Spectral Rigid Body Dynamics

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Overview

Rigid Body Dynamics

Limiting case of continuum dynamics where elastic modulus is infinite.

Pros:

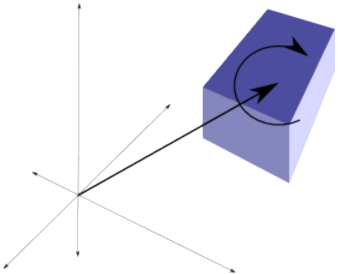
- Pretty accurate at human scales
- Good for materials which are stiff
- Efficient kinematic constraints (good for mechanism design)

Cons:

- Inaccurate at extremely small or large scales
- Bad for materials with low elastic modulus
- Not always solvable (See: Painleve's paradox)

Configuration Space of a Rigid Body

Must be a Euclidean isometry



Identified with translation + rotation, (ie $SE(d) \cong SO(d) \ltimes \mathbb{R}^d$)
Tangent space is isomorphic to $\mathfrak{so}(d+1)$ ($d+1 \times d+1$ skewsymmetric matrix)
Motion of a rigid body given by a curve, $q(t)$, in $SE(d)$. $\dot{q}(t)$ is the tangent curve.

Lagrangian Mechanics

Turns physics into an optimization problem.
For each path, $q : \mathbb{R} \to SE(d)$, define a functional

$$\mathcal{L}(q) = T(q) - U(q)$$

Where $T(q)$ is the total kinetic energy along q and U is the potential energy.
$\mathcal{L}$ measures the work done along q
Physical trajectories correspond to paths of minimal work.

Rigid Motions in 2D

For the sake of concreteness, let $d = 2$
Elements of $SE(2)$ are 3×3 matrices, parameterized by (θ, x, y):

$$(\theta, x, y) = \begin{pmatrix} \cos(\theta) & \sin(\theta) & x \\ -\sin(\theta) & \cos(\theta) & y \\ 0 & 0 & 1 \end{pmatrix}$$