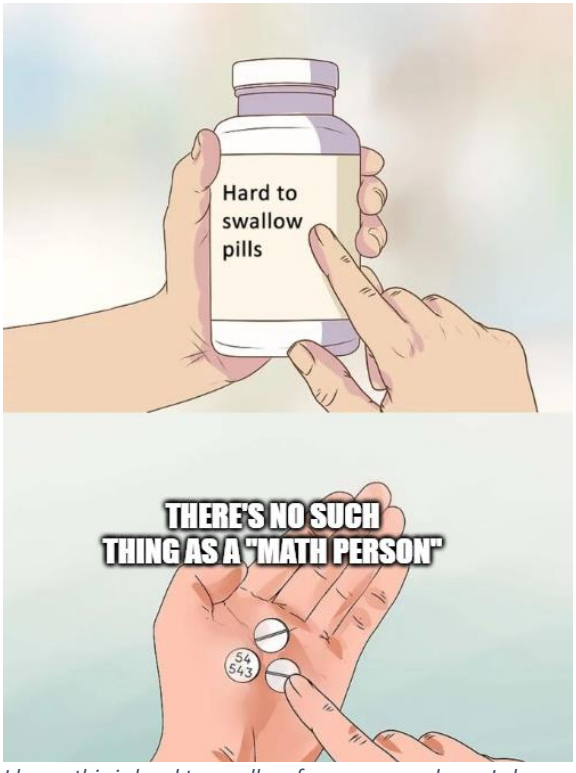
“Math is not my thing”? Think again…

Math is all fun and games until it’s not. I used to be pretty good at math in high school and found it to be a lot of fun. This was until I took AP Calculus in grade 12 and failed the course. Because of my grade in this one course, I was rejected from one of the most prestigious computer science schools in Canada, and this was CRUSHING. My parents were upset, my peers that expected me to be very good at the subject came to me for help only to find out I failed. All of a sudden, math just doesn’t seem so fun to me anymore; I became math anxious. I entered university fearing all the math courses I have to do in my curriculum, which translates to at least 30 units. To my surprise, my first university math course rekindled my love for math with the guidance of a very inspirational professor, and thus began my redemption arc in math. A quick spoiler, I now work part time as a highly qualified teacher/tutor for high school and university level math and programming.

We will look at how to overcome math anxiety and become better at math, from what the research says, and how I teach. As a disclaimer, research in this area is mostly proprietary, meaning publications only show that X works, but rarely how or why, or even what X really is. This is ~~why we can’t have nice things~~ what happens when money can be made off of research and people close source their word, showcasing heights in a field that no one else can reach.

Publication politics aside, let’s get on the same page with a meme I cooked up:



I know this is hard to swallow for some people, so I choose to use a meme to convey the message, glass of water not included; original creation Jack Zhou 2020

Now that we are on the same page, what is math anxiety? Math anxiety can be described as having aversive reactions to math and a tendency to avoid it. In the current age where numerical literacy is crucial to being competent in the workforce, math anxiety is seen as an undesirable trait by educators and parents alike.

Some quick facts: math anxiety is negatively correlated with mastery of mathematical concepts and motivation to do math (Ashcraft & Krause, 2007). Those that are mathematically anxious also show weaker working memory (Ashcraft & Kirk, 2001) and perceive math and their abilities to do math negatively (Hembree, 1990).

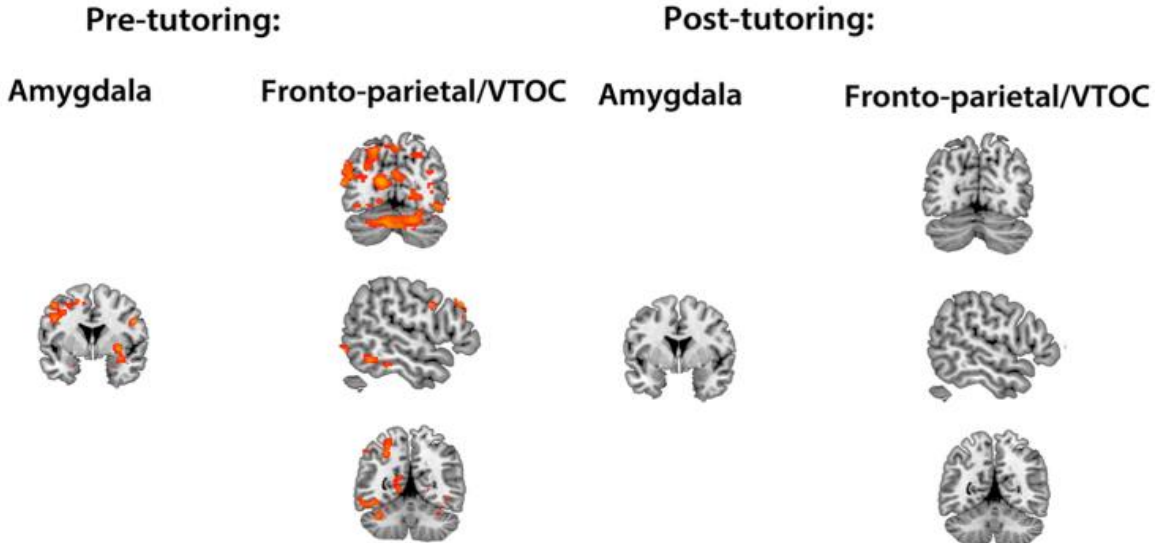
So why does mathematical anxiety happen? The academic world doesn’t really have a definitive answer. For me, it was because of the various consequences I faced for not performing well in a math course. Many people may have

childhood memories of their parents yelling at them while they are weeping in front of a multiplication worksheet figuring out why 5 times 7 isn't 32. Aversive experiences like these are often the cause behind math anxiety.

What can we do about this? An answer may simply be mere exposure. This is a common technique used in treating PTSD and phobias, where the mechanism is simply eliciting habituation (Abramowitz, 2019). Supekar et al. used this technique and found that after 8 weeks of 1 on 1 cognitive tutoring sessions[1], 3rd grade students who were math anxious improved their performance and attitudes toward math (Supekar, Iuculano, Chen & Menon, 2015). Each session starts with review before new concepts are taught, and synthesis between old and new material is emphasized towards the end.

In my teaching, I actively incorporate these techniques, but mostly synthesizing new information with old in the form of generalization. I have no control over my students' schedules after class, so the best I can do is come up with the most understanding in the least amount of time. A classic example of generalizing understanding is that basic arithmetic rules for fractions are the same for n^{th} radicals. If you know the former, you basically know the latter.

The neural mechanisms relating to math anxiety were also studied in the Supekar et al. study. They found that the amygdala, prefrontal and parietal cortices behave erratically when a mathematically anxious person is doing math. These areas contain regions that are critically responsible for emotion regulation (Phelps & LeDoux, 2005) and solving math problems[2] (Menon, 2014). The effect cognitive tutoring has can be seen in the fMRI image below.



Functional differences due to cognitive tutoring, the red part signals erratic behaviour while doing math; Supekar, Iuculano, Chen & Menon, 2015

After tutoring, the erratic behavior of these brain regions become normalized to a level that is similar to those with no math anxiety. The effect of mere exposure is now apparent.

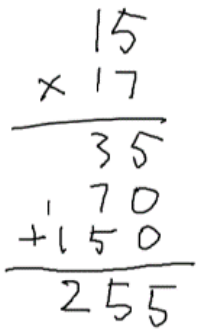
[1] roughly 3 hours a week

[2] When Einstein's brain was dissected, they found his parietal lobes to be very different!

We just mentioned abnormal amygdala activity, and as a matter of fact there is almost always an emotional connection that is involved with math anxiety. I find that a recurring notion with my students is the feeling of helplessness. Students would "thoroughly" understand how to do a problem, but as soon as the problem changes with slightly more complexity, their confidence is crushed because they don't know how to do it anymore. Thus, the presence of someone who is readily able point a bewildered student in the correct direction is important, to gradually instill and foster a fearless mentality.

Now that we know how to overcome math anxiety, how can we become better at math?

In a cognitive perspective, plain memorization/retrieval is not enough for doing math. Working memory is also needed. Math anxiety is also linked to weaker working memory, but this can easily be addressed. To gain an appreciative understanding on why working memory matters, consider the problem of "15x17=?". I have attached my hand-written work below, but you are encouraged to try it out for yourself first.



That wasn't too impressive of a problem, but I listed out the steps taken. We can see that there were intermediate results 35, 70, 150, and a carry digit when adding the numbers together. If you were doing this in your head, all these pieces of information will have to be stored in working memory. Now it doesn't seem as easy anymore.

There is plenty of research that backs up the notion that working memory can be improved with training, and that it is also linked to higher mathematical performance (ex. Dunning, Holmes & Gathercole, 2013). The general methods discussed by these kinds of studies are usually undisclosed because it's to show that some product that a company is trying to advertise actually works. These kinds of products come a dime a dozen with a google search.

However, I don't think working memory training is really necessary. Why? Let's go back to the 15x17 example: it looked easy on paper, right? The takeaway here is that paper IS your working memory! Don't hesitate with scratch work and don't worry about using too much space for scratchwork. With practice, you will know how to effectively use scratch space.

I myself am not very good with arithmetic, but this doesn't stop me from writing out everything (and making my students do so!). At school I always carry a sketchbook with me for scratchwork, of which I refuse to replace until EVERY page was filled on both sides.

Every. Single. Page in this stack of books is filled (both sides!) with scratchwork; original creation Jack Zhou 2020

Now we know how to empower our working memory, the next thing to do is to think and understand the concepts as deeply as possible. Unfortunately, this is easier said than done, and not much relevant empirical research can be found in this area. Often, the thinking needs to be expressed on paper in the form of writing out formulas and transforming formulas from one form to another[3]. This is what I would consider the meat of understanding math, and where most of the time SHOULD be spent while studying math instead of grinding out practice problems. Why is this needed? With a deep understanding, a lot of what people call memorization in math can be substituted with intuition and understanding.

One of the beautiful things about math is that everything can be built from the ground-up[4] with the bare minimums. The idea is that after learning about how to build these from the ground up, they can then be "compressed" into much less information. If you were to ask students who didn't do well in calculus about the course, they will tell you it's full of painful memorizations: there are two tables full of derivatives and integrals to memorize. If you forget one formula during an exam, then it's time to sign up for the course again next semester.

However, those who did well will instead tell you that everything is intuitive and there isn't much to memorize. Why is this? Take a quick look at these scary formulas:

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[cot^{-1} x] = -\frac{1}{1+x^2}$$

The four horsemen of the apocalypse: derivatives of inverse trigonometric functions; original creation Jack Zhou 2020

[3] more commonly known as derivations or reductions

[4] For our purpose, now of course there are things that can't be done this way in advanced math

They look very scary, even more so if I tell you to memorize them. I don't even have them memorized myself. However, because I understand, I can derive each of these from the ground-up[5] within a minute, using bare minimum calculus, basic trigonometry, and simple algebra.

So, memorization? Not needed. Practice? Absolutely. Time is required to understand and form pathways to go from the basic elements to a full-fledged result.

Now, we have covered quite a bit of ground. We talked about various techniques to perform well in math, and we talked about how some of these techniques can be used to treat mathematical anxiety. Hopefully, with this new knowledge, you as the reader may benefit or gain a new perspective.

What happened to me after I overcame math anxiety? I signed up for a math minor and enrolled in MATH310: Introduction to Ordinary Differential Equations. On the first lecture I introduced myself as a psychology major in front of 250+ engineers and mathematicians. I was instantly met with unanimous laughter. By the end of the semester, I was the one laughing as I walked out of the course with a shiny new A in my pocket and a track record of straight A's in my math courses ever since… although this may change this semester 🙃.

Word count: 1782

[5] For time and space's sake, I won't go through it here, but it can easily be found online

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