Notes on the Fundamental Equations of PHASTA

James Wright

October 21, 2019

1 Shorthand

$$\phi_{,t} = \frac{\partial \phi}{\partial t} \tag{1}$$

$$\phi_{,i} = \frac{\partial \phi}{\partial x_i} \tag{2}$$

$$u_{i,t} = \frac{\partial u_i}{\partial t} \tag{3}$$

$$u_{i,j} = \frac{\partial u_i}{\partial x_j} \tag{4}$$

$$[\phi u_i]_{,j} = \frac{\partial \phi u_i}{\partial x_j} \tag{5}$$

2 Fundamental Fluid Equations

Compressible Navier-Stokes equations:

2.1 Traditional Form

Continuity

$$\rho_{,t} + [\rho u_j]_{,j} = 0 \tag{6}$$

Momentum

$$[\rho u_i]_{,t} + [\rho u_i u_j]_{,j} + p_{,i} = \tau_{ij,j} + b_i \tag{7}$$

Energy

$$[\rho e_{tot}]_{,t} + [\rho e_{tot} u_j]_{,j} + [\rho u_j]_{,j} = [\tau_{ij} u_j]_{,j} + b_i u_j + r + q_{i,i} \tag{8}$$

2.2 Conservative Vectorized Form

$$\boldsymbol{U} \equiv \begin{Bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho e_{tot} \end{Bmatrix} = \begin{Bmatrix} \rho \\ \rho u_j \\ \rho e_{tot} \end{Bmatrix} \tag{9}$$

Flux Vector:

$$\begin{aligned} \boldsymbol{F}_i &= \underbrace{\begin{Bmatrix} \rho u_i \\ \rho u_i u_j \\ \rho u_i e_{tot} \end{Bmatrix} + \begin{Bmatrix} 0 \\ p\delta_{ij} \\ u_i p \end{Bmatrix}}_{\text{Advective Flux}} - \underbrace{\begin{Bmatrix} 0 \\ \tau_{ij} \\ \tau_{ij} u_j \end{Bmatrix} + \begin{Bmatrix} 0 \\ \boldsymbol{0} \\ q_{i,i} \end{Bmatrix}}_{\text{Diffusive Flux}} \\ &= \boldsymbol{F}_i^{\text{adv}} + \boldsymbol{F}_i^{\text{dif}} \end{aligned} \tag{10}$$

Also:

$$\boldsymbol{F}_i^{adv} = u_i \boldsymbol{U} + \begin{Bmatrix} 0 \\ p\delta_{ij} \\ u_i p \end{Bmatrix} \tag{11}$$

Source Vector:

$$\boldsymbol{\mathfrak{F}} = \begin{Bmatrix} 0 \\ b_j \\ b_j u_j + r \end{Bmatrix} \tag{12}$$

These terms combine together to form:

$$\boldsymbol{U}_{,t} + \boldsymbol{F}_{i,i} = \boldsymbol{\mathfrak{F}} \tag{13}$$