Funding NIMo

This is a specification of a toy balance sheet simulation model, an LP/NLP optimization problem formulation, and a numerically optimized execution for the constrained maximization of the Net Interest Margin/ Net Interest Revenue (NIM/NIR) over a given holding period using Federal Reserve historical data. Depending on the available computational resources and the availability of clean up-to-date accrual portfolio contract details, the NIM optimization can be run at the Firm or the entire market level. The novel idea investigated here is contract-by-contract balance and return rate quantitative modeling of an entire accrual portfolio used as a basis for a Firm or market wide numerical optimization of the NIM/NII function. The NIM/NII are optimized relative to the current market/econometric implied forward expectations and explicit forecasts. Clean CCAR portfolio data for worst case simulation and "free" (e.g., average sub 10 ps vectorized FMA FP execution) floating point arithmetic execution on commodity processors make this computation feasible in 2015. The broader idea is to swap out the worst case for average case simulation then use the numerical optimization to implement closed loop balance sheet control theory to automate Treasury capital allocation.

An asset or a liability in a Firm's accrual portfolio (or an aggregation/pool of a specific type of assets/liabilities) is represented by a 6-tuple: **ALM position = (time start, time end, cost(t), amount(t), runoff(t), default(t))**, where, assuming time start < time end is in the future, we define:

- Expected discrete holding period (time start, time end) or (ts, te);
- Expected holding cost function, **cost(t) (or c(t)),** is an amount in USD to be paid or received on (apportioned to) each business day in order to maintain the contractual position over the holding period ts < t < te. The cost may include fees and realized losses as well as scheduled contractual cashflows;
- Expected USD contractual notional amount is represented by the asset/liability function, **amount(t) (or a(t))**, over the holding period ts < t < te. The initial amount, a(ts), may change at time a(t) where t > ts due to contractual accretion or amortization cashflows, intra-period new business activity, or default loss realization;
- Expected daily retained accreted or amortized amount in USD is represented by the function, **runoff(t) (or r(t))**, for the holding period ts < t < te. r(t) tracks the daily expected change in amount due to contractual accretion and/or amortization (excluding new business, trading, or default realization); and
- Expected daily realized loss in USD is represented by the function **default(t) (or d(t))** , for the holding period ts < t < te.

If time t is in the past then we replace the Expected functions above with the Actual (or realized) function equivalents. The holding cost at time t or the realized default loss at time t are examples of the actual function equivalents. **The cost function, c(t),** allows the calculation of the excess capital available to be allocated, or the capital short fall, in multi-period balance sheet simulations. That cost is the sum of several components including: the expected credit loss on default, the contractual payment rate on the current face amount, and various contractual fees. The holding cost primarily determines the daily change in the balance sheet and the daily return on the balance sheet. The daily quantities can be aggregated to holding period balances and returns. All the remaining functions allow simply for attribution of the cost cashflows and the returns. **The default function, d(t),** allows the attribution of the collateral recovery and recognized loss during the holding period to be distinguished from cost changes or amortizing paydowns. In mortgage backed security terms, we are concerned mainly with the Constant Default Rate (CDR or probability of default) and the Loss Severity (LS or recovery rate in default). More detailed default modeling issues like Lockout Before Loss or loss Recovery Periods are deferred to higher fidelity models than this toy model. It seems important for the fidelity of the toy model under optimization to include some explicit notion of breakeven for the credit exposure so that the optimal NIM/NII does not simply converge on the nominally largest coupon loans funded by the least costly deposits. For example, Ruszczynski [*Nonlinear Optimization*] does this with Portfolio Optimization by retaining a portfolio return variance term in the problem formulation that forces the optimization to become quadratic. In this toy model we are going to instead rely on a no-arbitrage Credit model for the loans to enforce the non-triviality of the optimizer convergence. **The runoff function, r(t),** allows profit and loss attribution through the holding period for contracts/securities/pools in the accrual portfolio. So the runoff function allows one to determine how much of the current notional amount is due to contractual notional paydown schedules or position accretion versus new investment or liquidation. This should be useful in the case of contract pools with a variety of maturities and prepayment options. **The amount function, a(t),** establishes the base-line nominal face amount at the inception of the contract/security/pool and then altered in the accrual portfolio during the holding period by trading activity, market movement, new business, or for existing accruing or amortizing positions rolled from the previous holding period in the simulation.

The ratio of the holding period cost, c(t), and the amount, a(t), determines the rate return for the balance a(t).

$$\text{return}(t) = c(t)/a(t), \text{ where}$$

$$c(t) = r(t) + d(t) + \text{other terms}$$

the other terms could be brokerage fees, allocated charges, or Financial Control adjustments. For now let's label the 6-tuple with the generic designation of Asset or Liability. So we designate a simple accrual portfolio as:

$$\{\text{Asset}(ts, te, c(t), a(t), r(t), d(t)) , \text{Liability}(ts, te, c(t), a(t), r(t), d(t))\}, \text{ or}$$

$$\{\text{Credit Cards}(ts, te, c(t), a(t), r(t), d(t)) , \text{Deposits}(ts, te, c(t), a(t), r(t), d(t))\}.$$

In most circumstances you could use the sign of the holding period cost to determine if a position was a liability, but now you have recently seen negative interest rates in EUR deposits, so perhaps being a little more explicit in notation is good. The balance model outputs are recorded in a(t). The return model outputs are recorded by computing c(t)/a(t).

The idea is that the notational framework (developed throughout this paper) is sufficient to display the balance and return model outputs for the individual contracts in the accrual portfolio, as well as pools of similar contracts. The Balance and Return models are the key quantitative parameters representing the contract's contribution to the balance sheet over time. This is sort of interesting in that we are less interested in the valuation model for the contract mark-to-market than the model for the current face amount of the contract and a return model for the coupon accrual, fee payment, and principal prepayment. So in mortgage backed security terms, we are more interested in the prepayment model than the full valuation model. Non-callable US Treasury bonds, for example, do not amortize or accrete and thus have very simple balance models with known contractual maturity dates, $a(t)$ = par between settlement and maturity, 0 otherwise; and ignoring brokerage and other trade inception fees for the moment, $c(t)$ = accrued coupon between accrual start date and accrual end date, 0 otherwise.

Callable municipal bonds, most all deposit accounts, credit cards, mortgages, and commercial loans typically have more complex balance models because the contract account balances accrete and amortize over time. Their current face amount (or current notional) moves over time. Additionally, some contracts have no predefined maturity date making balance modeling harder to maintain accuracy over a simulation horizon. The notation is sufficiently precise that we can write code matching a specification presented in this notation. The notation allows us to then specify: the Treasury function of matching assets and liabilities, tracking the runoff cashflows from the accrual portfolio; entering new investments with (or due to) the accrual portfolio runoff; model the full balance sheet to a selected simulation horizon; calculate the NIM/NIR in a variety of market scenarios; and finally run a numerical optimization LP/NLP to find the maximum NIM/NIR over the holding period.

This paper is organized as follows. The Introduction defines the toy model and the notation for the Accrual portfolio securities. The One Period NIM section shows how compulsory capital allocation operations (e.g., assets must match liabilities and coupon cash flows sweep into fed funds, deposits or short term debt) are simulated. We cover the representation of accrual portfolio positions, the required market data, and the explanatories that measure the difference between model and actual. The Multi-Period NIM Section defines the toy model discretionary capital allocation operations and demonstrates how they are simulated. The idea is that the ultimate size of NIM/NII is driven by the implementation of the discretionary capital allocation plan and the market movement, assuming no significant gains or losses from compulsory capital allocation activities. We discuss the implementation of the capital allocation plan and the universe of new investments. The Expected Multi-Period NIM section shows how to introduce a stochastic market model to generate Monte Carlo paths to compute the expected NIM. For the purpose of simplicity we will restrict ourselves to historical US Federal Reserve data in this presentation. We discuss the Balance and Return models and the process of capital allocation performance attribution. The Net Interest Margin Optimization section shows how to formulate a Net Interest Margin optimization problem using the Expected Multi-Period NIM machinery. We discuss the problem formulation, present an example, examine the trade offs between LP and NLP formulations, and list several applications. The summary section concludes the toy model presentation, analyses the trade off between LP and NLP formulations, and outlines the next steps in NIMo research and development.

1. Introduction

The goal is to model portfolios of assets and liabilities with enough information to get meaningful numerical optimization results. In general, the newly feasible opportunity is to model the accrual portfolio at the security/contract level so that on a market expectations basis capital allocation decisions can be made optimally efficient.

For conceptual clarity we may temporarily assume some or all of the parameters listed as functions of t, are constants or, in the simplified case of a portfolio with one asset and one liability:

{Asset(ts, te, c, a, r, d) , Liability(ts, te, c, a, r, d)}.

In general and using short hand, our ALM positions for this simplified accrual portfolio look like this (assuming (ts < t < te):

{Asset(ts, te, c(t), a(t), r(t), d(t)) , Liability(ts, te, c(t), a(t), r(t), d(t))}.

The functions c(t), a(t), r(t), d(t) are defined on the discrete interval, [ts, te].

We use set notation { …} for a portfolio of assets and liabilities. So a portfolio or pool of k commercial loans is denoted by:

{Asset(ts_1, te_1, c_1(t), a_1(t), r_1(t), d_1(t)),

Asset(ts_2, te_2, c_2(t), a_2(t), r_2(t), d_2(t)),

…

Asset(ts_k, te_k, c_k(t), a_k(t), r_k(t), d_k(t))}

A portfolio or pool of m demand deposit accounts is denoted by:

{Liability(ts_1, te_1, c_1(t), a_1(t), r_1(t), d_1(t)),

Liability(ts_2, te_2, c_2(t), a_2(t), r_2(t), d_2(t)),

…

Liability(ts_m, te_m, c_m(t), a_m(t), r_m(t), d_m(t))}

We can model the Assets and Liabilities at the security, contract, or account level for a Depository Institution, an Insurance/Pension Fund, or other Financial Service Firms using a set of Asset and Liability portfolios denoted above. Given enough computational resources and portfolio detail we may choose to model the Assets and Liabilities (at the contract level) for an entire market e.g., Japanese Depository Institutions or USD Financial Service Firms.

Let’s ignore trading assets and liabilities in the toy model arguing they are relatively small balance sheets. Similarly let’s assume away any Available For Sale or Hold to Maturity portfolios assuming they do not use much capital in comparison with say Commercial Loans. We will come back for Trading and AFS/HTM in the NIM optimization context, but for this toy model they are small enough to be out of scope (even though their balance sheet returns can be significant). The securities in the toy model Assets include:

1. Commercial Loans,
2. Consumer Loans,
3. Bank Deposits, and
4. Fed Funds Sold and Resales.

Let’s assume the average maturity terms are a couple years for the Commercial Loans and under 5 years for the Consumer loans. Let’s also assume 70% of the Assets are Commercial and Consumer Loans. Assume all the loans are collateralized. Assume the Bank deposits are demand deposits. Assume the maturity on the Fed Funds is overnight. Independent of any new capital raise it looks like 30% to 40% of the Asset capital rolls off for reinvestment each year. Presumably there are some capital adequacy constraints mandating certain levels in Bank Deposits and Fed Funds Sold and Resales. Let’s call it $250 bn roll off annually per Trillion USD of Assets on the balance sheet. Informally, a quarter of the Assets roll off per year in the toy model with a discretionary capital plan. So we assume there is some managerial discretion for the optimizer to reallocate 25% of the Assets rolled off annually in capital planning to optimize the NIM.

The securities in the toy model Liabilities include:

1. Deposits,
2. Fed Funds Purchased,
3. Short Term Borrowing, and
4. Long Term Debt issued by Firm.

Assume Deposits are demand deposits with no contractual maturity date representing 50% of the outstanding Liabilities. Fed Funds Purchased have an overnight term. Assume Short Term Borrowing is collateralized with a term under one year. Assume Fed Funds and Short Term represent 40% of the outstanding Liabilities. Assume Long term debt issuance is out to 30 years with an average maturity of 10 years, representing perhaps 10% of Liabilities. Independent of any new capital 40% to 45% of the total funding rolls off annually, requiring discretionary recapitalization. Long term debt issuance does give you the means to move the average liability maturity out. On the other hand, the historically low US interest rates seem more likely to rise than drop further. In the toy model let’s assume 40% annual discretionary recapitalization.

There is quite a bit of balance sheet moving on any given year for capital planning.

We are focused on:

1. the Firm's accrual portfolio/balance sheet,
2. the quantitative model representation of the accrual portfolio positions and securities,
3. the computation of balances and returns from the quant models,
4. the potential portfolio of new business origination in the holding period,
5. the attribution of balance and return changes to market changes, realization of losses, and new business, and
6. the NLP optimization problem formulation and execution for the constrained maximization of the Net Interest Margin.

All this is in the context of the balance sheet simulation (matching assets and liabilities) resulting in the computation of the expected Firm NIM.

We will present a simple 4 period model for the NIM optimization formulation based on the ALM notation just introduced. Without loss of generality, we can extend the model to multiple currencies, more products/securities, OTC hedges, more periods, different countries or legal jurisdictions or tax jurisdictions later.

Examples:

1. A demand deposit of 1mm USD in time period (ts, ts+3M), for which the Firm pays 50 bps per anum and has no expected withdraws, drawdowns, or losses in the holding period is represented in the accrual portfolio as:

Liability(ts, te, -50bps*daycount(ts, ts+3M)* 1mm, 1mm, 1mm, 0).

For non-defined maturity contracts or pools we will simply push te over the simulation horizon to finesse the lack of a contractual (or maximum) maturity date.

2. A fully drawn down loan commitment of 1mm USD in time period (ts, ts+3Y) for which the Firm receives $USDLibor_{3M}$ + 500 bps per anum and has a 3 year term and a balloon principal payment at maturity is in the accrual portfolio as:

Asset(ts, ts+3Y, $(USDLibor_{3M}+500)^3$*1mm, 1mm, 1mm, default loss probability(ts, t+3Y)%*1mm)

3. If we need to cover asset funding in time period (ts, te) from the money market for 1mm USD we add to the accrual portfolio:

Liability(ts, ts+3M, -$USDlibor_{3M}$*1mm, 1mm, 1mm, 0)

We can extend to daily funding at some additional complexity of the notation and model processing and definition. We defer that discussion for now. If the accrual portfolio has a deficit of funds during a holding period the gap will be filled at the money market cost of funds.

4. If we have excess 1mm USD funds and invest the in the money market at 3M USDlibor we put in the accrual portfolio:

Asset(ts, ts+3M, $USDlibor_{3M}$*1mm, 1mm, 1mm, 0)

We recently saw the money market funds break the buck so the 100% constant retention function is probably not entirely accurate. Nor is the constant 0% default/loss function entirely accurate. We are just getting the ALM model notation down at the moment.

Time Period:

The holding period for the asset or liability (or their pools) in the accrual portfolio is from the starting time, ts, to the holding period ending time, te. In the case of pools, ts is the minimum starting time (accrual date) of any contract in the pool, and te is the maximum contractual maturity of any contract in the pool. Assume NY+LON Fixed Income calendar and a NY business day close.

Cost

The expected contractual cashflows over the holding period in USD. The spreads to USDLibor determining magnitude of the cashflows could vary over the time period. We also have the potential of fixed contractual coupons contributing to price levels. In the case of pools the cost could become a weighted average corresponding to the aggregated notional amounts. We will assume that the required resets are quoted at the NY close and defer discussion of other floating indices at the moment, wlg. For transient fees (e.g., on deposit or credit cards), we assume they are factored into the cost function c(t).

Amount

The expected inception nominal/notional amount invested in the contract/security/aggregate in USD over the holding period. In the case of deposits or pools of deposits the notional amount may increase or decrease over the holding period. In the case of loans, the undrawn amount in the loan facility is factored into the retention and default functions. The amount function, a(t), represents the drawn down portion of the loan facility at the later of 1. the inception of the holding period or 2. the inception of the security/contract as new origination to the accrual portfolio.

Runoff

The expected amount of notional accreted or amortized from the contract/security/aggregate over each business day of the holding period. The runoff function returns a USD amount at each time t.

Default

The amount of loss expected to be recognized in the contract/security/aggregate in the holding period. The default function returns a USD amount at each time t.

Pooled ALM

We can pool certain assets with other assets, similarly we can pool similar liabilities. We prohibit pooling of assets with liabilities in this notation. How do we represent aggregated asset or liability positions?

Basically we push all the pooling complexity into the functions: c(t), a(t), r(t), and d(t). Assume we have a pool loans denoted:

$$\{Asset(ts_1, te_1, c_1(t), a_1(t), r_1(t), d_1(t)) , Asset(ts_2, te_2, c_2(t), a_2(t), r_2(t), d_2(t))\}.$$

becomes in the pool notation:

$$\{Asset(min(ts_1, ts_2), max(te_1, te_2), c_1(t) + c_2(t), a_1(t)+ a_2(t), r_1(t) + r_2(t), d_1(t) + d_2(t))\}.$$

Example:

Assume we are given a portfolio of loans for a single bank branch that we want to represent in the pool notation for the holding period (ts, ts+3M). The first loan started prior to the holding period and is expected to default during the holding period, the second loan will start prior to the holding period and is contractually scheduled to mature with a full balloon payment, and the third loan starts during the holding period and is expected to mature after the holding period.

Assume that Loan 1 is a balloon repayment at maturity paying Libor + 330 and is expected to default with a 40% recovery rate.

$$Asset(ts, ts+3M, (USDlibor_{3M}+ 330bps)*400k, 400K, 1mm, 600k)$$

Assume that Loan 2 is a fully drawn balloon repayment scheduled for ts + 2M paying Libor + 350bps that is not expected to default in the current holding period.

$$Asset(ts, ts+3M, (USDlibor_{3M}+350bps)*1mm, 1mm, 1mm, 0)$$

Assume that Loan 3 starts at ts + 1M and pays Libor +200 with no default risk due to collateralization.

$$Asset(ts+1M, ts+3M, (USDlibor_{3M}+200bps)*1mm, 1mm, 1mm, 0)$$

The aggregate Loan Pool is then

$$Asset(ts_1, ts_1+3M,$$

Libor+330 * daycount(ts1, 3M)*$400K +

Libor + 350 * daycount(ts1+1M, 3M) *$1mm+

Libor + 200 * daycount(ts1+2, 3M)*$1mm),

, 2.4mm, 3mm , 600K)

Assume we have a pool of liabilities:

$$\{\text{Liability}(ts_1, te_1, c_1(t), a_1(t), r_1(t), d_1(t)) , \text{Liability}(ts_2, te_2, c_2(t), a_2(t), r_2(t), d_2(t))\}.$$

becomes in the pool notation:

$$\{\text{Liability}(\min(ts_1, ts_2), \max(te_1, te_2), c_1(t) + c_2(t), a_1(t)+ a_2(t), r_1(t) + r_2(t), d_1(t) + d_2(t))\}.$$

Example:

Assume we are given a portfolio of demand deposits from a single bank branch starting on consecutive days ts, ts+1, and ts+2. The deposits have no fixed maturity and pay a fixed return of 53bps + fees. The inception amounts deposited in each of the accounts are $100, $200, $300, respectively. The funds deposit and withdrawal functions for each of the accounts are captured in the functions r1, r2, r3, respectively. There are no defaults expected in the holding period ending at ts1 + 3M. If there were default amounts for each of the deposits they would be simply be additive in the pool notation, independent of the variation in notional size, since the breakdown is into dollars. We assume none of the demand deposits will be terminated until after ts1 + 3M.

The first demand deposit:

$$\text{Liability}(ts_1, ts_1\text{+3M}, \text{53bps*daycount(ts1, 3M)*\$100}, \$100, r_1(t), 0)$$

The second demand deposit:

$$\text{Liability}(ts_1\text{+1}, ts_1\text{+3M}, \text{53bps*daycount(ts1+1, 3M)*\$200}, \$200, r_2(t), 0)$$

The third demand deposit:

$$\text{Liability}(ts_1\text{+2}, ts_1\text{+3M}, \text{53bps* daycount(ts1+2, 3M)*\$300}, \$300, r_3(t), 0)$$

The pool of demand deposits

$$\text{Liability}(ts_1, ts_1\text{+3M},$$

$$\text{53bps*(daycount(ts1, 3M)*\$100 + daycount(ts1+1, 3M)*\$200 + daycount(ts1+2, 3M)*\$300)},$$

$$, \$600, r_1(t) + r_2(t) + r_3(t) , 0)$$

The complexity of aggregation in pooled positions is reflected in the complexity of the functions Cost(t), Amount(t), Retention(t), and Default(t).

Summarize and review the decisions made including the simplifying assumptions. Why might this toy model capture the required behavior to match market data?

The goal of studying such a toy model is to get estimates on how capital allocation is implemented and analyzed. Then we want to use this knowledge to automate the capital allocation process.

2. One Period NIM

In the last section we developed a notation for assets and liabilities, pools, and in general a generic accrual portfolio. In this section we make some dramatically simplifying assumptions about the composition of the accrual portfolio to move forward with the toy model. We will enumerate the compulsory capital allocation actions that must be implemented during each holding period. For example, daily funding of the Balance Sheet is a compulsory action as is sweeping realized cash flows into deposits or short-term debt. Although we may use longer holding periods at times, our goal is to model daily holding periods. We will discuss the position representation, the required market data, and the explanatories measuring the difference between the modeled NIM/NIR and the actual NIM/NIR. Finally we will review a one period NIR LP/Simplex optimization on the toy portfolio as the capital allocation plan is implemented.

For now, let's assume a bank accrual portfolio is composed of a limited set of representative security pools. The toy portfolio will simply have mortgages, credit cards, short term debt, and demand deposits. The contract's balance, return rate, and default cost realization over time will be represented by a set of time series functions (see equation 1) computed apriori. The composition of the inception face amount and type of contracts in the accrual portfolio follows the statistics available at the Fed. A better filled out set of accrual portfolio statistics can be seen at the Fed web site, here and here. I assume similar data can be found at the various centrals banks (e.g., ECB, BoE, BoJ, PBC). The challenge is to determine when, with some precision, the net cash flows first become available to implement the discretionary capital allocation plan so that the implementation of the plan, or even the capital allocation plan itself, can be optimized as a stochastic program [see Shapiro, et. al., Lectures on Stochastic Programming].

A key part driving almost all the Balance Sheet simulation analytics is the balance, return rate, and default modeling for the contracts in the accrual portfolio. These models are to be run against a stochastic market model, like LMM, to generate expected balances, return rates, and default costs to the simulation horizon. Separately, a stochastic market model, that we select, generates the market data that is fed to the balance model to output the corresponding balance. If the stochastic market model can use a random number generator to uniformly sample the whole space of future market/econometric data then you can use a Monte Carlo algorithm to derive the expected balances, rates, and default costs. Let's look at some example models from the simplest in UST to the more complex in deposits and mortgages.

For a US Treasury bond the balance model is a constant function from the start date of the bond to the maturity:

$$a(t) = 100.$$

The UST rate model, ignoring any up front brokerage fees, a constant function equal to the semiannually paid coupon

$$c(t) = \text{semi annual coupon rate } * a(t) \text{ on scheduled cash flow dates}$$

0 otherwise

The UST default model gives us d(t) = 0. The error in this set of UST models is small. The performance for the assignments will be in the picosecond range per contract. Your code should process millions of UST balance, rate, and default time series assignments for a full balance sheet in something like a millisecond.

For a deposit contract/account the balance, rate and default models are not so straightforward. Three main features of deposit contracts complicate the modeling:

1. many deposits have no contractual maturity date
2. the account balance fluctuates with individual deposits and withdrawals
3. the cost of the account may fluctuate with periodic fee assessments

In the US FDIC Insurance covers some checking, savings, and money market deposit accounts up to a posted maximum. So d(t)=0 for these insured accounts under the specified amounts. In the recent Cyprus and Greek banking crises there appears to be greater uncertainty about the possibility of default on deposit contracts and d(t) may be non zero and negative for deposit contracts in those institutions. Frequently deposit models pool many deposit contracts and then use single or multivariable regression to fit the aggregate historical balance, return rate, and default figures. The regression fit equations are then used with forecast data to provide forward estimates of the modeled parameters, consistent with the historical data. From Wikipedia:

Suppose there are n data points $\{(x_i, y_i), i = 1, ..., n\}$. The function that describes x and y is : $y_i = \alpha + \beta x_i + \varepsilon_i$.
The goal is to find the equation of the straight line
$y = \alpha + \beta x,$
which would provide a "best" fit for the data points. Here the "best" will be understood as in the least-squares approach: a line that minimizes the sum of squared residuals of the linear regression model. In other words, α (the y-intercept) and β (the slope) solve the following minimization problem:

$$\text{Find } \min_{\alpha,\,\beta} Q(\alpha, \beta), \qquad \text{for } Q(\alpha, \beta) = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \alpha - \beta x_i)^2$$

So if the known pooled balance at the conclusion of the last holding period is known, then assuming I have fit a regression based on Fed Funds:

Known Deposit Balance * (c1+ c2 * (projected Fed Funds – previous Fed Funds) = new Deposit Balance

I get a recursion defining a time series of projected deposit balances recorded in a(t). Return rate and default models can be similarly defined. The error in the regression fit can be large due to dispersion of the actual data to the fit as well as changes to observed behavior, contract definitions, or economic conditions causing a deviation from historically derived expectations. The execution time should be short, perhaps tens of picoseconds per contract time series on average. You are trading off accuracy for analytic simplicity and fast execution. The performance will not be that different from the UST case although the approximation error will be much higher.

Finally, what are the modeling alternatives if greater accuracy is required than is available through a pooled contract regression fit? These modeling problems arise when the contracts in the accrual portfolio have common features such as: embedded call options, amortization schedules, sink schedules, facility draw down agreements, prepayment options, structured product tranches, default protection, or cash flow waterfalls. Large portions of the accrual portfolio hold deposits, commercial loans, credit cards, and mortgages and all these contracts have some of these features that are difficult to model accurately. Any of these contract features can shape the future balance, return rate, and default costs aggregated into the accrual portfolio balance sheet. The timing of when there is expected to be sufficient runoff cash on hand to enter into a new investment can depend on how well these features are modeled. Fortunately, there is a mature literature on mortgage prepayment models used for mortgage backed security trading [see Hayre or Reel]. Almost all these features are encountered in the prepayment literature, with possible exceptions being the more credit oriented features like default protection. The modeling accuracy and the performance computing the models can vary widely. However, note that sell side mortgage desks have been trading in a competitive OTC market and in volume against these prepayment models for 30 years. The chances are good that there are solid accurate factor structure models for the expected behavior of all of these accrual contracts. The models may be proprietary, require frequent recalibrations, and be somewhat data intensive but it is implausible to assume they do not already exist.

At some level (not necessarily at the individual contract level), every bank has analytic models for these quantities, otherwise they would have no current capacity to forecast the NIR or NIM that their discretionary capital allocation plan is expected to achieve. In the absence of reasonable models, one would be hard pressed to offer a compelling argument asserting one capital allocation plan is better or worse than another. Moreover, in the absence of good models the bank's capacity to track how well the actual Treasury selected capital allocation plan was being implemented, would be limited. For a large bank with thousands of geographically dispersed centers and branches the lack of solid balance, return rate, and default models would amount to flying blind. Fortunately, with CCAR, the modeling fidelity is rapidly improving to an even more granular scale to provide greater forecasting ability and automated monitoring visibility.

For each asset and liability contract or pool in the target accrual portfolio the position is described using this notation:

$$\{\text{Asset}(ts, te, c(t), a(t), r(t), d(t)) , \text{Liability}(ts, te, c(t), a(t), r(t), d(t))\}. \qquad (1)$$

we need three main models to provide the time series functions cost(t) or c(t), amount(t) or a(t) and default(t) or d(t). The runoff function r(t) is a tracking function to enable the processing of pools of contracts rather than just single contracts and does not require a specific model. The purpose of the model is to provide a programmatic means to translate market and econometric data along with contract details and account activity into a set of quantities: cost, amount, and default that can be used to track the contract over the simulation horizon. For the most part the credit risk on the accrual portfolio is collateralized so the primary modeling focus is on the balance and the return rate. The expected default rate contribution to the cost of the position is expected to be small.

We take some pain to include the default cost in the toy model to increase the fidelity of the multi-period optimization problem. But the multi-period optimization problem with just the interest rate term structure exposure is reasonably complex by itself. There are two simulation outputs that are of general interest in automating this stochastic programming problem:

1. the optimal capital allocation plan based on market expectation and
2. the performance attribution of the current capital plan implementation (optimal or otherwise)

The sources of the optimization complexity include: the cost of collateralized credit default, the credit spread and interest rate risk over the term of the simulation horizon, and the precise timing of the availability of funds for reinvestment.

The sources of performance attribution complexity are somewhat standard. The typical idea is to use the first couple terms in the Taylor series expansion of the NIR/NIM function relative to the underlying market and econometric data. The hope is for a parsimonious common market/econometric data footprint across all the contracts in the accrual portfolio so that the capital allocation plan P&L realization is fully understood.

One interesting idea is the simulation horizon of the entire accrual portfolio can be recomputed daily. You still need balance, return rate, and default model going out 1-5 years to compute the expected NIR/NIM for the current capital plan (optimized or not). We are currently thinking to run daily balance sheet simulation for the first year, and quarterly or monthly thereafter.

One remaining big unknown at this point is the quality of the one-year horizon modeling of the balance, return rate, and default for standard accrual portfolio contracts. Even though there are hundreds or even thousands of types of accrual portfolio deposits, credit card accounts, commercial loans, and customer loans we tend to be optimistic because the modeling of these contracts seems to have much in common with the reasonably mature field of CMBS prepayment modeling. We expect there is a mature literature covering prepayment modeling that is straight-forward to extend to the full range of accrual portfolio contracts (see the Citi or JPM prepayment model paper). In the limit you are going to fit some factor model with some recalibration interval. One advantage in accrual portfolios is that they tend to be one directional with only macro rather than position level hedges. Banks do not typically trade demand deposits across a bid-ask spread. On the other hand pools of auto loans can be tranched up and traded across a bid-ask spread. We are reasonably certain that each bank has proprietary analytics models for these accrual portfolio contracts. The results that we review here leverage these proprietary models to implement a fast Balance Sheet simulator for Optimal Capital Allocation planning and control.

The one-period toy model simulates the Balance sheet over the course of a single holding period (ts, te). The simulator assures that the assets match liabilities within the holding period. For example, if on any given day of the holding period the level of deposits drop due to withdrawals, the simulation will secure funding using prearranged programmed assumptions (e.g., the toy model will cover with short term borrowing). If a loan payment is received during the holding period the cash flow will roll into bank deposits after a cash settlement period. These are default programmatic capital allocations. No new discretionary capital allocation decisions are initiated during the holding period. Discretionary capital allocation decisions are initiated between holding periods not during holding periods, in this model. Capital allocation decisions initiated at the start of the holding period may be implemented during that holding period and in subsequent holding periods. Discretionary capital allocation implementation plans may override default capital allocations within the holding period. In order to have greater discretion in the allocation of capital over a specific macro time horizon with the toy model, one needs to shorten the default holding period. That way there are more frequent assessments and opportunities to implement discretionary capital allocation strategies.

Within a given holding period the primary computational goal is to implement all compulsory capital allocations e.g., maintain an end-of-day match between assets and liabilities and process the holding period flow of funds. If all of these goals are met then we can compute the expected NIM prior to the end, $t < te$, of the holding period. We can compute the actual NIM for the holding period at time $t > te$, after the conclusion of the holding period, assuming all the market data is available.

The Net Interest Margin for the portfolio of an offsetting asset and liability

{ Asset(ts, te, s1, 1mm, 1mm, 0),

Liability(ts, te, s2, 1mm, 1mm, 0) }

in the simplest case is

NIM = -s2 + s1/1mm basis points

where the price, amount retention, and default time series functions are constant.

What if we have excess assets, what does our simple portfolio need for the balance sheet?

{ Asset(ts, te, s1, 1.1mm, 1mm, 0),

Liability(ts, te, s2, 1mm, 1mm, 0),

Liability(ts, ts+3M, usdlibor*0.1mm, 0.1mm, 0.1mm, 0) }

Where the NIM is then,

NIM = -s2 *1mm/1.1mm - Libor *0.1mm/1.1mm + s1 * 1.1mm/1.1mm basis points.

We can extend this to cover overnight funding w. Fed Funds, but we will not do so now, we will just borrow short term to maintain the simplicity of the toy optimization model.

What if we have a deficit of assets, what does out simple portfolio need for the balance sheet?

{ Asset(ts, te, s1, 1mm, 1mm, 0),

Liability(ts, te, s2, 1.1mm, 1mm, 0),

Asset(ts, te, s3, 0.1mm, 0.1mm, 0)}

Where the NIM is then,

NIM = =s2 *1.1mm/1.1mm + s3*0.1mm/1.1mm + s1 * 1.0mm/1.1mm basis points.

Toy Portfolio:

Let's make our toy portfolio have a common maturity of one-year post the start of the simulation. We define the runoff portfolio and the prospective new origination securities. We will run the toy portfolio through the balance sheet simulation to compute the expected NIM after one year. Then we will use the historic Fed data to compute the actual one period NIM. Comparing model with actual we will derive some performance attribution quantities. Finally we will use the expected and the historic market data to compute the optimal NIR using one period LP simplex. This simple one period case can be extended to the multi-period case.

The toy accrual portfolio is composed of a Toy liability portfolio:

[Deposit Liability(ts, ts+1Y, 50bps *10, 10, r(t), 0),

Short Term Debt(ts, ts+1Y, 66bps*0.5, 0.5, r(t), 0)}

and an asset portfolio:

{Cards Asset(ts, ts+1Y, (295 bps)*5 - a1*0.01*5, 5, 5, -0.01*5) ,

Mortgage Asset(ts, ts+1Y, (375 bps)*5.5 - a2*0.01*5.5, 5.5, 5.5, -0.01*5.5),

Short Term Loan Asset(ts, ts+1Y, 0, 0, r(t), 0)}.

Our portfolio has 10.5 of assets 10 of deposits and 0.5 of short term debt. On the liability side we have 10.5 of contracts, 5 of cards and 5.5 of mortgages. There is zero balance in short term loans in the toy portfolio. Note the d(t) gains/losses are realized through the holding period in the cost function. Let's assume the default functions are constants d1(t) = -0.05 and d2(t) = -0.055. Constants a1 and a2 determine how much of the expected default cost is realized at the end of the holding period.

In general, the balance models are going to be more complex than shown in this toy portfolio. The balances used in the toy model are simply constant functions. The Deposit Liability holds a balance of 10 for one year. The Cards Asset holds a balance of 5 and the Mortgage Asset holds a balance of 5.5 for one year. The notation allows the specification of the balance as a function of time for the holding period and the duration of the contract. The balance model can run against a market environment or scenario to produce a full time series of balances from ts to te where te is the maturity of the contract (or the end of the simulation horizon, which ever comes first). The portfolio notation represents the balance as the all in balance b(t) and the runoff balance r(t) so that new business during the holding period can be separated from the balance dynamics of the contracts booked prior to time ts. In the case of this toy example, the balances of the liabilities and the asset match at 10.5 with no new business during the one-year holding period.

The returns model will also in general be more complicated than this toy portfolio indicates. The Deposit Liability pays 50 bps and the Short term Debt pays 66 bps fixed. The Cards Asset contractually receives 295bps defined as a spread over Libor and the Mortgage Asset contractually receives 375bps similarly defined as a spread over Libor. The Libor fixing is assumed to be at time ts in this example. The Cards Asset is expected to take a default charge of -0.05 and the realized default charge at the end of the holding period is a fraction a1 of that. Similarly the Mortgage Asset is expected to take a default charge of -0.055 and the realized default charge at the end of the holding period is a fraction a2 of that. On an expectation/modeling basis a1 = a2 = 1.0.

Model NIM:

Let's assume the payment schedule is annual in this example. The NIM is computed by summing up the weighted average of the contract returns in the portfolio. The liabilities cost is negative and the asset cost is positive. At the end of the holding period we expect the liabilities will have had a cost of

$$50 \text{ bps} * 10 + 66 \text{ bps} * 0.5.$$

similarly we expect the assets to return

$$295 \text{ bps} * (5 - 0.05) + 375 \text{ bps} * (5.5 - 0.055)$$

thus the modeled or expected NIR is

$$\text{NIR} = 295 \text{ bps} * 4.95 + 375 \text{ bps} * 5.445 - (50 \text{ bps} * 10 + 66 \text{ bps} * 0.5)$$

and the modeled or expected NIM is the weighted average of the returns

$$\text{NIM} = 295 \text{ bps} * 4.95/10.5 + 375 \text{ bps} * 5.445/10.5 - (50 \text{ bps} * 10/10.5 + 66 \text{ bps} * 0.5/10.5)$$

or 282.7 bps. The expected loss is unrealized for the entire holding period and is accounted for in the returns for the end of the current holding period and also realized in the inception balance of the next holding period. Since we are in a single holding period example we ignore this.

Actual NIM:

At the conclusion of the holding period we assume all the market variables are known. We expected a NIM of 282.7 bps but in this toy model we might do better if realized losses are less than the expected losses. Let's assume a1 = 40% and a2 = 110% at realization. The default loss on the Cards is 40 % of what was expected but the realized loss on the Mortgage was 10 % greater than what was expected. Now we have asset actual returns or NIR

$$NIR = 295 \text{ bps} * (5 – 0.4*0.05) + 375 \text{ bps} * (5.5 – 1.1*0.055)$$

so that

$$NIM = 295 \text{ bps} *4.998/10.5 + 375 \text{ bps} * 5.4395/10.5 – (50 \text{ bps} * 10/10.5 + 66 \text{ bps} *0.5/10.5)$$

or 283.9 bps. We realized an additional 1.2 bps of NIM over the model value.

How do we address the performance attribution for measuring the distance between model NIM and actual NIM? Typically the way this is done is to make a list of all the market data components that were used in deriving the model NIM. Presumably the change in the listed market data components can explain the entire difference between the 282.7 Expected NIM and the 283.9 actual NIM. In this case the entire difference is attributable to the change between the expected default charge and the realized default charge on the Cards and Mortgages. We made almost 0.843 bps on the realized Card default:

$$295 \text{ bps} *(0.6 *0.05)/10.5 = 0.843 \text{ bps}$$

and we lost an additional 0.196 bps on the Mortgage Asset default realization:

$$375 \text{ bps} * (-0.1*0.055)/10.5 = -0.196 \text{ bps}$$

So that explains the move from the NIM model (the expected value at time ts and the actual NIM realized at time te:

$$282.7+ 0.843 – 0.196 = 283.4$$

In the toy model the performance attribution is very simple. As we move from the one period model to multiple periods the NIM performance attribution should take in a wider range of market variables interest rate and credit spread movements, interest rate volatility, and fx rates in addition to simple default charge realization.

Optimizing NIR:

If at time ts we knew the set of all prospective new investments that we could make in the holding period then we could run the accrual portfolio and the prospective new investments through an optimizing search algorithm and find the set of new investments that produces the maximum NIM or NIR. In capital allocation planning that would allow a Firm’s Treasurer to identify the optimial set of new investments to enter into during the holding period to produce the maximum expected NIM at the end of the holding period. Moreover, when extended to multiple periods, the optimizer can determine both the identity and the optimal timing of the new investments relative to the expected forward market expectations and the availability of cash-flows from the runoff portfolio. NIR optimization also provides a means for evaluating how the Firm’s capital allocation plan implementation is progressing relative to expectations. When Treasury decides to raise 100mm in deposits in Iowa, for example, it is not a deterministic process. Over a period of time, with the right advertising, and customer incentives the Iowa retail branches may increase new deposits to hit the 100mm target, or they may fall short. They cannot simply go to an exchange to get more deposit contracts, if they are willing to pay the spread. The NIR optimization formulation can factor in the probability that the Iowa retail branches hit the Treasury target, monitor in real time in how Iowa Retail is tracking expectations, and make changes to the capital allocation plan according to the unfolding realization. It is the basis for an automated control theory for Treasury Capital Allocation. The Treasurer can monitor performance and data quality globally for periodic attribution and guidance. Using performance attribution for the capital allocation plan you can allocate the difference between model and realization to both market factors and capital plan implementation efficiency. This can be done incrementally (daily or weekly) as well as at the end of quarter or year. Finally, the optimizer can use US Fed or central bank data to examine entire markets. Certainly at the Firm level you can expect to have very detailed knowledge about the timing of runoff cash-flow events because as a party to the contracts you presumably have access to the actual contracts. In general though, there appears to be sufficient government mandated disclosure throughout the ALM industry to make estimates, that can be improved over time, about the ALM contracts held in all the Firms in a given market. A Firm can conceivably use market wide expected NIM optimization to evaluate potential acquisitions relative to the existing runoff from the Firm's accrual portfolio as well as the Seller's accrual portfolio.

For NIR optimization we are given a set of new origination options in deposits and commercial loans. One interesting thing about the optimization problem formulation is that the variables and constraints are largely derived from the number of potential new investments as opposed to the number of runoff positions in the accrual portfolio. The runtime of the optimization problem depends on the number of holding periods and the number of potential new investments. The optimizer runtime is not very sensitive to the size of the runoff accrual portfolio.

Let's see how this all looks in the one period toy model example.

The toy model, at time ts, has a set of Prospective Assets:

{Deposit A Liability(ts, ts+1Y, f1 + 50bps a3*X, a3*X, r(t), 0),

Deposit B Liability(ts, ts+1Y, f2 + 51bps a4*X, a4*X, r(t), 0),

Short Term Debt Liability(ts, ts+1Y, (66bps)*X, X, r(t), 0)}

We can pay f1 to get amount X of Deposit A paying 50bps per anum for one year. We can pay f2 to get amount X of Deposit B paying 51 bps per anum for one year. In terms of realized amounts we historically expect to get Deposit A at a3 = 60% efficiency and Deposit B at a4 = 95% efficiency. We also have the option to fund for one Year for quantity X at Libor plus a spread depending on the size of X, s1(X) .

The toy model, at time ts, has a set of Prospective Liabilities:

{Cards Asset(ts, ts+1Y, (Libor+305)*X – 0.02*X, X, r(t), -0.02*X) , Mortgage Asset(ts, ts+1Y, (Libor+335)*X – 0.025 * X, X, r(t), -0.025*X),

Short Term Debt Asset(ts, ts+1Y, (Libor-s2(x))*X, X, r(t), 0)}.

The Firm can get more Cards assets for one year paying 305 bps over Libor with an expected default rate of 4%. The Firm can get more mortgage assets paying 335 bps over Libor for one year with an expect default rate of 7%. The Firm can invest excess cash in short term debt for one year at Libor – s2(X) with no expected default cost.

Here is a simple formulation for the one period case of the toy portfolio. Assume we have excess capital at time ts from activity in a previous holding period.

What is the optimal capital allocation of 1 to the existing accrual portfolio of 10.5 given that the relevant Libor fixing is 75 bps and given the Prospective Assets and Prospective Liabilities.

At time ts we have the following toy portfolio:

[Deposit Liability(ts, ts+1Y, 50bps *10, 10, r(t), 0),

Short Term Debt(ts, ts+1Y, Libor*0.5, 0.5, r(t), 0),

Cards Asset(ts, ts+1Y, (Libor+300)*5 + a1*d1(t), 5, 5, d1(t)) ,

Mortgage Asset(ts, ts+1Y, (Libor+350)*5.5 + a2*d2(t), 5.5, 5.5, d2(t)),

Short Term Loan Asset(ts, ts+1Y, Libor * 0, 0, r(t), 0)}.

This is the same as the previous toy portfolio. Similarly during the holding period the Firm can raise cash by taking deposits and invest cash by purchasing assets.

One Holding Period LP Formulation:

The standard LP formulation is:

maximize $\mathbf{c}^T x$, subject to $\mathbf{Ax = b}$ and $\mathbf{x} >= 0$.

We will maximize NIR

NIR = (interest payment on assets) – (interest payment on liabilities)

So $\mathbf{c}^T x$ is the runoff toy accrual portfolio return plus the parameterized return of the prospective new investments funded in part by the possible sale of the Short Term Loan.

NIR = RunoffAssetRate * RunoffAssetBalance + NewAssetRate * NewAssetBalance –

RunoffLiabilityRate * RunoffLiabilityBalance - NewLiabilityRate * NewLiabilityBalance.

- C1 – runoff asset rate
- C2 – runoff liability rate

- C3 - Deposit A rate
- C4 – Deposit B rate
- C5 - Short Term Debt rate
- C6 – Cards rate
- C7 – Mortgage rate
- C8 – Short Term Loan rate

and

- X1 runoff asset expected balance
- X2 runoff liability expected balance
- X3 Deposit A new balance
- X4 Deposit B new balance
- X5 Short Term Debt new balance
- X6 Cards new balance
- X7 Mortgage new balance
- X8 Short Term Loan new balance

So,

$$NIR = \sum_{i=1}^{8} ci * xi$$

The constraints determine the necessary and sufficient conditions to implement a capital plan over the holding period:

1. Balance Sheet: assets must equal liabilities at the conclusion of the holding period
2. Leverage: You may raise additional funding through liability contracts to purchase more assets than covered by available cash – let's assume up to 150% of available cash.
3. All cash must be assigned to an asset by the end of the holding period.
4. residual cash will lent out in a new Short Term Loan
5. additional funding as needed can be sourced from new Short Term Debt

The asset balance must equal the liability balance:

$$-x2 - x3 - x4 - x5 + x1 + x6 + x7 + x8 = 0 \text{ where } xi >= 0.$$

Introduce variables for exogenous or previous period runoff portfolio cash and funding requirements: X9 – excess cash for investment and X10 – deficit requiring funding The Firm may issue liabilities in order to maximize their incremental investment up to some prescribed limit:

$$-X3 - x4 - x5 + x6 + x7 + x8 <= 1.5 *(x9 – x10 - c1 * x1 + c2 *x2)$$

The Runoff Asset and Liability are known expectations at optimization time. In the one period model the size of the new Assets and Liabilities are determined through optimization time. The runoff equations are equalities.

$$x1 = 10.5$$

$$x2 = 10.5$$

Short term Debt and Loans must absorb the cash and funding on hand and not allocated to Deposits, Cards and Mortgages. Since all the scheduled payments are annual there are no cash flows from new investments to account for in the one-year holding period. The only returns we need to account for are the runoff portfolio returns that are paid at some point during the holding period.

$$x8 - x5 >= (x9 - x10 - c1*x1 + c2*x2) – (x6 + x7 – x3 –x4)$$

Deposit balances are capped to grow no more than 10%

$$X3 + x4 < = 1.1 *10$$

Card balances are capped to grow no more than 8%. But cards can also runoff to the expected default adjusted balance.

$$X6 <= 1.08 * 5$$

Mortgage balances are capped to grow no more than 10%. But mortgages can also runoff to the expected default adjusted balance.

$$X7 <= 1.1 * 5.5$$

Here is the solution to the LP formulation of NIR optimization in Excel.

Runoff	Balance	Rate	**Prospective Investments**		
ts Total Assets	**10.5**	**0.03369048**	**New Assets**	Balance	Rate
ts Cards	5	0.0295	ts Cards	0.127085	0.028
te Cards	4.95	0.0295	ts Mortgage	0.4	0.037
te Cards 100-def rate	NA	0.99	ST Loan	0	0.0066
ts Mortgage	5.5	0.0375	te Cards 100 - def rate	0	0.99
te Mortgage	5.445	0.0375	te Mortgage 100 - def rate	0	0.99
te Mortgage 100-def rate	NA	0.99	te Cards Timing	0	0.8
ST Loan	0	0.0066	te Mortgage Timing	NA	0.8
te Total Assets	**10.395**	**0.03403078**	te Cards Impl Factor	NA	0.97
ts Total Liabilities	**10.5**	**0.00507619**	te Mortgage Impl Factor	NA	0.95
Deposit	10	0.005	te Cards	0.12203973	0.028
ST Debt	0.5	0.0066	te Mortgage	0.3762	0.037
			New Liabilities	Balance	Rate
ts NIR	0.30045		ts Deposit A	0	0.00515
te NIR	0.31243352		ts Deposit B	0.39721184	0.0051
			ST Debt	0	0.0066
te Asset Sum	10.8932397		te Deposit A Impl Factor	NA	0.98
te Liab Sum	10.8932397		te Deposit B Impl Factor	NA	0.99
ts Cash	0.105		te Deposit A	0	0.00515
te Cash	0.40545		te Deposit B	0.39323973	0.0051
Max Leverage Cash	0.527085				
New Investment	0.527085		ts NIM (exp)	286.142857	
Max Leverage Rate	1.3		te NIM (exp)	286.814144	
			Net Short Term	0	
			te Excess Leverage	0	
			Deposit Limit	1.05	
			Cards Limit	0.3	
			Mortgage Limit	0.4	
			Sum Deposits	0.39721184	

Discussion and interpretation of the results

Answer

Sensitivity

	Scenario	te NIR	Delta to Base	te NIM	Delta to Base
	Base	0.312433522	0	286.814144	0
Runoff Model Sensitivity	Runoff Asset Rate +10 bps	0.323162583	0.010729061	296.306874	9.492730395
	Runoff Liability Rate + 10 bps	0.301704462	-0.010729061	277.29854	-9.515603577
	Runoff Asset Balance +10 bps	0.31251467	8.11471E-05	286.945018	0.130874489
	Runoff Liability Balance + 10 bps	0.312428076	-5.44628E-06	286.809319	-0.004824492
	Runoff Asset Default +10 bps	0.312713843	0.00028032	287.00345	0.189305803
	Runoff Liability Default + 10 bps	NA		NA	
New Invest Model Sensitivity	New Asset Rate + 10 bps	0.31283614	0.000402618	287.183747	0.369603507
	New Liability Rate +10 bps	0.312036311	-0.000397212	286.449503	-0.364640689
	New Asset Balance +10 bps	0.31243612	2.5976E-06	286.816737	0.00259313
	New Liability Balance + 10 bps	NA		NA	
	Cards def + 10bps	0.312434151	6.28689E-07	286.817934	0.003790431
	Mortgage def + 10bps	0.31243546	1.938E-06	286.825828	0.011684667
Capital Plan Sensitivity	Leverage + 10%	0.313318024	0.000884501	286.295829	-0.518314897
	Leverage - 10%	0.311549021	-0.000884501	287.337298	0.523154134
	ts cash - 10%	0.312204462	-0.000229061	286.949159	0.135014961
	Mortgage Limit +10%	0.312693282	0.00025976	287.073475	0.259331683
	Cards Implementation - 10%	0.312220261	-0.000213261	286.939836	0.125691945
	Mortgage Implementation -10%	0.311502522	-0.000931	286.950474	0.136330134
	Deposit B Implementation -10%	0.312392788	-4.07345E-05	286.776749	-0.037394275
	Cards Timing - 10%	0.312157392	-0.00027613	286.560656	-0.253487755
	Mortgage Timing -10%	0.311308722	-0.0011248	285.781577	-1.032567013

We showed how compulsory capital allocation actions work in the toy model for excess assets or excess liabilities. We discussed cashflows rolling to Fed Funds versus short-term debt. Get references for the analysis of treasury funding operations. We reviewed P&L attribution of capital allocation. How do we interpret inputs and how close to market observables are the outputs? Where does the toy model lose fidelity? What are the consequences of the simplifying assumptions we made? We discussed the matching of the contract maturity with the specific start and termination of the holding period.

3. Multi-period NIM

In the last section we defined the specific toy accrual portfolio and discussed how to manage cashflows so that assets match liabilities throughout a single holding period. We introduced the idea of p&l attribution for capital allocation. In this section, we provide the mechanism for implementing discretionary capital allocation policies over one or more holding periods using accumulated and runoff capital. We add to the toy model the new origination securities/contracts as well the capital allocation policy to be automated. This allows us to simulate the balance sheet for a fixed market environment and a fixed capital allocation. New investment policy and compute the resulting NIM. The range of possible future market environments and the variety of capital allocation policies will become the key variables in the NIM optimization formulation in section 4. This section will focus on the specification and implementation of a specific reinvestment capital allocation policy. We cover new investments, balance sheet simulation, and capital allocation planning.

In multi-period NIM you have the opportunity to reinvest the runoff or accumulated cash flows from one holding period in the next holding period, according to the toy model. Since the toy model assumes compulsory capital allocation operations are covered in the single period NIM case, the Multi-period NIM introduces discretionary capital allocation. Let's assume the holding periods are quarters, for simplicity. Then between quarters is presumably the time when discretionary capital allocation can be initialized and can ultimately make a difference in the magnitude of the terminal Net Interest Margin. Our toy model discretizes time so that the initiation of a new capital allocation strategy occurs between two holding periods. Investing in new originations; raising capital via new long term debt offerings; pooling cash for strategic initiatives; or adjusting the accrual portfolio security composition all fall into the set of discretionary capital allocation plans. The Net Interest Margin and any constraint levels are computed at the conclusion of each holding period. Like the single period model, the multi-period NIM can be computed on an expectations basis or an actual/realized basis.

Notice as the holding period length decrease the simulated time between changes to the discretionary capital allocation plan decreases.

Assume we have a pool of loans denoted in the accrual portfolio at time ts:

{Asset(ts, te, p(t), a(t), r(t), d(t)) }.

becomes in the pool notation assuming te > ts + 3M in the accrual portfolio at time ts+3M:

{Asset(ts+3M, te, p'(t), a'(t), r'(t), d'(t))}.

If te <= ts +3M we need to settle in the accrual portfolio at time ts+3M:

{Asset(ts+3M, te, p'(t), a'(t), r'(t), d'(t))}.

Example:

Assume we have a pool of liabilities denoted:

$$\{Liability(ts_1, te_1, p_1(t), a_1(t), r_1(t), d_1(t))\}.$$

becomes in the pool notation:

$$\{Liability(min(ts_1, ts_2), max(te_1, te_2), p_1(t) + p_2(t), a_1(t)+ a_2(t), r_1(t) + r_2(t), d_1(t) + d_2(t))\}.$$

What is the universe of capital allocation plans? Where are the references? Can these capital allocation plans be simply represented in a numerical optimization framework?

New Origination

The Firm has the option based on the assessment of the market conditions to enter into new contracts, accept deposits, adjust rates and fees, buy new securities, sell holdings, hedge interest rate/credit risk, and originate new loans over the course of the holding period.

Exogenous Cashflows

There may be gains or losses to the balance sheet not directly related to the market activity of the contracts in the portfolio. The Firm can issue stock to raise capital, the Firm may have to make payments to settle legal cases, etc.

Summary:

Implementing the capital allocation plan

Monitorint eh capital allocation performance attribution. How well is the capital allocation plan being implemented? How much of the market under/over performance is due to the plan, the implementation, or luck? What is the computational cost of explanatories for performance attribution?

4. Expected Multi-Period NIM

In the previous section we specified and implemented a reinvestment capital allocation policy for discretionary capital plans. For a fixed market environment and its implied forward rates we calculated the NIM over multiple holding periods. In this section we focus on the quantitative model of the securities and contracts in the accrual portfolio. The balance and return modeling decisions that we make here influence the choice of analytics through the entire process of NIM optimization. For example, if the balance and return models are linear regression fits then we can use LP algorithms (like simplex) to formulate the NIM optimization. If the underlying stochastic market model needed to find the max NIM on an expected basis requires higher fidelity models than linear regression then we may be forced to use NLP algorithms to formulate the NIM optimization problem. It looks like a trade off of simplicity and accuracy. For the toy model we need:

1. the market data environment (including any econometric variables)
2. the accrual portfolio security/contract balance and return models.

So this section specifies, for a given reinvestment policy, how to compute the market expectation of the Net Interest Margin. We cover the Balance models, Return models, and the capital allocationplan performance attribution.

The next section will formulate the optimization problem on top of the Expected Multiperiod NIM simulation to arrive at the expected Maximum Net Interest Margin.

Given an accrual portfolio, a set of market data, and a capital allocation strategy we can compute the Net Interest Margin expected to result from these inputs. If the capital allocation strategy is fixed, we generate a set of perturbed market data “paths” according to a selected stochastic market model and run the multi-period NIM model for each path. The single period NIM execution enforces the compulsory capital allocation processes in each holding period. The multi-period NIM execution implements the discretionary capital allocation strategy. The average of the resulting NIM over the stochastic model’s Monte Carlo paths will converge to the expected value of the Net Interest Margin (for that fixed capital allocation strategy).

The issue is how are the quantitative models for the securities in accrual portfolio depend on the perturbed market data determined by the stochastic market model.

There are tradeoffs. If the balance sheet security quant models are simple multivariate regression fits to econometric data then the balance sheet simulation can be made to execute fast and the numerical optimization formulation is linear at the cost of possible lack of precision in security valuation along the Monte Carlo paths. There may be no way to increase the number of Monte Carlo paths to converge to the correct expectation. If the balance sheet quant models are higher fidelity security balance and return models than simple regression fits then although the approximation error is controlled along each Monte Carlo path the time required for executing the balance model, limits the total number of Monte Carlo paths and hence the root of the number of paths convergence to the expectation. We may not be able to increase the number of Monte Carlo paths because there is not enough computer time available for execution.

Note in general the balance and return models can be considerably simpler that standard valuation models. For example with Non-callable fixed rate debt you simply need to know the cashflow payment schedule and the face amount of par notional. Loans and deposits have various draw down and repayment options that are not trivial to model. Fed Funds, standard short term borrowing contracts, and long term debt are likely to have simpler balance and return models.

General Idea for Loans and Deposits is to treat them like mortgage pools. Aggregate the set of similar deposits into pools and develop balance and return models for the pools. Kind of like you do for prepayment models for MBS. Ditto Loans. Aggregate into pools and employ prepayment and loan-facility-draw-down models.

Outline how we will proceed in this toy model:

1. Simulate the market – not one bank's accrual portfolio
2. Contract-by-contract modeling of accrual portfolio positions.
3. Use the Fed historical data rather than a stochastic market model
4. Review model inputs from Fed market data.

Discuss the actual balance and rate of return models plus the required inputs, Find specific references on the modeling required.

Balance Models for:

Commercial Loans

Consumer Loans

Bank Deposits Asset and Liability side

Fed Funds

Short Term Borrowing

Long Term Debt

Return Models

For simplicity we will use the Fed's market data rather than a stochastic market model like LMM.

Fed Superviosry stress scenario:
http://www.federalreserve.gov/newsevents/press/bcreg/bcreg20141023a1.pdf

Summary:

Balance and Return Models

Runoff Portfolio versus New Investment

If you back test NIMo with historical fed data. You can test the implied forwards versus the actual realized versus the incremental Control Theory implementation.

5. Net Interest Margin Optimization

The final step is formulating the Net Interest Margin Optimization problem. The goal is to parameterize the entire range of possible reinvestment policies in a single formulation of an optimization problem. The objective of the optimization is to maximize the Net Interest Margin, given the known and projected market expectations. In this section we present the generic problem formulation and present two toy model executions for LP and NLP.

The broad idea is to learn how the NIM function behaves over all capital allocation strategies, as opposed to a single fixed capital allocation strategy. In particular, where does the NIM function assume a maximal value and how sensitive to perturbations is the expected NIM function at its maximum value. Ultimately we are most interested in the Dynamic Optimization Programming problem of completely automating the capital allocation process, like factory floor automation.

The last step in formulating this toy model is to introduce a simple numerical optimization framework to evaluate the NIM function over a range of capital allocation strategies. We expect to be able to use methods ranging from Linear Programming to a gradient climbing search algorithm to find maximal values of the NIM function.

In this section we formulate the numerical optimization of the Net Interest Margin maximization corresponding to the models chosen for Expected Multi-period NIM. In order to get some familiarity with the trade offs we work out some small formations using the toy model.

Simple Optimization Problem Formulation

The standard LP formulation is:

$$\text{maximize } \mathbf{c}^T x\text{, subject to } \mathbf{Ax = b} \text{ and } \mathbf{x} >= 0.$$

The NIM function

NIM = interest rate on assets – interest rate on liabilities

forces us to lose linearity since getting the weighted average rate requires division rather than pure (addition and subtraction) to compute NIM. In the LP case we can use Net Interest Income (NII)

NII = (interest payments on assets) – (interest payments on liabilities)

as the purely linear function to maximize. Let's focus on the NII first. So, $\mathbf{c}^T x$ will be the NII summation function; the security by security sum of the holding period cashflows from the assets and liabilities reported on a cumulative end of holding period basis. The variable x_i represents the current face value amount (in USD) for the ith contract in the accrual portfolio. The variable c_i represents the holding period return from the ith contract (in USD – or set of contracts) as a percentage of the current face value. Not all security cash flows are specified as a percentage of the current face amount, for example, fee and penalty cash flow quantities are sometimes independent of x_i (or constant with respect to the x_i). All the cash flows from the security and apportioned to the security are aggregated into the c_i.

Note if we divide the simulation horizon into an integral number of holding periods then the NII function becomes

$$NII = \sum_{i=hp_1}^{hp_k} AssetRate(i) + \sum_{i=hp_1}^{hp_k} AssetBalance(i)$$

The interest payments on the assets term of the NII equation fall into two disjoint sets. We focus on Assets in this discussion by the Liabilities follows a similar course. The first set elements that we look at are the interest payments from the RunoffAssets. These are the Assets that were booked on the accrual portfolio prior to the start of the simulation interval. The second set contains the interest payments from the NewAssets. New Assets are positions booked into the accrual portfolio after the start of the simulation (and the simultaneous start of the first holding period). Some of these New Assets are new investments booked during the first holding period. Each of the NewAssets in the simulation horizon are booked in one of the k holding periods.

Look at the asset interest payment in first holding period, hp_1.

NII(hp1) = RunoffAssetRate(hp_1) * RunoffAssetBalance (hp_1) + NewAssetRate(hp_1) * NewAssetBalance(hp_1)

Where we define

$$\text{RunoffAssetRate(hp1)} * \text{ RunoffAssetBalance (hp1)} = \sum_{di\, \in hp_1} daily\ RunoffAsset\ cashflows(di)$$

and the RunoffAssetBalance(hp_1) is the aggregate face amount at the start of the holding period of the contracts in RunoffAsset set. Since all the contracts in the RunoffAsset set are on the books prior to the start of the simulation interval they are also on the books prior to the start of the first holding period.

For the NewAsset we have similarly,

$$\text{NewAssetRate(hp1)} * \text{ NewAssetBalance (hp1)} = \sum_{di\, \in hp_1} daily\ \ New\ Asset\ cashflows(di)$$

and the NewAssetBalance is defined as the inception face amount of the Asset contracts booked during the first holding period. Since all the NewAsset contracts got booked during the first holding period they each have a contractual inception face amount but not necessarily a face amount at the start of the holding period.

In the second and subsequent holding periods we need to account for newAsset contracts booked in previous holding periods. Their balances and returns have possibly moved with the market since their inception on the books. So for k > 1

$$\text{NII}(hp_k) = \text{RunoffAssetRate}(hp_k) * \text{RunoffAssetBalance}(hp_k) + \text{NewAssetRate}(hp_k) * \text{NewAssetBalance}(hp_k) + \textbf{NewRunoffAssetRate}(\mathbf{hp_k}) * \textbf{NewRunoffAssetBalance}(\mathbf{hp_k})$$

and

$$\text{NewRunoffAssetRate(hpk)} * \text{NewRunoffAssetBalance (hpk)} = \sum_{di \in hp_k} dailyNewRunoffAssetcfs(di)$$

The difference between contracts in NewRunoffAssets and RunoffAssets is subtle. The NewRunoffAsset contracts were selected by the optimization execution in a prior holding period. On the other hand the RunoffAssets were on the books prior to the start of the simulation horizon. As we move between holding periods we need to use the market data, econometric data, the balance model, and the return model to calculate the new expected balance and rate of return in the upcoming holding period.

Let's assume the set of NewRunoffAssets and NewRunoffLiabilites are drawn for a static set known prior to starting the entire simulation. Thus both the RunoffAsset, RunoffLiability and the potential contracts in NewRunoffAssets and NewRunoffLiabilties are known prior to simulation /optimization. The choices made in the optimization search do not add to or delete from the set of contracts eligible for new investment. If k is the number of holding periods and N is cardinality of PotentialNewAssets union PotentialNewLiabilities then the number of variables in the optimization formulation is O(kN). Offhand I would choose the holding period frequency to be daily for the first year then monthly for the remaining 4 years of the 5 year simulation horizon. Perhaps 300 total holding periods and about 1000 potential assets and liabilities. Shoot for under 500K total variables in the LP optimization formulation, all in. The {ci} and the {aij} are all computed statically once at initialization time. You can buy this LP code off the shelf, you do not even have to write it custom.

Move all the balance and rate modeling for the RunoffAssets and RunoffLiiabilities to a static one time computation prior to running the optimizer. The optimizer doesn't really know or care if the holding period data is expected case, worst case, or historical. That is all taken care of in up front static computation. The optimizer is simply sorting though all the possible combinations of new investments and timing of new investments to find the maximum NII over the simulation horizon. We may want to weigh the NII upfront to compensate for the uncertainty in the market movement.

The representation of the NewAsset and the NewRunoffAssets in the k*N PotentialNewAsset variables is interesting.

Next the formulation of the constraints:

$$\mathbf{Ax = b} \text{ and } \mathbf{x} >= 0.$$

Since the x_i are the current face value amount of a position in an accrual portfolio of assets and liabilities the constraint that the x_i is strictly positive is no problem (derivative hedges will require some additional thought, however). At the beginning of a holding period (think start of the fiscal year) the securities x_i fall into two sets:

Runoff Positions { x_i: where x_i are the current face amounts of contract positions held in the accrual portfolio from prior to the holding period}. Define two subsets **Runoff Assets** and **Runoff Liabilities** with the obvious definitions for each of the holding periods hp_k**.** Denote these subsets as

RunoffAssets(hp_k) and RunoffLiabilites(hp_k)

New Investments (x_i: where x_i are the current face amounts of positions first entered into during the holding period}. Define to additional subsets – **New Assets** and **New Liabilities** with the obvious definition and parameterize these by the holding period hp_k. Denote these subsets as

NewAssets(hp_k) and NewLiabilites(hp_k)

It is clear that the sets Runoff Assets, Runoff Liabilities, New Assets, New Liabilities are mutually disjoint.

We will augment the set **New Investments** to include prospective contracts that could go on the accrual book during the holding period. Call this set of parameterized current face amounts **Prospective New Investments** and the corresponding c_i are the expected returns over the holding period. The subsets Prospective Assets and Prospective Liabilities are defined in the obvious way. The sets Prospective Assets and Prospective Liabilities are disjoint. Per holding period then we have

ProspectiveAssets(hp_k) and ProspectiveLiabilites(hp_k)

For holding period hp_k the set of NewAssets(hp_k) is a subset of ProspectiveAssets(hp_k) and the set NewLiabilties(hp_k) is a subset of ProspectiveLiabilities (hp_k). We are abusing notation a bit in that a position in ProsepctiveAssets(hp_k) has a parametirized amount a(t) and the corresponding position in entered into and recorded in NewAssets(hp_k) has an actual amount. The NewAsset set element is an instantiated version of the ProspectiveAsset element.

Here is one hook. Computationally, the Runoff Positions are constant with respect to the optimization search. Once you know the expected timing of the cashflow runoff relative to the discretization induced by the toy model sequence of holding periods the only contribution that the Runoff Positions makes to the LP optimization is a base NII/NIM contribution.

That means for a k holding period optimization the entire accrual portfolio directly contributes 6k variables to the optimization problem formulation. Let $\{x_1, \ldots x_k\}$be the aggregate face amount of RunoffAssets and let $\{x_{k+1}, \ldots x_{2k}\}$be the aggregate face amount of RunoffLiabilities. Let $\{x_{2k+1}, \ldots x_{3kN}\}$ be the expected runoff cash flows for each of the k holding periods for the aggregated runoff assets. Let

(x_{3kN+1}, … x_{4kN}} be the expected runoff cashflows for each of the k holding periods for the aggregated runoff liabilities.

Variables	To	Description
1	K	RunoffAsset face amount at start of holding period
k+1	2k	RunoffLiability face amount at start of holding period
2k+1	3k	RunoffAsset cash flow during holding period
3k+1	4k	RunoffLiability cash flow during the holding period
4k+1	4k+kN	ProspectiveAsset agg face amount for holding period
4k+kN+1	4k+2kN	ProspectiveLiability agg face amount for holding period
4k+2kN+1	4K+3kN	ProspectiveAsset cash flow during holding period
4k+3kN+1	4k+4kN	ProspectiveLiability cash during holding period

After the Monte Carlo simulation (or historical simulation) these are known constants:

$a_{1,1} * x_1 = b_1$ // runoff asset return

$a_{k+1,k+1} * x_{k+1} = b_{k+1}$ // runoff liability return

$a_{3,3} * x_3 = b_3$ // runoff asset cashflow available in holding period 1

…

$a_{2k+3,2k+3} * x_{2k+3} = b_{2k+3}$ // runoff liability cashflow available in holding period k

For a large bank, the many millions of accrual portfolio positions can enter into the optimization in proportion to the number of holding periods (there will be other aspects of the accrual portfolio we will want to expose to the numerical optimizer, later). But the main point is that the dimensionality of the numerical optimizer problem formulation does not need to be proportional to the cardinality of the **Runoff Positions** set.

The number of variables in the numerical optimization formulation will be proportional to the cardinality of the set **Prospective New Investment** times the number of holding periods. It would make sense that the NII maximum increases with the number of simulated holding periods since runoff cashflows can roll to new investments earlier in the simulation period, presumably earning a greater expected net return. Let j = 2k +4 then we introduce slack variables to accommodate the following inequalitites:

$$a_{j,1} *x_j + a_{j,2} * x_{j+1} + a_{j,3} * x_{j+2} + … + a_{j,k} * x_{j+k-1} < = b_J$$

$$a_{j+!,1} *x_{j+k} + a_{j+1,2} * x_{j+k+1} + a_{j+1,3} * x_{j+k+2} + … + a_{j+!,k} * x_{j+2k-1} < = b_{j+1}$$

…

covering all the Prospective New Investment parameterized face amounts and the expected returns. The inception face amounts are selected by the optimizer relative to the runoff cashflow available. The Balance model determines how the face amount changes over the simulation horizon. The expected returns are generated by the market and simulation driven Returns model. Since these will be the dominant part of the formulation in terms of the number of variables, you can see that the matrix A in

Ax = b and **x** >= 0.

will be sparse.

The x_i values are determined by the numerical optimization execution. The values a_{ij} are computed once per expected (or historical) market. The a_{ij} are determined by the Monte Carlo paths and new investment balance and return model. They are constant with respect to the LP execution. In our examples in this presentation we will use Fed historical data as a proxy.

Additionally we need some constraints to keep the assets and liabilities balanced in each holding period:

Asset face amount must equal Liability face amount per holding period thus

$$\sum_{x_i \in RunoffAssets(hp)} x_i + \sum_{x_i \in NewAssets(hp)} x_i - \sum_{x_i \in RunoffLiabilities(hp)} x_i - \sum_{x_i \in newLiabilities(hp)} x_i = 0$$

Runoff cash flow reinvestment per discretionary capital allocation plan can only be used once in the holding period. Recall (x_3, x_{k+2}) are the runoff cashflows from RunoffAsset in each of the k holding periods. Similarly, (x_{k+3}, x_{2k+3}) are the runoff cashflows from the set RunoffLiabilities in each of the k holding periods.

$$x_3 - x_{k+3} - \sum_{x_i \in NewAsset(hp1)} x_i + \sum_{x_i \in newLiabilities(hp1)} x_i = 0$$

The first holding period is relatively simple because there were no accumulated new investments from previous holding periods to account for. In these next holding periods we need to introduce terms to account for the balance and return dynamics over the previous simulated holding periods.

$$x_4 - x_{k+4} - \sum_{x_i \in NewAsset(hp2)} x_i + \sum_{x_i \in newLiabilities(hp2)} x_i = 0$$

…

$$x_{k+2} - x_{2k+3} - \sum_{x_i \in NewAsset(hpk)} x_i + \sum_{x_i \in newLiabilities(hpk)} x_i = 0$$

You can only invest the runoff from the holding period. You could make it so that you can only invest the runoff that settled in the previous holding period as well.

We need equations for the balance model runoff and returns of the first holding period in the subsequent k-1 holding periods. What was new investment in holding period 1 becomes part of the runoff portfolio in holding periods 2, 3,… k. You cannot assume that the inception face amount is maintained as constant from the first holding period to the second holding period. The market changes thought he term of the first holding period must be reflected through the balance model in the face amount at the start of second holding period. Similarly for new investments entered in holding periods 2,3, .. , and k-1 they enter into the runoff portfolios in subsequent holding periods until we reach the simulation horizon.

Once in a new investment the capital is tied up to maturity of the new investment, subject to the runoff output from the Balance model. Notice the optimization search will select the holding period in which to enter into a new investment but for the subsequent holding periods in the simulation horizon the new investment balances and returns become subject to market moves. Therefore the c_i need to account for the inception runoff portfolio, the newly acquired holding period 1 runoff portfolio, the newly acquired holding period 2 runoff portfolio, …

Reiterate the form of the NII in terms of the variables just defined. In particular how are the {ci} computed.

We have a sparse matrix formulation with a simulation horizon long enough to allow the default function to effect the implementation of the capital allocation plan from the beginning.

Example:

We are given a balance sheet for the Accrual portfolio composed of a pool of deposits; a pool of short term borrowing; a pool of mortgages, and a pool of credit card accounts. The goal is to formulate a four quarter capital allocation plan where the capital allocation plan initiates in the first period (quarter) and is complete by the last quarter. The expected, or historical, market enters the problem formulation through the term of the contracts and the time series functions representing the balance and return.

Our market expectation is that interest rates will rise and credit spreads will deteriorate through the first two quarters. Interest rates will stabilize and credit spreads will tighten in the second half. This market view induces a set of optimal future portfolio toy asset configurations as well as New Asset and New Liability booking over the simulation horizon.

Given the Balance and return models we have several options:

1. Run the models with the implied forward market data
2. Run the models with the actual historical data
3. Run the models at a past as_of date for both the actual and the implied forward market

For now we will run with the actual historical Fed market data.

The toy accrual portfolio is composed of a Toy liability portfolio:

[Deposit Liability(ts, ts+10Y, 50bps *10, 10, r(t), 0),

Short Term Debt(ts, ts+3M, Libor*0.5, 0.5, r(t), 0)}

and an asset portfolio:

{Cards Asset(ts, ts+1Y, (Libor+300)*5 + d(t), 5, r(t), d(t)) ,

Mortgage Asset(ts, ts+5Y, (Libor+350)*5.5 + d(t), 5.5, r(t), d(t)),

Short Term Loan Asset(ts, ts+3M, p(t), a(t), r(t), 0)}.

Note the d(t) gains/losses are realized through the holding period.

We are given a set of new origination options in deposits and commercial loans.

Toy model ProspectiveAssets:

[Deposit Liability(ts, ts+10Y, 50bps *X, X, r(t), 0),

Short Term Debt Liability(ts, ts+3M, Libor*X, X, r(t), 0)}

Toy model ProspectiveLiabilties:

{Cards Asset(ts, ts+1Y, (Libor+300)*X + d(t), X, r(t), d(t)) ,

Mortgage Asset(ts, ts+5Y, (Libor+350)*X + d(t), X, r(t), d(t)),

Short Term Debt Asset(ts, ts+3M, Libor*X, X, r(t), 0)}.

I think you can force in the bid ask spread model depending on the size of the trade into the short term debt contract. I am not sure how meaningful the bid ask is for the deposits, card, and loans as the markets are sort of one-sided until you pool or tranche up assets to sell. The toy optimization problem is trivial if we ignore the credit risk. Select the amounts of new origination to maximize the aggregate NIM while staying within the Firm's prescribed risk limits.

The optimization will be non-trivial over multiple periods because we include the credit default risk in addition to the interest rate risk in the basic model of the asset and liability. Excel will run both the LP and NLP NIM Optimization toy model problem.

The accrual portfolio:

The new origination options:

The Market Scenario:

LP Toy Example

Use LP formation to get the approximate timing, location, and identity of the new investment capital plan relative to the runoff.

Formulation of the LP optimization problem for nimo divides the Accrual portfolio into the existing inventory and the new investment.

Assume the existing inventory enters the formulation as a series of equalities and the new investment as a series of inequalities.

The optimization search is limited to the combinations of the new investments with the various holding locations of the inventory. The idea is to find the timing and identity of the optimal new investments.

Note that we can use the FED portfolio, econometric, and market data for the entire market to set up the test formulation. Individual bank portfolios can be tagged in the universal portfolio. We can start the simulation in 2013 and roll forward to estimate the gain available to optimization for 2013-2014.

For all the expected case market valuations the runoff portfolio needs to be evaluated only once.

Push the formulated problem to the Excel LP optimizer

NLP Toy Example

Push the formulated problem to the Excel LP optimizer

Applications:

There are several applications of these sorts of optimizations. The first is simply optimizing the Firm capital allocation plan. The second is automation of the Treasury implementation of the dynamically optimized capital allocation process. The third application is the Whole Market Asset & Liability Optimization.

Application 1: Optimization of the Firm Capital Allocation Plan

Application 2: Automation of the Treasury

Application 3: Whole Market Asset & Liability Optimization

6. Summary

Provided a toy model and notation for Balance sheet simulation at the security/contract level.

Provided a simulation framework foe enforcing compulsory capital allocation activities and computing the single period P&L performance attribution.

Provided a mechanism for implementing discretionary capital allocation plans, Enumerated the common capital allocation alternatives and outlined how they might be automated in a numerical optimization formulation. Monitored P&L attribution

Integrated market data into multi-period NIM simulation to derive the model NIM/NII versus the actual NIM/NII.

Formulated the numerical optimization problem for NIM and NII. Showed that the optimization runtime will depend on the number of new investment alternatives and the number of holding periods simulated. Outlined the analytic trade offs between LP and NLP formulations and executions. Discussed three applications of the toy model: Balance Sheet Control Theory, Market Simulation, and Backward Analysis of outstanding positions.

Outline of Next Steps:

- Find the Current NIMo computational cost relative to Breakeven.
- Run entire market simulation and numerical optimization with Federal Reserve data. Explain the match to reported NIM/NII historical results.
- Application example in the toy model
- Show max NII behavior with exponentially shrinking holding periods.
- Try to match toy model to historical data – show fidelity of the match.

Estimate the computational resources for a full market NIM optimization in the Funding NIMo paper

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