Modeling and Analysis of a First-Order Temperature-Controlled System

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***Abstract* – This laboratory experiment aims to develop a mathematical model for the tested control system in the laboratory. The experiment results were used to derive the system's plant model. The actual experimental data was then compared to the model's data to evaluate if there is any error and how well the two data aligned with each other. The results of the experiment and with the error valuation indicate that the derived plant model closely aligns with the original data set, resulting in a very low percent error for the system.**

I. Introduction

In order for appliances such as air conditioners and refrigerators to maintain a certain output temperature, these appliances must utilize a temperature-control system that can adjust the output temperature to the desired levels through feedback. To accomplish this, the system must be able to read the temperature of the given environment. In the laboratory exercise, the students are tasked to create a temperature sensor system that can read the temperature of a brake lamp and output voltage that is equivalent to the said temperature.

II. Methodology

The circuit provided for the laboratory exercise is a simple temperature-controlled system. The BJT circuit is a common-emitter circuit such that the load is a 12V brake lamp, while the sensor used is an LM35 analog temperature sensor whose output is connected to the inverting input terminal of a non-inverting amplifier with a gain of 10. The output of the amplifier is read as V_{temp} as shown below.

The resistance of the resistor connected to the BJT Circuit is selected such that the voltage across the bulb is around 8V. Afterwards, the LM35 temperature sensor is connected to +12V DC and ground – and is mounted directly to the glass surface of the bulb. The output of the sensor is then amplified by the x10 non-inverting amplifier for easier data reading of V_{temp}. The resistor in the BJT circuit is then removed and the entire system is allowed to reach ambient temperature such that V_{temp} is 2.67V. The resistor is then reconnected such that an input step of 3V is emulated, and the system is allowed to reach steady state V_{temp} of around 4.75V which takes around 18 minutes.

II. Results And Discussion

A. Bulb Temperature vs Time

The ambient temperature V_{temp} is measured at 2.67V or 26.7°C which was measured such that the bulb had cooled down due to disconnecting the base resistor of the BJT circuit. Data points are collected in one-minute intervals up until the difference between the last three data points is less than or equal to 0.5°C. When the difference between the last three data points is now less than or equal to 0.5°C, it is safe to assume that the system has reached steady state. The plots of the bulb temperature in voltage Vtemp and the bulb temperature in Celsius versus time are shown in figures 1 and 2.

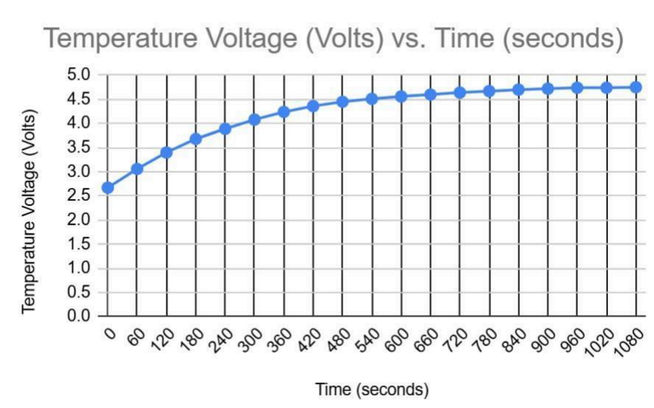

Fig. 1 Plot of temperature in voltage vs. time in seconds

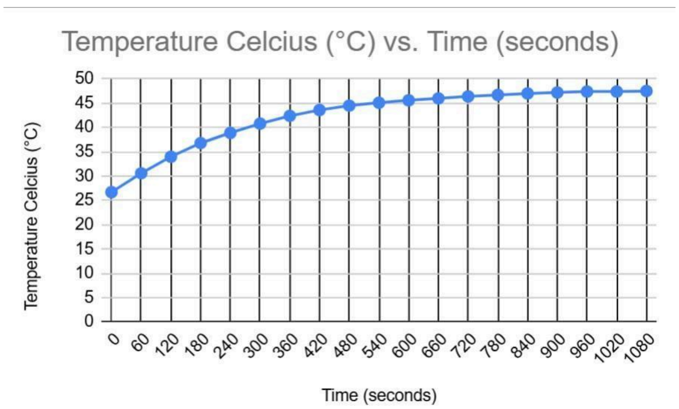

Fig. 2 Plot of temperature in celsius vs. time in seconds

B. Derivation of Plant Model

The system used for the laboratory exercise can be modeled as a first-order system such that k and a are constants in the transfer function:

$$G(s) = \frac{k}{s+a} \quad (1)$$

When applying the 3V step input to the system, the output will be:

$$Y(s) = X(s)G(s) \quad (2)$$

$$Y(s) = \left(\frac{3}{s}\right)\left(\frac{k}{s+a}\right) \quad (3)$$

$$Y(s) = \frac{3k}{s(s+a)} \quad (4)$$

Using inverse Laplace transform, to the output, the time-domain equation that describes the output is:

$$L^{-1}[Y(s)] = y(t) = \frac{3k}{a}\left(1 - e^{-at}\right) \quad (5)$$

To define the values of the constants k and a in the time-domain output equation, the voltages at 10% and 90% of the difference between the steady state voltage and ambient temperature voltage must be computed:

$$V_{0.9} = (0.9)(4.75V - 2.67V) = 1.872V \quad (6)$$

$$V_{0.1} = (0.1)(4.75V - 2.67V) = 0.208V \quad (7)$$

Given the 10% and 90% steady state voltage differences, adding the ambient temperature voltage 2.67V to the 10% and 90% steady state voltage differences yields the corresponding bulb temperature voltages which can be used to obtain the $t_{0.9}$ and $t_{0.1}$ from the Bulb Temperature vs Time plots – that is, ≈ 690s and ≈ 30s respectively. Using these values to solve for the rise time tr and constant a, we have:

$$t_r = t_{0.9} - t_{0.1} \approx \frac{2.2}{a} \quad (8)$$

$$690 - 30 \approx \frac{2.2}{a} \quad (9)$$

$$660 \approx \frac{2.2}{a} \quad (10)$$

$$a \approx 0.00333 \quad (11)$$

Considering our system which is a first-order system, we can analyze the response of the system [1] such that:

$$y(t) = y(\infty)\left(1 - e^{-at}\right) = \frac{3k}{a}\left(1 - e^{-at}\right) \quad (12)$$

$$y(\infty) = \frac{3k}{a} \quad (13)$$

Substituting y(∞) with the steady state difference 2.08V and a≈0.00333 and adding the ambient temperature voltage 2.67V to the output, we have:

$$y(t) = 2.08\left(1 - e^{-0.00333t}\right) + 2.67 \quad (14)$$

For the plot of the derived model from equation 14 see figure 3 below.

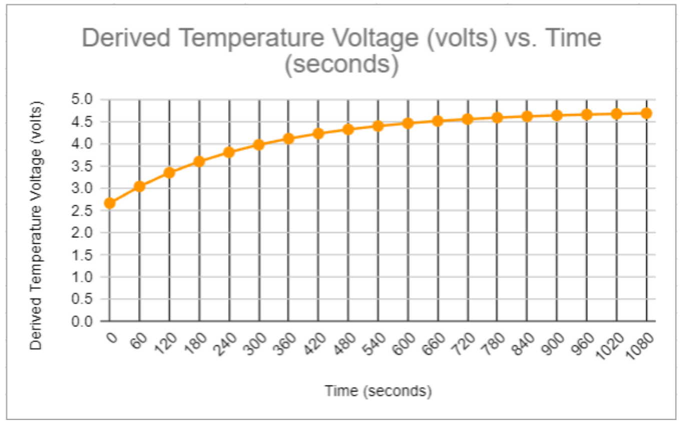

Fig. 3 Plot of derived temperature in voltage vs. time in seconds

For clarity, the following figure (4) is a superimposed plot of the actual temperature voltage vs time and the derived temperature voltage vs time.

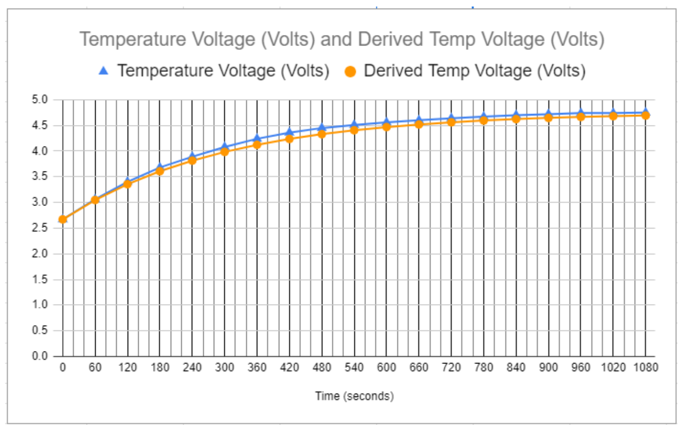

Fig. 4 Superimposed plot of actual and derived temperature in voltage vs. time in seconds

C. Relative Error Percentage Calculation

Considering the deviation of the actual V_{temp} from the derived plant model V_{temp} data points, the relative error percentage calculation can be computed. This will ensure that the actual data points are within acceptable range of error:

$$\varepsilon_t = \left|\frac{Actual\ Value - Derived\ Value}{Derived\ Value}\right| \times 100\% \quad (15)$$

Each data point equivalent in the actual bulb temperature voltage and derived bulb temperature voltages are computed for the corresponding relative error percentages, which are then plotted in figure 5.

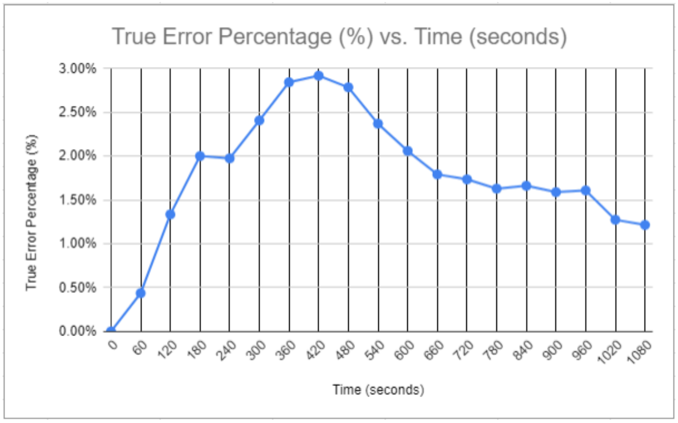

Fig. 5 True error percentage of the actual and derived model vs. time in seconds

The data plot shows that the typical value of percentage error is less than 3%, with the maximum percentage error of 2.92% occurring at around 420s. After the 420s mark, the error slowly decreases as the actual data points start to reach values near the derived data points.

IV. Conclusion

The temperature-controlled system actual data points were used to derive a plant model using relevant analog control system design concepts. The transfer function obtained was then converted into a time-domain equation that models the actual BJT circuit and sensor system. It can be concluded that the actual system in the laboratory exercise nearly matches the transfer function model since the data points of the actual system are very close to the values of the derived plant model data points such that the relative true error percentage peaks at 2.92%.

V. References

[1] T. W. Kerlin, L. F. Miller, H. M. Hashemian, W. P. Poore, M. Skorska, B. R. Upadhyaya, P. Cormault, and J. P. Jacquot, “Temperature sensor response characterization. Final report. [PWR],” 1980.

“We affirm that we have upheld the highest principles of integrity and honor in our academic work, We swear upon my honor that we have neither given nor received aid in this laboratory report. ”

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Appendix

Time (seconds)	Temperature Voltage (Volts)
0	2.67
60	3.06
120	3.4
180	3.68
240	3.89
300	4.08
360	4.24
420	4.36
480	4.45
540	4.51
600	4.56
660	4.6
720	4.64
780	4.67
840	4.7
900	4.72
960	4.74
1020	4.74
1080	4.75

Table 1: Actual Data of Temperature Voltage and Time in Seconds

Time (seconds)	Temperature Celsius (°C)
0	26.7
60	30.6
120	34
180	36.8
240	38.9
300	40.8
360	42.4
420	43.6
480	44.5
540	45.1
600	45.6
660	46
720	46.4
780	46.7
840	47
900	47.2
960	47.4
1020	47.4
1080	47.5

Table 2: Actual Data of Temperature in Celcius and Time in Seconds

Time (seconds)	Derived Temp Voltage (Volts)
0	2.67
60	3.046699179
120	3.355176111
180	3.607786214
240	3.814647269
300	3.984044676
360	4.122763301
420	4.236359239
480	4.329382341
540	4.40555846
600	4.467938674
660	4.519021496
720	4.560852944
780	4.595108493
840	4.623160177
900	4.646131551
960	4.664942685
1020	4.680347022
1080	4.692961551

Table 3: Derived Data of Temperature Voltage and Time in Seconds

Time (seconds)	True Error Percentage (%)
0	0.00%
60	0.44%
120	1.34%
180	2.00%
240	1.98%
300	2.41%
360	2.84%
420	2.92%
480	2.79%
540	2.37%
600	2.06%
660	1.79%
720	1.74%
780	1.63%
840	1.66%
900	1.59%
960	1.61%
1020	1.27%
1080	1.22%

Table 4: True Error Percentage of the Actual and Derived Temperature Voltage and Time in Seconds