Orbits of black holes in triaxial potentials

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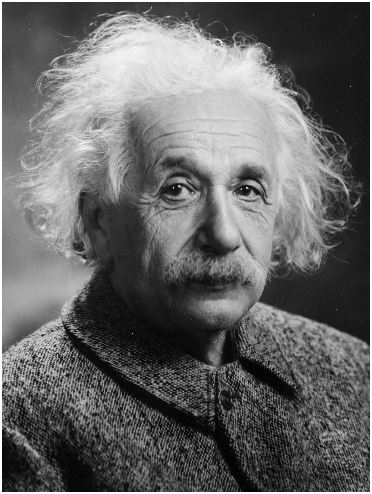
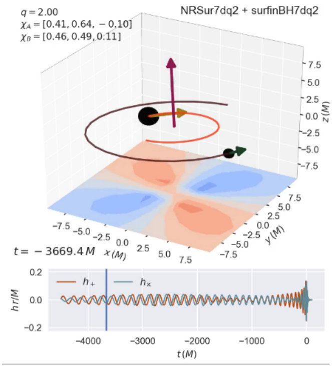
Introduction



- Theory of General Relativity, 1906
- Although more than 100 years have passed since the publication of the theory, even today there are gaps in the understanding and implications of Einstein's equations

Introduction



Binary black hole explorer

Objectives

Study the effect of different triaxial potentials, initial speeds and numerical integrators on the times required by a supermassive black hole to return to its initial position, after experiencing a recoil, as well as to quantify how chaotic its trajectory is.

- Obtain probability distributions for the return times based on each of the free parameters of the triaxial potential, the magnitude and direction of the initial velocity
- Quantify how chaotic is the trajectory of the black hole in each simulation, using exponents of Lyapunov
- Evaluate the performance of the numerical integrators using the information of the simulations

Methodology

Computer simulations are sensitive to rounding errors due to the lack of infinite precision when representing decimal numbers.

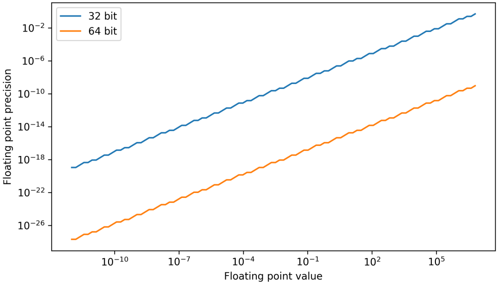


Figure: Floating point precision for different values, for a 32 bit and 64 bit holders.

Table: Units of measure used on the simulations.

Physical property	**unit**
Length	1 kilo-parsec (kpc)
Mass	10^5 solar masses ($10^5\ M_\odot$)
Time	1 giga-year (Gyr)

1. Universal gravitational constant:

$$G = 0.4493 \quad \frac{\mathrm{kpc}^3}{\mathrm{Gy}r^2 10^5 M_\odot} \tag{1}$$

2. Hubble parameter:

$$H \approx 1.023 H_0 \times 10^{-3}\ \mathrm{Gyr}^{-1} = 6.916 \times 10^{-2}\ \mathrm{Gyr}^{-1} \tag{2}$$

Mass distributions of the host galaxy, are divergent. The end of the host is taken at the virial radius.

$$\rho_{\text{crit}} = \frac{3H(t)^2}{8\pi G} \tag{3}$$

$$\frac{M(R_{\text{vir}})}{V(R_{\text{vir}})} = \bar{\rho}(R_{\text{vir}}) = 200\rho_{\text{crit}} = 75\frac{H(t)^2}{\pi G} \tag{4}$$

where $M(R_{\text{vir}})$ is the cumulative mass, and $V(R_{\text{vir}})$: the volume

Trajectories of the kicked black holes are obtained by numerically solving the equation of motion.

$$\ddot{\vec{x}} = -a_{\text{grav}}(\vec{x})\hat{x} + \left(a_{\text{DF}}(\vec{x}, \dot{\vec{x}}) - \dot{x}\frac{\dot{M}_\bullet(x, \dot{x})}{M_\bullet} \right) \dot{\hat{x}} \tag{5}$$

where $M_\bullet$ is the black hole mass

Dynamical friction

As the black hole travels through the galaxy, dark matter, stars and gaseous materials from the medium interact with the black hole adding a drag force due to friction.

$$a_{\mathrm{DF}}^{\mathrm{cl}}(\vec{x},\dot{\vec{x}}) = -\frac{4\pi G^2}{\dot{x}^2} M_{\bullet}\rho(\vec{x}) \ln\Lambda \left(\mathrm{erf}\,(X) - \frac{2}{\sqrt{\pi}} X e^{-X^2} \right) \quad (6)$$

$$X \equiv \frac{|\dot{x}|}{\sqrt{2}\sigma_{\mathrm{DM}}} \qquad \text{with } \sigma_{\mathrm{DM}} = \sqrt{\frac{GM_{\mathrm{DM}}}{2R_{\mathrm{vir}}}} \quad (7)$$

Drag generated by gas depends on local sound speed

$$\mathcal{M}(\dot{x}) \equiv \frac{|\dot{x}|}{c_s} = |\dot{x}|\sqrt{\frac{\mathcal{M}_w}{\gamma R T_{\text{vir}}}} = |\dot{x}|\sqrt{\frac{\mathcal{M}_w}{\gamma R}\left(\frac{2k_B R_{\text{vir}}}{\mu m_p G M_h}\right)} \quad (8)$$

$$\mathcal{M}(\dot{x}) = 1.63|\dot{x}|\sqrt{\frac{R_{\text{vir}}}{M_h}}$$

$$a^{\text{c}}_{\text{DF}}(\vec{x}, \dot{\vec{x}}) = -\frac{4\pi G^2}{\dot{x}^2} M_\bullet \rho_{\text{gas}}(\vec{x}) f(\mathcal{M}) \quad (9)$$

with

$$f(\mathcal{M}) = \begin{cases} 0.5 \ln\Lambda \left[\text{erf}\left(\frac{\mathcal{M}}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}}\mathcal{M}e^{-\mathcal{M}^2/2}\right] & \text{if } \mathcal{M} \leq 0.8 \\ 1.5 \ln\Lambda \left[\text{erf}\left(\frac{\mathcal{M}}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}}\mathcal{M}e^{-\mathcal{M}^2/2}\right] & \text{if } 0.8 < \mathcal{M} \leq \mathcal{M}_{eq} \\ 0.5 \ln\left(1 - \mathcal{M}^{-2}\right) + \ln\Lambda & \text{if } \mathcal{M} > \mathcal{M}_{eq} \end{cases} \quad (10)$$

Accretion into the black hole

As the black hole accretes matter from the surroundings, an acceleration appears, due to the second law of Newton.

$$\dot{M}_\bullet(\vec{x},\dot{\vec{x}}) = \begin{cases} \dot{M}_\bullet^{\mathrm{BHL}}(\vec{x},\dot{\vec{x}}) & \text{if } \dot{M}_\bullet^{\mathrm{BHL}} < \dot{M}_\bullet^{\mathrm{Edd}} \\ \dot{M}_\bullet^{\mathrm{Edd}} & \text{else} \end{cases} \tag{11}$$

$$\dot{M}_\bullet^{\mathrm{BHL}}(\vec{x},\dot{\vec{x}}) = \frac{4\pi G^2 \rho_G(\vec{x}) M_\bullet^2}{\left(c_s^2 + \dot{x}^2\right)^{3/2}} \tag{12}$$

$$\dot{M}_\bullet^{\mathrm{Edd}} = \frac{(1-\epsilon)M_\bullet}{\epsilon t_{\mathrm{Edd}}} \qquad \epsilon = 0.1, \quad t_{\mathrm{Edd}} = 0.44\ \mathrm{Gyr} \tag{13}$$

Spherical case

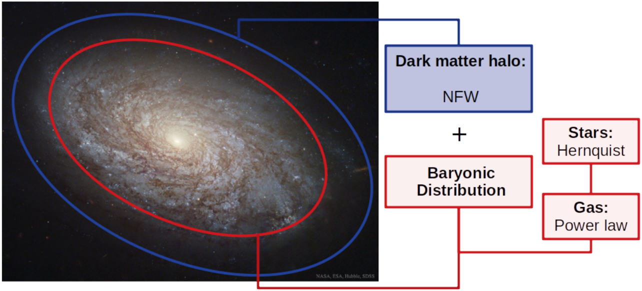


Figure: NGC4414 galaxy as seen by the Hubble telescope.

Mass distribution between components is given by:

$$M_{\mathrm{DM}}(R_{\mathrm{vir}}) = (1 - f_b) M_h \tag{14}$$

$$M_{\mathrm{stars}}(R_{\mathrm{vir}}) = f_s f_b M_h \tag{15}$$

$$M_{\mathrm{gas}}(R_{\mathrm{vir}}) = (1 - f_s) f_b M_h \tag{16}$$

with $f_b = 0.156$ and $M_h = 10^8\ M_\odot$

$$R_{\mathrm{vir}} = \left(\frac{M_h G}{100 H(t)^2} \right)^{1/3} \tag{17}$$

1. Dark matter (NWF):

$$\rho_{\mathrm{DM}}(r) = \frac{\rho_0^{\mathrm{DM}}}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2} \tag{18}$$

2. Stellar density (Hernquist):

$$\rho_s(r) = \frac{f_s f_b M_h \mathcal{R}_s}{2\pi r (r + \mathcal{R}_s)^3} \tag{19}$$

3. Gas density (Power law):

$$\rho_{\mathrm{gas}}(r) = \begin{cases} \rho_0^{\mathrm{gas}} & \text{if } r < r_0 \\ \rho_0^{\mathrm{gas}} \left(\frac{r_0}{r}\right)^{-n} & \text{if } r \geq r_0 \end{cases} \tag{20}$$

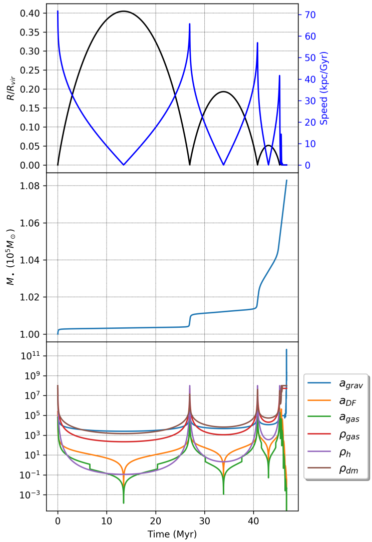


Figure: Properties of a single simulation.

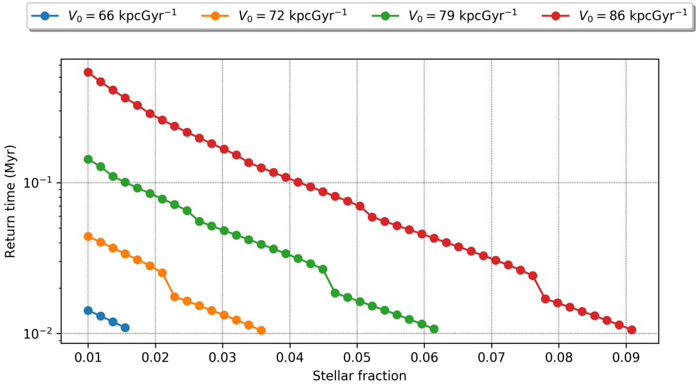


Figure: Return time for different stellar densities.

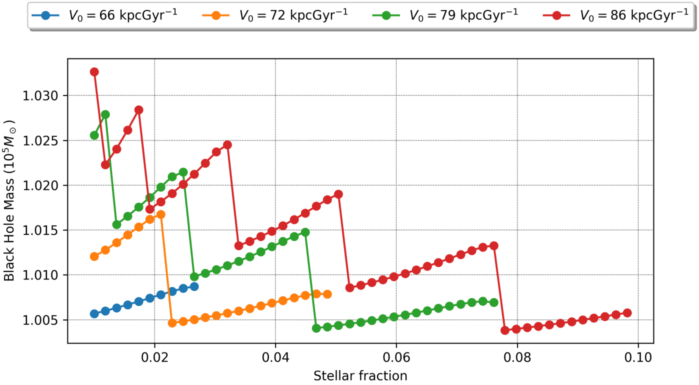
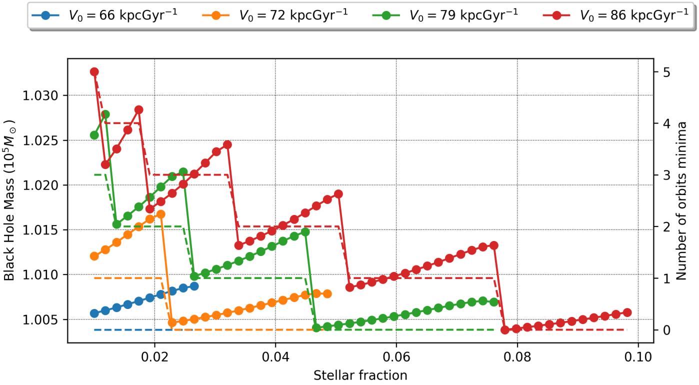
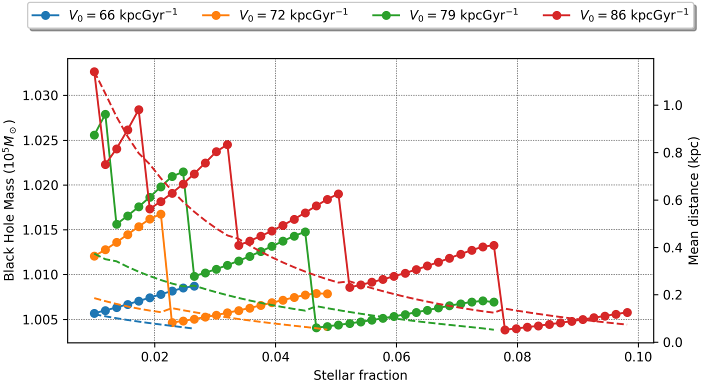


Figure: Black holes mass as function of initial speed and stellar fraction.


$V_0 = 66$ kpcGyr^{-1}
$V_0 = 72$ kpcGyr^{-1}
$V_0 = 79$ kpcGyr^{-1}
$V_0 = 86$ kpcGyr^{-1}
Black Hole Mass ($10^5 M_\odot$)
1.005
1.010
1.015
1.020
1.025
1.030
0.02
0.04
0.06
0.08
0.10
Stellar fraction
Number of orbits minima
0
1
2
3
4
5


$V_0 = 66$ kpcGyr^{-1}
$V_0 = 72$ kpcGyr^{-1}
$V_0 = 79$ kpcGyr^{-1}
$V_0 = 86$ kpcGyr^{-1}
Black Hole Mass ($10^5 M_\odot$)
Mean distance (kpc)
Stellar fraction
1.005
1.010
1.015
1.020
1.025
1.030
0.0
0.2
0.4
0.6
0.8
1.0
0.02
0.04
0.06
0.08
0.10

Schedule

Table: Activity schedule

Activities	Week															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Task 1	×															
Task 2	×	×														
Task 3		×	×	×												
Task 4					×	×										
Task 5							×									
Task 6								×	×							
Task 7										×	×	×				
Task 8													×	×	×	×

- **Task 1:** REBOUND instalación in HPC ✓
- **Task 2:** Understanding REBOUND examples ✓
- **Task 3:** Implementation of a Choksi simulation ✓
- **Task 4:** Implementation of a triaxial simulation
- **Task 5:** 30 % thesis dissertation
- **Task 6:** Optimization of the time step for *WHFast* y *IAS15*