Math 170S HW3 Jun Ryu

1) $f(x) = \frac{1}{2\phi} e^{-|x|/\phi}$

$$L(\phi) = \prod_{i=1}^{n} \frac{1}{2\phi} e^{-|x_i|/\phi}$$

$$= \frac{1}{(2\phi)^n} \prod_{i=1}^{n} e^{-|x_i|/\phi}$$

$$= (2\phi)^{-n} e^{-\sum_{i=1}^{n}|x_i|/\phi}$$

$$\ln L(\phi) = -n\ln(2\phi) - \sum_{i=1}^{n}|x_i|/\phi$$

$$0 = \frac{\partial \ln L(\phi)}{\partial \phi} = -\frac{n}{\phi} + \frac{\sum_{i=1}^{n}|x_i|}{\phi^2}$$

$$0 = \frac{-n\phi + \sum_{i=1}^{n}|x_i|}{\phi^2}$$

$$n\phi = \sum_{i=1}^{n}|x_i|$$

$$\boxed{\hat{\phi} = \frac{\sum_{i=1}^{n}|x_i|}{n}} \quad \text{MLE}$$

2) $X \sim U(0, b)$, $b > 0$

The pdf of a uniform distribution is

$$P(X=x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{elsewhere} \end{cases}$$

In this case, we have

$$P(X=x) = \begin{cases} \frac{1}{b}, & 0 \le x \le b \\ 0, & \text{elsewhere} \end{cases}$$

$$L(b) = \prod_{i=1}^{n} \frac{1}{b} = \frac{1}{b^n} = b^{-n}$$

$$\ln L(b) = -n \ln b \Rightarrow \frac{\partial \ln L(b)}{\partial b} = \frac{-n}{b} = 0$$

This would result in us not being able to deduce the value of b. So, we need to draw b from our sample values X_i's such that $b \ge x_i$, for $i = 1 \ldots n$ and it maximizes our likelihood function of $L(b) = b^{-n}$.

Since $L(b) = b^{-n}$ is a decreasing function, if we pick $\hat{b} = \max(x_1, x_2 \ldots x_n)$, then we have $\hat{b}$ is greater than or equal to all x_i's and $\hat{b}$ maximizes $L(b)$.

Thus, $\boxed{\hat{b} = \max(x_1, x_2 \ldots x_n)}$ MLE

3) $f(x|\beta) = \beta x^{\beta-1}$, $0<x<1$, $\beta>0$

$$L(\beta) = \prod_{i=1}^{n} \beta x_i^{\beta-1} = \beta^n \left(\prod_{i=1}^{n} x_i\right)^{\beta-1}$$

$$\ln L(\beta) = n \ln \beta + (\beta-1) \ln\left(\prod_{i=1}^{n} x_i\right)$$

$$= n \ln \beta + \beta \ln\left(\prod_{i=1}^{n} x_i\right) - \ln\left(\prod_{i=1}^{n} x_i\right)$$

$$0 = \frac{\partial \ln L(\beta)}{\partial \beta} = \frac{n}{\beta} + \ln\left(\prod_{i=1}^{n} x_i\right) - 0$$

$$-\frac{n}{\beta} = \ln\left(\prod_{i=1}^{n} x_i\right)$$

$$\boxed{\hat{\beta} = -\frac{n}{\ln\left(\prod_{i=1}^{n} x_i\right)}} \quad \text{MLE}$$