THE ULTRA AMPLIFIER FAMILY: CONSTRUCTIVE EQUALITY CLASSES OF ARITHMETIC KERNEL SYSTEMS IN THE ENTROPY–LANGLANDS FRAMEWORK

PU JUSTIN SCARFY YANG

ABSTRACT. We define and classify the Ultra Amplifier Family: a collection of arithmetic–spectral kernels that not only approximate target functions but reproduce them exactly via structured convolution operators. These amplifiers form constructive equality classes and are characterized by perfect spectral selectivity, entropy identity concentration, and stack-lifted trace invariance. We demonstrate their role in the Riemann Hypothesis trace convolution hierarchy, their categorification via Langlands sheaf convolution, and their use as identity kernels over entropy-zeta moduli stacks.

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1. INTRODUCTION

Amplifier kernels, as refined in the previous paper, selectively enhance spectral contributions toward a target. Traditionally, these are approximate operators. In

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this paper, we go further: we define the **Ultra Amplifier Family**, those kernels which satisfy:

- Spectral identity: the kernel acts as a projector onto a single spectral component or a complete spectral band with unity;
- Entropy exactness: the entropy-weighted convolution recovers the original function precisely;
- Langlands integration: the kernel arises naturally from Hecke–stack correspondence and maintains identity under trace convolution.

We formalize these kernels via three operator-theoretic and stack-theoretic criteria, and construct examples from exact Hecke amplifier classes, zeta-concentrated convolution families, and AI-regulated identity selectors. The resulting structure provides a canonical identity component of the entropy–Langlands–RH trace kernel lattice.

2. Definition and Characterization of Ultra Amplifier Kernels

2.1. Definition of the Ultra Amplifier Family.

Definition 2.1 (Ultra Amplifier Kernel)**.** Let $\mathcal{H}$ be a Hilbert space with orthonormal spectral basis $\{\phi_\lambda\}$. An *Ultra Amplifier Kernel* is a kernel of the form:

$$U(x,y) := \sum_{\lambda\in\Lambda} \delta_\lambda \cdot \phi_\lambda(x)\overline{\phi_\lambda(y)},$$

satisfying the following exactness conditions:

(UA1) **Spectral Identity**: $\delta_\lambda = 1$ for all $\lambda \in \Lambda$, and 0 elsewhere;

(UA2) **Convolution Idempotency**: $U * f = f$ for all f supported on $\mathrm{Span}\{\phi_\lambda \mid \lambda \in \Lambda\}$;

(UA3) **Entropy Sharpness**: There exists an entropy profile $H_Y(\lambda)$ such that $e^{-H_Y(\lambda)} = 1$ if $\lambda \in \Lambda$, 0 otherwise.

Remark 2.2. An Ultra Amplifier Kernel behaves as an identity operator on its spectral band. It perfectly isolates and reproduces a function supported on Λ with no smoothing or remainder, distinguishing it from classical approximating amplifiers.

2.2. Examples of Ultra Amplifiers in Arithmetic Spectra.

Example 2.3 (Spectral Projection Amplifier)**.** Let ϕ_λ be automorphic forms on $GL(2)$ and fix λ_0 an isolated spectral eigenvalue. Define:

$$U_{\lambda_0}(x,y) := \phi_{\lambda_0}(x)\overline{\phi_{\lambda_0}(y)}.$$

Then $U_{\lambda_0} * f = \langle f, \phi_{\lambda_0}\rangle \phi_{\lambda_0}$, and

$$U_{\lambda_0} * U_{\lambda_0} = U_{\lambda_0}.$$

This is a rank-one Ultra Amplifier projecting onto λ_0.

Example 2.4 (Ultra Hecke Amplifier)**.** Let T_p be the Hecke operator on $S_k(\Gamma_0(N))$, and ϕ_λ an eigenform with $T_p\phi_\lambda = \lambda_p\phi_\lambda$. Define:

$$U_p := \sum_{\phi \in \mathcal{B}} \delta_\phi \cdot \phi(x)\overline{\phi(y)},$$

where $\delta_\phi = 1$ if ϕ satisfies $T_p\phi = \lambda_p\phi$ for fixed λ_p, else 0.

Then U_p acts as identity on the Hecke eigenspace with eigenvalue λ_p:

$$U_p * \phi = \phi \quad \text{if } T_p\phi = \lambda_p\phi.$$

2.3. **Operator-Theoretic Equivalence Class.**

Definition 2.5 (Constructive Equality Class)**.** Two kernels K_1, K_2 belong to the same **constructive equality class** $\mathcal{C}_{\mathcal{E}}$ if:

$$K_1 * f = K_2 * f \quad \text{for all } f \in \mathcal{H}_\Lambda,$$

where $\mathcal{H}_\Lambda := \mathrm{Span}\{\phi_\lambda \mid \lambda \in \Lambda\}$ and K_1, K_2 are both Ultra Amplifiers supported on Λ.

Proposition 2.6. *All Ultra Amplifiers supported on a fixed Λ form a commutative idempotent algebra under convolution:*

$$K * K = K, \quad K * K' = K', \quad \forall K, K' \in \mathcal{C}_{\mathcal{E}}.$$

3. Applications to the Riemann Hypothesis and Langlands Stack Geometry

3.1. **Ultra Amplifiers in Zeta Trace Reconstruction.**

Recall that the Riemann zeta function admits spectral trace-like representations:

$$\zeta(s) = \mathrm{Tr}\left(K_s^{(zeta)}\right), \quad K_s^{(zeta)} := \sum_\rho \frac{1}{s-\rho}.$$

Let $\mathcal{U}_{\mathrm{RH}}$ be an Ultra Amplifier Kernel supported on the spectral set $\{\rho \mid \zeta(\rho) = 0\}$. Then:

$$\mathcal{U}_{\mathrm{RH}} * f = f \quad \text{if } f \text{ has zeta-zero spectral support.}$$

Theorem 3.1 (Ultra Amplifier Characterization of RH)**.** *RH holds if and only if there exists a kernel $\mathcal{U}_{RH}$ such that:*

$$\mathrm{Spec}(\mathcal{U}_{RH}) \subset \left\{ s \in \mathbb{C} \mid \Re(s) = \frac{1}{2} \right\}, \quad \textit{and} \quad \mathcal{U}_{RH} * f = f$$

for all f generated by ζ-zero test vectors.

3.2. **Langlands Stack Integration of Ultra Amplifiers.** Let $\mathcal{M}_{\mathrm{Bun}_G}$ be the moduli stack of G-bundles over a curve, and let π range over automorphic sheaves. Then define:

$$\mathcal{U}_\pi := \phi_\pi \boxtimes \phi_\pi^\vee$$

as a categorical kernel over $\mathcal{M}_{\mathrm{Bun}_G} \times \mathcal{M}_{\mathrm{Bun}_G}$. This acts via:

$$\mathcal{U}_\pi * \mathcal{F} := \langle \mathcal{F}, \phi_\pi \rangle \cdot \phi_\pi.$$

Proposition 3.2. *The sheaf-kernel $\mathcal{U}_\pi$ is a categorical Ultra Amplifier in the Langlands–Hecke convolution category. It satisfies:*

$$\mathcal{U}_\pi * \mathcal{U}_\pi \cong \mathcal{U}_\pi, \quad \mathcal{U}_\pi * \mathcal{U}_{\pi'} = 0 \quad \text{if } \pi \neq \pi'.$$

3.3. **Identity Realization over Entropy Period Stacks.** Let $\mathcal{M}_{\mathrm{period}}$ be a derived stack of periods or zeta-motives. Then any perfect spectral decomposition of $\mathcal{F} \in \mathrm{Sh}(\mathcal{M}_{\mathrm{period}})$ via Ultra Amplifiers yields:

$$\mathcal{F} = \sum_{\lambda \in \Lambda} \mathcal{U}_\lambda * \mathcal{F}.$$

Hence, Ultra Amplifier families serve as idempotent resolvers of period sheaves and automorphic cohomology classes.

4. Conclusion and Next Step

In this paper, we have:

- Defined the Ultra Amplifier Family via identity-level convolution properties;
- Constructed concrete arithmetic and stack-theoretic examples;
- Shown their use in RH zeta-reconstruction and Langlands sheaf convolution;
- Proposed a spectral categorification of kernel equality classes.

In the next article, we formalize the entropy–perfect mollifier family, study its inverse convolution with amplifiers, construct the Amplifier–Mollifier (inverse)* diagram, and realize Yang–Langlands integration via entropy convolution functors over moduli stacks.

References

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