# DINÁMICA DE GALAXIAS, UNA SIMULACIÓN CON N log N ITERACIONES.

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- LETTERSTONATURE -

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#### A hierarchical O(N log N) force-calculation algorithm

#### Josh Barnes & Piet Hut

The Institute for Advanced Study, School of Natural Sciences,

Listil recently the cravitational N-body problem has been modelled numerically either by direct integration, in which the computation needed increases as  $N^1$ , or by an iterative potential method in which the number of operations grows as N log N. Here we describe a novel method of directly calculating the force on Nbodies that grows only as N log N. The technique uses a treestructured hierarchical subdivision of space into cubic cells, each of which is recursively divided into eight subcells whenever more than one particle is found to occupy the same cell. This tree is constructed anew at every time step, avoiding ambiguity and tang ling. Advantages over potential-solving codes are: accurate local interactions; freedom from geometrical assumptions and restrictions; and applicability to a wide class of systems, including (proto-)planetary, stellar, galactic and cosmological ones. Advantages over previous hierarchical tree-codes include simplicity and the possibility of rigorous analysis of error. Although we concentrate here on stellar dynamical analications, our techniques of efficiently handling a large number of long-range interactions and concentrating computational effort where most needed have poten-

Until recently, the dynamics of a system of self-gravitating bodies (the gravitational N-body problem) has been modelled numerically in two fundamentally different ways. The first one N(N-1) forces between all pairs of particles. This allows an accurate description of the dynamical evolution but at a price that grows rapidly for increasing No. The second way involves a two-step approach; after fitting the global potential field to a special model with a number of free parameters, each particle propagated in this background field for a short time before the same procedure is reiterated. The potential method involves a number of operations that grow only as N log N. Thus calculations can be performed more quickly, but with a loss of accuracy and generality. The special nature of each potential is tuned to the geometry of the problem being considered (such as Fourier transforms or spherical or bispherical harmonics<sup>2</sup>) Recently, some of the advantages of both approaches have been combined by using direct integrations of force while group ing together increasingly large groups of particles at increasingly large distances. This corresponds to the way humans interact with neighbouring individuals, further villages and increasingly further and larger states and countries-driven by increasing cost and decreasing need to deal with more removed groups on an individual basis. The first implementation of such a hierar chical grouping of interactions was given by Appel', who used a tree structure to represent an N-body system, with the particles stored in the leaves of the tree. An independent implementation by Jernigan" and Porter" incorporated regularization of close

encounters. However, in both codes the logarithmic-growth gain

in efficiency comes at the price of introducing additional errors

the tree. Nearby particles may be counted as leaves of nearby branches, but the phase-space flow of realistic self-gravitating systems demands a continuous updating of the tree structure to avoid tangling and unphysical grouping, requiring complicated

book-keeping. It is not at all clear how to understand and estimate the errors caused by the process of approximating lumps of particles together as single pseudo-particles, because

Fig. 2. A partial term scheme for Fe III, showing the observed transitions, some lower-lying terms of the same even parity and the first two terms of odd parity. Term energies are given in ontial applications in other areas of astrophysics as well.

abilities found by Garstang' for lower-lying levels are in each case -1. Thus it is likely that the upper levels have populations close to their Boltzmann values with collisional de-excitation rates exceeding radiative decay rates. Then the ratio of the [Fe III] line fluxes to those of permitted transitions would be sensitive to the electron density. Detailed calculations of the regime for the individual lines. The upper levels may attain only a pseudo-Boltzmann population if collisional excitation to higher states of odd parity exceeds the rate for collisional deexcitation to lower levels. In any case the new [Fe ttt] identifications will provide more information on the structure of the solar chromosphere-corona transition region.

If the a'G, a'P and b'D levels are collisionally de-excited in the solar atmosphere, they could become stronger, relative to permitted transitions of species of similar excitation, in astrophysical sources of lower electron density. For this reason their presence is being investigated in such sources, including the Seyfert galaxy NGC 4151, which has unidentified emission features around 1,575, 1,581 and 1,518 Å (refs 13 and 14). [Fe III] emission is observed in the optical spectrum of NGC The relative intensities of the quintet transitions in NGC 4151 and other sources cannot be predicted until collision cross-sections and transition probabilities are known, and these are urgently required to establish whether or not the [Fe III] lines are of wider astronhysical significance.

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individual lumps can take more or less arbitrary shapes and sizes © 1999 Nature Publishing Group







Piet Hut



VAX 11/780



VAX 11/780

➤ Simulación con 4096 cuerpos.



VAX 11/780

- Simulación con 4096 cuerpos.
- ► Tiempo: 10 horas de CPU.

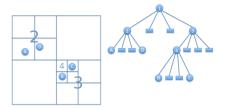
El algoritmo propuesto por Barnes y Hut consta de dos pasos fundamentales, la división recursiva del espacio y la forma como se calcula la fuerza sobre un cuerpo.

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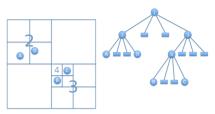
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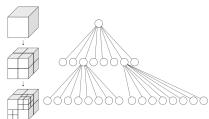
Árbol dos dimensional.

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1. División jerárquica del espacio.



Árbol dos dimensional.



Árbol tridimensional.

2. Fuerza sobre un cuerpo.

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El número de iteraciones se reduce al considerar centros de masa y distancias.

Cada caja tiene una longitud específica

2. Fuerza sobre un cuerpo.

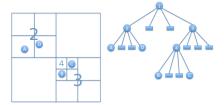
- ► Cada caja tiene una longitud específica
- Un centro de masa

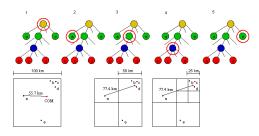
2. Fuerza sobre un cuerpo.

- Cada caja tiene una longitud específica
- Un centro de masa
- Y la masa contenida

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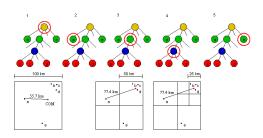
- ► Cada caja tiene una longitud específica
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- Y la masa contenida





Se define un coeficiente de precisión.

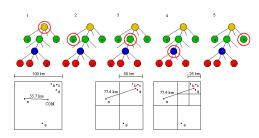
$$\theta = 0.5$$



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#### 1. Nodo principal

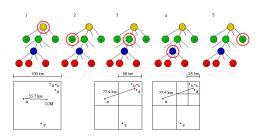


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$$\frac{s}{d} = \frac{100}{55,7} \approx 1.8 > \theta$$



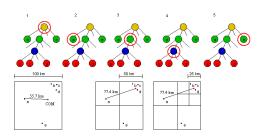
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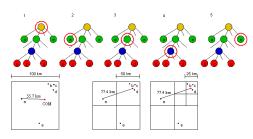
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- 3. Segundo nodo



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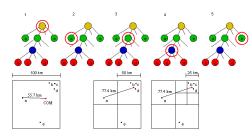
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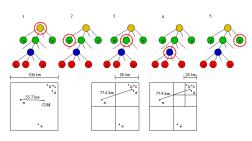
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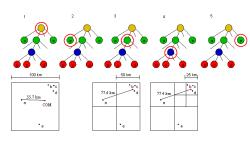
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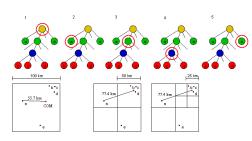
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 $Nodo\ externo,\ contribuye$ 

La construcción del arbol se realiza para cada instante de tiempo.

Observación

La construcción del arbol se realiza para cada instante de tiempo.

Todas las cajas

La construcción del arbol se realiza para cada instante de tiempo.

Cajas con una partícula

# CONSTRUCCIÓN DE UNA SIMULACIÓN

1. Descripción del sistema.

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- 3. Solución de las ecuaciones.

# Construcción de una simulación

- 1. Descripción del sistema.
- 2. Condiciones iniciales.
- 3. Solución de las ecuaciones.
- 4. Visualización.

#### DESCRIPCIÓN DEL SISTEMA

Usando la ley de gravitación universal:

$$\vec{F}_i = m_i \vec{a}_i = -\sum_{i \neq i}^N G \frac{m_i m_j}{|\vec{r}_{ij}|^3} (\vec{r}_i - \vec{r}_j)$$
 (1)

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$$\vec{r}_i = -\sum_{j \neq i}^N G \frac{m_j}{|\vec{r}_{ij}|^3} (\vec{r}_i - \vec{r}_j)$$
 (2)

#### CONDICIONES INICIALES

Suponiendo órbitas circulares y teniendo en cuenta la masa encerrada en las órbitas de menor tamaño:

$$v \approx \sqrt{\frac{GM(r)}{r}} \tag{3}$$

En coordenadas polares:

$$\begin{array}{ccc}
x = r\cos(\theta) & \longrightarrow & \dot{x} = -r\sin(\theta) = -y \\
y = r\sin(\theta) & \longrightarrow & \dot{y} = r\cos(\theta) = x
\end{array}$$
(4)