ME 397- ASBR

Week 5-Lecture 1

a Curiosity NASA/JPLCaltech; **b** Savioke Relay; **c** self driving car, Oxford Univ.;
d Cheetah legged robot, Boston Dynamics

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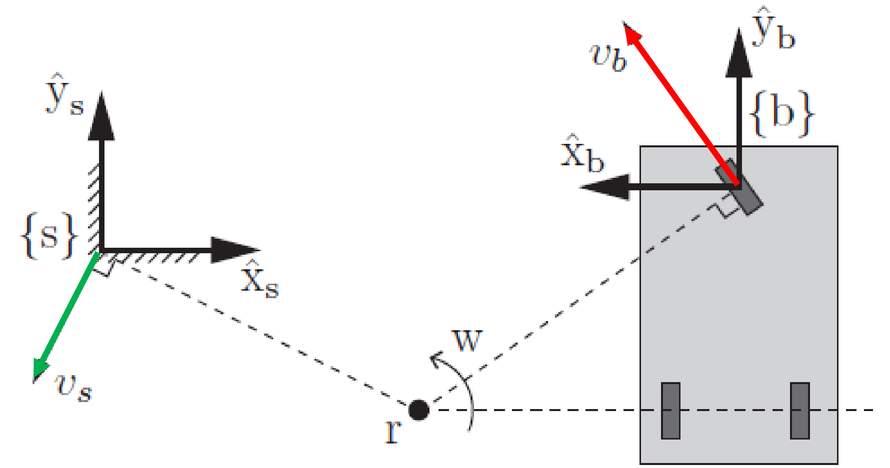
Example

Figure shows a top view of a car, with a single steerable front wheel, driving on a plane. The $\hat{\mathbf{z}}_b$**-axis** of **the body frame** {b} is **into the page** and the $\hat{\mathbf{z}}_s$**-axis** of the **fixed frame** {s} is **out of the page**.

- The angle of the front wheel of the car causes the car's motion to be a pure angular velocity w = 2 rad/s about an axis **out of the page** at the **point r** in the plane.



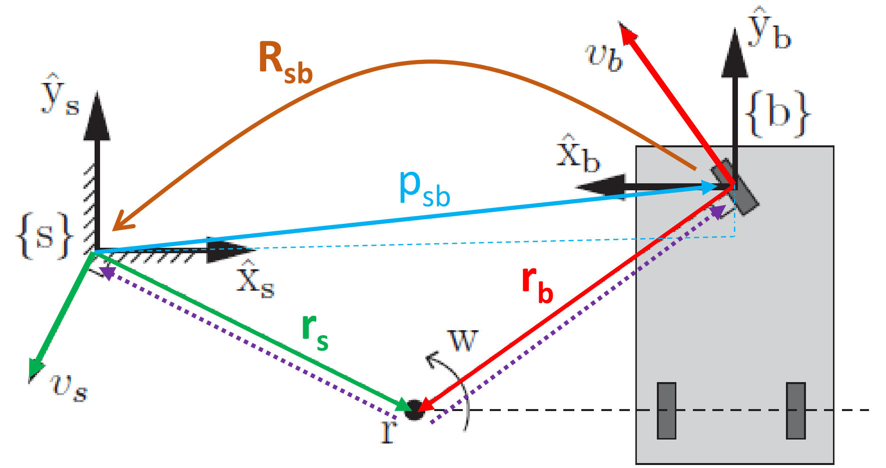
stack of angular and linear velocities

If $\mathbf{r}_s = (2;-1;0)$ or $\mathbf{r}_b = (2;-1.4;0)$, calculate twists $\mathcal{V}_s$ and $\mathcal{V}_b$ and verify them using corresponding adjoints.

Example

$\mathbf{r}_s = (2;-1;\ 0)$
$\mathbf{r}_b = (2;-1.4;\ 0)$

$\boldsymbol{\omega}_s = (0;\ 0;\ 2)$
$\boldsymbol{\omega}_b = (0;\ 0;\ -2)$



$$T_{sb} = \begin{bmatrix} \boxed{R_{sb}} & \boxed{p_{sb}} \\ 0 & 1 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} \hat{x}_b & \hat{y}_b & \hat{z}_b \end{array} \\ \left[\begin{array}{ccc|c} -1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0.4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

From the figure we get:

$$\boxed{v_s} = \omega_s \times \boxed{(-r_s)} = r_s \times \omega_s = (-2, -4, 0),$$
$$\boxed{v_b} = \omega_b \times \boxed{(-r_b)} = r_b \times \omega_b = (2.8, 4, 0),$$

$$\Rightarrow \mathcal{V}_s = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ -2 \\ -4 \\ 0 \end{bmatrix}, \quad \mathcal{V}_b = \begin{bmatrix} \omega_b \\ v_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -2 \\ 2.8 \\ 4 \\ 0 \end{bmatrix}$$

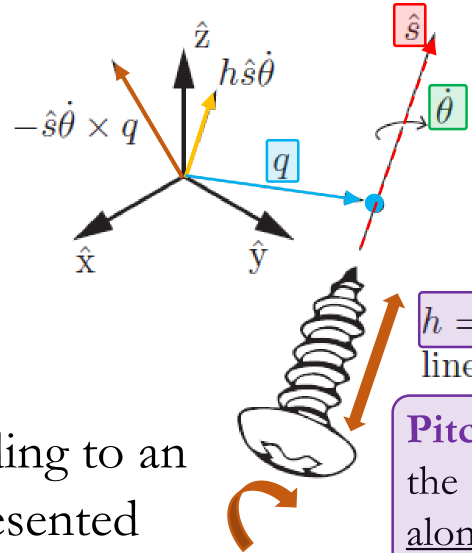
$$\mathcal{V}_s = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix} = \begin{bmatrix} \boxed{R} & 0 \\ \boxed{[p]}\boxed{R} & \boxed{R} \end{bmatrix} \begin{bmatrix} \omega_b \\ v_b \end{bmatrix} = \boxed{[\mathrm{Ad}_{T_{sb}}]}\mathcal{V}_b$$

$$[x] = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

[p] is the **skew-symmetric matrix** representation of vector p_{sb}

The Screw Interpretation of a Twist

- Just as an angular velocity $\boldsymbol{\omega} = \widehat{\boldsymbol{\omega}}\dot{\theta}$, where $\widehat{\boldsymbol{\omega}}$ is the **unit rotation axis** and $\dot{\theta}$, is the **rate of rotation about that axis**, a twist $\mathcal{V}= (\boldsymbol{\omega}; \mathrm{v})$ can be interpreted in terms of a **screw axis $\boldsymbol{S}$** and a **angular velocity $\dot{\theta}$** **about the screw axis** i.e., $\mathcal{V} = \boldsymbol{S}\,\dot{\theta}$.



h = pitch = linear speed/angular speed

Pitch is the ratio of the **linear velocity** along the screw axis to the **angular velocity $\dot{\theta}$** about the screw axis.

- We can write the twist $\mathcal{V} = (\boldsymbol{\omega}; \mathrm{v})$ corresponding to an angular velocity $\dot{\theta}$ about **Screw Axis $\boldsymbol{S}$** (represented by point q; a unit direction $\hat{\boldsymbol{s}}$; and a pitch **h**)

$$\mathcal{V} = \begin{bmatrix} \omega \\ v \end{bmatrix} = \begin{bmatrix} \hat{s}\dot{\theta} \\ \boxed{-\hat{s}\dot{\theta} \times q} + \boxed{h\hat{s}\dot{\theta}} \end{bmatrix}$$

\omega x r

Linear motion at the **origin** induced by rotation about the axis

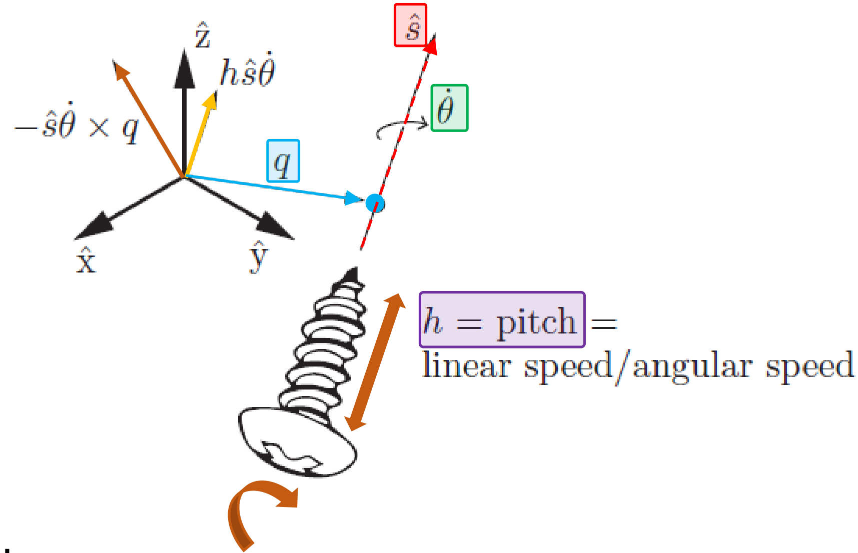
Translation along the screw axis

- The **second term** is in the direction of $\hat{\boldsymbol{s}}$, while the **first term** is in the plane orthogonal to $\hat{\boldsymbol{s}}$.

- For any $\mathcal{V} = (\boldsymbol{\omega}; \mathrm{v})$ where $\boldsymbol{\omega} \neq 0$, there exists an equivalent screw axis defined by {q; $\hat{\boldsymbol{s}}$; and h} and **velocity $\dot{\theta}$**.

The Screw Interpretation of a Twist

- Instead of defining the screw axis S using the cumbersome collection {q; $\hat{\mathbf{s}}$; and h}, we can **define the screw axis *S*** using a **normalized version** of <u>**any**</u> twist $\mathcal{V} = (\boldsymbol{\omega}; \mathrm{v})$ corresponding to motion along the screw:



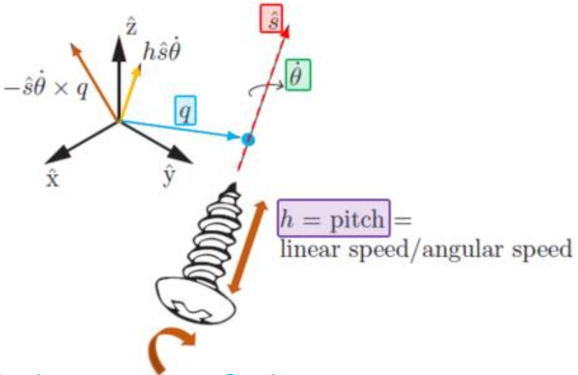
(a) If $\underline{\boldsymbol{\omega} \neq \mathbf{0}}$ then $\boldsymbol{S} = \underline{\boldsymbol{\mathcal{V}}/\|\boldsymbol{\omega}\|} = (\mathrm{v}/\underline{\|\boldsymbol{\omega}\|}, \underline{\boldsymbol{\omega}}/\underline{\|\boldsymbol{\omega}\|})$.

The **screw axis** $\boldsymbol{S}$ is simply **twist vector** $\boldsymbol{\mathcal{V}}$ **normalized by the length of the angular velocity vector**. The **angular velocity** about the screw axis is $\dot{\theta} = \|\boldsymbol{\omega}\|$, such that $S\dot{\theta} = \mathcal{V}$.

(b) If $\underline{\boldsymbol{\omega} = \mathbf{0}}$ then $\boldsymbol{S} = \underline{\boldsymbol{\mathcal{V}}/\|\mathbf{v}\|} = (\mathbf{0}; \underline{\mathbf{v}/\|\mathbf{v}\|})$.

The screw axis $\boldsymbol{S}$ is simply **twist vector** $\mathcal{V}$ **normalized by the length of the linear velocity vector** $\mathbf{v}$. The **linear velocity** along the screw axis is $\dot{\theta} = \|v\|$, such that $S\dot{\theta} = \mathcal{V}$.

The Screw Interpretation of a Twist



- For a **given reference frame**, a screw axis $\mathcal{S}$ is written as

$$\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$$

where either **(i)** $\|\omega\| = 1$ or **(ii)** $\|\omega\| = 0$ and $\|v\| = 1$.

- ✓ If **(i)** holds then $v = -\omega \times q + h\omega$, where **q** is a point on the axis of the screw and **h** is the pitch of the screw (**h = 0 for a pure rotation about the screw axis**).
- ✓ If **(ii)** holds then **the pitch h of the screw is infinite** and the twist is a **translation along the axis** defined by v **(Pure Translation)** .

- **Screw axis $\mathcal{S}$ simply is just a normalized twist**, the **4 × 4 matrix representation** $[\mathcal{S}]$ of $\mathcal{S} = (\omega; v)$ is

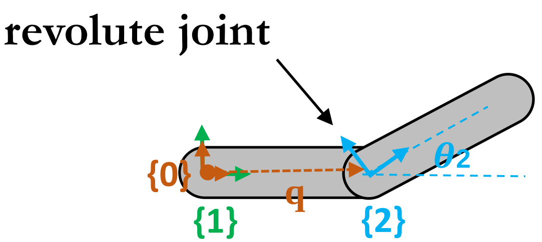
$$[\mathcal{S}] = \begin{bmatrix} [\omega] & v \\ 0 & 0 \end{bmatrix} \in se(3), \qquad [\omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \in so(3).$$

- A screw axis represented as $\mathcal{S}_a$ in a frame {a} is related to the representation $\mathcal{S}_b$ in a frame {b} by

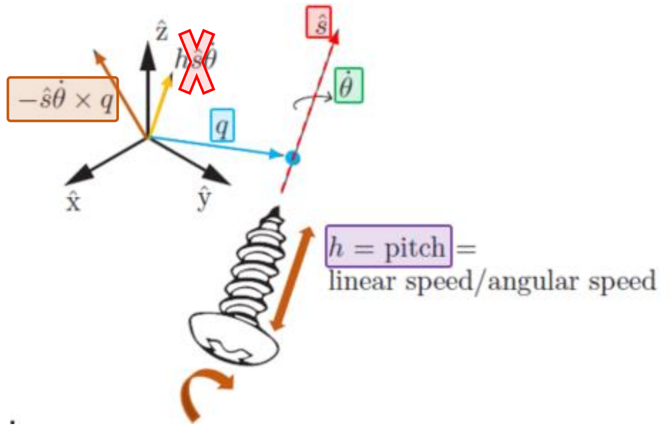
$$\mathcal{S}_a = [\mathrm{Ad}_{T_{ab}}]\mathcal{S}_b, \qquad \mathcal{S}_b = [\mathrm{Ad}_{T_{ba}}]\mathcal{S}_a.$$

Example: Screw axis of a revolute joint

$$\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$$



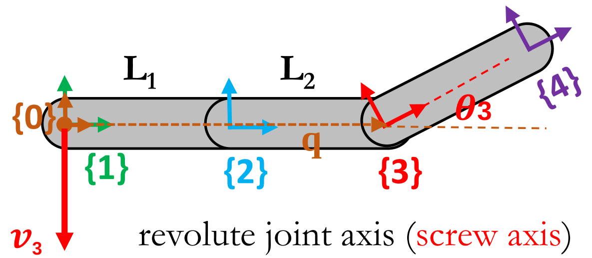
- Each **revolute joint axis** is:
  - ✓ a **zero-pitch** (2D motion) **screw axis**
  - ✓ and its location is **center of rotation** defining parameter **q**.



- If $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are held at their **zero position** then the **screw axis** corresponding to rotating about **joint 3** can be expressed in the {0} frame as

$$\mathcal{S}_3 = \begin{bmatrix} \omega_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -(L_1 + L_2) \\ 0 \end{bmatrix}$$

$$v_3 = -\omega_3 \times q_3$$
$$q_3 = (L_1 + L_2, 0, 0)$$



revolute joint axis (screw axis) is out of the plane.

Exponential Coordinates of Rigid-Body Motions

- The **Chasles-Mozzi** theorem states that **every rigid-body** displacement can be expressed as a **displacement** **along a fixed screw axis *S*** **in space** (compare it with ***Rodriguez theorem*** for rigid body **rotation**!).

- By analogy to the **three-dimensional exponential coordinates** $\hat{\omega}\theta \in \mathbb{R}^3$ for **rotations** (Rodriguez equation), we define the **six-dimensional** **exponential coordinates** of a **homogeneous transformation T** as

$$\boldsymbol{S\theta} \in \mathbb{R}^6$$

where ***S*** is the **screw axis** and $\boldsymbol{\theta}$ is the **distance** that **must be traveled along the screw axis** **to take a frame from the origin I to T**.

- If the **pitch** of the screw axis ***S*** = **(ω; v)** is **finite** then **$\|\omega\| = 1$** and $\boldsymbol{\theta} \in \mathbb{R}$ corresponds to the **angle of rotation** about the screw axis.

$$\mathbf{T} = e^{[\mathcal{S}]\theta} = \begin{bmatrix} \overset{\text{Rotation}}{\boxed{e^{[\omega]\theta}}} & \overset{\text{Translation vector}}{\boxed{(I\theta + (1-\cos\theta)[\omega] + (\theta - \sin\theta)[\omega]^2)\, v}} \\ 0 & 1 \end{bmatrix}$$

- (**Pure Translation**) If the **pitch** of the screw is **infinite** then $\omega = 0$ and $\|v\| = 1$ and θ corresponds to the linear distance traveled along the screw axis.

$$e^{[\mathcal{S}]\theta} = \begin{bmatrix} \boxed{I} & \boxed{v\theta} \\ 0 & 1 \end{bmatrix}$$

Exponential Coordinates of Rigid-Body Motions

- **Three-dimensional exponential coordinates $\hat{\omega}\theta \in \mathbb{R}^3$ for rotations, matrix exponential, and matrix logarithm (log):**

$$\begin{aligned} \exp: \quad [\hat{\omega}]\theta \in so(3) \quad &\rightarrow \quad R \in SO(3), \\ \log: \quad R \in SO(3) \quad &\rightarrow \quad [\hat{\omega}]\theta \in so(3). \end{aligned}$$

- **Six-dimensional exponential coordinates $\boldsymbol{S\theta} \in \mathbb{R}^6$ of a homogeneous transformation T, matrix exponential (exp) and matrix logarithm (log):**

$$\begin{aligned} \exp: \quad [S]\theta \in se(3) \quad &\rightarrow \quad T \in SE(3), \\ \log: \quad T \in SE(3) \quad &\rightarrow \quad [S]\theta \in se(3). \end{aligned}$$

Matrix Logarithm of Rigid-Body Motions

- **Given** an arbitrary **transformation (R; p)∈ SE(3),** one can always find a **screw axis** $\boldsymbol{S}$ = ($\boldsymbol{\omega}$; v) and a scalar $\boldsymbol{\theta}$ such that $e^{[S]\theta} = \begin{bmatrix} \boxed{R} & \boxed{p} \\ 0 & 1 \end{bmatrix}$

[$\boldsymbol{S}$] is the **4 × 4 matrix representation** of $\boldsymbol{S}$ = ($\boldsymbol{\omega}$; v)

i.e., the matrix [$\boldsymbol{S}$]$\boldsymbol{\theta}$= $\begin{bmatrix} [\omega]\theta & v\theta \\ 0 & 0 \end{bmatrix} \in se(3)$ is the matrix logarithm of T = (R; p).

- <u>**Given**</u> **(R; p)** written as T∈ SE(3), <u>find</u> a $\boldsymbol{\theta}$∈ [0; π] and a screw axis $\boldsymbol{S}$ = ($\boldsymbol{\omega}$; $\boldsymbol{v}$) ∈ $\mathbb{R}^6$ (where <u>at least one of</u> ||$\boldsymbol{\omega}$|| and ||$\boldsymbol{v}$|| is **unity**) such that $e^{[S]\theta}$ = T.

  The **vector** $\boldsymbol{S\theta}$ ∈ $\mathbb{R}^6$ comprises the **exponential coordinates for T** and the **matrix** [$\boldsymbol{S}$]$\boldsymbol{\theta}$ ∈ se(3) is the **matrix logarithm of T.**

(a) If **R = I** then set $\boldsymbol{\omega}$ = 0, $\boldsymbol{v}$ = p/||p||, and $\boldsymbol{\theta}$ = ||p||.

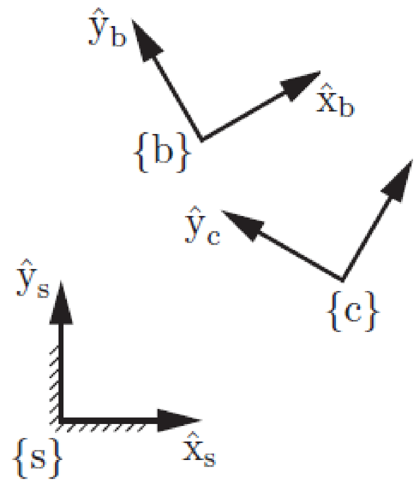
(b) Otherwise, first use the <u>**matrix logarithm on R∈ SO(3)**</u> to determine <u>$\boldsymbol{\omega}$</u> and <u>$\boldsymbol{\theta}$</u> for R. Next, $\boldsymbol{v}$ is calculated as:

$$\boxed{v} = G^{-1}(\theta)\boxed{p}$$

$$G^{-1}(\theta) = \frac{1}{\boxed{\theta}}I - \frac{1}{2}[\boxed{\omega}] + \left(\frac{1}{\boxed{\theta}} - \frac{1}{2}\cot\frac{\boxed{\theta}}{2}\right)[\boxed{\omega}]^2.$$

Example

The rigid-body motion is confined to the $\hat{\boldsymbol{x}}_s$-$\hat{\boldsymbol{y}}_s$ plane. The initial frame {b} and final frame {c} in the Figure can be represented by the following SE(3) matrices:



$$T_{sb} = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 0 & 1 \\ \sin 30^\circ & \cos 30^\circ & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_{sc} = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 & 2 \\ \sin 60^\circ & \cos 60^\circ & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

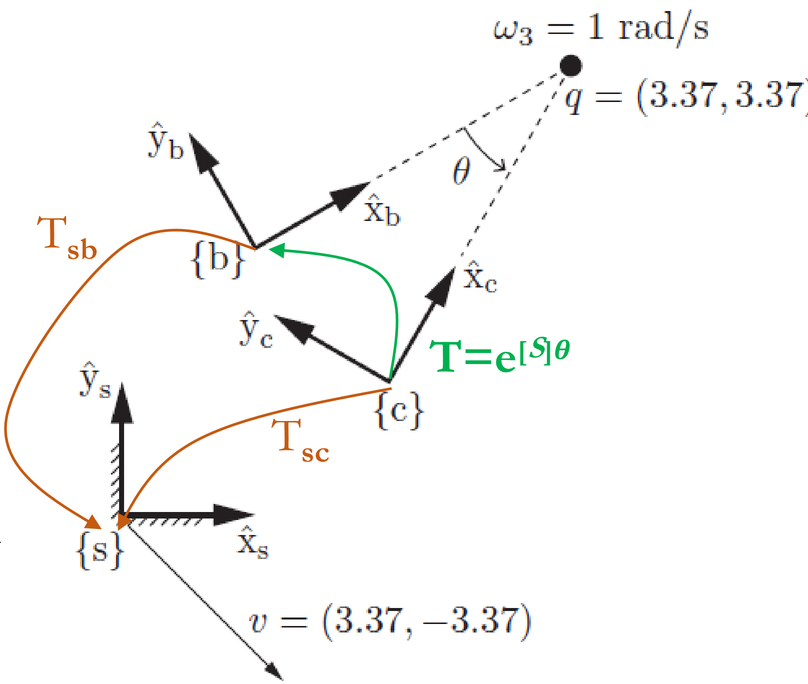
Find the **fixed frame screw motion** that displaces the frame at T_{sb} to T_{sc} .

Example

- We seek the **fixed frame screw motion** that displaces the frame at T_{sb} to T_{sc}:

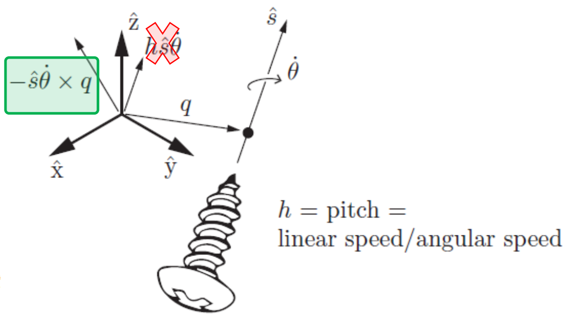
$$T_{sc} = e^{[\mathcal{S}]\theta} T_{sb} \Longrightarrow T_{sc} T^{-1}_{sb} = e^{[\mathcal{S}]\theta}$$

- Because the **motion is 2D and occurs in the $\hat{x}_s$-$\hat{y}_s$ plane**, the corresponding screw has an **axis of rotation** in the direction of the $\hat{z}_s$-axis and has **zero pitch (since it does not have a translation along the screw axis $\hat{z}_s$)**. The screw axis $\mathcal{S} = (\omega; v)$, expressed in {s}, therefore has the following form:



$$\boxed{\omega = (0, 0, \omega_3),} \qquad \boxed{v = (v_1, v_2, 0).}$$

$$[\mathcal{S}] = \begin{bmatrix} \boxed{[\omega]} & \boxed{v} \\ 0 & 0 \end{bmatrix} \in se(3),$$



$$[\mathcal{S}] = \begin{bmatrix} 0 & -\omega_3 & 0 & v_1 \\ \omega_3 & 0 & 0 & v_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad [\omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \in so(3)$$

- We can apply the **matrix logarithm algorithm** directly to $T_{sc}T^{-1}_{sb}$ to obtain $[S]$ (and therefore S) and θ as follows:

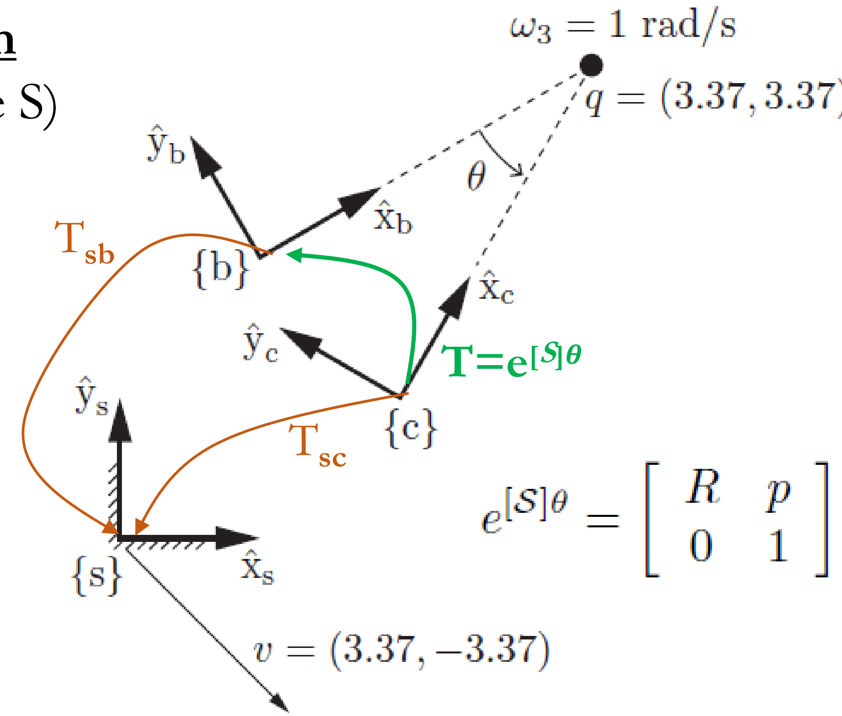
(i) We first use the matrix logarithm on **R∈ SO(3)** to determine ω and θ for R (Rodriguez formula: check W2-L1).

(ii) Then v is calculated as:

$$v = G^{-1}(\theta)p$$

$$G^{-1}(\theta) = \frac{1}{\theta}I - \frac{1}{2}[\omega] + \left(\frac{1}{\theta} - \frac{1}{2}\cot\frac{\theta}{2}\right)[\omega]^2.$$

(iii) The matrix $[S]\theta = \begin{bmatrix} [\omega]\theta & v\theta \\ 0 & 0 \end{bmatrix} \in se(3)$



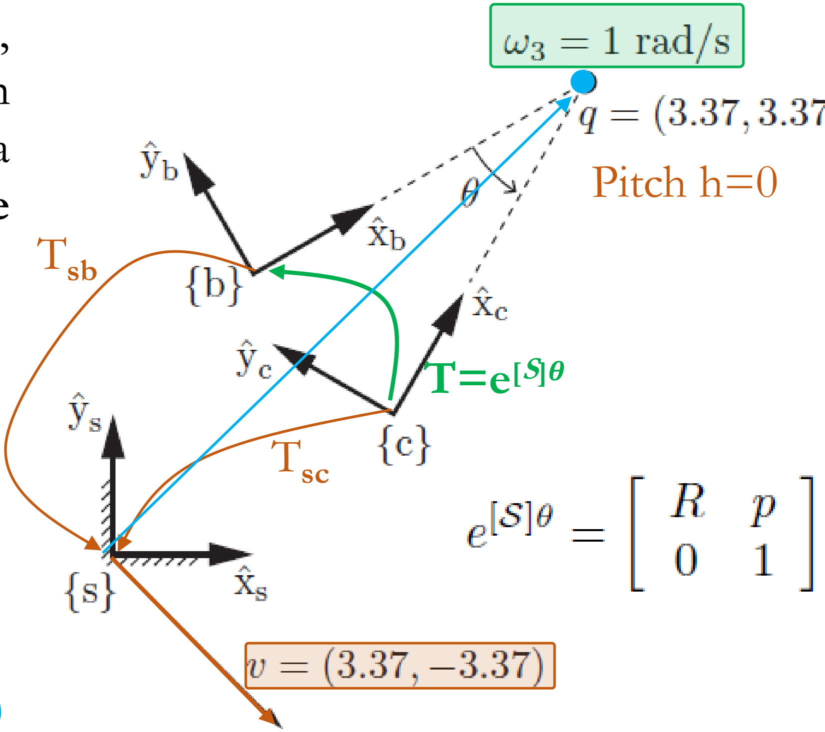
$T_{sc}T^{-1}_{sb} = e^{[S]\theta}$

$$[S] = \begin{bmatrix} 0 & -1 & 0 & 3.37 \\ 1 & 0 & 0 & -3.37 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{S} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3.37 \\ -3.37 \\ 0 \end{bmatrix}, \quad \boxed{\theta = \frac{\pi}{6} \text{ rad (or } 30^\circ).}$$

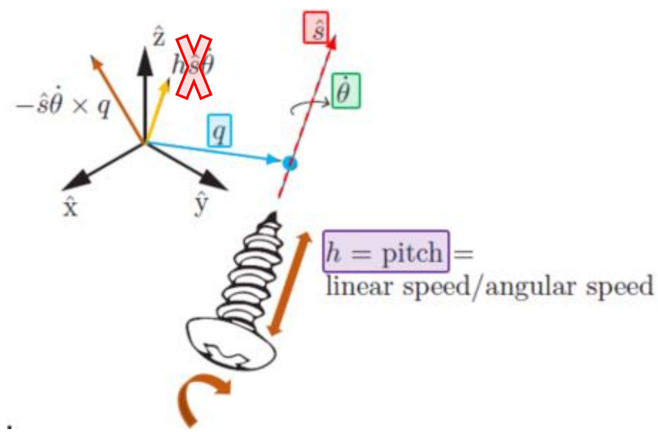
Example

- The value of $\mathcal{S}$ means that the **constant screw axis, expressed in the fixed frame {s},** is represented by an angular velocity of 1 rad/s about the $\hat{\mathbf{z}}_s$-axis and a linear velocity of (3.37;-3.37; 0) **expressed in the frame {s}.**

$$\mathcal{S} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3.37 \\ -3.37 \\ 0 \end{bmatrix}, \quad \theta = \frac{\pi}{6} \text{ rad (or } 30°).$$



- We can also <u>graphically</u> determine the point q = (q_x; q_y) in the $\hat{\boldsymbol{x}}_s$-$\hat{\boldsymbol{y}}_s$ plane <u>through which the screw axis passes</u>; for our example this point is given by q = (3.37; 3.37).



- Screw axis can either be defined by **point q**, **pitch h**, **and axis s** OR screw axis **S**.

- Transformation T can be defined using translation and rotation about screw axis!

Summary of Rigid Body Motion

Rotations	Rigid-Body Motions
$R \in SO(3)$: 3×3 matrices $R^{\mathrm{T}}R = I, \det R = 1$	$T \in SE(3)$: 4×4 matrices $T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$, where $R \in SO(3), p \in \mathbb{R}^3$
$R^{-1} = R^{\mathrm{T}}$	$T^{-1} = \begin{bmatrix} R^{\mathrm{T}} & -R^{\mathrm{T}}p \\ 0 & 1 \end{bmatrix}$
change of coordinate frame: $R_{ab}R_{bc} = R_{ac}, \quad R_{ab}p_b = p_a$	change of coordinate frame: $T_{ab}T_{bc} = T_{ac}, \quad T_{ab}p_b = p_a$
rotating a frame {b}: $R = \mathrm{Rot}(\hat{\omega}, \theta)$ $R_{sb'} = RR_{sb}$: rotate θ about $\hat{\omega}_s = \hat{\omega}$ $R_{sb''} = R_{sb}R$: rotate θ about $\hat{\omega}_b = \hat{\omega}$	displacing a frame {b}: $T = \begin{bmatrix} \mathrm{Rot}(\hat{\omega}, \theta) & p \\ 0 & 1 \end{bmatrix}$ $T_{sb'} = TT_{sb}$: rotate θ about $\hat{\omega}_s = \hat{\omega}$ (moves {b} origin), translate p in {s} $T_{sb''} = T_{sb}T$: translate p in {b}, rotate θ about $\hat{\omega}$ in new body frame
unit rotation axis is $\hat{\omega} \in \mathbb{R}^3$, where $\|\hat{\omega}\| = 1$	"unit" screw axis is $\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$, where either (i) $\|\omega\| = 1$ or (ii) $\omega = 0$ and $\|v\| = 1$
	for a screw axis $\{q, \hat{s}, h\}$ with finite h, $\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} = \begin{bmatrix} \hat{s} \\ -\hat{s} \times q + h\hat{s} \end{bmatrix}$
angular velocity is $\omega = \hat{\omega}\dot{\theta}$	twist is $\mathcal{V} = \mathcal{S}\dot{\theta}$

W1-L2
W2-L1
W3-L2
W3-L1
W5-L1

Summary of Rigid Body Motion

W4-L1 | W4-L2

Rotations (cont.)	Rigid-Body Motions (cont.)
for any 3-vector, e.g., $\omega \in \mathbb{R}^3$, $[\omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \in so(3)$ identities, $\omega, x \in \mathbb{R}^3$, $R \in SO(3)$: $[\omega] = -[\omega]^{\mathrm{T}}$, $[\omega]x = -[x]\omega$, $[\omega][x] = ([x][\omega])^{\mathrm{T}}$, $R[\omega]R^{\mathrm{T}} = [R\omega]$	for $\mathcal{V} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$, $[\mathcal{V}] = \begin{bmatrix} [\omega] & v \\ 0 & 0 \end{bmatrix} \in se(3)$ (the pair (ω, v) can be a twist $\mathcal{V}$ or a "unit" screw axis $\mathcal{S}$, depending on the context)
$\dot{R}R^{-1} = [\omega_s]$, $R^{-1}\dot{R} = [\omega_b]$	$\dot{T}T^{-1} = [\mathcal{V}_s]$, $T^{-1}\dot{T} = [\mathcal{V}_b]$
	$[\mathrm{Ad}_T] = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \in \mathbb{R}^{6\times 6}$ identities: $[\mathrm{Ad}_T]^{-1} = [\mathrm{Ad}_{T^{-1}}]$, $[\mathrm{Ad}_{T_1}][\mathrm{Ad}_{T_2}] = [\mathrm{Ad}_{T_1 T_2}]$
change of coordinate frame: $\hat{\omega}_a = R_{ab}\hat{\omega}_b$, $\omega_a = R_{ab}\omega_b$	change of coordinate frame: $\mathcal{S}_a = [\mathrm{Ad}_{T_{ab}}]\mathcal{S}_b$, $\mathcal{V}_a = [\mathrm{Ad}_{T_{ab}}]\mathcal{V}_b$
exp coords for $R \in SO(3)$: $\hat{\omega}\theta \in \mathbb{R}^3$	exp coords for $T \in SE(3)$: $\mathcal{S}\theta \in \mathbb{R}^6$
$\exp : [\hat{\omega}]\theta \in so(3) \to R \in SO(3)$ $R = \mathrm{Rot}(\hat{\omega}, \theta) = e^{[\hat{\omega}]\theta} =$ $I + \sin\theta[\hat{\omega}] + (1 - \cos\theta)[\hat{\omega}]^2$	$\exp : [\mathcal{S}]\theta \in se(3) \to T \in SE(3)$ $T = e^{[\mathcal{S}]\theta} = \begin{bmatrix} e^{[\omega]\theta} & * \\ 0 & 1 \end{bmatrix}$ where $* =$ $(I\theta + (1 - \cos\theta)[\omega] + (\theta - \sin\theta)[\omega]^2)v$
$\log : R \in SO(3) \to [\hat{\omega}]\theta \in so(3)$ algorithm in Section 3.2.3.3	$\log : T \in SE(3) \to [\mathcal{S}]\theta \in se(3)$ algorithm in Section 3.3.3.2

W2-L2 | W5-L1

References

- Murray, R.M., Li, Z., Sastry, S.S., "*A Mathematical Introduction to Robotic Manipulation.*", **Chapter 2**.

- Corke, Peter. "Robotics, vision and control: fundamental algorithms in MATLAB®" second, completely revised. Vol. 118. Springer, 2017, **Chapter 2.**

- Lynch and Park, "*Modern Robotics,*" Cambridge U. Press, 2017, **Chapter 3.**