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AMS 326

April 24, 2020

Homework 3 Report

PART 1:

Problem:

Calculate the trajectory of a boat with initial speed of 14 crossing a river of width 7777 with a water speed function of

$$w(x) = 4v_0\left(\frac{x}{a} - \frac{x^2}{a^2}\right)$$

for three different boat speeds: 7, 14, and 21.

Algorithm:

The boat's ***x*** and ***y*** component velocities can be represented as:

$$\begin{cases} \dfrac{dx}{dt} = -v_B\cos\alpha = -\, v_B\dfrac{x}{\sqrt{x^2+y^2}} \\ \dfrac{dy}{dt} = -v_B\sin\alpha + w(x) = -\, v_B\dfrac{y}{\sqrt{x^2+y^2}} + 4v_0\left(\dfrac{x}{a} - \dfrac{x^2}{a^2}\right) \end{cases}$$

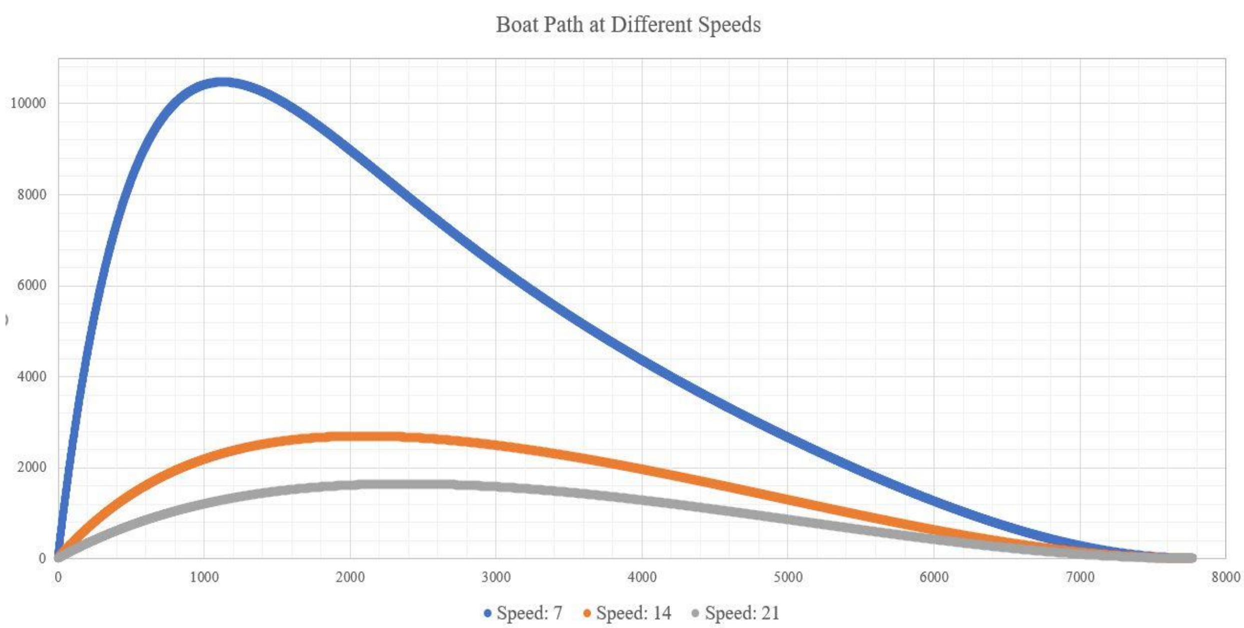
After eliminating the dependence of ***t*** in the two equations, we are left with the initial value problem:

$$
\begin{cases}
\dfrac{dy}{dx} = \dfrac{y}{x} - \dfrac{w(x)}{v_B\left(\dfrac{x}{\sqrt{x^2 + y^2}}\right)} \\
y(x = a) = 0
\end{cases}
$$

In my program, I used the Backward Euler Method to solve this IVP and generate data points that relate to the path of the boat.

Results:

My program outputs the data points into data.csv, a file that can be interpreted by Microsoft Excel. With Excel, I used the data to create a graph showing the boat's trajectory at different speeds:



The ***x*** and ***y*** axes indicate the boat's position. The curves make logical sense: the faster the boat is moving across the river, the less the water speed affects its path. Because of this, I am confident in the program's correctness. The runtime is also within seconds, so it is efficient too.

PART 2:

Problem:

Repurpose Buffon's Needle problem to calculate the probability of a disc (with a diameter of ***d*** = ¾) touching parallel lines distance ***w*** = 1 apart from each other after 1,984,444,444 tosses.

Algorithm:

To calculate this probability, I used a simple Monte Carlo algorithm. Each iteration, I generated a random value that represents the ***x*** coordinate of a disc thrown. I then found what two lines (I used a sample of 10 lines in my model) the center of the disc is between. If the difference between the center of the disc and either the left line or right line is less than or equal to the radius of the disk, then the disk is touching a line and I increase a counter. After all iterations, I the value of the counter divided by the number of tosses is the probability.

Results:

Here is example output of my program:

```
Generating 1984444444 tosses...
The probability of touching a line is 0.750002
Time Taken: 2.33 minutes
```

The probability, after several attempts, averages to 75%. I observed that this is the same value as the diameter of the disk. After experimenting with the parameters of the program, I have concluded that the general probability for a disk in this problem can be represented as:

$$P(disc) = \frac{d}{w}$$

NOTE: I worked with fellow student Gino Giacoio when discussing which algorithms and methods to use.