Chapter 1

Methodology

Some of the simulation parameters are dependent of the cosmological model used, unless otherwise specified, all data is acquired using the Λ-CDM model with a matter density parameter $\Omega_M = 0.309$, $\Omega_\Lambda = 0.6911$, and a baryonic fraction $f_b = 0.156$ [1].

1 Units

Computer simulations are sensitive to rounding errors due to the lack of infinite precision when representing decimal numbers. Really small numbers as well as really big ones tend to have bigger errors than those close to the unity, as can be seen on Figure 1.1.

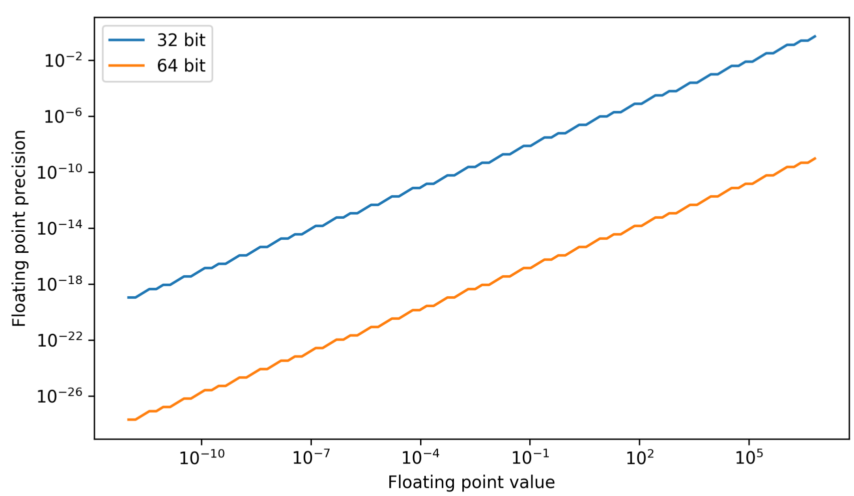


FIGURE 1.1: Floating point precision for different values, for a 32 bit and 64 bit holders.

Under the International System of Units, distances are measured on meters, times on seconds, and masses on kilograms, nevertheless black holes are too heavy to be measured on kilograms, galaxies sizes too big to be quantified on meters, and time scales too large for a second. Because of that, the following units will be used throughout this document:

TABLE 1.1: Units of measure used on the simulations.

Physical property	**unit**
Length	1 kilo-parsec (kpc)
Mass	10^5 solar masses ($10^5\ M_\odot$)
Time	1 giga-year (Gyr)

Along with the change of units, the universal gravitational constant and the Hubble parameter values are required to change.

1.1 Universal gravitational constant

First quantified by Henry Cavendish the gravitational constant has a value of $G_0 = 6.67408 \times 10^{-11}$ on SI units of $\mathrm{m^3 s^{-2} kg^{-1}}$. With the units of length, mass and time on Table 1.1, the constant of gravity used is given by:

$$\begin{aligned} G &= G_0 \left(\frac{1\ \mathrm{kpc}^3}{\left(3.0857 \times 10^{19}\right)^3\ \mathrm{m}^3} \right) \left(\frac{\left(3.154 \times 10^{16}\right)^2\ \mathrm{s}^2}{1\ \mathrm{Gyr}^2} \right) \left(\frac{1.98847 \times 10^{35}\ \mathrm{kg}}{10^5 M_\theta} \right) \\ &= 0.4493 \quad \frac{\mathrm{kpc}^3}{\mathrm{Gy}r^2 10^5 M_\odot} \end{aligned} \tag{1.1}$$

1.2 Hubble parameter

The Hubble constant is frequently used as $H_0 = 67.66 \pm 0.42\ \mathrm{kms^{-1}Mpc^{-1}}$ [2], stating the speed of an astronomical body on $\mathrm{kms^{-1}}$ at a distance of 1 Mpc. Nevertheless, the hubble constant has units of 1/time, thus, taking into account the units on Table 1.1 one gets:

$$\begin{aligned} H &= H_0 \left(\frac{1\ \mathrm{kpc}}{3.0857 \times 10^{16}\ \mathrm{km}} \right) \left(\frac{3.154 \times 10^{16}\ \mathrm{s}}{1\ \mathrm{Gyr}} \right) \left(\frac{1\ \mathrm{Mpc}}{1000\ \mathrm{kpc}} \right) \\ &\approx 1.023 H_0 \times 10^{-3}\ \mathrm{Gyr}^{-1} \\ &= 6.916 \times 10^{-2}\ \mathrm{Gyr}^{-1} \end{aligned} \tag{1.2}$$

Although the Hubble parameter is often called Hubble constant, its value changes with time as can be seen on Figure 1.2.

2 Critical density and Virial Radius

Mass distributions used for the simulation of the host galaxy, are divergent for distances up to infinity. Because of this, the cumulative mass of all bodies within a given distance is called the virial mass and its value is taken as the mass of the whole system. The distance taken to

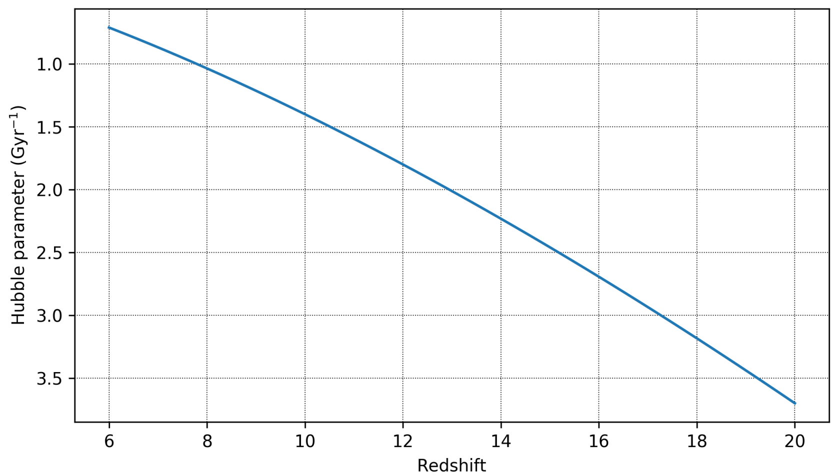


FIGURE 1.2: Dependency of the Hubble parameter with redshift.

calculate the virial mass is called virial radius (R_{vir}), and it is defined as the distance at which the average density of the galaxy is 200 times the critical density of the universe (ρ_{crit}).

$$\rho_{\text{crit}} = \frac{3H(t)^2}{8\pi G} \tag{1.3}$$

$$\frac{M(R_{\text{vir}})}{V(R_{\text{vir}})} = \bar{\rho}(R_{\text{vir}}) = 200\rho_{\text{crit}} = 75\frac{H(t)^2}{\pi G} \tag{1.4}$$

where $M(R_{\text{vir}})$ is the cumulative mass, and $V(R_{\text{vir}})$: the volume

The relation on Equation 1.3 is found by considering the case where the geometry of the universe is flat, as a consequence it is said that the critical density is the minimum density required to stop the expansion of the universe [3].

3 Equation of motion

Trajectories of the kicked black holes were obtained by numerically solving the equation of motion on Equation 1.5, where the first term on the right side of the equation is acceleration due to gravity, the second accounts for the drag of dynamical friction, while the third one is the deaceleration due to mass accretion of the black hole [1, 4].

$$\ddot{\vec{x}} = -a_{\text{grav}}(\vec{x})\hat{x} + \left(a_{\text{DF}}(\vec{x}, \dot{\vec{x}}) - \dot{x}\frac{\dot{M}_\bullet(x, \dot{x})}{M_\bullet}\right)\hat{\dot{x}} \qquad \text{where } M_\bullet \text{ is the black hole mass} \tag{1.5}$$

3.1 Dynamical friction

As the black hole travels through the galaxy, dark matter, stars and gaseous materials from the medium interact with the black hole adding a drag force due to friction. Drag force is different in nature depending on its source, collisionless components, such as dark matter and stars, apply a drag force to the black hole that follows the standard Chandrasekhar formula [1, 3–5].

$$a^{\mathrm{cl}}_{\mathrm{DF}}(\vec{x},\dot{\vec{x}}) = -\frac{4\pi G^2}{\dot{x}^2} M_\bullet \rho(\vec{x}) \ln\Lambda \left(\mathrm{erf}\,(X) - \frac{2}{\sqrt{\pi}} X e^{-X^2}\right), \quad \rho(\vec{x}) = \rho_{\mathrm{DM}}(\vec{x}) + \rho_{\mathrm{stars}}(\vec{x}) \tag{1.6}$$

$$X \equiv \frac{|\dot{x}|}{\sqrt{2}\sigma_{\mathrm{DM}}} \qquad \text{with } \sigma_{\mathrm{DM}} = \sqrt{\frac{GM_{\mathrm{DM}}}{2R_{\mathrm{vir}}}} \tag{1.7}$$

σ_{DM} is called the local velocity dispersion of the dark matter halo, and since varies little over the entire host, can be taken as constant [1, 4]. The Coulomb logarithm ($\ln\Lambda$) is not known but authors take it in the range of 2 - 4 [1]. Gas on the other hand is collisional, special care must be taken since gas can cool behind a passing object, such as a black hole [1]. A hybrid model for the drag force was proposed by Tanaka, in which both subsonic and supersonic velocities are possible. To do so, a mach number was defined as:

$$\mathcal{M}(\dot{x}) \equiv \frac{|\dot{x}|}{c_s} \tag{1.8}$$

where c_s is the local sound speed, which depends on local temperature. It was found that temperature inside the halo varies less than a factor of 3, thus on the simulation it is assumed that the entire halo is isothermal at the virial temperature (T_{vir}) [1]. The isothermal sound speed is [6]:

$$c_s = \sqrt{\frac{\gamma R}{\mathcal{M}_w} T_{\mathrm{vir}}} = \sqrt{\frac{\gamma R}{\mathcal{M}_w}\left(\frac{\mu m_p G M_h}{2k_B R_{\mathrm{vir}}}\right)} = \sqrt{\frac{\gamma R \mu m_p G}{2\mathcal{M}_w k_B}}\sqrt{\frac{M_h}{R_{\mathrm{vir}}}} \approx 0.614\sqrt{\frac{M_h}{R_{\mathrm{vir}}}}\ \mathrm{kpcGyr}^{-1} \tag{1.9}$$

where μ is the value of the mean molecular weight of the gas ($\mathcal{M}_w$), m_p is the proton mass and γ is the adiabatic index [6]. Approximating the gas to a monoatomic one $\gamma \approx 5/3$, yields the last expression on Equation 1.9. By knowing $\mathcal{M}$, the acceleration caused by gas can be written as [1, 4]:

$$a^{\mathrm{c}}_{\mathrm{DF}}(\vec{x},\dot{\vec{x}}) = -\frac{4\pi G^2}{\dot{x}^2} M_\bullet \rho_{\mathrm{gas}}(\vec{x}) f(\mathcal{M}) \tag{1.10}$$

with

$$f(\mathcal{M}) = \begin{cases} 0.5 \ln \Lambda \left[\operatorname{erf}\left(\frac{\mathcal{M}}{\sqrt{2}} \right) - \sqrt{\frac{2}{\pi}} \mathcal{M} e^{-\mathcal{M}^2/2} \right] & \text{if } \mathcal{M} \leq 0.8 \\ 1.5 \ln \Lambda \left[\operatorname{erf}\left(\frac{\mathcal{M}}{\sqrt{2}} \right) - \sqrt{\frac{2}{\pi}} \mathcal{M} e^{-\mathcal{M}^2/2} \right] & \text{if } 0.8 < \mathcal{M} \leq \mathcal{M}_{eq} \\ 0.5 \ln \left(1 - \mathcal{M}^{-2} \right) + \ln \Lambda & \text{if } \mathcal{M} > \mathcal{M}_{eq} \end{cases} \tag{1.11}$$

M_{eq} is the mach number that fulfills the following equation:

$$\ln \Lambda \left[1.5 \left(\operatorname{erf}\left(\frac{\mathcal{M}}{\sqrt{2}} \right) - \sqrt{\frac{2}{\pi}} \mathcal{M} e^{-\mathcal{M}^2/2} \right) - 1 \right] - 0.5 \ln \left(1 - \mathcal{M}^{-2} \right) = 0 \tag{1.12}$$

Numerically solving Equation 1.12, yields $M_{eq} \approx 1.731$ for a value of the Coulomb logarithm $\ln \Lambda = 2.3$.

3.2 Accretion onto the black hole

Chapter 2

Base case

1 Mass distributions

The host galaxy has two mass distributions that are superimposed, one for dark matter and the other one for all the luminous or baryonic matter.

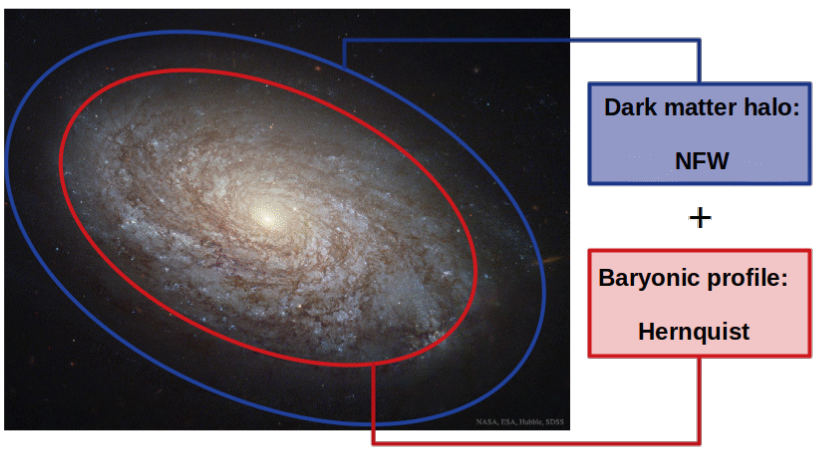


FIGURE 2.1: NGC4414 galaxy #TODO: cite

The dark matter halo used on the host, follows a NFW (Navarro–Frenk–White) profile, while the baryonic mass distribution of the galaxy depends on the redshift z. For high redshifts a gaseous disk with constant density followed by a density dependance with $r^{-2.2}$ is used, while for $z \approx 0$ a Hernquist model is applied [1].

2 Dark matter halo

For a dark matter halo following a NFW profile, the density at some distance r is given by the formula:

$$\rho_{\mathrm{DM}}(r) = \frac{\rho_0^{\mathrm{DM}}}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2} \tag{2.1}$$

Where R_s and ρ_0^{DM} are constants for a given dark matter halo. Using the density, the cumulative mass $M_{\mathrm{DM}}(r)$ within some radius r is given by the integral of the density over a volume, since Equation 2.1 is spherically symmetrical, the only dependance of the integral is with distance. On Equation 2.2 the r'^2 comes from the Jacobian of spherical coordinates, and the 4π from the solid angle.

$$M_{DM}(r) = \int_0^r 4\pi r'^2 \rho_{\mathrm{DM}}(r')dr' = 4\pi\rho_0^{\mathrm{DM}} R_s^3 \left[\ln\left(\frac{R_s + r}{R_s}\right) - \frac{r}{R_s + r}\right] \tag{2.2}$$

Since the mass of dark matter of a single galaxy diverges for $r \to \infty$ there is a radius called R_{vir}, at which the density of the NFW profile is 200 times the critical density ρ_{crit}, the minimum density for an expanding universe `#TODO: cite`.

$$R_{\mathrm{vir}} = 200\rho_{\mathrm{crit}} = 200\left(\frac{3H^2}{8\pi G}\right) = 0.2541 \text{ kpc} \tag{2.3}$$

Considering a concentration parameter $c(M_h, z)$ of dark matter in the halo, the following relation holds for the viral radius R_{vir} and the scale radius R_s:

$$R_{\mathrm{vir}} = c(M_h, z)R_s \tag{2.4}$$

Where the concentration parameter, dependence with the dark matter halo mass (M_h) and redshift is given by:

$$c(M_h, z) = c_0(z)\left(\frac{M_h}{10^{13}M_\theta}\right)^{\alpha(z)} \tag{2.5}$$

where $\alpha(z)$ and $c_0(z)$ were fitted using simulation data to the following functions [1]:

$$c_0(z) = \frac{4.58}{2}\left[\left(\frac{1+z}{2.24}\right)^{0.107} + \left(\frac{1+z}{2.24}\right)^{-1.29}\right] \tag{2.6}$$

$$\alpha(z) = -0.0965 \exp\left(-\frac{z}{4.06}\right) \tag{2.7}$$

For a fixed halo mass, as time passes (smaller redshift), concentration of dark matter will increase, as can be shown on Figure 2.2. Because of Equation 2.4 the value of R_{vir} will increase as well, this equation can be used to obtain the value of ρ_0^{DM} by evaluating Equation 2.2 at R_{vir}.

$$M_h \equiv M_{\mathrm{DM}}(R_{\mathrm{vir}}) = 4\pi\rho_0^{\mathrm{DM}} R_s^3 \left[\ln\left(\frac{R_s + c(M_h, z)R_s}{R_s}\right) - \frac{c(M_h, z)R_s}{R_s + c(M_h, z)R_s}\right] \tag{2.8}$$

$$\rho_0^{\mathrm{DM}} = \frac{M_h}{4\pi R_s^3\left[\ln\left(1 + c(M_h, z)\right) - \dfrac{c(M_h, z)}{1 + c(M_h, z)}\right]} \tag{2.9}$$

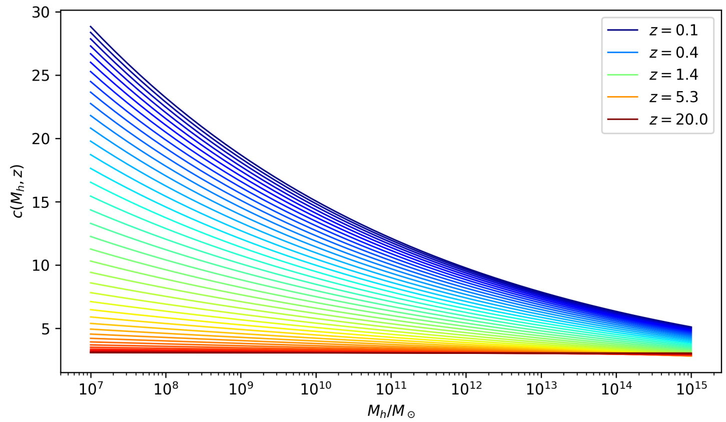


FIGURE 2.2: Dark matter concentration parameter as a function of the halo mass and the redshift.

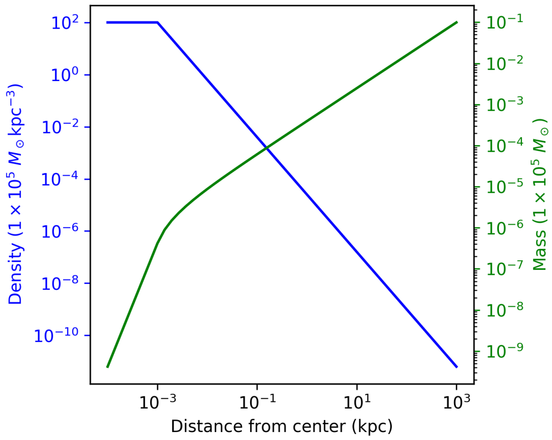
By considering a fixed concentration parameter $c(M_h, z) \equiv 4$, one obtains $R_s = 0.0635$ kpc, and $\rho_0^{\rm DM} = 383457\ 10^5 M_\odot {\rm kpc}^{-3}$.

2.1 Baryonic profile

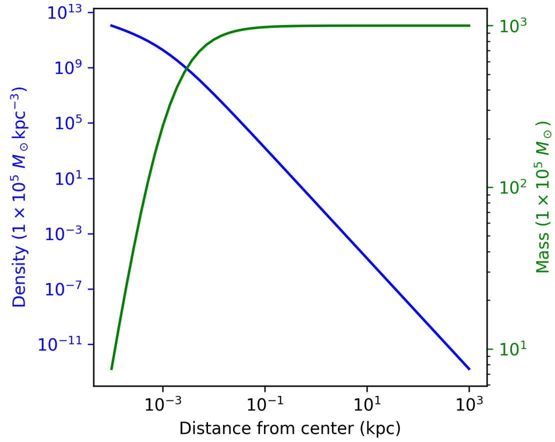
For high redshift the baryonic profile resembles that of a gaseous galaxy with a constant density for distances lower than $r_0 = 1 \times 10^{-3}$ kpc, followed by an $r^{-2.2}$ profile for greater distances as in Figure 2.3a.

$$\rho_B(r) = \begin{cases} \rho_0^B & \text{if } r < r_0 \\ \rho_0^B \left(\frac{r_0}{r}\right)^{2.2} & \text{if } r \geq r_0 \end{cases} \tag{2.10}$$

$$M_B(r) = \begin{cases} \frac{4}{3}\pi\rho_0^B r^3 & \text{if } r < r_0 \\ 4\pi\rho_0^B r_0^{2.2}\left(r^{0.8} - r_0^{0.8}\right) + \frac{4}{3}\pi\rho_0^B r_0^3 & \text{if } r \geq r_0 \end{cases} \tag{2.11}$$



(A) Constant density profile for $r < 1$ pc, followed by a fall as $r^{-2.2}$ for $z \gg 0$.



(B) Hernquist profile for mass distribution at $z \approx 0$.

FIGURE 2.3: Baryonic mass distributions for $z \gg 0$ and $z \approx 0$.

At low redshifts, stars have already been formed, and the baryonic profile is modeled as a Hernquist profile with half-mass radius $R_{1/2} = 0.01 R_{\text{vir}}$, as in Figure 2.3b. The half-mass radius, as the name implies, is the distance at which the cumulative mass is half the total mass [7].

$$\rho_B(r) = \frac{M_T^B \mathcal{R}_s}{2\pi r (r + \mathcal{R}_s)^3} \tag{2.12}$$

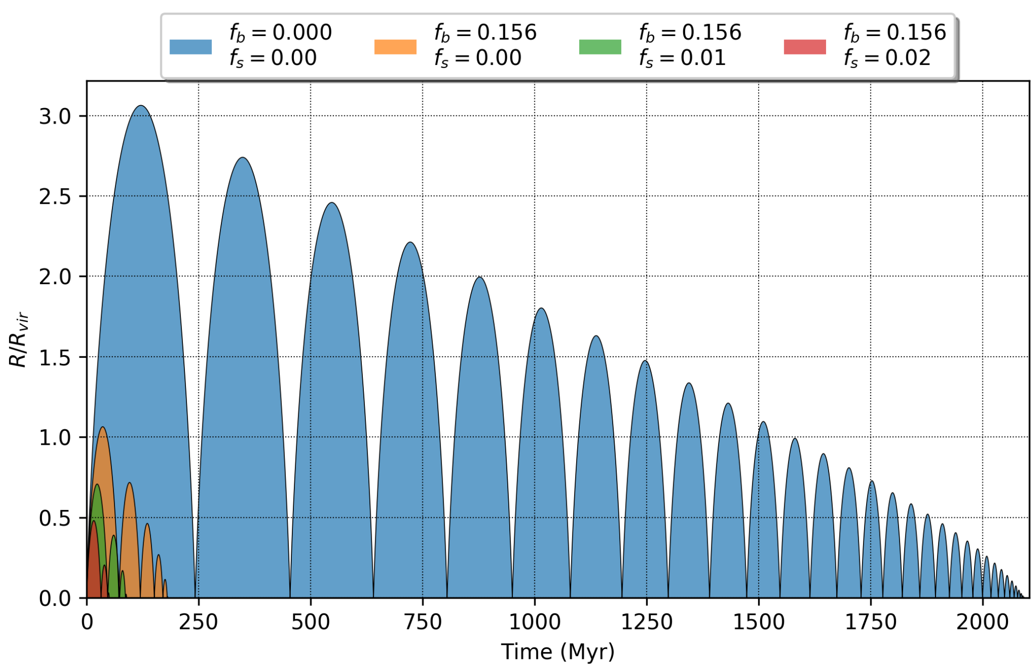
$$M_B(r) = \frac{M_T^B r^2}{(r + \mathcal{R}_s)^2} \tag{2.13}$$

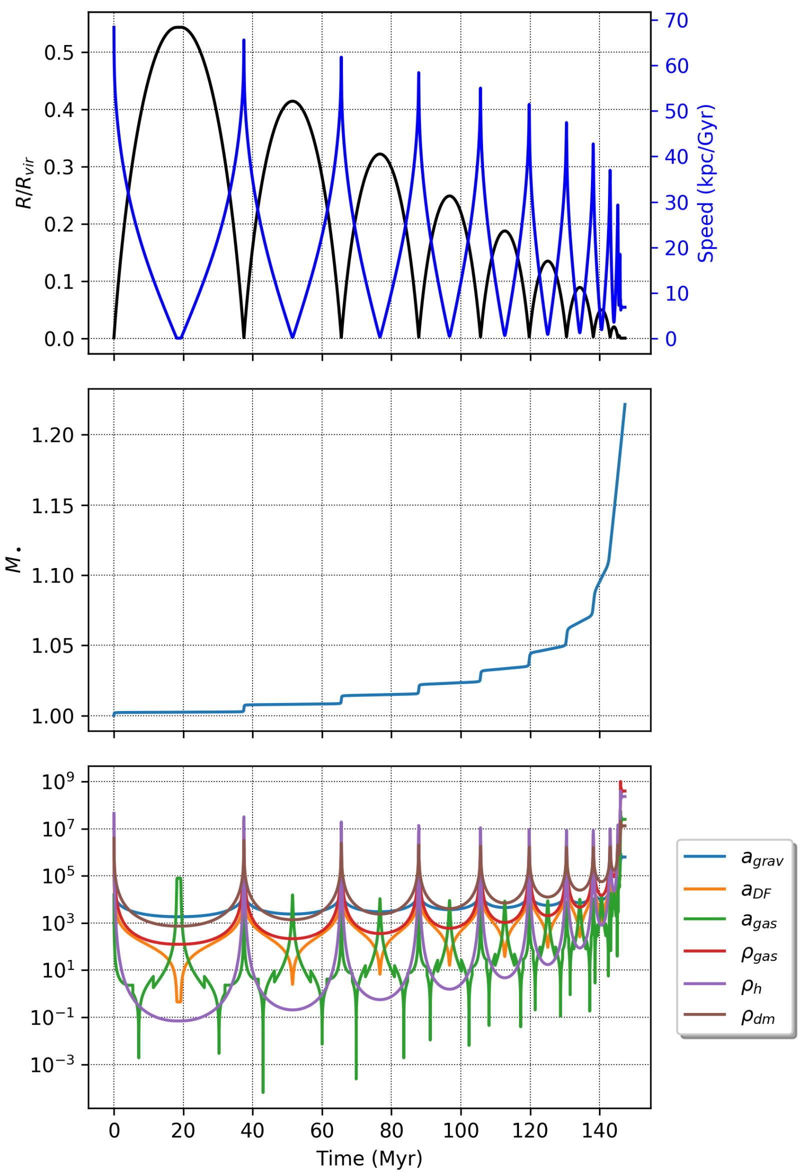
$$R_{1/2} = \left(1 + \sqrt{2}\right) \mathcal{R}_s \tag{2.14}$$

finally, the gas density is taken as a power-law profile, $\rho_{\text{gas}} = \rho_0^{\text{gas}} r^{-n}$, were $1 \leq n \leq 3$ [1].

TABLE 2.1: Constants used in the simulation

Constant	**Value (unit)**
n	2
h	0.678
Ω_M	0.309
$\ln \Lambda$	2.3
M_h	1×10^3 ($10^5 M_\odot$)
$M_\bullet$	1 ($10^5 M_\odot$)
M_T^B	158 ($10^5 M_\odot$)
R_s	1.3 (kpc)
$\mathcal{R}_s$	0.14 (kpc)
ρ_0^{DM}	1700 ($10^5 M_\odot \mathrm{kpc}^{-3}$)
ρ_0^{gas}	0.01 ($10^5 M_\odot \mathrm{kpc}^{-3}$)
f_b	baryon fraction $\Omega_b/\Omega_M = f_{\mathrm{gas}} + f_{\mathrm{gal}}$





R/R_{vir}
Speed (kpc/Gyr)
$M_\bullet$
Time (Myr)
a_{grav}
a_{DF}
a_{gas}
ρ_{gas}
ρ_h
ρ_{dm}