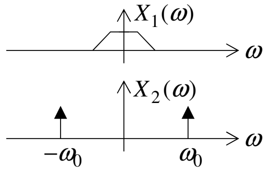
Modulation seen as convolution in frequency domain

Modulation (multiplication in time domain):
Let $x_1(t)$ be a band limited (information) signal and $y(t)$ the carrier frequency:

$$y(t) = x_1(t)\cdot x_2(t) = x_1(t)\cdot\cos(\omega_0 t)$$

What will the spectrum of $y(t)$ be?

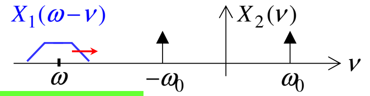


Multiplication in time domain $\Rightarrow$ Convolution in frequency domain

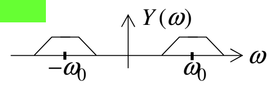
$$Y(\omega) = \mathfrak{F}\{y(t)\} = \mathfrak{F}\{x_1(t)\cdot x_2(t)\}$$

$$= \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$$

$$\propto \int_{-\infty}^{\infty} X_1(\omega-\nu)\cdot X_2(\nu)\, d\nu$$



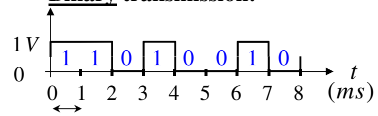
ω is a parameter in the integral



The baseband spectrum is moved up (upconverted) to around the carrier frequency

M-ary transmission (amplitude)

Binary transmission:



$T_b = T_S = 1\ ms$

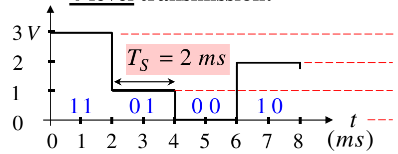
Bit rate (information rate):

$f_b = \frac{1}{T_b}$ $\quad \frac{8\text{ bit}}{8\ ms} = 1\,k\ \frac{\text{bit}}{\text{sec}} = 1\,k\ \text{bps}$

Symbol rate (Baud rate):

$f_B = \frac{1}{T_S}$ $\quad \frac{8\text{ symbols}}{8\ ms} = 1\,k\ \frac{\text{symbols}}{\text{sec}} = 1\,k\ baud$

4-level transmission:



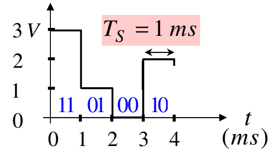
1 1 → 3 V
1 0 → 2 V
0 1 → 1 V
0 0 → 0 V

Bit rate (information):

$f_b = \frac{8\text{ bit}}{8\ ms} = 1\,k\ \text{bps}$

Symbol rate:

$f_B = \frac{4\text{ symbols}}{8\ ms} = 0.5\,k\ baud$



With 4 levels (states) one can either half the symbol rate or double the information rate.

Bit rate (information):

$f_b = \frac{8\text{ bit}}{4\ ms} = 2\,k\ \text{bps}$

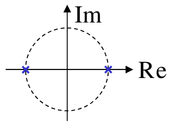
Symbol rate:

$f_B = \frac{4\text{ symbols}}{4\ ms} = 1\,k\ baud$

PSK and QAM in the complex plane

BPSK / 2-PSK

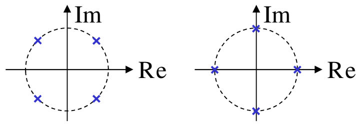
$n = 1\,\frac{\text{bit}}{\text{symbol}}$



One symbol corresponds to $n = log_2(M)$ bits.

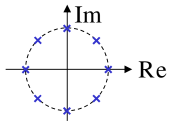
QPSK / 4-PSK

$n = 2\,\frac{\text{bit}}{\text{symbol}}$



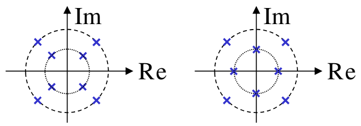
8-PSK

$n = 3\,\frac{\text{bit}}{\text{symbol}}$



8-QAM

$n = 3\,\frac{\text{bit}}{\text{symbol}}$



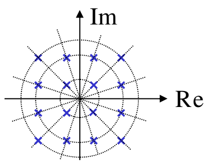
PSK } QAM
ASK }

4 phases
2 amplitudes

8 phases
2 amplitudes

16-QAM

$n = 4\,\frac{\text{bit}}{\text{symbol}}$



12 phases
3 amplitudes

Quadrature modulation and Complex envelope

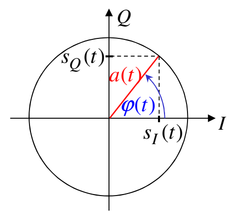
Phasor (baseband complex envelope): $u(t) = s_I(t) + j\cdot s_Q(t) = a(t)\,e^{j\varphi(t)}$

Real band-pass signal (high frequency):

$$s(t) = \text{Re}\left\{u(t)\cdot e^{j\omega_c t}\right\}$$

$$= \text{Re}\left\{a(t)\,e^{j\varphi(t)}\cdot e^{j\omega_c t}\right\}$$

$$= a(t)\cos\left(2\pi f_c t + \varphi(t)\right)$$



General expression: $s(t) = a(t)\cos\left(2\pi f(t)\cdot t + \varphi(t)\right)$

Amplitude

Frequency

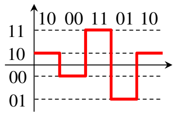
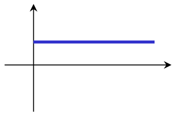
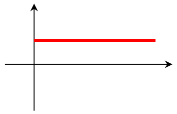
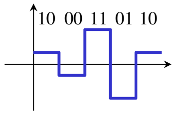
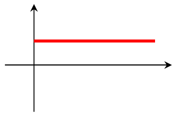
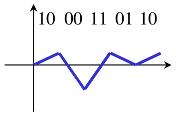
Phase

By letting the information signal control the amplitude, frequency or phase, we get the three basic types of digital modulation:

ASK – Amplitude Shift Keying
FSK – Frequency Shift Keying } Constant amplitude (or magnitude)!
PSK – Phase Shift Keying }

Basic Carrier modulation types

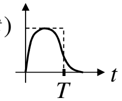
Transmitted radio signal: $s(t) = a(t)\cos\left(2\pi f_c t + \varphi(t)\right)$

	Amplitude $a(t)$	**Phase** $\varphi(t)$	Comments:
4-ASK			• Amplitude carries information • Phase constant (arbitrary)
4-PSK			• Amplitude constant (arbitrary) • Phase carries information
4-FSK			• Amplitude constant (arbitrary) • **Phase slope** (frequency) carries information

Baseband and RF bandwidths

$'0' \rightarrow \quad s_0(t) = 0$
$'1' \rightarrow \quad s_1(t) = p(t)\cos\omega_c t$

On-Off keying, used e.g. in optical fibers.

Pulse form: $p(t)$ 

Bandwidth to the base-band pulse $[P(\omega)] \rightarrow B_p$

It is important to distinguish between the baseband (or pulse) bandwidth B_p, and the carrier modulated (or RF) bandwidth B_{RF}.

As for ordinary AM, the RF bandwidth for on-off keying (or ASK) is double as large as the base-band bandwidth. This is (approximately) valid also for PSK (see later).

Bandwidth of the high frequency, RF, signal: $B_{ASK} = 2 \cdot B_p$
(compare with ordinary AM)

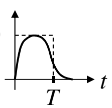
Note the factor 2!

For modulated (carrier) signal we get both upper and lower sideband!

ASK = Amplitude Shift Keying

$'0' \rightarrow \quad s_0(t) = 0$
$'1' \rightarrow \quad s_1(t) = p(t)\cos\omega_c t$

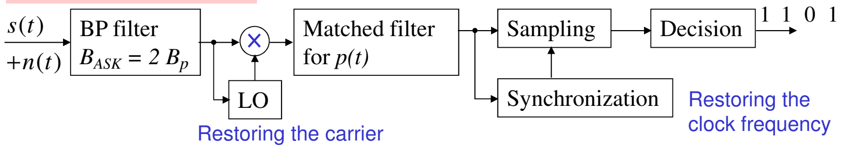
On-Off keying, used e.g. in optical fibers.

Pulse form: $p(t)$ 

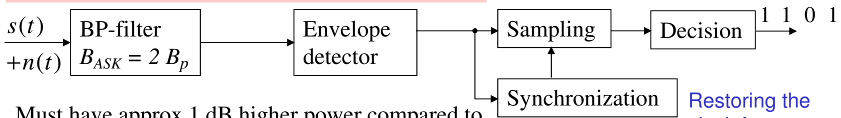
Bandwidth to the base band pulse $[P(\omega)] \rightarrow B_p$

Bandwidth to the high frequency signal: $B_{ASK} = 2 \cdot B_p$
(compare with ordinary AM)

Coherent demodulation:



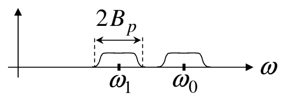
In practice an **envelope detector** is often used:



Must have approx 1 dB higher power compared to coherent demodulation to get the same error probability.

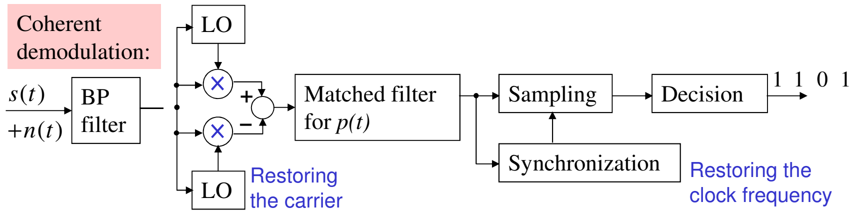
FSK = Frequency Shift Keying

$'0' \rightarrow \quad s_0(t) = p(t)\cdot\cos\omega_0 t$
$'1' \rightarrow \quad s_1(t) = p(t)\cdot\cos\omega_1 t$

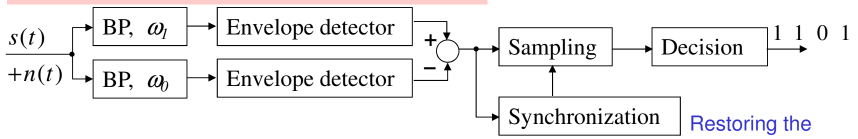


Bandwidth to the high frequency signal: $B_{FSK} = \omega_0 - \omega_1 + 2B_p$

Coherent demodulation:



In practice an **envelope detector** is often used:



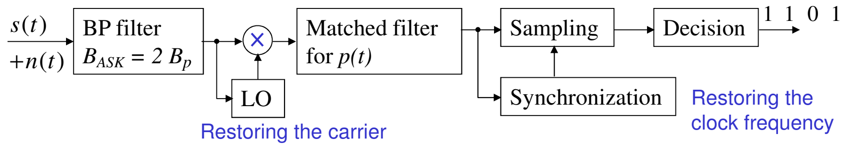
PSK = Phase Shift Keying

$'0' \rightarrow \quad s_0(t) = p(t)\cos(\omega_c t + \pi) = -p(t)\cos(\omega_c t)$

$'1' \rightarrow \quad s_1(t) = p(t)\cos(\omega_c t)$

High frequency bandwidth: $B_{PSK} = 2 \cdot B_p$

Only coherent demodulation possible: (Same block diagram as for ASK)



See *Young*
Ch 13-6 and 17-1

MODEM = MOdulator + DEModulator

$\Rightarrow$ MOdulation and DEModulation of digital data for analog channel.

Standard **analog** telephone channel: Bandwidth 4 kHz
Communication over large distances demands modulation with carrier.

The ordinary telephone system was made for analog speech with frequency content of 300 Hz – 3400 Hz (bandwidth $B = 3100\ Hz$, BP type).
The worst case scenario is (was) that also the channel has this limitation.

- FSK
- M-ary PSK (many levels)
- QAM (ASK + PSK)

} Constant envelope!
Many levels (states) used in order to
- increase the information content / get lower bandwidth.

Quadrature Amplitude Modulation (digital modulation)

Note that the same term is also used in analog modulation, where "in quadrature" implies that both the sine and cosine phases of the carrier is utilized.

MODEM

Old analog channel: 300 Hz – 3400 Hz ($B = 3100\ Hz$, BP-type).

- Example data rate: D = 1200 bps.
- Nyquist filter with $r = 1$ gives: $B_p = D\left.\dfrac{1+r}{2}\right|_{r=1} = D = 1200\ Hz$
- Bandwidth for RF signal: $B_{PSK} = 2 \cdot B_p = 2400\ Hz$

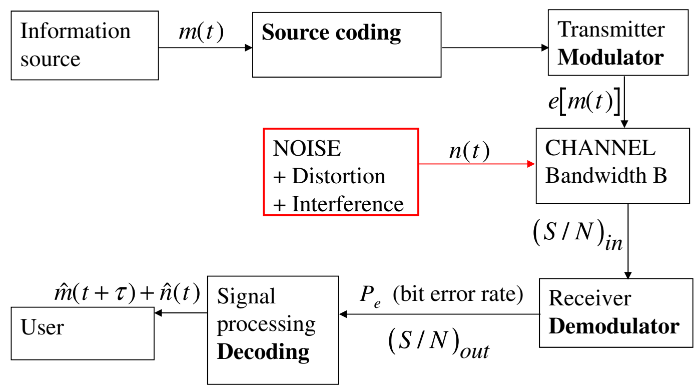
$\Rightarrow$ MODEM up to 1200 bps use binary FSK as modulation method, (sufficient bandwidth on the channel for binary coding).

$\Rightarrow$ High speed MODEMs have to use M-ary PSK or QAM !

E.g. a 9600 bps MODEM needs a 8-level modulation method.
In practice 16 levels are used.

However, modern communication over the telephone line, using e.g. ADSL, utilizes that the channel in practice has much larger bandwidth than the old standard of approx. 4 kHz.
The usable bandwidth is normally limited by the length of the analog channel.

Coding and Bandwidth requirements

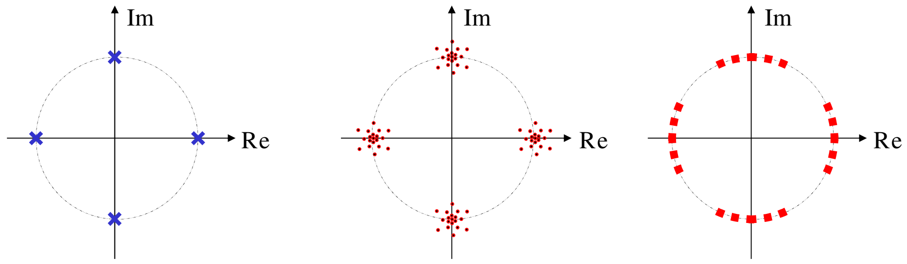


Remember that it is the coded bit or pulse stream that is modulated onto the carrier!

This means that for some codes, e.g. Manchestercode (Ethernet) with one transition for every information bit, we effectively get a coded bit stream that is approximately double as large as the source bit rate. This will of course also effect the bandwidth requirements.

Channel Distortions (ex QPSK)

QPSK / 4-PSK

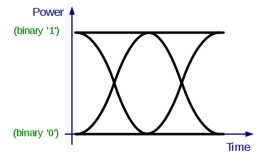


Perfect channel | White noise | Phase jitter

Compare with the Eye diagram

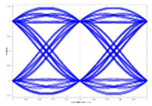
Eye diagram

https://en.wikipedia.org/wiki/Eye_pattern

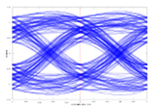


Graphical eye pattern showing an example of two power levels in an OOK modulation scheme.

Constant binary 1 and 0 levels are shown, as well as transitions from $0 \to 1$, $1 \to 0$, $0 \to 1 \to 0$, and $1 \to 0 \to 1$.

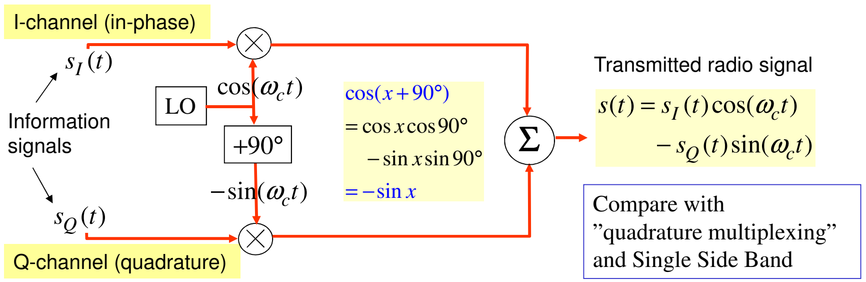


The eye diagram of a binary PSK system



The eye diagram of the same system with multipath interference (MI) effects added.

The IQ-modulator



$\cos(x+90°)$
$= \cos x \cos 90°$
$\quad - \sin x \sin 90°$
$= -\sin x$

Transmitted radio signal

$$s(t) = s_I(t)\cos(\omega_c t) - s_Q(t)\sin(\omega_c t)$$

Compare with "quadrature multiplexing" and Single Side Band

Remember the complex notation [Extra materiel]:

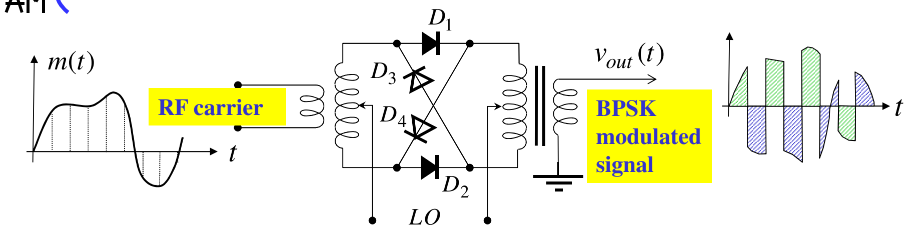
Complex envelope (Phasor):

$$u(t) = \left[s_I(t) + j \cdot s_Q(t) \right] = a(t)\, e^{j\varphi(t)}$$

Carrier factor: $e^{j\omega_c t}$

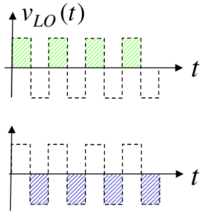
$$\Rightarrow \quad s(t) = \mathrm{Re}\left\{ u(t) \cdot e^{j\omega_c t} \right\} = a(t)\cos\left(2\pi f_c t + \varphi(t)\right)$$

Balanced Modulator, Binary PSK



Above is the balanced modulator for DSB-SC (when the output is filtered).

Digital information bits



$v_{LO} > 0 \Downarrow D_1$ and D_2 active

$v_{LO} < 0 \Downarrow D_3$ and D_4 active
The signal is inverted.

Now, let the high frequency analog carrier be to the left, and the digital information bits on the LO port below, then we get a BPSK modulator.