“Ex n.n.n” = Example”, “N n.n.n” = NUMBAS Practice Problem, “n.n.n” = Exercise

Learning Evidence – Jacob Booth 221530207

1.1.2

a) $3x+y=2$ let $x=t$

$\cancel{3t}+y=2-3t$ $(x,y)=(t,\ 2-3t)\in\mathbb{R},\ t$

let $y=s$

$\frac{3x+s}{3}=\frac{2-s}{3}$ $(x,y)=(\frac{2-s}{3},\ s)\in\mathbb{R},\ s$

b) $2x+3y=1$ let $x=t$

$\cancel{2t}+3y=1-2t$, $y=\frac{1-2t}{3}$ $(x,y)=(t,\ \frac{1-2t}{3})\in\mathbb{R},\ t$

let $y=s$

$2x+\cancel{3s}=1-3s$, $x=\frac{1-3s}{2}$ $(x,y)=(\frac{1-3s}{2},\ s)\in\mathbb{R},\ s$

1.1.2

c) $3x-y+2z=5$ let $y=s,\ z=t$ Introduce new vars to represent free params

$3x\ \cancel{-s+2t}=\frac{5+s-2t}{3}$ $(x,y,z)=(\frac{5+s-2t}{3},\ s,\ t)\in\mathbb{R}\ s,t$

These are in parametric form, free vars let $x=r,\ z=t$

$\cancel{3r}-y\ \cancel{+2t}=5-3r-2t$

$\times -1$ $y=3r+2t-5$ $(x,y,z)=(r,\ 3r+2t-5,\ t)\in\mathbb{R}\ r,t$

d) $x-2y+5z=1$ let $y=s,\ z=t$

$x\ \cancel{-2s+5t}=1+2s-5t$ $(x,y,z)=(1+2s-5t,\ s,\ t)\in\mathbb{R},\ s,t$

let $x=r,\ y=s$

$\cancel{r-2s}+5z=\frac{1-r+2s}{5}$ $(x,y,z)=(r,\ s,\ \frac{1-r+2s}{5})\in\mathbb{R}\ r,s$

1.1.7

a) $\begin{matrix}x-3y=5\\2x+y=1\end{matrix}\Rightarrow\left|\begin{matrix}1&-3&5\\2&1&1\end{matrix}\right|$

b) $\begin{matrix}x+2y=0\\y=1\end{matrix}\Rightarrow\left|\begin{matrix}1&2&0\\0&1&1\end{matrix}\right|$

c) $\begin{matrix}x-y+z=2\\x\quad -z=1\\2x+y\quad=0\end{matrix}\Rightarrow\left|\begin{matrix}1&-1&1&2\\1&0&-1&1\\2&1&0&0\end{matrix}\right|$

d) $\begin{matrix}x+y=1\\y+z=0\\z-x=2\end{matrix}\Rightarrow\left|\begin{matrix}1&1&0&1\\0&1&1&0\\-1&0&1&2\end{matrix}\right|$

Ex 1.2.2

$\begin{matrix}3x+y-4z=-1\\x\quad+10z=5\\4x+y+6z=1\end{matrix}\Rightarrow\begin{matrix}1&0&10&5\\0&1&-34&-16\\0&1&-34&-19\end{matrix}\xrightarrow{-R2}0\ 0\ 0\ -3$

$-3R1$, $-4R1$

$0x+0y+0z\neq-3$

$\therefore$ no solution

Ex 1.2.3

$\begin{matrix}x_1-2x_2-x_3+3x_4=1\\2x_1-4x_2+x_3\quad=5\\x_1-2x_2+2x_3-3x_4=4\end{matrix}\quad\left|\begin{matrix}1&-2&-1&3&1\\0&0&3&-6&3\\0&0&3&-6&3\end{matrix}\right|$ $-2R1$, $-R1$

$\xrightarrow{+R2}1\ -2\ 0\ 1\ 2$

$\xrightarrow{/R3}0\ 0\ 1\ -2\ 1$

$\xrightarrow{-R2}0\ 0\ 0\ 0\ 0$

let $x_2=s,\ x_4=t$

$x_3\ \cancel{-2t}=1+2t$

$x_1\ \cancel{-2s+t}=2+2s-t$

$\therefore (x_1,\dots,x_4)\ (2+2s-t,\ s,\ 1+2t,\ t)\in\mathbb{R}\ s,t$

Ex 1.2.4

$\begin{matrix}x_1+3x_2+x_3=a\\-x_1-2x_2+x_3=b\\3x_1+7x_2-x_3=c\end{matrix}\quad\left|\begin{matrix}1&3&1&a\\0&1&2&a+b\\0&-2&-4&c-3a\end{matrix}\right|$ $+R1$, $-3R1$

$\xrightarrow{+2R2}0\ 0\ 0\ c-3a+2(a+b)$

$=c-3a$

$0+0+0=c-a+2b$

$\therefore c=a-2b$

$\left|\begin{matrix}1&3&1&a\\0&1&2&a+b\\0&0&0&0\end{matrix}\right|$

$x_1\ \cancel{+3x_2+x_3}=a-3x_2-x_3$

$x_2\ \cancel{+2x_3}=a+b-2x_3$

$x_1=a-3(a+b-2x_3)-x_3$

let $x_3=t$

$=-2a-3b+\cancel{6x_3-x_3}\ 5x_3$

$\therefore (x_1,x_2,x_3)=(-2a-3b+5t,\ a+b-2t,\ t)\in\mathbb{R},t$

when $c=a-2b$

Ex 1.2.5

$A=\left|\begin{matrix}1&1&-1&4\\2&1&3&0\\0&1&-5&8\end{matrix}\right|$ $\xrightarrow{-2R1}0\ -1\ 5\ -8$ $\xrightarrow{+R2}0\ 0\ 0\ 0$ $\therefore$ Rank(A) = 2

Important moving forward for quicker interpretation

Theorem 1.2.2

A system of m equations in n variables is consistent, with rank r:

1) The solution set uses $n-r$ params

2) $r<n$, infinitely many solutions

3) $r=n$, a unique solution

N1.3.5

$\left|\begin{matrix}2&2&-5&-4&20\\2&2&-1&0&6\\2&2&-2&3&7\\1&1&1&0&5\end{matrix}\right|$

$\xrightarrow{\div2}1\ 1\ -5/2\ -2\ 10$

$\xrightarrow{-2R1}0\ 0\ 4\ 6\ -14$ $\div4$ $\rightarrow 0\ 0\ 1\ 3/2\ -7/2$

$\xrightarrow{-2R1}0\ 0\ 3\ 7\ -13$ $-3R2$ $\rightarrow 0\ 0\ 0\ 5/2\ -5/2$

$\xrightarrow{-R1}0\ 0\ 7/2\ 2\ -5$ $-3/2R2$ $\rightarrow 0\ 0\ 0\ -1/4\ 1/4$

1 1 -5/2 -2 10

0 0 1 3/2 -7/2

$\cancel{0\ 0\ 0\ 5/2\ -5/2}$ $\div\frac{5}{2}$

0 0 0 1 -1

$\cancel{0\ 0\ 0\ -1/4\ 1/4}$ $+\frac{1}{4}R3$

0 0 0 0 0

$x_3+\frac{3}{2}x_4=-7/2+3/2$

$=-4/2=-2$

$x_3=-2$

$x_4=-1$

let $x_2=t$, so $x_1=3-t$

$x_1+x_2-\frac{5}{2}x_3-2x_4=10$

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Ex 1.3.2 $ax^2+bxy+cy^2+dx+ey+f=0$

$(x_i, y_i) = 1, 2, 3, 4, 5$

$$ax_i^2+bx_iy_i+cy_i^2+dx_i+ey_i+f=0$$

When i is $1, \dots, 5$ there are 5 equations m,
six variables $a, \dots, f$ n,
since for $(x_i, y_i) = 1, \dots, 5$ $m < n$, there is
a nontrivial solution

Ex 1.3.4

$x = \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$ $y = \begin{vmatrix} 2 \\ 1 \\ 0 \end{vmatrix}$ $z = \begin{vmatrix} 3 \\ 1 \\ 1 \end{vmatrix}$ $v = \begin{vmatrix} 0 \\ -1 \\ 2 \end{vmatrix}$ $w = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$

Scalars a, b, c where $ax+by+cz=v$ gives

$a|1,0,1| + b|2,1,0| + c|3,1,1| = \overset{v}{|0,-1,2|}$

$a1+b2+c3=0$
$a0+b1+c1=-1$ $\Rightarrow$ $\cancel{a}+\cancel{2b}+3c=0$, $b\,\cancel{+c} = -1-c$ for any c
$a1+b0+c1=2$ $a\,\cancel{+c} = 2-c$

Let $c=t \Rightarrow b=-1-t$, $a=2-t$ $\therefore$ ∞ solutions

$a|1,0,1|+b|2,1,0|+c|3,1,1| = \overset{w}{|1,1,1|}$

$\left.\begin{matrix} \cancel{x}+2b+3c = 1 \end{matrix}\right\}$ $3+0c \neq 1$ $\therefore$ no solution

$b\,\cancel{+c} = 1-c$
$a\,\cancel{+c} = 1-c$

only v is a combination of x, y, z

Ex 1.3.5

Homogeneous system $A = \begin{vmatrix} 1 & -2 & 3 & -2 & | & 0 \\ 0 & 0 & 10 & -6 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{vmatrix}$ $+3R1$, $-R2$, $+2R1$

$x_1 = 2x_2 + \frac{1}{5}x_4$

$\cancel{10}x_3 \cancel{-6x_4} = \cancel{0}\ \frac{6x_4}{10} = \frac{3}{5}x_4$

$x_1 = 2x_2 + \frac{1}{5}x_4$

$x_2 = s$

$x_3 = \frac{3}{5}t$ $\Rightarrow$ $\begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} = s\begin{vmatrix} 2 \\ 1 \\ 0 \\ 0 \end{vmatrix} + t\begin{vmatrix} \frac{1}{5} \\ 0 \\ \frac{3}{5} \\ 1 \end{vmatrix}$

$x_4 = t$

for all $x \in \mathbb{R}$

Ex 1.3.6

Homogeneous System $A = \begin{vmatrix} 1 & -3 & 0 & 2 & 2 & | & 0 \\ 0 & 0 & 1 & 6 & -1 & | & 0 \\ 0 & 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & 0 & | & 0 \end{vmatrix}$ $+2R1$, $-3R1$, $+3R1$, $+R2$, $-2R2$

$x_1 \cancel{-3x_2} \cancel{+2x_4} \cancel{+2x_5} = \cancel{0}\ 3x_2 - 2x_4 - 2x_5$

$x_3 \cancel{+6x_4} \cancel{-x_5} = \cancel{0} -6x_4 + x_5$

Let $x_2 = s$, $x_4 = t$, $x_5 = u$

$x_3 = -6t+u$

$x_1 = 3s-2t-2u$ $\therefore$ $\begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{vmatrix} = \begin{vmatrix} 3s \\ s \\ 0 \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} -2t \\ 0 \\ -6t \\ t \\ 0 \end{vmatrix} + \begin{vmatrix} -2u \\ 0 \\ u \\ 0 \\ u \end{vmatrix}$

Basic Solutions:

$s\begin{vmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{vmatrix}$ $t\begin{vmatrix} -2 \\ 0 \\ -6 \\ 1 \\ 0 \end{vmatrix}$ $u\begin{vmatrix} -2 \\ 0 \\ 1 \\ 0 \\ 1 \end{vmatrix}$

Theorem 1.2.2

A consistent system of m equations in n variables with augmented rank r:
1) $n-r$ param solution set
2) $r<n$, ∞ solutions
3) $r=n$, unique solution

① This establishes the number of free vars in any consistent system

Theorem 1.3.2

For an $m \times n$ matrix A of rank r, Homogeneous System in n variables with coefficient matrix A:
1) There are $n-r$ basic solutions, params
2) Every solution is a linear combination of the basic solutions

② The theorem is then applied to homogeneous systems outlining how each free variable forms a basic solution

Conclusion: The general solution to a homogeneous system is a linear combination of $n-r$ basic solutions, reflecting the solution set param count outlined previously in 1.2.2

Here I make the link between Theorem 1.2.2 and 1.3.2 to establish the nature of solutions by observing rank, and how the rank/var balance implies the number of combinations of basic solutions for homogeneous systems, creating the general solution.

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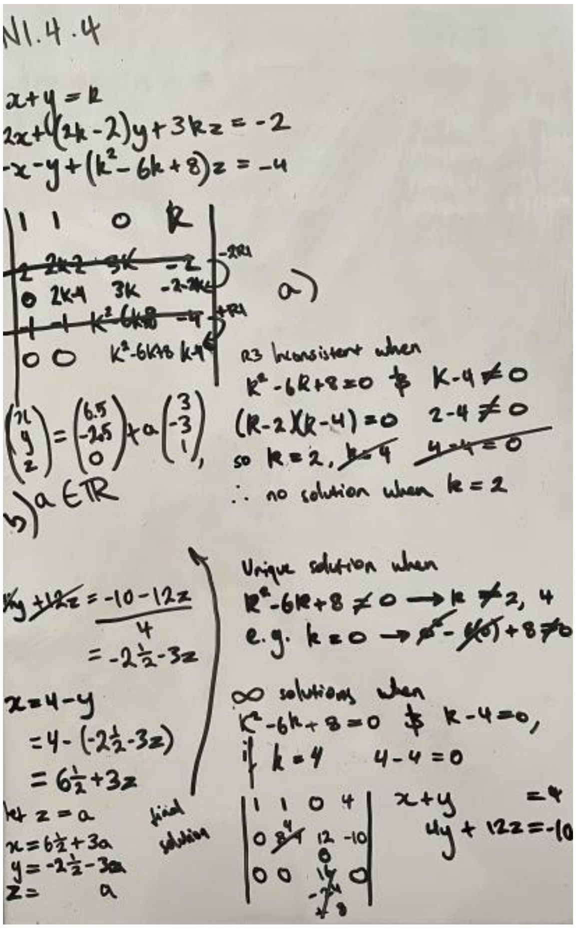

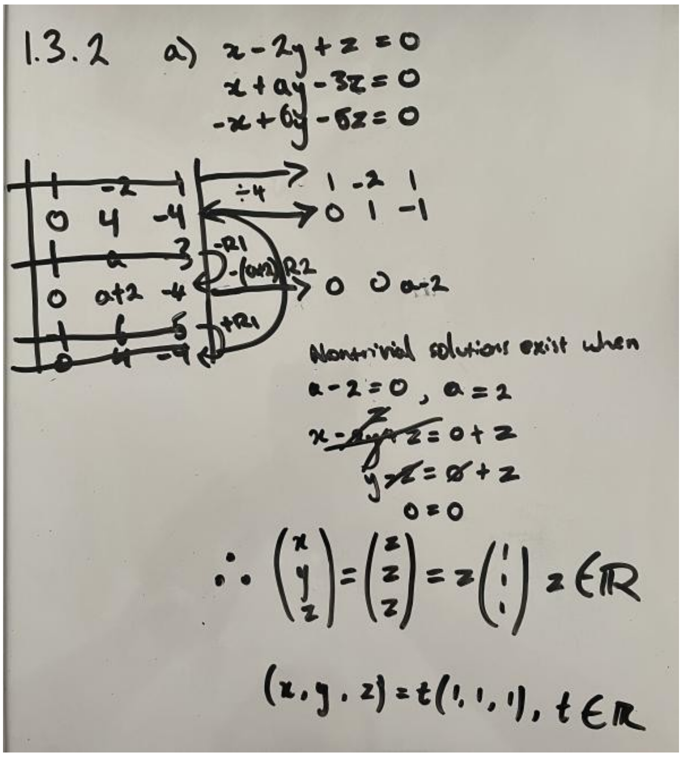

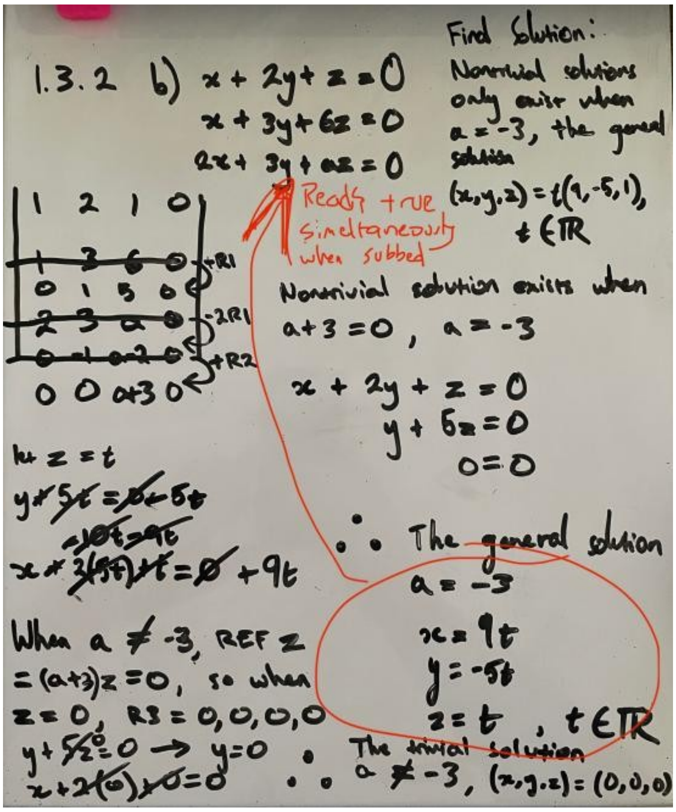

Exercises N1.4.4, 1.3.3 a and b were particularly good examples of the essence of this module. We have a good mix of unique, trivial/nontrivial (homogeneous), and a general solution in parametric form. These 3 problems highlight the largest amount of underlying prerequisite concepts as proof of clarifying the solution.

The areas where I made most mistakes for this module were the slightly larger 3x4 + matrices, since my train-of-thought and problem-depth-awareness become a little fuzzy the longer I spend on a single question. I relied on row-swapping for initiating Gaussian Elimination as smoothly as possible, and I had to take my time to avoid mini-mistakes leading to cascading errors. Although the speed that I can solve these problems is on the slower side, it’s been incredibly important for ensuring I don’t get lost in the middle of my problem. Taking the time to clarify where I’m up to and documenting each step by hand has gone a long way toward helping me keep my head on straight through longer multi-step problems. The shift over to a whiteboard-workflow has been much more effective for this whole process than my previous pen/paper approach, so I will definitely lean into this style of note-taking and solving going forward.

Additionally, my solution for problem 6 in the Self Assessment could have been done a little more gracefully. Part of balancing stress with timeliness is taking the approach which strikes me as intuitive first, but had I considered the matrix properties more carefully, I would have noticed the inter-dependencies that popped up as nested sub-problems while I was substituting. The second phase required substitution, but the first phase could have been more gracefully solved had I opted straight for Gaussian Elimination. Reflectively, maintaining stress and taking confidence in my capabilities to implement the module concepts is increasingly important. Moving forward, I’ll be considering more thoroughly how I can approach each problem based on the emphasis of the module, and work more on addressing my conceptual-toolkit before getting stuck into a solution.