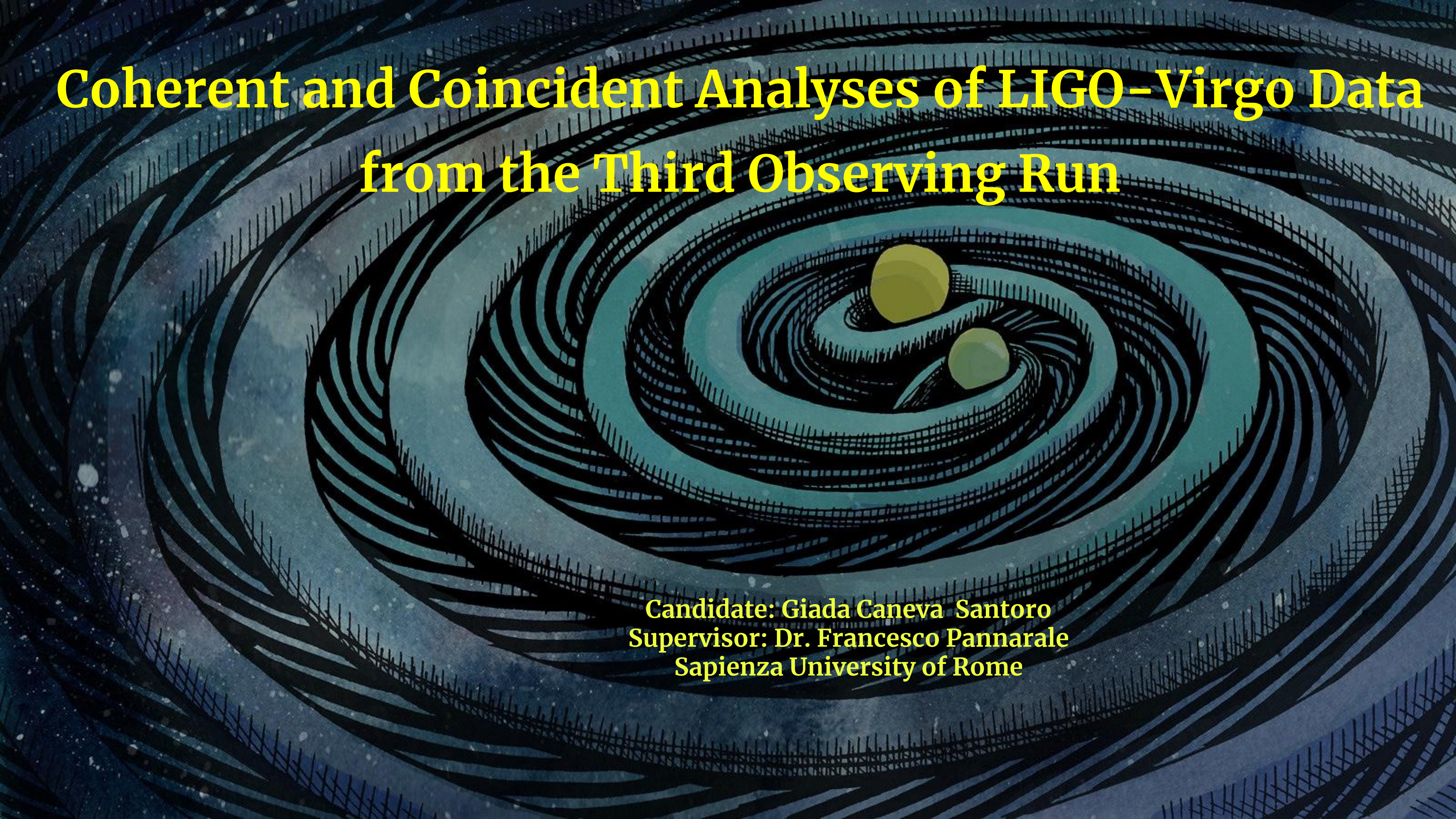

Coherent and Coincident Analyses of LIGO-Virgo Data from the Third Observing Run

Candidate: Giada Caneva Santoro
Supervisor: Dr. Francesco Pannarale
Sapienza University of Rome

Gravity: from apples to ripples

GENERATION OF SPACETIME METRIC WAVES

Newton's law of universal gravitation

$$\boxed{F = G\frac{m_1 \times m_2}{d^2}}$$

Einstein field equations (EFE)

$$\boxed{G_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}}$$

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$$\boxed{\Box \bar{h}_{\mu\nu} = 0}$$

WAVE EQUATION WITH PROPAGATION VELOCITY *c*

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...whose solution is a superposition of plane waves:

$$\bar{h}_{\mu\nu}(t, \mathbf{x}) = \int d^3\mathbf{k}\, A_{\mu\nu}(\mathbf{k}) \cos(c|\mathbf{k}|t - \mathbf{k} \cdot \mathbf{x})$$

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By imposing gauge conditions its 10 components reduce to two degrees of freedom, thus for a plane wave along the *z* axis:

$$h_{\mu\nu}^{TT} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cos \omega(t - z/c)$$

h_+ and $h_\times$ are the two GW polarisations perpendicular to the direction of propagation

GRAVITATIONAL PERTURBATION ON MATTER

A GW generates periodic distortions in flat spacetime, which can be described in terms of the Riemann tensor:

$$\boxed{R^{\mu}_{\nu\rho\sigma} = \frac{1}{2}\left(\partial_\rho \partial_\nu h^{\mu}_{\nu\rho} + \partial_\sigma \partial^\mu h^{\nu\rho} - \partial_\rho \partial^\mu h^{\nu\sigma} - \partial_\sigma \partial_\nu h^{\mu}_{\rho}\right)}$$

- Measures the curvature of spacetime.
- Embodies the tidal force field and describes the relative acceleration between two particles in free fall.

Gravitational Wave Detection

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- Measures the curvature of spacetime.
- Embodies the tidal force field and describes the relative acceleration between two particles in free fall.

How does the curvature of spacetime affect the motion of matter?

Geodesic deviation: relative motion of nearby particles in terms of a tidal force determined by the Riemann tensor.

$$\boxed{\frac{D^2\zeta^\mu}{D\tau^2} = -R^{\mu}_{\nu\rho\sigma}\zeta^\rho \frac{dx^\nu}{d\tau}\frac{dx^\sigma}{d\tau}}$$

Gravitational Wave Detection

GRAVITATIONAL PERTURBATION ON MATTER

Geodesic deviation for two test masses in a near local-Lorentz frame, moving non-relativistically:

$$\ddot{\zeta}^j = -R^i_{0j0}\zeta^j$$

$$R^i_{0j0} = R_{i0j0} = -\frac{1}{2}\ddot{h}^{TT}_{ij}$$

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Gravitational Wave Detection

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GW travelling in the z-direction towards a ring of test masses:

- The plus polarisation is

$$h^{TT}_{ab} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} h_+ \cos\omega t$$

- The displacement between geodesic is

$$\zeta_a(t) = (x_0 + \delta x(t), y_0 + \delta y(t))$$

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$$\zeta_a(t) = (x_0 + \delta x(t), y_0 + \delta y(t))$$

$$\delta\ddot{x} = -\frac{h_+}{2}(x_0 + \delta x)\omega^2 \sin\omega t$$

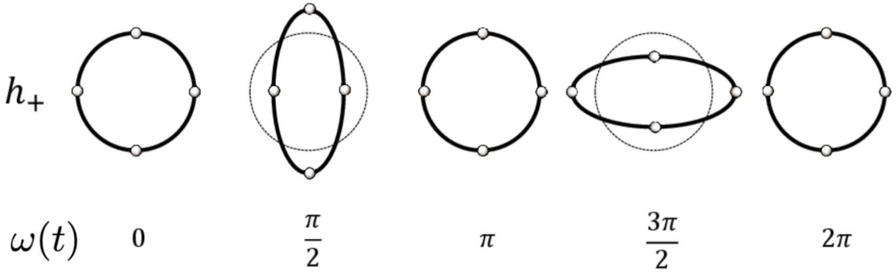
$$\delta\ddot{y} = \frac{h_+}{2}(y_0 + \delta y)\omega^2 \sin\omega t$$

GRAVITATIONAL PERTURBATION ON MATTER

Integrating for the plus and cross polarisation leads to two set of equations:

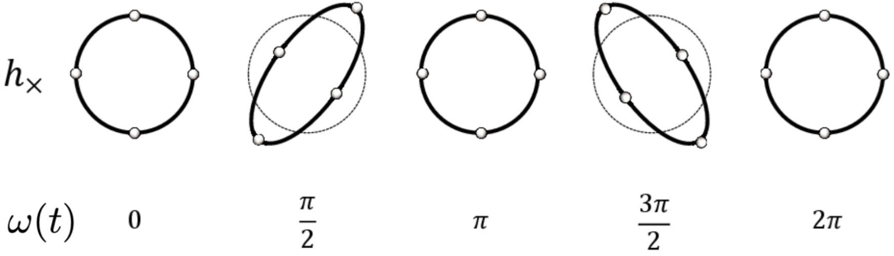
$$\delta x(t) = \frac{h_+}{2} x_0 \sin \omega t$$

$$\delta y(t) = -\frac{h_+}{2} y_0 \sin \omega t$$



$$\delta x(t) = \frac{h_\times}{2} y_0 \sin \omega t$$

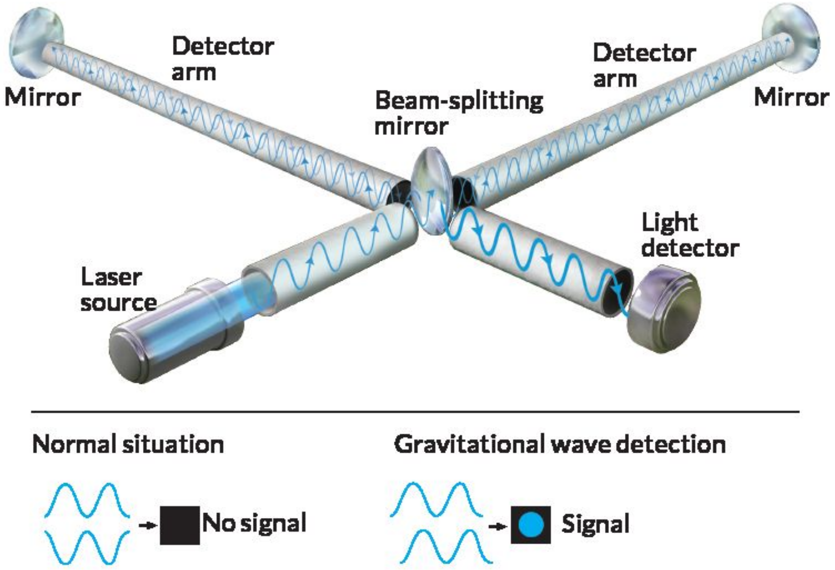
$$\delta y(t) = \frac{h_\times}{2} x_0 \sin \omega t$$



Gravitational Wave Detection

INTERFEROMETRIC DETECTORS

- Laser beam is sent through a beam splitter and ends up in two resonant cavities, in which luminosity is built up.
- The light is allowed to interfere where the arms join together.



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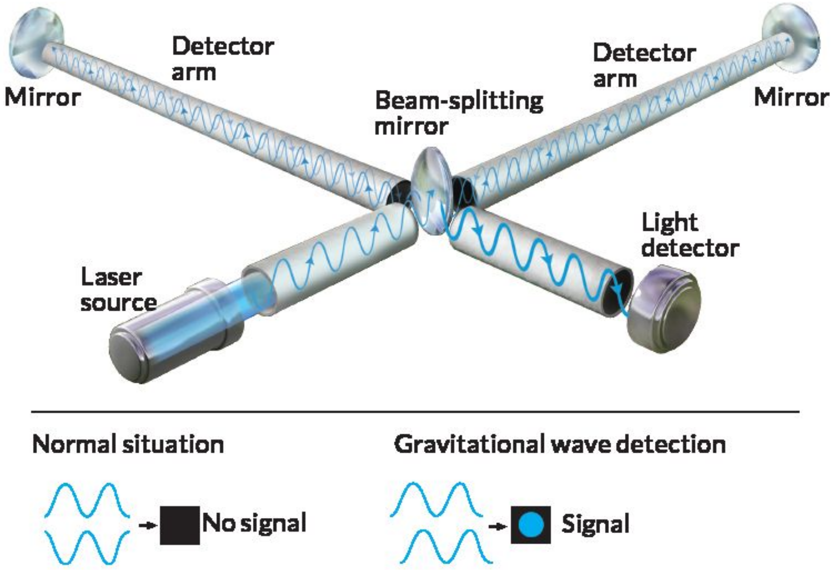

No GW → Interference is destructive, no light hits the photodetector at the output.

GW → Light path periodically shorten in one direction and lengthen it in the other.

Gravitational Wave Detection

INTERACTION WITH TEST MASSES

The distance of a mirror with respect to the beam splitter is described by the geodesic deviation equation:

$$\ddot{L}^i = \frac{1}{2}\ddot{h}^{TT}_{ij} L^j$$

The proper distance along the *x* axis expands and shrinks periodically, with a fractional length change

$$\frac{\delta L}{L_c} \simeq \frac{1}{2} h^{TT}_{xx}(t, z = 0)$$

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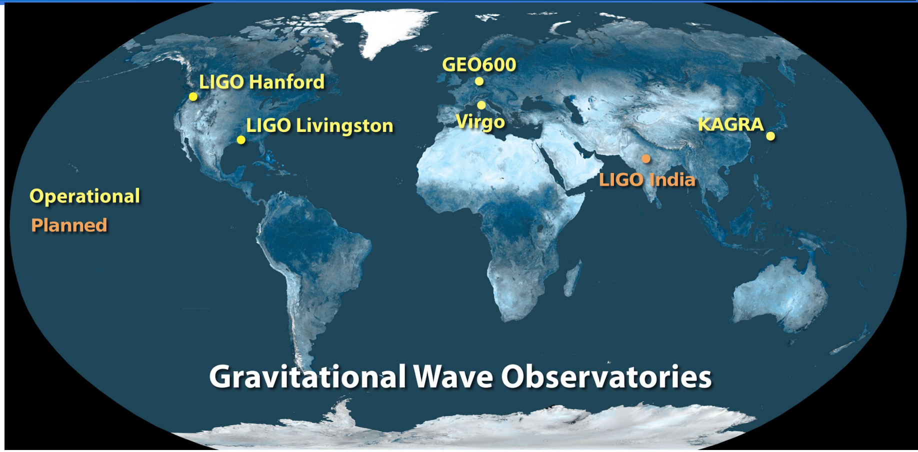
Since h(t) is linear in h_+ and $h_\times$, it can be written through the *beam pattern functions*

$$h(t) = F_+ h_+(t) + F_\times h_\times(t)$$

Gravitational Wave Detection

Network of detector is necessary to

- provide directional information about the source
- separate a signal from noise confidently



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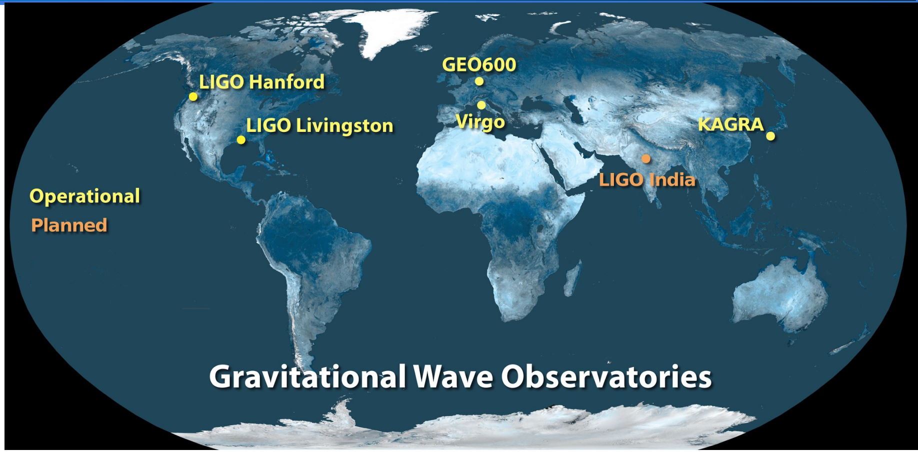
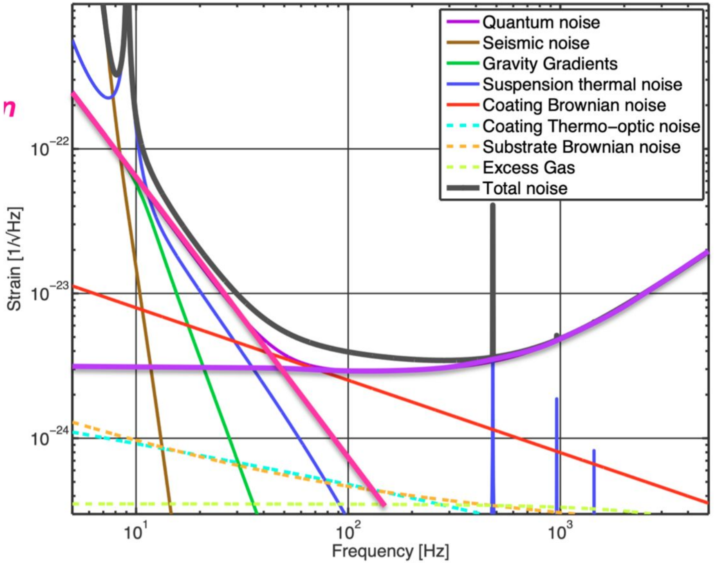




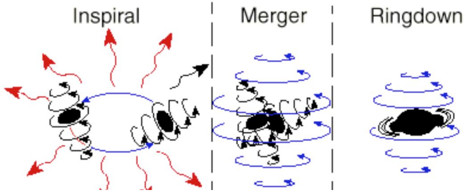
If there is a signal: measured strain is a sum of noise and signal:

$$s(t) = \underbrace{n(t)}_{\text{instrumental noise arising from naturally occurring random processes}} + h(t)$$

Gravitational Wave Sources

INSPIRALLING BINARY SYSTEMS:

- Compact objects: neutron stars and/or black holes.
- Three stages evolution that leads to coalescence.
- Inspiral needs to be in its final stages in order for the GWs to be detectable by Earth-based interferometers.



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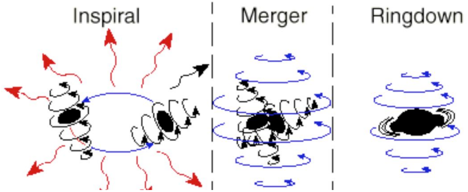
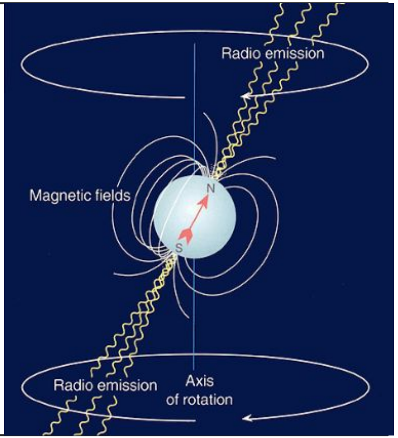


PERIODIC SOURCES:

- Continuous sources of GWs such as pulsars with a non-trivial quadrupole moment.
- Evolution time is longer, GW emission weaker
- GW signal almost constant in amplitude and frequency



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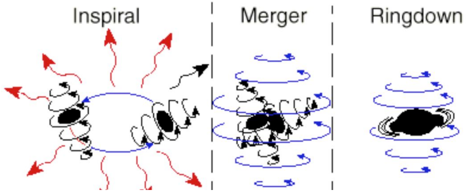

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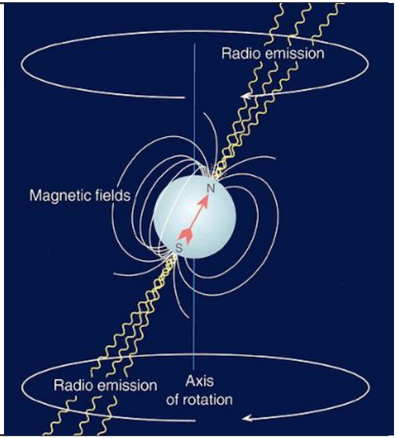

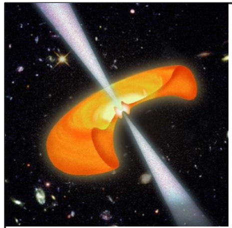

UNMODELED BURST

- Short-duration unknown or unanticipated sources.
- Potential progenitors: systems such as supernovae or gamma ray bursts
- Little is known about the details of these systems to anticipate an accurate waveform

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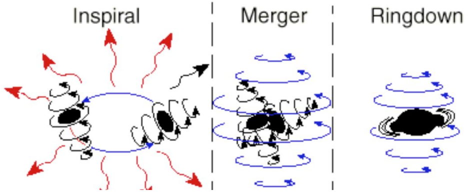

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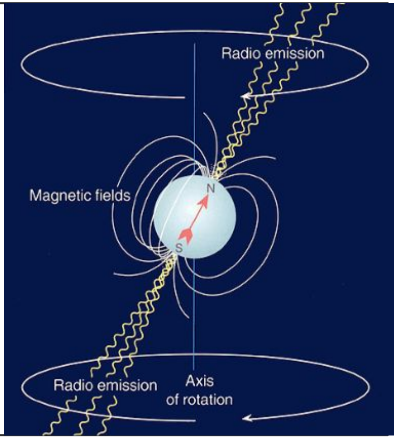

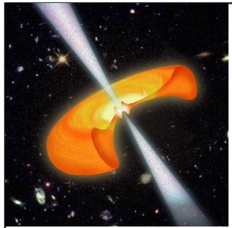

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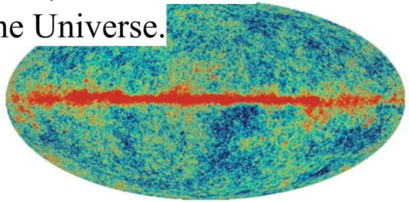
STOCHASTIC GWs

- Remnant GWs from the early evolution of the universe.
- Superposition of GWs produced by many simultaneous inspirals, bursts, or continuous signals from throughout the Universe.
- cosmic GW background



Gravitational Wave Data Analysis

CBC GW SEARCHES

- **UNMODELLED:** minimal information about GW waveforms assumed
- **MODELLED:** theoretical GW waveforms predicted by GR

Gravitational Wave Data Analysis

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Gravitational Wave Data Analysis

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Signal from CBC buried deep in noise

$$s(t) = n(t) + h(t)$$

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$$(s|h)(t) = 4Re \int_{f_{low}}^{f_{high}} \frac{\tilde{s}(f)\tilde{h}^*_{\text{template}}(f)}{S_n(f)} e^{2\pi i f t} df \,,$$

Gravitational Wave Data Analysis

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optimal filter

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PSD

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FILTER THAT MAXIMISES THE OUTPUT OPTIMALLY EXTRACTS THE SIGNAL FROM NOISE!!

Signal candidate or not?

↓

Thresholding statistic necessary: SNR

$$\boxed{\rho(t) = \frac{|(s|h)|}{\sigma_h}}$$

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observed filter output

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root-mean-square fluctuations of the optimal filter

Gravitational Wave Data Analysis

Signal candidate or not?

↓

Thresholding statistic necessary: SNR

$$\boxed{\rho(t) = \frac{|(s|h)|}{\sigma_h}}$$

- Proportional to the amplitude of the signal buried in the noise.
- Threshold needs to be accurately chosen to avoid high FAR while detecting signals

Gravitational Wave Data Analysis

- Parameters of astrophysical systems will not be known a-priori.
- We know approximate *waveforms* of various sources (BBH, BNS and NSBH).
- To cover the full parameter space is computationally prohibitive.
  - maximise over an overall amplitude and phase of the signal.
  - use inverse Fourier transform to evaluate the statistic as a function of time.
    - **Only intrinsic parameters - masses and spins - remain.**

Match-filter data against a predetermined collection of waveform models:
the template bank.

Gravitational Wave Data Analysis

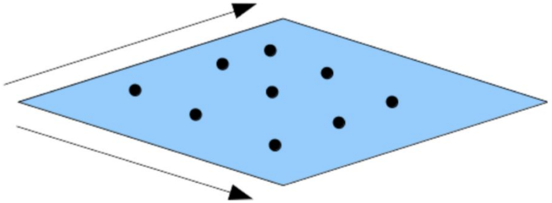
TEMPLATE BANKS

Waveforms manifold: continuous space in the component masses and spins.

Lay out template banks over parameter space: points $\lambda = \{\lambda_{intr}, \lambda_{extr}\}$ waveforms $h(f;\lambda)$

NORMALIZED OVERLAP

$$(\hat{h}_i|\hat{h}_j) = \frac{(h_i|h_j)}{\sqrt{(h_i|h_i)(h_j|h_j)}} \,.$$



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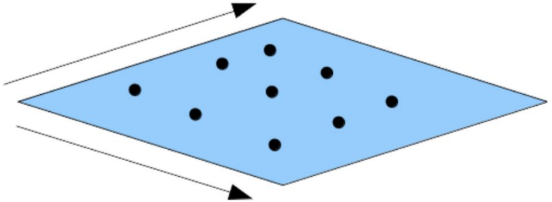

MATCH BETWEEN NEARBY TEMPLATES

$$\mathcal{M}(\lambda, \Delta\lambda) \equiv \max_{\Delta\lambda_{extr}} \left(\hat{h}(f;\lambda), \hat{h}(f;\lambda+\Delta\lambda)\right)$$

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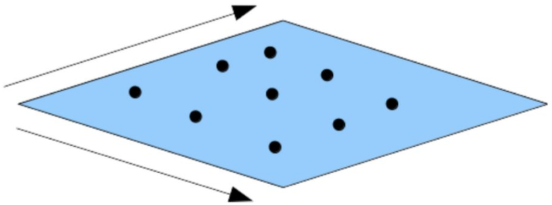

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EXPAND

$$\mathcal{M}(\lambda,\Delta\lambda) \simeq 1 - g_{ij}\Delta\lambda^i\Delta\lambda^j\,,$$

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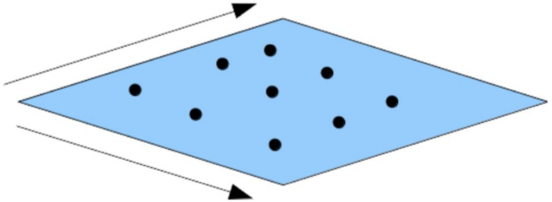

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METRIC WAVEFORM MANIFOLD

EXPAND

$$\mathcal{M}(\lambda, \Delta\lambda) \simeq 1 - g_{ij}\Delta\lambda^i\Delta\lambda^j ,$$

$$g_{ij} \equiv -\frac{1}{2}\frac{\partial^2\mathcal{M}}{\partial\Delta\lambda^i\partial\Delta\lambda^j}|_{\Delta\lambda=0}$$

Gravitational Wave Data Analysis

TEMPLATE BANKS

Templates placement in parameter space such that metric distance is never larger than a preset mismatch:

$$1 - \mathcal{M} = g_{ij}\Delta\lambda^i\Delta\lambda^j$$

Choice of the set of waveform models based on *Fitting Factor*:

$$FF(h_*) = \max_{\Lambda} \mathcal{M}(\tilde{h}_*, \tilde{h}(\Lambda))\,.$$

Template discrete placement $\longrightarrow$ Fractional loss of SNR in capturing the signal with the template bank.

$$\boxed{1 - FF(h_*)}$$

TEMPLATE BANKS

Effectualness: template bank is built such that no putative signal anywhere in the parameter space has

FF < the minimal match

- Reasonable FF value for effectual bank~97%
- FF distribution as indicator of parameter space regions with poor sensitivity.
- Systems whose GW is weaker corresponds to lowest FF:

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$$\boxed{FF_{\text{eff}} = \alpha^{1/3}}$$

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$$\boxed{FF_{\text{eff}} = \alpha^{1/3}}$$

$$\alpha = \frac{\int d\vec{x} FF^3(\vec{x})\sigma^3(\vec{x})}{\int d\vec{x}\sigma^3(\vec{x})}$$

★ Intrinsically bright sources are favoured.

★ Poorly aligned and intrinsically low GW luminosity disfavoured.

CANDIDATE RANKING STATISTIC

Detector calibrated strain data contains both Gaussian and non-Gaussian noise.

↓

Background under control

- **Data quality investigations and vetoes**
- **Consistency checks**
  - Mitigate the effect of noise transients.
    - **Chi-square test:** is signal distribution across frequency band as expected?
  - Assign an accurate statistical significance to candidate signals.

CANDIDATE RANKING STATISTIC

- **Chi-square test**

Background estimation

- Estimate search background from the data themselves.
- Time-shift technique: repeat analysis with time offset applied between detectors.
  - Background noise sample: coincidences resulting in this time-shifted data.
  - Each coincident trigger is assigned a FAR.
- Divide search spaces into different classes to deal with non-uniform background.
  - Candidate events is measured against the background from the same class.
- Hierarchical removal of confirmed signals from background to assess significance of other events.

COINCIDENT ANALYSIS

- Data from individual detectors are processed separately and then combined.
- Signals must be seen in both detectors within an allowed time window.
- Matched filtering done once per detector per template.
- Non-coincident triggers immediately rejected.
- Found coincidences are ranked by

$$\boxed{\hat{\rho}_{\mathrm{coinc}} = \sqrt{\hat{\rho}_1^2 + \hat{\rho}_2^2}.}$$

- Background estimation analysis repeated, FAR computed as function of the detection statistic.

COINCIDENT ANALYSIS

Detection is claimed

COHERENT ANALYSIS

- Data from the individual detectors are combined coherently in phase and then processed.
- Computationally expensive for modelled searches.
  - Externally triggered searches: time and sky-location are known, computational cost is decreased.
- Single statistic for the full network as to construct just one more sensitive detector.
  - New ranking statistic.

COHERENT ANALYSIS

The multi-detector log-likelihood

$$\boxed{\ln \lambda = (\mathbf{s}|\mathbf{h}) - \frac{1}{2}(\mathbf{h}|\mathbf{h})}$$

- Search a 7D parameter space of signals for binaries on circular orbits with and aligned spin components.
  - Maximising log-likelihood over the values of the 4 waveform amplitudes leads to:

$$\boxed{\rho^2_{\mathrm{coh}} \equiv 2 \ln \lambda|_{\mathrm{MAX}} \equiv (\mathbf{s}|\mathbf{h}_\mu)\mathcal{M}^{\mu\nu}(\mathbf{s}|\mathbf{h}_\nu)}$$

- More sensitive search: more effective than the coincident search in removing background noise.

COHERENT ANALYSIS

- Sensitivity increases with the number of the detectors in the network.
- Other improvements:
  - Ranking statistic.
  - Noise reduction and rejection mechanism.

Null stream consistency: stream that contains transients which do not contribute power to the two-polarisation signal space.

$$\boxed{\rho^2_{\text{null}} = \rho^2_{\text{coinc}} - \rho^2_{\text{coh}}}$$

COHERENT ANALYSIS

- Chi-square test

The PyGRB pipeline