

# **Dyadic Structural Equation Modeling in R**

John Kitchener Sakaluk

2024-10-10

# Table of contents

<b>Preface</b>	<b>5</b>
<b>I Act I: Getting Started</b>	<b>7</b>
1 Introduction	8
2 The Big Picture	11
3 Dyadic Data Structure	12
4 Latent Variables in SEM	13
<b>II Act II: SEM Fundamentals</b>	<b>14</b>
5 Models (and Their Elements)	15
6 Model Identification	16
7 Scale-Setting	17
8 Model Fit Indexes	18
9 Model Comparison Strategies	19
10 A Simple (Non-Dyadic) Example in {lavaan}	20
<b>III Intermission 1: Meet Your Dyadic Models</b>	<b>21</b>
11 Design, Scope, and Form of Dyadic Models	22
<b>IV Act III: Uni-Construct Cross-Sectional Dyadic Models</b>	<b>23</b>
12 The One-Factor (Surprisingly Dyadic) Model	24

13 The Correlated Two-Factor Model	25
14 The Bifactor Model	26
15 The Hierarchical Factor Model	27
16 The Causal Relations Between Univariate Cross-Sectional Dyadic SEMs	28
17 Dyadic Invariance Testing in Univariate Cross-Sectional Dyadic SEMs	29
<b>V Act IV: Bi-Construct Cross-Sectional Dyadic Models</b>	<b>30</b>
18 The Couple Interdependence Model	31
19 The Actor-Partner Interdependence Model	32
20 The Bifactor Structural Model	33
21 The Common Fate Model	34
22 Mixing Univariate Structures in Bivariate Models	35
<b>VI Intermission 2: What Is ‘(In)distinguishability’?</b>	<b>36</b>
23 (In)distinguishability Is More Complicated than You Think	37
<b>VII Act V: Advanced Applications of Dyadic SEM</b>	<b>38</b>
24 “Third Variable” Cross-Sectional Dyadic Models	39
25 Multi-Group Cross-Sectional Dyadic Models	40
26 Longitudinal Dyadic Models	41
27 Monte Carlo Simulations of Dyadic Modeling Strategies	42
28 Contributing to Open-Source Software Development for Dyadic SEM (and Beyond)	43
<b>VIII Fin</b>	<b>44</b>
29 Conclusion	45



# Preface

This is a book about analyzing dyadic data with latent variables using the structural equation modeling (SEM) framework.

Academics often find reading their own writing to be a “cringe-worthy” experience, and reading that leading sentence is a particularly strange experience for me. I never once thought I’d write a book on something so statistics- and programming-oriented like dyadic SEM:

*I dropped out of 11th grade programming course*

*I nearly failed my 12th grade introductory statistics course*

*My formal training in dyadic data analysis consists entirely of one weeklong workshop.*

And yet...

This is a book about analyzing dyadic data with latent variables using the SEM framework, and I am writing it. And I hope it will prove to be a book worth reading, if you are interested in dyadic data analysis. Why? Because despite my aforementioned reasons to feel surprised that I am writing this book, I *do* know a thing or two about SEM. And maybe in spite of my earlier challenges with statistics and programming—actually, perhaps *because* of them—I will be able to provide you a different way of thinking about dyadic data analysis. These challenges have shaped the way I teach SEM-related content, and directly informed many of the design choices for how I have programmed the *R* package *dySEM*

Indeed, cast through the lens of SEM, the analysis of dyads affords unique opportunities to ask—and answer—some particularly interesting questions. Moreover, dyadic SEM is used so infrequently, as of the writing of this initial draft, so that I think the “market” to make an impact with this approach to data analysis is *wide open*. I hope, in time, through writing this book and developing open-source software like *dySEM*, that this “market” will become a bit more lively.

**The good news**, if I have your interest, is that I plan to write and maintain this book open-access. That means it will be legally free for you to use whenever you want. **The bad news** is that I plan to write and maintain this book open-access. That means that I will write it in bursts—not always in a straightforward linear way—and things will change. Don’t be surprised to come back between versions and find I’ve totally reorganized a section, or a chapter, or even an “Act” of the book. And of course, all of my embarrassing mistakes and gaps in knowledge will be on display for all to see.

With that in mind, if you notice conceptual or code-based errors, or have requests for features or area of content coverage, you'd be doing me a favour by submitting these to the “Issues” page ([LINK](#)) of the GitHub repo, where I'll be maintaining the book.

Otherwise, I hope you enjoy this book and find something useful in it.

## **Part I**

# **Act I: Getting Started**

# 1 Introduction

This is a book about analyzing dyadic data with latent variables using the SEM framework. As such, it occupies a unique location within the market of available books across several domains. Kenny, Kashy, and Cook (2006), for example, is an excellent book focused exclusively on dyadic data analysis. It describes the challenges of managing dyadic data, as well as many of the most popular models of dyadic data. But though Kenny, Kashy, and Cook (2006) describe the SEM framework and its application to some models, they don't really mean "SEM" in the way that I do. That is, their consideration of SEM isn't particularly engaged with the prospect of modeling latent variables, while I am *exclusively* focused on the use of latent variables.

A book like Little (2013), meanwhile, describes the use of latent variable modeling for dependent data *like* dyadic data, albeit in the context of longitudinal research designs, where the source of dependency is repeated observation. As such, though this book possesses some wisdom for those wishing to use latent variables in their models of dyadic data, the book is not written with dyadic data or dyadic models in mind. And as it turns out, things get *weird* when you are analyzing dependent groups of two (as opposed to, say, individuals over the span of three, four, or five waves of repeated assessment).

There are many other excellent books, too, that offer additional guidance—to some (in)direct degree or another—for applying latent variable models to dyadic data, such as Brown (2015) and Kline (2023) and Bolger and Laurenceau (2013). By writing this book, I mean to take nothing away from their value propositions; they are great resources. But they either do not consider the unique features of dyadic data and/or do not substantively engage in the distinctive benefits to modeling dyadic data with latent variables.

This is a book about analyzing dyadic data with latent variables using the SEM framework. And so I will exclusively discuss the analysis of dyadic data, **and** I will exclusively discuss its analysis with latent variable models.

In **Act I** of the book, I will first lay out "The Big Picture" (Chapter 2) of the what's, why's, and how's of dyadic SEM with latent variables, as well as introducing some of the technical jargon I will use throughout the rest of the book. I then discuss some of the unique considerations of data management for dyadic data analysis (Chapter 3), before providing an overview of latent variable theory in the context of modeling dyadic data (Chapter 4).

In **Act II** of the book, I attempt to provide what I consider to be a "sufficient" overview of the conceptual and applied specifics of modeling latent variables, without yet engaging with how to extend this framework to the analysis of dyadic data. Chapter 5 is essential reading if you are



unaccustomed to the statistical features of latent variables models (e.g., the visual depictions, notation and interpretation for particular features, etc.). I then discuss two related problems that must be resolved in fitting SEMs: model identification (Chapter 6) and setting the scale of the latent variable(s) (Chapter 7). These topics then set the table for describing how we evaluate (Chapter 8) and compare (Chapter 9) SEMs. I then work through all of this (and more) in an applied non-dyadic example (Chapter 10).

I then pause in **Intermission 1** to provide some foreshadowing about the dyadic SEMs we will encounter in the subsequent chapters (Chapter 11). For those approaching this book while having some familiarity of the models described in Kenny, Kashy, and Cook (2006), this chapter will help to transition you to thinking about these models recast in latent space.

In **Act III**, we *finally* get into the specification of models for dyadic data with latent variables, beginning with the simplest cross-sectional models possible: those intended to capture only one *construct* (i.e., “uni-construct”) shared somehow between dyad members. These include the dyadic one-factor model (Chapter 12), the correlated two-factor model (Chapter 13), the bifactor model (Chapter 14), and the hierarchical factor model (Chapter 15). These models have a surprisingly interesting (and complex) relationship to one another, which I discuss in the subsequent chapter (Chapter 16). I also describe how to use dyadic invariance testing within these models, in order to evaluate the generalizability of latent variable model parameters across partners (Chapter 17), which plays an important role in many other comparisons in dyadic data analysis, as well as being an (unappreciated) interesting phenomenon in its own right.

In **Act IV**, we move to discussing dyadic SEMs that are latent embodiments of the kinds of models that may seem more prototypical in dyadic data analysis (i.e., those covered in Kenny, Kashy, and Cook (2006)). These models involve the prediction of one dyad-related construct by another (i.e., are bi-construct). I first describe bi-construct models where the predictor construct and outcome construct share the same uni-construct dyadic model, including the Couple Interdependence Model (Chapter 18), the Actor-Partner Interdependence Model (Chapter 19), the Bifactor Structural Model (Chapter 20), and the Common Fate Model (Chapter 21). I conclude this section with a discussion (and some encouragement) of how different uni-construct models could be combined in more boutique bi-construct models (Chapter 22).

We then pause once more, in **Intermission 2**, in order to discuss—with the knowledge of uni-construct and bi-construct dyadic SEMs under our belts—just how complicated the concept of “distinguishability” is, when cast through the SEM lens.

Finally, in **Act V**, we delve into even more complex applications of dyadic SEM, including some themes of practice that are not yet done, yet which I hope will be on the (near) horizon of analytic practice in our field. These include the modeling of so-called “third variable” processes (Chapter 24), testing dyadic SEMs across groups (Chapter 25), and the modeling of both dyadic and longitudinal dependency with latent variables (Chapter 26). I also discuss the application of (and need for more) Monte Carlo simulation studies (Chapter 27), to evaluate the performance of dyadic SEMs (and other modeling strategies); in this chapter, I also discuss

how these simulations can help to inform sample size planning. I then end this section with some encouragement and guidance of how to contribute to open-source dyadic data modeling tools (Chapter 28), for those so inclined.

I'll also (eventually) write a Conclusion to this book (Chapter 29), and I'm sure it'll be very meaningful and impressive. But for now, I need to generate some content, before I can realize what it is I ought to conclude.

What I will *not* write about, however, is the basics of using *R*—the open-source cross-platform statistical programming language that I use in my dyadic SEM work (and upon which this book and its applications currently rely). If you are entirely new to *R*, the good news is that most dyadic SEM modeling instantiations require precious little of fiddling around with basic data management in *R*. That is, you're often "good to go" soon after data importation. If you need additional scaffolding for using *R*, however, I encourage you to check out "R for Data Science", or "R4DS" as it's sometimes known (**NEED REFERENCE**). It's a gold mine of useful information for *R* users of all levels of comfort.

And no: I will not provide analytic resources for other programming/statistics languages (e.g., SPSS/AMOS, SAS, MPlus). Though I have sometimes done this in the past (e.g., John K. Sakaluk and Short (2017), John Kitchener Sakaluk (2019)), and some of the models I describe herein are possible to specify in these other softwares, I have decided that I am done supporting proprietary software. This may lose me some readers; so be it. I want to create learning resources and tools that are available to anyone and everyone, for free, and the increasing expense of these other software packages threatens what I see as a necessary mandate to democratize access to learning.

## 2 The Big Picture

### **3 Dyadic Data Structure**

## **4 Latent Variables in SEM**

## **Part II**

# **Act II: SEM Fundamentals**

## **5 Models (and Their Elements)**

## 6 Model Identification



## **7 Scale-Setting**

## 8 Model Fit Indexes

## 9 Model Comparison Strategies

## 10 A Simple (Non-Dyadic) Example in {lavaan}

## **Part III**

### **Intermission 1: Meet Your Dyadic Models**

## **11 Design, Scope, and Form of Dyadic Models**

## **Part IV**

# **Act III: Uni-Construct Cross-Sectional Dyadic Models**

## **12 The One-Factor (Surprisingly Dyadic) Model**



## **13 The Correlated Two-Factor Model**

## 14 The Bifactor Model

## **15 The Hierarchical Factor Model**

## **16 The Causal Relations Between Univariate Cross-Sectional Dyadic SEMs**

## **17 Dyadic Invariance Testing in Univariate Cross-Sectional Dyadic SEMs**

## **Part V**

# **Act IV: Bi-Construct Cross-Sectional Dyadic Models**

## **18 The Couple Interdependence Model**

## **19 The Actor-Partner Interdependence Model**



## **20 The Bifactor Structural Model**

## **21 The Common Fate Model**

## **22 Mixing Univariate Structures in Bivariate Models**

## **Part VI**

### **Intermission 2: What Is '(In)distinguishability'?**

## **23 (In)distinguishability Is More Complicated than You Think**

## **Part VII**

# **Act V: Advanced Applications of Dyadic SEM**

## **24 “Third Variable” Cross-Sectional Dyadic Models**

## **25 Multi-Group Cross-Sectional Dyadic Models**



## **26 Longitudinal Dyadic Models**

## **27 Monte Carlo Simulations of Dyadic Modeling Strategies**

## **28 Contributing to Open-Source Software Development for Dyadic SEM (and Beyond)**

## **Part VIII**

### **Fin**

## 29 Conclusion

## References

- Bolger, Niall, and Jean-Philippe Laurenceau. 2013. *Intensive Longitudinal Methods: An Introduction to Diary and Experience Sampling Research*. New York, NY: Guilford Press.
- Brown, Timothy A. 2015. *Confirmatory Factor Analysis for Applied Research*. 2nd Ed. New York, NY: Guilford Press.
- Kenny, David A., Deborah A. Kashy, and William L. Cook. 2006. *Dyadic Data Analysis*. New York, NY: Guilford Press.
- Kline, Rex B. 2023. *Principles and Practice of Structural Equation Modeling*. 5th Ed. New York, NY: Guilford Press.
- Little, Todd D. 2013. *Longitudinal Structural Equation Modeling*. New York, NY: Guilford Press.
- Sakaluk, John Kitchener. 2019. “Expanding Statistical Frontiers in Sexual Science: Taxometric, Invariance, and Equivalence Testing.” *The Journal of Sex Research* 56 (4-5): 475–510. <https://doi.org/10.1080/00224499.2019.1568377>.
- Sakaluk, John K., and Stephen D. Short. 2017. “A Methodological Review of Exploratory Factor Analysis in Sexuality Research: Used Practices, Best Practices, and Data Analysis Resources.” *The Journal of Sex Research* 54 (1): 1–9. <https://doi.org/10.1080/00224499.2015.1137538>.