1 To do this, we introduce the following preconditions and postconditions on the fragment, C

$$\frac{}{\{x = a, y = b\}C\{x = b, y = a\}}$$

We now apply the assignment, consequence and composition axiom schemas several times to verify the expression:

$$\frac{\frac{}{\{x = a, y = b\}\, x = x \oplus y; y = x \oplus y; \{x = a \oplus b, y = a\}}, \frac{}{\{x = a \oplus b, y = a\}\, x = y \oplus x; \{x = b, y = a\}}}{\{x = a, y = b\}\, x = x \oplus y; y = x \oplus y; x = x \oplus y; \{x = b, y = a\}}$$

$$\frac{\frac{}{\{x = a, y = b\}\, x = x \oplus b\, \{x = a \oplus b, y = b\}}, \frac{}{\{x = a \oplus b, y = b\}\, y = x \oplus y\, \{x = a \oplus b, y = a\}}}{\{x = a, y = b\}\, x = x \oplus y; y = x \oplus y; \{x = a \oplus b, y = a\}},$$

(Note that $\oplus$ is used in place of $\wedge$ to denote exclusive-or to remove ambiguity.)

2 Using the same pre/post conditions from problem 1, we proceed with the modified assignment axiom schema:

$$\frac{}{\{x = a, y = b\}\, t = x; x = y; y = t; \{x = b, y = a\}}$$

Working from the left, we get the following:

$$\frac{\{x = a, y = b\}\, t = x; \{t = a, x = a, y = b\}\, , \{t = a, x = a, y = b\}\, x = y; y = t; \{x = b, y = a\}}{\{x = a, y = b\}\, t = x; x = y; y = t; \{x = b, y = a\}}$$

Taking the toprightmost expression, we further refine our derivation:

$$\frac{\{t = a, x = a, y = b\}\, x = y; \{t = a, x = b, y = t\}\, , \{t = a, x = b, y = t\}\, y = t; \{x = b, y = a\}}{\{t = a, x = a, y = b\}\, x = y; y = t; \{x = b, y = a\}}$$

And to complete the derivation, we observe that:

$$\frac{\{t = a, x = b, y = t\}\, y = t; \{t = a, x = b, y = a\}\, , (t = a, y = a) \implies y = a}{\{t = a, x = b, y = t\}\, y = t; \{x = b, y = a\}}$$

And so the segment correctly implements swap.

3 To prove the correctness of this segment, we introduce the following loop invariant:

$$\{x = y * q + r, 0 \le r\}\, \text{while } r \ge y \text{ do } r = r - y; q = q + 1; \text{ od}\, \{x = yq + r, 0 \le r, r < y\}$$

Applying the while axiom

$$\frac{\frac{\frac{}{\{x = yq + r, 0 \le r, r \ge y\}\, r = r - y; \{x = y, (q + 1)+, 0 \le r\}}, \frac{}{\{x = y(q + 1) + r, 0 \le r, r < y\}\, q = q + 1; \{x = yq + r, 0 \le}}{\{x = yq + r, 0 \le r, r \ge y\}\, r = r - y; q = q + 1; \{x = yq + r, 0 \le r\}}}{\{x = yq + r, 0 \le r\}\, \text{while } r \ge y \text{ do } r = r - y; q = q + 1; \text{ od}\, \{x = yq + r, 0 \le r, r < y\}}$$

Which we complete using the following:

$$\frac{(x = yq + r \implies x = y(q - 1) + r - y), \{x = y(q - 1) + (r - y), 0 \le r, r \ge y\}\, r = r - y\, \{x = y(q - 1) + r, 0 \le r, r < y\}}{\{x = yq + r, 0 \le r, r \ge y\}\, r = r - y; \{x = y, (q + 1)+, 0 \le r\}}$$

To finish the proof, we must check the conditions on the intro fragment:

$$\frac{\frac{0 \le x, x = r \implies 0 \le r}{\{0 \le x, 0 < y, r = x\}\, r = x; \{x = r, 0 \le r\}}, \{x = r, 0 \le r\}\, q = 0; \{x = yq + r, 0 \le r\}}{\{0 \le x, 0 < y\}\, r = x; q = 0; \{x = yq + r, 0 \le r\}}$$

And so the code correctly implements swap.

4

a Take the following configuration:

```
px     = &py
py     = 0x100
*0x100 = 0
```

If we execute the code, then we get the following sequence of states:
1:

```
PC    = 1
temp   = &py
px     = &py
py     = 0x100
*0x100 = 0
```

2.

```
PC    = 2
temp   = &py
px     = &py
py     = 0
*0x100 = 0
```

3. Segmentation fault

```
PC    = 2
temp   = &py
px     = &py
py     = 0
*0x100 = 0
```

b Here are my steps; the left column is the line number (from the bottom), the middle column is the Hoare predicate and the right column is the state of the store.

Step	Condition	Store
1	$\left(\left(c_X \neq F_\rho^0\left(F_\rho^0\left(c_{\&py}\right)\right)\right) \vee \left(c_Y \neq F_\rho^0\left(F_\rho^0\left(c_{\&px}\right)\right)\right)\right)$	F_ρ^0
2	$\left(\left(c_X \neq c_{temp}\right) \vee \left(c_Y \neq F_\rho^0\left(F_\rho^0\left(c_{\&px}\right)\right)\right)\right)$	$F_\rho^1 = F_\rho^0\left[F_\rho^0\left(c_{\&py}\right) \mapsto c_{temp}\right]$
3	$\left(\left(c_X \neq c_{temp}\right) \vee \left(c_Y \neq F_\rho^1\left(F_\rho^1\left(c_{\&py}\right)\right)\right)\right)$	$F_\rho^2 = F_\rho^1\left[F_\rho^0\left(c_{\&py}\right) \mapsto c_{temp}, F_\rho^1\left(c_{\&px}\right) \mapsto F_\rho^2\left(F_\rho^2\left(c_{\&py}\right)\right)\right]$
4	$\left(\left(c_X \neq F_\rho^2\left(F_\rho^2\left(c_{\&px}\right)\right)\right) \vee \left(c_Y \neq F_\rho^1\left(F_\rho^1\left(c_{\&py}\right)\right)\right)\right)$	$F_\rho^3 = F_\rho^2\left[F_\rho^0\left(c_{\&py}\right) \mapsto F_\rho^2\left(F_\rho^2\left(c_{\&px}\right)\right), F_\rho^1\left(c_{\&px}\right) \mapsto F_\rho^1\left(F_\rho^1\left(c_{\&py}\right)\right)\right]$
5	$\left(\left(c_X \neq F_\rho^2\left(F_\rho^2\left(c_{\&px}\right)\right)\right) \vee \left(F_\rho^3\left(F_\rho^3\left(c_{\&py}\right)\right) \neq F_\rho^1\left(F_\rho^1\left(c_{\&py}\right)\right)\right)\right)$	F_ρ^3
6	$\left(\left(F_\rho^3\left(F_\rho^3\left(c_{\&px}\right)\right) \neq F_\rho^2\left(F_\rho^2\left(c_{\&px}\right)\right)\right) \vee \left(F_\rho^3\left(F_\rho^3\left(c_{\&py}\right)\right) \neq F_\rho^1\left(F_\rho^1\left(c_{\&py}\right)\right)\right)\right)$	F_ρ^3