

Homework I Part I

1. Convert decimal number 0.15 into binary and write it in binary scientific notation

$$0.15 \times 2 = 0.3 + 0$$

$$0.3 \times 2 = 0.6 + 0$$

$$0.6 \times 2 = 0.2 + 1$$

$$0.2 \times 2 = 0.4 + 0$$

$$0.4 \times 2 = 0.8 + 0$$

$$0.8 \times 2 = 0.6 + 1$$

$$0.6 \times 2 = 0.2 + 1$$

$$0.2 \times 2 = 0.4 + 0$$

$$0.4 \times 2 = 0.8 + 0$$

$$0.8 \times 2 = 0.6 + 1$$

$$0.6 \times 2 = 0.2 + 1$$

$$0.4 \times 2 = 0.8 + 0$$

$$0.8 \times 2 = 0.6 + 0$$

⋮

⋮

$$0.15 = 0.0010011$$
$$= (1.0011)_2 \times (10)^{(-11)}_2$$

2. Convert binary number to decimal

$$101.011_2$$

$$101_2 = (2^2 \times 1) + (2^1 \times 0) + (2^0 \times 1) = 5_{10}$$

$$0.011_2 = \frac{011_2}{111_2} = \frac{3_{10}}{7_{10}} = 0.428571429$$

$$101.011_2 = 5.428571429$$

3. Write decimal 100 in IEEE 754 double precision standard

| | |
|-------|--------|
| 100/2 | 50 R 0 |
| 50/2 | 25 R 0 |
| 25/2 | 12 R 1 |
| 12/2 | 6 R 0 |
| 6/2 | 3 R 0 |
| 3/2 | 1 R 1 |
| 1/2 | 0 R 1 |

$$\begin{aligned} 100_{10} &= 1100100_2 \\ &= 1.100100 \times 2^6 \\ &= 1.100100 \times 2^{1029-1023} \\ &= (-1)^0 (1.100100) \times 2^{1029-1023} \end{aligned}$$

| | |
|--------|---------|
| 1029/2 | 514 R 1 |
| 514/2 | 257 R 0 |
| 257/2 | 128 R 1 |
| 128/2 | 64 R 0 |
| 64/2 | 32 R 0 |
| 32/2 | 16 R 0 |
| 16/2 | 8 R 0 |
| 8/2 | 4 R 0 |
| 4/2 | 2 R 0 |
| 2/2 | 1 R 0 |
| 1/2 | 0 R 1 |

$$1029_{10} = 100000000101_2$$

$$100_{10} = 0 \mid 100000000101 \mid 100100 \dots 0$$

4. What is the result of the following operations in IEEE 754

$$2^{-51} + 10 + 2^{-51} - 10,$$

$$= 2^{-51} + 2^{-51} = \frac{1}{2^{51}} + \frac{1}{2^{51}} = \frac{2}{2^{51}} = \frac{1}{2^{50}} = 2^{-50}$$

5. Consider $\pi \approx \frac{355}{113}$. Find relative error. How many significant figures of π does this approximation contain?

$$RE = \frac{\left| \frac{355}{113} - \pi \right|}{\left| \frac{355}{113} \right|} = \left| \frac{355}{113\pi} - 1 \right| = 8.49137e-8$$

$\frac{355}{113}$ contains 7 significant figures of π .