



Homework/Programming Assignment #2

Homework/midterm Due: 04/05/2022- 5:00 PM

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Signature (required)

I/We have followed the rules in completing this Assignment.

Bryant Zhou

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Signature (required)

I/We have followed the rules in completing this Assignment.

Jonathan Salfity

Question	Points	Total
HA 1	25	
HA 2	25	
HA 3	25	
HA 4	25	
PA	100	
PA. k (Bonus)	15	
PA. m (Bonus)	30	
Presentation* (Bonus)	20	

Instruction:

- Remember that this is a graded assignment. It is the equivalent of a **midterm take-home exam**.
- * You should present the results of the PA in the class** and receive extra bonus depending on the quality of your presentation!
- For PA questions, you need to write a report showing how you derived your equations, describes your approach, test functions, and discusses the results.** You should show your test results for each function.
- You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
- It is open book, notes, and web. But you should cite any references you consult.
- Unless I say otherwise in class, it is due before the start of class on the due date mentioned in the Assignment.
- Sign and append** this score sheet as the first sheet of your assignment.
- Remember to submit your assignment in Canvas.

HA: 4.8, 4.11, 5.12, 5.13

$$M = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & L_3 + L_4 \\ 0 & 0 & 1 & -L_5 - L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	w_i	q_i	v_i
1	$(1, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
2	$(0, 0, -1)$	$(L_1, 0, 0)$	$(0, L_1, 0)$
3	$(0, 1, 0)$	$(L_1, 0, L_2)$	$(-L_2, 0, L_1)$
4	$(1, 0, 0)$	$(L_1, L_3, 0)$	$(0, 0, -L_3)$
5	$(0, 0, 0)$	$(L_1, L_3 + L_4, 0)$	$(0, 1, 0)$
6	$(0, 1, 0)$	$(L_1, L_3 + L_4, -L_5)$	$(L_5, 0, L_1)$

$$s_1 = [1, 0, 0, 0, 0, 0]^T$$

$$s_2 = [0, 0, -1, 0, L_1, 0]^T$$

$$s_3 = [0, 1, 0, -L_2, 0, L_1]^T$$

$$s_4 = [1, 0, 0, 0, 0, -L_3]^T$$

$$s_5 = [0, 0, 0, 0, 1, 0]^T$$

$$s_6 = [0, 1, 0, L_5, 0, L_1]^T$$

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 & -L_1 \\ 0 & 1 & 0 & -L_3 - L_4 \\ 0 & 0 & 1 & L_5 + L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ad_{M^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -L_5 - L_6 & -L_3 - L_4 & 1 & 0 & 0 & 0 \\ L_5 + L_6 & 0 & L_1 & 0 & 1 & 0 & 0 \\ L_3 + L_4 & -L_1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_i = Ad_{M^{-1}} \zeta_i$$

$$B_1 = [1, 0, 0, 0, L_5 + L_6, L_3 + L_4]^T$$

$$B_2 = [0, 0, -1, L_3 + L_4, 0, 0]^T$$

$$B_3 = [0, 1, 0, -L_2 - L_3 - L_6, 0, 0]^T$$

$$B_4 = [1, 0, 0, 0, L_5 + L_6, L_4]^T$$

$$B_5 = [0, 0, 0, 0, 1, 0]^T$$

$$B_6 = [0, 1, 0, -L_6, 0]^T$$

4.11

$$M = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	w_i	z_i	v_i
1	$(0, 0, 1)$	$(0, 0, 0)$	$(0, 0, 0)$
2	$(0, 0, 0)$	$(0, 0, 0)$	$(1, 0, 0)$
3	$(0, 0, 1)$	$(1, 0, 0)$	$(0, -1, 0)$
4	$(0, -1, 0)$	$(1, 0, -1)$	$(-1, 0, -1)$
5	$(-1/\sqrt{2}, 0, 1/\sqrt{2})$	$(2, 0, -1)$	$(0, -1/\sqrt{2}, 0)$

$$s_1 = [0, 0, 1, 0, 0, 0]^T$$

$$s_2 = [0, 0, 0, 1, 0, 0]^T$$

$$s_3 = [0, 0, 1, 0, -1, 0]^T$$

$$s_4 = [0, -1, 0, -1, 0, -1]^T$$

$$s_5 = [-1/\sqrt{2}, 0, 1/\sqrt{2}, 0, -1/\sqrt{2}, 0]^T$$

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ad_{n^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 1 & 0 \\ 0 & -3 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_i = Ad_{n^{-1}} s_i$$

$$B_1 = [0, 0, 1, 0, 3, 0]^T$$

$$B_2 = [0, 0, 0, 1, 0, 0]^T$$

$$B_3 = [0, 0, 1, 0, 2, 0]^T$$

$$B_4 = [0, -1, 0, -1, 0, 2]^T$$

$$B_5 = [-1/\sqrt{2}, 0, 1/\sqrt{2}, 0, \sqrt{2}, 0]^T$$

5.12a) i	w_i	z_i	v_i
1	$(0, 0, 1)$	$(0, -L-\theta_4, -L)$	$(-L-\theta_4, 0, 0)$
2	$(1, 0, 0)$	$(0, -L-\theta_4, 0)$	$(0, 0, L+\theta_4)$
3	$(0, 0, 1)$	$(0, -\theta_4, 0)$	$(-\theta_4, 0, 0)$
4	$(0, 0, 0)$		$(0, 1, 0)$

$$J_{B_4} = B_4 = [0, 0, 0, 0, 1, 0]^T$$

$$J_{B_3} = \text{Ad}_{e^{-[B_4]\theta_4}} B_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -L & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ L & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -\theta_4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -L-\theta_4 \\ 0 \\ 0 \end{bmatrix}$$

$$J_{B_2} = \text{Ad}_{e^{-[B_4]\theta_4} e^{-[B_3]\theta_3}} B_2$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -L-\theta_4 & 0 & 1 & 0 \\ 0 & 0 & \theta_4 & -1 & 0 & 0 \\ L+\theta_4 & -\theta_4 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ L+\theta_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 2L+2\theta_4 \end{bmatrix}$$

$$J_{B_1} = Ad_{e^{-[B_4]\theta_4}} e^{-[B_3]\theta_3} e^{-[B_2]\theta_2} B_1$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -L-\theta_4 & 0 & 1 & 0 \\ 0 & 0 & \theta_4 & -1 & 0 & 0 \\ L+\theta_4 & -\theta_4 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -L-\theta_4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -L-\theta_4 \\ L+2\theta_4 \\ 0 \end{bmatrix}$$

$$J_b = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -L-\theta_4 & 0 & -L-\theta_4 & 0 \\ L+2\theta_4 & 0 & 0 & 1 \\ 0 & 2L+2\theta_4 & 0 & 0 \end{bmatrix}$$

b)

$$V_b = J_b \dot{\theta} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -L-\theta_4 & 0 & -L-\theta_4 & 0 \\ L+2\theta_4 & 0 & 0 & 1 \\ 0 & 2L+2\theta_4 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \\ -2L-2\theta_4 \\ L+2\theta_4+1 \\ 2L+2\theta_4 \end{bmatrix}$$

$$\dot{p} = Ad_{T_{sb}} V_b$$

$$T_{sb} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L+\theta_f \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{p} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -L & L+\theta_f & 1 & 0 & 0 & 0 \\ L & 0 & 0 & 0 & 1 & 0 & 0 \\ -L-\theta_f & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 2 \\ -2L-2\theta_f \\ L+2\theta_f+1 \\ 2L+2\theta_f \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \\ L \\ L+2\theta_f+1 \\ -2L+2\theta_f \end{bmatrix}$$

5.13(a)	i	ω_i	q_i	v_i
	1	$(0,0,1)$		$(0,0,0)$
	2	$(0,1,0)$		$(0,0,0)$
	3	$(-1,0,0)$		$(0,0,0)$
	4	$(-1,0,0)$	$(0,2L,0)$	$(0,0,2L)$
	5	$(-1,0,0)$	$(0,3L,0)$	$(0,0,3L)$
	6	$(0,1,0)$		$(0,0,0)$

$$[S_1] = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S_1 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[S_2] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[S_3] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[S_4] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 2L \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2L \end{bmatrix}$$

$$[S_5] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 3L \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S_5 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 3L \end{bmatrix}$$

$$[S_6] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S_6 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Each column of the Jacobian matrix can be calculated as below with each joint angle

$$J_{s_1} = S_1$$

$$J_{s_2} = Ad_{e^{[S_1]\theta_1}} S_2$$

$$J_{s_3} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3$$

$$J_{s_4} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3}} S_4$$

$$J_{s_5} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} e^{[S_4]\theta_4}} S_5$$

$$J_{s_6} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} e^{[S_4]\theta_4} e^{[S_5]\theta_5}} S_6$$

b) Kinematic singularities

① Colinear :

When $\theta_3 = \theta_4 = \theta_5 = 0$, joints 2 and 6 are colinear. From the above calculation,

$$J_{s_3} = \text{Ad}_{e^{[s_1]\theta_1}} e^{[s_2]\theta_2} s_3$$

$$J_{s_4} = \text{Ad}_{e^{[s_1]\theta_1}} e^{[s_2]\theta_2} s_4$$

$$J_{s_5} = \text{Ad}_{e^{[s_1]\theta_1}} e^{[s_2]\theta_2} s_5$$

$$J_{s_6} = \text{Ad}_{e^{[s_1]\theta_1}} e^{[s_2]\theta_2} s_6$$

Based on the representation of s_3, \dots, s_6 , it's easy to see that J is singular.

② Coplanar & Parallel :

When $\theta_3 = \theta_4 = \theta_5 = 0$, joints 3, 4, and 5 are coplanar and parallel

$$J_{s_{3-5}} = \begin{bmatrix} \omega & \omega & \omega \\ -\omega \times q_3 & -\omega \times q_4 & -\omega \times q_5 \end{bmatrix}$$

This Jacobian cannot be full rank.

Thus, when $\theta_3 = \theta_4 = \theta_5 = 0$, the end effector loses freedom in the y - and z -directions