# 3H Inference
# Tutorial 1

*These questions relate to lectures 1-6.*

1. Suppose we have data $x_1, \ldots, x_n$ obtained as a random sample from the Ga$(2, \theta)$ distribution, with density function
$$f(x; \theta) = \theta^2 x e^{-\theta x}, x > 0.$$
   (a) Write down the log-likelihood function for $\theta$, and use this to find the maximum likelihood estimate of $\theta$ in terms of the observed values $x_1, \ldots, x_n$, making sure to check that you have found a maximum likelihood estimate.
   (b) Construct an approximate 95% confidence interval for $\theta$ by the Wald method.
   (c) Suppose that your interval for $\theta$ in part (b) has been calculated in the form $(a, b)$, where $a$ and $b$ are constants. Explain how you would construct an approximate 95% Wald interval for the probability that the random variable $X$ is greater than $t$, where $X$ follows a Ga$(2, \theta)$ distribution.

2. (a) We have 10 observations from a U$(0, \alpha)$ distribution, given as:
$$13.0, 14.9, 12.0, 13.5, 13.4, 13.0, 12.6, 10.3, 11.6, 11.4.$$
   Find the maximum likelihood estimate of $\alpha$. (Hint: think carefully about what happens when $x_i$ lies outside the range of the distribution.)
   (b) An ornithologist is studying the number of eggs $X$ laid by a particular seabird. A clutch of eggs is identified through the incubation behaviour of the mother but this means that birds which have failed to lay any eggs cannot easily be identified. As a result a truncated Poisson model for $X$ has been proposed, with probability mass function
$$p_x(x) = \frac{1}{1 - e^{-\theta}} \frac{e^{-\theta}\theta^x}{x!},$$
   where $x$ is a positive integer and $\theta > 0$.
      i. Data $x_1, \ldots, x_n$ have been observed. Write down the log-likelihood function for $\theta$ and find its first and second derivatives. Comment on whether it is possible to find the maximum likelihood estimate explicitly.
      ii. Explain how Newton's method can be used to locate the maximum likelihood estimate numerically. In particular, write down the updating equation which constructs a new proposal for the maximum likelihood estimate from the current one and the first and second derivatives of the log-likelihood.