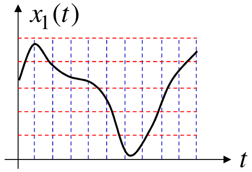
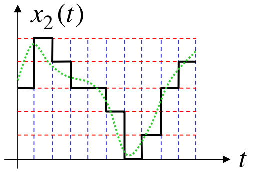
Analog, Time discrete and Digital signals

Analog signal



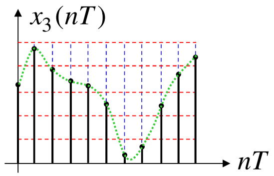
- Continuous time
- Continuous amplitude

Quantized signal



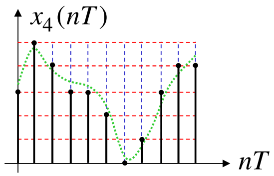
- Continuous time
- Quantized amplitude

Sampled signal:



- Quantized time (time discrete signal)
- Continuous amplitude

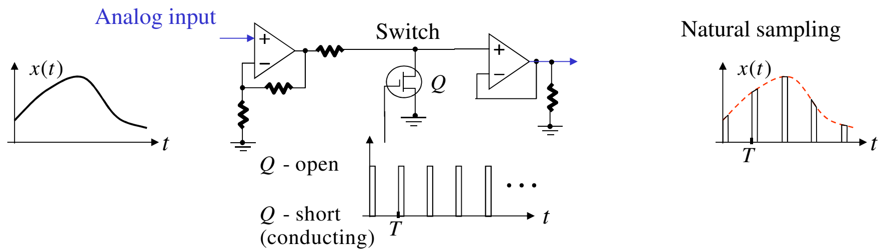
Digital signal:

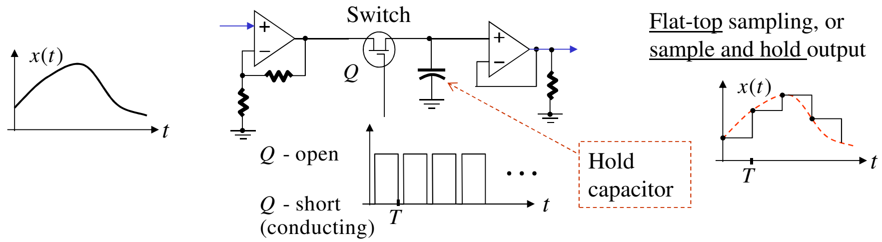


- Quantized time
- Quantized amplitude (e.g. 8 bit/sample)

Jfr *Young* p ???

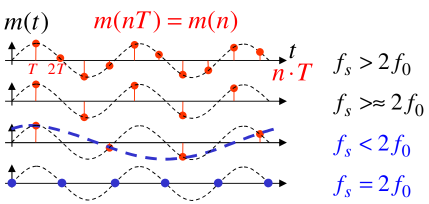
Pulse Amplitude Modulation (PAM)





The Sampling Theorem

How often must a time continuous signal be sampled to preserve all information (so that it is possible to accurately reconstruct it)?

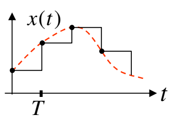


The sampling theorem:

A band limited time continuous signal *x(t)* can be fully reconstructed by an ideal LP filter from its sampled values *x(nT)* if the sampling frequency is at least two times the highest frequency component f_0 in the signal (i.e. two times the baseband bandwidth) $f_s > 2f_0$.

Reconstruction by an ideal LP filter is of course not possible. Normally the reconstruction is done by a hold circuit and a LP filter (anti-imaging filter).

Zero order hold D/A conversion:

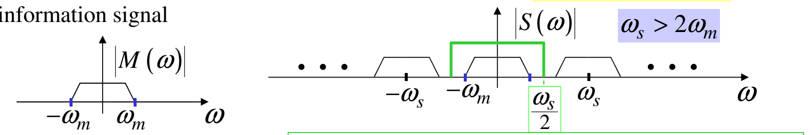


The hold circuit gives a staircase representation, and this signal is then smoothed by an analog LP filter, removing what is left of the higher frequency replica from the periodic spectrum.

Aliasing

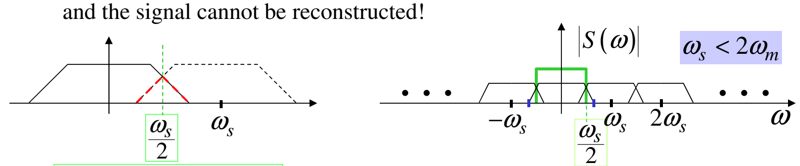
Band limited information signal

Sampled signal: Periodic spectrum!



The signal can be reconstructed by an ideal LP-filter.

When the sampling frequency is too low, we get aliasing (vikningsdistortion) and the signal cannot be reconstructed!



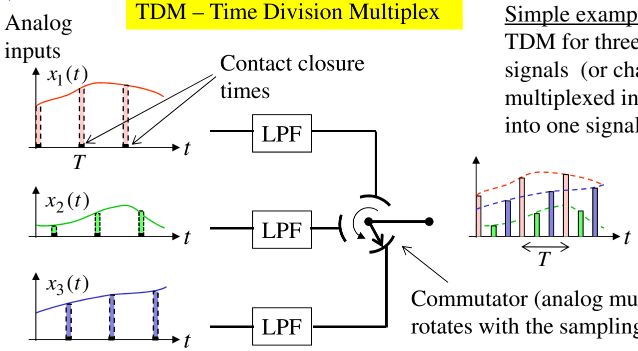
Foldover-frequency (Viknings-frekvens)

Examples of aliasing:
- Propeller (airplane)
- Spokes in a wheel
- Stroboscope

The velocity seen by the eye is often not correct!

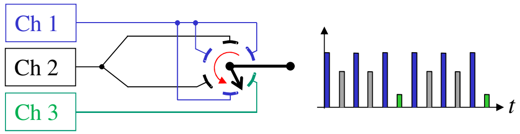
Time discrete signals and TDM

TDM – Time Division Multiplex



Simple example:
TDM for three information signals (or channels) multiplexed in time into one signal.

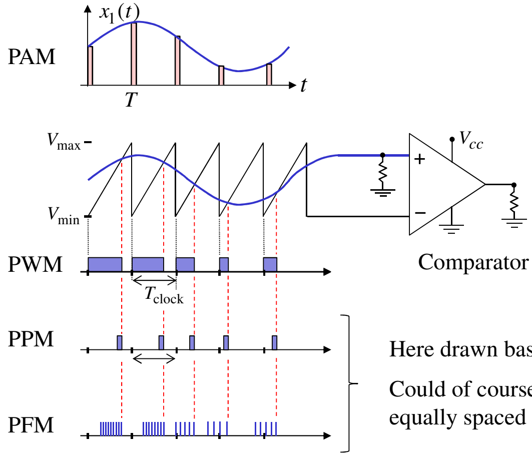
The commutator can be more complex to ensure minimum pulse rate if the analog input signal have different bandwidths and thus different sampling frequency requirements.



(However, Ch 2 does NOT get a constant pulse rate.)

Pulse Modulation methods

PAM – Pulse <u>Amplitude</u> Modulation
PWM – Pulse <u>Width</u> Modulation (or number of equally spaced pulses)
PPM – Pulse <u>Position</u> Modulation (needs a reference – synchronization)
PFM – Pulse <u>Frequency</u> Modulation



<u>Simple PWM modulator</u>: Here the amplitudes and thus the sample times are given by the end of the pulses, and thus they are NOT equally spaced.

Here drawn based on the PWM signal.

Could of course also be based on equally spaced sample values.

PCM and Baseband Coding

- PCM – Pulse Code Modulation
- DPCM – Differential PCM
- DM – Delta Modulation
- Baseband Coding

Speech (telephone), PCM-modulator

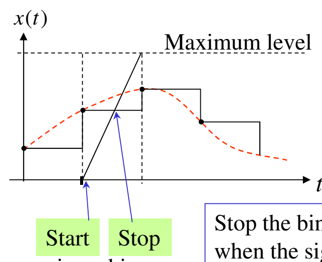
<u>Speech:</u> $300\,Hz - 3400\,Hz \Rightarrow B = 3100\,Hz$

$f_s \geq 2 f_{max} \Rightarrow f_s = 8000\,Hz$

sufficient quality with $2^8 = 256$ levels
(- found from fysiological testing)

PCM = Pulse Code Modulation

<u>PCM-modulator, Ramp encoder:</u>



Start Stop using a binary counter with 8 bits

Stop the binary counter when the signal has reached the level of the sample and hold circuit.

Basic building block in classical telecommunication:

$$D = 8\left[\frac{bit}{samples}\right]\cdot 8k\left[\frac{samples}{sec}\right]$$

$$= 64k\left[\frac{bit}{sec}\right] \quad \text{Bitrate} = 64\,k\ bps$$

The term PCM is often used with this <u>specific definition of 64 k bps</u>.
However, PCM is also often used <u>generally</u> for any digital signal from a sampled analog signal like above (with arbitrary bitrate).

Comparison of SNR due to quantization

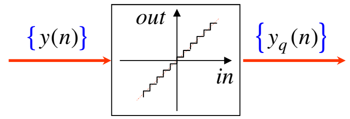
PCM = Pulse Code Modulation

Ordinary PCM has *n = 8 bits*, and number of levels: $M = 2^n = 2^8 = 256$

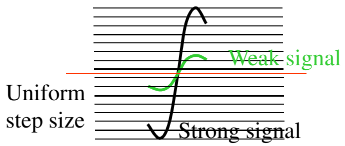
The introduced quantization error gives: $\left(\frac{S}{N}\right)_q \approx M^2 = \left(2^n\right)^2 = 2^{2n}$
(See end of file for the derivation!)

	Phone (USA)	Phone (Europe)	CD (Compact Disk)
Sampling frequency	f_S = 8 kHz	8 kHz	44.1 kHz
bits	n = 7	8	16
levels	M = 128	256	2^{16} = 65536
(S/N)q	$2^{(2n)} = 2^{14}$	2^{16}	2^{32}
(S/N)q in dB	42 dB	48 dB	96 dB

Quantization

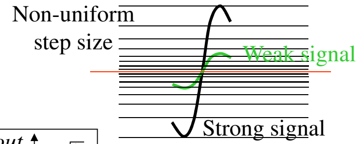


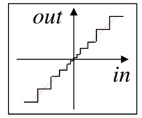
Linear quantizer
n-bits give $M = 2^n$ levels



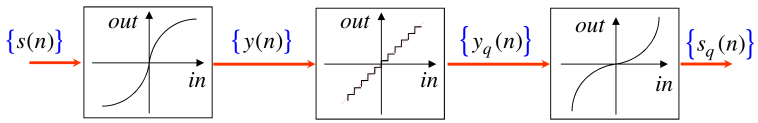
Two major drawbacks of linear quantization:
- The uniform step size means that weak analog signals will have much poorer $(S/N)_q$ than strong signals.
- Systems with wide dynamic range require many bits and thus large bandwidth.

Non-linear quantization:
- Decrease the step size for weak analog signals and increase it for strong signals.
- This gives a more uniform relative quantization error or uniform $(S/N)_q$!





Compression/Expansion = Companding



A-law or µ-law **compressor**

Linear quantizer (b-bits)

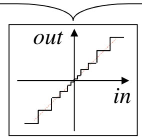
A-law or µ-law **expander**

An analog voltage compressor (which also limits the amplitude) precedes the linear quantizer.

A complementary expander (after decoding) restores the original dynamic range.

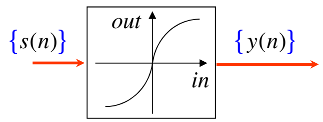
The analog compression / expansion could also be done digitally, in order to save bits in e.g. a digital transmission, starting with a high sampling rate signal.

The above is equivalent to a **non-linear quantizer**.



See also: http://www.dspguide.com/ch22/5.htm

PCM A-law (or µ-law)



A-law or µ-law **compressor**

North America uses AT&T's so-called ***µ-255*** companding shape (the 'mu'-law).

sign function

input

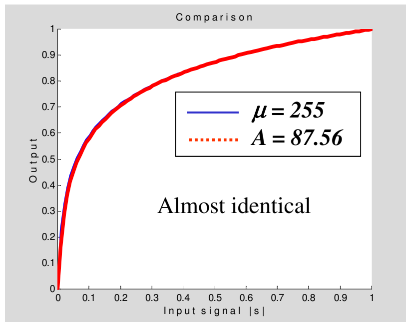
output

$$y = \frac{\ln(1+\mu|s|)}{\ln(1+\mu)}\operatorname{sgn}(s) \quad , \quad |s| \le 1 \; , \quad |y| \le 1$$

µ-law expander: $|s| = \frac{(1+\mu)^{|y|}-1}{\mu}$

Europe uses the CCITT (now ITU) specification, the so-called 'A'-law (***A = 87.56***)

$$y = \begin{cases} \frac{1+\ln(A|s|)}{1+\ln A}\operatorname{sgn}(s) \quad , & \frac{1}{A} \le |s| \le 1 \\ \frac{A|s|}{1+\ln A}\operatorname{sgn}(s) \quad , & 0 \le |s| \le \frac{1}{A} \end{cases}$$



Differential PCM and Delta Modulation

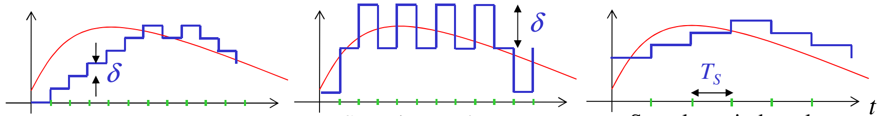
Differentiell PCM

Use the available number of bits to represent the difference in amplitude between two sample points, instead of representing the full amplitude. Then the quantization error is much smaller.

Delta Modulation (DM)

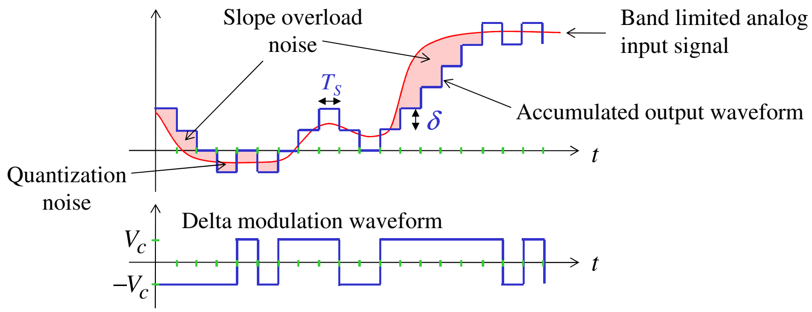
By increasing the sampling frequency the difference amplitude will be small. Go to the extreme and use only one bit to represent this difference.

The design parameters sampling frequency (or sampling period T_S) and step size (delta) need to be chosen carefully.

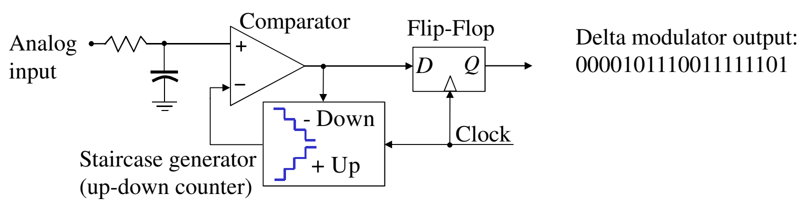


Step size too small | Step size too large Granular noise | Sample period too long

Delta Modulation (DM)



Delta modulator principle:



Analog ↔ Time discrete ↔ Digital

Analog signal

Sampling (Nyquist)

Time discrete signal → PAM, PWM, PPM. TDM (Time Div. Multiplex)

Quantization

Digital signal

The use of the terms are a bit "sloppy"!

- PCM
- DPCM
- DM

Base band "modulation"

Coding:
- NRZ
- Bipolar
- RZ
- AMI
- Manchester
- Differential code

"Matching" to the baseband channel. (See Young Fig 12-10)

- ASK
- FSK
- PSK
- QAM
- MSK

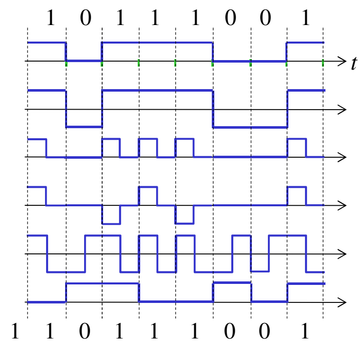
Digital modulation (High frequency modulation using carrier)

(Minimum Shift Keying, used in mobile phone GSM)

See *Young* p 488
Bengt Wedelin p 97

Baseband modulation or Channel coding

1 = "mark"
0 = "hole"



NRZ – Non Return to Zero

Bipolar

RZ – Return to Zero (50 % duty cycle)

AMI – Alternate Mark Inversion

Manchestercode (Ethernet)
(one transition in each bit interval)

Differential code
'Hi' = change, 'Low' = no change

This is modulation in the wide sense of the word: to match the signal to the channel

Important aspects:
- Avoid DC levels in the signal (problem for many channels)
- Synchronization (clock recovery) necessary (problem with many codes)

Noise and Error probability

- Noise
- ISI
- Eye diagram
- Error probability

Limiting factor: Noise and ISI

Analog system: The noise is always accumulative!
(The signal becomes worse and worse)

Digital system: Use regenerators! Detect '1', '0' and resend.
⇒ May have a perfect signal through a bad channel (many regenerators)!

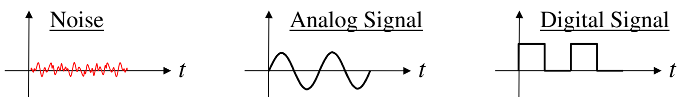
Wired systems: Limiting factor is ISI - (Inter) Symbol Interference
Radio systems: Thermal noise is the limiting factor.

BER (Bit Error Rate): The probability of error is given by the thermal noise and ISI

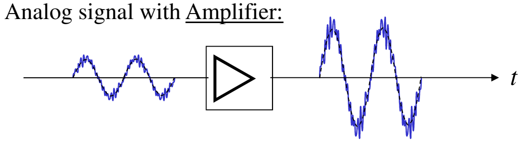
The total noise is often given as the sum of the thermal noise and the quantization noise.
However, the quantization noise is introduced by the D/A and does **not** give any BER!!!

$$N_{tot} = N_q + N_{term}$$

Amplifier - Regenerator

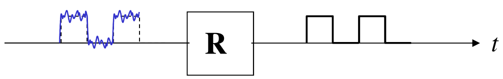


Analog signal with Amplifier:



The **noise** is accumulative!
(- noise on input is amplified together with the signal, and the amplifier adds some additional noise. S/N is always lower afterwards.)

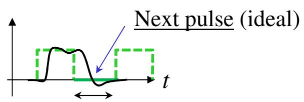
Digital signal with Regenerator:



Using a sufficient number of regenerators, a digital signal can be perfectly transmitted through a noisy channel!

ISI – Inter Symbol Interference

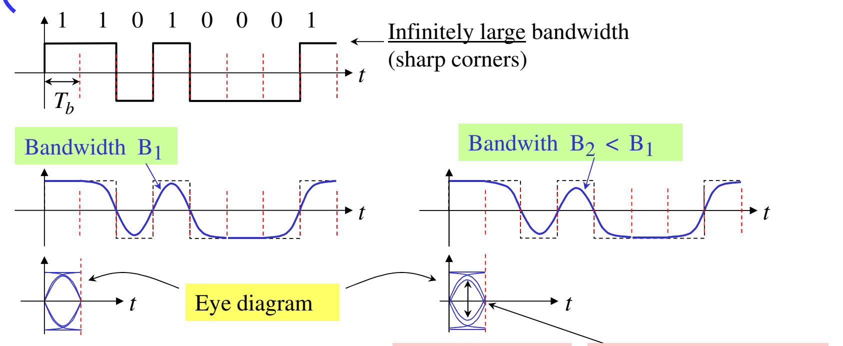
Limited bandwidth and/or Transmissions channel distortion give ISI:



The pulse has a "stretching delay" that interferes with the next pulse.

The pulses are "floating" into each other.

Eye diagram



The eye opening is smaller

The zero crossings are more varying

Cut the curve and put all bit intervals on top of each other.

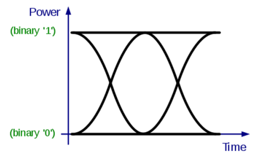
Use a sufficiently long pseudo random sequence so that all possible bit combinations (transitions) are encountered.

The eye opening is smaller in both directions

$\Rightarrow$ The probability of error is larger
(smaller margin left for thermal noise)

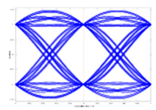
Eye diagram

https://en.wikipedia.org/wiki/Eye_pattern

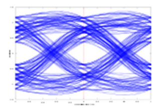


Graphical eye pattern showing an example of two power levels in an OOK modulation scheme.

Constant binary 1 and 0 levels are shown, as well as transitions from $0 \to 1$, $1 \to 0$, $0 \to 1 \to 0$, and $1 \to 0 \to 1$.



The eye diagram of a binary PSK system



The eye diagram of the same system with multipath interference (MI) effects added.

Noise and Error probability

Quantization noise $\left(\frac{S}{N}\right)_q = M^2 - 1 \approx M^2 = \left(2^n\right)^2, \quad \begin{cases} M = \text{number of levels} \\ n = \text{number of bits} \end{cases}$

Thermal noise $\rightarrow$ Matched filter

Cause a probability of error $\rightarrow$ P_e
$\Rightarrow$ Bit Error Rate $\rightarrow$ BER

ISI (depends on the bandwidth) $\rightarrow$ Nyquist filter (compare with RC filter and ideal LP filter)
when modeling the total transfer chain: $H_T(\omega) \cdot H_C(\omega) \cdot H_R(\omega)$

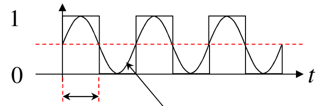
The pulse form gives power spectrum
(and thereby the bandwidth)!

Bandwidth Considerations

- Transfer function
- Bandwidth considerations
- Pulse shape

Nyquist bandwidth (minimum bandwidth)

If we had always changing 1's and 0's
$\Rightarrow$ sufficient to transmit the fundamental frequency only.



Bit time: T_b

Fundamental frequency $\Big\}$ $f_0 = \dfrac{1}{2T_b}$

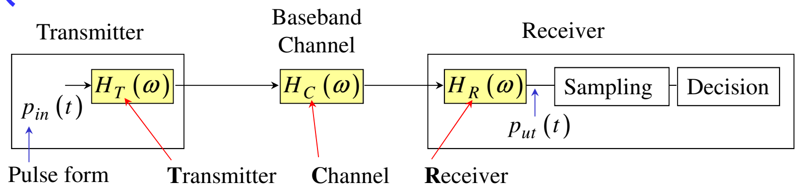
Data rate $D = f_b$

Bit rate $f_b = \dfrac{1}{T_b}$

Lowest possible bandwidth: $B_{\min} = \dfrac{1}{2} f_b = \dfrac{1}{2T_b}$ = **Nyquist bandwidth**

This corresponds to the theoretical lowest possible sample frequency for the fundamental frequency (2 times/period).

Modeling of the transfer chain



$$P_{ut}(\omega) = P_{in}(\omega) \cdot \underbrace{H_T(\omega) \cdot H_C(\omega) \cdot H_R(\omega)}_{H(\omega)}$$

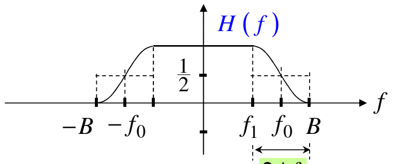
The square pulses (on input) are band limited through the "filters" $H_T \cdot H_C \cdot H_R$ and thus "smudged out" so they interfere with each other $\Rightarrow$ ISI.
(See the eye diagram).

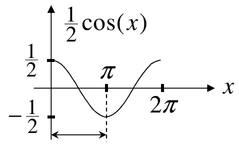
What is the "best" transfer function? $H(\omega) = H_T(\omega) \cdot H_C(\omega) \cdot H_R(\omega)$

Compromise between good detection and small bandwidth.

Harry Nyquist suggested a cosine roll-off filter as a good compromize. See next slide.

Nyquist filter, (Cosine shaped filter)





$2\Delta f$ $\quad$ π

$$H(f) = \begin{cases} 1 & , \ |f| \le f_1 \\ \frac{1}{2} + \frac{1}{2}\cos\left(\pi \dfrac{f - f_1}{2\Delta f}\right) & , \ f_1 \le f \le B \\ 0 & , \ |f| \ge B \end{cases}$$

Roll-off factor: $r = \dfrac{\Delta f}{f_0} \quad , \quad 0 \le r \le 1$

Total bandwidth: $B = f_0 + \Delta f = f_0(1 + r)$

Impulse response: $r \cdot f_0 = \Delta f$ $\quad$ $2\pi f_0 = \omega_0 \quad , \quad r \cdot \omega_0 = \Delta\omega$

$$h(t) = 2 f_0 \cdot \frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \cdot \frac{\cos(2\pi r f_0 t)}{1 - (4 r f_0 t)^2}$$

$$h(t) = \frac{\omega_0}{\pi} \cdot \frac{\sin(\omega_0 t)}{\omega_0 t} \cdot \frac{\cos(r\omega_0 t)}{1 - \left(2r \dfrac{\omega_0}{\pi} t\right)^2}$$

Nyquist filter, Bandwidth

The bandwidth corresponds to the data rate: $f_0 = \dfrac{1}{2} f_b = \dfrac{1}{2} D = \dfrac{1}{2T_b}$

$B = f_0(1 + r) = f_b \dfrac{1 + r}{2} = D \dfrac{1 + r}{2}$ $\quad$ $r = 0$ gives an ideal LP filter.

$r = 0$	$B = f_0(1 + r) = f_0 = \dfrac{1}{2T_b} = 0.5 f_b$	Theoretical minimum. Not achievable!
$r = 0.5$	$B = 1.5 f_0 = 1.5 \dfrac{1}{2T_b} = 0.75 f_b$	Pretty good! (Now also $r < 0.5$ is being used!)
$r = 1$	$B = 2 f_0 = \dfrac{1}{T_b} = f_b$	"Luxury!"

PCM signal with *64 kbps*:
$(r = 0 \quad B = 32\,kHz)$
$r = 0.5 \quad B = 48\,kHz$
$(r = 1 \quad B = 64\,kHz)$

PCM – demands larger bandwidth than e.g. PAM, but is not so sensitive to noise.
(Compare with FM where robustness to noise also was paid for by bandwidth.)

See *Young* p 487

Pulse shape and Bandwidth

The Nyquist bandwidth (or the Nyquist filter) gives a good indication of the necessary bandwidth to transmit a certain bitrate of data.

However, the signal (or pulse) bandwidth is also very important.
A square pulse has the sinc-function as Fourier transform, and thus the squared sinc-function as power spectral density (PSD).
This power spectrum is much wider than the Nyquist filter, and thus a lot of power will be lost (filtered away in the channel) and will not reach the receiver.

Therefore we want the binary pulse to have a shape so that most of the power actually comes through the channel and can be received (to maximize the SNR).

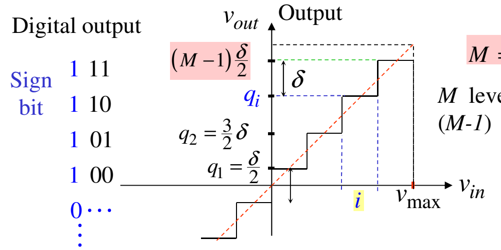
Hint for the RF bandwidth:
Multiplication in time domain corresponds to convolution in frequency domain.
(Base-band pulse bandwidth times a factor 2. Compare upper and lower side-band.)

Extra: Use MATLAB (fft) to calculate the PSD for some simple pulse shapes, e.g. a rectangular pulse, raised-cosine or cosine squared pulse.

Also, use MATLAB (fft) to calculate the impulse response for some filter shapes, e.g. Nyquist filter with different roll-off factors.
(Check you result using r = 0 which corresponds to an ideal LP filter.)

See *Bengt Wedelin* p ??-??
Young p 494

[Extra] The quantized signal's average power



$M = 2^n$

M levels, $(M\text{-}1)$ steps } Total for positive and negative levels.

p_i is the probability that $v_{in}(t)$ is within the interval i so that the output signal is q_i

The quantized signals average power: $S_q = \frac{1}{R}\sum_{i=1}^{M} p_i \cdot q_i^2$

Assume all signal levels equally probable: $p_i = \frac{1}{M}$

$$S_q = \frac{1}{R}2\frac{1}{M}\left[\left(\frac{\delta}{2}\right)^2 + \left(\frac{3}{2}\delta\right)^2 + \cdots + \left((M-1)\frac{\delta}{2}\right)^2\right] = \frac{1}{R}2\frac{1}{M}\left(\frac{\delta}{2}\right)^2\left[1^2+3^2+5^2+\cdots+(M-1)^2\right]$$

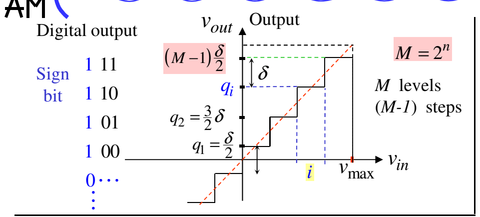
$$= \frac{1}{R}2\frac{1}{M}\left(\frac{\delta}{2}\right)^2\left[\frac{M(M-1)(M+1)}{6}\right] = \frac{1}{R}2\left(\frac{\delta}{2}\right)^2\cdot\frac{M^2-1}{6} = \frac{1}{R}\delta^2\cdot\frac{M^2-1}{12}$$

positive and negative (sign bit)

The sum of the series can be shown by "induction".

See *Bengt Wedelin* p ??-??

[Extra] Quantization noise: $v(t) - v_q(t)$.



- $p(x)$ is the probability density function for the signal amplitude.
- The average quantization noise power for the interval i becomes :

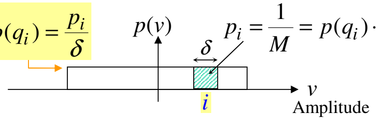
$$N_{qi} = \frac{1}{R}\int_{q_i-\delta/2}^{q_i+\delta/2}(x-q_i)^2\, p(x)dx$$

- If δ is small, the probability can be treated as constant over the small interval: $p(x) \approx p(q_i) = \text{constant}$
- Substitute: $y = x - q_i \quad , \quad dy = dx$

$$N_{qi} = \frac{1}{R}p(q_i)\int_{-\delta/2}^{\delta/2}(y)^2\,dy = \frac{1}{R}p(q_i)\left[\frac{1}{3}y^3\right]_{-\delta/2}^{\delta/2} = \frac{1}{R}p(q_i)\frac{\delta^3}{12}$$

Assume that all levels are equally probable - Uniform probability density function :

$p(q_i) = \frac{p_i}{\delta}$



$p_i = \frac{1}{M} = p(q_i)\cdot\delta$

$\boxed{\sum_{i=1}^{M} p_i = 1}$ The sum of all probabilities has to be unity.

[Extra] Signal to quantization noise ratio

We had the uniform probability density function $p(q_i) = \frac{p_i}{\delta}$

and that the average power for the quantization noise for the interval i was :

$N_{qi} = \frac{1}{R}p(q_i)\frac{\delta^3}{12}$ Thus, the total power for the quantization noise becomes:

$$N_q = \sum_i N_{qi} = \frac{1}{R}\sum_i \frac{p_i}{\delta}\frac{\delta^3}{12} = \frac{1}{R}\frac{\delta^2}{12}\sum_i p_i = \frac{1}{R}\frac{\delta^2}{12}$$

Average signal power:

Signal to quantization noise ratio: $\frac{S_q}{N_q} = \left(\frac{S}{N}\right)_q = \frac{\frac{1}{R}\delta^2\cdot\frac{M^2-1}{12}}{\frac{1}{R}\frac{\delta^2}{12}} = M^2 - 1 \approx M^2$

Some authors use the maximum signal power (for a sinusoidal signal) to get: $\left(\frac{S}{N}\right)_q \approx \frac{3}{2}M^2$