1 To do this, we introduce the following preconditions and postconditions on the fragment, C

$$\frac{}{\{x=a, y=b\}C\{x=b, y=a\}}$$

We now apply the assignment, consequence and composition axiom schemas several times to verify the expression:

$$\frac{\dfrac{}{\{x=a, y=b\}\, x = x \oplus y; y = x \oplus y; \{x = a \oplus b, y = a\}}, \dfrac{}{\{x = a \oplus b, y = a\}\, x = y \oplus x; \{x = b, y = a\}}}{\{x=a, y=b\}\, x = x \oplus y; y = x \oplus y; x = x \oplus y; \{x=b, y=a\}}$$

$$\frac{\dfrac{}{\{x=a, y=b\}\, x = x \oplus b\, \{x = a \oplus b, y = b\}}, \dfrac{}{\{x = a \oplus b, y = b\}\, y = x \oplus y\, \{x = a \oplus b, y = a\}}}{\{x=a, y=b\}\, x = x \oplus y; y = x \oplus y; \{x = a \oplus b, y = a\}},$$

(Note that $\oplus$ is used in place of $\wedge$ to denote exclusive-or to remove ambiguity.)

2 Using the same pre/post conditions from problem 1, we proceed with the modified assignment axiom schema:

$$\frac{}{\{x=a, y=b\}\, t = x; x = y; y = t; \{x=b, y=a\}}$$

Working from the left, we get the following:

$$\frac{\{x=a, y=b\}\, t = x; \{t=a, x=a, y=b\}\,, \{t=a, x=a, y=b\}\, x = y; y = t; \{x=b, y=a\}}{\{x=a, y=b\}\, t = x; x = y; y = t; \{x=b, y=a\}}$$

Taking the toprightmost expression, we further refine our derivation:

$$\frac{\{t=a, x=a, y=b\}\, x = y; \{t=a, x=b, y=t\}\,, \{t=a, x=b, y=t\}\, y = t; \{x=b, y=a\}}{\{t=a, x=a, y=b\}\, x = y; y = t; \{x=b, y=a\}}$$

And to complete the derivation, we observe that:

$$\frac{\{t=a, x=b, y=t\}\, y = t; \{t=a, x=b, y=a\}\,, (t=a, y=a) \implies y=a}{\{t=a, x=b, y=t\}\, y = t; \{x=b, y=a\}}$$

And so the segment correctly implements swap.

3 To prove the correctness of this segment, we introduce the following loop invariant:

$$\{x = y * q + r, 0 \le r\}\, \text{while } r \ge y \text{ do } r = r - y; q = q + 1;\ \text{od}\, \{x = yq + r, 0 \le r, r < y\}$$

Applying the while axiom

$$\frac{\dfrac{\dfrac{}{\{x = yq + r, 0 \le r, r \ge y\}\, r = r - y; \{x = y, (q+1)+, 0 \le r\}}, \dfrac{}{\{x = y(q+1) + r, 0 \le r, r < y\}\, q = q + 1; \{x = yq + r,}}{\{x = yq + r, 0 \le r, r \ge y\}\, r = r - y; q = q + 1; \{x = yq + r, 0 \le r\}}}{\{x = yq + r, 0 \le r\}\, \text{while } r \ge y \text{ do } r = r - y; q = q + 1;\ \text{od}\, \{x = yq + r, 0 \le r, r < y\}}$$

Which we complete using the following:

$$\frac{(x = yq + r \implies x = y(q-1) + r - y), \{x = y(q-1) + (r - y), 0 \le r, r \ge y\}\, r = r - y\, \{x = y(q-1) + r, 0 \le r, r < y\}}{\{x = yq + r, 0 \le r, r \ge y\}\, r = r - y; \{x = y, (q+1)+, 0 \le r\}}$$

To finish the proof, we must check the conditions on the intro fragment:

$$\frac{\dfrac{0 \le x, x = r \implies 0 \le r}{\{0 \le x, 0 < y, r = x\}\, r = x; \{x = r, 0 \le r\}}, \{x = r, 0 \le r\}\, q = 0; \{x = yq + r, 0 \le r\}}{\{0 \le x, 0 < y\}\, r = x; q = 0; \{x = yq + r, 0 \le r\}}$$

And so the code correctly implements swap.