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CS704
Problem Set 3

**2.a** Here is a version written in C++:

```
void DotProduct_gen(vector<int> x)
{
    printf("int DotProduct_sp(vector<int> y) { return ");
    if(x.size() == 0)
        printf("0");
    for(int i=0; i<x.size(); i++)
    {
        if(i > 0)
            printf("+");
        printf("%d*y[%d]", x[i], i);
    }
    printf("; }");
}
```

**2.b** Based on the principle that we are given more information sooner, it ought to be possible to apply greater human insight and domain specific knowledge to the optimization problem.

**2.c**

$$||[cogen]||p =_{df} ||[pe]||[pe, pe, p] = ||[pe]||[pe, p] = ||[pe]||p = p - gen$$

**3** The time complexity for multiplying an $m \times n$ matrix by a $n$ dimensional row-vector is $O(mn)$ (which is clearly optimal). So, the complexity of the unevaluated algorithm is $O(mn + np)$, which when applied to $k$ vectors gives the cost:

$$T_{uneval}(m, n, p, k) = O(k(mn + np))$$

The time complexity of evaluating the symbolic composition depends on the precise value of, $\omega$, the matrix multiplication exponent. It is true that for multiplying two $n \times n$ matrices, $2 \leq \omega \leq \log_2(5)$. It is conjectured that this should carry through to the multiplication of a pair of $m \times n$, $n \times p$ matrices giving the asymptotically optimal cost of multiplication, $T(m, n, p)$ as:

$$T(m, n, p) = O(mn + np + mn^{\omega-2}p)$$

And so the time complexity of computing the product for $k$ vectors is:

$$T_{symbolic}(m, n, p, k) = O(mn + np + mn^{\omega-2}p + kmp)$$

We now asymptotically compute the bounds on $m, n, p, k$ such that $T_{symbolic} < T_{uneval}$. Cancelling terms, we get the asymptotic expansion:

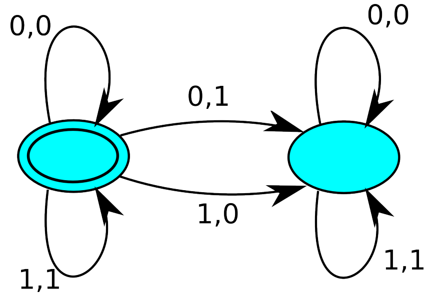
$$k(mn + np - mp) > mn^{\omega-2}p$$

The complete analysis breaks down into 2 cases:

i. $n \in o(m+p)$: in which case it is always true that $T_{uneval} > T_{symbolic}$ (by the rearrangement inequality).

ii. $n \in \Omega(m + p)$: Without loss of generality, assume that $m > p$ (the other case is simply a transposition of $m, p$). Then the leading order term is $kmn$ on the left hand side, and so we must require that $k \in \Omega(pn^{\omega-3})$. Note that if $\omega = 2$, then this is true for all values of $k$.

**4.a**




**4.b** Given a pair of nondeterministic transducers $M = (Q^M, \Sigma, \Delta, \lambda^M, q_0^M), N = (Q^N, \Delta, \Gamma, \lambda^N, q_0^N)$; we give a naïve construction to obtain a new transduver $P = (Q^P, \Sigma, \Gamma, \lambda^P, q_0^P)$ which is the composition of $M, N$. To do this, pick:

$$\begin{aligned} Q^P &= Q^M \otimes Q^N \\ q_0^P &= q_0^M \otimes q_0^N \\ \lambda^P &= \{(s_0 \otimes t_0, \sigma, \gamma, s_1 \otimes t_1))|(s_0, \sigma, \delta, s_1) \in \lambda^M, (t_0, \delta, \gamma, t_1) \in \lambda^N\} \end{aligned}$$

To prove that $P$ is the composition of $M, N$, we show a bisimulation between the labelled transition systems $P$ and $N \circ M$. Indeed the states in $N \circ M$ are naturally isomorphic to $Q^M \otimes Q^N = Q^P$. Moreover, for any transition in $P$, it is obvious that there is a pair of transitions in $\lambda^M \otimes \lambda^N$ and so $P$ is properly simulated by $N \circ M$. So, all that remains is to show that the valid transitions in $N \circ M$ are simulated in $\lambda^P$. But, this must be the case since for all transitions $(s_0, \sigma, \delta, s_1) \in \lambda^M$, only transitions of the form $(t_0, \delta, \gamma, t_1) \in \lambda^N$ may occur; and so $\lambda^P \cong \lambda^{N \circ M}$.

**4.c**