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Quant Report

Building a Diversified Portfolio using the Markowitz Method and HRP

Jung Bin HONG (BEng in IEDA @ HKUST) & Chae Hyun SONG (BEng in IEDA & BBA in GBM)

Introduction

Abstract

For the past several decades, portfolio construction and optimization have been critical studies in the finance industry. Modern-day portfolio optimization is coined as the task of selecting assets that maximize the return from the investment while minimizing the risk. Each of these tasks involves balancing risk and return, where risk generally refers to the fluctuations in asset value. An assumption for portfolio theories is that investors are rational and want to achieve the most significant possible returns in the long run without taking extreme levels of market risk. Under this assumption, many portfolio algorithms have improved to aid investors in constructing robust portfolios. This paper explores whether the Hierarchical Risk Parity (HRP) algorithm, a risk-based portfolio optimization method applying modern graph theory and machine learning techniques, can outperform the conventional Markowitz Mean-Variance (MVP) model.

Theoretical Background:

Markowitz's Mean-Variance Portfolio (MVP)

The MVP is based on the process of weighing risk (expressed as variance) against expected return (expressed as a probability on estimated returns). If two different assets, A and B have similar expected returns, but asset A has lower variance, asset A is selected. Similarly, if the same assets have approximately the same variance, the asset with higher expected returns is preferred. The algorithm is structured so that if an asset with high variance is combined with diverse assets with low correlation, the resultant portfolio will have a lower variance.[1] Shown in *figure 1*, Markowitz's method can be visualized as an 'Efficient Frontier,' a graphical representation of all possible portfolio combinations that show the optimal returns given a particular level of risk.[2]

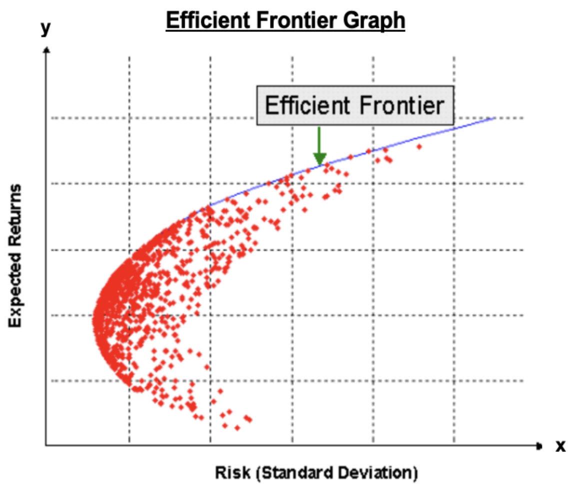

Figure 1. Efficient Frontier

The formulation of the efficient frontier is as follows:

$$Minimize: \sigma_p^2 = \omega^T \Sigma \omega$$
$$Subject\ to: \ r^T \omega = r_p (a\ determined\ return)$$
$$\sum_{i=0}^{N} w_i = 1, \ w_i \geq 0\ \forall\ i$$

[1] Peter Westfall. (Sept 2021). Modern Portfolio Theory: What MPT Is and How Investors Use It. Retrieved Oct 16, 2022 from https://www.investopedia.com/terms/m/modernportfoliotheory.asp

[2] Nicolás Besser. (Apr 2021). Python for Finance: Portfolio Optimization and the value of Diversifying. Retrieved Oct 16, 2022, from https://nicobesser.medium.com/python-for-finance-portfolio-optimization-and-the-value-of-diversifying-99ef8e5cfbdc

[3]We define λ as the risk aversion constant that depends on individual investors.[4] This constant will be greater the more the investor is risk averse, despite the lower expected return. The formulation of the mean-variance optimization is:

$$Maximize: \omega^T r - \frac{\lambda}{2}\omega^T \Sigma\omega = r_p - \frac{\lambda}{2}\sigma_p$$
$$Subject\ to: \sum_{i=0}^{N} \omega_i = 1, w_i \geq 0\ \forall\ i$$

Hierarchical Risk Parity(HRP)

The HRP Algorithm consists of 3 major stages. The details in each step will be explained in the following sections.

Hierarchical Tree Clustering

Hierarchical Risk Parity classifies the securities into different hierarchical clusters. The algorithm will establish a hierarchical tree that depicts the recursive process that allocates the universe of assets in clusters, containing similar assets with a higher correlation, that will then be divided too. Agglomerative clustering(AGNES) executes the cluster analysis. Initially, from a single element cluster at the bottom, it sequentially merges with the most synchronized cluster from bottom to top, until one cluster encloses all securities. Given a T x N matrix containing T observed stock returns for each N asset, the algorithm first calculates the correlation of each stock's return into a N x N matrix. The correlation N x N matrix is converted to a correlation distance matrix D. Then, it formulates the other matrix D, which contains the Euclidean distance between the pairs of columns in the matrix $\overline{D}$. Using the two matrices, different clusters of assets are assembled in a recursive process. Finally, a form of a diagram called a den dendrogram visualizes the clusters. The figure below represents a dendrogram of our datase

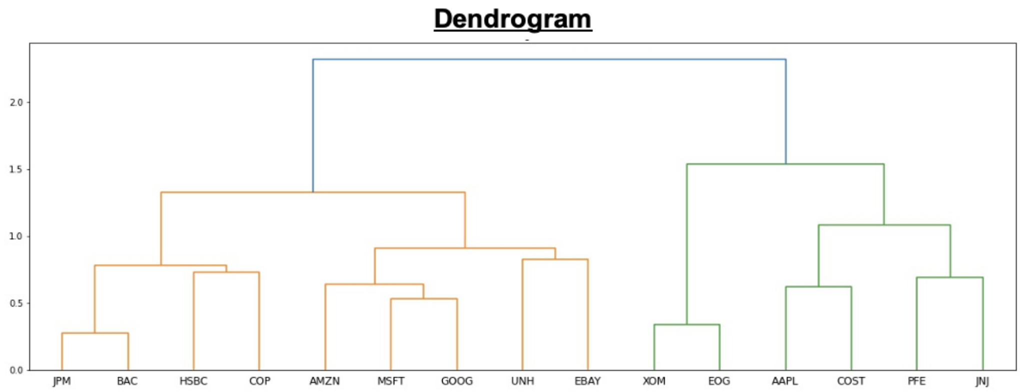

Figure 2. Dendrogram of Dataset

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[3] Prado M., Building Diversified Portfolios that Outperform Out-of-Sample (May 23, 2016). Journal of Portfolio Management, 2016, Retrieved Oct 25, 2022, from https://ssrn.com/abstract=2708678

[4] Mikel Mercader Pérez. (May 2021). Hierarchical Risk Parity: portfolio optimization. Retrieved Oct 25, 2022, from https://upcommons.upc.edu/bitstream/handle/2117/350200/TFG.pdf?sequence=1&isAllowed=y

Quasi-Diagonalization

This step rearranges the columns and rows of the covariance matrix of stocks so that the most significant correlation values lie along the diagonal. Synchronized investments are clustered close together, and unrelated assets are located further apart. The figures below represent how a correlation matrix of our dataset could look before and after undergoing this quasi-diagonalization. After undergoing quasi-diagonalization, it is noticeable that the tickers along the x-axis of the darker blue rectangles match the tickers clustered in orange in the dendrogram shown above.

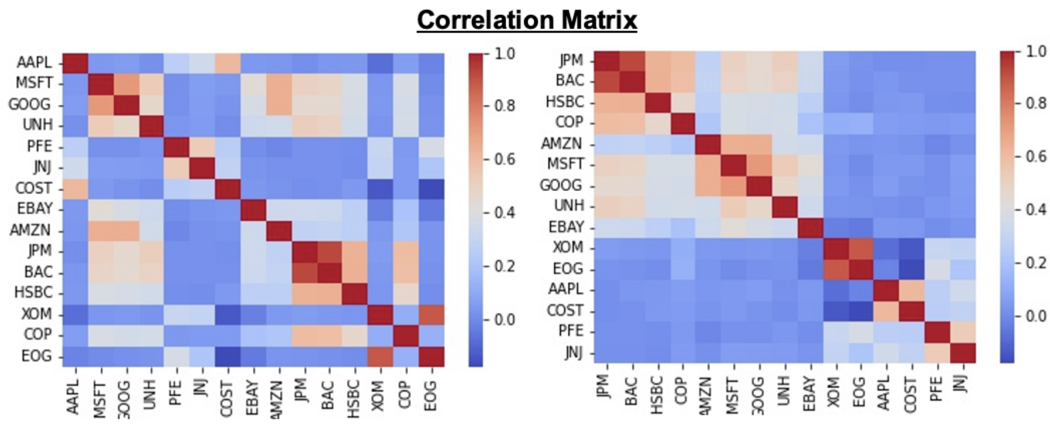

Figure 3. Correlation Matrix before and after Quasi-Diagonalization

Recursive Bisection

The final stage of the HRP algorithm determines how to allocate the weight of assets in the optimal portfolio. First, it initializes the weights for all n assets equally to one. Next, in a top-down manner, it splits the weights inversely proportional to the aggregated variance of each cluster between adjacent subsets. The step is where this algorithm makes use of the quasi-diagonalized covariance matrix for defining the variance of the recursing clusters and allocating the assets. [5]The primary strength of the allocation method is that similar assets with higher correlation compete for the weight distribution on behalf of challenging all the securities in the portfolio.

[5] Aditya V., The Hierarchical Risk Parity Algorithm: An Introduction, Hudson & Thames, Retrieved Oct 25, 2022, from https://hudsonthames.org/an-introduction-to-the-hierarchical-risk-parity-algorithm/

Methodology

Due to the complex nature of the stock market and the overwhelming amount of financial data, this report presents an algorithmic approach for building simplified, efficient portfolios by selecting three representative stocks from five sectors of the U.S. stock market. The selected sectors are technology, healthcare, retail, finance, and energy.

1. Technology: Google (GOOG), Apple (AAPL), Microsoft (MSFT)
2. Healthcare: United Healthcare (UNH), Pfizer (PFE), Johnson & Johnson (JNJ)
3. Retail: Amazon (AMZN), eBay (EBAY), Costco (COST)
4. Finance: JPMorgan Chase & Co (JPM), Bank of America (BAC), HSBC Holding (HSBC)
5. Energy: Exxon Mobil Corp (XOM), ConocoPhillips (COP), EOG Resources (EOG)

Portfolios were built using a total investment amount of $10,000 USD, using the MVP and HRP algorithms for the historical prices of the stocks between 1 Jan 2015 to 19 Jan 2021. The portfolios were then back-tested on the sample data of the stock prices from 20 Jan 2021 to 29 Aug 2022.

Portfolio Selection

The efficient frontier module in python allows us to use the maximum Sharpe statistic to find the maximum Sharpe ratio for each generated portfolio. This translates to the portfolio with the highest return and the lowest risk. For each portfolio selected in this report, the red cross on the curve denotes the portfolio that has been selected. Using this statistic, the portfolios that are generated are updated every year on 1 Jan. The following figure is an example of the efficient frontier curve and the portfolio selected for the year 2020.

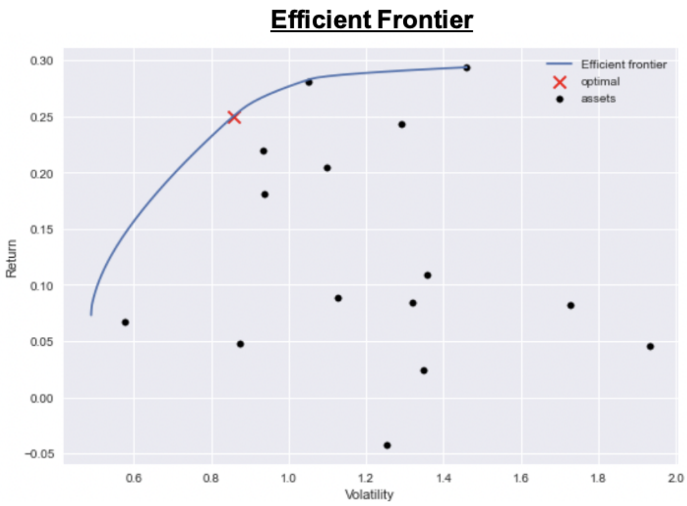

Figure 4. Efficient Frontier for 2020 MVP Portfolio

Portfolio Construction

As expected, the HRP algorithm constructs a much more diverse portfolio than the traditional MVP algorithm. The following are two stacked charts for the two portfolios constructed using the Mean-Variance and HRP algorithms.

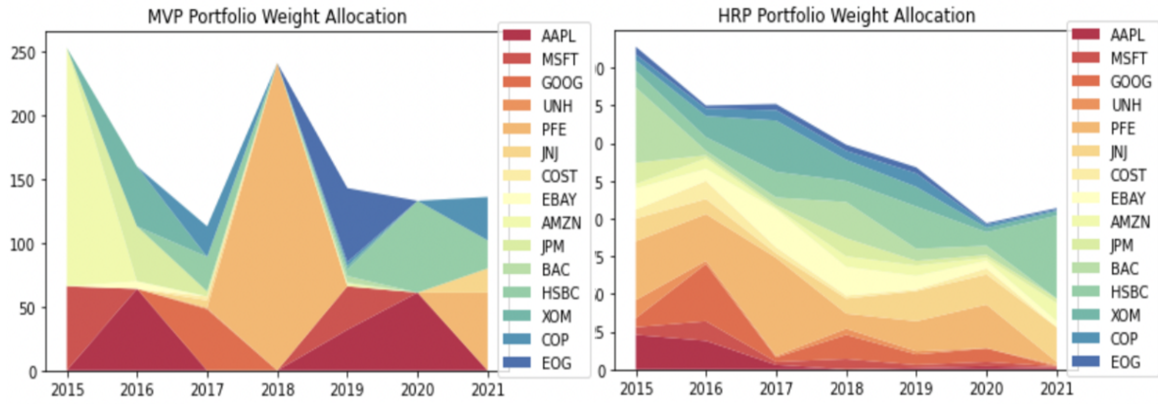

Figure 5. Stacked Area Chart of MVP and HRP Portfolios

From the generated stacked area charts, the MVP concentrates majority of its assets on two stocks: AAPL, and PFE. Part of the reason for this extreme concentration in specific stocks derives from the MVP algorithm's aggressive tendency to minimize variance. Meanwhile, HRP attempts to allocate more evenly distributed weight to diverse securities, and less weight to the assets with higher correlation by clustering assets concerning correlation data. This can be noticed in the HRP stacked chart – the area under each color is nowhere as thick as the stacked areas under the MVP chart. In the HRP, the risk is diversified into more sub-categories allowing these uncorrelated assets to hedge and protect the portfolio from unprecedented fluctuation. Below is a comparison chart demonstrating the two algorithms' performance by plotting the expected returns and annual volatility.

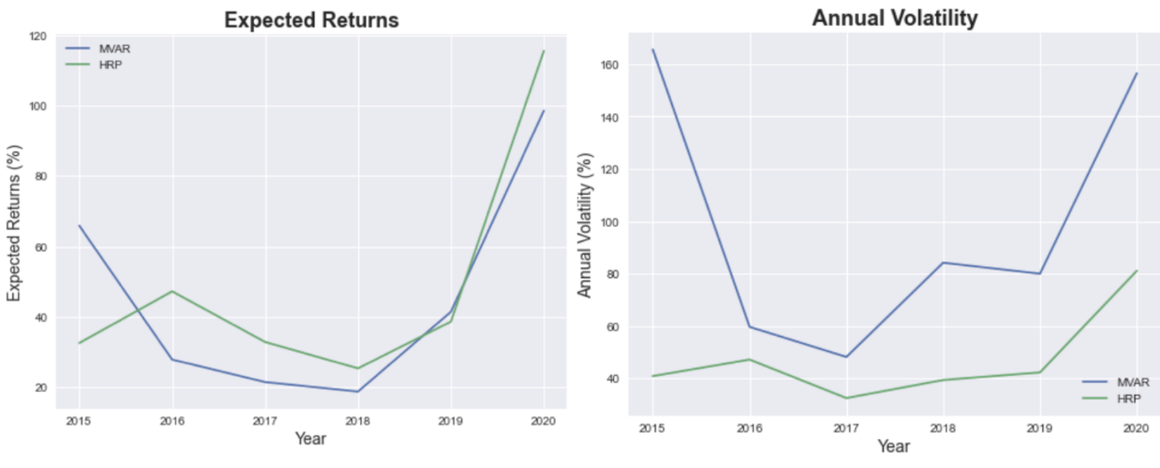

Figure 6. Expected Returns and Annual Volatility Comparison Chart

The apparent trend to be noted is that although there is a slight difference in the expected returns for the two algorithms, there is a significant difference in the annual volatility.This is due to another major weakness of the MVP, which is the assumption that historical returns will ultimately reflect future returns.[6] From a pragmatic perspective stock returns are very difficult to estimate, making the method very unstable – susceptible to high volatility. A slight deviation in predictions could cause the mean variance to produce widely different portfolios. The outcomes of the assumptions are better illustrated with the following backtesting results.

Backtesting Results

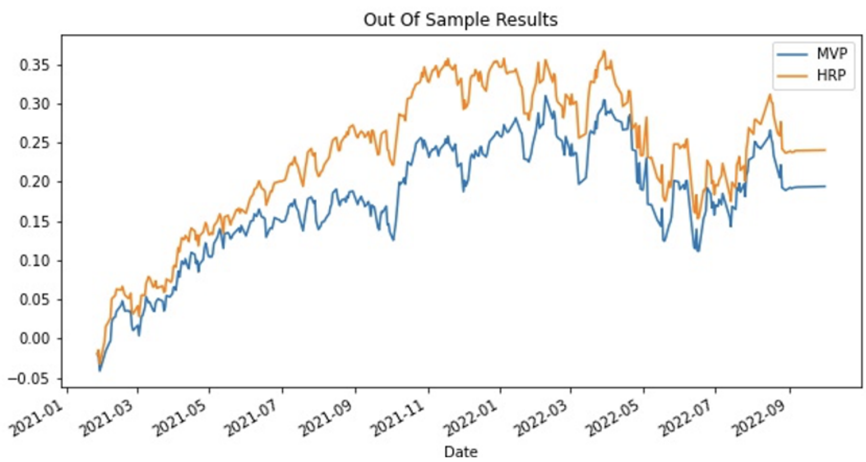

Figure 7. Backtesting Results

From the backtesting results, both the MVP and HRP performed moderately despite the highly volatile stock market in 2022. The period of the training data was set between 2015 to 2021 January, when the U.S. stock market in this time frame had continuous stable growth. When the portfolios were tested against the training data, the current stock market in 2022 was an outlier. From the aforementioned assumptions for the MVP, the portfolio had more trouble in hedging to a highly volatile market. Hence, the Sharpe ratio of HRP is greater than that of MVP. This was somewhat expected from the testing data results, where the HRP outperformed the MVP.

	standard_deviation_testing_data	sharpe_ratio_testing_data
MVP	0.209250	0.611922
HRP	0.219142	0.723422

Figure 8. Table of Standard Deviation and Sharpe Ratio for Testing Data

[6] Aditya V., The Hierarchical Risk Parity Algorithm: An Introduction, Hudson & Thames, Retrieved Oct 25, 2022, from https://hudsonthames.org/an-introduction-to-the-hierarchical-risk-parity-algorithm/

Conclusion

To conclude, the HRP presents specific characteristics that surpass Markowitz's model in several situations. The HRP provides better protection against both idiosyncratic shocks affecting a specific investment and shocks involving several correlated investments. While both portfolios produced similar expected returns, the difference in volatility was significantly lower for the portfolio generated by the HRP algorithm. The HRP algorithm generally constructs a more diverse portfolio, allocating weights uniformly across all the assets with lower correlation. However, from the backtesting results, both portfolios produced lower Sharpe ratios, indicating that weaknesses still exist when dealing with unstable stock markets – even if the risk was minimized.

The scope of the investigation can be further extended if a more diverse set of asset classes were considered in the data collection stage. Such actions would potentially produce a more robust portfolio with better results as selecting three stocks from five sectors – accounting for a total of only 15 stocks – dramatically limits the algorithm's range to pick and choose stocks with the best statistical measures.

Additionally, implementing a shrinkage estimator could have improved Markowitz's model to make a fair comparison with HRP. [7]In fact, the several limitations of Markowitz's model are known for its poor out-of-sample performance and the extreme weights of the resulting optimal portfolios. The errors associated with estimating the means, variances, and correlations of asset returns maximize these limitations. The most popular approach to managing the problems of estimation risk is using Bayesian shrinkage estimators, which identify a new estimate by shrinking a raw estimate. The shrinkage estimator combines two extreme means to converge into one centralized mean, revising all means in a sample.

[7] Roberto O., Mauricio C. & Cristhian M. (2022) Improving the volatility of the optimal weights of the Markowitz model, Economic Research-Ekonomska Istraživanja Retrieved Oct 25, 2022, from https://doi.org/10.1080/1331677X.2021.1981963

Appendix

Below are the portfolios generated each year using the MVP and HRP algorithms.