

1 Galactic setup

1.1 Units

Computer simulations are sensitive to rounding errors due to the lack of infinite precision when representing decimal numbers. Really small numbers as well as really big ones tend to have bigger errors than those close to the unity, as can be seen on [Figure 1](#).

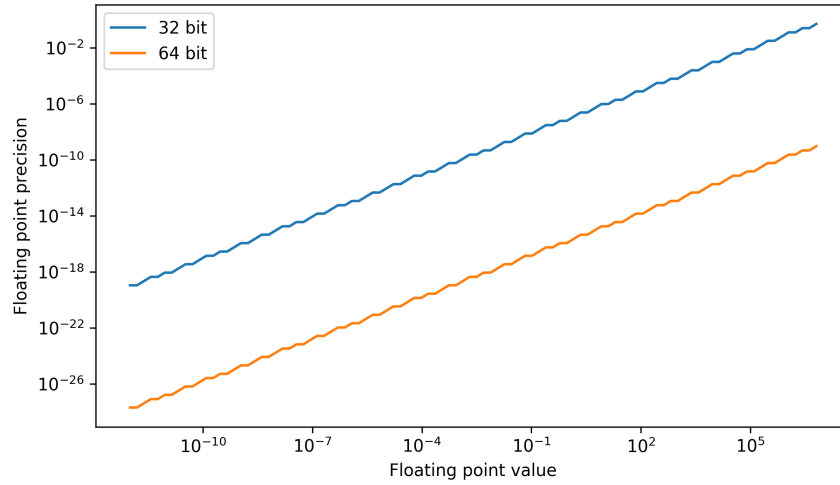


FIGURE 1: Floating point precision for different values, for a 32 bit and 64 bit holders.

Under the International System of Units, distances are measured on meters, times on seconds, and mass on kilograms, nevertheless black holes are too heavy to be measured on kilograms, galaxies sizes too big to be quantified on meters, and time scales too large for a second. Because of that, the following units will be used throughout this document:

TABLE 1: Natural units

Physical property	unit
Length	1 kilo-parsec (kpc)
Mass	10^5 solar masses ($10^5 M_{\odot}$)
Time	1 giga-year (Gyr)

Universal gravitational constant

First quantified by Henry Cavendish the gravitational constant has a value of $G_0 = 6.67408 \times 10^{-11}$ on SI units of $\text{m}^3\text{s}^{-2}\text{kg}^{-1}$. With the units of length, mass and time on [Table 1](#), the constant

of gravity to be used is given by:

$$G = G_0 \left(\frac{1 \text{ kpc}^3}{(3.0857 \times 10^{19})^3 \text{ m}^3} \right) \left(\frac{(3.154 \times 10^{16})^2 \text{ s}^2}{1 \text{ Gyr}^2} \right) \left(\frac{1.98847 \times 10^{35} \text{ kg}}{10^5 M_\odot} \right) = 0.4493 \frac{\text{kpc}^3}{\text{Gyr}^2 10^5 M_\odot} \quad (1)$$

Hubble constant

The hubble constant is frequently used as $H_0 = 67.66 \pm 0.42 \text{ kms}^{-1} \text{Mpc}^{-1}$, stating the speed of an astronomical body on kms^{-1} at a distance of 1 Mpc.

$$H = H_0 \left(\frac{1 \text{ kpc}}{3.0857 \times 10^{16} \text{ km}} \right) \left(\frac{1 \text{ s}}{3.154 \times 10^{13} \text{ Gyr}} \right) \left(\frac{1 \text{ Mpc}}{1000 \text{ kpc}} \right) \quad (2)$$

1.2 Mass distributions

The host galaxy has two mass distributions that are superimposed, one for dark matter and the other one for all the luminous or baryonic matter.

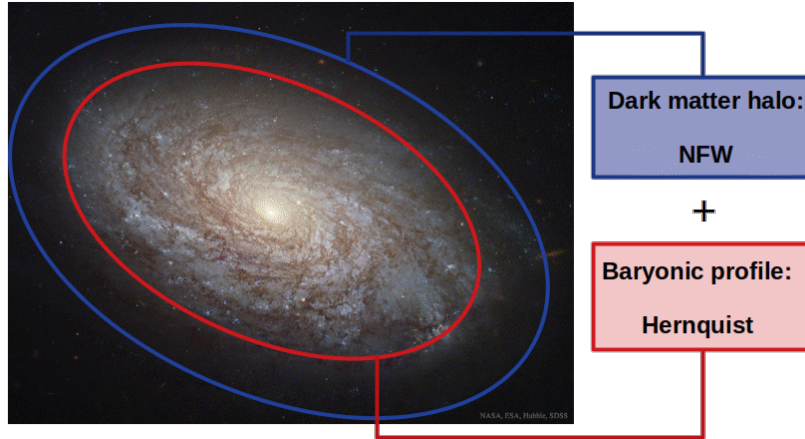


FIGURE 2: NGC4414 galaxy #TODO: cite image

The dark matter halo used on the host, follows a NFW (Navarro–Frenk–White) profile, while the baryonic mass distribution of the galaxy depends on the redshift z . For high redshifts a gaseous disk with constant density followed by a density dependance with $r^{-2.2}$ is used, while for $z \approx 0$ a Hernquist model is applied [1].

1.3 Dark matter halo

For a dark matter halo following a NFW profile, the density at some distance r is given by the formula:

$$\rho_{\text{DM}}(r) = \frac{\rho_0^{\text{DM}}}{\frac{r}{R_s} \left(1 + \frac{r}{R_s} \right)^2} \quad (3)$$

Where R_s and ρ_0^{DM} are constants for a given dark matter halo. Using the density, the cumulative mass $M_{\text{DM}}(r)$ within some radius r is given by the integral of the density over a volume, since Equation 3 is spherically symmetrical, the only dependance of the integral is with distance. On Equation 4 the r'^2 comes from the Jacobian of spherical coordinates, and the 4π from the solid angle.

$$M_{\text{DM}}(r) = \int_0^r 4\pi r'^2 \rho_{\text{DM}}(r') dr' = 4\pi \rho_0 R_s^3 \left[\ln \left(\frac{R_s + r}{R_s} \right) - \frac{r}{R_s + r} \right] \quad (4)$$

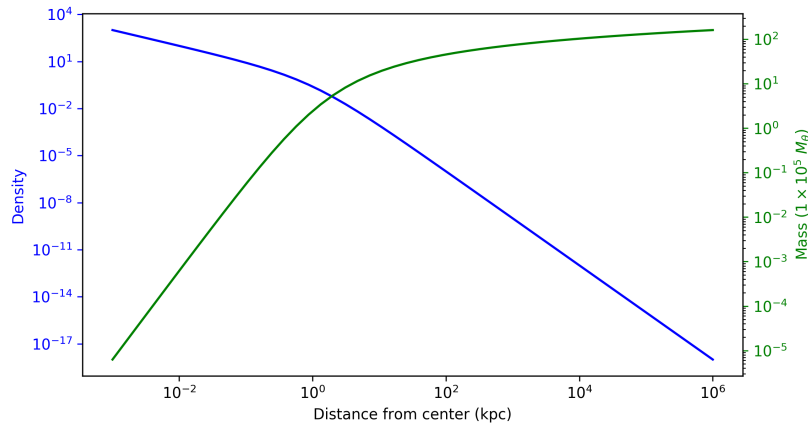


FIGURE 3: Density and cumulative mass for a NFW profile with $\rho_0^{\text{DM}} =$ and $R_s =$.

Since the mass of dark matter of a single galaxy diverges for $r \rightarrow \infty$ there is a radius called R_{vir} , at which the density of the NFW profile is 200 times the critical density ρ_{crit} the minimum density for an expanding universe.

$$R_{\text{vir}} = 200 \rho_{\text{crit}} = 200 \left(\frac{3H^2}{8\pi G} \right) \quad (5)$$

Considering a concentration parameter $c(M_h, z)$ of dark matter in the halo, given by:

$$c(M_h, z) = c_0(z) \left(\frac{M_h}{10^{13} M_\odot} \right)^{\alpha(z)} \quad (6)$$

where $\alpha(z)$ and $c_0(z)$ were fitted using simulation data to the following functions [1]:

$$c_0(z) = \frac{4.58}{2} \left[\left(\frac{1+z}{2.24} \right)^{0.107} + \left(\frac{1+z}{2.24} \right)^{-1.29} \right] \quad (7)$$

$$\alpha(z) = -0.0965 \exp \left(-\frac{z}{4.06} \right) \quad (8)$$

The concentration parameter of dark matter relates the viral radius R_{vir} and the scale radius R_s as:

$$R_{\text{vir}} = c(M_h, z) R_s \quad (9)$$

The relation of R_{vir} with the concentration parameter is particularly useful to the simulation. As the system evolves on time, so does the viral radius due to the existing relation between time and redshift.

$$t = \frac{2H^{-1}}{1 + (1 + z)^2} \quad (10)$$

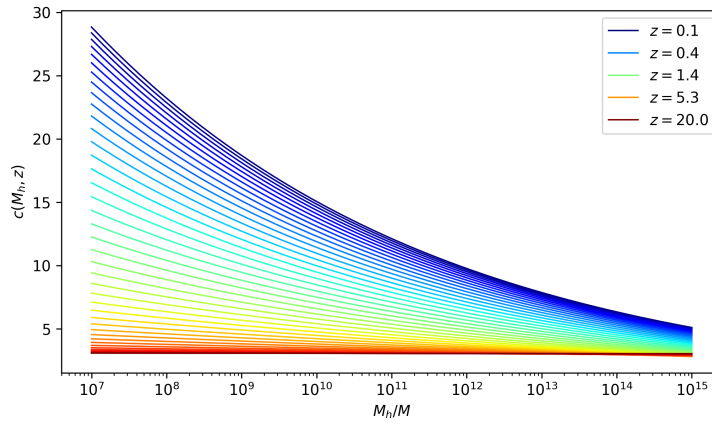
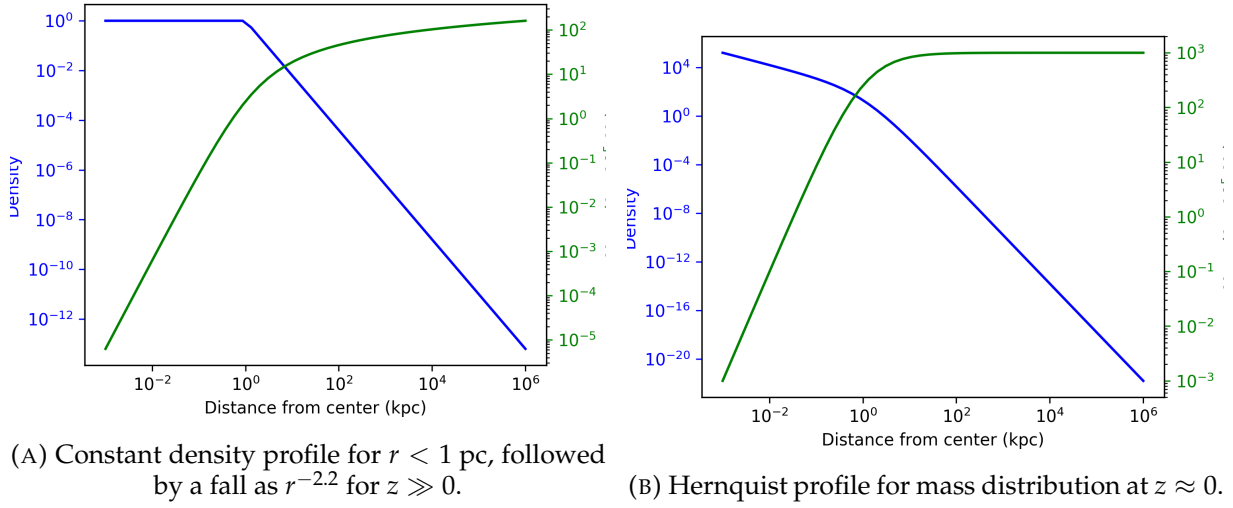


FIGURE 4: Dark matter concentration parameter as a function of the halo mass and the redshift.

Baryonic profile

FIGURE 5: Baryonic mass distributions for $z \gg 0$ and $z \approx 0$.