# Fundamental groups in Topology

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# Contents

- Let X be a top. space. A **space over** $X$ is a topological space $Y$ together with a continuous map $p : Y \to X$.

  A morphism between two spaces $p_i : Y_i \to X$ (i=1,2) over $X$ is given by a continuous map $f : Y_1 \to Y_2$ making the following diagram commutes :

$$\begin{tikzcd} Y_1 \arrow[r, "f"] \arrow[dr, "p_1"'] & Y_2 \arrow[d, "p_2"] \\ & X \end{tikzcd}$$

- A **cover** of X is a space Y over X where the projection $p : Y \to X$ satisfies the following condition :
  Each point of $X$ has an open neighbourhood $V$ for which $p^{-1}(V)$ decomposes as a disjoint union of open subsets $U_i \subseteq Y$ such that each restricted map $p_{U_i} : U_i \to Y$ induces a homeomorphism of $U_i$ with $V$.

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- **Example**(*Trivial cover*) : Let $I$ be a nonempty discrete top. space. The first projection

$$X \times I \to X$$

turns $X \times I$ into a cover of $X$.

- **Proposition 2.1.3 :** A space $Y$ over $X$ is a cover iff each point of $X$ has an open neighbourhood $V$ such that the restriction of the projection $p : Y \to X$ to $p^{-1}(V)$ is isomorphic as a space ver $V$ to a trivial cover.
- The points of X over which the fibre of $p$ equals $I$ form an open subset of X.
- **Corollary 2.1.4 :** It $X$ is connected, the fires of $p$ are all homeomorphic to the same discrete space $I$.
- Let $G$ be a group acting continuously from the left on a topological space $Y$. The action of $G$ is **even** (or **properly discontuonus**) if each point $y \in Y$ has some open neighbourhood $U$ such that $g_1 U \cap g_2 U = \varnothing$ if $g_1 \neq g_2$.

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- **Example 2.1.8 :** 1. Let $\mathbb{Z}$ act on $\mathbb{R}$ by translations. We obtain a cover $\mathbb{R} \to \mathbb{R}/\mathbb{Z} \simeq S^1$.
  2. For an integer $n > 1$, we get an even action of $\mu_n$ on $\mathbb{C}^*$ from that a cover $p_n : \mathbb{C}^* \to \mathbb{C}^*/\mu_n$. The map $z \mapsto z^n$ defines a natural homeomorphism (isomorphism) of $\mathbb{C}^*/\mu_n$ onto $\mathbb{C}^*$. So this induces a cover $\mathbb{C}^* \to \mathbb{C}^*$.
- For now on, assume that $X$ is locally connected.
- $\mathrm{Aut}(Y/X) := \{\phi : Y \to Y : p \circ \phi = p\}$.
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- Apply the above proposition to $Z = Y,\ f = id$ and $g = \phi$ to get :
  **Lemma 2.2.1** An automorphism $\phi$ of a connected cover $p : Y \to X$ having a fixed point must be trivial.
- If $p : Y \to X$ is a connected cover, the action of $\mathrm{Aut}(Y/X)$ on $Y$ is even.
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- Given a connected cover $p : Y \to X$, form the quotient of $Y$ by the action of $\mathrm{Aut}(Y/X)$. The projection factors as :

$$Y \longrightarrow \mathrm{Aut}(Y/X)\backslash Y \xrightarrow{\;\overline{p}\;} X$$

- A cover $p : Y \to X$ is said to be **Galois** if $Y$ is connected and the induved map $\overline{p}$ above is a homeomorphism.
- **Proposition 2.2.7** A connected cover $p : Y \to X$ is Galois iff $\mathrm{Aut}(Y/X)$ acts transitively on each fibre of $p$.
- We will need the following lemma :
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- Let $p : Y \to X$ be a Galois cover and $G = \mathrm{Aut}(Y|X)$. The map $H \mapsto H\backslash Y$, $Z \mapsto \mathrm{Aut}(Y|Z)$ induce a bijection between subgroups of $G$ and intermediate covers $Z$ as above.

$$\begin{tikzcd} Y \arrow[r, "f"] \arrow[dr, "p"'] & Z \arrow[d, "q"] \\ & X \end{tikzcd}$$

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# The Monodromy Action

- **Lemma 2.3.2** Let $p : Y \to X$ be a cover, $y \in Y$ and $x = p(y)$.
  1. Given a path $f : [0,1] \to X$ with $f(0) = x$, there is a unique path $\tilde{f} : [0,1] \to Y$ with $p \circ \tilde{f} = f$.
  2. Assume moreover given a second path $g : [0,1] \to X$ homotopic to $f$. Then the unique $\tilde{g} : [0,1] \to Y$ with $\tilde{g}(0) = y$ and $p \circ \tilde{g} = g$ has the same endpoint as $\tilde{f}$, i.e. $\tilde{f}(1) = \tilde{g}(1)$.
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- The cover $\widetilde{X}_x$ depends on the choice of the base point $x$.
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- *For an arbitrary cover* $p : Y \to X$ *and element* $y \in \pi^{-1}(x)$*, the cover map* $\pi^{-1}(y) : \widetilde{X}_x \to Y$ *corresponding to* $y$ *via the isomorphism* $Fib_x(Y) \cong Hom(\widetilde{X}_x, Y)$ *maps* $\widetilde{x}$ *to* $y$*.*

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- **Remark** : The cover $\widetilde{X}_x$ plays the role of a separable closure $k_s$. In other words, the choice of x corresponds to the choice of the separable closure.
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- ***Corollary 2.3.9*** *: For $X$ and $x$ as before, the functor $Fib_x$ induces an equivalence of the category of finite covers of $X$ with the category of finite continuous left $\widehat{\pi_1(X,x)}$-sets. Connected covers correspond to finite $\widehat{\pi_1(X,x)}$-sets with transitive action and Galois covers to coset spaces of open normal subgroups.*
- *Here $\widehat{\pi_1(X,x)}$ denotes the profinite completion of $\pi_1(X,x)$ :*

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