

Probability and Statistics

Assignment 1 (Due: 18/08)

1 Probability and Counting

1. (3 points) How many 5-digit numbers can be formed from the integers $1, 2, \dots, 9$ if no digit can appear more than twice?
2. (3 points) If 12 people are to be divided into 3 committees of respective sizes 3, 4, and 5, how many divisions are possible?
3. (5 points) Delegates from 10 countries, including Russia, France, England, and the United States, are to be seated in a row. How many different seating arrangements are possible if the French and English delegates are to be seated next to each other and the Russian and U.S. delegates are not to be next to each other?
4. (2 points) A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment that consists of taking 1 marble from the box and then replacing it in the box and drawing a second marble from the box. Describe the sample space. Repeat when the second marble is drawn without replacing the first marble.
5. (2 points) Two dice are thrown. Let E be the event that the sum of the dice is odd, let F be the event that at least one of the dice lands on 1, and let G be the event that the sum is 5. Describe the events EF , $E \cup F$, FG , EF^c , and EFG .
6. (3 points) A system is comprised of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed
 - How many outcomes are in the sample space of this experiment?
 - Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W
 - Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A ?

7. (3 points) A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.
- Give the sample space of this experiment
 - Let A be the event that the patient is in serious condition. Specify the outcomes in A .
 - Let B be the event that the patient is uninsured. Specify the outcomes in B
 - Give all the outcomes in the event $B^c \cup A$
8. (2 points) Suppose that A and B are mutually exclusive events for which $P(A) = .3$ and $P(B) = .5$. What is the probability that
- either A or B occurs?
 - A occurs but B does not?
 - both A and B occur?
9. (2 points) Sixty percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring and 30 percent wear a necklace. If one of the students is chosen randomly, what is the probability that this student is wearing
- a ring or a necklace?
 - a ring and a necklace?
10. (3 points) The following data were given in a study of a group of 1000 subscribers to a certain magazine: In reference to job, marital status, and education, there were 312 professionals, 470 married persons, 525 college graduates, 42 professional college graduates, 147 married college graduates, 86 married professionals, and 25 married professional college graduates. Show that the numbers reported in the study must be incorrect.
11. (2 points) A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a 5 occurs first.
12. (2 points) Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?

13. (5 points) There are hundred people in queue to enter a train. Everyone has ticket with assigned seat. Unfortunately, the first traveler in queue lost his ticket and he decides to take a random seat. Remaining people take their assigned seat if it is unoccupied. If they see their seat occupied, then they too take one random seat. Calculate the probability that the last person sits in the seat assigned to him.
14. (5 points) X , Y , and Z recently appeared for 10th board exam. However, all three were poor students, and they did not have courage to check results themselves. They decided to ask one of their friend to check their result. Their friend after checking result told them that one of them failed but did not mention the name. X thought of asking about Y first thinking he can estimate his probability; X thought that if Y passes then his probability of pass would reduce to $1/2$. So, X decides not to reduce his chances by asking. Is X mistaken in his calculations? How does this relate to Monty-Hall problem?
15. (5 points) A box contains red and blue pens. When two pens are drawn randomly, then it is given that the probability that both are red is $1/2$.
- What could be the minimum number of pens in the box?
 - What could be the minimum number of pens if it is given that number of blue pens is even?