Homework 4

1. $3a + 12b + 1c = 2.36$

$12a + \quad 2c = 5.26$

$2b + 3c = 2.77$

$$\left[\begin{array}{ccc|c} 3 & 12 & 1 & 2.36 \\ 12 & 0 & 2 & 5.26 \\ 0 & 2 & 3 & 2.77 \end{array}\right]$$

$R_2: -4R_1 + R_2$

$$\begin{array}{cccc} -12 & -48 & -4 & -9.44 \\ 12 & 0 & 2 & 5.26 \\ \hline 0 & -48 & -2 & -4.18 \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 12 & 1 & 2.36 \\ 0 & -48 & -2 & -4.18 \\ 0 & 2 & 3 & 2.77 \end{array}\right]$$

$R_3: R_2 + 24R_3$

$$\begin{array}{cccc} 0 & -48 & -2 & -4.18 \\ 0 & 48 & 72 & 66.48 \\ \hline 0 & 0 & 70 & 62.30 \end{array}$$

$R_3: \frac{1}{70}R_3$

$$\left[\begin{array}{ccc|c} 3 & 12 & 1 & 2.36 \\ 0 & -48 & -2 & -4.18 \\ 0 & 0 & 1 & 0.89 \end{array}\right]$$

$R_2: R_2 + 2R_3$

$$\begin{array}{cccc} 0 & -48 & -2 & -4.18 \\ 0 & 0 & 2 & 1.78 \\ \hline 0 & -48 & 0 & -2.40 \end{array}$$

$R_2: \frac{-1}{48}R_2$

$$\left[\begin{array}{ccc|c} 3 & 12 & 1 & 2.36 \\ 0 & 1 & 0 & 0.05 \\ 0 & 0 & 1 & 0.89 \end{array}\right]$$

$R_1: R_1 - R_3$

$$\begin{array}{cccc} 3 & 12 & 1 & 2.36 \\ 0 & 0 & -1 & -0.89 \\ \hline 3 & 12 & 0 & 1.47 \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 12 & 0 & 1.47 \\ 0 & 1 & 0 & 0.05 \\ 0 & 0 & 1 & 0.89 \end{array}\right]$$

$R_1: R_1 - 12R_2$

1. cont

$$\left[\begin{array}{cccc} 3 & 12 & 0 & 1.47 \\ 0 & 1 & 0 & 0.05 \\ 0 & 0 & 1 & 0.89 \end{array}\right] \quad R_1 : R_1 - 12R_2 \quad \begin{array}{cccc} 3 & 12 & 0 & 1.47 \\ 0 & -12 & 0 & -0.60 \\ \hline 3 & 0 & 0 & 0.87 \end{array}$$

$R_1 : \frac{1}{3}R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.29 \\ 0 & 1 & 0 & 0.05 \\ 0 & 0 & 1 & 0.89 \end{array}\right] \quad a = 0.29 \quad b = 0.05 \quad c = 0.89$$

2. Computer completes back substitution for $n = 5000$ system of equations in 0.005 sec. Estimate time to complete Gaussian Elimination for the $n = 5000$ system of equations.

$$\frac{(5000)^2}{0.005} \frac{\text{op}}{\text{sec}} = (5000)^2(200) \frac{\text{op}}{\text{sec}}$$

$$\frac{\frac{2}{3}(5000)^3}{(5000)^2(200)} = \frac{\frac{2}{3}(5000)}{200} = \frac{10000}{600} \approx 17 \text{ seconds}$$

3. Given the 1000×1000 matrix, your computer can solve the 500 problems $Ax = b_1, \ldots, Ax = b_{500}$ in exactly one min using $A = LU$ factorization. How much time was the computer working on the $A = LU$ factorization.

$$\frac{\frac{2}{3}(1000)^3 + 2(500)(1000)^2}{60 \text{ sec}} \frac{\text{op}}{}$$

$$= \frac{\frac{2}{3}(1000)^3 + (1000)^3}{} = (1000)^3\left(\frac{5}{3}\right)$$

$$\frac{\frac{2}{3}(1000)^3 \, 60 \frac{\text{op}}{1}}{\frac{5}{3}(1000)^3 \frac{\text{op}}{5 \text{ sec}}} = \frac{120}{3} \cdot \frac{3}{5} = 24 \text{ sec}$$