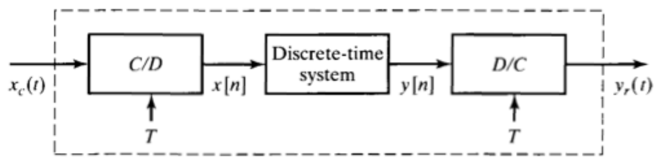
Homework Set #4

1. (**4 pts**) A bandlimited continuous-time signal is known to contain 60 Hz component which we want to remove by processing with the system shown below. Assume that $T = 10^{-4}$.

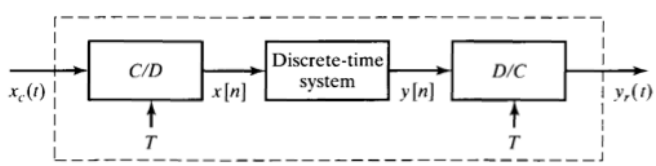


(a) (**1 pts**) What is the highest frequency that the continuous time signal can contain if aliasing is to be avoided?
(b) (**2 pts**) The discrete-time system to be used has the following frequency response:

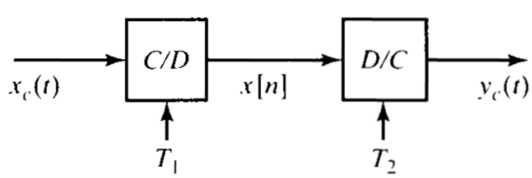
$$H(e^{j\omega}) = \frac{[1 - e^{-j(\omega-\omega_0)}][1 - e^{-j(\omega+\omega_0)}]}{[1 - 0.9e^{-j(\omega-\omega_0)}][1 - 0.9e^{-j(\omega+\omega_0)}]}$$

Using Python, plot the magnitude and phase of $H(e^{j\omega})$ for $\omega_0 = \pi/2$. Please submit your code and plots.
(c) (**1 pts**) Which value of ω_0 would you choose to eliminate the 60 Hz component?

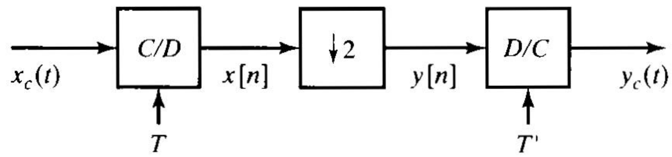
2. (**3 pts**) Consider the system in the figure below with $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi(1000)$ and the discrete time system a squarer, i.e. $y[n] = x^2[n]$. What is the largest value of T such that $y_c(t) = x^2(t)$?



3. (**3 pts**) Assume that in figure below $X_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T_1$. For the general case of $T_1 \neq T_2$, express $y_c(t)$ in terms of $x_c(t)$.



4. (**4 pts**) In the figure below, $x[n] = x_c(nT)$ and $y[n] = x[2n]$.

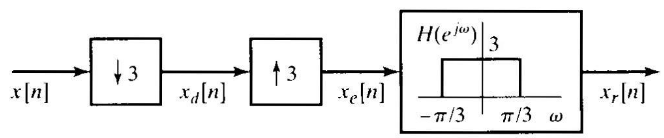


(**a**) Assume that $x_c(t)$ has a Fourier transform such that $X_c(j\Omega) = 0,\ |\Omega| > 2\pi(100)$. What value of T is required so that

$$X(e^{j\omega}) = 0, \quad \frac{\pi}{2} < |\omega| \leq \pi?$$

(**b**) How should T' be chosen so that $y_c(t) = x_c(t)$?

5. (**4 pts**) Consider the system shown below. For each of the following input signals $x[n]$, indicate whether the output $x_r[n] = x[n]$.



(**a**) (**1 pts**) $x[n] = cos(\pi n/4)$
(**b**) (**1 pts**) $x[n] = cos(\pi n/2)$
(**c**) (**2 pts**)

$$x[n] = \left[\frac{sin(\pi n/8)}{\pi n}\right]^2$$

Hint: Use the modulation property of the Fourier transform to find $X(e^{j\omega})$.