Homework 3.

1. How many roots are there for $f(x) = \cos x - 0.1x$? Write an interval for each unique root.

There are seven roots for $f(x) = \cos x - 0.1x$.

$[-10, -9]$, $[-9, -8]$, $[-5, -4]$, $[-2, -1]$, $[1, 2]$, $[5, 6]$, $[7, 8]$

2. For $g(x) = \sqrt[3]{x^3 + x - 2} = x$

a) find the fixed point analytically

b) does the iteration method converge

a) $(x^3 + x - 2)^{1/3} = x \rightarrow x^3 + x - 2 = x^3 \rightarrow x - 2 = 0$

$x = 2$

b) $|g'(x)| = \left|\frac{1}{3}(x^3 + x - 2)^{-2/3}(3x^2 + 1)\right|$

$|g'(2)| = \left|\frac{1}{3}(2^3 + 2 - 2)^{-2/3}(3(2^2) + 1)\right|$

$= \left|\frac{1}{3}\left(\frac{1}{4}\right)(13)\right| = \left|\frac{13}{12}\right| > 1$

The FPI does not converge.

3. Do two steps of Newtons method for $f(x) = -x^3 + x + 6$ starting at $x_0 = 1$

$P_1 = 1 - \frac{f(1)}{f'(1)} \rightarrow P_1 = 1 - \left(\frac{6}{-2}\right) \rightarrow P_1 = 1 + 3 \rightarrow \underline{P_1 = 4}$

$P_2 = 4 - \frac{f(4)}{f'(4)} \rightarrow P_2 = 4 - \left[\frac{-54}{-47}\right] \rightarrow P_2 = 4 - \frac{54}{47} \rightarrow P_2 = \frac{188}{47} - \frac{54}{47} \rightarrow$

$\underline{P_2 = \frac{134}{47}}$

I could not figure out how to use the equation from Liu to iteratively analyize the error.