Homework/Programming Assignment #2 **Homework/midterm Due: 04/05/2022- 5:00 PM**

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Signature (*required*)
I/We have followed the rules in completing this Assignment.

Bryant

Name/EID:
Email:

Signature (*required*)
I/We have followed the rules in completing this Assignment.

Question	Points	Total
HA 1	25	
HA 2	25	
HA 3	25	
HA 4	25	
PA	100	
PA. k (Bonus)	15	
PA. m (Bonus)	30	
Presentation* (Bonus)	20	

Instruction:

1. Remember that this is a graded assignment. It is the equivalent of a **midterm take-home exam**.
2. *** You should present the results of the PA in the class** and receive extra bonus depending on the quality of your presentation!
3. **For PA questions**, you need to write a report showing how you derived your equations, describes your approach, test functions, and discusses the results. You should show your test results for each function.

3. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
4. It is open book, notes, and web. But you should cite any references you consult.
5. Unless I say otherwise in class, it is due before the start of class on the due date mentioned in the Assignment.
6. **Sign and append** this score sheet as the first sheet of your assignment.
7. Remember to submit your assignment in Canvas.

HA: 4.8, 4.11, 5.12, 5.13

4.8

$$M = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & L_3+L_4 \\ 0 & 0 & 1 & -L_5-L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	ω_i	q_i	v_i
1	$(1,0,0)$	$(0,0,0)$	$(0,0,0)$
2	$(0,0,-1)$	$(L_1,0,0)$	$(0,L_1,0)$
3	$(0,1,0)$	$(L_1,0,L_2)$	$(-L_2,0,L_1)$
4	$(1,0,0)$	$(L_1,L_3,0)$	$(0,0,-L_3)$
5	$(0,0,0)$	$(L_1,L_3+L_4,0)$	$(0,1,0)$
6	$(0,1,0)$	$(L_1,L_3+L_4,-L_5)$	$(L_5,0,L_1)$

$S_1 = [1,0,0,0,0,0]^T$

$S_2 = [0,0,-1,0,L_1,0]^T$

$S_3 = [0,1,0,-L_2,0,L_1]^T$

$S_4 = [1,0,0,0,0,-L_3]^T$

$S_5 = [0,0,0,0,1,0]^T$

$S_6 = [0,1,0,L_5,0,L_1]^T$

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 & -L_1 \\ 0 & 1 & 0 & -L_3-L_4 \\ 0 & 0 & 1 & L_5+L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ad_{M^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -L_5-L_6 & -L_3-L_4 & 1 & 0 & 0 \\ L_5+L_6 & 0 & L_1 & 0 & 1 & 0 \\ L_3+L_4 & -L_1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_i = Ad_{M^{-1}} S_i$$

$B_1 = [\ 1, 0, 0, 0,\ \ L_5+L_6,\ \ L_3+L_4\ ]^T$

$B_2 = [0, 0, -1,\ L_3+L_4,\ 0, 0]^T$

$B_3 = [0, 1,\ 0,\ -L_2-L_5-L_6\ , 0, 0]^T$

$B_4 = [1, 0, 0, 0, L_5+L_6,\ L_4]^T$

$B_5 = [0, 0, 0, 0, 1, 0]^T$

$B_6 = [0, 1, 0,\ -L_6,\ 0\ ]^T$

4.11

$$M = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	ω_i	q_i	v_i
1	$(0,0,1)$	$(0,0,0)$	$(0,0,0)$
2	$(0,0,0)$	$(0,0,0)$	$(1,0,0)$
3	$(0,0,1)$	$(1,0,0)$	$(0,-1,0)$
4	$(0,-1,0)$	$(1,0,-1)$	$(-1,0,-1)$
5	$(-1/\sqrt{2},0,1/\sqrt{2})$	$(2,0,-1)$	$(0,-1/\sqrt{2},0)$

$S_1 = [0,0,1,0,0,0]^T$

$S_2 = [0,0,0,1,0,0]^T$

$S_3 = [0,0,1,0,-1,0]^T$

$S_4 = [0,-1,0,-1,0,-1]^T$

$S_5 = [-1/\sqrt{2},0,1/\sqrt{2},0,-1/\sqrt{2},0]^T$

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
Ad_{M^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 1 & 0 \\ 0 & -3 & 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$B_i = Ad_{M^{-1}} S_i$$

$B_1 = [0, 0, 1, 0, 3, 0]^T$
$B_2 = [0, 0, 0, 1, 0, 0]^T$
$B_3 = [0, 0, 1, 0, 2, 0]^T$
$B_4 = [0, -1, 0, -1, 0, 2]^T$
$B_5 = [-1/\sqrt{2}, 0, 1/\sqrt{2}, 0, \sqrt{2}, 0]^T$

5.12 a)

i	ω_i	q_i	v_i
1	$(0,0,1)$	$(0,-L-\theta_4,-L)$	$(-L-\theta_4,0,0)$
2	$(1,0,0)$	$(0,-L-\theta_4,0)$	$(0,0,L+\theta_4)$
3	$(0,0,1)$	$(0,-\theta_4,0)$	$(-\theta_4,0,0)$
4	$(0,0,0)$		$(0,1,0)$

$$J_{B_4} = B_4 = [0,0,0,0,1,0]^T$$

$$J_{B_3} = Ad_{e^{-[B_4]\theta_4}} B_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -L & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ L & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -\theta_4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -L-\theta_4 \\ 0 \\ 0 \end{bmatrix}$$

$$J_{B_2} = Ad_{e^{-[B_4]\theta_4} e^{-[B_3]\theta_3}} B_2$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -L-\theta_4 & 0 & 1 & 0 \\ 0 & 0 & \theta_4 & -1 & 0 & 0 \\ L+\theta_4 & -\theta_4 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ L+\theta_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 2L+2\theta_4 \end{bmatrix}$$

$$J_{B_1} = Ad_{e^{-[B_4]\theta_4} e^{-[B_3]\theta_3} e^{-[B_2]\theta_2}} B_1$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -L-\theta_4 & 0 & 1 & 0 \\ 0 & 0 & \theta_4 & -1 & 0 & 0 \\ L+\theta_4 & -\theta_4 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -L-\theta_4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -L-\theta_4 \\ L+2\theta_4 \\ 0 \end{bmatrix}$$

$$J_b = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -L-\theta_4 & 0 & -L-\theta_4 & 0 \\ L+2\theta_4 & 0 & 0 & 1 \\ 0 & 2L+2\theta_4 & 0 & 0 \end{bmatrix}$$

b)

$$\mathcal{V}_b = J_b \dot{\theta} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -L-\theta_4 & 0 & -L-\theta_4 & 0 \\ L+2\theta_4 & 0 & 0 & 1 \\ 0 & 2L+2\theta_4 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \\ -2L-2\theta_4 \\ L+2\theta_4+1 \\ 2L+2\theta_4 \end{bmatrix}$$

$$\dot{p} = Ad_{T_{sb}} \mathcal{V}_b$$

$$T_{sb} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L+\theta_4 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{p} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -L & L+\theta_4 & 1 & 0 & 0 \\ L & 0 & 0 & 0 & 1 & 0 \\ -L-\theta_4 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 2 \\ -2L-2\theta_4 \\ L+2\theta_4+1 \\ 2L+2\theta_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \\ L \\ L+2\theta_4+1 \\ 2L+2\theta_4 \end{bmatrix}$$

5.13(a)

i	ω_i	q_i	v_i
1	$(0,0,1)$		$(0,0,0)$
2	$(0,1,0)$		$(0,0,0)$
3	$(-1,0,0)$		$(0,0,0)$
4	$(-1,0,0)$	$(0,2L,0)$	$(0,0,2L)$
5	$(-1,0,0)$	$(0,3L,0)$	$(0,0,3L)$
6	$(0,1,0)$		$(0,0,0)$

$$[S_1] = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[S_2] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[S_3] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad S_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[S_4] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 2L \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad S_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2L \end{bmatrix}$$

$$[S_5] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 3L \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S_5 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 3L \end{bmatrix}$$

$$[S_6] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Each column of the Jacobian matrix can be calculated as below with each joint angle

$$J_{S_1} = S_1$$

$$J_{S_2} = Ad_{e^{[S_1]\theta_1}} S_2$$

$$J_{S_3} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3$$

$$J_{S_4} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3}} S_4$$

$$J_{S_6} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} e^{[S_4]\theta_4}} S_5$$

$$J_{S_6} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} e^{[S_4]\theta_4} e^{[S_5]\theta_5}} S_6$$

b) Kinematic singularities

① Colinear:

when $\theta_3 = \theta_4 = \theta_5 = 0$, joints 2 and 6 are colinear. From the above calculation,

$$J_{S_3} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3$$

$$J_{S_4} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_4$$

$$J_{S_6} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_5$$

$$J_{S_6} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_6$$

Based on the representation of $S_3, \cdots, S_6$, it's easy to see that J is singular.

② Coplanar & Parallel:

when $\theta_3 = \theta_4 = \theta_5 = 0$, joints 3, 4, and 5 are coplanar and parallel

$$J_{S_{3\text{-}5}} = \begin{bmatrix} \omega & \omega & \omega \\ -\omega \times q_3 & -\omega \times q_4 & -\omega \times q_5 \end{bmatrix}$$

This Jacobian cannot be full rank.

Thus, when $\theta_3 = \theta_4 = \theta_5 = 0$, the end effector losses freedom in the y- and z-directions