Chapter 1

Methodology

Some of the simulation parameters are dependent of the cosmological model used, unless otherwise specified, all data is acquired using the Λ-CDM model with a matter density parameter $\Omega_M = 0.309$, $\Omega_\Lambda = 0.6911$, and a baryonic fraction $f_b = 0.156$ [1].

1 Units

Computer simulations are sensitive to rounding errors due to the lack of infinite precision when representing decimal numbers. Really small numbers as well as really big ones tend to have bigger errors than those close to the unity, as can be seen on Figure 1.1.

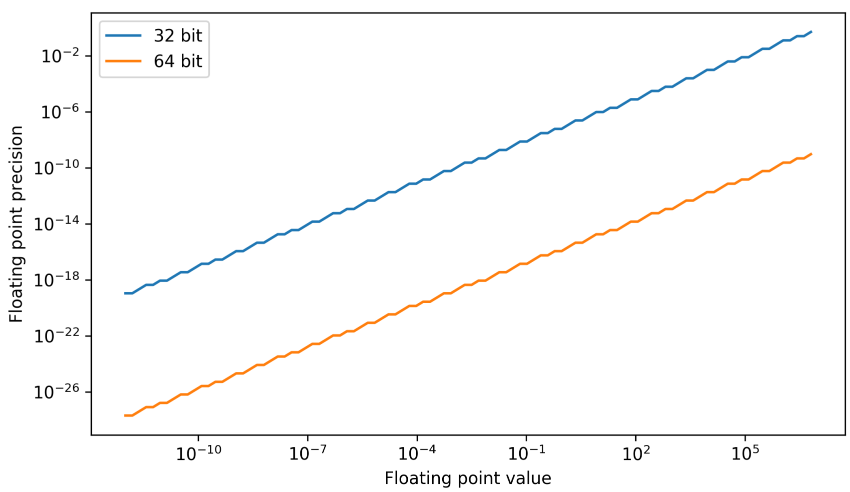


FIGURE 1.1: Floating point precision for different values, for a 32 bit and 64 bit holders.

Under the International System of Units, distances are measured on meters, times on seconds, and masses on kilograms, nevertheless black holes are too heavy to be measured on kilograms, galaxies sizes too big to be quantified on meters, and time scales too large for a second. Because of that, the following units will be used throughout this document:

TABLE 1.1: Units of measure used on the simulations.

Physical property	**unit**
Length	1 kilo-parsec (kpc)
Mass	10^5 solar masses ($10^5\ M_\odot$)
Time	1 giga-year (Gyr)

Along with the change of units, the universal gravitational constant and the Hubble parameter values are required to change.

1.1 Universal gravitational constant

First quantified by Henry Cavendish the gravitational constant has a value of $G_0 = 6.67408 \times 10^{-11}$ on SI units of $\mathrm{m^3 s^{-2} kg^{-1}}$. With the units of length, mass and time on Table 1.1, the constant of gravity used is given by:

$$\begin{aligned} G &= G_0 \left(\frac{1\ \mathrm{kpc}^3}{\left(3.0857 \times 10^{19}\right)^3\ \mathrm{m}^3} \right) \left(\frac{\left(3.154 \times 10^{16}\right)^2\ \mathrm{s}^2}{1\ \mathrm{Gyr}^2} \right) \left(\frac{1.98847 \times 10^{35}\ \mathrm{kg}}{10^5 M_\theta} \right) \\ &= 0.4493 \quad \frac{\mathrm{kpc}^3}{\mathrm{Gy}r^2 10^5 M_\odot} \end{aligned} \tag{1.1}$$

1.2 Hubble parameter

The Hubble constant is frequently used as $H_0 = 67.66 \pm 0.42\ \mathrm{kms^{-1}Mpc^{-1}}$ [2], stating the speed of an astronomical body on $\mathrm{kms^{-1}}$ at a distance of 1 Mpc. Nevertheless, the hubble constant has units of 1/time, thus, taking into account the units on Table 1.1 one gets:

$$\begin{aligned} H &= H_0 \left(\frac{1\ \mathrm{kpc}}{3.0857 \times 10^{16}\ \mathrm{km}} \right) \left(\frac{3.154 \times 10^{16}\ \mathrm{s}}{1\ \mathrm{Gyr}} \right) \left(\frac{1\ \mathrm{Mpc}}{1000\ \mathrm{kpc}} \right) \\ &\approx 1.023 H_0 \times 10^{-3}\ \mathrm{Gyr}^{-1} \\ &= 6.916 \times 10^{-2}\ \mathrm{Gyr}^{-1} \end{aligned} \tag{1.2}$$

Although the Hubble parameter is often called Hubble constant, its value changes with time as can be seen on Figure 1.2.

2 Critical density and Virial Radius

Mass distributions used for the simulation of the host galaxy, are divergent for distances up to infinity. Because of this, the cumulative mass of all bodies within a given distance is called the virial mass and its value is taken as the mass of the whole system. The distance taken to

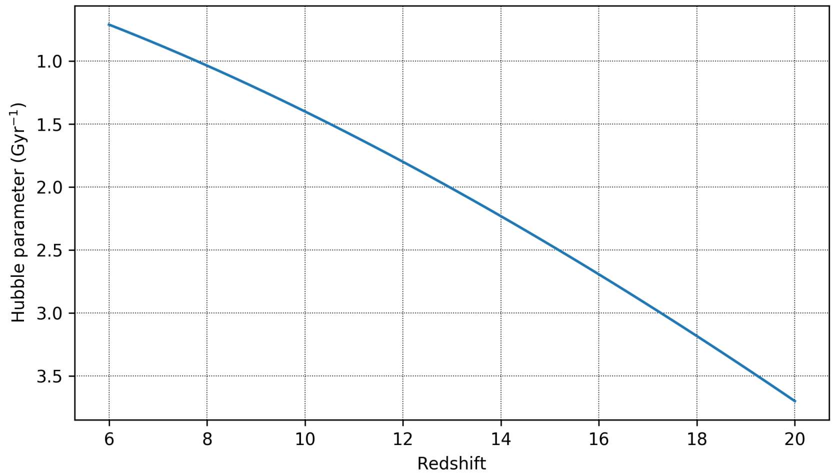


FIGURE 1.2: Dependency of the Hubble parameter with redshift.

calculate the virial mass is called virial radius ($R_{\rm vir}$), and it is defined as the distance at which the average density of the galaxy is 200 times the critical density of the universe ($\rho_{\rm crit}$).

$$\rho_{\rm crit} = \frac{3H(t)^2}{8\pi G} \tag{1.3}$$

$$\frac{M(R_{\rm vir})}{V(R_{\rm vir})} = \bar{\rho}(R_{\rm vir}) = 200\rho_{\rm crit} = 75\frac{H(t)^2}{\pi G} \tag{1.4}$$

where $M(R_{\rm vir})$ is the cumulative mass, and $V(R_{\rm vir})$: the volume

The relation on Equation 1.3 is found by considering the case where the geometry of the universe is flat, as a consequence it is said that the critical density is the minimum density required to stop the expansion of the universe [3].

3 Equation of motion

Trajectories of the kicked black holes were obtained by numerically solving the equation of motion on Equation 1.5, where the first term on the right side of the equation is acceleration due to gravity, the second accounts for the drag of dynamical friction, while the third one is the deaceleration due to mass accretion of the black hole [1, 4].

$$\ddot{\vec{x}} = -a_{\rm grav}(\vec{x})\hat{x} + \left(a_{\rm DF}(\vec{x}, \dot{\vec{x}}) - \dot{x}\frac{\dot{M}_\bullet(x, \dot{x})}{M_\bullet}\right)\hat{\dot{x}} \qquad \text{where } M_\bullet \text{ is the black hole mass} \tag{1.5}$$

3.1 Dynamical friction

As the black hole travels through the galaxy, dark matter, stars and gaseous materials from the medium interact with the black hole adding a drag force due to friction. Drag force is different in nature depending on its source, collisionless components, such as dark matter and stars, apply a drag force to the black hole that follows the standard Chandrasekhar formula [1, 3–5].

$$a_{\mathrm{DF}}^{\mathrm{cl}}(\vec{x},\dot{\vec{x}}) = -\frac{4\pi G^2}{\dot{x}^2} M_{\bullet}\rho(\vec{x}) \ln\Lambda \left(\mathrm{erf}\,(X) - \frac{2}{\sqrt{\pi}} X e^{-X^2} \right), \quad \rho(\vec{x}) = \rho_{\mathrm{DM}}(\vec{x}) + \rho_{\mathrm{stars}}(\vec{x}) \tag{1.6}$$

$$X \equiv \frac{|\dot{x}|}{\sqrt{2}\sigma_{\mathrm{DM}}} \qquad \text{with } \sigma_{\mathrm{DM}} = \sqrt{\frac{GM_{\mathrm{DM}}}{2R_{\mathrm{vir}}}} \tag{1.7}$$

σ_{DM} is called the local velocity dispersion of the dark matter halo, and since varies little over the entire host, can be taken as constant [1, 4]. The Coulomb logarithm ($\ln\Lambda$) is not known but authors take it in the range of 2 - 4 [1]. Gas on the other hand is collisional, special care must be taken since gas can cool behind a passing object, such as a black hole [1]. A hybrid model for the drag force was proposed by Tanaka and Haiman, in which both subsonic and supersonic velocities are possible. To do so, a mach number was defined as:

$$\mathcal{M}(\dot{x}) \equiv \frac{|\dot{x}|}{c_s} \tag{1.8}$$

where c_s is the local sound speed, which depends on local temperature. It was found that temperature inside the halo varies less than a factor of 3, thus on the simulation it is assumed that the entire halo is isothermal at the virial temperature (T_{vir}) [1]. The isothermal sound speed is [6]:

$$c_s = \sqrt{\frac{\gamma R}{\mathcal{M}_w} T_{\mathrm{vir}}} = \sqrt{\frac{\gamma R}{\mathcal{M}_w}\left(\frac{\mu m_p G M_h}{2k_B R_{\mathrm{vir}}}\right)} = \sqrt{\frac{\gamma R \mu m_p G}{2\mathcal{M}_w k_B}}\sqrt{\frac{M_h}{R_{\mathrm{vir}}}} \approx 0.614\sqrt{\frac{M_h}{R_{\mathrm{vir}}}}\ \mathrm{kpcGyr}^{-1} \tag{1.9}$$

where μ is the value of the mean molecular weight of the gas ($\mathcal{M}_w$), m_p is the proton mass and γ is the adiabatic index [6]. Approximating the gas to a monoatomic one $\gamma \approx 5/3$, yields the last expression on Equation 1.9. By knowing $\mathcal{M}$, the acceleration caused by gas can be written as [1, 4]:

$$a_{\mathrm{DF}}^{\mathrm{c}}(\vec{x},\dot{\vec{x}}) = -\frac{4\pi G^2}{\dot{x}^2} M_{\bullet}\rho_{\mathrm{gas}}(\vec{x}) f(\mathcal{M}) \tag{1.10}$$

with

$$f(\mathcal{M}) = \begin{cases} 0.5 \ln \Lambda \left[\operatorname{erf}\left(\frac{\mathcal{M}}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}} \mathcal{M} e^{-\mathcal{M}^2/2} \right] & \text{if } \mathcal{M} \leq 0.8 \\ 1.5 \ln \Lambda \left[\operatorname{erf}\left(\frac{\mathcal{M}}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}} \mathcal{M} e^{-\mathcal{M}^2/2} \right] & \text{if } 0.8 < \mathcal{M} \leq \mathcal{M}_{eq} \\ 0.5 \ln \left(1 - \mathcal{M}^{-2}\right) + \ln \Lambda & \text{if } \mathcal{M} > \mathcal{M}_{eq} \end{cases} \tag{1.11}$$

M_{eq} is the mach number that fulfills the following equation:

$$\ln \Lambda \left[1.5 \left(\operatorname{erf}\left(\frac{\mathcal{M}}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}} \mathcal{M} e^{-\mathcal{M}^2/2} \right) - 1 \right] - 0.5 \ln \left(1 - \mathcal{M}^{-2}\right) = 0 \tag{1.12}$$

Numerically solving Equation 1.12, yields $M_{eq} \approx 1.731$ for a value of the Coulomb logarithm $\ln \Lambda = 2.3$. The full acceleration due to dynamical friction is given by the sum of the noncollisional drag on Equation 1.6 and Equation 1.10:

$$a_{\mathrm{DF}}(\vec{x}, \dot{\vec{x}}) = a_{\mathrm{DF}}^{\mathrm{cl}}(\vec{x}, \dot{\vec{x}}) + a_{\mathrm{DF}}^{\mathrm{c}}(\vec{x}, \dot{\vec{x}}) \tag{1.13}$$

3.2 Accretion onto the black hole

As the black hole accretes matter from the surroundings, an acceleration appears, due to the second law of Newton:

$$\vec{F} = \frac{d\vec{P}}{dt} = \dot{\vec{x}} \dot{M}_\bullet + M_\bullet \ddot{\vec{x}} \tag{1.14}$$

By considering conservation of momentum:

$$\ddot{\vec{x}} = -\dot{\vec{x}} \frac{\dot{M}_\bullet}{M_\bullet} \tag{1.15}$$

Two schemes describe the speed at which the black hole gains mass, on the first one the black hole undergoes Bondi-Hoyle-Littleton accretion [1, 4]:

$$\dot{M}_\bullet^{\mathrm{BHL}}(\vec{x}, \dot{\vec{x}}) = \frac{4\pi G^2 \rho_B(\vec{x}) M_\bullet^2}{(c_s^2 + \dot{x}^2)^{3/2}} \qquad \text{with } \rho_B(\vec{x}) = \rho_{\mathrm{stars}}(\vec{x}) + \rho_{\mathrm{gas}}(\vec{x}) \tag{1.16}$$

There is a limit of accretion for the black hole given by the Eddington luminosity:

$$\dot{M}_\bullet^{\mathrm{Edd}} = \frac{(1-\epsilon) M_\bullet}{\epsilon t_{\mathrm{Edd}}} \qquad \epsilon = 0.1, \quad t_{\mathrm{Edd}} = 0.44\ \mathrm{Gyr} \tag{1.17}$$

Final accretion rate is given by:

$$\dot{M}_\bullet(\vec{x},\dot{\vec{x}}) = \begin{cases} \dot{M}_\bullet^{\rm BHL}(\vec{x},\dot{\vec{x}}) & \text{if } \dot{M}_\bullet^{\rm BHL} < \dot{M}_\bullet^{\rm Edd} \\ \dot{M}_\bullet^{\rm Edd} & \text{else} \end{cases} \tag{1.18}$$

3.3 Initial conditions and numerical integration

For all simulations the virial radius remains constant through the simulation. The virial radius is fixed at the start of every simulation depending on the redshift at which the kick occurs, the chosen densities profiles and the mass of the host galaxy. Sound speed also remains constant for a simulation, as it depends on $R_{\rm vir}$ and the mass of the host. Cosmological acceleration is ignored at all times as in Tanaka and Haiman, as it has been found that it only marginally affects black hole orbits [1]. The initial position of the black hole is always $\vec{x} = (0,0,0)$ kpc.

Numerical integration is carried out using a leapfrog scheme on REBOUND with the C programming language [7], with time steps of a thousand years, simulations are stopped when the system destabilizes and starts gaining energy, due to singularities at $x \to 0$ and $\dot{x} \to 0$, or if they simply last more than the age of the universe.

4 Definitions

4.1 Escape velocity

Minimum initial velocity required for the maximum distance of a single orbit of the black hole to stay outside $0.1R_{\rm vir}$ after $z = 0$, $z = 6$ or 10 % of the age of the universe at the moment of the kick [1, 4].

4.2 Time of return

Time required by the black hole to orbit with maximum distances of less than $0.1R_{\rm vir}$.

Chapter 2

Spherical study

1 Setup

The host galaxy has two mass distributions that are superimposed, one for dark matter and the other one for all the luminous or baryonic matter.

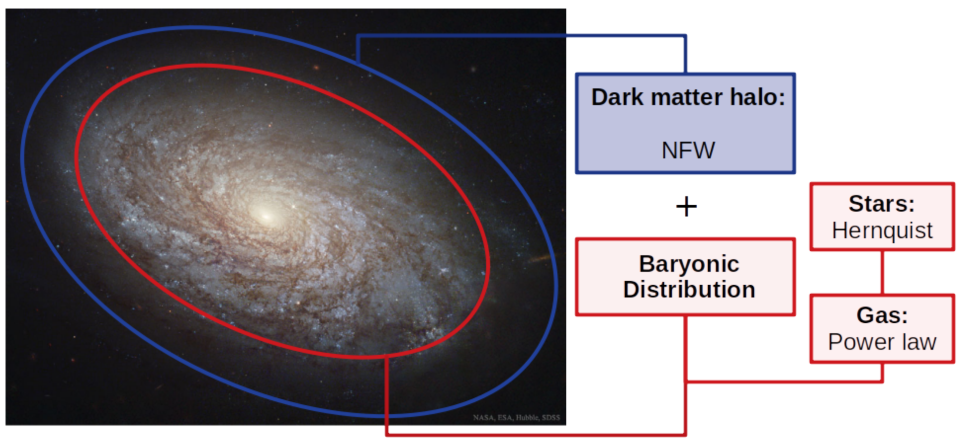


FIGURE 2.1: NGC4414 galaxy as seen by the Hubble telescope.

The dark matter halo used, follows a NFW (Navarro–Frenk–White) profile, baryonic matter is divided in stars and gas, for gas a power law profile with $r^{-2.2}$ is used, while for stars a Hernquist model is applied [1, 4]. The sum of all these components accounts for the total mass of the host (M_h), which remains constant through a simulation. The amount of baryonic matter is given by the baryonic fraction parameter (f_b), and the mass of stars by the stellar fraction parameter (f_s). Cumulative masses at the virial radius are defined as follows:

$$M_{\rm DM}(R_{\rm vir}) = (1 - f_b)M_h \tag{2.1}$$

$$M_{\rm stars}(R_{\rm vir}) = f_s f_b M_h \tag{2.2}$$

$$M_{\rm gas}(R_{\rm vir}) = (1 - f_s)f_b M_h \tag{2.3}$$

1.1 Virial radius

Since all of the density profiles are spherically symmetrical, it follows from Equation 1.4 that:

$$\frac{M_h}{4/3\pi R_{\text{vir}}^3} = 75\frac{H(t)^2}{\pi G} \tag{2.4}$$

$$R_{\text{vir}} = \left(\frac{M_h G}{100H(t)^2}\right)^{1/3} \tag{2.5}$$

1.2 Dark matter halo

For a dark matter halo following a NFW profile, the density at some distance r is given by the formula:

$$\rho_{\text{DM}}(r) = \frac{\rho_0^{\text{DM}}}{\frac{r}{R_s}\left(1+\frac{r}{R_s}\right)^2} \tag{2.6}$$

Where R_s and ρ_0^{DM} are constants for a given dark matter halo. Using the density, the cumulative mass $M_{\text{DM}}(r)$ within some radius r is given by the integral of the density over a volume, since Equation 2.6 is spherically symmetrical, the only dependance of the integral is with distance. On Equation 2.7 the r'^2 comes from the Jacobian of spherical coordinates, and the 4π from the solid angle.

$$M_{DM}(r) = \int_0^r 4\pi r'^2 \rho_{\text{DM}}(r')dr' = 4\pi\rho_0^{\text{DM}} R_s^3 \left[\ln\left(\frac{R_s+r}{R_s}\right) - \frac{r}{R_s+r}\right] \tag{2.7}$$

Considering a concentration parameter $c(M_h, z)$ of dark matter in the halo, the following relation holds for the viral radius R_{vir} and the scale radius R_s:

$$R_{\text{vir}} = c(M_h, z)R_s \tag{2.8}$$

Where the concentration parameter, dependence with the dark matter halo mass (M_h) and redshift is given by:

$$c(M_h, z) = c_0(z)\left(\frac{M_h}{10^{13}M_\theta}\right)^{\alpha(z)} \tag{2.9}$$

where $\alpha(z)$ and $c_0(z)$ were fitted using simulation data to the following functions [1]:

$$c_0(z) = \frac{4.58}{2}\left[\left(\frac{1+z}{2.24}\right)^{0.107} + \left(\frac{1+z}{2.24}\right)^{-1.29}\right] \tag{2.10}$$

$$\alpha(z) = -0.0965\exp\left(-\frac{z}{4.06}\right) \tag{2.11}$$

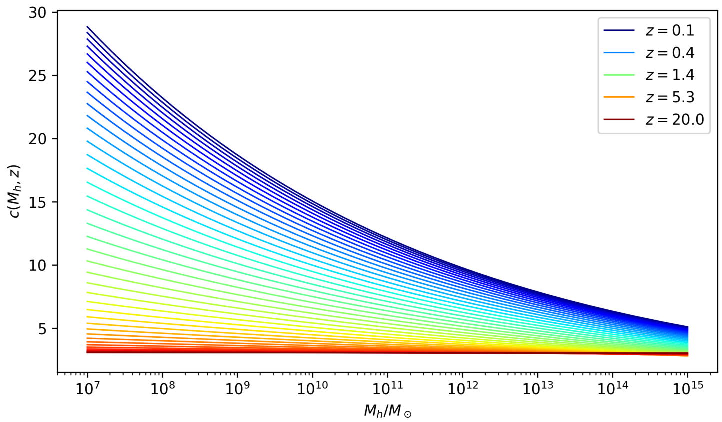


FIGURE 2.2: Dark matter concentration parameter as a function of the halo mass and the redshift.

For a fixed halo mass, as time passes (smaller redshift), concentration of dark matter will increase, as can be shown on Figure 2.2, nevertheless for high redshifts concentration is approximately constant. By using Equation 2.8 one can obtain the value of ρ_0^{DM} by evaluating Equation 2.7 at R_{vir}.

$$M_{\mathrm{DM}}(R_{\mathrm{vir}}) = 4\pi\rho_0^{\mathrm{DM}} R_s^3 \left[\ln\left(\frac{R_s + c(M_h,z)R_s}{R_s} \right) - \frac{c(M_h,z)R_s}{R_s + c(M_h,z)R_s} \right] = (1 - f_b)M_h \tag{2.12}$$

$$\rho_0^{\mathrm{DM}} = \frac{(1-f_b)M_h}{4\pi \left(\frac{R_{\mathrm{vir}}}{c(M_h,z)} \right)^3 \left[\ln\left(1 + c(M_h,z)\right) - \frac{c(M_h,z)}{1 + c(M_h,z)} \right]} \tag{2.13}$$

1.3 Stellar profile

Stellar density is modeled as a Hernquist profile with half-mass radius $R_{1/2} = 0.01R_{\mathrm{vir}}$, as in Choksi et al. Density for a Hernquist profile is given by [8]:

$$\rho_s(r) = \frac{f_s f_b M_h \mathcal{R}_s}{2\pi r(r + \mathcal{R}_s)^3} \qquad \mathcal{R}_s \text{ is known as scale length} \tag{2.14}$$

Integrating from 0 to r yields:

$$M_s(r) = \frac{f_s f_b M_h r^2}{(r + \mathcal{R}_s)^2} \tag{2.15}$$

The half-mass radius, as the name implies, is the distance at which the cumulative mass is half the total mass [8].

$$R_{1/2} = \left(1 + \sqrt{2}\right)\mathcal{R}_s = 0.01\left(\frac{M_h G}{100 H(t)^2} \right)^{1/3} \tag{2.16}$$

From which the scale length can be set as a function of the time when the kick occurs, and the mass of the host, as:

$$\mathcal{R}_s = \frac{0.01}{\left(1+\sqrt{2}\right)} \left(\frac{M_h G}{100 H(t)^2}\right)^{1/3} \approx 6.835 \times 10^{-4} \left(\frac{M_h}{H(t)^2}\right)^{1/3} \tag{2.17}$$

1.4 Gas profile

For high redshift the baryonic profile resembles that of a gaseous galaxy, Choksi et al. use a constant density gas core of $r_0 = 1$ pc, followed by a power law of $r^n = r^{-2.2}$. The complete density is described as follows:

$$\rho_{\text{gas}}(r) = \begin{cases} \rho_0^{\text{gas}} & \text{if } r < r_0 \\ \rho_0^{\text{gas}} \left(\frac{r_0}{r}\right)^{-n} & \text{if } r \geq r_0 \end{cases} \tag{2.18}$$

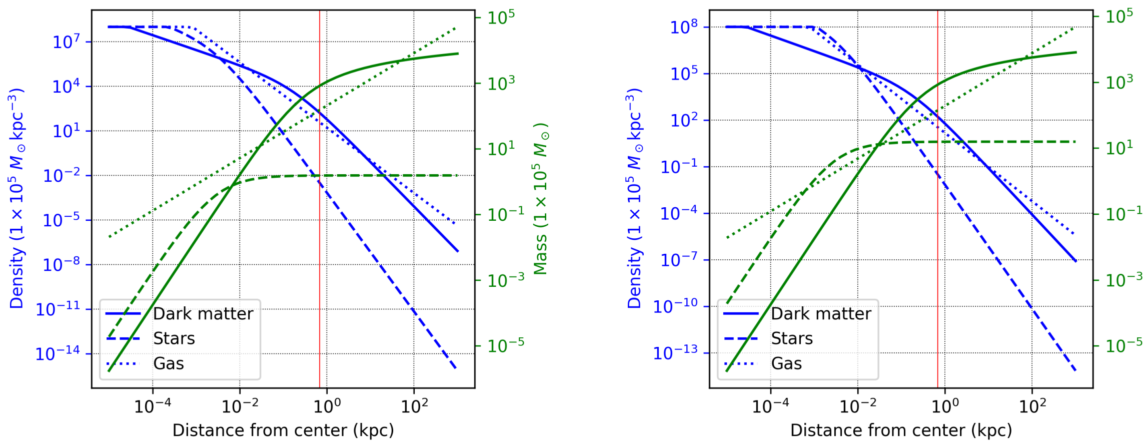
The cumulative mass is found by integrating the density in spherical coordinates, which for $n \neq -3$ is equal to:

$$M_{\text{gas}}(r) = \begin{cases} \frac{4}{3}\pi\rho_0^{\text{gas}} r^3 & \text{if } r < r_0 \\ 4\pi\rho_0^{\text{gas}} \left(\frac{\left(r^{3+n} - r_0^{3+n}\right)}{(3+n) r_0^n} + \frac{r_0^3}{3}\right) & \text{if } r \geq r_0 \end{cases} \tag{2.19}$$

The value of the constant ρ_0^{gas} is found using a similar process as in Equation 2.12 and 2.13.

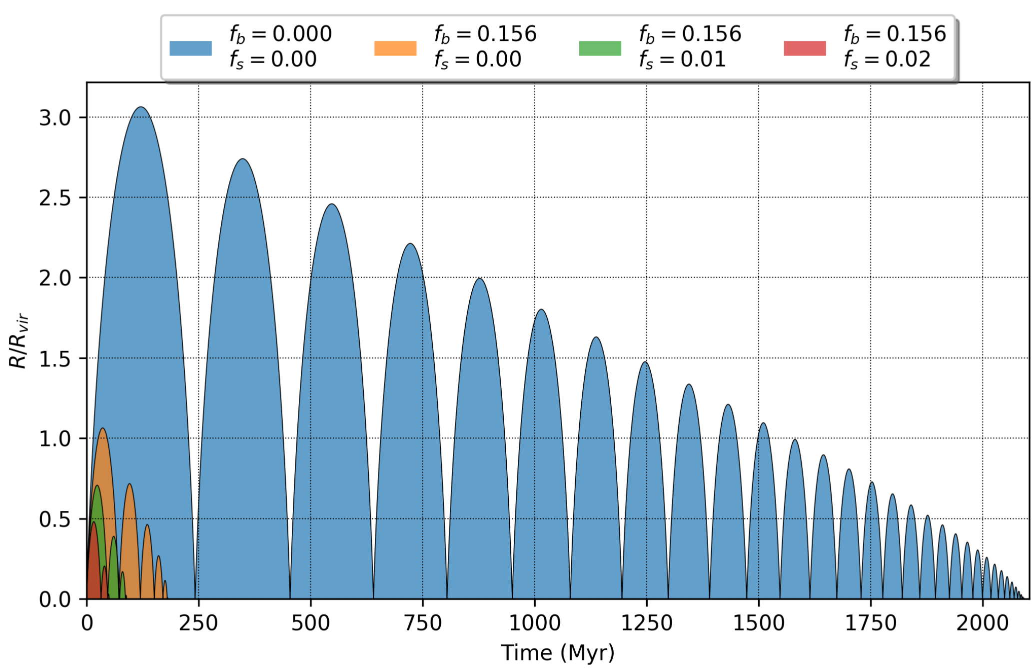
$$M_{\text{gas}}(R_{\text{vir}}) = 4\pi\rho_0^{\text{gas}} \left(\frac{\left(R_{\text{vir}}^{3+n} - r_0^{3+n}\right)}{(3+n) r_0^n} + \frac{r_0^3}{3}\right) = (1 - f_s) f_b M_h \tag{2.20}$$

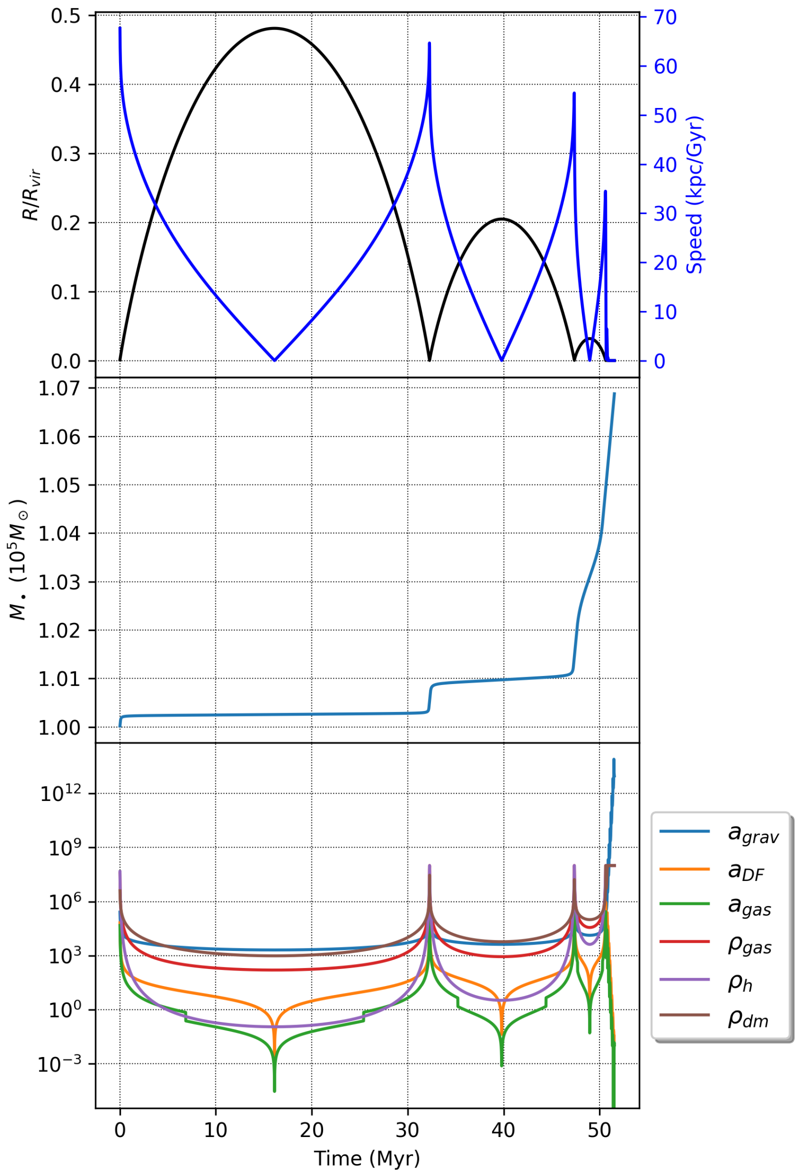
$$\rho_0^{\text{gas}} = \frac{(1 - f_s) f_b M_h}{4\pi \left(\frac{\left(R_{\text{vir}}^{3+n} - r_0^{3+n}\right)}{(3+n) r_0^n} + \frac{r_0^3}{3}\right)} \qquad \text{if } R_{\text{vir}} > r_0 \tag{2.21}$$



(A) Stellar fraction $f_s = 0.01$. (B) Stellar fraction $f_s = 0.1$.

FIGURE 2.3: Baryonic mass distributions for $z \gg 0$ and $z \approx 0$.





R/R_vir
Speed (kpc/Gyr)
M• (10^5 M⊙)
a_grav
a_DF
a_gas
ρ_gas
ρ_h
ρ_dm
Time (Myr)

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