computer graphics, we will show you some of these computer graphics. Soon you will be producing these, or similar, graphics for yourself. Whether they can be described as computer art you must decide for yourself.

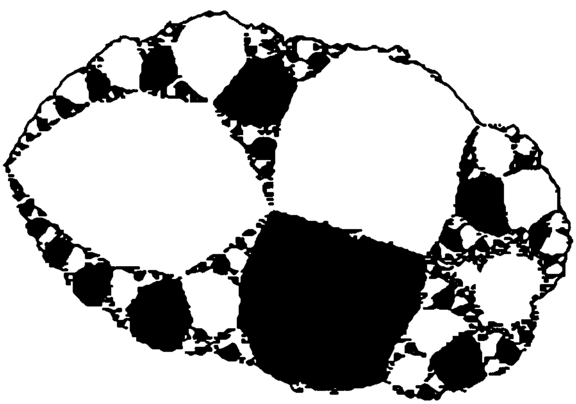

Figure 1.2-1 Rough Diamond.

Figure 1.2-2 Vulcan's Eye.

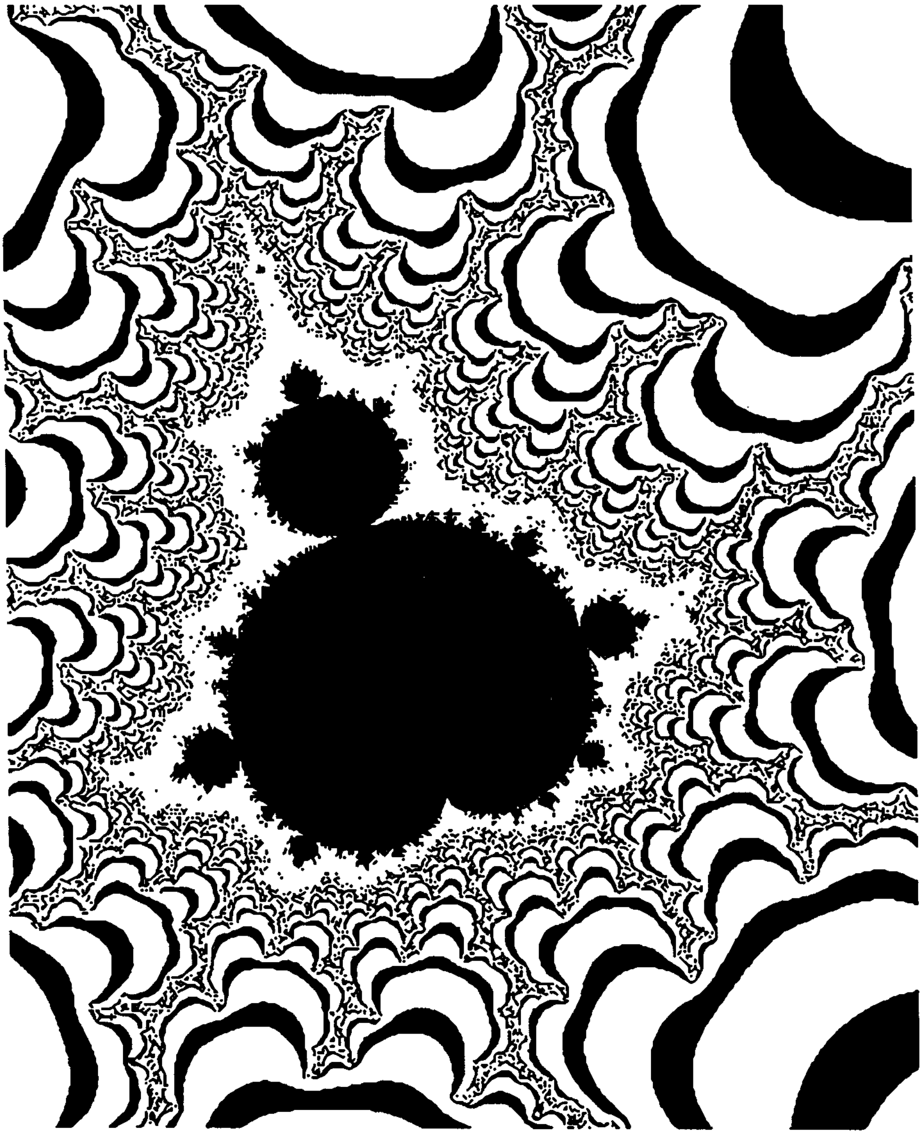

Figure 1.2–3 Gingerbread Man.

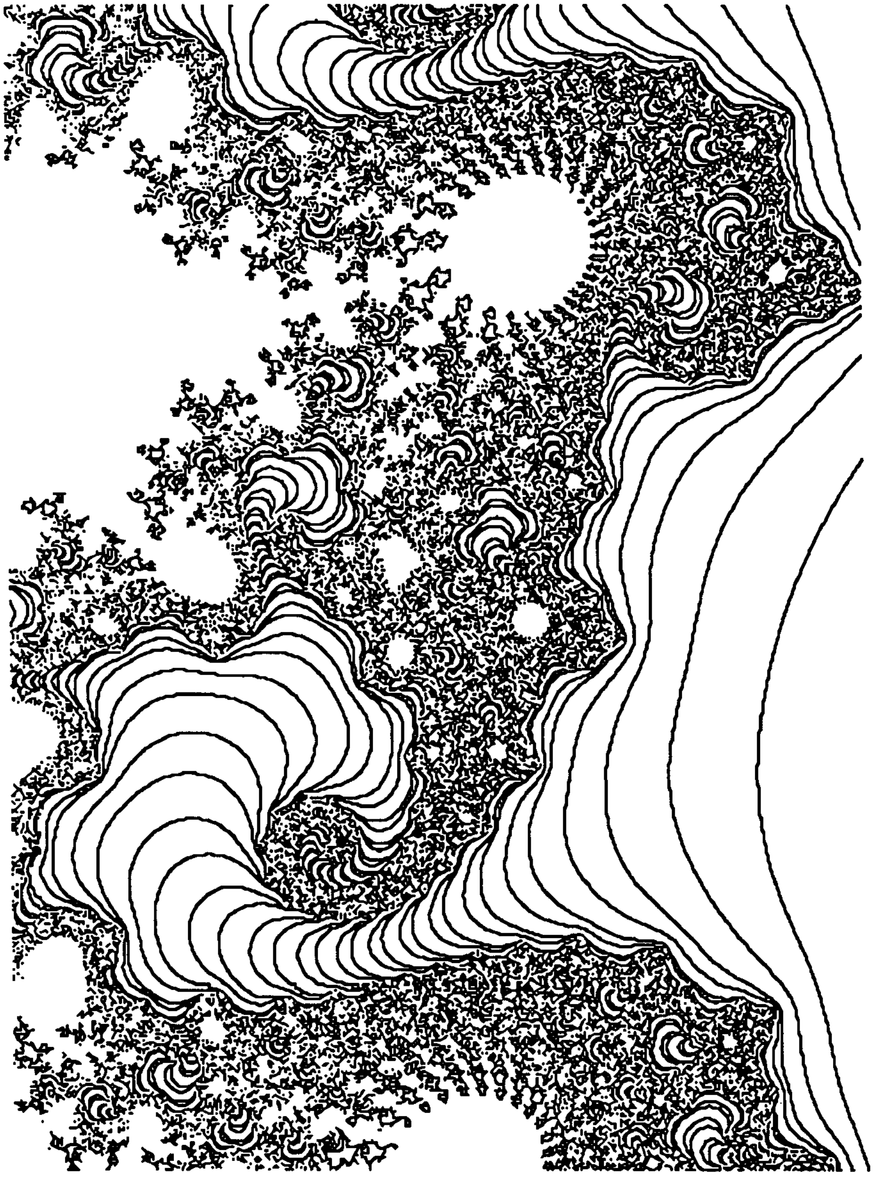

Figure 1.2-4 Tornado Convention.[2]

[2]This picture was christened by Prof. K. Kenkel of Dartmouth College.

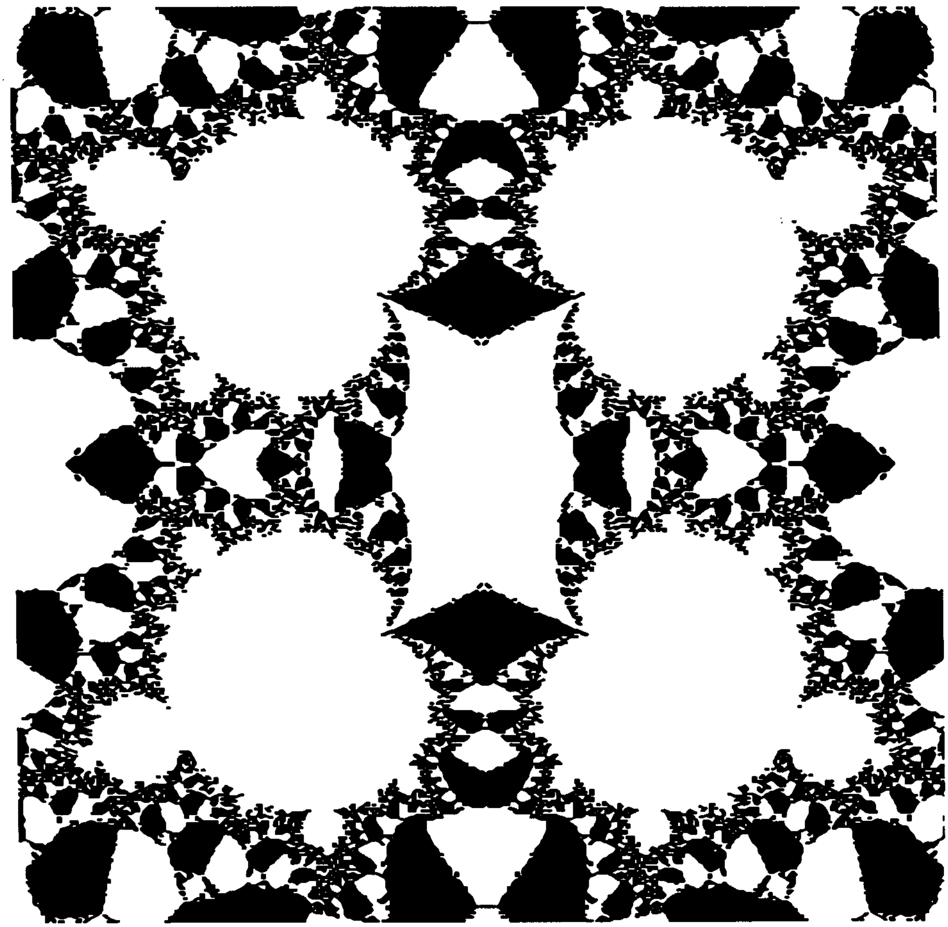

Figure 1.2–5 Quadruple Alliance.

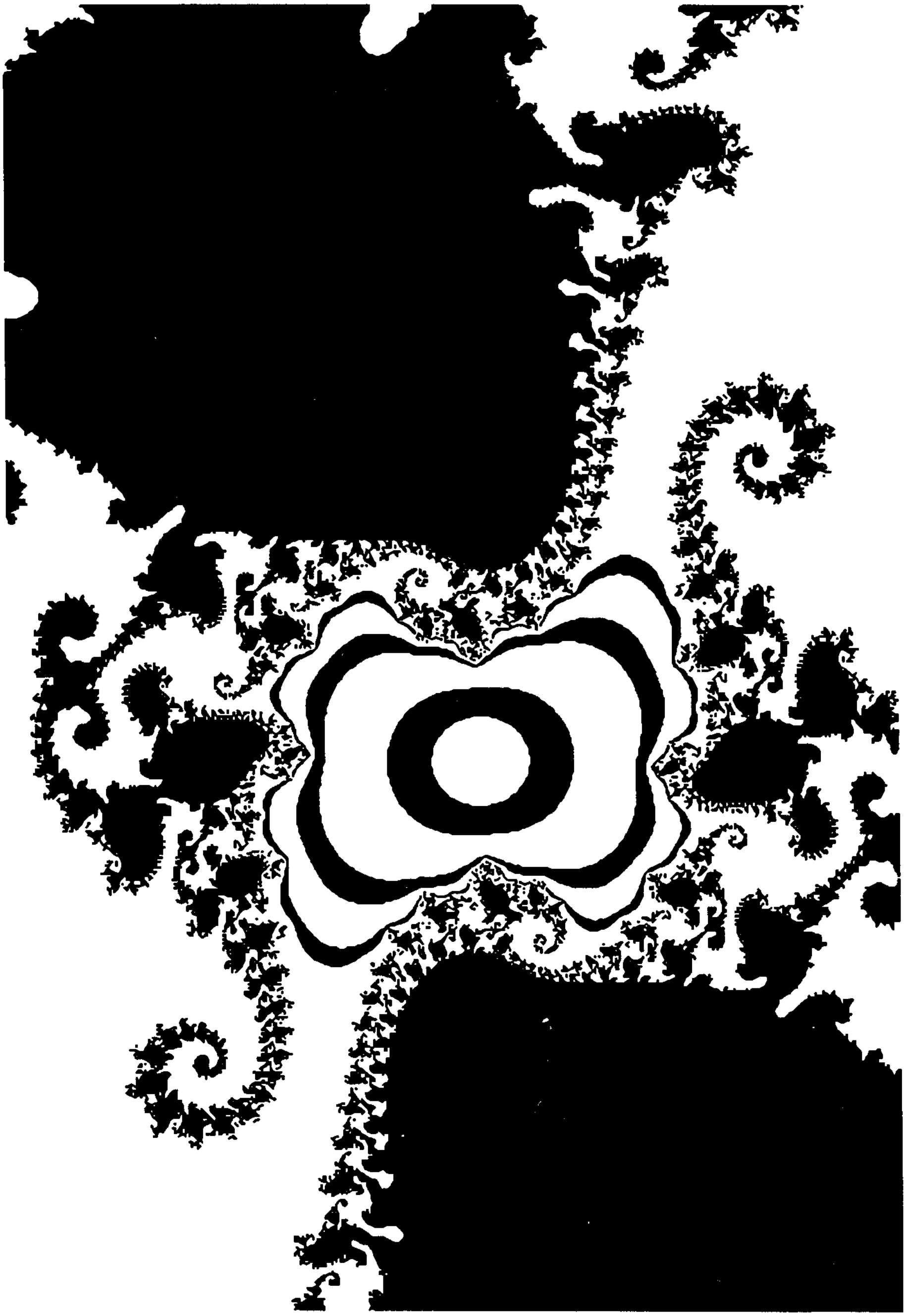

Figure 1.2-6 Seahorse Roundelay.

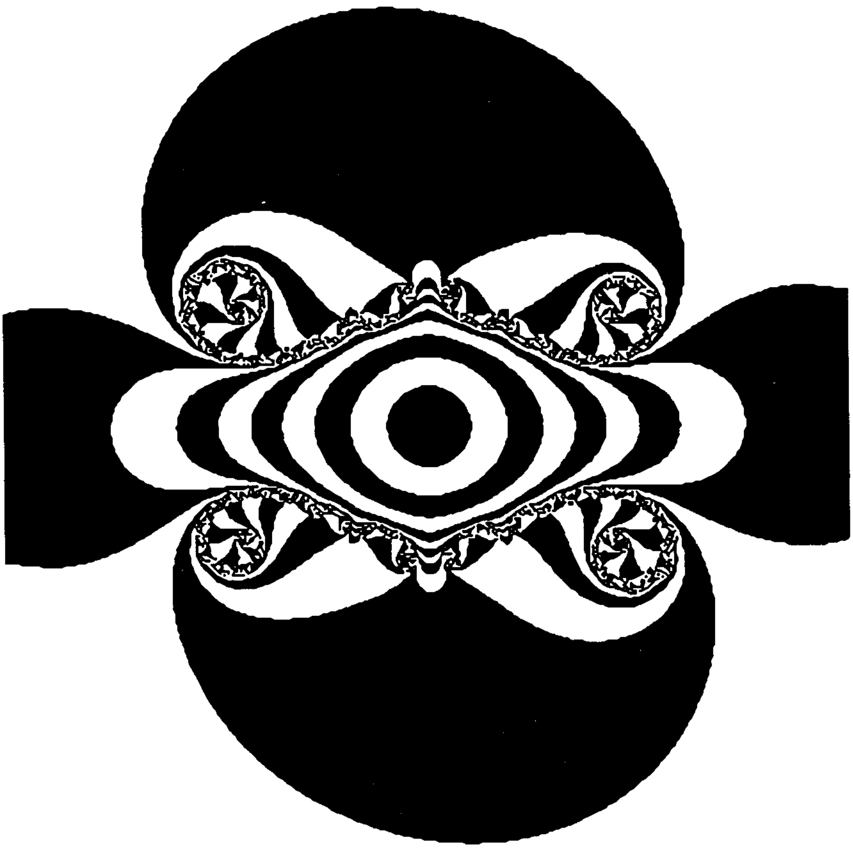

Figure 1.2–7 Julia Propeller.

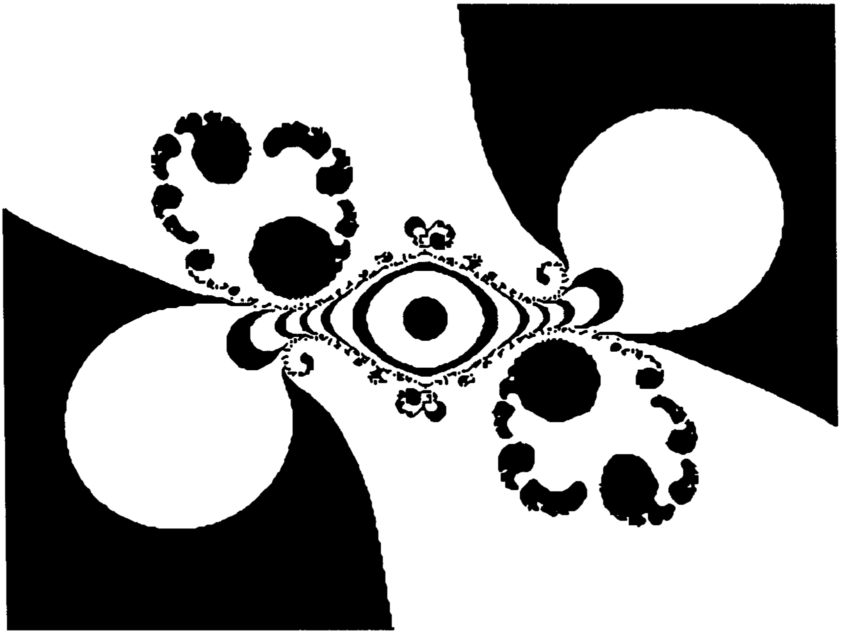

Figure 1.2–8 Variation 1.

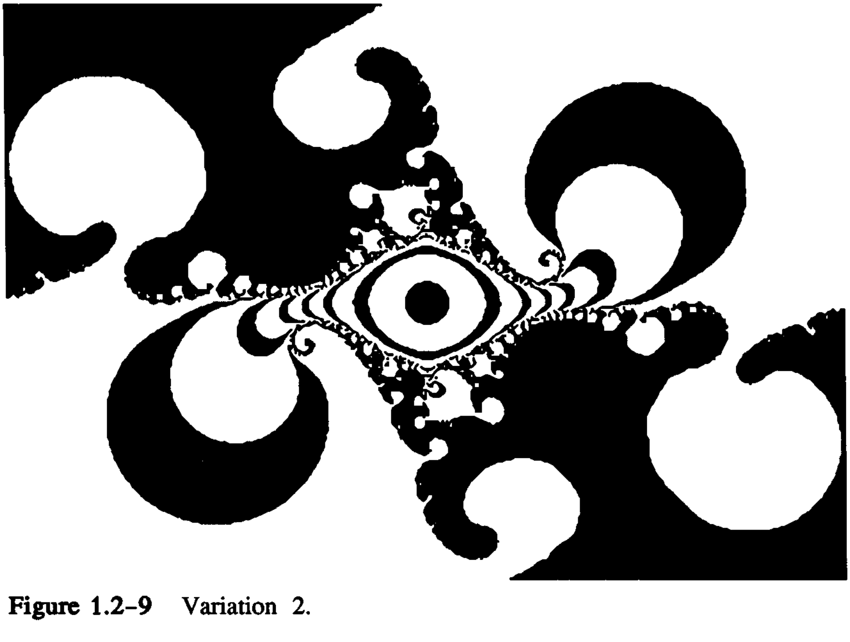

Figure 1.2–9 Variation 2.

Figure 1.2-10 Variation 3.

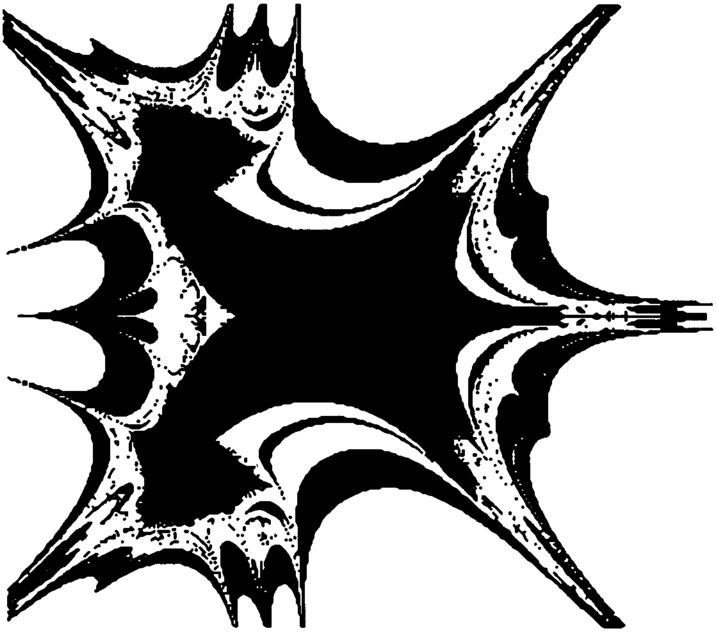

Figure 1.2-11 Explosion.

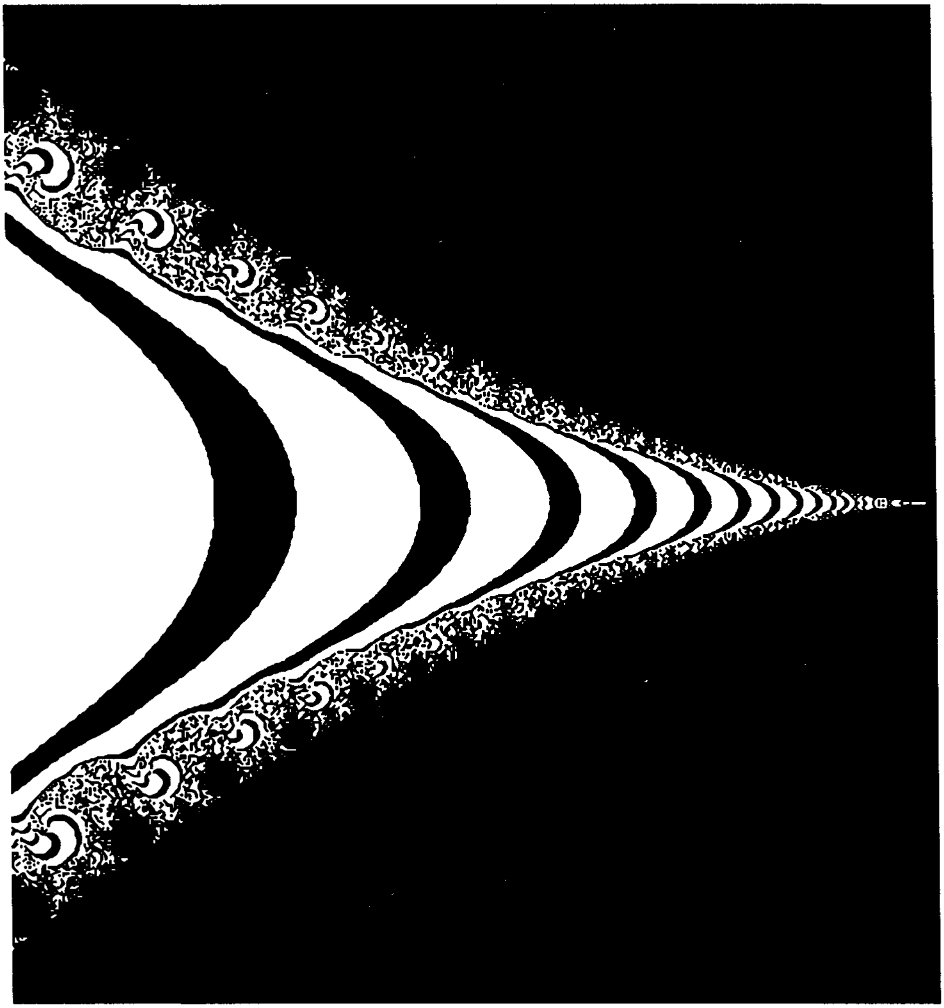

Figure 1.2-12 Mach 10.

Computer graphics in, computer art out. In the next chapter we will explain the relation between experimental mathematics and computer graphics. We will generate our own graphics and experiment for ourselves.