# Homework 5

Jacob Sachs

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## Problem 1

Prove: $X \to Y,\ Z \to W \Rightarrow X\,Z \to Y\,W$

$X \to Y \Rightarrow X\,Z \to Y\,Z$
$Z \to W \Rightarrow Y\,Z \to Y\,W$
$X\,Z \to Y\,Z,\ Y\,Z \to Y\,W \Rightarrow X\,Z \to Y\,W\ \blacksquare$

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## Problem 2

Prove: $W \to Y,\ X \to Z \Rightarrow W\,X \to Y$

$W \to Y \Rightarrow W\,X \to Y\,X$
$Y\,X \supset Y \Rightarrow Y\,X \to Y$
$W\,X \to Y\,X,\ Y\,X \to Y \Rightarrow W\,X \to Y\ \blacksquare$

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## Problem 3

Prove: $X \to Y,\ X \to W,\ W\,Y \to Z \Rightarrow X \to Z$

Using the result of problem 1,
$X \to Y,\ X \to W \Rightarrow X\,X \to Y\,W \Rightarrow X \to Y\,W$
$X \to Y\,W,\ W\,Y \to Z \Rightarrow X \to Z\ \blacksquare$

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## Problem 4

$F = \{A \to C,\ A\,C \to D,\ E \to A\,D,\ E \to H\}$
$G = \{A \to C\,D,\ E \to A\,H\}$

Let's try to decompose G to get the FDs in F.
For G:
$A \to C\,D \Rightarrow \boxed{A \to C}$, $A \to D$ (assuming we have proved decomposition from Armstrong's axioms)

$A \to D \Rightarrow A\,C \to D\,C,\ D\,C \supset D \Rightarrow D\,C \to D \Rightarrow \boxed{A\,C \to D}$

$E \to A\,H \Rightarrow E \to A,\ \boxed{E \to H}$

$E \to A,\ A \to C\,D \Rightarrow E \to C\,D$
$E \to C\,D \Rightarrow E \to C,\ E \to D$

$E \to A,\ E \to D \Rightarrow \boxed{E \to A\,D}$

This gives us all four of the FDs in F.

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# Problem 5

$R = \{A, B, C, D, E, F, G, H, I, J\}$
$F = \{A\,B \to C,\ B\,D \to E\,F,\ A\,D \to G\,H,\ A \to I,\ H \to J\}$

The key must be $A\,B\,D$, as these are the three attributes not determined by a functional dependency.
Furthermore, these three attributes are necessar to determine every attribute in R.

Now we decompose. First we find a canonical cover.

$F = \{A\,B \to C,\ B\,D \to E\,F,\ A\,D \to G\,H,\ A \to I,\ H \to J\}$ appears to already be a canonical cover, since all left-hand sides are unique and there are no extraneous attributes.

Now we create a relation for each FD in the cover.

$R_1 = \{A, B, C\}$, $R_2 = \{B, D, E, F\}$, $R_3 = \{A, D, G, H\}$, $R_4 = \{A, I\}$, $R_5 = \{H, J\}$

There are no schemas that are subsets of one another, so we keep them all.
However, there is no schema with the key of R.
We add $R_6 = \{A, B, D\}$, and we're done.