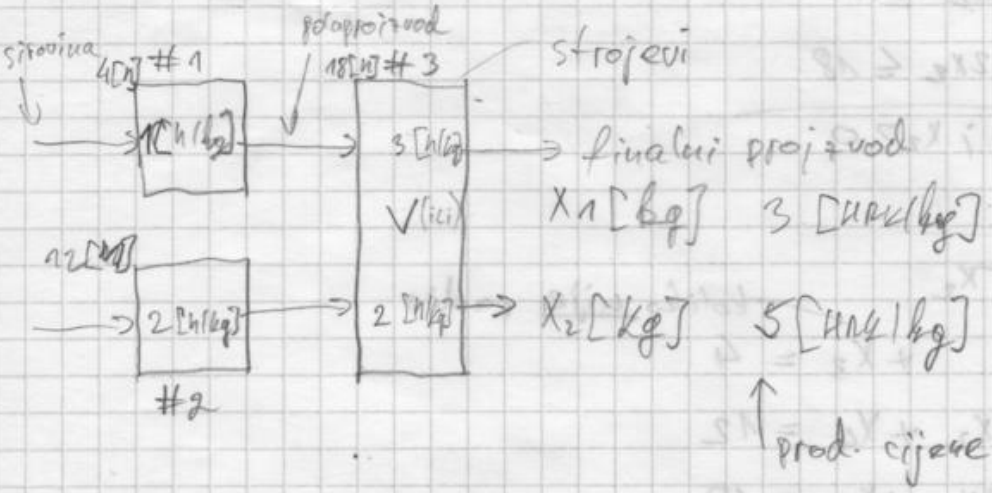
LINEARNO PROGRAMIRANJE 18. 9. 2009.

- LINEAR PROGRAMMING (LP); 1947. Dantzig



kg/h - produktivnost
h/kg - normativ

$\max z = 3[\text{nov/kg}] \cdot x_1[\text{kg}] + 5 \cdot x_2$ ← prihod
ciljna

~~$4[h] \cdot 1[h/kg]$~~

(x_1, x_2) - rješenje

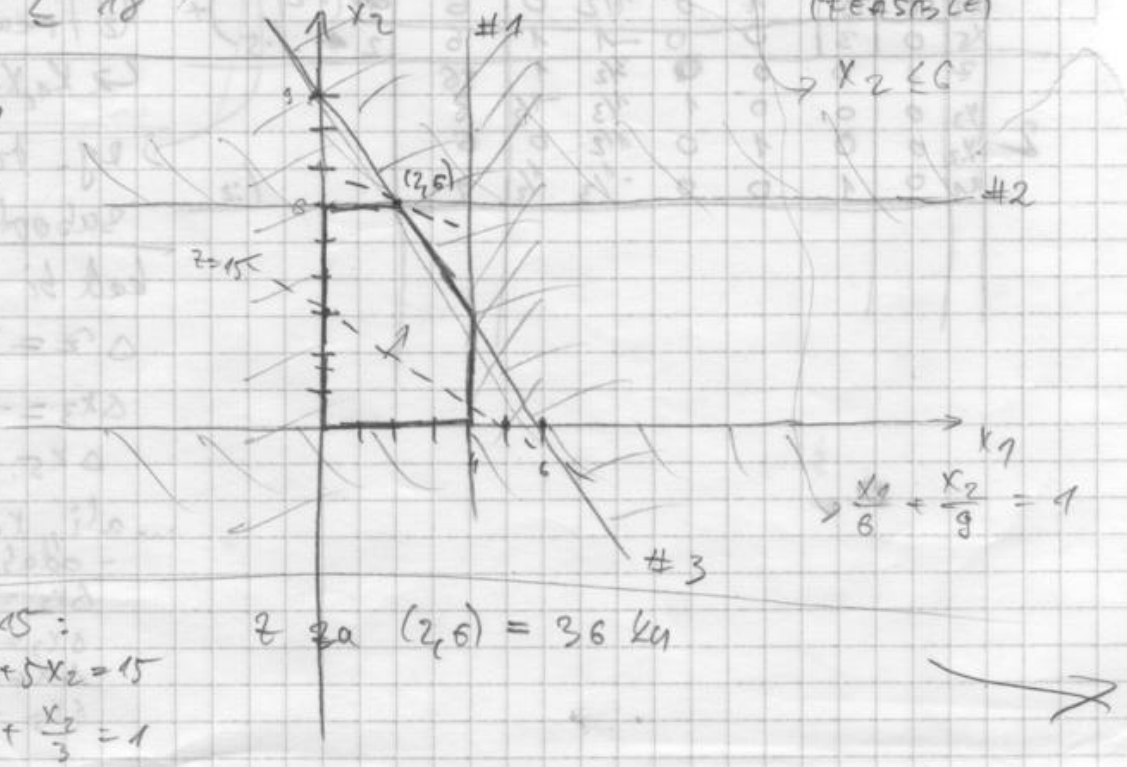
$1[h/kg] \cdot x_1[kg] \leq 4[h]$ → $x_1 \leq 4$

$2[h/kg] \cdot x_2[kg] \leq 12[h]$ = s lijeve strane su moguća prihvatljiva, dopustiva (feasible)

$3x_1 + 2x_2 \leq 18$ → $x_2 \leq 6$

$x_1 \geq 0$

$x_2 \geq 0$



$\frac{x_1}{6} + \frac{x_2}{9} = 1$

upr. $z = 15$:
$3x_1 + 5x_2 = 15$
$\frac{x_1}{5} + \frac{x_2}{3} = 1$

z za (2,6) = 36 nov

$\max \quad z = 3x_1 + 5x_2$

$x_1 \le 4$

$2x_2 \le 12$

$3x_1 + 2x_2 \le 18$

$x_1 \ge 0 \; ; \; x_2 \ge 0$

Kanonski oblik

x_1, x_2 – strukturne varijable

$z = 3x_1 + 5x_2$

$1x_1 + x_3 = 4$ ← koliko nije radio

$2x_2 + x_4 = 12$

$3x_1 + 2x_2 + x_5 = 18$

$x_1 \ge 0 \; ; \; x_2 \ge 0 \; ; \; x_3 \ge 0 \; ; \; x_4 \ge 0 \; ; \; x_5 \ge 0$

baz. var. — 5 nepozn. — (RHS – right hand side)

Iterac.		z	x_1	x_2	x_3	x_4	x_5	Desne strane $= b$	Q
0	z	1	−3	−5	0	0	0	0	
	x_3	0	1	0	1	0	0	4	$\frac{4}{0} = \infty$ ned.
	x_4	0	0	2	0	1	0	12	$\frac{12}{2} = 6$ ←
	x_5	0	3	2	0	0	1	18	$\frac{18}{2} = 9$
1	z	1	−3	0	0	5/2	0	30	
	x_3	0	1	0	1	0	0	4	4
	x_2	0	0	1	0	1/2	0	6	∞ (·−2)
	x_5	0	3	0	0	−1	1	6	2 ← (·5)
2	z	1	0	0	0	3/2	1	36	
	x_3	0	0	0	1	1/3	−1/3	8	
	x_2	0	0	1	0	1/2	0	6	
	x_1	0	1	0	0	−1/3	1/3	2	/:2

$z - 3x_1 - 5x_2 = 0$

SIMPLEKS

– nebazične var. $\equiv \emptyset$

$\binom{5}{2}$ – od 5 var. odabiremo 2 nebaz.

↳ x_1, x_2

Rj. trenutno suboptimalno

kad bi $x_4 = 1$; $x_2 = 0$

$\Delta z = 3$

$\Delta x_3 = -1 \; | \; \Delta x_4 = 0$

$\Delta x_5 = -3$

…ali, x_2 nudi $\Delta z = 5$!

– odabiremo x_2

$\delta x_2 = 1$

$\delta x_3 = 0$

$\Delta x_4 = -2$

$\delta x_5 = -2$

OPEIST

25. 9. 2009.

$$\max / \min \quad z = \sum_{j=1}^{n} c_j x_j$$

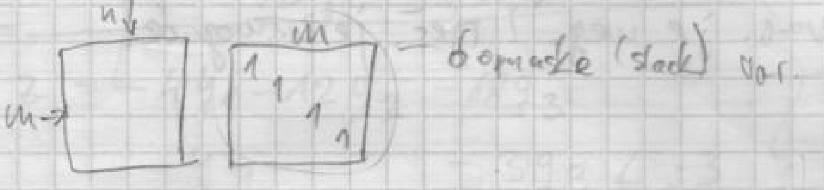
uz

$$\sum a_{ij} x_j \le b_i \qquad i = 1, m$$

$$x_j \ge 0 \qquad j = 1, n$$

n jedn.

m+n nepozn.



— dopunske (slack) var.

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n nebazičnih $\equiv 0$

$$O\left(\binom{m+n}{n}\right)$$

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zad / $\min z = 3x_1 - 4x_2 \rightarrow \max z' = -3x_1 + 4x_2$

$$\max z' = -3x_1 + 4x_2$$

$$2x_1 + x_2 + x_3 = 4$$

$$-3x_1 - 2x_2 + x_4 = -8 \qquad x_1, x_2, x_3, x_4 \ge 0$$

	iter.	baz var.	z	x_1	x_2	x_3	x_4	DESNE STRANE	
Dualni simpleks	0	z'	1	3	−4	0	0	0	nemoguće (infeasible) |:(−3)
		x_3	0	2	1	1	0	4	
		x_4	0	−3	−2	0	1	−8 ←	
	1	z'	1	0	−6	0	1	−8	x_4 se mijenja s x_1 || x_2 s kojom? mora biti neg. pa najmanji kvocijent!
		x_3	0	0	−1/3	1	2/3	−4/3 ←	
		x_1	0	1	2/3	0	−1/3	8/3	|:(−3) |·(−3)
	2	z'	1	0	0	−18	−11	16	moguće
		x_2	0	0	1	−3	−2	4	subopt. |·(−2/3) |·(−6)
		x_1	0	1	0	(2)	1	0 ←	degeneracija → jer je baz. var. (x_1) = 0!
Primalni simpl.	3	z'	1	[illegible]	0	0	−2	16	
		x_2	0	3/2	1	0	−1/2	4	
		x_3	0	1/2	0	1	(1/2)	0	|·3 |·18
	4	z'	1	11	0	4	0	16	
		x_2	0	2	1	0	0	4	
		x_4	0	1	0	2	1	0	|·2

$$z + 3x_1 - 4x_2 = 0$$

svrha izbora neb. var. je da odmah vidimo rješ. (bez rač.)

— treba z izbaciti → x_3, x_4

$$3x_1 + 2x_2 \ge 8$$

$$3x_1 + 2x_2 - x_3 + \bar{x} = 8$$

umjetna var.

Dualna simpleksna metoda
- dobra kad je rješenje nemoguće!

- u prethodnom primjeru, rješenje je nemoguće i suboptimalno.
- čim imamo u desne strane neg. broj, npr. var. je neg. i rješ. je nemoguće.

OPTIMALNOST:
- tak dugo dok ima minusa u fji cilja, rješenje je suboptimalno!

KONAČNA RJEŠENJA:
- nemoguće
- neograničeno (moguće je, a ulazna nebazična var. raste a što se vidi da su svi koeficijenti $Q \leq 0$)

2. 10. 2008.

ad. s. 28 (zadnja iteracija):

$\min \boxed{y_0 = \sum_{i=1}^{m} y_i \cdot b_i}$ vrijednost fje. cilja

uvjet optimalnosti $\begin{cases} z_j - c_j \geq 0 & j = 1, n \\ \boxed{y_i \geq 0} & i = 1, m \end{cases}$

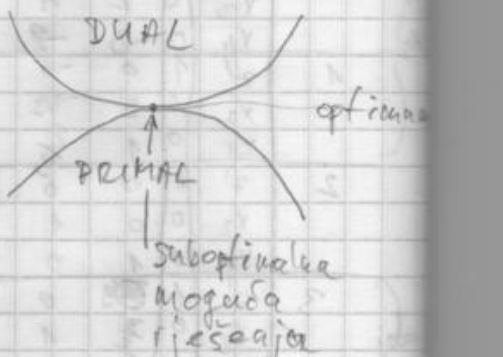
$z_j \geq c_j \; ; \; j = 1, n$

$z_1 = a_{11} y_1 + a_{21} y_2 + \ldots + a_{m1} y_m$

- općenito:

$z_j = \sum_{i=1}^{m} a_{ij} y_i$

$\boxed{\sum_{i=1}^{m} a_{ij} y_i \geq c_j \; ; \; j = 1, n}$



Primjer sa slajda 9.:

- tamo je primal
- dual je:

$$\min z = 4y_1 + 12y_2 + 18y_3$$
$$y_1 + 3y_3 \geq 3$$
$$2y_2 + 2y_3 \geq 5$$

	x_1	x_2	
	≥ 3	≥ 5	$\leq$
y_1	1	0	4
y_2	0	2	12
y_3	3	2	18

PRIMAL / DUAL — max / MIN

$$\max z = -4y_1 - 12y_2 - 18y_3$$
$$-y_1 - 3y_3 \leq -3$$
$$-2y_2 - 2y_3 \leq -5$$

LIN. PROGRAM U KANONSKOM OBLIKU!

$\left|\frac{18}{-3}\right| < \left|\frac{12}{-2}\right|$

Red.	B.V.	z	y_1	y_2	y_3	y_4	y_5	
	z	1	4	12	18	0	0	0
	y_4	0	−1	0	−3	1	0	−3
	y_5	0	0	(−2)	−2	0	1	−5 ←
	z	1	4	0	6	0	8	−30
	y_4	0	−1	0	(−3)	1	0	−3 ← 1:(−3)
	y_2	0	0	1	1	0	$-\frac{1}{2}$	$\frac{5}{2}$ 1·(−12)
	z	1	2	0	0	2	6	−36
	y_3	0	$\frac{1}{3}$	0	1	$-\frac{1}{3}$	0	1 1·(−6) 1·(−1)
	y_2	0	$-\frac{1}{3}$	1	0	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{3}{2}$

uvodimo dop. var.!

ima < 0 pa je neg.!

← dualni simpl., y_5 je gori pa prvo njega izlazimo!

$$\max z = 2x_1 + 5x_2$$
$$3x_1 + 4x_2 \leq 12$$
$$6x_1 + x_2 \leq 15$$
$$2x_1 + 3x_2 \leq 8$$
$$7x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

m; a empir. složenost je $\sim km^3$

a dual →

min $12y_1 + 15y_2 + 8y_3 + 6y_4 + 3y_5$

$3y_1 + 6y_2 + 2y_3 + 7y_4 + y_5 \geq 2$

$4y_1 + y_2 + 3y_3 + 2y_4 + y_5 \geq 5$

$y_i \geq 0$

...pa se s dualom problem riješi u bitno manje iteracij[a] nego primalom (početno)

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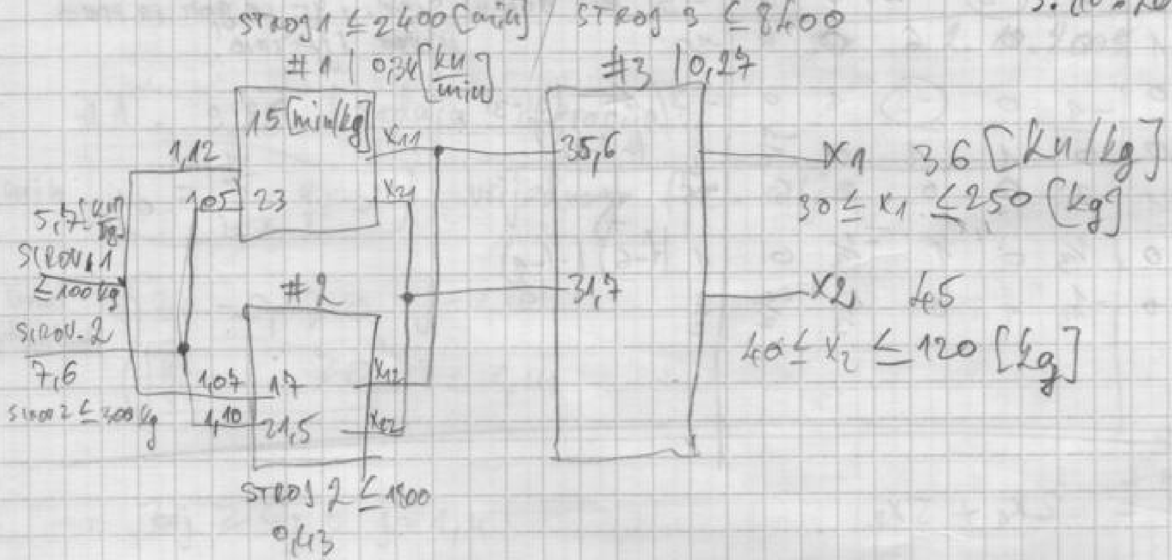
max $z = 3x_1 + 5x_2$

$x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

3.10.2008.



VF Proizv[...]
u LPE vel[...]

DOHODAK — TROŠAK

$\sum PC_i X_i$ — prod. cijena

$FX + VT$

FX: NAJAM, fiksni dio, plaća, osiguranje, amortizacija

VT: utrošci strojeva, cijena sirovine, habanje, rad

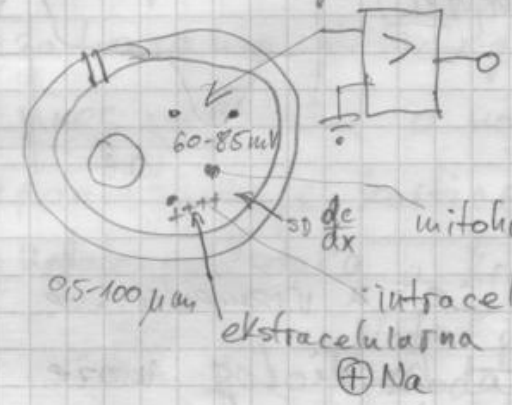
KONTRIBUCIJA = IZLAZNOS - VAR. TROŠAK ← funkcija cilja
(DOPRINOS ZA POKRIĆE FIKSNIH troškova)

BIOELE

7.10.2009.

17 trilijuna E I

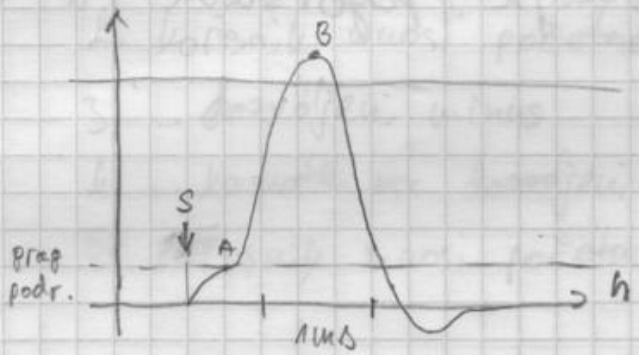
STANICA u stanju mirovanja N U



Na^+ 140 : 1

K^+ 5 : 140 ← Anioni 150 : 1

A-B - depol. stanice



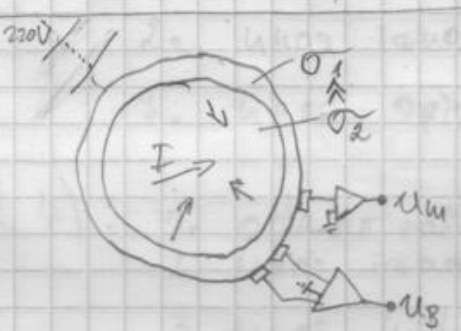
→ N A-C rep. stanica

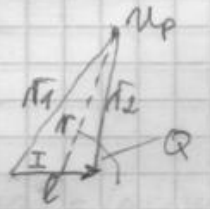
Na-K pumpa

Σ akcijski potencijal

MEH. | 1. gradijent konc.
KEM. | 2. električko polje
ELEK. |

⇒ D Fickov - zakon





$$U_p = \frac{I l \cos\Theta}{4\pi\sigma r^2}$$

$$U_p = U_s + U_A + U_{50}$$

(AKTIV.)