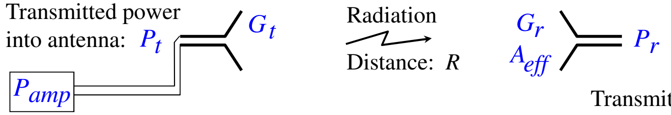
Friis' Transmission equation

Friis' Transmission Equation



<u>Branch</u> (e.g. circulators) and <u>feeding</u> (e.g. cables) losses are included by:

$$P_t = \frac{P_{amp}}{L_B \cdot L_f} \quad \Rightarrow \quad P_{amp}(dB) - L_B(dB) - L_f(dB)$$

Transmitted power density at the receiver:

$$W_t = \frac{P_t G_t}{4\pi R^2} = \frac{EIRP}{4\pi R^2}$$

EIRP =
Effective
Isotropic
Radiated
Power

Relationship between effective area and gain (valid for all antennas):

$$A_{eff} = \frac{\lambda^2}{4\pi} G_r$$

<u>Received power:</u>

$$P_r = W_t A_{eff} = \frac{P_t}{4\pi R^2} G_t \cdot \frac{\lambda^2}{4\pi} G_r = P_t\, G_t\, G_r \left(\frac{\lambda}{4\pi R}\right)^2 = EIRP \cdot G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

Friis' transmission equation (dB)

Friis' transmission equation:

$$P_r = P_t\, G_t\, G_r \left(\frac{\lambda}{4\pi R}\right)^2 = EIRP\, G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

$$\Downarrow$$

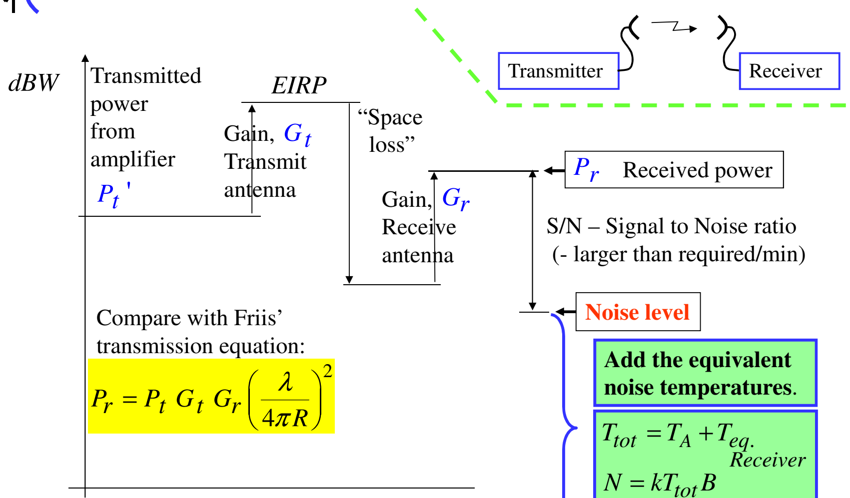
$$P_r(dB) = P_t(dB) + G_t(dB) + G_r(dB) - L_S(dB) - L_{\text{Fading Margin}}(dB)$$

Additional term to account for <u>unknown</u> fading.

<u>Space</u> "**<u>loss</u>**"
- normally defined as the inverse of the linear power expression
- thus it gives a positive dB value (loss) and
- must be subtracted in dB expressions

$$L_S = \left(\frac{4\pi R}{\lambda}\right)^2$$

Link budget



Compare with Friis' transmission equation:

$$P_r = P_t\, G_t\, G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

Add the equivalent noise temperatures.

$$T_{tot} = T_A + T_{eq.\,Receiver}$$

$$N = kT_{tot}B$$

Always correct result!

Both the received signal and the noise are calculated at the input of the receiver.

Energy/bit and G/T

$\frac{S}{N} = \frac{C}{N}$ Carrier to Noise = Signal to Noise
(e.g. for FSK and PSK – constant envelope signals)

Signal power = Received power (Friis' transmission equation)

$$\overbrace{S = P_r}^{} = E_b \cdot f_b$$

$$\Downarrow$$

$$[\text{Watt}] = \left[\frac{\text{Energy}}{\text{bit}}\right] \cdot \left[\frac{\text{bit}}{\text{sec}}\right]$$

In digital transmission it is convenient to use **energy/bit**, E_b instead of signal power, S.

Noise power: $N = \underbrace{k \cdot T_{sys}}_{} \cdot B = \underbrace{N_0}_{} \cdot B$

Noise power density

$$\frac{E_b}{N_0} = \frac{P_r}{f_b k T_{sys}} = \frac{P_t G_t \cdot G_r \left(\frac{\lambda}{4\pi R}\right)^2}{f_b k T_{sys}} = \frac{EIRP}{L_S f_b k} \cdot \frac{G_r}{T_{sys}}$$

G/T is a quality measure for a (satellite) receiver.

Example, Link budget and SNR calculations

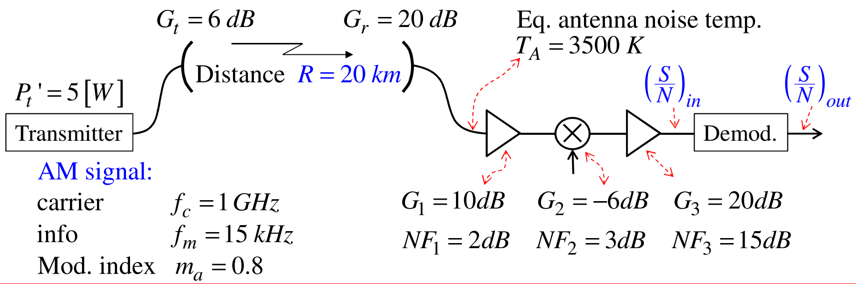
A transmitter with output power of 5 W at the carrier frequency of 1 GHz is connected to a transmit antenna with 6 dB gain.
At a distance of 15 km there is a receive antenna with 20 dB gain.
The signal is amplitude modulated with an information frequency of 15 kHz and a modulation index of 80 %. The noise bandwidth thus becomes 30 kHz.
The receive antenna (including the cable) has an equivalent noise temperature of $T_A = 3500\ K$.
The receiver consists of an RF amplifier with 10 dB gain and noise figure (NF) of 2 dB, a mixer with 6 dB losses and NF = 3 dB, an IF amplifier with 20 dB gain and NF = 15 dB and at last a demodulator (considered ideal, without internal noise).

a) What is the received power (after the receive antenna)?
   What is the voltage level if the impedance is 50 ohm?
   [Alternative: If the receiver require a voltage level of *1 mV (rms)*, determine the maximum distance.]

b) Determine the S/N at the input of the demodulator.
   Determine the S/N after the AM demodulator.

Example, Solution (1)

Draw a figure and collect / organize all information given in the text:



We need linear values (linear power gains) for the calculations:

$$G_{dB} = 10 \cdot \log_{10}(G_{lin}) \quad \Rightarrow \quad G_{lin} = 10^{\frac{G_{dB}}{10}}$$

Linear values for the antenna gains: $G_{tl} = 3.981, \quad G_{rl} = 100$

See next slide for answers. Do the detailed calculations yourself!

Example, Solution (2)

Received power (Friis transmission equation):

$P_r \approx 5\ nW \quad \rightarrow \quad -83\ dBW \quad \text{or} \quad -53\ dBm$

Alternative:
convert all values to dB and add/subtract dB values.

Within 50 ohm this corresponds to a voltage level (rms) of:

$V_r \approx 0.5\ mV_{rms}$

$$\boxed{NR = NR_1 + \frac{NR_2 - 1}{G_1} + \frac{NR_3 - 1}{G_1 \cdot G_2} \approx 1.58 + 0.10 + 12.20 = 13.88}$$

Add the equivalent noise temperatures.

Noise bandwidth = AM bandwidth: $B = 30\ kHz$

Input and output SNR for the AM demodulator:

$$\left(\frac{S}{N}\right)_{in} = \frac{P_r}{k \cdot T_{sys} \cdot B} \rightarrow 62.3\ dB \qquad \left(\frac{S}{N}\right)_{out} = \frac{m_a^2}{1 + \frac{m_a^2}{2}} \left(\frac{S}{N}\right)_{in} \rightarrow 59.1\ dB$$

dB calculations

dB calculations

dB – calculations

dB is a **relative** measure between **power relations**.

$$dB \Rightarrow 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right)$$

$$P = \frac{(V)^2}{2R}$$

$$\Downarrow$$

$$P_{dB} \sim 10 \cdot \log\left(V^2\right) = 20 \cdot \log(V)$$

Power expression

Amplitude expression

dBm and **dBW** etc. are **absolute** measures, (ie. Relative to a fixed level).

$$P_{dBm} = 10 \cdot \log_{10}\left(\frac{P}{1mW}\right)$$

$$P_{dBW} = 10 \cdot \log_{10}\left(\frac{P}{1W}\right)$$

$1mW = 0\,dBm$

$2mW = 3\,dBm$

$10mW = 10\,dBm$

$100mW = 20\,dBm$

$0.1mW = -10\,dBm$

$1W = 0\,dBW = 30\,dBm$

(x)	$10 \cdot \log(x)$	$20 \cdot \log(x)$
1	0	0
2	3.01	6.02
4	6.02	12.04
10	10	20
100	20	40
1000	30	60

dB – calculations (2)

(x)	$10 \cdot \log(x)$	$20 \cdot \log(x)$
1	0	0
2	3.01	6.02
4	6.02	12.04
10	10	20
100	20	40
1000	30	60

power amplification

$$G_{dB} = 10 \cdot \log G_{lin}$$

$$G_{lin} = 10^{\left(\frac{G_{dB}}{10}\right)}$$

amplitude amplification

$$G_{dB} = 20 \cdot \log A_{lin}$$

$$A_{lin} = 10^{\left(\frac{G_{dB}}{20}\right)} = \sqrt{G_{lin}}$$

G_{dB}	G_{lin}	A_{lin}
1	1.2589	1.1220
2	1.5849	1.2589
3	1.9953 ≈ 2	1.4125
4	2.5119	1.5849
6	3.9811 ≈ 4	1.9953 ≈ 2
10	10	3.1623
20	100	10

dB – calculations (3)

$V_{in} = 10\,mV_{pk}$ in 50Ω $\rightarrow$ $\boxed{G = 30dB}$ $\rightarrow$ $V_{out} = ?$, $S_{out} = ?$

$$S_{in} = P = \frac{(V_{in})^2}{2R} = \frac{\left(10 \cdot 10^{-3}\right)^2}{2 \cdot 50} = 10^{-6} \Rightarrow 1\mu W$$

$$S_{in,dBm} = 10 \cdot \log\left(\frac{1\mu W}{1mW}\right) = 10 \cdot \log\left(10^{-3}\right) = -30\ dBm$$

Power amplification: $G_{dB} = 10 \cdot \log G_{lin} \Leftrightarrow G_{lin} = 10^{\left(\frac{G_{dB}}{10}\right)} = 10^{\left(\frac{30}{10}\right)} = \underline{1000}$

Amplitude amplification: $G_{dB} = 20 \cdot \log A_{lin} \Leftrightarrow A_{lin} = 10^{\left(\frac{G_{dB}}{20}\right)} = 10^{\left(\frac{30}{20}\right)} = \sqrt{1000} = \underline{31.623}$

$$S_{out} = S_{in} \cdot G_{lin} = 1\mu W \cdot 1000 = \underline{1\,mW}$$

Same result!

$$S_{out,dBm} = S_{in,dBm} + G_{dB} = -30\,dBm + 30\,dB = \underline{0\,dBm}$$

Check that you get the same result by:

$$S_{out} = \frac{(V_{out})^2}{2R} = \frac{(V_{in} \cdot A_{lin})^2}{2R}$$