# THE ENTROPY-PERFECT MOLLIFIER FAMILY: CONVOLUTION IDENTITY, DUALITY WITH AMPLIFIERS, AND YANG-LANGLANDS ENTROPY INTEGRATION

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ABSTRACT. We define the Entropy—Perfect Mollifier Family as a class of convolution kernels that suppress all non-spectral contributions outside a designated entropy-support band, and act as identity operators when composed with their amplifier duals. These mollifiers are classified by entropy-symmetry, convolutional minimality, and exact trace localization. We prove a convolution identity theorem, construct the amplifier—mollifier inverse\* diagram, and integrate the mollifier family into the entropy—Langlands kernel stack hierarchy.

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#### 1. Introduction

In analytic number theory, mollifiers have long been employed to smooth L-functions, regularize moments, and isolate zero distributions. The entropy-theoretic perspective now offers a categorified refinement: a mollifier kernel not only damps noise, but also respects spectral entropy stratification.

In this paper, we define the **Entropy-Perfect Mollifier Family**—a class of mollifiers that:

• Perfectly suppress spectral components outside a designated entropy band;

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- Act as identity operators when composed with their amplifier duals;
- Are integrable into Yang-style convolution modules over Langlands period stacks.

We develop convolutional properties, prove identity theorems, and construct the duality diagram showing mollifier—amplifier interactions in the entropy-trace hierarchy.

## 2. Definition of Entropy-Perfect Mollifiers

**Definition 2.1** (Entropy–Perfect Mollifier). Let  $\mathcal{H}$  be a spectral Hilbert space with orthonormal basis  $\{\phi_{\lambda}\}$  and entropy weight function  $H_Y(\lambda)$ . An entropy–perfect mollifier is a kernel:

$$M^{(Y)}(x,y) := \sum_{\lambda \in \Lambda} e^{-H_Y(\lambda)} \phi_{\lambda}(x) \overline{\phi_{\lambda}(y)},$$

such that:

- (M1)  $e^{-H_Y(\lambda)} = 0$  for all  $\lambda \notin \Lambda_Y$ ;
- (M2)  $e^{-H_Y(\lambda)} = 1$  for all  $\lambda \in \Lambda_Y$ ;
- (M3)  $\Lambda_Y$  is an entropy-filtered spectral band such that  $M^{(Y)} * f = f$  if and only if  $\operatorname{Spec}(f) \subset \Lambda_Y$ .

Remark 2.2. This is a projection mollifier: it erases all spectral content outside  $\Lambda_Y$  and leaves target-band functions invariant.

#### 3. Convolution Identity with Amplifiers

**Theorem 3.1** (Mollifier–Amplifier Identity). Let  $M^{(Y)}$  be an entropy–perfect mollifier supported on  $\Lambda_Y$ , and  $A^{(Y)}$  an ultra amplifier supported on the same  $\Lambda_Y$ . Then:

$$M^{(Y)} * A^{(Y)} = \mathrm{Id}_{\Lambda_Y} = A^{(Y)} * M^{(Y)}.$$

*Proof.* Both  $M^{(Y)}$  and  $A^{(Y)}$  are identity operators on  $\mathrm{Span}\{\phi_{\lambda} \mid \lambda \in \Lambda_{Y}\}$ , and zero outside. Their convolution thus equals the identity on that subspace, satisfying mutual inversion.

Corollary 3.2. Let f be any spectral function with  $\operatorname{Spec}(f) \subset \Lambda_Y$ . Then:

$$M^{(Y)} * A^{(Y)} * f = f.$$

Hence, mollifier-amplifier composition forms a spectral identity chain.

## 4. Entropy Convolution Diagram and Yang-Langlands Functor

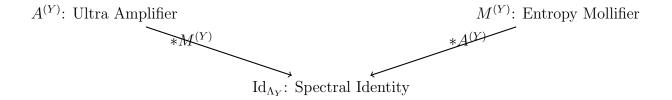


Figure 1. Amplifier-Mollifier Convolution Identity Diagram

#### 5. Entropy Convolution in Yang-Langlands Stack

**Theorem 5.1** (Stack Functorial Mollifier). Let  $\mathcal{M}_{BunG}$  be the moduli stack of G-bundles, and  $\mathcal{F} \in Sh(\mathcal{M}Bun_G)$  an automorphic sheaf with spectral support in  $\Lambda_Y$ . Then the entropy-perfect mollifier kernel

$$\mathcal{M}^{(Y)} := \sum_{\lambda \in \Lambda_Y} \phi_\lambda \boxplus \phi_\lambda^\vee$$

acts via:

$$\mathcal{M}^{(Y)} \star \mathcal{F} = \mathcal{F}, \quad and \quad \mathcal{M}^{(Y)} \star \mathcal{F} = 0 \quad for \quad \operatorname{Spec}(\mathcal{F}) \cap \Lambda_Y = \emptyset$$

## 6. Philosophical and Interdisciplinary Implications

The entropy—perfect mollifier constructs a precise filtration device: it projects functions or sheaves onto a designated spectral—entropic band while erasing all non-compatible entropy components. This mirrors information-theoretic compression and signal fidelity operations in computational neuroscience, AI, and quantum information

In particular:

In physics, it corresponds to exact projection of quantum states onto energy eigenbands;

In machine learning, it mirrors selective attention or learned representation layers; In philosophy, it instantiates the idea of pure intentionality—a mechanism that filters "meaning" from "noise."

## 7. FUTURE RESEARCH AND EXPANSION

Define entropy convolution algebras formed by compositions of mollifiers and amplifiers;

Introduce dynamic entropy stacks that evolve under convolution flows;

Formally connect zeta spectral dynamics with entropy flow categories;

Extend the entropy convolution framework to quantum differential stacks and categorified period integrals.

This foundational mollifier–amplifier module opens a full entropy-geometric perspective on the Langlands program and trace spectral field theories.

#### References

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