See HP product note 85150-4

Noise classification

See e.g. http://en.wikipedia.org/wiki/Noise_(electronics)

Examples:
- Low frequency ”hum” (50 Hz, e.g. in music)
- Random ”speckles” (e.g. in old CRT screens)
- Sporadically interference from storms (lightning) machinery, lights etc.
  - avoided by shielding, grounding, filtering
- Sources inside the system or circuit itself
  - due to the discrete electron charge.

- Thermal noise / white noise / Johnson-Nyquist noise (resistors) $\langle i^2 \rangle = \frac{4kT}{R} \cdot B$
- Shot noise / Partition noise (diods, BJT’s) [white] $\langle i^2 \rangle = 2q \cdot I_{DC} \cdot B$
- Flicker noise / *1/f* -noise (BJT’s, MOSFET’s) $\langle i^2 \rangle = \left(K_f \cdot I_{DC} \right) \cdot \frac{1}{f} \cdot B$
- Burst noise (BJT’s) $\langle i^2 \rangle \propto \frac{1}{f^2} \cdot B$
- Channel noise (MOSFET’s) [white] $\langle i^2 \rangle = \frac{8kT \cdot g_m}{3q} \cdot B$
- Avalanche noise (junction phenomenon in semiconductors)

The noise bandwidth (B) is normally the same as the filter or system bandwidth.

Shot noise, Partition noise

Shot noise:
Electrons crossing a potential barrier in a semiconductor junction (diode, transistor etc).
[*The name comes from the sound of vacuum tubes when electrons were crossing the tube and striking the metal anode*.]

- purely random, with a flat power spectrum in frequency (“white”).
- proportional to the bias current (DC current).

$$\langle i^2 \rangle = 2q \cdot I_{DC} \cdot B$$

Partition noise (for multi-electrode devices e.g. transistors):
- random partition or selection between e.g. base and collector for the electron movements.
- has a random shot noise effect.

http://en.wikipedia.org/wiki/Phase_noise

Flicker noise, Burst noise

Flicker noise:
Due to crystal surface defects in semiconductors.
Important in oscillators (where it contributes to phase noise near the carrier)

- proportional to the bias current (DC current)
- decreases with frequency
  - approximately *1/f* for low frequencies
  - for higher frequencies the noise power is weak and essentially flat.

$$\langle i^2 \rangle = \left(K_f \cdot I_{DC} \right) \cdot \frac{1}{f} \cdot B$$

For Phase noise and time jitter see e.g.
http://en.wikipedia.org/wiki/Phase_noise

Burst noise (popcorn noise):
Associated with heavy-metal ion contamination
A sudden change of current lasting a short time only

- proportional to the bias current (DC current)
- decreases with frequency, approximately $1/f^2$
- very device specific

$$\langle i^2 \rangle \propto \frac{1}{f^2} \cdot B$$

Noise and Feedback

Noise is a random process!
- can NOT utilize negative feedback for cancelling the noise!
- the amount of noise is not affected by negative feedback
- the contribution from each component **adds on power basis** (not amplitude basis)

BUT, certain interfering signals such as
- 50 Hz ”hum” can be filtered out, and
- strong noise impulses can be detected and shorted out in the time domain, e.g. by rapid switches.

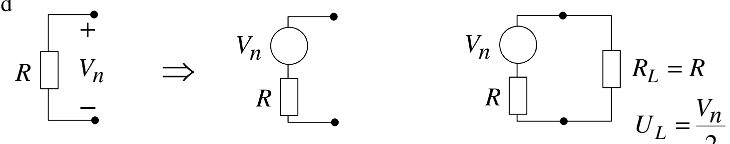
Smart filtering (adaptive) can be used to reduce the effect of some type of noise.

Thermal (White) Noise

The noise power (voltage) from a resistor can be measured

Equivalent model with ideal resistor

Max power transfer using matched load



$$U_L = \frac{V_n}{2}$$

$$I_L = \frac{U_L}{R_L}$$

Harry Nyquist: $V_n = \sqrt{4kTBR}$ (**RMS-value**)

$$P_L = U_L \cdot I_L{}^* = \frac{|U_L|^2}{R_L} = \frac{|V_n|^2}{4R} = \frac{4kTBR}{4R} = kTB$$

Thermal noise power (time average): $N = k \cdot T \cdot B$

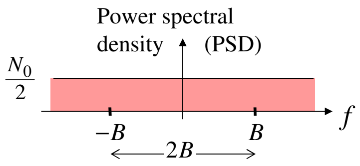
$k = 1.38 \cdot 10^{-23} \left[\frac{W}{Hz \cdot K}\right]$ Boltzmann constant

T Absolute temperature in Kelvin $[K]$

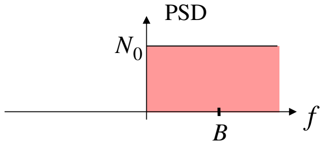
B Bandwidth in Hertz $[Hz]$

Power Spectral Density (PSD), White noise

Double sided:



Single sided:



Noise power:

$$N = \frac{N_0}{2} \cdot 2B = N_0 B = kTB$$

$$N = N_0 B = kTB$$

Noise power spectral density (single sided)

$k = 1.38 \cdot 10^{-23} \left[\frac{W}{Hz \cdot K}\right]$ Boltzmann constant

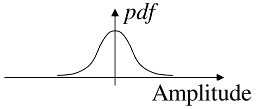
T Absolute temperature in Kelvin $[K]$

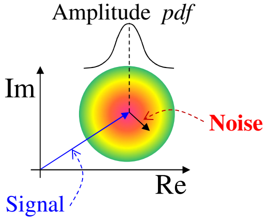
B Bandwidth in Hertz $[Hz]$

$$N = k \cdot T \cdot B$$

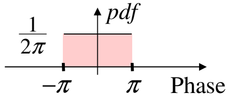
White noise in the complex plane

Gaussian Amplitude distribution
(probability density function *pdf*)





Flat Phase distribution
(all phases equal probable)



[Extra] White noise, statistical description

The power spectral density (PSD) of a wide sense stationary (WSS) process is defined as the Fourier transform of its autocorrelation function with respect to τ. $\Big\}$ $S_x(f) = \int_{-\infty}^{\infty} A_x(\tau) \cdot e^{-j2\pi f_c \tau} d\tau$

By the inverse transform we have that: $A_x(\tau) = E[x(t)x(t+\tau)] = \int_{-\infty}^{\infty} S_x(f) \cdot e^{j2\pi f_c \tau} df$

Thus the mean power can be expressed as: $\overline{P} = E\left[x^2(t)\right] = A_x(0) = \int_{-\infty}^{\infty} S_x(f)\, df$

The expected power of a random process $x(t)$ is in the frequency domain given by the integral of its PSD (compare with Parseval's theorem).

Hence $S_x(f)$ must be the power spectral density!

White noise is defined by its constant PSD (double sided): $S_n(f) = \frac{N_0}{2}$

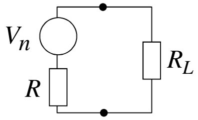
Autocorrelation: $A_n(\tau) = E[n(t)n(t+\tau)] = \int_{-\infty}^{\infty} S_n(f) \cdot e^{j2\pi f_c \tau} df = \frac{N_0}{2} \int_{-\infty}^{\infty} e^{j2\pi f_c \tau} df = \frac{N_0}{2} \delta(\tau)$

Validation of the improper integral: $\boxed{FT\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) \cdot e^{j2\pi f t} dt = 1 \quad \Leftrightarrow \quad FT^{-1}\{1\} = 1 \cdot \int_{-\infty}^{\infty} e^{j2\pi f t} df = \delta(t)}$

For a zero mean process we have: $\sigma^2 = E\left[(n(t) - \mu)^2\right]\Big|_{\mu=0} = E\left[n^2(t)\right]$

Thus the white noise power is given by: $\overline{P_n} = E\left[n^2(t)\right] = A_n(0) = \frac{N_0}{2} = \sigma^2$

Max power transfer using matched load



$$U_L = V_n \frac{R_L}{R_L+R}$$

$$I_L = \frac{U_L}{R_L} = \frac{V_n}{R_L+R}$$

$$P_L = U_{L_{rms}} \cdot I_{L_{rms}}{}^{*} = \frac{|U_L|^2}{R_L} = \frac{|V_n|^2 R_L}{(R_L+R)^2}$$

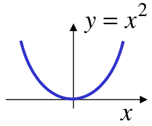
$$= |V_n|^2 R_L (R_L+R)^{-2}$$

$$\frac{\partial P_L}{\partial R_L} = |V_n|^2\left[(R_L+R)^{-2} + R_L(-2)(R_L+R)^{-3}\right] = |V_n|^2 \frac{R_L+R-2R_L}{(R_L+R)^3} = |V_n|^2 \frac{R-R_L}{(R_L+R)^3}$$

Max power transfer: $\frac{\partial P_L}{\partial R_L} = 0 \quad \Rightarrow \quad \underline{R_L = R}$

$$\frac{\partial^2 P_L}{\partial R_L{}^2} \sim -(R_L+R)^{-3} + (R-R_L)(-3)(R_L+R)^{-4} = \frac{-(R_L+R)-3(R-R_L)}{(R_L+R)^4} = \frac{-4R+2R_L}{(R_L+R)^4}$$

$$\left.\frac{\partial^2 P_L}{\partial R_L{}^2}\right|_{R_L=R} < 0 \quad \Rightarrow \quad \underline{\text{Max-point}}$$

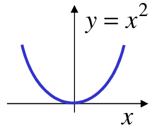


$$\frac{\partial y}{\partial x} = 2x$$

$$\frac{\partial^2 y}{\partial x^2} = 2 \quad \underline{> 0}$$

Minimum and Maximum point

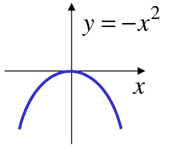
Simple example:



$$\frac{\partial y}{\partial x} = 2x$$

$$\frac{\partial^2 y}{\partial x^2} = 2 \quad \underline{> 0}$$

Minimum point
(Second derivative is positive)



$$\frac{\partial y}{\partial x} = -2x$$

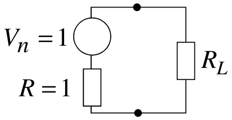
$$\frac{\partial^2 y}{\partial x^2} = -2 \quad \underline{< 0}$$

Maximum point
(Second derivative is negative)

Maximum power transfer

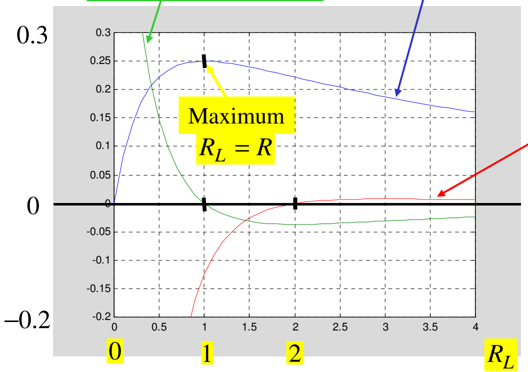
Power as a function of R_L

$$P_L = \frac{|V_n|^2 R_L}{(R_L+R)^2}$$



First derivative:

$$\frac{\partial P_L}{\partial R_L} = |V_n|^2 \frac{R-R_L}{(R_L+R)^3}$$



Second derivative:

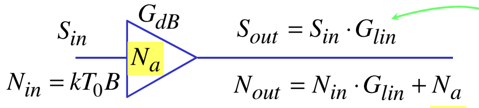
$$\frac{\partial^2 P_L}{\partial R_L{}^2} = |V_n|^2 \cdot \frac{-4R+2R_L}{(R_L+R)^4}$$

$$\frac{\partial^2 P_L}{\partial R_L{}^2} = 0 \quad \text{when} \quad R_L = 2R$$

Noise calculations

Noise calculations

Noise Ratio – Noise Figure



Input noise given at reference temperature. $T_0 = 290\ K$

Linear power amplification

Noise Ratio: $NR = \dfrac{\left(S/N\right)_{in}}{\left(S/N\right)_{out}} = \dfrac{\frac{\cancel{S_{in}}}{N_{in}}}{\frac{\cancel{S_{in}} \cdot G_{lin}}{N_{in} \cdot G_{lin} + N_a}} = \dfrac{N_{in} \cdot G_{lin} + N_a}{N_{in} \cdot G_{lin}} = 1 + \dfrac{N_a}{N_{in} \cdot G_{lin}}$

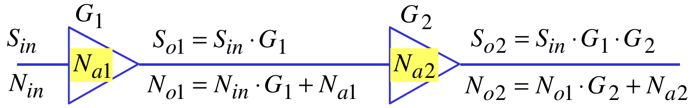
The inherent noise from the amplifier can be expressed as: $N_a = N_{in} \cdot G_{lin} \cdot (NR - 1)$

This is used in the development for cascaded amplifiers (next slide).

Note that the Noise Ratio is defined for approximately room temperature,
i.e. at the **reference temperature $T_0 = 290$ K** .

Noise Figure (dB): $NF = 10 \cdot \log_{10}(NR)$

Noise Figure Equation, Friis' formula



$N_{a1} = N_{in} \cdot G_1 \cdot (NR_1 - 1)$ $N_{a2} = N_{in} \cdot G_2 \cdot (NR_2 - 1)$

$$NR = \frac{\frac{S_{in}}{N_{in}}}{\frac{S_{o2}}{N_{o2}}} = \frac{\frac{\cancel{S_{in}}}{N_{in}}}{\frac{\cancel{S_{in}} \cdot G_1 \cdot G_2}{(N_{in} \cdot G_1 + N_{a1}) \cdot G_2 + N_{a2}}} = \frac{(N_{in} \cdot G_1 + N_{a1}) \cdot G_2 + N_{a2}}{N_{in} \cdot G_1 \cdot G_2} = 1 + (NR_1 - 1) + \frac{NR_2 - 1}{G_1}$$

$$NR = NR_1 + \frac{NR_2 - 1}{G_1}$$

Generally we get the so called "Friis' formula "

$$NR = NR_1 + \frac{NR_2 - 1}{G_1} + \frac{NR_3 - 1}{G_1 G_2} + \cdots + \frac{NR_n - 1}{G_1 G_2 \cdots G_{n-1}}$$

Valid also for attenuators, i.e. when G is less than one (negative in dB).
Note that the **first stage is most important**! (- use LNA with high gain.)

Equivalent Noise Temperature

We had that the internally generated noise *seen at the output* could be expressed as:
(- where the input noise N_{in} is defined at the reference temperature T_0)

$$N_{a,\,out} = N_{in} \cdot G_{lin} \cdot (NR - 1) = (kT_0B) \cdot G_{lin} \cdot (NR - 1)$$

Now, change to an ideal model (without any internal noise).
To get the same noise on output, introduce an **equivalent noise temperature**, T_{eq} on input instead.

$N_{in,eq} = kT_{eq}B$ → Ideal model with no inherent noise. Amplification G_{lin} →

$$N_{a,\,out} = (kT_0B) \cdot G_{lin}\,(NR - 1) = k\,\underbrace{T_0\,(NR - 1)}\,B \cdot G_{lin} = k \cdot T_{eq} \cdot B \cdot G_{lin}$$

Use equivalent noise at input and an ideal model when calculating the noise on output.
The equivalent noise temperature is said to be **referred to the input**.

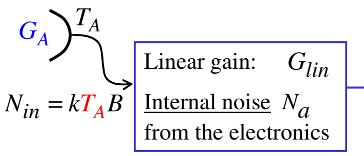
Equivalent noise temperature (for the device itself – referred to the input)

$$T_{eq} = T_0\,(NR - 1)$$

Noise calculations

Real system:

The input noise is given by an equivalent noise temperature from the antenna T_A .



$N_{out} = N_{in} \cdot G_{lin} + N_a$

$N_{a,\,out} = kT_{eq}B \cdot G_{lin}$

$T_{eq} = T_0\,(NR - 1)$

Equivalent system (for noise calculations):
The internal noise is modeled as an equivalent noise temperature on input together with an ideal (noise free) system.

$N_{in} = kT_AB$ →
$N_{eq}\text{(internal)} = kT_{eq}B$ →
Linear gain: G_{lin} Ideal model with no inherent noise →

$$N_{out} = k\,(T_A + T_{eq}) \cdot B \cdot G_{lin} = kT_{sys} \cdot B \cdot G_{lin}$$

Using NR when calculating the noise, is correct ONLY when $T_A = T_0$:

$$T_{sys} = T_A + T_{eq} = T_0 + T_0\,(NR - 1) = T_0 \cdot NR$$

Adding the equivalent noise temperatures (when referred to the same point) always gives the correct answer!

Equivalent Noise Temperature

Define the **equivalent** noise temperature: $T_{eq} = T_0\,(NR - 1)$

$$NR = \frac{T_{eq}}{T_0} + 1$$

Noise Figure *(NF)* **/ Noise Ratio** *(NR)*
is defined at the reference temperature $T_0 = 290\ K$
(approx. room temperature).

Combine this with the Friis' formula

$$NR = NR_1 + \frac{NR_2 - 1}{G_1} + \frac{NR_3 - 1}{G_1 G_2} + \ \bullet\bullet\bullet \ + \frac{NR_n - 1}{G_1 G_2 \cdots G_{n-1}}$$

$$\frac{T_{sys}}{T_0} + 1 \ = \ \frac{T_{eq1}}{T_0} + 1 + \frac{T_{eq2}}{T_0 G_1} + \frac{T_{eq3}}{T_0 G_1 G_2} + \ \bullet\bullet\bullet \ + \frac{T_{eq\,n}}{T_0 G_1 G_2 \cdots G_{n-1}}$$

$$T_{sys} = T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2} + \ \bullet\bullet\bullet \ + \frac{T_{eq\,n}}{G_1 G_2 \cdots G_{n-1}}$$