Orbits of black holes in triaxial potentials

Juan Barbosa

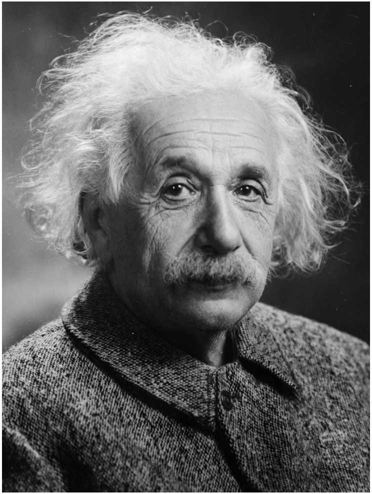
Jaime Forero
Advisor

Departamento de Física
Facultad de Ciencias
Universidad de los Andes

Overview

1. Introduction
2. Methodology
   - Units
   - Equation of motion
3. Studies
   - Symmetrical
     - Results
   - Triaxial
     - Results

Introduction



- Theory of General Relativity, 1906
- Although more than 100 years have passed since the publication of the theory, even today there are gaps in the understanding and implications of Einstein's equations

Introduction

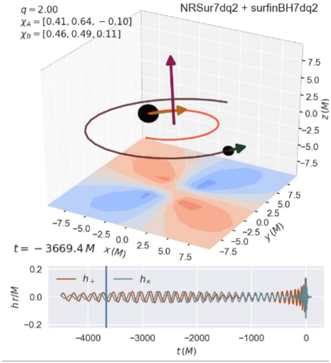


Figure: Binary black hole explorer

Vijay Varma, Leo C Stein, and Davide Gerosa. "The binary black hole explorer: on-the-fly visualizations of precessing binary black holes". In: *arXiv preprint arXiv:1811.06552* (2018)

Objectives

Study the effect of different triaxial potentials, initial speeds and numerical integrators on the times required by a supermassive black hole to return to its initial position, after experiencing a recoil, as well as to quantify how chaotic its trajectory is.

- Obtain probability distributions for the return times based on each of the free parameters of the triaxial potential, the magnitude and direction of the initial velocity
- Quantify how chaotic is the trajectory of the black hole in each simulation, using exponents of Lyapunov
- Evaluate the performance of the numerical integrators using the information of the simulations

Galactic setup

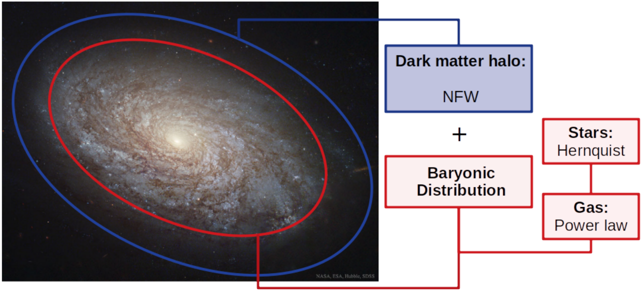


Figure: NGC4414 galaxy as seen by the Hubble telescope.

1. Dark matter (NWF):

$$\rho_{\rm DM}(r) = \frac{\rho_0^{\rm DM}}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2} \tag{1}$$

2. Stellar density (Hernquist):

$$\rho_s(r) = \frac{f_s f_b M_h \mathcal{R}_s}{2\pi r(r + \mathcal{R}_s)^3} \tag{2}$$

3. Gas density (Power law):

$$\rho_{\rm gas}(r) = \begin{cases} \rho_0^{\rm gas} & \text{if } r < r_0 \\ \rho_0^{\rm gas}\left(\frac{r_0}{r}\right)^{-n} & \text{if } r \geq r_0 \end{cases} \tag{3}$$

Mass distributions of the host galaxy, are divergent. The end of the host is taken at the virial radius.

$$\rho_{\mathrm{crit}} = \frac{3H(t)^2}{8\pi G} \tag{4}$$

$$\frac{M(R_{\mathrm{vir}})}{V(R_{\mathrm{vir}})} = \bar{\rho}(R_{\mathrm{vir}}) = 200\rho_{\mathrm{crit}} = 75\frac{H(t)^2}{\pi G} \tag{5}$$

where $M(R_{\mathrm{vir}})$ is the cumulative mass, and $V(R_{\mathrm{vir}})$: the volume

Computer simulations are sensitive to rounding errors due to the lack of infinite precision when representing decimal numbers.

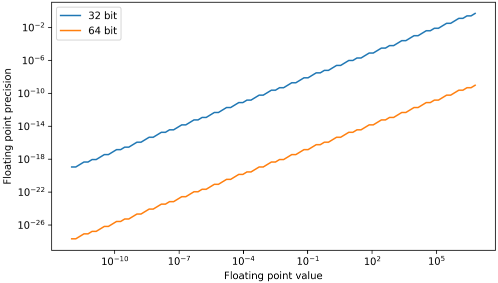


Figure: Floating point precision for different values, for a 32 bit and 64 bit holders.

Table: Units of measure used on the simulations.

Physical property	**unit**
Length	1 kilo-parsec (kpc)
Mass	10^5 solar masses ($10^5\ M_\odot$)
Time	1 giga-year (Gyr)

1. Universal gravitational constant:

$$G = 0.4493 \quad \frac{\mathrm{kpc}^3}{\mathrm{Gy}r^2 10^5 M_\odot} \tag{6}$$

2. Hubble parameter:

$$H \approx 1.023 H_0 \times 10^{-3}\ \mathrm{Gyr}^{-1} = 6.916 \times 10^{-2}\ \mathrm{Gyr}^{-1} \tag{7}$$

Equation of motion

Trajectories of the kicked black holes are obtained by numerically solving the equation of motion.

$$\ddot{\vec{x}} = -a_{\rm grav}(\vec{x})\hat{x} + \left(a_{\rm DF}(\vec{x}, \dot{\vec{x}}) - \dot{x}\frac{\dot{M}_{\bullet}(x, \dot{x})}{M_{\bullet}} \right) \dot{\hat{x}} \qquad (8)$$

where $M_{\bullet}$ is the black hole mass

Dynamical friction

As the black hole travels through the galaxy, dark matter, stars and gaseous materials from the medium interact with the black hole adding a drag force.

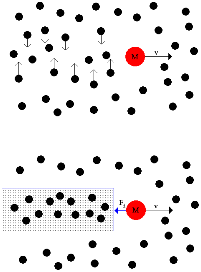


Figure: Collisionless dynamical friction

Dynamical Friction. http://abyss.uoregon.edu/~js/lectures/cannibalism/cannibalism_7.html.

- Collisionless matter interacts with the black hole gravitational only

$$a_{\mathrm{DF}}^{\mathrm{cl}}(\vec{x},\dot{\vec{x}}) = -\frac{4\pi G^2}{\dot{x}^2} M_\bullet \rho(\vec{x}) \ln\Lambda \left(\mathrm{erf}(X) - \frac{2}{\sqrt{\pi}} X e^{-X^2} \right) \quad (9)$$

- On the other side gas is in direct contact with the black hole

$$a_{\mathrm{DF}}^{\mathrm{c}}(\vec{x},\dot{\vec{x}}) = -\frac{4\pi G^2}{\dot{x}^2} M_\bullet \rho_{\mathrm{gas}}(\vec{x}) f(\mathcal{M}) \quad (10)$$

Accretion into the black hole

As the black hole accretes matter from the surroundings, an acceleration appears, due to the second law of Newton.

$$\dot{M}_\bullet(\vec{x}, \dot{\vec{x}}) = \begin{cases} \dot{M}_\bullet^{\mathrm{BHL}}(\vec{x}, \dot{\vec{x}}) & \text{if } \dot{M}_\bullet^{\mathrm{BHL}} < \dot{M}_\bullet^{\mathrm{Edd}} \\ \dot{M}_\bullet^{\mathrm{Edd}} & \text{else} \end{cases} \tag{11}$$

$$\dot{M}_\bullet^{\mathrm{BHL}}(\vec{x}, \dot{\vec{x}}) = \frac{4\pi G^2 \rho_G(\vec{x}) M_\bullet^2}{\left(c_s^2 + \dot{x}^2\right)^{3/2}} \tag{12}$$

$$\dot{M}_\bullet^{\mathrm{Edd}} = \frac{(1-\epsilon) M_\bullet}{\epsilon t_{\mathrm{Edd}}} \qquad \epsilon = 0.1, \quad t_{\mathrm{Edd}} = 0.44 \text{ Gyr} \tag{13}$$

Results

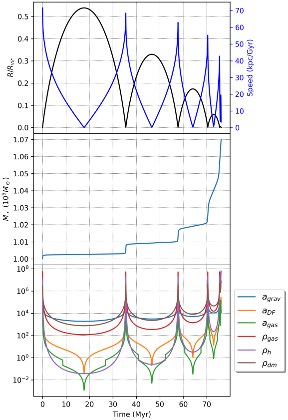
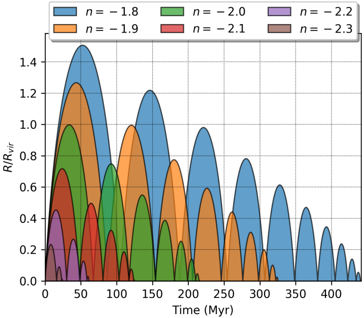


Figure: Properties of a single simulation.

Effect of the power law exponent



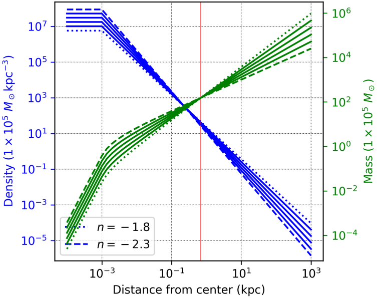


Figure: Smaller exponents increase the return time.

Effect of the baryonic fraction

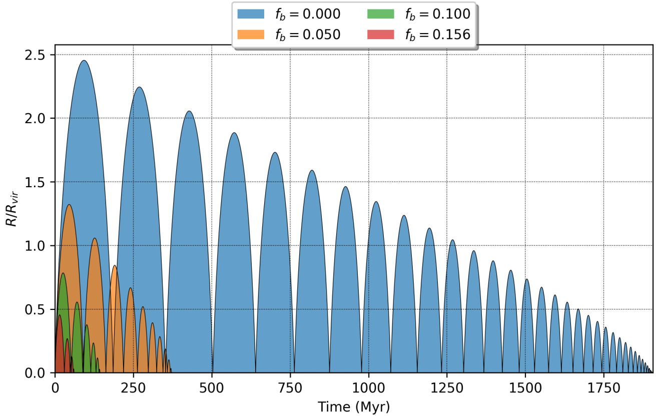


Figure: 5 % increase on baryonic fraction decreases return times.

Effect of the stellar fraction

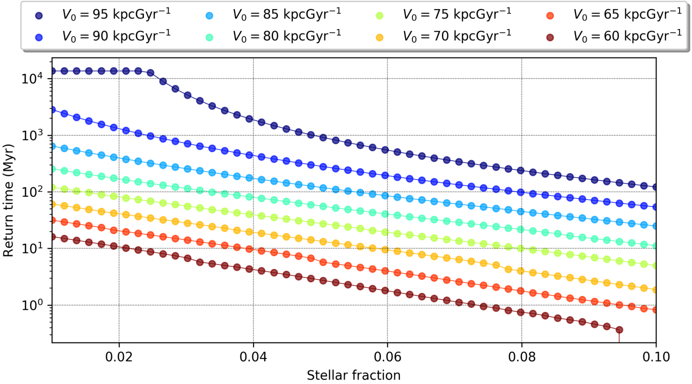


Figure: Return time for different stellar densities.

Triaxial setup

Density shells for each profile are ellipsoids

$$m^2(\vec{x}) \equiv x_1^2 + \left(\frac{a_1}{a_2}\right)^2 x_2^2 + \left(\frac{a_1}{a_3}\right)^2 x_3^2 \tag{14}$$

A thin shell, whose inner and outer skins are the surfaces m and $m + \delta m$ is described by:

$$m^2(\vec{x}, \tau) = a_1^2 \left(\frac{x_1^2}{\tau + a_1^2} + \frac{x_2^2}{\tau + a_2^2} + \frac{x_3^2}{\tau + a_3^2}\right) \tag{15}$$

where $\tau \geq 0$ labels the surfaces

James Binney and Scott Tremaine. *Galactic dynamics*. Vol. 20. Princeton university press, 2011

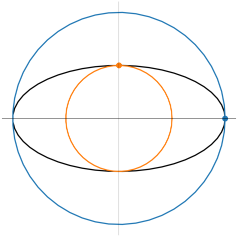


Figure: Effective gravitational mass

The contributions of all ellipsoidal shells that make up the profile are taken into account

$$\psi(m) \equiv \int_0^{m^2} \rho(m^2) dm^2 = \int_0^{k=m^2} \rho(k) dk \qquad (16)$$

The potential of any body in which $\rho = \rho(m^2)$ is:

$$\Phi(\vec{x}) = -\pi G \frac{a_2 a_3}{a_1} \int_0^{\infty} \frac{\psi(\infty) - \psi(m)}{\sqrt{(\tau + a_1^2)(\tau + a_2^2)(\tau + a_3^2)}} d\tau \qquad (17)$$

Binney and Tremaine, *Galactic dynamics*

Numerical integration of the gradients of the potentials is made with Simpson 3/8 rule.

$$\nabla\Phi_{\mathrm{DM}}(\vec{x}) = 2\pi G R_s^3 \rho_0 a_1 a_2 a_3 \int_0^\infty \frac{\vec{\phi}(\vec{x},\tau)d\tau}{m(\vec{x},\tau)\left(R_s + m(\vec{x},\tau)\right)^2} \tag{18}$$

$$\nabla\Phi_{\mathrm{S}}(\vec{x}) = G M_s a_1 a_2 a_3 \int_0^\infty \frac{\vec{\phi}(\vec{x},\tau)d\tau}{m(\vec{x},\tau)\left(\mathcal{R}_\int + m(\vec{x},\tau)\right)^3} \tag{19}$$

$$\nabla\Phi_{\mathrm{G}}(\vec{x}) = 2\pi G \rho_0 a_1 a_2 a_3 \begin{cases} \int_0^\infty \vec{\phi}(\vec{x},\tau)d\tau & \text{for } m(\vec{x},\tau) < r_0 \\ r_0^{-n} \int_0^\infty m(\vec{x},\tau)^n \vec{\phi}(\vec{x},\tau)d\tau & \text{else} \end{cases} \tag{20}$$

Results

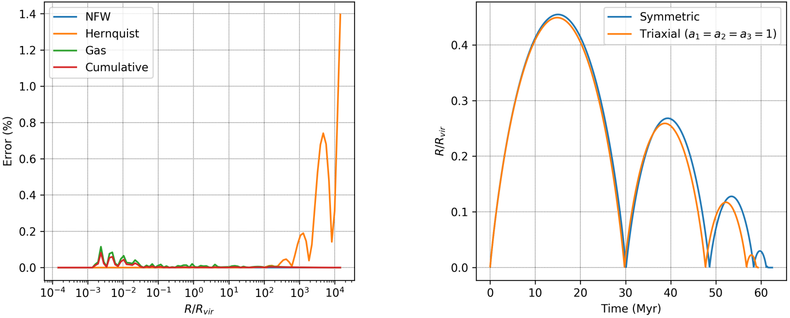
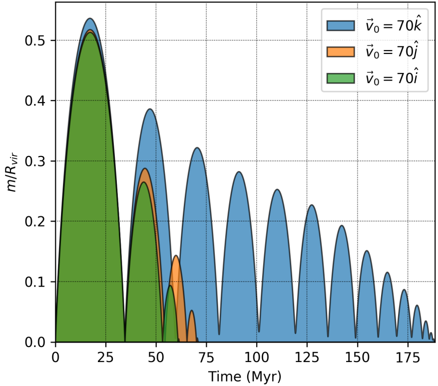


Figure: Differences for analytical and numerical integration of the potentials. Analytical is taken as: $-GM(r)/r^2$



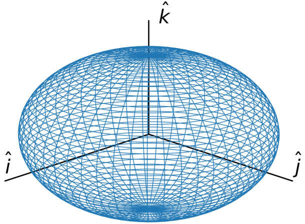


Figure: Orthogonal launches for a triaxial profile with semi-axis $(a_1{:}a_2{:}a_3) = (1{:}0.99{:}0.95)$

Schedule

Table: Activity schedule

Activities	Week						
	1	2	3	4	5	6	7
Task 1	x						
Task 2	x	x					
Task 3		x	x	x			
Task 4					x	x	
Task 5							x

Task 1: REBOUND instalación in HPC ✓
Task 2: Understanding REBOUND examples ✓
Task 3: Implementation of a Choksi simulation ✓
Task 4: Implementation of a triaxial simulation ✓
Task 5: 30 % thesis dissertation ✓

Schedule

Table: Activity schedule

Activities	Week								
	8	9	10	11	12	13	14	15	16
Task 6	×	×							
Task 7			×	×	×				
Task 8						×	×	×	×

Task 6: Optimization of the time step for *WHFast* y *IAS15*
Task 7: Implementation of an automated algorithm for the space of parameters of initial velocities and parameters of the triaxial potential, for the integrators *Leapfrog*, *WHFast* and *IAS15*
Task 8: Analysis and writing of the document

Thank You

Mass distributions between components are given by:

$$M_{\mathrm{DM}}(R_{\mathrm{vir}}) = (1 - f_b)M_h \tag{21}$$

$$M_{\mathrm{stars}}(R_{\mathrm{vir}}) = f_s f_b M_h \tag{22}$$

$$M_{\mathrm{gas}}(R_{\mathrm{vir}}) = (1 - f_s) f_b M_h \tag{23}$$

with $f_b = 0.156$ and $M_h = 10^8\ M_\odot$

$$R_{\mathrm{vir}} = \left(\frac{M_h G}{100 H(t)^2}\right)^{1/3} \tag{24}$$

Drag generated by gas depends on local sound speed

$$\mathcal{M}(\dot{x}) \equiv \frac{|\dot{x}|}{c_s} = |\dot{x}|\sqrt{\frac{\mathcal{M}_w}{\gamma R T_{\rm vir}}} = |\dot{x}|\sqrt{\frac{\mathcal{M}_w}{\gamma R}\left(\frac{2k_B R_{\rm vir}}{\mu m_p G M_h}\right)} \tag{25}$$

$$\mathcal{M}(\dot{x}) = 1.63|\dot{x}|\sqrt{\frac{R_{\rm vir}}{M_h}}$$

$$a^{\rm c}_{\rm DF}(\vec{x},\dot{\vec{x}}) = -\frac{4\pi G^2}{\dot{x}^2} M_\bullet \rho_{\rm gas}(\vec{x}) f(\mathcal{M}) \tag{26}$$

with

$$f(\mathcal{M}) = \begin{cases} 0.5\ln\Lambda\left[\mathrm{erf}\left(\frac{\mathcal{M}}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}}\mathcal{M}e^{-\mathcal{M}^2/2}\right] & \text{if } \mathcal{M} \leq 0.8 \\ 1.5\ln\Lambda\left[\mathrm{erf}\left(\frac{\mathcal{M}}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}}\mathcal{M}e^{-\mathcal{M}^2/2}\right] & \text{if } 0.8 < \mathcal{M} \leq \mathcal{M}_{eq} \\ 0.5\ln\left(1-\mathcal{M}^{-2}\right) + \ln\Lambda & \text{if } \mathcal{M} > \mathcal{M}_{eq} \end{cases} \tag{27}$$

$$\omega = \frac{\tau^\gamma}{\tau^\gamma + 1}, \qquad \tau = \left(\frac{\omega}{1-\omega}\right)^{\frac{1}{\gamma}}, \qquad d\tau = \frac{\left(-\frac{\omega}{\omega-1}\right)^{\frac{1}{\gamma}}}{\gamma\omega\left(-\omega+1\right)} \tag{28}$$

$$\phi_i(x_i, \tau) = \frac{x_i}{\left(\tau + a_i^2\right)^{\frac{3}{2}} \prod_{i \neq j}^{3} \sqrt{\tau + a_j^2}} \tag{29}$$

$$\vec{\phi}(\vec{x}, \tau) = (\phi_1(x_1, \tau), \phi_2(x_2, \tau), \phi_3(x_3, \tau)) \tag{30}$$