Assigned: Thursday Feb. 26, 2010
Due: Tuesday March 18, 2010

MATH/CS 715: HOMEWORK 3
SPRING 2010

PAGE LIMIT: 20 pages (single-sided).

NOTE: Please include a cover page – this will not count toward the total page limit.

NOTE: Don't forget to include MATLAB code where appropriate – this does count toward your total pages.

Chebyshev Spectral Methods

1. Perform a study of the relative efficiencies of `cheb` and `chebfft` as a function of N. Do not count the time taken by `cheb` to form D_N, just the time take to multiply D_N by a vector.

2. Download the MATLAB code for solving the 2D wave equation:

   `http://www.comlab.ox.ac.uk/nick.trefethen/p20.m`

   Modify this program to make use of matrices instead of the FFT. Make sure you do this elegantly, using matrix-matrix multiplications rather than explicit loops. You will find that the code gets much shorter, and faster, too. How much faster is it? How large do we have to make N in order for the FFT to be faster than matrix approach?

3. Write a code `chebfft2` for second-order differentiation by the FFT, and show by examples that it matches the results obtained by matrices, apart from rounding errors.

Korteweg-deVries equation

4. The KdV equation discussed in class is closely related to *Burgers equation*:

$$u_t + \left(\frac{1}{2}u^2\right)_x = \epsilon u_{xx},$$

   where $\epsilon > 0$ is a constant. Download the MATLAB code for solving the KdV equation:

   `http://www.comlab.ox.ac.uk/nick.trefethen/p27.m`

   and modify to solve the Burger's equation on $x \in [-\pi, \pi]$ with $\epsilon = 0.25$. Take $u(x, 0)$ equal to $\sin^2(x)$ in $[-\pi, 0]$ and zero in $[0, \pi]$. Produce plots at times 0, 0.5, 1, 1.5, 2, 2.5, and 3 with a sufficiently small time step for $N = 64, 128$, and 256. How small a value of ϵ can you take without obtaining unphysical oscillations?

5. Another related PDE is the *Kuramoto-Sivashinsky equation*:

$$u_t + (u^2)_x = -u_{xx} - u_{xxxx},$$

whose solutions evolve chaotically. Write a program to solve it with periodic BCs on $[-20, 20]$ for initial data:

$$u(x, 0) = e^{-x^2}.$$

Use RK4 for the time-stepping. Can you get results for $0 \leq t \leq 50$ that you trust?