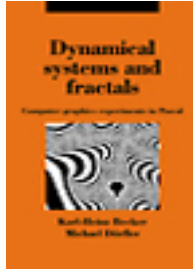


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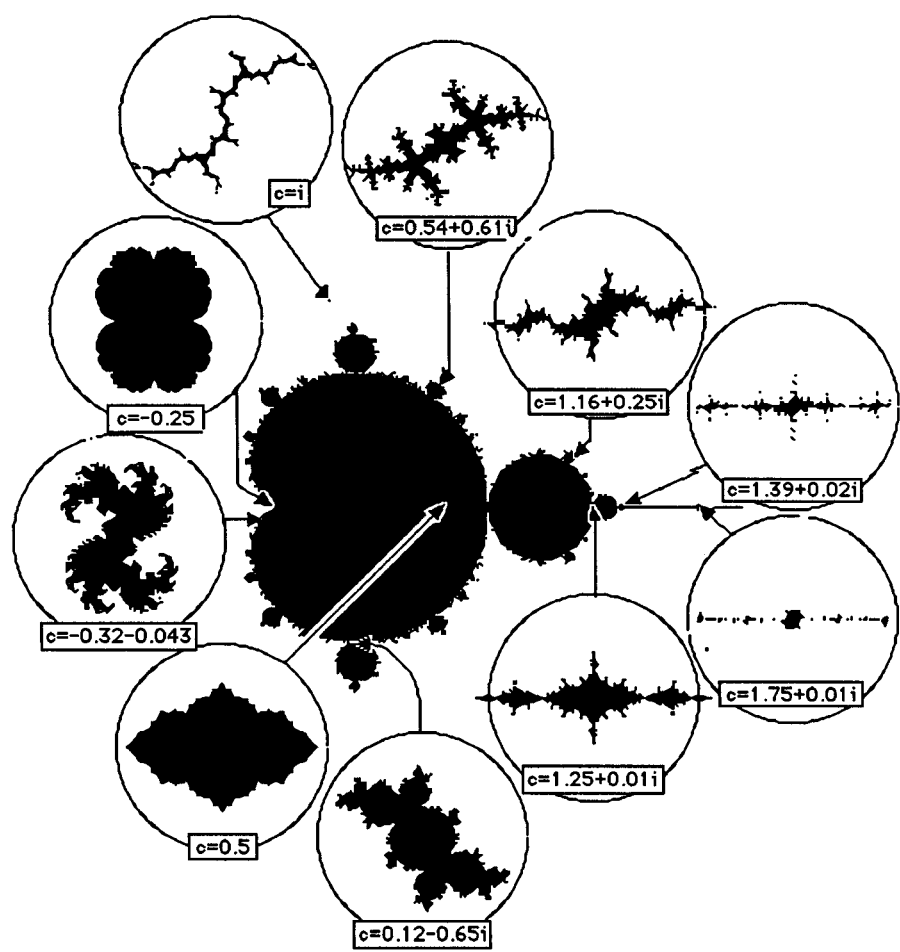
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# 13 Appendices



### 13.1 Data for Selected Computer Graphics

Especially in your first investigations, it is useful to know where the interesting sections occur. Then you can test out your own programs there. Some of the more interesting Julia sets, together with their parameters in the Mandelbrot set, are collected together below.

In addition to the title picture for this Chapter, an 'Atlas' of the Mandelbrot set, we also give a table of the appropriate data for many of the pictures in this book. For layout reasons many of the original pictures have been cropped. Some data for interesting pictures may perhaps be missing: they come from the early days of our experiments, where there were omissions in our formulas or documentation.

Table 13-1 shows for each figure number the type of picture drawn, as this may not be clear from the caption. We use the following abbreviations:

G	Gingerbread man (Mandelbrot set) or variation thereon
J	Julia set, or variation thereon
C	Set after Curry, Garnett, Sullivan
N1	Newton development of the equation $f(x) = (x-1)*x*(x+1)$
N3	Newton development of the equation $f(x) = x^3-1$
N5	Newton development of the equation $f(x) = x^5-1$
T	Tomogram picture, see Chapter 6
F	Feigenbaum diagram
*	See text for further information

Near the (approximate) boundaries of the picture sections you will see the maximal number of iterations and the quantity that determines the spacing of the contour lines. The last two columns give the initial value for Mandelbrot sets, and the complex constant  $c$  for Julia sets.

Picture	Type	Left	Right	Top	Bottom	Maximal iteration	Bound	Complex constant or initial value $c_r$ or $x_0$ $c_i$ or $y_0$	
5.1-2ff	N3	-2.00	2.00	-1.50	1.50	*	*	-	-
5.1-7	N5	-2.00	2.00	-1.50	1.50	25	5	-	-
5.2-1	J	-1.60	1.60	0.00	1.20	20	20	0.10	0.10
5.2-2	J	-1.60	1.60	0.00	1.20	20	20	0.20	0.20
5.2-3	J	-1.60	1.60	0.00	1.20	20	20	0.30	0.30
5.2-4	J	-1.60	1.60	-1.20	1.20	20	20	0.40	0.40
5.2-5	J	-1.60	1.60	-1.20	1.20	20	20	0.50	0.50
5.2-6	J	-1.60	1.60	0.00	1.20	20	20	0.60	0.60
5.2-7	J	-1.60	1.60	0.00	1.20	20	20	0.70	0.70
5.2-8	J	-1.60	1.60	0.00	1.20	20	20	0.80	0.80
5.2-9	J	-1.75	1.75	-1.20	1.20	200	0	0.745 405 4	0.113 006 3
5.2-10	J	-1.75	1.75	-1.20	1.20	200	0	0.745 428	0.113 009
5.2-11	J	0.15	0.40	-0.322	-0.15	200	40	0.745 405 4	0.113 006 3
5.2-12	J	0.29	0.316 4	-0.209 1	-0.191 4	400	140	0.745 405 4	0.113 006 3
5.2-13	J	0.295 11	0.298 14	-0.203 3	-0.201 3	400	140	0.745 405 4	0.113 006 3

Picture	Type	Left	Right	Top	Bottom	Maximal iteration	Bound	Complex or initial value $c_r$ or $x_0$	constant or $c_j$ or $y_0$
5.2-14	J	0.295 11	0.298 14	-0.2033	-0.2013	400	140	0.745 428 4	0.113 009
5.2-15	J	0.296 26	0.296 86	-0.2024	-0.202	600	200	0.745 405 4	0.113 006 3
5.2-16	J	0.296 26	0.296 86	-0.2024	-0.202	600	200	0.745 428	0.113 009
5.2-17	J	-1.75	1.75	-1.20	1.20	50	0	0.745 428	0.113 009
5.2-18	J	-2.00	1.50	-1.20	1.20	1000	12	0.745 428	0.113 009
5.2-19	J	-0.08	0.07	-0.1	0.1	200	60	0.745 428	0.113 009
5.2-20	J	-0.08	0.07	-0.1	0.1	300	0	0.745 428	0.113 009
5.2-23	J	-1.75	1.75	-1.20	1.20	60	10	1.25	0.011
6.1-1	J	-0.05	0.05	-0.075	0.075	400	150	0.745 405 4	0.113 006 3
6.1-2	J	-0.05	0.05	-0.075	0.075	400	150	0.745 428	0.113 009
6.1-3	G	-1.35	2.65	-1.50	1.50	4	0	0.00	0.00
6.1-4	G	-1.35	2.65	-1.50	1.50	6	0	0.00	0.00
6.1-5	G	-1.35	2.65	-1.50	1.50	8	0	0.00	0.00
6.1-6	G	-1.35	2.65	-1.50	1.50	10	0	0.00	0.00
6.1-7	G	-1.35	2.65	-1.50	1.50	20	0	0.00	0.00
6.1-8	G	-1.35	2.65	-1.50	1.50	100	16	0.00	0.00
6.1-9	G	-1.35	2.65	-1.50	1.50	60	0	0.00	0.00
6.1-10	G	-0.45	-0.25	-0.10	0.10	40	40	0.00	0.00
6.1-11	G	1.934 68	1.949 3	-0.005	0.009	100	20	0.00	0.00
6.1-12	G	0.74	0.75	0.108	0.115 5	120	100	0.00	0.00
6.1-13	G	0.74	0.75	0.115 5	0.123	120	100	0.00	0.00
6.1-14	G	-0.465	-0.45	0.34	0.35	200	60	0.00	0.00
6.2-1ff	T	-2.10	2.10	-2.10	2.10	100	7	*	*
6.2-11	T	0.62	0.64	0.75	0.80	250	100	$y_0=0.1$	$c_j=0.4$
6.3-4ff	F	0.60	0.90	0.00	1.50	50	250	-	-
6.4-1	C	-2.50	2.00	-2.00	2.00	250	0	0.00	0.00
6.4-2	C	-0.20	0.40	1.50	1.91	100	0	0.00	0.00
6.4-3ff	G*	-2.10	2.10	-2.10	2.10	100	7	0.00	0.00
6.4-6	C	0.90	1.10	-0.03	0.10	100	0	0.00	0.00
6.4-7	J*	-2.00	2.00	-2.00	2.00	225	8	-0.50	0.44
7.1-1	N1	-2.00	2.00	-1.50	1.50	20	0	-	-
7.1-2	N3	1.00	3.40	-4.50	-2.70	20	0	-	-
7.1-3	J	-2.00	2.00	-1.50	1.50	10	0	0.50	0.50
7.1-4	G	-1.35	2.65	-1.50	1.50	15	0	0.00	0.00
7.1-5	J	-2.00	2.00	-1.50	1.50	20	0	0.745	0.113
7.1-6	G	-1.35	2.65	-1.50	1.50	20	0	0.00	0.00
7.2-1	G	-4.00	1.50	-2.00	2.00	40	12	0.00	0.00
7.2-2	G*	-1.50	1.50	-0.10	1.50	40	7	0.00	0.00
7.2-3	G*	-3.00	3.00	-2.25	2.25	30	10	0.00	0.00
7.2-4	N3	-2.00	2.00	-1.00	1.50	40	3	-	-
7.2-5	J	-2.00	2.00	-1.50	1.50	30	10	1.39	-0.02
7.2-6	J	-18.00	18.00	-13.50	13.50	30	10	1.39	-0.02
7.2-7	J	-2.00	2.00	-1.50	1.50	30	30	-0.35	-0.004
7.2-8	J	-3.20	3.20	-2.00	4.80	30	30	-0.35	-0.004
7.4-1	G	-1.35	2.65	-1.50	1.50	20	0	0.00	0.00
7.4-2	G	-1.35	2.65	-1.50	1.50	20	0	0.00	0.00
7.4-3	J	-2.00	2.00	-1.50	1.50	30	0	0.50	0.50
7.4-4	J	-2.00	2.00	-1.50	1.50	30	5	-0.35	0.15
9.5	J	-1.00	1.00	-1.20	1.20	100	0	-0.30	-0.005
10-1	G	-1.35	2.65	-1.50	1.50	20	0	0.00	0.00

Picture	Type	Left	Right	Top	Bottom	Maximal iteration	Bound	Complex or initial value $c_r$ or $x_0$	constant $c_i$ or $y_0$
10-2	G	0.80	0.95	-0.35	-0.15	25	0	0.00	0.00
10-3	G	0.80	0.95	-0.35	-0.15	25	15	0.00	0.00
10-4	G	0.85	0.95	-0.35	-0.25	25	0	0.00	0.00
10-5	G	0.85	0.95	-0.35	-0.25	25/50	15/21	0.00	0.00
10-6	G	0.857	0.867	-0.270	-0.260	50	0	0.00	0.00
10-7	G	0.857	0.867	-0.270	-0.260	100	40	0.00	0.00
10-8	G	0.915	0.940	-0.315	-0.305	100	40	0.00	0.00
10-9	G	0.935	0.945	-0.305	-0.295	100	40	0.00	0.00
10-10	G	0.925	0.935	-0.295	-0.285	100	40	0.00	0.00
10-11	G	0.857	0.867	-0.270	-0.260	100	40	0.00	0.00
10-12	G	0.900	0.92	-0.255	-0.275	150	60	0.00	0.00
10-13	G	1.044	1.172	-0.299 2	-0.211 6	60	30	0.00	0.00
10-14	G	1.044	1.172	-0.299 2	-0.211 6	60	30	0.00	0.00
10-15	G	1.044	1.172	-0.299 2	-0.211 6	60	30	0.00	0.00
10-16	G	0.75	0.74	0.108	0.115 5	120	99	0.00	0.00
10-19	G	0.745 05	0.745 54	0.112 91	0.113 24	400	100	0.00	0.00
10-20	G	0.745 34	0.745 90	0.112 95	0.113 05	400	140	0.00	0.00
10-21	G	0.015 36	0.015 40	1.020 72	1.020 75	300	60	0.00	0.00
12.1-1	F	1.80	3.00	0.00	1.50	50	100	-	-
12.4-2	F	1.80	3.00	0.00	1.50	50	100	-	-

Table 13-1 Data for selected pictures.

## 13.2 Figure Index

Figure	Page	Caption
1.1-1	4	Feedback cycle of weather research
1.1-2	6	General feedback scheme
1.2-1	7	Rough Diamond
1.2-2	8	Vulcan's Eye
1.2-3	9	Gingerbread Man
1.2-4	10	Tomado Convention
1.2-5	11	Quadruple Alliance
1.2-6	12	Seahorse Roundelay
1.2-7	13	Julia Propeller
1.2-8	14	Variation 1
1.2-9	14	Variation 2
1.2-10	15	Variation 3
1.2-11	15	Explosion
1.2-12	16	Mach 10
2.1-1	19	Feedback scheme for 'measles'
2.1-2	20	Development of the disease for $p_0 = 0.3$ and $k = 0.5$
2.1-3	20	Development of the disease for $p_0 = 0.3$ and $k = 1.0$
2.1-4	21	Development of the disease for $p_0 = 0.3$ and $k = 1.5$
2.1-5	21	Development of the disease for $p_0 = 0.3$ and $k = 2.0$
2.1-6	21	Development of the disease for $p_0 = 0.3$ and $k = 2.5$
2.1-7	22	Development of the disease for $p_0 = 0.3$ and $k = 3.0$
2.1-8	22	List of formulas
2.1-9	23	Discrete series of 6 $(k, p_i)$ -values after 10 iterations
2.1-10	26	Calculation of measles values
2.1.1-1	28	Two coordinate systems
2.1.1-2	33	Representation of the measles epidemic on the screen, IterationNo = 10
2.1.2-1	36	Initial value $p = 0.1$ , $k = 1.99$ , a limit point, with screen dialogue
2.1.2-2	37	Initial value $p = 0.1$ , $k = 2.4$ , two limiting points
2.2-1	38	Situation after the onset of oscillations, iteration number = 50
2.2-2	40	Print-out from Program 2.2-1
2.2-3	43	Section from the Feigenbaum diagram
2.2-4	44	Fig-tree with data: 2.5, 2.8, 0.9, 1.4, 50, 100
2.2-5	44	Fig-tree with data: 2.83, 2.87, 0.5, 0.8, 50, 100
2.2.1-1	46	Logarithmic representation from $k = 1.6$ to $k = 2.569$
2.2.2-1	50	Basin of attraction for the Feigenbaum diagram
2.2.3-1	52	Feigenbaum landscape with the data 0, 3, 0, 1.4, 50, 500

Picture	Page	Caption
2.2.3-2	53	Feigenbaum landscape with the data 3, 2.4, 1.4, 0, 50, 500
3.1-1	57	'Trace' of the parabola-attractor in the $p, f(p)$ -plane
3.1-2	58	The Verhulst attractor for $k = 1.60$
3.1-3	60	Self-similarity at each step (top left projections of the attractor)
3.1-4	61	Self-similarity at each step (detail from the 'node' to the right of centre)
3.2-1	62	The Hénon attractor
3.3-1	65	Lorenz attractor for $a = 10, b = 28, c = 8/3$ , and screen dimensions $-30, 30, -30, 80$
3.3-2	65	Lorenz attractor for $a = 20, b = 20, c = 8/3$ , and screen dimensions $-30, 30, -30, 80$
4.1-1	72	The graph of the function $f(x) = (x+1)*x*(x-1)$
4.1-2	73	How Newton's method leads to a zero
4.1-3	74	Initial value $x_6$ leads to attractor $x_1$
4.1-4	75	Initial value $x_7$ leads to attractor $x_2$
4.1-5	75	Initial value $x_8$ leads to attractor $x_3$
4.1-6	76	Initial value $x_9$ leads to attractor $x_2$
4.1-7	76	Initial value $x_{10}$ leads to attractor $x_1$
4.1-8	77	Initial value $x_{11}$ leads to attractor $x_3$
4.1-9	78	Graphical representation of the basins of attraction
4.1-10	78	Basins of attraction (detail of Fig. 4.1-9)
4.1-11	79	Basins of attraction (detail of Fig. 4.1-10)
4.2-1	84	A point in the Gaussian plane and its polar coordinates
4.3-1	87	The basins of attraction in the complex plane
4.3-2	88	Section from Fig. 4.3-1 left of centre
4.3-3	89	At the boundary between two basins of attraction
4.3-4	89	The boundary between two basins of attraction
4.3-5	90	'Stripes approaching the boundary'
5.1-1	92	Position of three point attractors in the Gaussian plane
5.1-2	98	Boundary between the three basins after 15 iterations
5.1-3	99	Basin of the attractor $z_C$
5.1-4	100	'Contour lines'
5.1-5	101	Every third contour line, and the boundaries, in a single picture
5.1-6	102	The basins of the three attractors $z_A, z_B, z_C$
5.1-7	104	A Julia set with fivefold symmetry
5.1-8	107	Grey shades, normal size (left) and magnified, to show individual pixels
5.2.1-4	111	Julia sets
5.2.5-8	112	Julia sets
5.2-9/10	114	Julia sets for $c_1$ and $c_2$

Picture	Page	Caption
5.2-11	115	Julia set for $c_1$ . Section from Fig. 5.2-9
5.2-12	115	Julia set for $c_1$ . Section from Fig. 5.2-11
5.2-13	116	Julia set for $c_1$ . Section from Fig. 5.2-12
5.2-14	116	Julia set for $c_2$ . Section corresponding to Fig. 5.2-13
5.2-15	117	Julia set for $c_1$ . Section from Fig. 5.2-13
5.2-16	117	Julia set for $c_2$ . Section from Fig. 5.2-14
5.2-17	118	Julia sets with small iteration number
5.2-18	119	Julia set with higher iteration number also acts as a boundary
5.2-19	120	Section from the centre of Fig. 5.2-17
5.2-20	121	Section from the centre of Fig. 5.2-19
5.2-21	124	Backwards iteration, 20 seconds' computing time
5.2-22	124	Backwards iteration, 4 hours' computing time
5.2-23	126	Yet another Julia set (just to whet your appetite)
6.1-1	129	Julia set for $c_1$ , section near the origin
6.1-2	130	Julia set for $c_2$ , section near the origin
6.1-3	136	Mandelbrot set (4 repetitions)
6.1-4	136	Mandelbrot set (6 repetitions)
6.1-5	136	Mandelbrot set (8 repetitions)
6.1-6	137	Mandelbrot set (10 repetitions)
6.1-7	137	Mandelbrot set (20 repetitions)
6.1-8	138	Mandelbrot set (100 repetitions, 'contour lines' up to 16)
6.1-9	139	Mandelbrot set (60 repetitions)
6.1-10	140	Mandelbrot set (section left of the origin)
6.1-11	141	A Mandelbrot set of the second order
6.1-12	142	A section between the main body and a bud
6.1-13	142	A section directly below Figure 6.1-12
6.1-14	143	A satellite fairly far left
6.2-1	146	Quasi-Mandelbrot sets for different initial values
6.2-2	147	Quasi-Mandelbrot sets for different initial values
6.2-3	150	Diagram for case 2
6.2-4	151	Diagram for case 2
6.2-5	152	Diagram for case 3
6.2-6	153	Diagram for case 3
6.2-7	154	Diagram for case 4
6.2-8	155	Diagram for case 4
6.2-9	156	Diagram for case 5
6.2-10	157	Diagram for case 5
6.2-11	158	Detail with 7-fold spiral
6.3-1	162	Feigenbaum diagram from the Mandelbrot set



Picture	Page	Caption
6.3-2	163	Direct comparison: Gingerbread Man / Fig-tree
6.3-3	163	A parameter path in the Mandelbrot set
6.3-4	164	Quasi-Feigenbaum diagram, real part
6.3-5	165	Quasi-Feigenbaum diagram, imaginary part
6.3-6	165	Pseudo-three-dimensional representation of trifurcation (oblique view from the front)
6.4-1	171	Basin of the attractor $z = 1$
6.4-2	172	Section from Figure 6.4-1 (with a surprise!)
6.4-3	174	Generalised Mandelbrot set for powers from 1 to 2
6.4-4	175	Generalised Mandelbrot set for powers from 2 to 3
6.4-5	176	Generalised Mandelbrot set for powers from 3 to 5
6.4-6	177	Section from Figure 6.4-1 near $c = 1$
6.4-7	178	Generalised Julia set
7.1-1	181	Boundaries between three attractors on the real axis
7.1-2	184	Julia set for Newton's method applied to $z^3 - 1$
7.1-3	185	Julia set for $c = 0.5 + i \cdot 0.5$
7.1-4	185	Gingerbread Man
7.1-5	186	Julia set, top and bottom interchanged, $c = 0.745 + i \cdot 0.113$
7.1-6	186	Gingerbread Man, inverse iteration height
7.2-1	188	Inverted Mandelbrot set
7.2-2	188	Gingerbread Man for the third power (top half, compare Figure 6.4-5)
7.2-3	189	Gingerbread Man for the third power, inverted (cropped on right)
7.2-4	189	Inverted Julia set for Newton's method applied to $z^3 - 1$
7.2-5	190	Julia set for $c = 1.39 - i \cdot 0.02$
7.2-6	190	Inverted Julia set for $c = 1.39 - i \cdot 0.02$
7.2-7	191	Julia set for $c = -0.35 - i \cdot 0.004$
7.2-8	191	Inverted Julia set for $c = -0.35 - i \cdot 0.004$
7.3-1	192	The entire complex plane in two unit circles
7.3-2	193	Projection from plane to sphere
7.3-3	198	Examples of Mandelbrot sets on the Riemann sphere
7.3-4	198	Gingerbread Man rotated $60^\circ$ , front and back
7.4-1	200	Gingerbread Man with interior structure (insideFactor = 2)
7.4-2	200	Gingerbread Man with interior structure (insideFactor = 10)
7.4-3	201	Julia set for $c = 0.5 + i \cdot 0.5$ with interior structure
7.4-4	201	Julia set for $c = -0.35 + i \cdot 0.15$ with interior structure
8.1-1	204	'Genesis' of the Hilbert curve
8.1-2	205	'Genesis' of the Sierpiński curve
8.1-3	205	Different dragon curves

Picture	Page	Caption
8.1-4	206	Different Koch curves
8.1-5	206	A C-curve
8.2-1	212	Grass and twigs
8.2-2	213	Two-dimensional cloud formation
8.2-3	214	Different grasses
8.2-4	215	Fractal landscape with lake and mountains
8.3-1	217	The structure $111\{11\}11\{11\}1$
8.3-2	220	Construction of a graftal
8.3-3	221	Graftal-plant
8.3-4	221	Development of a graftal from the 4th to the 12th generation
8.3-5	222	A graftal from the 4th to the 13th generation
8.4-1	224	Interference pattern 1
8.4-2	225	Interference pattern 2
8.4-3	225	Interference pattern 3
8.4-4	226	Garland
8.4-5	227	Spiderweb with $a = -137$ , $b = 17$ , $c = -4$ , $n = 1\ 898\ 687$
8.4-6	228	Cell culture
9-1	235	Complex weather boundaries around Bremen: 23/24.7 1985
9-2	236	Gingerbread weather?
9-3	239	Water drop experiment
9-4	241	EEG curves: normal action and fibrillation
9-5	245	Revolving, self-modifying patterns on the screen
10-1-21	248-55	From the 'Land of Infinite Structures': no title
11.2-1	273	Fractal mountains
11.2-2	281	Input dialogue for Graftal
11.3-1	285	Rössler attractor
11.5-1	304	Three methods of data compression
12.1-1	328	Feigenbaum reference picture
12.4-1	353	Turbo Pascal reference picture
12.4-2	360	Screen dialogue
-	379	'Atlas' of the Mandelbrot set.

### 13.3 Program Index

We list here both programs and program fragments (without distinguishing them). Each represents the algorithmic heart of the solution to some problem. By embedding these procedures in a surrounding program you obtain a runnable Pascal program. It is left to you as an exercise to declare the requisite global variables, to change the initialisation procedure appropriately, and to fit together the necessary fragments (see hints in Chapters 11 and 12). The heading 'Comments' states which problem the procedures form a solution to.

Program	Page	Comments
2.1-1	23	Measles numeric
2.1.1-1	29	Measles graphical
2.1.2-1	35	Graphical iteration
2.2-1	38	Feigenbaum iteration
2.2-2	42	Print-out of $k$ in a running program
3.1-1	58	Verhulst attractor
3.2-1	63	Hénon attractor
3.3-1	66	Lorenz attractor
5.1-1	93	Assignment for iteration equation
5.1-2	94	Belongs to $z_C$
5.1-3	94	Mapping
5.1-4	95	JuliaNewtonComputeAndTest
5.1-5	96	StartVariableInitialisation
5.1-6	96	Compute
5.1-7	97	Test
5.1-8	97	Distinguish (does the point belong to the boundary?)
5.1-9	98	Distinguish (does the point belong to the basin?)
5.1-10	100	Distinguish (iteration number MOD 3 etc.)
5.2-1	109	Formulas in Pascal
5.2-2	113	JuliaNewtonComputeAndTest
5.2-3	122	Backwards iteration
5.2-4	123	Roots
6.1-1	135	MandelbrotComputeAndTest
6.3-1	159	Equality test for real numbers
6.3-2	159	Equality test for complex numbers
6.3-3	160	Mapping
6.3-4	164	Working part of Mapping
6.3-5	164	Drawing commands, real part
6.3-6	164	Drawing commands, imaginary part

Program	Page	Comments
6.3-7	165	Drawing commands, pseudo-3D
6.4-1	168	Complex arithmetic
6.4-2	170	Curry-Garnett-Sullivan method
6.4-3	173	Compute for higher powers
7.1-1	181	D3-Mapping
7.1-2	182	D3ComputeAndTest
7.1-3	183	D3Draw
7.2-1	187	ComputeAndTest, inverted
7.3-1	194	Mapping for Riemann sphere
7.4-1	199	Distinguish for structures in interior
8.1-1	207	Turtle graphics
8.4-1	224	Conett method
8.4-2	227	Martin method
11.1-1	258	Empty Application Shell, shell program for graphics
11.1-2	265	Global constants, types, variables
11.2-1	267	Turtle graphics - dragon
11.2-2	269	Fractal landscapes
11.2-3	273	Graftals
11.3-1	281	Feigenbaum iteration, logarithmic
11.3-2	282	Feigenbaum landscape
11.3-3	285	Rössler attractor
11.3-4	286	Newton demonstration
11.4-1	288	Mapping for Newton development of $x^3-1$
11.4-2	291	Mapping for quadratic iteration, Julia sets
11.4-3	293	Mapping for Gingerbread Man
11.4-4	296	Mapping for tomogram
11.4-5	297	Mapping for quasi-Feigenbaum diagram
11.4-6	299	Compute for high-powered Gingerbread Man
11.4-7	300	Mapping and Initialise for pseudo-3D
11.5-1	305	Integer encoding (Berkeley Pascal)
11.5-2	310	Integer encoding to paint
11.5-3	312	Compress int to int
11.5-4	314	Run length encoding integer to paint
11.5-5	315	Transfer int to char
11.5-6	317	Run length encoding char to paint
12.2-1	331	Turbo Pascal Reference Program for MS-DOS
12.3-1ff	339	Program examples in C
12.4-1	347	Turbo Pascal Reference Program for Macintosh

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<b>Program</b>	<b>Page</b>	<b>Comments</b>
<hr/>		
12.4-2	354	Lightspeed Pascal Reference Program for Macintosh
12.5-1	361	ST Pascal Plus Reference Program for Atari
12.6-1	366	UCSD Pascal Reference Program for Apple II
12.6-2	367	Include-File of useful subroutines
12.6-3	371	TMLPascal Reference Program for Apple IIGS

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