

# Homework 7

$$1. \left| (x-x_1) \dots (x-x_n) \right| \leq \frac{\left( \frac{b-a}{2} \right)^n}{2^{n-1}}$$

a)  $[-1, 1] \quad n=6$

$$\left| (x-x_1) \dots (x-x_6) \right| \leq 0.03125$$

b)  $[-2, 2] \quad n=4$

$$\left| (x-x_1) \dots (x-x_4) \right| \leq 2$$

c)  $[4, 12] \quad n=6$

$$\left| (x-x_1) \dots (x-x_6) \right| \leq 128$$

d)  $[-0.3, 0.7] \quad n=5$

$$\left| (x-x_1) \dots (x-x_5) \right| \leq 0.001953125$$

2.  $\left| e^x - Q_5(x) \right| \leq \frac{\left| (x-x_1) \dots (x-x_6) \right|}{6!} e^{1.0}$  for  $[0.6, 1.0]$

see notebook

3. 
$$S(x) = \begin{cases} a_1 + b_1 x + c_1 x^2 + d_1 x^3 & [0, 1] \\ a_2 + b_2(x-1) + c_2(x-1)^2 + d_2(x-1)^3 & [1, 2] \\ a_3 + b_3(x-2) + c_3(x-2)^2 + d_3(x-2)^3 & [2, 3] \end{cases} \quad n=4$$

$(0, 1), (1, 2), (2, 2), (3, 0)$

a)  $a_1=1, a_2=2, a_3=2$

$$a_1 + b_1 + c_1 + d_1 = 2, \quad a_2 + b_2 + c_2 + d_2 = 2, \quad a_3 + b_3 + c_3 + d_3 = 0$$

$$b_1 + 2c_1 + 3d_1 = b_2, \quad b_2 + 2c_2 + 3d_2 = b_3$$

b) natural spline

$$2c_1 + 6d_1 = 2c_2, \quad 2c_2 + 6d_2 = 2c_3$$

c) clamped

$$b_2 = 1, \quad b_3 = -2$$

d) natural end point conditions

$$2c_1 = 2c_3 + 6d_3$$

clamped end point conditions

$$b_1 = b_3 + 2c_3 + 3d_3$$

not-a-knot end point condition

$$6d_1 = 6d_2 = 6d_3$$

4) Find and plot the not-a-knot cub. spline

that interpolates a)  $(0,3), (1,5), (2,4), (3,1)$

$$S(x) = \begin{cases} 3 + \frac{23}{6}x - 2x^2 + \frac{1}{6}x^3 & [0,1] \\ 5 + \frac{1}{3}(x-1) - \frac{3}{2}(x-1)^2 + \frac{1}{6}x^3 & [1,2] \\ 4 - \frac{13}{6}(x-2) - (x-2)^2 + \frac{1}{6}(x-2)^3 & [2,3] \end{cases}$$

5. see notebook.