Homework 2

1. Rewrite function to avoid the cancellation of leading digits.

$$f(x) = 1 - \cos x \left(\frac{1+\cos x}{1+\cos x}\right) = \frac{1-\cos^2 x}{1+\cos x} = \frac{\sin^2 x}{1+\cos x}$$

$$f_1(x) = \frac{\sin^2 x}{1+\cos x}$$

2. $g(x) = (1+x^2)^{1/2} - 1 \left(\frac{(1+x^2)^{1/2}+1}{(1+x^2)^{1/2}+1}\right) = \frac{(1+x^2)-1}{(1+x^2)^{1/2}+1}$

$$g_1(x) = \frac{x^2}{(1+x^2)^{1/2}+1}$$

3. Use Taylor polynomial to approx. the numerator and denominator of

$$r(x) = \frac{1-\cos x}{(1+x^2)^{1/2}-1} \qquad r(x) \approx \frac{P(x)}{Q(x)}$$

$$P(x) = 0 + \frac{0}{1}x + \frac{1}{2}x^2 + \frac{0}{6}x^3 - \frac{1}{24}x^4 = \frac{1}{2}x^2 - \frac{1}{24}x^4 + \cdots$$

$$Q(x) = 0 + \frac{0}{1}x + \frac{1}{2}x^2 + \frac{0}{6}x^3 - \frac{3}{24}x^4 = \frac{1}{2}x^2 - \frac{1}{8}x^4 + \cdots$$

$$r(x) \approx \frac{P(x)}{Q(x)} = \frac{\frac{x^2}{2} - \frac{x^4}{24}}{\frac{x^2}{2} - \frac{x^4}{8}} = \frac{1 - \frac{x^2}{12}}{1 - \frac{x^2}{4}} = \frac{12 - x^2}{12 - 3x^2}$$

$$r(x) \approx \frac{12 - x^2}{12 - 3x^2}$$