Spectral Rigid Body Dynamics

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Pros:

- + Pretty accurate at human energy scales
- + Good for stiff materials (ie metals, plastics etc.)
- + Easy kinematic constraints (useful for mechanisms)
- + Standard animation tool (videogames!)

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- $+$ Good for stiff materials (ie metals, plastics etc.)
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Cons:

- - Inaccurate at extremely large energies
- - Bad for materials with low elastic modulus
- - Not always solvable! (See: Painleve's paradox)

What is a Rigid Body?

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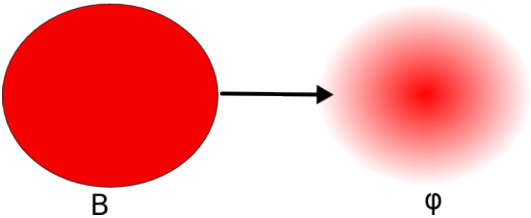
We identify a body B with a scalar field, $\varphi : \mathbb{R}^d \to \mathbb{R}^+$



φ represents the mass distribution of B

$\varphi(x) = 0$ indicates B does not occupy the space at x

W.L.O.G. Assume center of mass at origin:

$$\int_{\mathbb{R}^d} x\varphi(x)dx = 0$$

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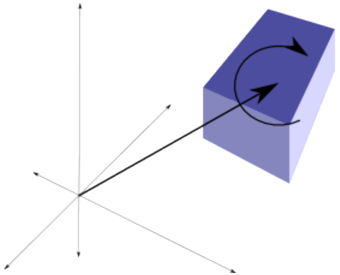

Can be parameterized by a translation t and a rotation R

Matrix: $\begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix}$

$d + \binom{d}{2}$ degrees of freedom

Tangent space: $\mathfrak{so}(d+1)$

Motions of rigid objects $\cong$ **paths** $q(t) \subset SE(d)$

Newton's Equations for Rigid Body Dynamics

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A:High-school physics:

$$\begin{aligned} \frac{dq(t)}{dt} &= \dot{q}(t) \\ M\frac{d\dot{q}(t)}{dt} &= F(t) \end{aligned}$$

$F(t)$ is the force vector and M is the mass matrix for the rigid body:

$$M = \int_{\mathbb{R}^d} \varphi(x)dx$$