

Homework 5

1. $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix}$

a) 1-norm

$$\|A\|_1 = \max(1+|-1|+2, 2+2+0, 3+3+2) \\ = \max(4, 4, 8) = 8$$

b) ∞ -norm

$$\|A\|_\infty = \max(1+2+3, |-1|+2+3, 2+0+2) \\ = \max(6, 6, 4) = 6$$

c) Frobenius norm

$$\|A\|_F = (\langle A, A \rangle_F)^{1/2} \\ = (1+4+9+1+4+9+4+0+4)^{1/2} \\ = (36)^{1/2} = 6$$

2. Find the condition number using the infinity norm

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A|I = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix} \quad \begin{array}{r} -3 \quad -6 \quad -3 \quad 0 \\ + (3 \quad 4 \quad 0 \quad 1) \\ \hline 0 \quad -2 \quad -3 \quad 1 \end{array}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \quad \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{bmatrix} \quad \begin{array}{r} 1 \quad 2 \quad 1 \quad 0 \\ 0 \quad -2 \quad -3 \quad 1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{bmatrix} \quad \begin{array}{r} 1 \quad 2 \quad 1 \quad 0 \\ 0 \quad -2 \quad -3 \quad 1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \quad \begin{array}{r} 1 \quad 0 \quad -2 \quad 1 \end{array}$$

$$= I|A^{-1}$$

$$\|A\|_\infty = \max(1+2, 3+4) = 7$$

$$\|A^{-1}\|_\infty = \max(|-2|+1, |\frac{3}{2}|+|-\frac{1}{2}|) = 3$$

$$\text{cond}(A) = \|A\|_\infty \|A^{-1}\|_\infty = 7 \cdot 3 = 21$$

$$b) \vec{X}_a = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$FE = \|\vec{X} - \vec{X}_a\|_{\infty} = \left\| \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\|_{\infty} = \left\| \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\|_{\infty} = \max(|-2|, |1|) = 2$$

$$BE = \|\vec{b} - A\vec{X}_a\|_{\infty} = \left\| \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 4.01 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\|_{\infty} = \left\| \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 2.01 \end{bmatrix} \right\|_{\infty} = \left\| \begin{bmatrix} 0 \\ -0.01 \end{bmatrix} \right\|_{\infty} = \max(0, |-0.01|) = 0.01$$

$$EMF = \frac{RFE}{RBE} = \frac{\frac{2}{1}}{\frac{0.01}{2}} = \frac{2}{1} \cdot \frac{2}{0.01} = 400$$

$$c) \vec{X}_a = \begin{bmatrix} 2 \\ -\frac{1}{2} \end{bmatrix}$$

$$FE = \|\vec{X} - \vec{X}_a\|_{\infty} = \left\| \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ -\frac{1}{2} \end{bmatrix} \right\|_{\infty} = \left\| \begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix} \right\|_{\infty} = \max(|-1|, |\frac{1}{2}|) = 1$$

$$BE = \|\vec{b} - A\vec{X}_a\|_{\infty} = \left\| \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 4.01 \end{bmatrix} \begin{bmatrix} 2 \\ -\frac{1}{2} \end{bmatrix} \right\|_{\infty} = \left\| \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1.995 \end{bmatrix} \right\|_{\infty} = \left\| \begin{bmatrix} 0 \\ 0.005 \end{bmatrix} \right\|_{\infty} = \max(0, 0.005) = 0.005$$

$$EMF = \frac{RFE}{RBE} = \frac{\frac{1}{1}}{\frac{0.005}{2}} = \frac{1}{1} \cdot \frac{2}{0.005} = \frac{2}{0.005} = 400$$

4. Put the rows of a 5×5 matrix in the following order $(5, 4, 1, 3, 2)$ with a matrix P .

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$