Q-1

digits = {1, 2, 3, ..., 9}

5-digit No is to be found.

Atmost 2 times repetation allowed.

Break it into 3 subproblems.

(i) E = All no are unique no repitation allowed.

$\underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \Rightarrow 15,120$

(ii) F = Only one digit can be taken 2 times others are unique

$\_\ \_\ \_\ \_\ \_ \Rightarrow \frac{5C_2 \times 9C_1 \times 2!}{2!} \times 8C_3 \times 3!$

$\Rightarrow \frac{5 \times 4 \times 9 \times 2 \times 8 \times 7 \times 6 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 2}$

$\Rightarrow 30,240$

Q-1 Continued

(iii) G = Only 2 digit can be ~~rep~~ taken 2 times.

$$\frac{{}^9C_2 \times {}^5C_4 \times {}^1C_1 \times 4! \times {}^7C_1}{2! \times 2!}$$

$$= \frac{9 \times 8 \times 5 \times 1 \times \overset{2}{\cancel{4}} \times 3 \times \cancel{2} \times 7}{\cancel{2} \times \cancel{2} \times \cancel{2}}$$

$$= 9 \times 8 \times 5 \times \cancel{2} \times 3 \times 7$$

$$= 7560$$

no of ways = $n(E) + n(F) + n(G)$

$= 15,120 + 30,240 + 7,560$

$= 52,920$

$\boxed{\text{So, 5-digits No's possible} = 52,920}$ //

Q - 2

N = 12 people,

Committes = 3, C1 = 3, C2 = 4, C3 = 5.

Sample space ⇒ distributing the peoples into 3 committes is

$SS = 12!$

for committe 1 ⇒ $^{12}C_3$ ways

for committe 2 ⇒ $^{9}C_4$ ways

for committe 3 ⇒ $^{5}C_5$ ways.

finally it comes to ⇒ $^{12}C_3 \times {}^{9}C_4 \times {}^{5}C_5$

So, total No of divisions possible

$\Rightarrow {}^{12}C_3 \times {}^{9}C_4 \times {}^{5}C_5$

$\Rightarrow \dfrac{12!}{3! \times 4! \times 5!}$

Q-3

$N = 10$

E = Event where french and English are seated togather.

$n(E) = 9! \times 2!$

F = Event where Russian & U.S. are not seating togather.

Now,

$n(E \text{ and } F) = 9! \times 2! - 8! \times 2! \times 2!$

↓ there is subtracted as it show that how many arrangments are there for seating Russian and US togather

$\Rightarrow 725760$
$\quad - 161280$

$\Rightarrow 564480$

The No of seating arrangments $\Rightarrow$ 564480

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Q-4

Red = 1, Blue = 1, Green = 1

Consider, Red = R, Blue = B, Green = G.

(i) With replacement

$SS = \{ (R,R), (R,B), (R,G),$
$(B,R), (B,B), (B,G),$
$(G,R), (G,B), (G,G) \}.$

$n(SS) = 9$

(ii) Without Replacement

$SS = \{ (R,B), (R,G),$
$(B,R), (B,G),$
$(G,R), (G,B) \}.$

$n(SS) = 6.$

Q-5

E = sum of dice is odd

$SS_E = \{(1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6), (4,1), (4,3), (4,5), (5,2), (5,4), (5,6), (6,1), (6,3), (6,5)\}$

$n(E) = 18$

F = Atleast one side is 1.

$SS_F = \{(1,2), (1,3), (1,4), (1,5), (1,6), (1,1), (2,1), (3,1), (4,1), (5,1), (6,1)\}$

$n(F) = 11$

G = sum is 5.

$SS_G = \{(1,4), (4,1), (2,3), (3,2)\}$.

$n(G) = 4$.

Q-5 continued

$n(EF) = \{(1,2), (1,4), (1,6), (2,1), (4,1), (6,1)\}$

$\boxed{n(EF) = 6}$ —— 1

$n(E \cup F) = n(E) + n(F) - n(E \cap F)$
$= 18 + 11 - 6$
$= 23$

$\boxed{n(E \cup F) = 23}$ —— 2

$n(FG) = \{(1,4); (4,1)\}$
$= 2$

$\boxed{n(FG) = 2}$ —— 3

$n(EF^c) = n(E) - n(E \cap F)$
$= 18 - 6$
$= 12.$

$\boxed{n(EF^c) = 12}$ —— 4

$n(EFG) = \{(1,4), (4,1)\} = 2$

$\boxed{n(EFG) = 2}$ —— 5

Q-6

$n = 5$ components

(i)

Sample space $\Rightarrow \underset{C1}{\underline{2}} \times \underset{C2}{\underline{2}} \times \underset{C3}{\underline{2}} \times \underset{C4}{\underline{2}} \times \underset{C5}{\underline{2}}$

$$\boxed{SS = 2^5 = 32} \quad — ①$$

(ii)

E = component 1 and 2 are working

$$\underset{C1}{\underline{1}} \times \underset{C2}{\underline{1}} \times \underset{C3}{\underline{2}} \times \underset{C4}{\underline{2}} \times \underset{C5}{\underline{2}} \Rightarrow 2^3 \Rightarrow 8$$

F = component 3 and 4 are working

$$\underset{C1}{\underline{2}} \times \underset{C2}{\underline{2}} \times \underset{C3}{\underline{1}} \times \underset{C4}{\underline{1}} \times \underset{C5}{\underline{2}} \Rightarrow 2^3 \Rightarrow 8$$

G = component 1, 3, and 5 are workin

$$\underset{C1}{\underline{1}} \times \underset{C2}{\underline{2}} \times \underset{C3}{\underline{1}} \times \underset{C4}{\underline{2}} \times \underset{C5}{\underline{1}} = 2^2 = 4.$$

Q-6 Continued

E and F $\Rightarrow$ 1, 2, 3 and 4 are working

$$\underset{C1}{\underline{1}} \times \underset{C2}{\underline{1}} \times \underset{C3}{\underline{1}} \times \underset{C4}{\underline{1}} \times \underset{C5}{\underline{2}} \Rightarrow 2$$

F and G $\Rightarrow$ 1, 3, 4, 5 are working

$$\underset{C1}{\underline{1}} \times \underset{C2}{\underline{2}} \times \underset{C3}{\underline{1}} \times \underset{C4}{\underline{1}} \times \underset{C5}{\underline{1}} \Rightarrow 2$$

E and G $\Rightarrow$ 1, 2, 3, 5 are working

$$\underset{C1}{\underline{1}} \times \underset{C2}{\underline{1}} \times \underset{C3}{\underline{1}} \times \underset{C4}{\underline{2}} \times \underset{C5}{\underline{1}} \Rightarrow 2$$

E, F and G $\Rightarrow$ All are working

$$\underset{C1}{\underline{1}} \times \underset{C2}{\underline{1}} \times \underset{C3}{\underline{1}} \times \underset{C4}{\underline{1}} \times \underset{C5}{\underline{1}} \Rightarrow 1$$

$$n(E \cup F \cup G) = n(E) + n(F) + n(G) - n(E \cap F) - n(E \cap G) - n(F \cap G) + n(E \cap F \cap G)$$

$$= 8 + 8 + 4 - 2 - 2 - 2 + 1$$

$$= 15$$

$$\boxed{n(E \cup F \cup G) = 15}$$ //.

Q-6 Continued

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(iii)

A = 4, 5 are failed.

$$\frac{2}{C1}\ \frac{2}{C2}\ \frac{2}{C3}\ \frac{0}{C4}\ \frac{0}{C5} \Rightarrow 2^3 \Rightarrow 8$$

$$\boxed{n(A) = 8}$$

Q-7

(i)

Sample space ⇒ {good (coding 0),
good (coding 1),
fair (coding 0), fair (coding 1),
serious (coding 0), serious (coding 1)}

⇒ 8.

(ii) A = patient is serious.

outcomes ⇒ {serious (coding 0),
serious (coding 1)}

(iii) B = patient is uninsured.

outcomes ⇒ {good (coding 0), fair (coding 0), serious (coding 0)}

(iv) $B^c \cup A$ = {good (coding 0), fair (coding 0),
serious (coding 0)} ∪
{serious (coding 0), serious
(coding 1)}

= {good (coding 0), fair (coding 0),
serious (coding 0), serious (coding 1)}

⇒ 4

Q - 8

$p(A) = 0.3$, $p(B) = 0.5$

As A and B are mutually exclusive events

$$p(A \cap B) = 0.$$

(i) $p(A \cup B) = p(A) + p(B) - p(A \cap B)$
$= 0.3 + 0.5 - 0$
$= 0.8$

$$\boxed{p(A \cup B) = 0.8}$$

(ii) $p(A - B) = p(A) - p(A \cap B)$
$= 0.3 - 0 = 0.3$

$$\boxed{p(A - B) = 0.3}$$

(iii) $p(A \cap B) = 0$

As, A and B are mutually exclusive.

Q - 9

n(Neither ring nor necklace) = 60

n(either ring or necklace) = 40

(i) $p(A \cup B) = \frac{n(\text{either ring or necklace})}{\text{Total}}$

$= \frac{40}{100} = 0.4$

$\boxed{p(A \cup B) = 0.4}$

(ii) A = Wearing a ring
B = Wearing a necklace

$p(A \cap B) = p(A) + p(B) - p(A \cup B)$

$= \frac{20}{100} + \frac{30}{100} - 0.4$

$= 0.2 + 0.3 - 0.4$

$= 0.1$

$\boxed{p(A \cap B) = 0.1}$

Q-10

n(professionals) = 312
n(married) = 470
n(college graduates) = 525
n(professionals + college grad) = 42
n(married + professionals) = 86
n(married + college grad) = 147
n(married + college grad + prof) = 25.

~~n(professionals or married or grad~~

Consider, professionals = P, married = M,
college grad = CG.

$$n(P \cup M \cup CG) = n(P) + n(M) + n(CG) - n(P \cap M) - n(P \cap CG) - n(M \cap CG) + n(P \cap CG \cap M)$$
$$= 312 + 470 + 525 - 86 - 42 - 147 + 25$$
$$= 1057$$

As, given is 1000 and $n(P \cup M \cup CG)$ is 1057 hence proved that numbers stored in study are incorrect.

Q-11

~~sample~~

sample space for [5 or 7]

$= \{ (1,4), (2,3), (3,2), (4,1),$
$(1,6), (2,5), (3,4), (4,3), (5,2),$
$(6,1) \}$

$Pr[5 \text{ or } 7] = \frac{10}{36} = \frac{5}{18}$

$Pr[\text{only } 5] = \frac{4}{36} = \frac{1}{9}$

$Pr[\text{only } 7] = \frac{6}{36} = \frac{1}{6}$

$Pr[\text{Neither } 5 \text{ nor } 7] = \frac{26}{36} = \frac{13}{18}.$

Pr[5 comes first]

$= \frac{1}{9} + \frac{13}{18} \times \frac{1}{9} + \left(\frac{13}{18}\right)^2 \times \frac{1}{9}$

$+ \ldots \infty.$

Q-11 continued

$$S_n = \frac{a}{1-r}.$$

$$a = \frac{1}{9}, \quad r = 13/18$$

$$S = \frac{\frac{1}{9}}{1-\frac{13}{18}} = \frac{18}{9 \times 5} = 2/5$$

$$\boxed{Pr[5 \text{ comes first}] = \frac{2}{5}}$$

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Q-12

A = Event where both no on dice are different

$SS_A = \{(1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5)\}$

$= 30$

B' = Atleast one on 6 after A is occured

$SS_B = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5)\}$

$= 10$

$$P(B|A) = \frac{10}{30} = \frac{1}{3}$$

$$\boxed{P(B|A) = 1/3}$$

Q - 13

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Given, $n = 100$.
1^{st} person lost the ticket.

Pr [last person seats on assigned seat] = ?

Lets, say $n = 3$ people are there

Sample space

1	2	3
2	1	3
3	1	2
3	2	1

Pr [last person on assigned seat]
$= 2/4 = 1/2$
$\Rightarrow \boxed{2^{n-2}/2^{n-1}}$ — 1

say $n = 4$ people are there.

1	2	3	4
2	1	3	4
3	1	2	4
4	1	2	3
4	1	3	2
4	2	1	3
3	2	1	4
4	2	3	1

Pr [last person at assigned place]
$\Rightarrow 4/8 = 1/2 \Rightarrow \boxed{2^{n-2}/2^{n-1}}$ — 2

Q-13 Continued

So, using eqn -① & ② we can generalize that for, n people probability of last person to ~~seat~~ sit at assigned seat is $2^{n-2} / 2^{n-1} = 1/2$

So, for $n = 100$

Pr[last person on its assigned place]

$$= \frac{2^{n-2}}{2^{n-1}} = \frac{2^{98}}{2^{99}} = \frac{1}{2}$$

$$\boxed{Pr = 1/2}$$

Q-14

Solⁿ :-

Yes, X is mistaken. Because if he is asking the status of Y then also his probability does not decreases it remains same with 2/3.

Relation With Monty hall problem :-

If, X asks for the Y to the host then there are th possibilities :

X	Y	Z
P	P	F
P	F	P
F	P	P

$$Pr(X_P) = P(Y_P) \cdot P(X_P / Y_P) + P(Y_F) \cdot P(X_P / Y_F)$$

$$= \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} + \frac{1}{3} \times 1$$

$$\boxed{Pr(X_P) = 2/3}$$

Q-14 Continued

So, in the same way in Monty hall problem also, there 2 goats and 1 car are there.

So, $Pr(car) = 2/3$

So, these way it relates to the Monty hall problem.

Q - 15

Total pens = n
Red pens = x

(a) $Pr(Red = 2) = \frac{\binom{x}{2}}{\binom{n}{2}}$

$\Rightarrow \frac{x(x-1)}{n(n-1)} = \frac{1}{2}$

$$\boxed{2x^2 - 2x = n^2 - n} \quad — (1)$$

Now, for above equation n = 4, x = 3 satisfies the condition: equ^n — (1)

By Keeping n = 4, x = 3

$2(9) - 2(3) = 16 - 4$

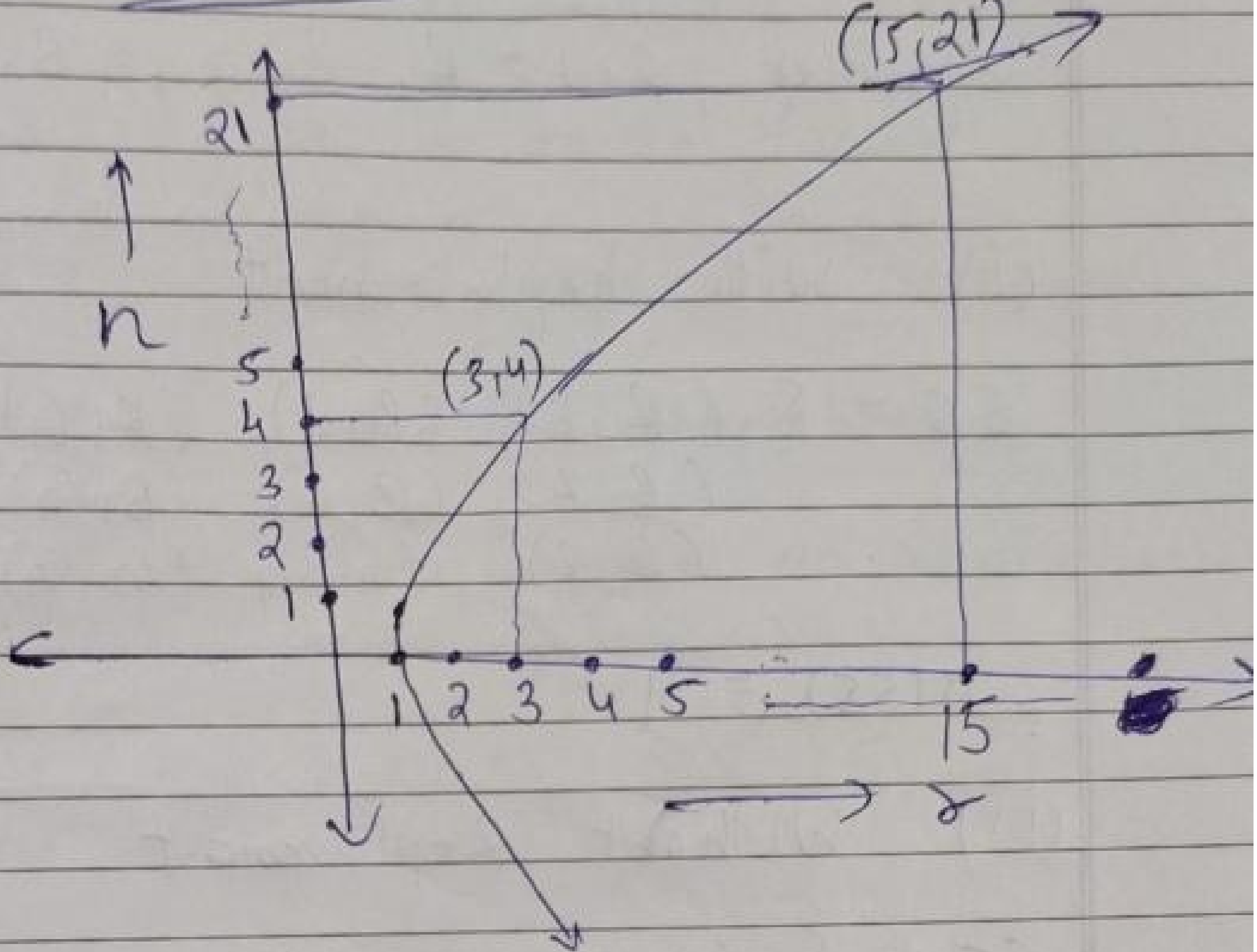
$18 - 6 = 16 - 4$

$\boxed{12 = 12}$

Hence, proved that for n=4, x=3 it satisfies the equ^n.

$\boxed{\text{Minimum no of pens in box} \Rightarrow 4}$ //

Q -15 (b)



So, using above graph we can see that no of red balls = 15, and total n = 21 balls are there.

So, for blue balls $n - r = 21 - 15 = 6$

$\boxed{\text{So, blue balls are} = 6 \quad \text{Red balls are} = 15}$