Homework 6

1. Calculate $P_3(x)$ for $(-3,3),(-1,5),(1,-9),(3,9)$

a) Lagrange

$$P_3(x) = 3\frac{(x+1)(x-1)(x-3)}{(-3+1)(-3-1)(-3-3)} + (5)\frac{(x+3)(x-1)(x-3)}{(-1+3)(-1-1)(-1-3)}$$
$$+(-9)\frac{(x+3)(x+1)(x-3)}{(1+3)(1+1)(1-3)} + (9)\frac{(x+3)(x+1)(x-1)}{(3+3)(3+1)(3-1)}$$
$$= \frac{3}{(-2)(-4)(-6)}(x+1)(x-1)(x-3) + \frac{5}{(2)(-2)(-4)}(x+3)(x-1)(x-3)$$
$$+ \frac{-9}{(4)(2)(-2)}(x+3)(x+1)(x-3) + \frac{9}{(6)(4)(2)}(x+3)(x+1)(x-1)$$
$$= \frac{-1}{16}(x+1)(x-1)(x-3) + \frac{5}{16}(x+3)(x-1)(x-3)$$
$$+ \frac{9}{16}(x+3)(x+1)(x-3) + \frac{3}{16}(x+3)(x+1)(x-1)$$

$(x^2-1)(x-3) = x^3-3x^2-x+3$ $\quad(x^2-9)(x-1) = x^3-x^2-9x+9$

$(x^2-9)(x+1) = x^3+x^2-9x-9$ $\quad(x^2-1)(x+3) = x^3+3x^2-x-3$

$$P_3(x) = \frac{-1}{16}(x^3-3x^2-x+3) + \frac{5}{16}(x^3-x^2-9x+9)$$
$$+ \frac{9}{16}(x^3+x^2-9x-9) + \frac{3}{16}(x^3+3x^2-x-3)$$
$$\left(\frac{-1+5+9+3}{16}\right)x^3 + \left(\frac{3-5+9+9}{16}\right)x^2 + \left(\frac{1-45-81-3}{16}\right)x + \left(\frac{-3+45-81-9}{16}\right)$$

$$P_3(x) = x^3 + x^2 - 8x - 3$$

b) Newtons DD

$$P_3(x) = f[-3] + f[-3\ -1](x+3)$$
$$+ f[-3\ -1\ 1](x+3)(x+1)$$
$$+ f[-3\ -1\ 1\ 3](x+3)(x+1)(-1)$$

c) power form

$$P_3(x) = x^3 + x^2 - 8x - 3$$

4. Given $(1,0), (2, \ln 2), (4, \ln 4)$

a) $P_2(x) = 0 + \ln 2 \frac{(x-1)(x-4)}{(2-1)(2-4)} + \ln 4 \frac{(x-1)(x-2)}{(4-1)(4-2)}$

$= \ln 2 \frac{x^2-5x+4}{-2} + \ln 4 \frac{x^2-3x+2}{6}$

$= \frac{-1}{2} \ln 2 (x^2-5x+4) + \frac{1}{6} \ln 4 (x^2-3x+2)$

$= \frac{-1}{2} \ln 2 (x^2) + \frac{5}{2} \ln 2 (x) - 2 \ln 2$

$+ \frac{1}{6} \ln 4 (x^2) - \frac{1}{2} \ln 4 (x) + \frac{1}{3} \ln 4$

$= \ln(2^{-1/2}) x^2 + \ln(2^{5/2}) x + \ln(2^{-2}) + \ln(4^{1/6}) x^2 + \ln(4^{-1/2}) + \ln(4^{1/3})$

$P_2(x) = (\ln(2^{-1/2}) + \ln(4^{1/6})) x^2 + (\ln(2^{5/2}) + \ln(4^{-1/2})) x + (\ln 2^{-2} + \ln 4^{1/3})$

$= [\ln(2^{-1/2})(2^{1/3})] x^2 + [\ln(2^{5/2})(2^{-1})] x + [\ln(2^{-2})(2^{2/3})]$

$P_2(x) = \ln 2^{-1/6} x^2 + \ln 2^{3/2} x + \ln 2^{-4/3}$

b) $P_2(3) = \ln 2^{-1/6}(9) + \ln 2^{3/2}(3) + \ln 2^{-4/3}$

$= \ln(2^{-3/2}) + \ln(2^{9/2}) + \ln(2^{-4/3})$

$= \ln(2^{-3/2})(2^{9/2}) + \ln(2^{-4/3})$

$= \ln(2^{6/2}) + \ln(2^{-4/3}) = \ln(2^{9/3})(2^{-4/3})$

$P_2(3) = \ln 2^{5/3} = \frac{5}{3} \ln 2$

c) Find an error bound

$f(x) = \ln x$
$f'(x) = \frac{1}{x}$
$f''(x) = \frac{-1}{x^2}$
$f^{(3)}(x) = \frac{2}{x^3}$

$f(x) - P(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{n!} f^{(n)}(c)$

$\ln x - P(x) = \frac{(x-1)(x-2)(x-4)}{6} f^{(3)}(c)$

$1 < c < 4 \quad |f^{(3)}| \leq 2$

$|\ln x - P(x)| \leq \frac{|(x-1)(x-2)(x-4)|}{6} |2|$

$|\ln(3) - P(3)| \leq \frac{|(3-1)(3-2)(3-4)|}{6} |2| = \frac{1}{3} |(2)(1)(-1)|$

$|\ln(3) - P(3)| \leq \frac{2}{3}$

d) $|\ln(3) - P(3)| = |\ln 3 - \ln 2^{5/3}| = \ln \frac{3}{2^{5/3}}$

$= |-0.05663\ldots| = 0.05663\ldots \leq \frac{2}{3}$