1 Galatic setup

1.1 Units

Computer simulations are sensitive to rounding errors due to the lack of infinite precision when representing decimal numbers. Really small numbers as well as really big ones tend to have bigger errors than those close to the unity. `#TODO: include IEEE 754`

Under the International System of Units, distances are measured on meters, times on seconds, and mass on kilograms, nevertheless black holes are too heavy to be measured on kilograms, galaxies sizes too big to be quantified on meters, and time scales too large for a second. Because of that, the following units will be used throughout this document:

TABLE 1: Natural units

Physical property	**unit**
Length	1 kilo-parsec (kpc)
Mass	10^5 solar masses (10^5 $M_\odot$)
Time	1 giga-year (Gyr)

Universal gravitational constant

First quantified by Henry Cavendish the gravitational constant has a value of $G_0 = 6.67408 \times 10^{-11}$ on SI units of $\mathrm{m^3 s^{-2} kg^{-1}}$. With the units of length, mass and time on Table 1 the constant of gravity to be used is given by:

$$G = G_0 \left(\frac{1\ \mathrm{kpc}^3}{\left(3.0857 \times 10^{19}\right)^3\ \mathrm{m}^3} \right) \left(\frac{\left(3.154 \times 10^{16}\right)^2\ \mathrm{s}^2}{1\ \mathrm{Gyr}^2} \right) \left(\frac{1.98847 \times 10^{35}\ \mathrm{kg}}{10^5 M_\theta} \right) = 0.4493 \frac{\mathrm{kpc}^3}{\mathrm{Gyr} r^2 10^5 M_\odot} \tag{1}$$

Hubble constant

1.2 Mass distributions

1.3 Dark matter halo

For a dark matter halo following a NFW profile, the density at some distance r is given by the formula:

$$\rho_{\mathrm{DM}}(r) = \frac{\rho_0^{\mathrm{DM}}}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2} \tag{2}$$

Where R_s and ρ_0^{DM} are constants for a given dark matter halo.

The cumulative mass within some radius r is:

$$M_{DM} = \int_0^r 4\pi r'^2 \rho_{\mathrm{DM}}(r') dr' = 4\pi\rho_0 R_s^3 \left[\ln\left(\frac{R_s + r}{R_s}\right) - \frac{r}{R_s + r} \right] \tag{3}$$

Since the mass of dark matter of a single galaxy diverges for $r \to \infty$ there is a radius called R_{vir}, at which the density of the NFW profile is 200 times the critical density ρ_{crit} the minimum density for an expanding universe.

$$R_{\text{vir}} = 200\rho_{\text{crit}} = 200\left(\frac{3H^2}{8\pi G}\right) \tag{4}$$

Considering a concentration parameter $c(M_h, z)$ of dark matter in the halo, given by:

$$c(M_h, z) = c_0(z)\left(\frac{M_h}{10^{13}M_\theta}\right)^{\alpha(z)} \qquad \text{where } z \text{ is the redshift} \tag{5}$$

where $\alpha(z)$ and $c_0(z)$ were fitted using simulation data to the following functions:

$$c_0(z) = \frac{4.58}{2}\left[\left(\frac{1+z}{2.24}\right)^{0.107} + \left(\frac{1+z}{2.24}\right)^{-1.29}\right] \tag{6}$$

$$\alpha(z) = -0.0965\exp\left(-\frac{z}{4.06}\right) \tag{7}$$

The concentration parameter of dark matter relates the viral radius R_{vir} and the scale radius R_s as:

$$R_{\text{vir}} = c(M_h, z)R_s \tag{8}$$

2 Metodología

$$\ddot{\vec{x}} = \left(-\frac{GM_h(x)}{x^2} + a_{DF} - \ddot{x}\frac{\dot{M}_\bullet}{M_\bullet} - qH^2x \right)\hat{x} \tag{9}$$