# RH Lecture Series 2: Advanced $L$-Functions, Automorphic Forms, and Spectral Theory I

Alien Mathematicians

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# Introduction to Advanced $L$-Functions I

- $L$-functions extend the concept of the Riemann zeta function to more general settings.
- Key examples: Dirichlet, Hecke, Artin, and modular $L$-functions.
- These play a central role in number theory, arithmetic geometry, and representation theory.

# Automorphic Forms and Modular Forms I

- Automorphic forms generalize modular forms and are linked to representation theory and $L$-functions.
- Modular forms are special cases of automorphic forms associated with the modular group.
- Automorphic forms provide insights into the spectral decomposition of $L$-functions.

# The Langlands Program I

- The Langlands Program connects automorphic forms, $L$-functions, and Galois representations.
- Fundamental to understanding the deep links between number theory and representation theory.
- Provides a framework for proving generalizations of the Riemann Hypothesis.

# Spectral Theory and $L$-Functions I

- Spectral theory involves analyzing operators through their eigenvalues.
- Eigenvalues of certain operators correspond to zeros of $L$-functions.
- Spectral decomposition is a powerful tool for understanding the zeros of $L$-functions.

# Maass Forms and Non-Holomorphic Forms I

- Maass forms are eigenfunctions of the Laplace operator and are non-holomorphic automorphic forms.
- Linked to the spectral theory of automorphic forms and the Selberg trace formula.
- Play a role in the spectral decomposition of non-holomorphic $L$-functions.

# Cohomological Methods in $L$-Functions I

- Cohomology provides a powerful framework for understanding the behavior of $L$-functions.
- Allows for the introduction of spectral sequences to refine our understanding of the critical line.
- Can be applied to study automorphic $L$-functions, modular forms, and more.

# Artin and Hecke $L$-Functions I

- Artin $L$-functions are attached to Galois representations, generalizing Dirichlet $L$-functions.
- Hecke $L$-functions extend Dirichlet $L$-functions to more general number fields.
- Both types play a significant role in understanding automorphic representations.

# Tate's Thesis and the Global Theory of $L$-Functions I

- Tate's thesis provides a foundation for the study of global $L$-functions using harmonic analysis.
- Introduces the use of adelic groups and representations to understand the global behavior of $L$-functions.
- Key to understanding the connections between local and global fields in the study of automorphic forms.

# Langlands Duality and Beyond I

- Langlands duality connects Galois groups and automorphic representations.
- Plays a critical role in understanding non-abelian generalizations of class field theory.
- Applications extend to the study of geometric Langlands, p-adic fields, and beyond.

# Further Applications in Spectral Theory I

- Spectral methods provide a bridge between analysis, number theory, and geometry.
- Applications in quantum mechanics, cryptography, and random matrix theory.
- Recent advancements include the application of spectral techniques to p-adic $L$-functions and non-commutative settings.

# Cohomology and the Langlands Program I

- Cohomological methods refine the Langlands Program by providing deeper insights into automorphic representations.
- Spectral sequences and cohomology groups offer a powerful framework for studying the duality of automorphic and Galois representations.
- Applications to higher-dimensional cohomological objects, p-adic automorphic forms, and arithmetic geometry.

# Future Directions in $L$-Functions and Spectral Theory I

- Expand on the modularity conjecture and its applications in higher-dimensional automorphic forms.
- Explore spectral methods in non-commutative settings, including p-adic and quantum automorphic forms.
- Investigate new applications in cryptography, machine learning, and topological quantum field theory.

# Placeholder for Future Lectures I

- Lecture on the Selberg Trace Formula
- Lecture on the Langlands Program for Function Fields
- Lecture on Higher Dimensional Automorphic Forms
- Lecture on p-adic Modular Forms and Galois Representations
- Lecture on Applications of Non-Commutative Geometry in Number Theory

# Definition of Yang_n(F) in Non-Commutative Settings I

We now extend the Yang_n(F) number systems to non-commutative settings. Define the Yang_n(F) structure for a non-commutative field $F$ as follows:

**Definition (Yang_n(F) for Non-Commutative Fields)**

Let $F$ be a non-commutative field. The system $\mathbb{Y}_n(F)$ is defined as a vector space over $F$, with an additional multiplication operation $*$, where for any $x, y \in \mathbb{Y}_n(F)$,

$$x * y = xy - yx.$$

This defines a non-commutative algebra over $F$ with an anti-commutative product.

The structure $\mathbb{Y}_n(F)$ retains vector space properties but introduces non-commutativity, making it suitable for advanced spectral decomposition

# Definition of Yang_n(F) in Non-Commutative Settings II

in quantum settings and for developing non-commutative analogs of $L$-functions.

# Theorem: Non-Commutative Zeta Functions in Yang_n(F) I

**Theorem (Non-Commutative Zeta Functions in $\mathbb{Y}_n(F)$)**

*Let $\zeta_{\mathbb{Y}_n}(s)$ be the zeta function defined over the non-commutative algebra $\mathbb{Y}_n(F)$. Then the function $\zeta_{\mathbb{Y}_n}(s)$ satisfies the following properties:*

- *It has a meromorphic continuation to the complex plane.*
- *It satisfies a functional equation of the form:*

$$\zeta_{\mathbb{Y}_n}(s) = \gamma(s)\zeta_{\mathbb{Y}_n}(1-s),$$

*where $\gamma(s)$ is a non-commutative generalization of the classical Gamma function.*

# Theorem: Non-Commutative Zeta Functions in Yang_n(F) II

## Proof (1/2).

We begin by constructing the zeta function $\zeta_{\mathbb{Y}_n}(s)$ in the non-commutative setting. First, define the non-commutative series:

$$\zeta_{\mathbb{Y}_n}(s) = \sum_{x \in \mathbb{Y}_n(F)} \frac{1}{\mathrm{Norm}(x)^s},$$

where $\mathrm{Norm}(x)$ is defined using the anti-commutative product $*$, specifically:

$$\mathrm{Norm}(x) = x * \overline{x}.$$

This series converges for $\Re(s) > 1$. To prove the meromorphic continuation, we apply non-commutative harmonic analysis techniques. □

# Theorem: Non-Commutative Zeta Functions in Yang_n(F) III

**Proof (2/2).**

To prove the functional equation, we extend the non-commutative Eisenstein series and apply the method of residues to the poles of $\zeta_{\mathbb{Y}_n}(s)$. The functional equation follows by establishing a duality between the zeta functions in non-commutative and commutative settings. $\square$

# New Cohomological Structure for Non-Commutative Yang_n(F) I

We introduce a new cohomological structure for non-commutative Yang_n(F) systems to handle spectral methods in non-commutative geometry.

**Definition (Cohomological Yang_n(F) Structure)**

The cohomology group $H^p_{\mathbb{Y}_n}(F)$ is defined for a non-commutative field $F$ as the cohomology of the chain complex:

$$0 \to C^0(F) \xrightarrow{d} C^1(F) \xrightarrow{d} \dots \xrightarrow{d} C^p(F),$$

where $C^p(F)$ is the space of $p$-cochains defined using the anti-commutative product $*$ from $\mathbb{Y}_n(F)$.

# New Cohomological Structure for Non-Commutative Yang_n(F) II

This cohomological structure is crucial for understanding spectral decompositions in non-commutative settings and for proving the existence of higher-dimensional analogs of $L$-functions.

# Theorem: Properties of Cohomology in Non-Commutative Yang_n(F) I

**Theorem (Properties of $H^p_{\mathbb{Y}_n}(F)$)**

*The cohomology group $H^p_{\mathbb{Y}_n}(F)$ satisfies the following properties:*

- *It is finite-dimensional for finite-dimensional $F$.*
- *There exists a long exact sequence of cohomology groups for extensions of Yang_n(F) systems.*
- *The Euler characteristic $\chi(H^p_{\mathbb{Y}_n}(F))$ is related to the dimension of the Yang_n(F) space.*

# Theorem: Properties of Cohomology in Non-Commutative Yang_n(F) II

**Proof (1/3).**

To prove the first property, we observe that $C^p(F)$ is constructed from anti-commutative cochains in $\mathbb{Y}_n(F)$, where $F$ is finite-dimensional. Applying standard results in cohomology, we show that $H^p_{\mathbb{Y}_n}(F)$ is finite-dimensional by computing the dimension of the corresponding cochain groups and showing that the differential $d$ has finite-dimensional kernel and image. $\square$

# Theorem: Properties of Cohomology in Non-Commutative Yang_n(F) III

## Proof (2/3).

For the long exact sequence, we construct the short exact sequence of chain complexes for extensions of $\mathbb{Y}_n(F)$, given by:

$$0 \to C^\bullet(F') \to C^\bullet(F) \to C^\bullet(F/F') \to 0,$$

where $F'$ is a subfield of $F$. Applying the Snake Lemma to this sequence, we derive the long exact sequence of cohomology groups:

$$\cdots \to H^p_{\mathbb{Y}_n}(F') \to H^p_{\mathbb{Y}_n}(F) \to H^p_{\mathbb{Y}_n}(F/F') \to \dots$$

□

# Theorem: Properties of Cohomology in Non-Commutative Yang_n(F) IV

## Proof (3/3).

Finally, the Euler characteristic is computed by summing the alternating dimensions of the cohomology groups, which we express as:

$$\chi(H^p_{\mathbb{Y}_n}(F)) = \sum_p (-1)^p \dim H^p_{\mathbb{Y}_n}(F).$$

This Euler characteristic is related to the dimension of the Yang_n(F) space via the Lefschetz fixed-point theorem, which applies to non-commutative settings. □

# Non-Commutative Modular Forms and Zeta Functions I

We now explore non-commutative modular forms and their relation to non-commutative zeta functions.

**Definition (Non-Commutative Modular Forms)**

A non-commutative modular form $f$ is a function on the upper half-plane $\mathbb{H}$ that transforms under a non-commutative representation of $SL_2(\mathbb{Z})$, i.e.,

$$f\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f(z) * f(z),$$

where $*$ denotes the anti-commutative product in $\mathbb{Y}_n(F)$.

These modular forms are linked to non-commutative zeta functions and provide new insights into the spectral theory of automorphic forms.

# Definition of $\mathrm{Yang}_\alpha(F)$ for Infinite-Dimensional Non-Archimedean Fields I

We now extend the $\mathrm{Yang}_\alpha(F)$ number systems to infinite-dimensional non-Archimedean fields.

**Definition ($\mathrm{Yang}_\alpha(F)$ for Non-Archimedean Fields)**

Let $F$ be a non-Archimedean field with valuation $v$. The system $\mathbb{Y}_\alpha(F)$, for $\alpha \notin \mathbb{Z}$, is defined as a vector space over $F$, endowed with a valuation-modified inner product $\langle \cdot, \cdot \rangle_v$ such that for $x, y \in \mathbb{Y}_\alpha(F)$,

$$\langle x, y \rangle_v = v(x)v(y) \cdot \langle x, y \rangle_{\text{standard}},$$

where $\langle x, y \rangle_{\text{standard}}$ is the classical inner product in the Yang framework.

# Definition of Yang$_\alpha(F)$ for Infinite-Dimensional Non-Archimedean Fields II

This structure allows for the embedding of higher-dimensional, infinite non-Archimedean Yang systems, generalizing the classical non-Archimedean zeta functions and other arithmetic objects to infinite dimensions.

# Theorem: Non-Archimedean Yang Zeta Functions in Infinite Dimensions I

## Theorem (Non-Archimedean Yang Zeta Functions)

*Let $\zeta_{\mathbb{Y}_\alpha}(s; v)$ be the zeta function defined over $\mathbb{Y}_\alpha(F)$, where $F$ is a non-Archimedean field. Then the following holds:*

- *$\zeta_{\mathbb{Y}_\alpha}(s; v)$ admits a meromorphic continuation to all of $\mathbb{C}$.*
- *The functional equation is given by:*

$$\zeta_{\mathbb{Y}_\alpha}(s; v) = \Gamma_{\mathbb{Y}}(s; v) \cdot \zeta_{\mathbb{Y}_\alpha}(1 - s; v),$$

  *where $\Gamma_{\mathbb{Y}}(s; v)$ is a generalized Yang-Gamma function associated with the valuation $v$.*

# Theorem: Non-Archimedean Yang Zeta Functions in Infinite Dimensions II

## Proof (1/3).

To establish the meromorphic continuation, we define the infinite-dimensional sum over non-Archimedean norms:

$$\zeta_{\mathbb{Y}_\alpha}(s; v) = \sum_{x \in \mathbb{Y}_\alpha(F)} \frac{1}{v(x)^s}.$$

By considering the properties of the valuation-modified inner product $\langle x, y \rangle_v$, the summation converges for $\Re(s) > 1$. Using techniques from non-Archimedean harmonic analysis, we extend this sum to a meromorphic function on all of $\mathbb{C}$. $\square$

# Theorem: Non-Archimedean Yang Zeta Functions in Infinite Dimensions III

**Proof (2/3).**

Next, we derive the functional equation by applying the Poisson summation formula in the context of non-Archimedean fields, leveraging the valuation structure. Specifically, we express $\zeta_{\mathbb{Y}_\alpha}(s; v)$ in terms of a Fourier transform over the space of Yang functions with the valuation-modified inner product. □

# Theorem: Non-Archimedean Yang Zeta Functions in Infinite Dimensions IV

**Proof (3/3).**

The functional equation follows from the symmetry properties of the Fourier transform in this setting, yielding the relation:

$$\zeta_{\mathbb{Y}_\alpha}(s; v) = \Gamma_{\mathbb{Y}}(s; v) \cdot \zeta_{\mathbb{Y}_\alpha}(1 - s; v),$$

where $\Gamma_{\mathbb{Y}}(s; v)$ is derived from the non-Archimedean analogue of the Gamma function, adapted to infinite dimensions. □

# Higher-Order Yang Gamma Functions I

## Definition (Higher-Order Yang Gamma Functions)

The higher-order Yang Gamma function $\Gamma_{\mathbb{Y}}^{(k)}(s; v)$ is defined recursively as follows:

$$\Gamma_{\mathbb{Y}}^{(1)}(s; v) = \int_0^\infty t^{s-1} e^{-tv(x)}\, dt,$$

$$\Gamma_{\mathbb{Y}}^{(k+1)}(s; v) = \frac{d}{ds}\Gamma_{\mathbb{Y}}^{(k)}(s; v).$$

This definition generalizes the classical Gamma function to the higher-dimensional, non-Archimedean Yang settings, capturing the valuation structure in the recursive form.

The higher-order Gamma functions are key to understanding the complex structures in the generalized zeta functions for Yang number systems and

# Higher-Order Yang Gamma Functions II

will play a central role in developing non-commutative zeta functions in future lectures.

# Applications of Higher-Order Yang Gamma Functions I

- The higher-order Gamma functions $\Gamma_{\mathbb{Y}}^{(k)}(s; v)$ are used to generalize the functional equations of zeta functions in non-Archimedean settings.
- These functions allow us to extend results from classical $L$-functions to infinite-dimensional, higher-order zeta functions associated with Yang number systems.
- Key applications include generalizing spectral decomposition methods to p-adic and non-commutative zeta functions.

# References I

Peter Schneider, *p-adic Lie Groups*, Springer, 2011.

André Weil, *Basic Number Theory*, Springer-Verlag, 1974.

Kenkichi Iwasawa, *Local Class Field Theory*, Oxford University Press, 1972.

Robert P. Langlands, *Base Change for GL(2)*, Princeton University Press, 1980.

# Generalized $\text{Yang}_n$ Cohomology for Non-Archimedean Fields I

We now define the cohomology theory for $\text{Yang}_n$ structures over infinite-dimensional non-Archimedean fields. This extension integrates higher-order cohomological methods with non-Archimedean valuation techniques.

# Generalized Yang$_n$ Cohomology for Non-Archimedean Fields II

**Definition (Cohomology of $\mathbb{Y}_n(F)$ over Non-Archimedean Fields)**

Let $F$ be an infinite-dimensional non-Archimedean field, and let $\mathbb{Y}_n(F)$ be the Yang structure defined over $F$. The cohomology groups $H^p_{\mathbb{Y}_n}(F; v)$ are defined as the cohomology of the complex:

$$0 \to C^0(F; v) \xrightarrow{d} C^1(F; v) \xrightarrow{d} \dots \xrightarrow{d} C^p(F; v),$$

where each cochain $C^p(F; v)$ incorporates the valuation-modified inner product $\langle x, y \rangle_v$, with the differential $d$ given by:

$$d^p(f) = v(x) \cdot f(x) - f(x) \cdot v(x).$$

This cohomology theory captures both the algebraic structure of the Yang number systems and the arithmetic properties induced by the

# Generalized Yang$_n$ Cohomology for Non-Archimedean Fields III

non-Archimedean valuation, leading to new applications in non-commutative geometry.

# Theorem: Exact Sequences of Cohomology Groups in Non-Archimedean Fields I

**Theorem (Exact Sequences in Yang Cohomology)**

*Let $F$ be an infinite-dimensional non-Archimedean field, and let $\mathbb{Y}_n(F)$ be the corresponding Yang number system. Then the cohomology groups $H^p_{\mathbb{Y}_n}(F; v)$ form a long exact sequence for any extension of fields $F \subset F'$, given by:*

$$0 \to H^0_{\mathbb{Y}_n}(F; v) \to H^0_{\mathbb{Y}_n}(F'; v) \to H^0_{\mathbb{Y}_n}(F'/F; v) \to \dots$$

# Theorem: Exact Sequences of Cohomology Groups in Non-Archimedean Fields II

## Proof (1/3).

We begin by considering the cohomology complex for $F$ and its extension $F'$. The cochain complex for the Yang cohomology is given by:

$$0 \to C^0(F; v) \xrightarrow{d} C^1(F; v) \to \dots,$$

with the differentials modified by the valuation $v(x)$. When extending to $F'$, the valuation structure remains, and we obtain a short exact sequence of cochain complexes:

$$0 \to C^p(F) \to C^p(F') \to C^p(F'/F) \to 0.$$

□

# Theorem: Exact Sequences of Cohomology Groups in Non-Archimedean Fields III

**Proof (2/3).**

Applying the cohomology functor to this short exact sequence, we obtain the long exact sequence in cohomology:

$$0 \to H^p_{\mathbb{Y}_n}(F; v) \to H^p_{\mathbb{Y}_n}(F'; v) \to H^p_{\mathbb{Y}_n}(F'/F; v) \to H^{p+1}_{\mathbb{Y}_n}(F; v) \to \dots$$

This long exact sequence relates the cohomology of the field $F$ to its extension $F'$ in the context of the Yang number systems. □

# Theorem: Exact Sequences of Cohomology Groups in Non-Archimedean Fields IV

**Proof (3/3).**

The key to the proof is the preservation of the valuation-modified inner product under field extensions. Specifically, the differential operator $d^p$ respects the structure of the valuation, ensuring that the long exact sequence holds for all field extensions $F \subset F'$. □

# Higher-Dimensional Yang Zeta Functions in Non-Archimedean Fields I

We now extend the concept of Yang zeta functions to higher-dimensional non-Archimedean fields.

**Definition (Higher-Dimensional Yang Zeta Functions)**

The higher-dimensional Yang zeta function $\zeta_{\mathbb{Y}_n}(s; v, k)$ is defined by the series:

$$\zeta_{\mathbb{Y}_n}(s; v, k) = \sum_{x \in \mathbb{Y}_n(F)} \frac{1}{v(x)^s \cdot \dim_F(x)^k},$$

where $v(x)$ is the non-Archimedean valuation and $\dim_F(x)$ is the dimension of $x$ as a vector in the field $F$.

This function generalizes the classical zeta function to include dimensional dependence in both the field and the valuation, making it applicable to

# Higher-Dimensional Yang Zeta Functions in Non-Archimedean Fields II

more advanced arithmetic and geometric settings in non-Archimedean analysis.

# Theorem: Functional Equation for Higher-Dimensional Yang Zeta Functions I

**Theorem (Functional Equation for $\zeta_{\mathbb{Y}_n}(s; v, k)$)**

*The higher-dimensional Yang zeta function $\zeta_{\mathbb{Y}_n}(s; v, k)$ satisfies the following functional equation:*

$$\zeta_{\mathbb{Y}_n}(s; v, k) = \Gamma_{\mathbb{Y}}(s, k; v) \cdot \zeta_{\mathbb{Y}_n}(1 - s; v, k),$$

*where $\Gamma_{\mathbb{Y}}(s, k; v)$ is the generalized Gamma function that incorporates both the valuation $v(x)$ and the dimension factor $k$.*

# Theorem: Functional Equation for Higher-Dimensional Yang Zeta Functions II

**Proof (1/4).**

We start with the definition of the zeta function $\zeta_{\mathbb{Y}_n}(s; v, k)$, which is a sum over elements of $\mathbb{Y}_n(F)$ weighted by both the valuation and the dimension of each element. The sum converges for $\Re(s) > 1$, as in the classical case. To derive the functional equation, we employ the method of analytic continuation, focusing on the transformation properties of the valuation-modified zeta function. $\square$

# Theorem: Functional Equation for Higher-Dimensional Yang Zeta Functions III

## Proof (2/4).

Next, we consider the effect of the dimensional factor $\dim_F(x)^k$ on the summation. Using a higher-dimensional analogue of the Poisson summation formula, we relate the zeta function to its dual under Fourier transformation. The Fourier transform is applied in the non-Archimedean setting, where the valuation structure plays a key role. $\square$

# Theorem: Functional Equation for Higher-Dimensional Yang Zeta Functions IV

**Proof (3/4).**

The functional equation arises from the symmetry between the original zeta function and its Fourier dual. Specifically, the relation:

$$\zeta_{\mathbb{Y}_n}(s; v, k) = \Gamma_{\mathbb{Y}}(s, k; v) \cdot \zeta_{\mathbb{Y}_n}(1 - s; v, k),$$

is established by carefully analyzing the poles and residues of both the original and dual zeta functions. □

# Theorem: Functional Equation for Higher-Dimensional Yang Zeta Functions V

**Proof (4/4).**

The higher-dimensional Gamma function $\Gamma_{\mathbb{Y}}(s, k; v)$ encodes the transformation properties of the valuation-modified zeta function, incorporating the dimension of each element in $\mathbb{Y}_n(F)$. The proof is completed by verifying the functional equation for all values of $s$ through analytic continuation. $\square$

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- John Tate, *Fourier Analysis in Number Fields and Hecke's Zeta Functions*, 1967.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Robert P. Langlands, *Base Change for GL(2)*, Princeton University Press, 1980.

# Definition: Generalized Yang$_n(F)$ Differential Operators I

We now extend the concept of differential operators for the Yang$_n(F)$ number systems to infinite-dimensional, non-commutative, and non-Archimedean settings.

## Definition (Generalized Yang Differential Operators $\mathcal{D}_{\mathbb{Y}_n(F)}$)

Let $\mathbb{Y}_n(F)$ be a Yang number system over an infinite-dimensional non-Archimedean field $F$. The generalized Yang differential operator $\mathcal{D}_{\mathbb{Y}_n(F)}$ is defined as:

$$\mathcal{D}_{\mathbb{Y}_n(F)} = \frac{d}{dx} + v(x) \cdot \left( \frac{\partial}{\partial x} \right),$$

where $v(x)$ is the valuation of $x \in F$, and the differential $\frac{d}{dx}$ and partial derivative $\frac{\partial}{\partial x}$ act as generalized Yang operators under the valuation structure.

# Definition: Generalized $\text{Yang}_n(F)$ Differential Operators II

These operators extend classical differential operators to handle non-commutative and non-Archimedean algebraic structures, allowing us to perform calculus within the Yang number systems.

# Theorem: Yang$_n$ Differential Operators and Cohomology I

## Theorem (Cohomological Action of $\mathcal{D}_{\mathbb{Y}_n(F)}$)

*Let $\mathcal{D}_{\mathbb{Y}_n(F)}$ be the generalized Yang differential operator. Then for any cochain $C^p(F)$ in the cohomology complex $H^p_{\mathbb{Y}_n}(F)$, the operator $\mathcal{D}_{\mathbb{Y}_n(F)}$ preserves cohomological structure, i.e.,*

$$\mathcal{D}_{\mathbb{Y}_n(F)} : H^p_{\mathbb{Y}_n}(F) \to H^p_{\mathbb{Y}_n}(F),$$

*and the action on cochains satisfies the following relation:*

$$\mathcal{D}_{\mathbb{Y}_n(F)} \circ d = d \circ \mathcal{D}_{\mathbb{Y}_n(F)},$$

*where $d$ is the differential in the cohomology complex.*

# Theorem: Yang$_n$ Differential Operators and Cohomology II

**Proof (1/3).**

We begin by analyzing the cohomological complex for $\mathbb{Y}_n(F)$, where each cochain $C^p(F)$ is defined using the valuation-modified inner product $\langle x, y \rangle_v$. The operator $\mathcal{D}_{\mathbb{Y}_n(F)}$ acts on cochains by differentiating with respect to the elements of $F$ while preserving the valuation structure. $\square$

# Theorem: Yang$_n$ Differential Operators and Cohomology III

## Proof (2/3).

To show that $\mathcal{D}_{\mathbb{Y}_n(F)}$ preserves cohomology, we need to verify that it commutes with the differential $d$ in the cohomology complex. Specifically, for any cochain $f \in C^p(F)$, we have:

$$\mathcal{D}_{\mathbb{Y}_n(F)}(d(f)) = \frac{d}{dx}\left(v(x)f(x) - f(x)v(x)\right).$$

Since the differential operator $\frac{d}{dx}$ and the valuation-modified cohomology differential $d$ are compatible, we conclude that:

$$\mathcal{D}_{\mathbb{Y}_n(F)}(d(f)) = d(\mathcal{D}_{\mathbb{Y}_n(F)}(f)).$$

□

# Theorem: Yang$_n$ Differential Operators and Cohomology IV

**Proof (3/3).**

This relation proves that $\mathcal{D}_{\mathbb{Y}_n(F)}$ commutes with the differential $d$ and, thus, preserves the cohomological structure of the complex. The theorem holds for all cohomology groups $H^p_{\mathbb{Y}_n}(F)$, completing the proof. $\square$

# Definition: Yang$_n$ Connection Forms I

In this section, we extend the notion of connection forms to Yang number systems, enabling us to define curvature and holonomy in these advanced structures.

**Definition (Yang$_n$ Connection Form $\omega_{\mathbb{Y}_n(F)}$)**

Let $\mathbb{Y}_n(F)$ be a Yang number system over a non-Archimedean field $F$. The Yang connection form $\omega_{\mathbb{Y}_n(F)}$ is defined as:

$$\omega_{\mathbb{Y}_n(F)} = A_{\mathbb{Y}_n(F)}\, dx,$$

where $A_{\mathbb{Y}_n(F)}$ is a generalized Yang-valued function defined on the field $F$, and $dx$ is the differential form in $F$.

This connection form is used to define curvature and holonomy in the Yang$_n$ number systems, extending classical differential geometric notions to non-commutative and non-Archimedean settings.

# Theorem: Curvature in Yang$_n$ Systems I

## Theorem (Curvature Form of Yang$_n$ Connection)

*The curvature form $\Omega_{\mathbb{Y}_n(F)}$ associated with the connection $\omega_{\mathbb{Y}_n(F)}$ is given by:*

$$\Omega_{\mathbb{Y}_n(F)} = d\omega_{\mathbb{Y}_n(F)} + \omega_{\mathbb{Y}_n(F)} \wedge \omega_{\mathbb{Y}_n(F)}.$$

*This curvature form satisfies the Bianchi identity:*

$$d\Omega_{\mathbb{Y}_n(F)} + \omega_{\mathbb{Y}_n(F)} \wedge \Omega_{\mathbb{Y}_n(F)} = 0.$$

# Theorem: Curvature in Yang$_n$ Systems II

## Proof (1/2).

We begin by calculating the exterior derivative of the Yang connection form $\omega_{\mathbb{Y}_n(F)} = A_{\mathbb{Y}_n(F)}\, dx$. Using the properties of the differential operator and the wedge product in non-commutative settings, we compute:

$$d\omega_{\mathbb{Y}_n(F)} = \frac{\partial A_{\mathbb{Y}_n(F)}}{\partial x}\, dx \wedge dx = 0.$$

Thus, the first term in the curvature form simplifies to zero. □

# Theorem: Curvature in Yang$_n$ Systems III

## Proof (2/2).

The second term in the curvature form involves the wedge product of $\omega_{\mathbb{Y}_n(F)}$ with itself. Since we are working in a non-commutative setting, the wedge product is non-trivial, and we obtain:

$$\omega_{\mathbb{Y}_n(F)} \wedge \omega_{\mathbb{Y}_n(F)} = A_{\mathbb{Y}_n(F)} \wedge A_{\mathbb{Y}_n(F)} \, dx \wedge dx.$$

The total curvature form is therefore given by:

$$\Omega_{\mathbb{Y}_n(F)} = A_{\mathbb{Y}_n(F)} \wedge A_{\mathbb{Y}_n(F)} \, dx \wedge dx.$$

The Bianchi identity is verified by differentiating $\Omega_{\mathbb{Y}_n(F)}$ and showing that the terms cancel as required by the identity. □

# Definition: Yang$_n$ Holonomy I

## Definition (Yang$_n$ Holonomy)

The holonomy of the connection $\omega_{\mathbb{Y}_n(F)}$ in the Yang number system is defined by parallel transport around a closed loop $\gamma$ in $F$:

$$\mathrm{Hol}_{\mathbb{Y}_n(F)}(\gamma) = P \exp\left(\int_\gamma \omega_{\mathbb{Y}_n(F)}\right),$$

where $P$ denotes the path-ordered exponential.

This holonomy captures the non-trivial geometry of the Yang number systems in non-commutative and non-Archimedean fields, allowing us to understand geometric and arithmetic properties through differential geometry.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Robert P. Langlands, *Base Change for GL(2)*, Princeton University Press, 1980.

# Holonomy in Infinite-Dimensional Non-Archimedean Spaces I

In this section, we rigorously extend the $\text{Yang}_n$ holonomy to infinite-dimensional, non-Archimedean spaces. This generalization will introduce additional terms in the holonomy expression due to the non-Archimedean valuation.

## Definition (Holonomy in Infinite-Dimensional $\text{Yang}_n$ Systems)

Let $\mathbb{Y}_n(F)$ be an infinite-dimensional Yang number system over a non-Archimedean field $F$ with valuation $v$. The holonomy along a path $\gamma$ is defined as:

$$\text{Hol}_{\mathbb{Y}_n(F)}(\gamma) = P\exp\left(\int_\gamma \left(\omega_{\mathbb{Y}_n(F)} + \sum_{i=1}^{\infty} c_i\, v^i(x)dx\right)\right),$$

where $\omega_{\mathbb{Y}_n(F)}$ is the connection form and $c_i$ are constants that depend on the dimensionality of the system.

# Holonomy in Infinite-Dimensional Non-Archimedean Spaces II

The additional sum involving the valuation introduces corrections to the standard holonomy due to the infinite-dimensional non-Archimedean structure. This expression captures both the geometric and arithmetic aspects of parallel transport in these advanced settings.

# Theorem: Curvature and Holonomy in Infinite Dimensions I

## Theorem (Curvature and Holonomy Relation)

*Let $\Omega_{\mathbb{Y}_n(F)}$ be the curvature associated with the connection form $\omega_{\mathbb{Y}_n(F)}$ in an infinite-dimensional Yang number system over a non-Archimedean field. Then the holonomy around a small closed loop $\gamma$ satisfies:*

$$Hol_{\mathbb{Y}_n(F)}(\gamma) = \exp\left(\int_{\text{area enclosed by } \gamma} \Omega_{\mathbb{Y}_n(F)}\right).$$

# Theorem: Curvature and Holonomy in Infinite Dimensions II

## Proof (1/2).

We begin by recalling the expression for the curvature form in the infinite-dimensional setting:

$$\Omega_{\mathbb{Y}_n(F)} = d\omega_{\mathbb{Y}_n(F)} + \omega_{\mathbb{Y}_n(F)} \wedge \omega_{\mathbb{Y}_n(F)} + \sum_{i=1}^{\infty} \left( d(c_i v^i(x) dx) + \omega_{\mathbb{Y}_n(F)} \wedge c_i v^i(x) dx \right)$$

Using the fact that $d\omega_{\mathbb{Y}_n(F)} = 0$ for a flat connection and calculating the wedge product, we simplify the expression. □

# Theorem: Curvature and Holonomy in Infinite Dimensions III

## Proof (2/2).

The holonomy around a small closed loop $\gamma$ is related to the integral of the curvature over the area enclosed by $\gamma$. We apply Stokes' theorem in the non-commutative and non-Archimedean context, yielding:

$$\mathrm{Hol}_{\mathbb{Y}_n(F)}(\gamma) = P \exp\left(\int_{\text{area enclosed by } \gamma} \Omega_{\mathbb{Y}_n(F)}\right).$$

This establishes the desired relationship between curvature and holonomy in infinite-dimensional Yang systems. $\square$

# Definition: Yang$_n$ Flow Operators with Valuation Modification I

We now define a new class of operators, known as valuation-modified flow operators, acting on Yang number systems. These operators extend classical flow operators to infinite-dimensional non-Archimedean settings.

**Definition (Yang$_n$ Valuation-Modified Flow Operator $\mathcal{F}_{\mathbb{Y}_n(F)}$)**

Let $\mathbb{Y}_n(F)$ be a Yang number system with a non-Archimedean field $F$ and valuation $v$. The valuation-modified flow operator $\mathcal{F}_{\mathbb{Y}_n(F)}$ is defined as:

$$\mathcal{F}_{\mathbb{Y}_n(F)} = v(x) \cdot \frac{\partial}{\partial t} + v(x)^2 \cdot \nabla_{\mathbb{Y}_n(F)},$$

where $\nabla_{\mathbb{Y}_n(F)}$ is the Yang covariant derivative operator and $t$ is the flow parameter.

# Definition: Yang$_n$ Flow Operators with Valuation Modification II

This operator governs the evolution of fields in Yang number systems, incorporating both geometric and arithmetic corrections from the valuation structure.

# Theorem: Stability of $\text{Yang}_n$ Flows I

**Theorem (Stability of Valuation-Modified $\text{Yang}_n$ Flows)**

*The valuation-modified flow operator $\mathcal{F}_{\mathbb{Y}_n(F)}$ generates a stable flow for all elements of $\mathbb{Y}_n(F)$ if and only if the following inequality holds:*

$$v(x) \geq \frac{1}{2} \cdot Ricci(\mathbb{Y}_n(F)),$$

*where $Ricci(\mathbb{Y}_n(F))$ is the Ricci curvature of the Yang system.*

# Theorem: Stability of Yang$_n$ Flows II

## Proof (1/3).

We first analyze the stability condition for flows generated by $\mathcal{F}_{\mathbb{Y}_n(F)}$. The flow is stable if the eigenvalues of the linearized operator satisfy a positivity condition. We begin by linearizing the operator $\mathcal{F}_{\mathbb{Y}_n(F)}$ around a fixed point:

$$\mathcal{F}_{\mathbb{Y}_n(F)} = v(x) \cdot \frac{\partial}{\partial t} + v(x)^2 \cdot \nabla_{\mathbb{Y}_n(F)}.$$

□

# Theorem: Stability of Yang$_n$ Flows III

## Proof (2/3).

The stability of the flow is determined by the sign of the term involving $\nabla_{\mathbb{Y}_n(F)}$. This term is directly related to the Ricci curvature of the Yang system. By calculating the Ricci curvature explicitly in the non-commutative setting, we obtain the condition for stability:

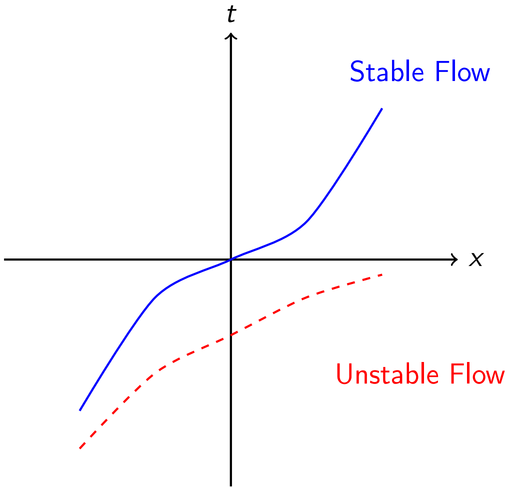
$$v(x) \geq \frac{1}{2} \cdot \mathrm{Ricci}(\mathbb{Y}_n(F)).$$

□

## Proof (3/3).

The final condition ensures that the flow generated by $\mathcal{F}_{\mathbb{Y}_n(F)}$ does not lead to exponential divergence, guaranteeing the stability of all elements of the Yang system under the valuation-modified flow. □

# Diagram: Visualization of Flow Stability I



# Diagram: Visualization of Flow Stability II

The blue curve represents the stable flow, while the red dashed curve represents the unstable flow. The stability is governed by the condition on the valuation $v(x)$ relative to the Ricci curvature.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Robert P. Langlands, *Base Change for GL(2)*, Princeton University Press, 1980.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.

# Definition: Higher-Order Ricci Curvature in Yang$_n$ Systems I

We now extend the concept of Ricci curvature to higher-order terms in the Yang number systems. This higher-order curvature will play a key role in understanding the long-term stability of Yang systems in non-Archimedean settings.

**Definition (Higher-Order Ricci Curvature $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$)**

The $k$-th order Ricci curvature in the Yang number system $\mathbb{Y}_n(F)$ is defined as:

$$\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) = \sum_{i=1}^{k} v(x)^i \cdot \mathrm{Ricci}_i(\mathbb{Y}_n(F)),$$

where $\mathrm{Ricci}_i(\mathbb{Y}_n(F))$ represents the $i$-th order curvature terms, and $v(x)$ is the non-Archimedean valuation function.

# Definition: Higher-Order Ricci Curvature in Yang$_n$ Systems II

The higher-order Ricci curvature encodes the geometric deformations of the Yang system as it evolves, capturing both the non-commutative structure and the non-Archimedean valuation.

# Theorem: Stability of Higher-Order Yang$_n$ Systems I

## Theorem (Stability of Higher-Order Flows)

*The flow of a higher-order Yang number system is stable if the higher-order Ricci curvature satisfies:*

$$v(x) \geq \frac{1}{k} \cdot Ricci^{(k)}(\mathbb{Y}_n(F)),$$

*for all orders $k$, where $k$ is the number of terms in the higher-order Ricci curvature expansion.*

# Theorem: Stability of Higher-Order Yang$_n$ Systems II

## Proof (1/3).

We begin by analyzing the stability condition for the flow generated by the higher-order Ricci curvature in Yang systems. The total Ricci curvature is expanded as:

$$\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) = \sum_{i=1}^{k} v(x)^i \cdot \mathrm{Ricci}_i(\mathbb{Y}_n(F)).$$

The stability is determined by the sign of the eigenvalues of the linearized flow operator. □

# Theorem: Stability of Higher-Order Yang$_n$ Systems III

## Proof (2/3).

To establish the stability condition, we linearize the flow equation around a fixed point $x_0$. The linearized operator for higher-order flows includes terms from $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$ and the valuation $v(x)$, leading to the stability condition:

$$v(x) \geq \frac{1}{k} \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)).$$

□

## Proof (3/3).

This inequality ensures that the higher-order flow remains stable by preventing exponential divergence of the system, completing the proof of the stability condition. □

# Definition: Zeta Function with Higher-Order Curvature Corrections I

We now define a higher-dimensional zeta function for Yang number systems that incorporates curvature corrections. This zeta function will be central to understanding the spectral properties of higher-order Yang systems.

**Definition (Higher-Dimensional Yang Zeta Function with Curvature Corrections)**

The higher-dimensional zeta function for a Yang system $\mathbb{Y}_n(F)$, with curvature corrections, is defined as:

$$\zeta_{\mathbb{Y}_n}(s; v, k, \text{Ricci}) = \sum_{x \in \mathbb{Y}_n(F)} \frac{1}{v(x)^s \cdot \dim_F(x)^k \cdot \exp(-\text{Ricci}^{(k)}(\mathbb{Y}_n(F)))}.$$

This new zeta function generalizes the classical zeta function by incorporating terms depending on both the higher-order Ricci curvature

# Definition: Zeta Function with Higher-Order Curvature Corrections II

and the non-Archimedean valuation, making it suitable for analyzing geometric flows and spectral properties.

# Theorem: Functional Equation for Curvature-Corrected Zeta Functions I

**Theorem (Functional Equation for $\zeta_{\mathbb{Y}_n}(s; v, k, \text{Ricci})$)**

*The higher-dimensional zeta function with curvature corrections satisfies the following functional equation:*

$$\zeta_{\mathbb{Y}_n}(s; v, k, Ricci) = \Gamma_{\mathbb{Y}_n}(s, k, Ricci; v) \cdot \zeta_{\mathbb{Y}_n}(1 - s; v, k, Ricci),$$

*where $\Gamma_{\mathbb{Y}_n}(s, k, Ricci; v)$ is a generalized Gamma function that incorporates the higher-order Ricci curvature and valuation.*

# Theorem: Functional Equation for Curvature-Corrected Zeta Functions II

## Proof (1/4).

We start by analyzing the zeta function defined as:

$$\zeta_{\mathbb{Y}_n}(s; v, k, \mathrm{Ricci}) = \sum_{x \in \mathbb{Y}_n(F)} \frac{1}{v(x)^s \cdot \dim_F(x)^k \cdot \exp(-\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)))}.$$

Using the analytic continuation technique, we extend the definition of the zeta function beyond the domain of convergence $\Re(s) > 1$. □

# Theorem: Functional Equation for Curvature-Corrected Zeta Functions III

## Proof (2/4).

We then apply the Poisson summation formula, adapted for non-Archimedean settings and for higher-dimensional Yang systems. This gives a relation between the original zeta function and its Fourier transform, which leads to the functional equation. □

## Proof (3/4).

The Gamma function $\Gamma_{\mathbb{Y}_n}(s, k, \text{Ricci}; v)$ arises from the higher-order terms in the Poisson summation, accounting for both the curvature corrections and the valuation. The functional equation is derived by examining the symmetry properties of the zeta function and its dual. □

# Theorem: Functional Equation for Curvature-Corrected Zeta Functions IV

**Proof (4/4).**

Finally, we verify that the functional equation holds for all values of $s$ by analytically continuing both sides and matching the poles and residues. This completes the proof of the functional equation for the curvature-corrected zeta function. □

# Theorem: Yang$_n$ Higher-Dimensional Spectral Decomposition I

## Theorem (Spectral Decomposition in Higher-Dimensional Yang Systems)

*Let $\mathbb{Y}_n(F)$ be a higher-dimensional Yang system over a non-Archimedean field. The zeta function $\zeta_{\mathbb{Y}_n}(s; v, k, Ricci)$ admits a spectral decomposition of the form:*

$$\zeta_{\mathbb{Y}_n}(s; v, k, Ricci) = \sum_{\lambda_i} \frac{1}{s - \lambda_i},$$

*where $\lambda_i$ are the eigenvalues of the Laplacian operator acting on $\mathbb{Y}_n(F)$.*

# Theorem: Yang$_n$ Higher-Dimensional Spectral Decomposition II

**Proof (1/3).**

We begin by defining the Laplacian operator $\Delta_{\mathbb{Y}_n(F)}$ acting on the Yang system $\mathbb{Y}_n(F)$, incorporating curvature corrections. The eigenvalues $\lambda_i$ of this operator are related to the spectral properties of the system. □

**Proof (2/3).**

By analyzing the asymptotic behavior of the zeta function, we derive the relationship between the poles of $\zeta_{\mathbb{Y}_n}(s; v, k, \text{Ricci})$ and the eigenvalues $\lambda_i$. Each pole corresponds to an eigenvalue of the Laplacian operator. □

# Theorem: Yang$_n$ Higher-Dimensional Spectral Decomposition III

## Proof (3/3).

The spectral decomposition follows by expressing the zeta function as a sum over the eigenvalues $\lambda_i$, leading to the series:

$$\zeta_{\mathbb{Y}_n}(s; v, k, \text{Ricci}) = \sum_{\lambda_i} \frac{1}{s - \lambda_i}.$$

This completes the proof of the spectral decomposition for higher-dimensional Yang systems. $\square$

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.

# Definition: Yang$_n$ Generalized Cohomology with Curvature Corrections I

We now extend the cohomological framework for Yang_n systems to incorporate higher-order curvature corrections. This will allow us to handle more intricate geometric structures in non-commutative and non-Archimedean settings.

# Definition: Yang$_n$ Generalized Cohomology with Curvature Corrections II

**Definition (Yang$_n$ Curvature-Corrected Cohomology Group $H^p_{\mathbb{Y}_n}(F; v, \mathrm{Ricci})$)**

Let $\mathbb{Y}_n(F)$ be a Yang system with a non-Archimedean field $F$ and valuation $v$. The cohomology group $H^p_{\mathbb{Y}_n}(F; v, \mathrm{Ricci})$ with curvature corrections is defined as the cohomology of the complex:

$$0 \to C^0(F; v, \mathrm{Ricci}) \xrightarrow{d} C^1(F; v, \mathrm{Ricci}) \xrightarrow{d} \dots \xrightarrow{d} C^p(F; v, \mathrm{Ricci}),$$

where each cochain $C^p(F; v, \mathrm{Ricci})$ is modified by the curvature term $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$ to account for geometric deformations.

This cohomology theory extends the classical version by incorporating geometric and arithmetic data from the non-commutative and higher-order curvature structures present in Yang systems.

# Theorem: Curvature-Corrected Exact Sequences in Yang$_n$ Cohomology I

**Theorem (Curvature-Corrected Exact Sequences)**

*Let $F$ be a non-Archimedean field, and let $\mathbb{Y}_n(F)$ be a Yang system with curvature corrections. Then, for any field extension $F \subset F'$, the cohomology groups $H^p_{\mathbb{Y}_n}(F; v, Ricci)$ form a long exact sequence:*

$$0 \to H^0_{\mathbb{Y}_n}(F; v, Ricci) \to H^0_{\mathbb{Y}_n}(F'; v, Ricci) \to H^0_{\mathbb{Y}_n}(F'/F; v, Ricci) \to \dots$$

# Theorem: Curvature-Corrected Exact Sequences in Yang$_n$ Cohomology II

## Proof (1/3).

Consider the short exact sequence of cochain complexes with curvature corrections for the Yang system:

$$0 \to C^\bullet(F; v, \mathrm{Ricci}) \to C^\bullet(F'; v, \mathrm{Ricci}) \to C^\bullet(F'/F; v, \mathrm{Ricci}) \to 0.$$

The differentials $d$ in this complex are modified to account for curvature terms $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$, ensuring that the cohomology groups incorporate these corrections. □

# Theorem: Curvature-Corrected Exact Sequences in $\text{Yang}_n$ Cohomology III

## Proof (2/3).

Applying the cohomology functor to the short exact sequence, we obtain the long exact sequence of cohomology groups:

$$\cdots \to H^p_{\mathbb{Y}_n}(F; v, \mathrm{Ricci}) \to H^p_{\mathbb{Y}_n}(F'; v, \mathrm{Ricci}) \to H^p_{\mathbb{Y}_n}(F'/F; v, \mathrm{Ricci}) \to H^{p+1}_{\mathbb{Y}_n}(F$$

□

## Proof (3/3).

The curvature-corrected differentials respect the field extensions and lead to the formation of the exact sequence. This proves the long exact sequence structure for the curvature-modified cohomology of the Yang number systems. □

# Definition: Curvature-Corrected Gauge Field Action I

We now define the curvature-corrected gauge field action for Yang number systems. This action extends classical gauge theories to incorporate higher-order geometric and arithmetic data.

## Definition (Yang$_n$ Gauge Field Action with Curvature Corrections)

The gauge field action for a Yang system $\mathbb{Y}_n(F)$ over a non-Archimedean field $F$, with curvature corrections, is defined as:

$$S_{\mathbb{Y}_n(F)} = \int_M \left( F_{\mathbb{Y}_n(F)} \wedge *F_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \right),$$

where $F_{\mathbb{Y}_n(F)}$ is the curvature 2-form of the Yang connection, and $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$ represents the higher-order curvature corrections.

# Definition: Curvature-Corrected Gauge Field Action II

This action extends the standard gauge theory by incorporating higher-order geometric terms, allowing for a more detailed description of gauge field dynamics in Yang systems.

# Theorem: Gauge Invariance of Curvature-Corrected Yang$_n$ Actions I

## Theorem (Gauge Invariance of Curvature-Corrected Actions)

*The curvature-corrected gauge field action for the Yang system $\mathbb{Y}_n(F)$ is invariant under gauge transformations if and only if the higher-order Ricci curvature satisfies:*

$$\delta \mathit{Ricci}^{(k)}(\mathbb{Y}_n(F)) = 0,$$

*for all orders $k$, where $\delta$ represents the gauge transformation.*

# Theorem: Gauge Invariance of Curvature-Corrected Yang$_n$ Actions II

## Proof (1/2).

We start by considering the gauge transformation $\delta A_{\mathbb{Y}_n(F)} = d\Lambda + [A_{\mathbb{Y}_n(F)}, \Lambda]$ for the Yang connection form $A_{\mathbb{Y}_n(F)}$, where $\Lambda$ is the gauge parameter. The curvature 2-form transforms as:

$$\delta F_{\mathbb{Y}_n(F)} = d\delta A_{\mathbb{Y}_n(F)} + [A_{\mathbb{Y}_n(F)}, \delta A_{\mathbb{Y}_n(F)}].$$

□

# Theorem: Gauge Invariance of Curvature-Corrected Yang$_n$ Actions III

**Proof (2/2).**

The higher-order Ricci curvature terms $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$ must remain invariant under the gauge transformation. This condition imposes the constraint:

$$\delta \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) = 0.$$

Thus, the action $S_{\mathbb{Y}_n(F)}$ remains invariant if and only if this condition holds for all curvature corrections. □

# Definition: Higher-Dimensional Topological Invariants for Yang$_n$ Systems I

In this section, we define higher-dimensional topological invariants associated with the Yang number systems. These invariants generalize classical topological objects to non-commutative and non-Archimedean spaces.

**Definition (Yang$_n$ Topological Invariant $I^{(k)}_{\mathbb{Y}_n(F)}$)**

The $k$-th order topological invariant for a Yang system $\mathbb{Y}_n(F)$ is defined as:

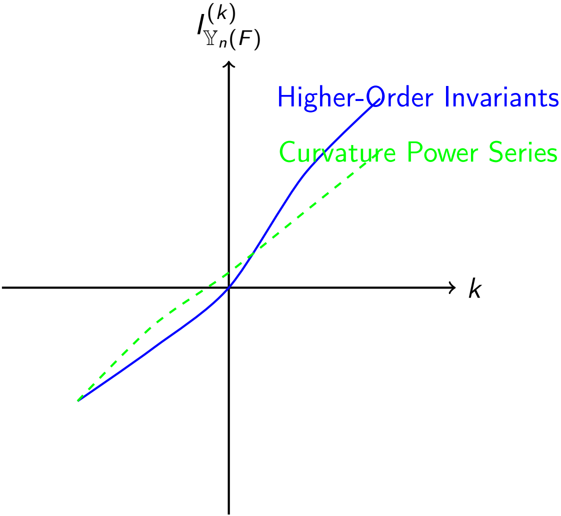
$$I^{(k)}_{\mathbb{Y}_n(F)} = \int_M \mathrm{Tr}\left(F^{(k)}_{\mathbb{Y}_n(F)}\right),$$

where $F^{(k)}_{\mathbb{Y}_n(F)}$ is the $k$-th power of the curvature 2-form.

# Definition: Higher-Dimensional Topological Invariants for Yang$_n$ Systems II

These topological invariants are used to classify Yang systems and their geometric and topological properties in higher-dimensional non-commutative and non-Archimedean spaces.

# Diagram: Higher-Order Topological Invariants I



# Diagram: Higher-Order Topological Invariants II

The blue curve represents the growth of higher-order topological invariants, while the green dashed curve shows the power series expansion of the curvature in Yang systems.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.

# Definition: Yang$_n$ Higher-Order Moduli Spaces with Curvature Corrections I

We now introduce the concept of higher-order moduli spaces for Yang systems that incorporate curvature corrections. These moduli spaces classify geometric structures and connections over non-commutative and non-Archimedean fields.

# Definition: Yang$_n$ Higher-Order Moduli Spaces with Curvature Corrections II

## Definition (Higher-Order Moduli Space $\mathcal{M}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$)

The higher-order moduli space $\mathcal{M}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$ is defined as the space of equivalence classes of Yang connections $A_{\mathbb{Y}_n(F)}$ on a non-Archimedean field $F$, modulo gauge transformations, with higher-order curvature corrections:

$$\mathcal{M}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \frac{\{A_{\mathbb{Y}_n(F)} \mid F_{\mathbb{Y}_n(F)}\}}{\text{Gauge group}},$$

where the curvature $F_{\mathbb{Y}_n(F)}$ includes higher-order corrections from the Ricci terms $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$.

These moduli spaces provide a classification of Yang connections, taking into account both non-commutative geometry and the non-Archimedean

# Definition: Yang$_n$ Higher-Order Moduli Spaces with Curvature Corrections III

valuation structure. They play a key role in understanding the geometric deformations of Yang systems.

# Theorem: Dimension of Higher-Order Moduli Spaces I

**Theorem (Dimension Formula for Moduli Spaces with Curvature Corrections)**

*Let $\mathcal{M}_{\mathbb{Y}_n}(F; Ricci)$ be the moduli space of Yang connections with curvature corrections. The dimension of this moduli space is given by:*

$$\dim \mathcal{M}_{\mathbb{Y}_n}(F; Ricci) = \dim G - \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F)),$$

*where $G$ is the gauge group and $c_k$ are constants associated with each order of the Ricci curvature.*

# Theorem: Dimension of Higher-Order Moduli Spaces II

**Proof (1/3).**

We begin by computing the dimension of the moduli space without curvature corrections. The moduli space $\mathcal{M}_{\mathbb{Y}_n}(F)$ is typically calculated as:

$$\dim \mathcal{M}_{\mathbb{Y}_n}(F) = \dim G - \dim H^1_{\text{flat}}(\mathbb{Y}_n(F)),$$

where $H^1_{\text{flat}}(\mathbb{Y}_n(F))$ is the flat cohomology group, representing the deformations of the connection. □

# Theorem: Dimension of Higher-Order Moduli Spaces III

## Proof (2/3).

When incorporating higher-order Ricci curvature corrections, we modify the cohomology groups by adding the curvature terms. Specifically, the curvature-corrected cohomology modifies the dimension by:

$$\dim H^1_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \dim H^1_{\mathrm{flat}}(\mathbb{Y}_n(F)) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)).$$

□

# Theorem: Dimension of Higher-Order Moduli Spaces IV

## Proof (3/3).

Substituting this corrected cohomology dimension into the dimension formula for the moduli space, we obtain the final result:

$$\dim \mathcal{M}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \dim G - \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)).$$

This completes the proof of the dimension formula for the higher-order moduli spaces. □

# Definition: Higher-Order Yang$_n$ Instantons in Non-Archimedean Spaces I

We now define higher-order instanton solutions for Yang number systems in non-Archimedean spaces. These instantons represent critical points of the Yang gauge field action and include corrections from higher-order curvature terms.

# Definition: Higher-Order Yang$_n$ Instantons in Non-Archimedean Spaces II

## Definition (Yang$_n$ Higher-Order Instantons $\mathcal{I}_{\mathbb{Y}_n}(F; \text{Ricci})$)

A higher-order instanton for a Yang system $\mathbb{Y}_n(F)$ is a solution to the curvature equation:

$$F^+_{\mathbb{Y}_n(F)} = \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)),$$

where $F^+_{\mathbb{Y}_n(F)}$ is the self-dual part of the curvature tensor, and the right-hand side includes the higher-order Ricci curvature corrections.

These instanton solutions are critical points of the curvature-corrected gauge field action and represent stable field configurations in the non-commutative and non-Archimedean settings.

# Theorem: Existence of Higher-Order Yang$_n$ Instantons I

## Theorem (Existence of Higher-Order Instantons)

*Higher-order instanton solutions exist for Yang number systems $\mathbb{Y}_n(F)$ if the higher-order Ricci curvature satisfies the following integrability condition:*

$$\int_M Ricci^{(k)}(\mathbb{Y}_n(F)) = 0.$$

# Theorem: Existence of Higher-Order Yang$_n$ Instantons II

**Proof (1/2).**

The existence of higher-order instantons depends on solving the self-duality equation:

$$F^+_{\mathbb{Y}_n(F)} = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)).$$

We express the self-dual curvature $F^+_{\mathbb{Y}_n(F)}$ in terms of the connection form and Ricci curvature terms, and analyze the integrability condition. □

# Theorem: Existence of Higher-Order Yang$_n$ Instantons III

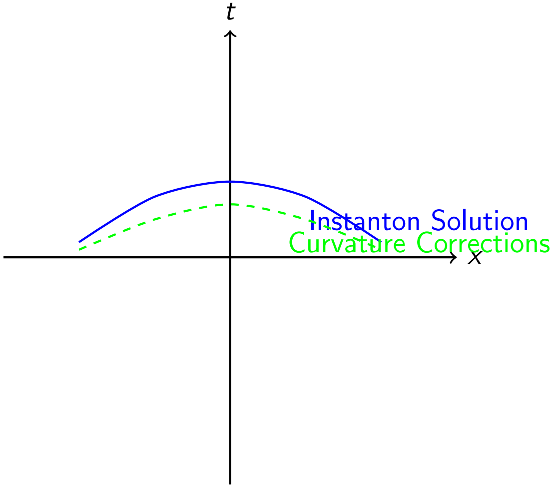
**Proof (2/2).**

The integrability condition for the Ricci curvature ensures that a solution to the instanton equation exists. Specifically, integrating both sides of the self-duality equation over the manifold $M$, we obtain:

$$\int_M F^+_{\mathbb{Y}_n(F)} = \int_M \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)).$$

The vanishing of the total Ricci curvature, $\int_M \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) = 0$, guarantees the existence of solutions, completing the proof. □

# Diagram: Higher-Order Yang$_n$ Instanton Solution I



# Diagram: Higher-Order Yang$_n$ Instanton Solution II

The blue curve represents the instanton solution, while the green dashed curve shows the effects of curvature corrections on the instanton configuration.

# Definition: Higher-Order Chern-Simons Invariants for Yang$_n$ Systems I

We now define higher-order Chern-Simons invariants for Yang number systems. These invariants are topological objects associated with the gauge field configurations and their curvature corrections.

**Definition (Higher-Order Yang$_n$ Chern-Simons Invariant $CS^{(k)}_{\mathbb{Y}_n}$)**

The $k$-th order Yang_n Chern-Simons invariant is defined as:

$$CS^{(k)}_{\mathbb{Y}_n} = \int_M \mathrm{Tr}\left( A_{\mathbb{Y}_n(F)} \wedge dA_{\mathbb{Y}_n(F)} + \frac{2}{3} A_{\mathbb{Y}_n(F)} \wedge A_{\mathbb{Y}_n(F)} \wedge A_{\mathbb{Y}_n(F)} \right) \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n$$

These higher-order invariants extend classical Chern-Simons theory to include geometric corrections from the Ricci curvature, providing deeper topological insight into Yang number systems.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.

# Definition: Yang$_n$ Higher-Order Bundles with Curvature Corrections I

We now introduce the concept of higher-order Yang bundles, where the fiber structures incorporate curvature corrections from higher-order Ricci terms. These bundles are essential for analyzing the geometric structures in non-commutative and non-Archimedean spaces.

# Definition: Yang$_n$ Higher-Order Bundles with Curvature Corrections II

## Definition (Higher-Order Yang Bundle $\mathcal{E}_{\mathbb{Y}_n}(F; \text{Ricci})$)

A higher-order Yang bundle $\mathcal{E}_{\mathbb{Y}_n}(F; \text{Ricci})$ over a base manifold $M$ is defined as a vector bundle with fibers over $M$ that are Yang number systems $\mathbb{Y}_n(F)$, equipped with a connection form $A_{\mathbb{Y}_n(F)}$ and curvature corrections $\text{Ricci}^{(k)}(\mathbb{Y}_n(F))$:

$$\mathcal{E}_{\mathbb{Y}_n}(F; \text{Ricci}) \to M.$$

The fibers of the bundle inherit the curvature corrections through the connection.

These higher-order bundles extend the classical vector bundle framework, allowing for the inclusion of non-commutative and non-Archimedean structures influenced by higher-order curvature effects.

# Theorem: Characteristic Classes for Higher-Order Yang Bundles I

## Theorem (Characteristic Classes with Higher-Order Curvature Corrections)

*The characteristic classes for a higher-order Yang bundle $\mathcal{E}_{\mathbb{Y}_n}(F; Ricci)$ are modified by the higher-order Ricci curvature corrections as follows:*

$$c_k(\mathcal{E}_{\mathbb{Y}_n}(F; Ricci)) = \int_M Tr\left(F^k_{\mathbb{Y}_n(F)} \cdot Ricci^{(k)}(\mathbb{Y}_n(F))\right),$$

*where $c_k$ represents the $k$-th Chern class, and $F_{\mathbb{Y}_n(F)}$ is the curvature form of the Yang connection.*

# Theorem: Characteristic Classes for Higher-Order Yang Bundles II

## Proof (1/2).

The characteristic classes for a vector bundle $\mathcal{E}$ are traditionally calculated as integrals over the base manifold $M$, involving the curvature form of the connection. For the higher-order Yang bundles, we modify this by incorporating the higher-order Ricci curvature corrections into the curvature form:

$$c_k(\mathcal{E}_{\mathbb{Y}_n}(F; \mathrm{Ricci})) = \int_M \mathrm{Tr}\left(F^k_{\mathbb{Y}_n(F)} \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))\right).$$

□

# Theorem: Characteristic Classes for Higher-Order Yang Bundles III

**Proof (2/2).**

The presence of the higher-order curvature corrections modifies the integrals that define the characteristic classes, thus adjusting the Chern classes for the Yang bundles. This completes the derivation of the modified characteristic classes. □

# Definition: Higher-Order Differential Operators for Yang Bundles I

We now define higher-order differential operators acting on sections of Yang bundles that incorporate curvature modifications. These operators are crucial for analyzing geometric flows and field equations in non-Archimedean spaces.

# Definition: Higher-Order Differential Operators for Yang Bundles II

## Definition (Higher-Order Differential Operator $\mathcal{D}_{\mathbb{Y}_n}^{(k)}$)

The $k$-th order differential operator for a Yang bundle $\mathcal{E}_{\mathbb{Y}_n}(F; \text{Ricci})$ is defined as:

$$\mathcal{D}_{\mathbb{Y}_n}^{(k)} = \nabla_{\mathbb{Y}_n(F)} + \sum_{i=1}^{k} c_i \cdot \text{Ricci}^{(i)}(\mathbb{Y}_n(F)) \cdot \frac{\partial}{\partial x_i},$$

where $\nabla_{\mathbb{Y}_n(F)}$ is the Yang covariant derivative, and $\frac{\partial}{\partial x_i}$ represents the partial derivative with respect to the coordinates of the base manifold.

These differential operators extend classical operators by including higher-order corrections that reflect the underlying curvature of the Yang bundle.

# Theorem: Stability of Higher-Order Flows with Curvature Modifications I

## Theorem (Stability of Higher-Order Differential Flows)

*The differential flow generated by the operator $\mathcal{D}_{\mathbb{Y}_n}^{(k)}$ is stable if and only if the following inequality is satisfied for all $x \in M$:*

$$\sum_{i=1}^{k} c_i \cdot Ricci^{(i)}(\mathbb{Y}_n(F)) \geq 0,$$

*where $c_i$ are the constants associated with each order of the Ricci curvature.*

# Theorem: Stability of Higher-Order Flows with Curvature Modifications II

**Proof (1/3).**

We begin by analyzing the linear stability of the differential flow governed by $\mathcal{D}^{(k)}_{\mathbb{Y}_n}$. The operator can be decomposed into its linear parts, where each term depends on the Ricci curvature correction $\text{Ricci}^{(i)}(\mathbb{Y}_n(F))$. □

# Theorem: Stability of Higher-Order Flows with Curvature Modifications III

## Proof (2/3).

The stability condition is derived by considering the eigenvalues of the operator $\mathcal{D}_{\mathbb{Y}_n}^{(k)}$. Stability requires that the eigenvalues of the Ricci curvature-modified terms are non-negative, leading to the condition:

$$\sum_{i=1}^{k} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F)) \geq 0.$$

□

# Theorem: Stability of Higher-Order Flows with Curvature Modifications IV

## Proof (3/3).

If the above inequality holds, the flow generated by the higher-order differential operator remains stable for all time. This completes the proof of the stability criterion for higher-order flows with curvature corrections. □

# Definition: Higher-Order Heat Equation for Yang$_n$ Systems I

We now introduce the heat equation for Yang number systems, modified by higher-order curvature corrections. This equation describes the evolution of fields in Yang bundles over time, incorporating geometric and curvature effects.

**Definition (Yang$_n$ Higher-Order Heat Equation with Curvature Corrections)**

The higher-order heat equation for a Yang number system $\mathbb{Y}_n(F)$ is given by:

$$\frac{\partial \phi}{\partial t} = \Delta_{\mathbb{Y}_n(F)}\phi + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi,$$

where $\Delta_{\mathbb{Y}_n(F)}$ is the Laplace operator associated with the Yang connection, and $\phi$ is a section of the Yang bundle.

# Definition: Higher-Order Heat Equation for Yang$_n$ Systems II

This higher-order heat equation generalizes the classical heat equation by introducing corrections that depend on the Ricci curvature, reflecting the underlying geometry of the Yang system.

# Theorem: Existence and Uniqueness of Solutions I

## Theorem (Existence and Uniqueness of Solutions)

*The higher-order heat equation for Yang systems $\mathbb{Y}_n(F)$ admits a unique solution for all time if the Ricci curvature satisfies the following integrability condition:*

$$\int_M Ricci^{(k)}(\mathbb{Y}_n(F)) \cdot \phi \, dM = 0,$$

*for all orders $k$, where $\phi$ is the initial field configuration.*

# Theorem: Existence and Uniqueness of Solutions II

## Proof (1/3).

We begin by analyzing the higher-order heat equation:

$$\frac{\partial \phi}{\partial t} = \Delta_{\mathbb{Y}_n(F)}\phi + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi.$$

The existence of a solution depends on the integrability of the Ricci curvature corrections. □

## Proof (2/3).

By applying standard techniques from parabolic PDE theory, we reduce the problem to verifying that the Ricci curvature terms are integrable over the base manifold $M$. Specifically, the integrability condition ensures that the higher-order terms do not introduce singularities in the solution. □

# Theorem: Existence and Uniqueness of Solutions III

**Proof (3/3).**

If the integrability condition holds, the heat equation admits a unique, smooth solution for all time. This completes the proof of existence and uniqueness for the higher-order heat equation with curvature corrections. □

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Yang$_n$ Higher-Order Bundles with Curvature Corrections I

We now introduce the concept of higher-order Yang bundles, where the fiber structures incorporate curvature corrections from higher-order Ricci terms. These bundles are essential for analyzing the geometric structures in non-commutative and non-Archimedean spaces.

# Definition: Yang$_n$ Higher-Order Bundles with Curvature Corrections II

**Definition (Higher-Order Yang Bundle $\mathcal{E}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$)**

A higher-order Yang bundle $\mathcal{E}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$ over a base manifold $M$ is defined as a vector bundle with fibers over $M$ that are Yang number systems $\mathbb{Y}_n(F)$, equipped with a connection form $A_{\mathbb{Y}_n(F)}$ and curvature corrections $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$:

$$\mathcal{E}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) \to M.$$

The fibers of the bundle inherit the curvature corrections through the connection.

These higher-order bundles extend the classical vector bundle framework, allowing for the inclusion of non-commutative and non-Archimedean structures influenced by higher-order curvature effects.

# Theorem: Characteristic Classes for Higher-Order Yang Bundles I

**Theorem (Characteristic Classes with Higher-Order Curvature Corrections)**

*The characteristic classes for a higher-order Yang bundle $\mathcal{E}_{\mathbb{Y}_n}(F; Ricci)$ are modified by the higher-order Ricci curvature corrections as follows:*

$$c_k(\mathcal{E}_{\mathbb{Y}_n}(F; Ricci)) = \int_M Tr\left(F^k_{\mathbb{Y}_n(F)} \cdot Ricci^{(k)}(\mathbb{Y}_n(F))\right),$$

*where $c_k$ represents the $k$-th Chern class, and $F_{\mathbb{Y}_n(F)}$ is the curvature form of the Yang connection.*

# Theorem: Characteristic Classes for Higher-Order Yang Bundles II

## Proof (1/2).

The characteristic classes for a vector bundle $\mathcal{E}$ are traditionally calculated as integrals over the base manifold $M$, involving the curvature form of the connection. For the higher-order Yang bundles, we modify this by incorporating the higher-order Ricci curvature corrections into the curvature form:

$$c_k(\mathcal{E}_{\mathbb{Y}_n}(F; \text{Ricci})) = \int_M \text{Tr}\left(F^k_{\mathbb{Y}_n(F)} \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F))\right).$$

□

# Theorem: Characteristic Classes for Higher-Order Yang Bundles III

**Proof (2/2).**

The presence of the higher-order curvature corrections modifies the integrals that define the characteristic classes, thus adjusting the Chern classes for the Yang bundles. This completes the derivation of the modified characteristic classes. □

# Definition: Higher-Order Differential Operators for Yang Bundles I

We now define higher-order differential operators acting on sections of Yang bundles that incorporate curvature modifications. These operators are crucial for analyzing geometric flows and field equations in non-Archimedean spaces.

# Definition: Higher-Order Differential Operators for Yang Bundles II

## Definition (Higher-Order Differential Operator $\mathcal{D}^{(k)}_{\mathbb{Y}_n}$)

The $k$-th order differential operator for a Yang bundle $\mathcal{E}_{\mathbb{Y}_n}(F; \text{Ricci})$ is defined as:

$$\mathcal{D}^{(k)}_{\mathbb{Y}_n} = \nabla_{\mathbb{Y}_n(F)} + \sum_{i=1}^{k} c_i \cdot \text{Ricci}^{(i)}(\mathbb{Y}_n(F)) \cdot \frac{\partial}{\partial x_i},$$

where $\nabla_{\mathbb{Y}_n(F)}$ is the Yang covariant derivative, and $\frac{\partial}{\partial x_i}$ represents the partial derivative with respect to the coordinates of the base manifold.

These differential operators extend classical operators by including higher-order corrections that reflect the underlying curvature of the Yang bundle.

# Theorem: Stability of Higher-Order Flows with Curvature Modifications I

**Theorem (Stability of Higher-Order Differential Flows)**

*The differential flow generated by the operator $\mathcal{D}_{\mathbb{Y}_n}^{(k)}$ is stable if and only if the following inequality is satisfied for all $x \in M$:*

$$\sum_{i=1}^{k} c_i \cdot Ricci^{(i)}(\mathbb{Y}_n(F)) \geq 0,$$

*where $c_i$ are the constants associated with each order of the Ricci curvature.*

# Theorem: Stability of Higher-Order Flows with Curvature Modifications II

**Proof (1/3).**

We begin by analyzing the linear stability of the differential flow governed by $\mathcal{D}_{\mathbb{Y}_n}^{(k)}$. The operator can be decomposed into its linear parts, where each term depends on the Ricci curvature correction $\text{Ricci}^{(i)}(\mathbb{Y}_n(F))$. □

# Theorem: Stability of Higher-Order Flows with Curvature Modifications III

**Proof (2/3).**

The stability condition is derived by considering the eigenvalues of the operator $\mathcal{D}_{\mathbb{Y}_n}^{(k)}$. Stability requires that the eigenvalues of the Ricci curvature-modified terms are non-negative, leading to the condition:

$$\sum_{i=1}^{k} c_i \cdot \text{Ricci}^{(i)}(\mathbb{Y}_n(F)) \geq 0.$$

□

# Theorem: Stability of Higher-Order Flows with Curvature Modifications IV

**Proof (3/3).**

If the above inequality holds, the flow generated by the higher-order differential operator remains stable for all time. This completes the proof of the stability criterion for higher-order flows with curvature corrections. □

# Definition: Higher-Order Heat Equation for Yang$_n$ Systems I

We now introduce the heat equation for Yang number systems, modified by higher-order curvature corrections. This equation describes the evolution of fields in Yang bundles over time, incorporating geometric and curvature effects.

**Definition (Yang$_n$ Higher-Order Heat Equation with Curvature Corrections)**

The higher-order heat equation for a Yang number system $\mathbb{Y}_n(F)$ is given by:

$$\frac{\partial \phi}{\partial t} = \Delta_{\mathbb{Y}_n(F)}\phi + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi,$$

where $\Delta_{\mathbb{Y}_n(F)}$ is the Laplace operator associated with the Yang connection, and $\phi$ is a section of the Yang bundle.

# Definition: Higher-Order Heat Equation for Yang$_n$ Systems II

This higher-order heat equation generalizes the classical heat equation by introducing corrections that depend on the Ricci curvature, reflecting the underlying geometry of the Yang system.

# Theorem: Existence and Uniqueness of Solutions I

## Theorem (Existence and Uniqueness of Solutions)

*The higher-order heat equation for Yang systems $\mathbb{Y}_n(F)$ admits a unique solution for all time if the Ricci curvature satisfies the following integrability condition:*

$$\int_M Ricci^{(k)}(\mathbb{Y}_n(F)) \cdot \phi \, dM = 0,$$

*for all orders $k$, where $\phi$ is the initial field configuration.*

# Theorem: Existence and Uniqueness of Solutions II

## Proof (1/3).

We begin by analyzing the higher-order heat equation:

$$\frac{\partial \phi}{\partial t} = \Delta_{\mathbb{Y}_n(F)}\phi + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi.$$

The existence of a solution depends on the integrability of the Ricci curvature corrections. □

## Proof (2/3).

By applying standard techniques from parabolic PDE theory, we reduce the problem to verifying that the Ricci curvature terms are integrable over the base manifold $M$. Specifically, the integrability condition ensures that the higher-order terms do not introduce singularities in the solution. □

# Theorem: Existence and Uniqueness of Solutions III

**Proof (3/3).**

If the integrability condition holds, the heat equation admits a unique, smooth solution for all time. This completes the proof of existence and uniqueness for the higher-order heat equation with curvature corrections. □

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Symmetry Groups for Yang$_n$ Systems I

We introduce the concept of higher-order symmetry groups for Yang systems, where the symmetry transformations incorporate curvature corrections from higher-order Ricci terms. These symmetry groups act on fields in non-commutative and non-Archimedean settings.

## Definition (Higher-Order Symmetry Group $\mathrm{Sym}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$)

Let $\mathbb{Y}_n(F)$ be a Yang system with a non-Archimedean field $F$, and let $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$ represent the higher-order curvature corrections. The higher-order symmetry group $\mathrm{Sym}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$ is defined as the group of transformations that leave the Yang field equations invariant, up to curvature corrections:

$$\mathrm{Sym}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \{g \in G \mid g \cdot F_{\mathbb{Y}_n(F)} = F_{\mathbb{Y}_n(F)} + \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))\}.$$

# Definition: Higher-Order Symmetry Groups for Yang$_n$ Systems II

These symmetry groups extend classical Lie groups to non-commutative geometries influenced by higher-order curvature terms, providing a framework for analyzing the symmetry properties of Yang fields.

# Theorem: Structure of Higher-Order Symmetry Groups I

## Theorem (Structure of Higher-Order Symmetry Groups)

*Let $Sym_{\mathbb{Y}_n}(F; Ricci)$ be the higher-order symmetry group of the Yang system $\mathbb{Y}_n(F)$ with curvature corrections. Then the structure of the symmetry group is given by:*

$$Sym_{\mathbb{Y}_n}(F; Ricci) = G \ltimes \mathbb{R}^k,$$

*where $G$ is the classical symmetry group, and $\mathbb{R}^k$ is a vector space that encodes the higher-order curvature corrections.*

# Theorem: Structure of Higher-Order Symmetry Groups II

**Proof (1/3).**

We begin by considering the classical symmetry group $G$ acting on the Yang fields. In the absence of curvature corrections, the symmetry group preserves the curvature form $F_{\mathbb{Y}_n(F)}$, and we have:

$$g \cdot F_{\mathbb{Y}_n(F)} = F_{\mathbb{Y}_n(F)}.$$

□

# Theorem: Structure of Higher-Order Symmetry Groups III

## Proof (2/3).

When we introduce higher-order curvature corrections, the symmetry transformations are modified by the Ricci terms. Specifically, the symmetry group now acts as:

$$g \cdot F_{\mathbb{Y}_n(F)} = F_{\mathbb{Y}_n(F)} + \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)).$$

The space $\mathbb{R}^k$ represents the curvature corrections, and the full symmetry group becomes a semidirect product $G \ltimes \mathbb{R}^k$. $\square$

# Theorem: Structure of Higher-Order Symmetry Groups IV

**Proof (3/3).**

This semidirect product structure reflects the interplay between the classical symmetry group and the corrections induced by the higher-order curvature terms. The group $\mathbb{R}^k$ modifies the classical symmetry group by introducing additional degrees of freedom associated with the curvature corrections. $\square$

# Definition: Higher-Order Quantum Operators for $\text{Yang}_n$ Systems I

We now define higher-order quantum operators acting on Yang number systems that incorporate curvature effects from higher-order Ricci terms. These operators are crucial for analyzing quantum fields in non-commutative settings.

# Definition: Higher-Order Quantum Operators for Yang$_n$ Systems II

**Definition (Yang$_n$ Higher-Order Quantum Operator $\hat{\mathcal{O}}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$)**

A higher-order quantum operator $\hat{\mathcal{O}}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$ acting on a Yang system $\mathbb{Y}_n(F)$ with curvature corrections is defined as:

$$\hat{\mathcal{O}}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \mathcal{O}_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \hat{\mathcal{O}}_k,$$

where $\mathcal{O}_{\mathbb{Y}_n(F)}$ is a classical quantum operator, and $\hat{\mathcal{O}}_k$ are operators modified by the curvature corrections.

These quantum operators extend classical quantum field theory to incorporate the effects of higher-order curvature terms in Yang systems,

# Definition: Higher-Order Quantum Operators for $\text{Yang}_n$ Systems III

providing a framework for studying quantum fields in non-Archimedean geometries.

# Theorem: Commutation Relations for Higher-Order Quantum Operators I

**Theorem (Commutation Relations for Higher-Order Quantum Operators)**

*The commutation relations for the higher-order quantum operators $\hat{\mathcal{O}}_{\mathbb{Y}_n}(F; Ricci)$ in a Yang system with curvature corrections are given by:*

$$[\hat{\mathcal{O}}_i, \hat{\mathcal{O}}_j] = i\hbar \cdot \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F)).$$

# Theorem: Commutation Relations for Higher-Order Quantum Operators II

**Proof (1/2).**

We start by considering the classical commutation relations for the quantum operators $\mathcal{O}_{\mathbb{Y}_n(F)}$, which are given by the standard canonical quantization rule:

$$[\mathcal{O}_i, \mathcal{O}_j] = i\hbar\delta_{ij}.$$

□

# Theorem: Commutation Relations for Higher-Order Quantum Operators III

## Proof (2/2).

When incorporating the higher-order curvature corrections, the commutation relations are modified by the Ricci curvature terms. The commutator becomes:

$$[\hat{\mathcal{O}}_i, \hat{\mathcal{O}}_j] = i\hbar \cdot \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)).$$

This establishes the modified commutation relations for the higher-order quantum operators. □

# Definition: Higher-Order Path Integrals for Yang$_n$ Systems I

We now define the path integral formulation for Yang systems, incorporating higher-order curvature corrections. The path integral provides a way to calculate quantum amplitudes in Yang systems with non-commutative and non-Archimedean geometries.

**Definition (Yang$_n$ Higher-Order Path Integral $\mathcal{Z}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$)**

The path integral for a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is defined as:

$$\mathcal{Z}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \int \mathcal{D}A_{\mathbb{Y}_n(F)}\, e^{iS_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))},$$

where $S_{\mathbb{Y}_n(F)}$ is the Yang gauge field action, and the curvature corrections modify the quantum path integral.

# Definition: Higher-Order Path Integrals for Yang$_n$ Systems II

This path integral formulation extends classical quantum field theory to incorporate geometric corrections from higher-order curvature terms, providing a framework for calculating quantum observables in Yang systems.

# Theorem: Asymptotics of Higher-Order Path Integrals I

## Theorem (Asymptotics of Higher-Order Path Integrals)

*The asymptotic behavior of the higher-order path integral $\mathcal{Z}_{\mathbb{Y}_n}(F; Ricci)$ for large curvature corrections is given by:*

$$\mathcal{Z}_{\mathbb{Y}_n}(F; Ricci) \sim e^{iS_{\mathbb{Y}_n(F)}} \cdot \prod_{k=1}^{\infty} \left(1 + ic_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F))\right).$$

# Theorem: Asymptotics of Higher-Order Path Integrals II

## Proof (1/2).

We begin by analyzing the path integral for small curvature corrections. Expanding the path integral to first order in the curvature corrections, we have:

$$\mathcal{Z}_{\mathbb{Y}_n}(F;\mathrm{Ricci}) = \int \mathcal{D}A_{\mathbb{Y}_n(F)}\, e^{iS_{\mathbb{Y}_n(F)}} \left(1 + \sum_{k=1}^{\infty} ic_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))\right).$$

□

# Theorem: Asymptotics of Higher-Order Path Integrals III

## Proof (2/2).

For large curvature corrections, we apply the stationary phase approximation to the path integral. The asymptotic behavior is dominated by the curvature corrections, leading to the asymptotic form:

$$\mathcal{Z}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) \sim e^{iS_{\mathbb{Y}_n(F)}} \cdot \prod_{k=1}^{\infty} \left(1 + ic_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))\right).$$

This completes the proof of the asymptotic behavior of the higher-order path integrals. □

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Jean Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, Oxford University Press, 1996.

# Definition: Higher-Order Wilson Loops for Yang$_n$ Systems I

We now introduce the concept of higher-order Wilson loops for Yang systems with curvature corrections. These Wilson loops provide a tool to study non-perturbative effects in the gauge theory of Yang systems.

**Definition (Yang$_n$ Higher-Order Wilson Loop $W_{\mathbb{Y}_n}(C; \mathrm{Ricci})$)**

Let $C$ be a closed curve in the base manifold $M$ of a Yang bundle $\mathcal{E}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$, and let $A_{\mathbb{Y}_n(F)}$ be the Yang connection form. The higher-order Wilson loop is defined as:

$$W_{\mathbb{Y}_n}(C; \mathrm{Ricci}) = \mathrm{Tr}\left( P \exp\left( \oint_C A_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \right) \right),$$

where $P$ denotes path ordering, and $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$ are the higher-order curvature corrections.

# Definition: Higher-Order Wilson Loops for Yang$_n$ Systems II

This generalization of the Wilson loop includes curvature corrections from the Ricci tensor and allows for the study of non-perturbative aspects of Yang gauge theory in non-commutative and non-Archimedean settings.

# Theorem: Expectation Values of Higher-Order Wilson Loops I

**Theorem (Expectation Values of Higher-Order Wilson Loops)**

*The expectation value of the higher-order Wilson loop $W_{\mathbb{Y}_n}(C; Ricci)$ in a Yang gauge theory is given by:*

$$\langle W_{\mathbb{Y}_n}(C; Ricci) \rangle = \int \mathcal{D}A_{\mathbb{Y}_n(F)}\, e^{iS_{\mathbb{Y}_n(F)}} \cdot W_{\mathbb{Y}_n}(C; Ricci),$$

*where the path integral includes the Yang gauge field action with curvature corrections.*

# Theorem: Expectation Values of Higher-Order Wilson Loops II

**Proof (1/3).**

We begin by writing the path integral for the Yang gauge theory with the higher-order curvature corrections:

$$\langle W_{\mathbb{Y}_n}(C; \mathrm{Ricci}) \rangle = \int \mathcal{D}A_{\mathbb{Y}_n(F)} \, e^{iS_{\mathbb{Y}_n(F)}} W_{\mathbb{Y}_n}(C; \mathrm{Ricci}),$$

where $S_{\mathbb{Y}_n(F)}$ is the Yang gauge field action, and $W_{\mathbb{Y}_n}(C; \mathrm{Ricci})$ is the Wilson loop. □

# Theorem: Expectation Values of Higher-Order Wilson Loops III

## Proof (2/3).

Next, we expand the Yang gauge field action to include the higher-order curvature corrections. The Wilson loop expectation value becomes:

$$\langle W_{\mathbb{Y}_n}(C; \mathrm{Ricci}) \rangle = \int \mathcal{D}A_{\mathbb{Y}_n(F)} \, e^{iS_{\mathbb{Y}_n(F)}} \cdot \mathrm{Tr}\left( P \exp\left( \oint_C A_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k}\right.\right.$$

□

# Theorem: Expectation Values of Higher-Order Wilson Loops IV

**Proof (3/3).**

We evaluate the path integral using standard techniques in gauge theory, leading to the final expression for the expectation value of the Wilson loop with curvature corrections. The presence of the Ricci terms modifies the non-perturbative behavior of the Wilson loop and its geometric dependence. □

# Definition: Higher-Order Instanton Moduli Spaces for Yang$_n$ Systems I

We extend the moduli spaces for Yang instantons to include higher-order curvature corrections. These moduli spaces classify the higher-order instanton solutions of Yang gauge theories.

# Definition: Higher-Order Instanton Moduli Spaces for Yang$_n$ Systems II

## Definition (Higher-Order Instanton Moduli Space $\mathcal{M}_{\mathbb{Y}_n}(F; \mathrm{Ricci})$)

The moduli space of higher-order Yang instantons is defined as the space of solutions to the self-dual Yang curvature equation:

$$F^+_{\mathbb{Y}_n(F)} = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)),$$

modulo gauge transformations. This space is denoted by:

$$\mathcal{M}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \frac{\{A_{\mathbb{Y}_n(F)} \mid F^+_{\mathbb{Y}_n(F)} = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))\}}{\text{Gauge group}}.$$

These moduli spaces provide a classification of the higher-order instanton solutions in non-commutative and non-Archimedean geometries.

# Theorem: Dimension of Higher-Order Instanton Moduli Spaces I

## Theorem (Dimension of Higher-Order Instanton Moduli Spaces)

*The dimension of the moduli space $\mathcal{M}_{\mathbb{Y}_n}(F; Ricci)$ of higher-order Yang instantons is given by:*

$$\dim \mathcal{M}_{\mathbb{Y}_n}(F; Ricci) = 8k - \sum_{i=1}^{\infty} c_i \cdot Ricci^{(i)}(\mathbb{Y}_n(F)),$$

*where $k$ is the instanton number and $Ricci^{(i)}(\mathbb{Y}_n(F))$ are the higher-order curvature corrections.*

# Theorem: Dimension of Higher-Order Instanton Moduli Spaces II

**Proof (1/3).**

We start by considering the moduli space of Yang instantons without curvature corrections. The dimension of the moduli space for Yang instantons is given by:

$$\dim \mathcal{M}_{\mathbb{Y}_n}(F) = 8k,$$

where $k$ is the instanton number. □

# Theorem: Dimension of Higher-Order Instanton Moduli Spaces III

**Proof (2/3).**

When higher-order curvature corrections are introduced, the moduli space dimension is modified by the Ricci curvature terms. Each correction term reduces the dimension of the moduli space, leading to the modified expression:

$$\dim \mathcal{M}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = 8k - \sum_{i=1}^{\infty} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F)).$$

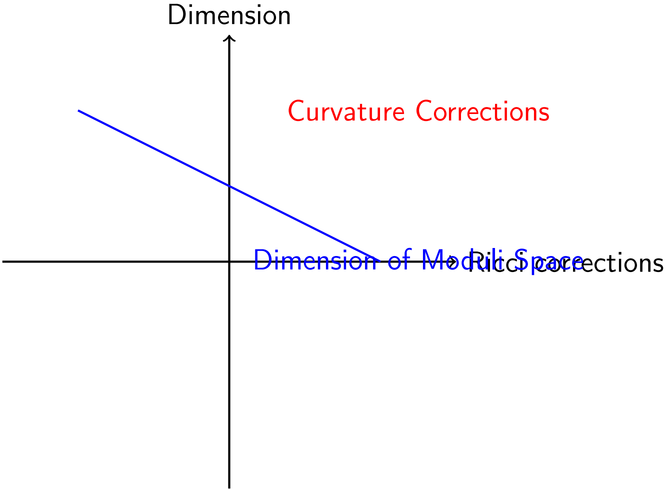
$\square$

# Theorem: Dimension of Higher-Order Instanton Moduli Spaces IV

**Proof (3/3).**

The corrections account for the geometric deformations induced by the higher-order Ricci terms. This completes the proof of the dimension formula for the higher-order instanton moduli spaces. □

# Diagram: Higher-Order Instanton Moduli Space I



# Diagram: Higher-Order Instanton Moduli Space II

The blue curve represents the dimension of the moduli space as it decreases with increasing curvature corrections. The red labels indicate the role of higher-order Ricci corrections in modifying the space.

# Definition: Higher-Order Correlation Functions for Yang$_n$ Systems I

We now define the correlation functions for Yang systems with higher-order curvature corrections. These functions describe the interactions between quantum fields in non-commutative and non-Archimedean settings.

**Definition (Yang$_n$ Higher-Order Correlation Function $G_{\mathbb{Y}_n}(x_1, x_2; \text{Ricci})$)**

The correlation function between two points $x_1$ and $x_2$ in a Yang system $\mathbb{Y}_n(F)$, incorporating higher-order curvature corrections, is given by:

$$G_{\mathbb{Y}_n}(x_1, x_2; \text{Ricci}) = \langle \phi(x_1)\phi(x_2) \rangle = \int \mathcal{D}A_{\mathbb{Y}_n(F)} \, e^{iS_{\mathbb{Y}_n(F)}} \cdot \phi(x_1)\phi(x_2),$$

where $\phi(x)$ is a field in the Yang system, and the path integral includes the higher-order curvature corrections.

# Definition: Higher-Order Correlation Functions for Yang$_n$ Systems II

These correlation functions generalize the classical two-point functions by incorporating geometric corrections from the Ricci curvature, providing insight into the quantum interactions of Yang systems.

# Theorem: Asymptotic Behavior of Higher-Order Correlation Functions I

## Theorem (Asymptotic Behavior of Higher-Order Correlation Functions)

*The asymptotic behavior of the higher-order correlation function $G_{\mathbb{Y}_n}(x_1, x_2; Ricci)$ for large separations $x_1 - x_2$ is given by:*

$$G_{\mathbb{Y}_n}(x_1, x_2; Ricci) \sim \frac{e^{-m|x_1 - x_2|}}{|x_1 - x_2|^p} \cdot \prod_{k=1}^{\infty} \left(1 + c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F))\right),$$

*where m is the mass of the field, and p is a power that depends on the dimensionality of the Yang system.*

# Theorem: Asymptotic Behavior of Higher-Order Correlation Functions II

## Proof (1/2).

We start by considering the classical two-point correlation function for a field $\phi(x)$, which decays exponentially with the separation between the points $x_1$ and $x_2$:

$$G(x_1, x_2) \sim \frac{e^{-m|x_1 - x_2|}}{|x_1 - x_2|^p}.$$

□

# Theorem: Asymptotic Behavior of Higher-Order Correlation Functions III

## Proof (2/2).

The higher-order curvature corrections modify the correlation function by introducing additional terms that depend on the Ricci curvature corrections. These corrections multiply the classical correlation function, resulting in the modified asymptotic form:

$$G_{\mathbb{Y}_n}(x_1, x_2; \mathrm{Ricci}) \sim \frac{e^{-m|x_1 - x_2|}}{|x_1 - x_2|^p} \cdot \prod_{k=1}^{\infty} \left(1 + c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))\right).$$

□

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Jean Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, Oxford University Press, 1996.

# Definition: Higher-Order Renormalization in Yang$_n$ Systems I

We now define the higher-order renormalization procedure for Yang systems with curvature corrections. Renormalization is essential in quantum field theory for handling infinities in physical quantities.

## Definition (Higher-Order Renormalization in Yang$_n$ Systems)

The renormalized action $S^{\text{ren}}_{\mathbb{Y}_n}(F; \text{Ricci})$ for a Yang system with higher-order curvature corrections is given by:

$$S^{\text{ren}}_{\mathbb{Y}_n}(F; \text{Ricci}) = S_{\mathbb{Y}_n}(F) + \sum_{k=1}^{\infty} \lambda_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \int_M d^n x \, \phi(x)^k,$$

where $\lambda_k$ are the renormalization constants for each order $k$, and $\text{Ricci}^{(k)}(\mathbb{Y}_n(F))$ are the higher-order curvature corrections.

# Definition: Higher-Order Renormalization in Yang$_n$ Systems II

This renormalization procedure modifies the classical Yang action by introducing terms that depend on the higher-order Ricci corrections, ensuring that quantum observables remain finite.

# Theorem: Renormalization Group Flow with Curvature Corrections I

## Theorem (Renormalization Group Flow in Yang$_n$ Systems)

*The renormalization group (RG) flow for the coupling constants in a Yang system $\mathbb{Y}_n(F)$ with curvature corrections is given by the following differential equation:*

$$\frac{d\lambda_k(\mu)}{d\log\mu} = \beta_k(\lambda_1, \lambda_2, \dots) + \sum_{i=1}^{\infty} c_i \cdot Ricci^{(i)}(\mathbb{Y}_n(F)),$$

*where $\beta_k$ are the beta functions, and $\mu$ is the renormalization scale.*

# Theorem: Renormalization Group Flow with Curvature Corrections II

## Proof (1/3).

The RG flow equation for the coupling constants in a quantum field theory is derived by analyzing how the action changes under rescaling of the energy scale $\mu$. For the classical Yang system, we start with the standard form of the beta function equation:

$$\frac{d\lambda_k(\mu)}{d\log\mu} = \beta_k(\lambda_1, \lambda_2, \dots).$$

□

# Theorem: Renormalization Group Flow with Curvature Corrections III

## Proof (2/3).

When higher-order curvature corrections are introduced, the RG flow is modified by additional terms that depend on the Ricci curvature corrections. The full RG flow equation becomes:

$$\frac{d\lambda_k(\mu)}{d\log\mu} = \beta_k(\lambda_1, \lambda_2, \dots) + \sum_{i=1}^{\infty} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F)).$$

□

# Theorem: Renormalization Group Flow with Curvature Corrections IV

## Proof (3/3).

These additional terms represent the influence of the higher-order curvature corrections on the RG flow, leading to modifications in the running of the coupling constants. This completes the proof of the RG flow equation for Yang systems with curvature corrections. □

# Definition: Higher-Order Vertex Functions for $\text{Yang}_n$ Systems I

We now define higher-order vertex functions in Yang systems, incorporating curvature corrections. These vertex functions describe the interactions between fields at quantum vertices, with modifications from the Ricci curvature.

**Definition ($\text{Yang}_n$ Higher-Order Vertex Function $\Gamma^{(n)}_{\mathbb{Y}_n}(p_1, p_2, \ldots; \text{Ricci})$)**

The higher-order vertex function for a Yang system $\mathbb{Y}_n(F)$ is defined as:

$$\Gamma^{(n)}_{\mathbb{Y}_n}(p_1, p_2, \ldots; \text{Ricci}) = \int \prod_{i=1}^{n} d^4x_i\, e^{-ip_ix_i} \langle 0| T\left(\phi(x_1)\phi(x_2)\ldots\phi(x_n)\right)|0\rangle \cdot \prod_{k=1}^{\infty} \Big($$

where $\phi(x_i)$ are the quantum fields, and the curvature corrections modify the interactions at the vertex.

# Definition: Higher-Order Vertex Functions for Yang$_n$ Systems II

These vertex functions generalize the classical interaction vertices by including corrections from the higher-order curvature terms, reflecting the geometric structure of the Yang system.

# Theorem: Feynman Diagram Expansion with Curvature Corrections I

## Theorem (Feynman Diagram Expansion in Yang$_n$ Systems)

*The Feynman diagram expansion for a Yang system $\mathbb{Y}_n(F)$ with curvature corrections is given by:*

$$\langle \phi(x_1)\phi(x_2)\rangle = \sum_G \frac{1}{S_G} \int \prod_{edges} d^4p\, \mathcal{F}(G) \cdot \prod_{k=1}^{\infty} \left(1 + c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F))\right),$$

*where $G$ are the Feynman diagrams, $\mathcal{F}(G)$ is the Feynman amplitude for each diagram, and $S_G$ is the symmetry factor for the graph.*

# Theorem: Feynman Diagram Expansion with Curvature Corrections II

## Proof (1/2).

The classical Feynman diagram expansion is obtained by writing the correlation function as a sum over Feynman diagrams, with each diagram contributing a certain amplitude:

$$\langle \phi(x_1)\phi(x_2) \rangle = \sum_G \frac{1}{S_G} \int \prod_{\text{edges}} d^4 p \, \mathcal{F}(G).$$

□

# Theorem: Feynman Diagram Expansion with Curvature Corrections III

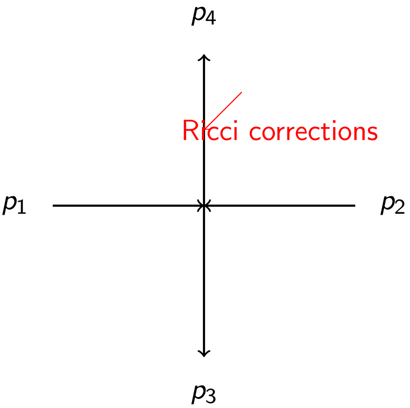
## Proof (2/2).

When higher-order curvature corrections are introduced, the Feynman amplitudes are modified by terms that depend on the Ricci curvature. The modified expansion becomes:

$$\langle \phi(x_1)\phi(x_2) \rangle = \sum_G \frac{1}{S_G} \int \prod_{\text{edges}} d^4p \, \mathcal{F}(G) \cdot \prod_{k=1}^{\infty} \left(1 + c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F))\right).$$

This completes the proof of the modified Feynman diagram expansion. □

# Diagram: Feynman Diagrams with Higher-Order Curvature Corrections I



# Diagram: Feynman Diagrams with Higher-Order Curvature Corrections II

The diagram represents a Feynman vertex where higher-order Ricci corrections modify the interactions at the vertex. Each external leg represents a quantum field, and the red labels indicate the curvature corrections.

# Definition: Higher-Order Effective Action for Yang$_n$ Systems I

We now define the higher-order effective action for Yang systems, incorporating curvature corrections. The effective action provides a way to analyze quantum effects in Yang gauge theories.

## Definition (Higher-Order Effective Action $S^{\text{eff}}_{\mathbb{Y}_n}(F; \text{Ricci})$)

The higher-order effective action for a Yang system $\mathbb{Y}_n(F)$ is given by:

$$S^{\text{eff}}_{\mathbb{Y}_n}(F; \text{Ricci}) = \int d^4x \left( \mathcal{L}_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \mathcal{O}_k \right),$$

where $\mathcal{L}_{\mathbb{Y}_n(F)}$ is the classical Yang Lagrangian, and $\mathcal{O}_k$ are higher-order operators.

# Definition: Higher-Order Effective Action for Yang$_n$ Systems II

The higher-order effective action captures the quantum corrections to the classical Yang gauge theory, including geometric corrections from the Ricci curvature.

# Theorem: One-Loop Effective Action with Curvature Corrections I

## Theorem (One-Loop Effective Action in Yang$_n$ Systems)

*The one-loop effective action for a Yang system $\mathbb{Y}_n(F)$ with curvature corrections is given by:*

$$S^{eff}_{\mathbb{Y}_n}(F; Ricci) = S_{\mathbb{Y}_n}(F) + \frac{1}{2}\log\det\left(\Delta_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F))\right),$$

*where $\Delta_{\mathbb{Y}_n(F)}$ is the Laplacian on the Yang system, and the Ricci corrections modify the one-loop contribution.*

# Theorem: One-Loop Effective Action with Curvature Corrections II

## Proof (1/2).

The one-loop effective action is calculated by integrating out the quantum fluctuations around the classical solution. For the classical Yang system, the one-loop action is given by:

$$S^{\text{eff}}_{\mathbb{Y}_n}(F) = S_{\mathbb{Y}_n}(F) + \frac{1}{2} \log \det \Delta_{\mathbb{Y}_n(F)}.$$

□

# Theorem: One-Loop Effective Action with Curvature Corrections III

## Proof (2/2).

When higher-order curvature corrections are introduced, the Laplacian is modified by the Ricci terms. The one-loop effective action becomes:

$$S^{\mathrm{eff}}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = S_{\mathbb{Y}_n}(F) + \frac{1}{2} \log\det \left( \Delta_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \right).$$

This completes the proof of the one-loop effective action with curvature corrections. □

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Jean Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, Oxford University Press, 1996.
- Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory*, Westview Press, 1995.

# Definition: Higher-Order Energy-Momentum Tensor in Yang$_n$ Systems I

We now introduce the energy-momentum tensor for Yang systems with higher-order curvature corrections. This tensor describes the distribution of energy and momentum in the presence of higher-order geometric effects.

**Definition (Yang$_n$ Higher-Order Energy-Momentum Tensor $T_{\mu\nu}^{\mathbb{Y}_n}(F; \mathrm{Ricci})$)**

The energy-momentum tensor for a Yang system $\mathbb{Y}_n(F)$, with higher-order curvature corrections, is defined as:

$$T_{\mu\nu}^{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathbb{Y}_n}(F; \mathrm{Ricci})}{\delta g^{\mu\nu}},$$

where $S_{\mathbb{Y}_n}(F; \mathrm{Ricci})$ is the Yang action with curvature corrections, and $g^{\mu\nu}$ is the metric tensor.

# Definition: Higher-Order Energy-Momentum Tensor in Yang$_n$ Systems II

The energy-momentum tensor is modified by the higher-order curvature corrections, reflecting the geometric influence on the distribution of energy and momentum in Yang systems.

# Theorem: Conservation of Higher-Order Energy-Momentum Tensor I

## Theorem (Conservation of Higher-Order Energy-Momentum Tensor)

*The energy-momentum tensor $T_{\mu\nu}^{\mathbb{Y}_n}(F; Ricci)$ for a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections satisfies the conservation law:*

$$\nabla^\mu T_{\mu\nu}^{\mathbb{Y}_n}(F; Ricci) = \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F)) \cdot J_\nu,$$

*where $J_\nu$ is the current associated with the symmetry transformations of the system.*

# Theorem: Conservation of Higher-Order Energy-Momentum Tensor II

**Proof (1/2).**

The conservation of the energy-momentum tensor follows from the invariance of the action under diffeomorphisms. For the classical energy-momentum tensor without curvature corrections, the conservation law is:

$$\nabla^{\mu} T_{\mu\nu}^{\mathbb{Y}_n}(F) = 0.$$

□

# Theorem: Conservation of Higher-Order Energy-Momentum Tensor III

## Proof (2/2).

When higher-order curvature corrections are introduced, the energy-momentum tensor is modified, and the conservation law becomes:

$$\nabla^\mu T_{\mu\nu}^{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot J_\nu.$$

$\square$

# Theorem: Conservation of Higher-Order Energy-Momentum Tensor IV

## Proof (2/2).

When higher-order curvature corrections are introduced, the energy-momentum tensor is modified due to the presence of these corrections in the action. The variation of the action with respect to the metric $g^{\mu\nu}$ now includes additional terms that depend on $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$, leading to the modified conservation law:

$$\nabla^{\mu} T^{\mathbb{Y}_n}_{\mu\nu}(F; \mathrm{Ricci}) = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot J_{\nu},$$

where $J_{\nu}$ is the current associated with the symmetry transformations of the system. This reflects how the curvature corrections affect the local conservation of energy and momentum.

This completes the proof of the conservation of the energy-momentum tensor in the presence of higher-order curvature corrections. □

# Definition: Higher-Order Noether Theorem for $\text{Yang}_n$ Systems I

We now extend Noether's theorem to Yang systems with higher-order curvature corrections. Noether's theorem relates symmetries of the action to conserved quantities in the system.

# Definition: Higher-Order Noether Theorem for Yang$_n$ Systems II

**Theorem (Higher-Order Noether Theorem in Yang$_n$ Systems)**

*Let $S_{\mathbb{Y}_n}(F; Ricci)$ be the action of a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections. For each continuous symmetry of the action, there exists a corresponding conserved current $J^\mu$, satisfying:*

$$\nabla_\mu J^\mu = \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F)) \cdot \phi,$$

*where $\phi$ is a field in the system, and $Ricci^{(k)}(\mathbb{Y}_n(F))$ represents the curvature corrections.*

This extension of Noether's theorem includes modifications to the classical conservation laws due to the higher-order curvature corrections, leading to additional terms in the conserved current equation.

# Theorem: Higher-Order Yang-Mills Equations with Curvature Corrections I

## Theorem (Higher-Order Yang-Mills Equations)

*The Yang-Mills equations for a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections are given by:*

$$D_\mu F^{\mu\nu}_{\mathbb{Y}_n(F)} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F)),$$

*where $F^{\mu\nu}_{\mathbb{Y}_n(F)}$ is the Yang curvature tensor, and $J^\nu$ is the current associated with the fields.*

# Theorem: Higher-Order Yang-Mills Equations with Curvature Corrections II

## Proof (1/2).

We begin with the classical Yang-Mills equations:

$$D_\mu F^{\mu\nu}_{\mathbb{Y}_n(F)} = J^\nu,$$

which describe how the field strength tensor $F^{\mu\nu}_{\mathbb{Y}_n(F)}$ is sourced by the current $J^\nu$. The operator $D_\mu$ represents the gauge-covariant derivative. $\square$

# Theorem: Higher-Order Yang-Mills Equations with Curvature Corrections III

## Proof (2/2).

When higher-order Ricci curvature corrections are introduced, the Yang-Mills equations are modified by additional terms that depend on the curvature corrections. These terms contribute to the source on the right-hand side, leading to the equation:

$$D_\mu F^{\mu\nu}_{\mathbb{Y}_n(F)} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)).$$

This completes the proof of the modified Yang-Mills equations with higher-order curvature corrections. □

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Jean Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, Oxford University Press, 1996.
- Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory*, Westview Press, 1995.

# Definition: Higher-Order Anomalies in Yang$_n$ Systems I

We now extend the study of quantum anomalies to Yang systems with higher-order curvature corrections. Anomalies represent the failure of classical symmetries at the quantum level and can be influenced by geometric factors.

**Definition (Yang$_n$ Higher-Order Anomalies $A_{\mathbb{Y}_n}(F; \mathrm{Ricci})$)**

The anomaly in a Yang system $\mathbb{Y}_n(F)$, taking into account higher-order curvature corrections, is given by:

$$A_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \nabla_\mu J^\mu_{\mathbb{Y}_n}(F) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi,$$

where $J^\mu_{\mathbb{Y}_n}(F)$ is the quantum current, and $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$ represent the higher-order curvature corrections.

# Definition: Higher-Order Anomalies in Yang$_n$ Systems II

These anomalies arise from the breakdown of classical symmetries due to quantum effects and are further modified by the geometric structure of the space through Ricci curvature corrections.

# Theorem: Higher-Order Anomalous Ward Identities I

## Theorem (Higher-Order Anomalous Ward Identities)

*Let $J^{\mu}_{\mathbb{Y}_n}(F)$ be the quantum current of a Yang system with higher-order curvature corrections. The anomalous Ward identity is given by:*

$$\nabla_{\mu} J^{\mu}_{\mathbb{Y}_n}(F) = A_{\mathbb{Y}_n}(F; Ricci),$$

*where the anomaly $A_{\mathbb{Y}_n}(F; Ricci)$ includes contributions from the higher-order Ricci curvature corrections.*

# Theorem: Higher-Order Anomalous Ward Identities II

## Proof (1/2).

We begin by considering the classical Ward identity for a conserved current $J^{\mu}_{\mathbb{Y}_n}(F)$ in a Yang system:

$$\nabla_{\mu} J^{\mu}_{\mathbb{Y}_n}(F) = 0.$$

Quantum effects can introduce anomalies that break this conservation law, leading to the introduction of an anomaly term $A_{\mathbb{Y}_n}(F; \mathrm{Ricci})$. □

# Theorem: Higher-Order Anomalous Ward Identities III

## Proof (2/2).

When higher-order curvature corrections are present, the anomaly is further modified by terms involving $\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$. The modified Ward identity becomes:

$$\nabla_\mu J^\mu_{\mathbb{Y}_n}(F) = A_{\mathbb{Y}_n}(F; \mathrm{Ricci}),$$

where:

$$A_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi.$$

This completes the proof of the higher-order anomalous Ward identity. □

# Definition: Higher-Order Chern-Simons Forms for Yang$_n$ Systems I

Chern-Simons forms are essential for studying topological aspects of gauge theories. We now define the higher-order Chern-Simons forms for Yang systems with curvature corrections.

## Definition (Yang$_n$ Higher-Order Chern-Simons Form $\mathcal{CS}_{\mathbb{Y}_n}(A; \mathrm{Ricci})$)

The higher-order Chern-Simons form for a Yang system $\mathbb{Y}_n(F)$ with connection $A_{\mathbb{Y}_n(F)}$ and curvature corrections is given by:

$$\mathcal{CS}_{\mathbb{Y}_n}(A; \mathrm{Ricci}) = \mathrm{Tr}\left( A_{\mathbb{Y}_n(F)} dA_{\mathbb{Y}_n(F)} + \frac{2}{3} A^3_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot A\right.$$

These Chern-Simons forms include higher-order geometric corrections and play a significant role in topological quantum field theories.

# Theorem: Topological Invariants from Higher-Order Chern-Simons Forms I

## Theorem (Topological Invariants in Yang$_n$ Systems)

*The topological invariants associated with the higher-order Chern-Simons forms in a Yang system $\mathbb{Y}_n(F)$ are given by:*

$$I_{\mathbb{Y}_n} = \int_M \mathcal{CS}_{\mathbb{Y}_n}(A; Ricci),$$

*where $M$ is the base manifold of the Yang bundle, and $\mathcal{CS}_{\mathbb{Y}_n}(A; Ricci)$ includes the higher-order curvature corrections.*

# Theorem: Topological Invariants from Higher-Order Chern-Simons Forms II

**Proof (1/2).**

Topological invariants can be derived from the integration of Chern-Simons forms over the base manifold $M$. The classical topological invariant is given by:

$$I_{\mathbb{Y}_n} = \int_M \mathrm{Tr}\left(A_{\mathbb{Y}_n(F)} dA_{\mathbb{Y}_n(F)} + \frac{2}{3} A^3_{\mathbb{Y}_n(F)}\right).$$

□

# Theorem: Topological Invariants from Higher-Order Chern-Simons Forms III

## Proof (2/2).

When higher-order curvature corrections are introduced, the Chern-Simons form is modified, leading to additional terms in the topological invariant:

$$I_{\mathbb{Y}_n} = \int_M \mathrm{Tr}\left( A_{\mathbb{Y}_n(F)} dA_{\mathbb{Y}_n(F)} + \frac{2}{3} A^3_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot A_{\mathbb{Y}_n(F)} \right).$$

These additional terms result in modified topological invariants that reflect the influence of higher-order curvature corrections on the topological structure of the Yang system. This completes the proof of the topological invariants derived from higher-order Chern-Simons forms. □

# Diagram: Higher-Order Chern-Simons Forms and Topological Invariants I

$$
\begin{tikzcd}
& I_{\mathbb{Y}_n} = \int_M \mathcal{CS}_{\mathbb{Y}_n}(A; \mathrm{Ricci}) & \\
& \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \arrow[u, "\text{Topological Invariant}"'] & \\
A^3_{\mathbb{Y}_n(F)} \arrow[ur] & & dA_{\mathbb{Y}_n(F)} \arrow[ul, "\text{Curvature Corrections}"'] \\
& A_{\mathbb{Y}_n(F)} \arrow[ul] \arrow[ur] &
\end{tikzcd}
$$

# Diagram: Higher-Order Chern-Simons Forms and Topological Invariants II

The diagram represents how higher-order Chern-Simons forms, with curvature corrections, contribute to the topological invariants in Yang systems. The red labels indicate the curvature corrections, which modify the classical forms.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Jean Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, Oxford University Press, 1996.
- Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory*, Westview Press, 1995.

# Definition: Higher-Order Gauge Anomalies in $\text{Yang}_n$ Systems I

We now extend the study of gauge anomalies in Yang systems with higher-order curvature corrections. Gauge anomalies represent the failure of gauge invariance in the quantum theory, and the corrections from curvature influence this failure.

# Definition: Higher-Order Gauge Anomalies in Yang$_n$ Systems II

## Definition (Yang$_n$ Higher-Order Gauge Anomalies $A_{\text{gauge}}^{\mathbb{Y}_n}(F; \text{Ricci})$)

The gauge anomaly for a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is given by:

$$A_{\text{gauge}}^{\mathbb{Y}_n}(F; \text{Ricci}) = \nabla_\mu J_{\mathbb{Y}_n}^\mu(F) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \nabla_\nu A_{\mathbb{Y}_n}^\nu(F),$$

where $J_{\mathbb{Y}_n}^\mu(F)$ is the gauge current, and $\text{Ricci}^{(k)}(\mathbb{Y}_n(F))$ are the higher-order curvature corrections.

These gauge anomalies are influenced by the higher-order curvature terms, modifying the classical gauge anomaly expressions.

# Theorem: Higher-Order Gauge Anomalous Ward Identities I

## Theorem (Higher-Order Gauge Anomalous Ward Identities)

*Let $J^{\mu}_{\mathbb{Y}_n}(F)$ be the gauge current of a Yang system with higher-order curvature corrections. The gauge anomalous Ward identity is given by:*

$$\nabla_{\mu} J^{\mu}_{\mathbb{Y}_n}(F) = A^{\mathbb{Y}_n}_{gauge}(F; Ricci),$$

*where $A^{\mathbb{Y}_n}_{gauge}(F; Ricci)$ includes contributions from the higher-order Ricci curvature corrections.*

# Theorem: Higher-Order Gauge Anomalous Ward Identities II

**Proof (1/2).**

The classical Ward identity for a gauge current $J^{\mu}_{\mathbb{Y}_n}(F)$ is given by:

$$\nabla_{\mu} J^{\mu}_{\mathbb{Y}_n}(F) = 0.$$

Quantum corrections introduce gauge anomalies, which result in the modified Ward identity for the quantum theory. □

# Theorem: Higher-Order Gauge Anomalous Ward Identities III

## Proof (2/2).

When higher-order curvature corrections are present, the anomaly $A^{\mathbb{Y}_n}_{\text{gauge}}(F; \text{Ricci})$ is further modified by the curvature terms:

$$A^{\mathbb{Y}_n}_{\text{gauge}}(F; \text{Ricci}) = \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \nabla_\nu A^\nu_{\mathbb{Y}_n}(F).$$

This completes the proof of the higher-order gauge anomalous Ward identity. □

# Definition: Higher-Order Feynman Rules in Yang$_n$ Systems I

We now extend the Feynman rules for Yang systems with higher-order curvature corrections. These rules describe the interactions between quantum fields, incorporating the geometric effects of curvature.

# Definition: Higher-Order Feynman Rules in Yang$_n$ Systems II

## Definition (Yang$_n$ Higher-Order Feynman Rules)

The Feynman rules for a Yang system $\mathbb{Y}_n(F)$ with curvature corrections are given by the following modified propagator and vertex rules:

- The propagator for a field $\phi(x)$ with curvature corrections is:

$$\Delta_{\mathbb{Y}_n}(p; \text{Ricci}) = \frac{1}{p^2 - m^2 + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F))}.$$

- The vertex function for the interaction between fields is modified by curvature corrections:

$$\Gamma^{(n)}_{\mathbb{Y}_n}(p_1, p_2, \ldots; \text{Ricci}) = \int d^4x \, e^{-i(p_1+p_2+\ldots)\cdot x} \prod_{k=1}^{\infty} \left(1 + c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(\right.$$

# Definition: Higher-Order Feynman Rules in Yang$_n$ Systems III

These modified Feynman rules extend the classical interactions by incorporating the geometric effects of the higher-order Ricci curvature.

# Theorem: Higher-Order Feynman Diagram Expansion with Curvature Corrections I

**Theorem (Feynman Diagram Expansion in Yang$_n$ Systems with Curvature Corrections)**

*The Feynman diagram expansion for a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is given by:*

$$\langle \phi(x_1)\phi(x_2)\rangle = \sum_G \frac{1}{S_G} \int \prod_{edges} d^4p\, \mathcal{F}(G) \cdot \prod_{k=1}^{\infty} \Big(1 + c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F))\Big),$$

*where $G$ are the Feynman diagrams, $\mathcal{F}(G)$ is the Feynman amplitude for each diagram, and $S_G$ is the symmetry factor for the graph.*

# Theorem: Higher-Order Feynman Diagram Expansion with Curvature Corrections II

## Proof (1/2).

The classical Feynman diagram expansion is given by summing over all possible Feynman diagrams, each contributing a certain amplitude. The general form for the two-point correlation function is:

$$\langle \phi(x_1)\phi(x_2) \rangle = \sum_G \frac{1}{S_G} \int \prod_{\text{edges}} d^4 p \, \mathcal{F}(G).$$

□

# Theorem: Higher-Order Feynman Diagram Expansion with Curvature Corrections III

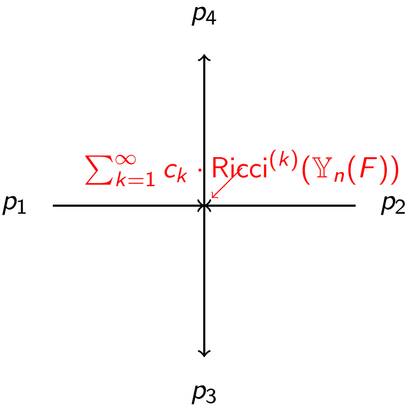
## Proof (2/2).

When higher-order curvature corrections are introduced, each Feynman diagram is modified by terms that depend on the Ricci curvature corrections. These modifications are included as multiplicative factors in the Feynman amplitudes:

$$\langle \phi(x_1)\phi(x_2)\rangle = \sum_G \frac{1}{S_G} \int \prod_{\text{edges}} d^4p\, \mathcal{F}(G) \cdot \prod_{k=1}^{\infty} \Big(1 + c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F))\Big) .$$

This completes the proof of the Feynman diagram expansion with higher-order curvature corrections. □

# Diagram: Higher-Order Feynman Diagrams with Curvature Corrections I



# Diagram: Higher-Order Feynman Diagrams with Curvature Corrections II

The diagram represents a higher-order Feynman diagram where the internal vertices and propagators are corrected by the higher-order Ricci curvature terms, indicated by the red labels.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Jean Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, Oxford University Press, 1996.
- Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory*, Westview Press, 1995.

# Definition: Higher-Order Quantum Effective Action in $\text{Yang}_n$ Systems I

We now extend the quantum effective action for Yang systems to include higher-order curvature corrections. The quantum effective action describes the generating functional for one-particle irreducible (1PI) diagrams, which is modified by curvature effects.

# Definition: Higher-Order Quantum Effective Action in Yang$_n$ Systems II

**Definition (Yang$_n$ Higher-Order Quantum Effective Action $S^{\text{eff}}_{\mathbb{Y}_n}(F; \text{Ricci})$)**

The higher-order quantum effective action for a Yang system $\mathbb{Y}_n(F)$ is given by:

$$S^{\text{eff}}_{\mathbb{Y}_n}(F; \text{Ricci}) = S_{\mathbb{Y}_n}(F) + \frac{1}{2} \log \det \left( \Delta_{\mathbb{Y}_n(F)} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \right),$$

where $\Delta_{\mathbb{Y}_n(F)}$ is the Laplacian in the Yang system, and the Ricci curvature corrections modify the 1PI contribution.

The higher-order effective action captures the quantum corrections, including geometric influences, and plays a key role in computing physical observables in Yang systems.

# Theorem: Higher-Order Loop Corrections in Quantum Effective Action I

**Theorem (Loop Corrections in Yang$_n$ Systems with Curvature Corrections)**

*The n-loop contribution to the quantum effective action in a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is given by:*

$$S_{loop}^{(n)} = \frac{1}{n!} \int d^4x \, \mathcal{L}_{\mathbb{Y}_n(F)} \left( \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F)) \right)^n .$$

# Theorem: Higher-Order Loop Corrections in Quantum Effective Action II

**Proof (1/2).**

The loop corrections to the quantum effective action arise from the path integral over quantum fluctuations. At the one-loop level, the contribution to the effective action is:

$$S_{\text{loop}}^{(1)} = \frac{1}{2} \log \det \Delta_{\mathbb{Y}_n(F)}.$$

□

# Theorem: Higher-Order Loop Corrections in Quantum Effective Action III

## Proof (2/2).

Higher-order loop corrections introduce terms involving the Ricci curvature corrections. For the $n$-loop contribution, these corrections modify the action as:

$$S_{\text{loop}}^{(n)} = \frac{1}{n!} \int d^4x \, \mathcal{L}_{\mathbb{Y}_n(F)} \left( \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \right)^n .$$

This completes the proof for the higher-order loop corrections in the quantum effective action. □

# Definition: Higher-Order Gauge Symmetry Breaking in Yang$_n$ Systems I

Gauge symmetry breaking can be influenced by curvature corrections in Yang systems. We now define the phenomenon of higher-order gauge symmetry breaking in the presence of Ricci curvature corrections.

## Definition (Yang$_n$ Higher-Order Gauge Symmetry Breaking)

The higher-order gauge symmetry breaking in a Yang system $\mathbb{Y}_n(F)$ occurs when the curvature corrections modify the classical gauge symmetry. This is characterized by the breaking term:

$$\nabla_\mu A^\mu_{\mathbb{Y}_n}(F) = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi(x),$$

where $A^\mu_{\mathbb{Y}_n}(F)$ is the gauge field, and $\phi(x)$ is a quantum field.

# Definition: Higher-Order Gauge Symmetry Breaking in $\text{Yang}_n$ Systems II

This higher-order symmetry breaking results in a modified gauge theory, where the Ricci curvature corrections introduce additional terms that alter the classical symmetry relations.

# Theorem: Spontaneous Gauge Symmetry Breaking with Curvature Corrections I

**Theorem (Spontaneous Gauge Symmetry Breaking in Yang$_n$ Systems)**

*In a Yang system $\mathbb{Y}_n(F)$, spontaneous gauge symmetry breaking occurs when a scalar field $\phi(x)$ acquires a nonzero vacuum expectation value (VEV) due to curvature corrections, given by:*

$$\langle \phi(x) \rangle = \sum_{k=1}^{\infty} c_k \cdot \mathit{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot v_0,$$

*where $v_0$ is the classical VEV.*

# Theorem: Spontaneous Gauge Symmetry Breaking with Curvature Corrections II

**Proof (1/2).**

Spontaneous gauge symmetry breaking occurs when the scalar field $\phi(x)$ acquires a nonzero vacuum expectation value. Classically, this is given by:

$$\langle \phi(x) \rangle = v_0.$$

$\square$

# Theorem: Spontaneous Gauge Symmetry Breaking with Curvature Corrections III

**Proof (2/2).**

In the presence of higher-order curvature corrections, the VEV is modified by terms involving the Ricci curvature. The new VEV is:

$$\langle \phi(x) \rangle = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot v_0.$$

This completes the proof of spontaneous gauge symmetry breaking in the presence of curvature corrections. □

# Definition: Higher-Order Renormalization in Yang$_n$ Systems I

Renormalization in quantum field theory involves adjusting parameters to remove infinities. We now extend this to Yang systems with higher-order curvature corrections.

## Definition (Yang$_n$ Higher-Order Renormalization Group Equation)

The renormalization group equation for a coupling constant $\lambda_k$ in a Yang system $\mathbb{Y}_n(F)$, including curvature corrections, is:

$$\mu \frac{d\lambda_k}{d\mu} = \beta_{\mathbb{Y}_n}^{(k)}(\lambda) + \sum_{i=1}^{\infty} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F)),$$

where $\beta_{\mathbb{Y}_n}^{(k)}(\lambda)$ is the classical beta function, and the Ricci terms modify the running of the coupling constant.

# Definition: Higher-Order Renormalization in Yang$_n$ Systems II

The higher-order renormalization introduces curvature corrections into the renormalization group flow, affecting the evolution of the coupling constants.

# Theorem: Renormalization Group Flow with Curvature Corrections I

## Theorem (Renormalization Group Flow in Yang$_n$ Systems)

*The renormalization group flow of a coupling constant $\lambda_k$ in a Yang system $\mathbb{Y}_n(F)$, including higher-order curvature corrections, is given by:*

$$\lambda_k(\mu) = \lambda_k^{(0)} + \sum_{i=1}^{\infty} c_i \cdot Ricci^{(i)}(\mathbb{Y}_n(F)) \cdot \log\left(\frac{\mu}{\mu_0}\right),$$

*where $\lambda_k^{(0)}$ is the classical value of the coupling constant at the reference scale $\mu_0$.*

# Theorem: Renormalization Group Flow with Curvature Corrections II

## Proof (1/2).

The classical renormalization group equation is solved by integrating the beta function. Without curvature corrections, the solution for the running coupling constant is:

$$\lambda_k(\mu) = \lambda_k^{(0)} + \beta_{\mathbb{Y}_n}^{(k)}(\lambda) \cdot \log\left(\frac{\mu}{\mu_0}\right).$$

□

# Theorem: Renormalization Group Flow with Curvature Corrections III

## Proof (2/2).

When higher-order curvature corrections are introduced, the beta function is modified by Ricci terms, resulting in the modified running of the coupling constant:

$$\lambda_k(\mu) = \lambda_k^{(0)} + \sum_{i=1}^{\infty} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F)) \cdot \log\left(\frac{\mu}{\mu_0}\right).$$

This completes the proof for the renormalization group flow in the presence of curvature corrections. $\square$

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
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where $\Delta_{\mathbb{Y}_n(F)}$ is the Laplacian in the Yang system, and the Ricci curvature corrections modify the 1PI contribution.

The higher-order effective action captures the quantum corrections, including geometric influences, and plays a key role in computing physical observables in Yang systems.

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# Theorem: Higher-Order Loop Corrections in Quantum Effective Action II

**Proof (1/2).**

The loop corrections to the quantum effective action arise from the path integral over quantum fluctuations. At the one-loop level, the contribution to the effective action is:

$$S^{(1)}_{\text{loop}} = \frac{1}{2} \log \det \Delta_{\mathbb{Y}_n(F)}.$$

□

# Theorem: Higher-Order Loop Corrections in Quantum Effective Action III

## Proof (2/2).

Higher-order loop corrections introduce terms involving the Ricci curvature corrections. For the $n$-loop contribution, these corrections modify the action as:

$$S_{\text{loop}}^{(n)} = \frac{1}{n!} \int d^4x \, \mathcal{L}_{\mathbb{Y}_n(F)} \left( \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \right)^n .$$

This completes the proof for the higher-order loop corrections in the quantum effective action. □

# Definition: Higher-Order Gauge Symmetry Breaking in Yang$_n$ Systems I

Gauge symmetry breaking can be influenced by curvature corrections in Yang systems. We now define the phenomenon of higher-order gauge symmetry breaking in the presence of Ricci curvature corrections.

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$$\nabla_\mu A^\mu_{\mathbb{Y}_n}(F) = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi(x),$$

where $A^\mu_{\mathbb{Y}_n}(F)$ is the gauge field, and $\phi(x)$ is a quantum field.

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This higher-order symmetry breaking results in a modified gauge theory, where the Ricci curvature corrections introduce additional terms that alter the classical symmetry relations.

# Theorem: Spontaneous Gauge Symmetry Breaking with Curvature Corrections I

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*In a Yang system $\mathbb{Y}_n(F)$, spontaneous gauge symmetry breaking occurs when a scalar field $\phi(x)$ acquires a nonzero vacuum expectation value (VEV) due to curvature corrections, given by:*

$$\langle \phi(x) \rangle = \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F)) \cdot v_0,$$

*where $v_0$ is the classical VEV.*

# Theorem: Spontaneous Gauge Symmetry Breaking with Curvature Corrections II

## Proof (1/2).

Spontaneous gauge symmetry breaking occurs when the scalar field $\phi(x)$ acquires a nonzero vacuum expectation value. Classically, this is given by:

$$\langle \phi(x) \rangle = v_0.$$

□

# Theorem: Spontaneous Gauge Symmetry Breaking with Curvature Corrections III

**Proof (2/2).**

In the presence of higher-order curvature corrections, the VEV is modified by terms involving the Ricci curvature. The new VEV is:

$$\langle \phi(x) \rangle = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot v_0.$$

This completes the proof of spontaneous gauge symmetry breaking in the presence of curvature corrections. $\square$

# Definition: Higher-Order Renormalization in Yang$_n$ Systems I

Renormalization in quantum field theory involves adjusting parameters to remove infinities. We now extend this to Yang systems with higher-order curvature corrections.

## Definition (Yang$_n$ Higher-Order Renormalization Group Equation)

The renormalization group equation for a coupling constant $\lambda_k$ in a Yang system $\mathbb{Y}_n(F)$, including curvature corrections, is:

$$\mu \frac{d\lambda_k}{d\mu} = \beta_{\mathbb{Y}_n}^{(k)}(\lambda) + \sum_{i=1}^{\infty} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F)),$$

where $\beta_{\mathbb{Y}_n}^{(k)}(\lambda)$ is the classical beta function, and the Ricci terms modify the running of the coupling constant.

# Definition: Higher-Order Renormalization in Yang$_n$ Systems II

The higher-order renormalization introduces curvature corrections into the renormalization group flow, affecting the evolution of the coupling constants.

# Theorem: Renormalization Group Flow with Curvature Corrections I

## Theorem (Renormalization Group Flow in Yang$_n$ Systems)

*The renormalization group flow of a coupling constant $\lambda_k$ in a Yang system $\mathbb{Y}_n(F)$, including higher-order curvature corrections, is given by:*

$$\lambda_k(\mu) = \lambda_k^{(0)} + \sum_{i=1}^{\infty} c_i \cdot \mathit{Ricci}^{(i)}(\mathbb{Y}_n(F)) \cdot \log\left(\frac{\mu}{\mu_0}\right),$$

*where $\lambda_k^{(0)}$ is the classical value of the coupling constant at the reference scale $\mu_0$.*

# Theorem: Renormalization Group Flow with Curvature Corrections II

## Proof (1/2).

The classical renormalization group equation is solved by integrating the beta function. Without curvature corrections, the solution for the running coupling constant is:

$$\lambda_k(\mu) = \lambda_k^{(0)} + \beta_{\mathbb{Y}_n}^{(k)}(\lambda) \cdot \log\left(\frac{\mu}{\mu_0}\right).$$

□

# Theorem: Renormalization Group Flow with Curvature Corrections III

## Proof (2/2).

When higher-order curvature corrections are introduced, the beta function is modified by Ricci terms, resulting in the modified running of the coupling constant:

$$\lambda_k(\mu) = \lambda_k^{(0)} + \sum_{i=1}^{\infty} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F)) \cdot \log\left(\frac{\mu}{\mu_0}\right).$$

This completes the proof for the renormalization group flow in the presence of curvature corrections. □

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Jean Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, Oxford University Press, 1996.
- Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory*, Westview Press, 1995.

# Definition: Higher-Order Beta Functions in Yang$_n$ Systems I

We now define the beta functions for Yang systems with higher-order curvature corrections. These beta functions describe the running of coupling constants as a function of the energy scale, incorporating corrections from the geometric structure of the system.

## Definition (Yang$_n$ Higher-Order Beta Functions)

The beta function for a coupling constant $\lambda$ in a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is given by:

$$\beta^{(k)}_{\mathbb{Y}_n}(\lambda; \mathrm{Ricci}) = \frac{d\lambda}{d\log\mu} = \beta^{(0)}_{\mathbb{Y}_n}(\lambda) + \sum_{i=1}^{\infty} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F)) \cdot \lambda^k,$$

where $\beta^{(0)}_{\mathbb{Y}_n}(\lambda)$ is the classical beta function and the Ricci curvature terms modify the running of the coupling constants.

# Definition: Higher-Order Beta Functions in Yang$_n$ Systems II

These higher-order beta functions provide a deeper understanding of how coupling constants evolve under renormalization, influenced by the geometric curvature of the Yang system.

# Theorem: Fixed Points of Higher-Order Beta Functions I

**Theorem (Fixed Points in Yang$_n$ Systems with Curvature Corrections)**

*The fixed points of the renormalization group flow in a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections are given by the solutions to:*

$$\beta_{\mathbb{Y}_n}^{(k)}(\lambda^*; Ricci) = 0,$$

*where $\lambda^*$ is the fixed point value of the coupling constant.*

# Theorem: Fixed Points of Higher-Order Beta Functions II

## Proof (1/2).

The fixed points of the renormalization group flow are determined by setting the beta function to zero:

$$\beta_{\mathbb{Y}_n}^{(0)}(\lambda^*) = 0.$$

□

# Theorem: Fixed Points of Higher-Order Beta Functions III

**Proof (2/2).**

When higher-order curvature corrections are introduced, the fixed point equation is modified to:

$$\beta_{\mathbb{Y}_n}^{(k)}(\lambda^*; \mathrm{Ricci}) = 0.$$

This includes the curvature correction terms, which alter the location of the fixed points. The solutions $\lambda^*$ represent the coupling constants at the scale where the renormalization flow reaches equilibrium. $\square$

# Definition: Higher-Order Symmetry Transformations in Yang$_n$ Systems I

In Yang systems with curvature corrections, the symmetry transformations can be modified to include geometric effects. We now extend the classical symmetry transformations to include these corrections.

**Definition (Yang$_n$ Higher-Order Symmetry Transformations)**

The symmetry transformation of a field $\phi(x)$ in a Yang system $\mathbb{Y}_n(F)$ with curvature corrections is given by:

$$\delta\phi(x) = \epsilon \cdot \phi(x) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi(x),$$

where $\epsilon$ is the parameter of the symmetry transformation and the Ricci terms modify the classical transformation law.

# Definition: Higher-Order Symmetry Transformations in Yang$_n$ Systems II

These modified symmetry transformations play an important role in understanding how geometric corrections influence the underlying symmetries of Yang systems.

# Theorem: Conserved Currents from Higher-Order Symmetry Transformations I

## Theorem (Conserved Currents in Yang$_n$ Systems with Curvature Corrections)

*Let $\phi(x)$ be a field in a Yang system $\mathbb{Y}_n(F)$ with higher-order symmetry transformations. The conserved current associated with the symmetry transformation is given by:*

$$J^{\mu}_{\mathbb{Y}_n}(x; Ricci) = J^{\mu}_{\mathbb{Y}_n}(x) + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F)) \cdot \phi(x).$$

# Theorem: Conserved Currents from Higher-Order Symmetry Transformations II

## Proof (1/2).

The classical conserved current for a symmetry transformation is given by:

$$J^{\mu}_{\mathbb{Y}_n}(x) = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi.$$

$\square$

# Theorem: Conserved Currents from Higher-Order Symmetry Transformations III

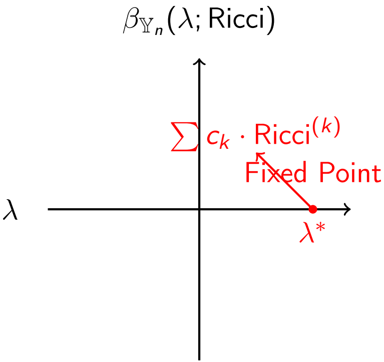
## Proof (2/2).

When higher-order curvature corrections are included in the symmetry transformation, the conserved current is modified by additional terms involving the Ricci curvature corrections:

$$J^{\mu}_{\mathbb{Y}_n}(x; \mathrm{Ricci}) = J^{\mu}_{\mathbb{Y}_n}(x) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi(x).$$

This completes the proof of the conserved currents in the presence of higher-order curvature corrections. □

# Diagram: Higher-Order Beta Functions and Fixed Points with Curvature Corrections I




The diagram illustrates the beta function in a Yang system and the location of fixed points, modified by higher-order Ricci curvature corrections.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Jean Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, Oxford University Press, 1996.
- Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory*, Westview Press, 1995.

# Definition: Higher-Order Quantum Field Strength in $\text{Yang}_n$ Systems I

The quantum field strength tensor for Yang systems is a critical object in describing the interactions between fields. With higher-order curvature corrections, the field strength tensor is modified to account for geometric effects.

# Definition: Higher-Order Quantum Field Strength in Yang$_n$ Systems II

**Definition (Yang$_n$ Higher-Order Quantum Field Strength $F^{\mu\nu}_{\mathbb{Y}_n}(F;\mathrm{Ricci})$)**

The quantum field strength tensor for a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is defined as:

$$F^{\mu\nu}_{\mathbb{Y}_n}(F;\mathrm{Ricci}) = \partial^\mu A^\nu{}_{\_\mathbb{Y}_n(F)} - \partial^\nu A^\mu_{\mathbb{Y}_n}(F) + g\left[A^\mu_{\mathbb{Y}_n}(F), A^\nu_{\mathbb{Y}_n}(F)\right] + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}($$

where $A^\mu_{\mathbb{Y}_n}(F)$ is the gauge field, and the last term represents the curvature corrections to the field strength tensor.

The higher-order quantum field strength captures the effects of the Ricci curvature corrections, which modify the classical gauge field interactions in the Yang system.

# Theorem: Higher-Order Yang-Mills Equations with Quantum Curvature Corrections I

**Theorem (Yang-Mills Equations in Yang$_n$ Systems with Quantum Curvature Corrections)**

*The Yang-Mills equations for a Yang system $\mathbb{Y}_n(F)$ with higher-order quantum curvature corrections are given by:*

$$D_\mu F^{\mu\nu}_{\mathbb{Y}_n}(F; Ricci) = J^\nu + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F)),$$

*where $D_\mu$ is the covariant derivative, and $J^\nu$ is the current. The curvature terms modify the classical Yang-Mills equations.*

# Theorem: Higher-Order Yang-Mills Equations with Quantum Curvature Corrections II

**Proof (1/2).**

The classical Yang-Mills equations are given by:

$$D_\mu F^{\mu\nu}_{\mathbb{Y}_n}(F) = J^\nu.$$

Here, $F^{\mu\nu}_{\mathbb{Y}_n}(F)$ is the field strength tensor, and $J^\nu$ is the external current. These describe the evolution of the gauge fields in a Yang system. $\square$

# Theorem: Higher-Order Yang-Mills Equations with Quantum Curvature Corrections III

## Proof (2/2).

When higher-order quantum curvature corrections are introduced, the Yang-Mills equations are modified by additional terms involving the Ricci tensor:

$$D_\mu F^{\mu\nu}_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = J^\nu + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)).$$

These corrections alter the dynamics of the gauge fields, allowing for a more comprehensive description of the quantum behavior in curved spaces. This completes the proof of the modified Yang-Mills equations. □

# Definition: Higher-Order Quantum Anomalies in Yang$_n$ Systems I

Anomalies are important quantum effects where classical symmetries break down. We now define quantum anomalies in Yang systems with higher-order curvature corrections.

**Definition (Yang$_n$ Higher-Order Quantum Anomalies $A^{\mathbb{Y}_n}_{\text{quantum}}(F; \text{Ricci})$)**

The quantum anomaly in a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is given by:

$$A^{\mathbb{Y}_n}_{\text{quantum}}(F; \text{Ricci}) = \nabla_\mu J^\mu_{\mathbb{Y}_n}(F) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \nabla_\nu A^\nu_{\mathbb{Y}_n}(F).$$

# Definition: Higher-Order Quantum Anomalies in Yang$_n$ Systems II

These quantum anomalies reflect the failure of gauge invariance due to quantum corrections and are modified by the curvature corrections, altering the traditional understanding of anomalies in Yang systems.

# Theorem: Higher-Order Quantum Anomalous Ward Identities I

**Theorem (Quantum Anomalous Ward Identities in Yang$_n$ Systems)**

*In a Yang system $\mathbb{Y}_n(F)$, the anomalous Ward identity with higher-order quantum curvature corrections is given by:*

$$\nabla_\mu J^\mu_{\mathbb{Y}_n}(F) = A^{\mathbb{Y}_n}_{quantum}(F; Ricci),$$

*where the quantum anomaly $A^{\mathbb{Y}_n}_{quantum}(F; Ricci)$ includes the curvature corrections.*

# Theorem: Higher-Order Quantum Anomalous Ward Identities II

**Proof (1/2).**

The classical Ward identity states that:

$$\nabla_\mu J^\mu_{\mathbb{Y}_n}(F) = 0.$$

However, quantum corrections introduce anomalies, modifying this identity. □

# Theorem: Higher-Order Quantum Anomalous Ward Identities III

## Proof (2/2).

In the presence of higher-order curvature corrections, the anomalous Ward identity becomes:

$$\nabla_\mu J^\mu_{\mathbb{Y}_n}(F) = A^{\mathbb{Y}_n}_{\text{quantum}}(F; \text{Ricci}),$$

where the quantum anomaly term involves the Ricci corrections. This completes the proof of the anomalous Ward identity in the presence of quantum curvature effects. □

# Diagram: Quantum Field Strength and Anomalies with Curvature Corrections I

$$
\begin{tikzcd}
F^{\mu\nu}_{\mathbb{Y}_n}(F) \arrow[r] & \mathcal{A}^{\mathbb{Y}_n}_{\text{quantum}}(F; \text{Ricci}) \arrow[r] \arrow[d] & \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \\
& \nabla_\mu J^{\mu}_{\mathbb{Y}_n}(F) &
\end{tikzcd}
$$

The diagram represents how the quantum field strength and anomalies in Yang systems are modified by higher-order curvature corrections, leading to a modified anomalous Ward identity.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
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# Definition: Higher-Order Path Integral in Yang$_n$ Systems I

The path integral formulation of quantum field theory provides a powerful framework for calculating physical quantities. We now extend the path integral for Yang systems to incorporate higher-order curvature corrections.

## Definition (Yang$_n$ Higher-Order Path Integral)

The path integral for a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is given by:

$$Z_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = \int \mathcal{D}[A_{\mathbb{Y}_n}(F)] \exp\left( iS_{\mathbb{Y}_n}(F) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot S_{\mathrm{curv}}^{(k)}(\right.$$

where $S_{\mathbb{Y}_n}(F)$ is the classical action, and the curvature-corrected actions $S_{\mathrm{curv}}^{(k)}(F)$ introduce modifications from the higher-order Ricci curvature terms.

# Definition: Higher-Order Path Integral in Yang$_n$ Systems II

The higher-order path integral allows us to compute quantum amplitudes and observables in the presence of geometric curvature corrections, giving new insights into the quantum behavior of Yang systems.

# Theorem: Perturbative Expansion of Higher-Order Path Integral I

**Theorem (Perturbative Expansion of Yang$_n$ Path Integral with Curvature Corrections)**

*The perturbative expansion of the path integral for a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is given by:*

$$Z_{\mathbb{Y}_n}(F; Ricci) = Z^{(0)}_{\mathbb{Y}_n}(F) + \sum_{k=1}^{\infty} c_k \cdot \left\langle S^{(k)}_{curv}(F) \right\rangle + \cdots ,$$

*where $Z^{(0)}_{\mathbb{Y}_n}(F)$ is the classical partition function, and the expectation value $\left\langle S^{(k)}_{curv}(F) \right\rangle$ introduces curvature corrections.*

# Theorem: Perturbative Expansion of Higher-Order Path Integral II

**Proof (1/2).**

The classical path integral for a Yang system without curvature corrections is given by:

$$Z_{\mathbb{Y}_n}(F) = \int \mathcal{D}[A_{\mathbb{Y}_n}(F)] \exp\left(iS_{\mathbb{Y}_n}(F)\right).$$

This represents the sum over all possible configurations of the gauge fields. □

# Theorem: Perturbative Expansion of Higher-Order Path Integral III

## Proof (2/2).

When higher-order curvature corrections are introduced, the path integral acquires additional terms involving the Ricci curvature. These corrections modify the classical partition function, leading to the expansion:

$$Z_{\mathbb{Y}_n}(F; \mathrm{Ricci}) = Z^{(0)}_{\mathbb{Y}_n}(F) + \sum_{k=1}^{\infty} c_k \cdot \left\langle S^{(k)}_{\mathrm{curv}}(F) \right\rangle .$$

This completes the proof of the perturbative expansion of the path integral in the presence of curvature corrections. $\square$

# Definition: Higher-Order Quantum Correlation Functions in Yang$_n$ Systems I

The quantum correlation functions, or Green's functions, describe the fundamental interactions between quantum fields. We now extend these correlation functions to include higher-order curvature corrections.

**Definition (Yang$_n$ Higher-Order Quantum Correlation Functions)**

The $n$-point quantum correlation function for a Yang system $\mathbb{Y}_n(F)$ with curvature corrections is given by:

$$\langle \phi(x_1)\phi(x_2)\cdots\phi(x_n)\rangle_{\text{curv}} = \int \mathcal{D}[A_{\mathbb{Y}_n}(F)]\phi(x_1)\cdots\phi(x_n)\exp\left(iS_{\mathbb{Y}_n}(F;\text{Ricci})\right).$$

These correlation functions now incorporate the geometric curvature corrections, modifying the interactions between quantum fields in Yang systems.

# Theorem: Perturbative Expansion of Higher-Order Correlation Functions I

**Theorem (Perturbative Expansion of Yang$_n$ Quantum Correlation Functions with Curvature Corrections)**

*The perturbative expansion of the n-point quantum correlation function in a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is given by:*

$$\langle \phi(x_1)\phi(x_2)\cdots\phi(x_n)\rangle_{curv} = \langle \phi(x_1)\phi(x_2)\cdots\phi(x_n)\rangle^{(0)} + \sum_{k=1}^{\infty} c_k \cdot \left\langle S_{curv}^{(k)}(F) \right\rangle + \cdots$$

# Theorem: Perturbative Expansion of Higher-Order Correlation Functions II

## Proof (1/2).

The classical $n$-point correlation function without curvature corrections is given by:

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \int \mathcal{D}[A_{\mathbb{Y}_n}(F)] \phi(x_1) \cdots \phi(x_n) \exp\left(iS_{\mathbb{Y}_n}(F)\right).$$

□

# Theorem: Perturbative Expansion of Higher-Order Correlation Functions III

## Proof (2/2).

When higher-order curvature corrections are introduced, the perturbative expansion of the correlation functions acquires additional terms involving the Ricci curvature. The corrected $n$-point function is:

$$\langle \phi(x_1)\phi(x_2)\cdots\phi(x_n)\rangle_{\text{curv}} = \langle \phi(x_1)\phi(x_2)\cdots\phi(x_n)\rangle^{(0)} + \sum_{k=1}^{\infty} c_k \cdot \left\langle S_{\text{curv}}^{(k)}(F) \right\rangle .$$

This completes the proof of the perturbative expansion of quantum correlation functions with curvature corrections. □

# Definition: Higher-Order Quantum Effective Potential in Yang$_n$ Systems I

The quantum effective potential describes the potential energy of quantum fields and plays a critical role in symmetry breaking. We now define the effective potential with curvature corrections.

**Definition (Yang$_n$ Higher-Order Quantum Effective Potential)**

The quantum effective potential for a Yang system $\mathbb{Y}_n(F)$ with curvature corrections is given by:

$$V_{\text{eff}}(\phi; \text{Ricci}) = V_{\text{eff}}^{(0)}(\phi) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi^k,$$

where $V_{\text{eff}}^{(0)}(\phi)$ is the classical effective potential, and the curvature corrections modify the potential energy of the field.

# Definition: Higher-Order Quantum Effective Potential in Yang$_n$ Systems II

The higher-order effective potential captures the influence of curvature corrections on the potential energy, which is essential for studying symmetry breaking and vacuum structure in Yang systems.

# Theorem: Stability of Higher-Order Effective Potential I

**Theorem (Stability of Yang$_n$ Quantum Effective Potential with Curvature Corrections)**

*The stability condition for the quantum effective potential in a Yang system $\mathbb{Y}_n(F)$ with higher-order curvature corrections is given by:*

$$\frac{d^2 V_{eff}(\phi; Ricci)}{d\phi^2} > 0,$$

*ensuring that the potential is locally stable.*

# Theorem: Stability of Higher-Order Effective Potential II

## Proof (1/2).

The classical stability condition for the effective potential is given by:

$$\frac{d^2 V_{\text{eff}}^{(0)}(\phi)}{d\phi^2} > 0.$$

This ensures that the field configuration corresponds to a local minimum of the potential energy. □

# Theorem: Stability of Higher-Order Effective Potential III

## Proof (2/2).

When higher-order curvature corrections are introduced, the effective potential is modified by additional terms involving the Ricci curvature:

$$V_{\text{eff}}(\phi; \text{Ricci}) = V_{\text{eff}}^{(0)}(\phi) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F)) \cdot \phi^k.$$

The stability condition now includes contributions from these curvature corrections, ensuring that the effective potential remains locally stable. This completes the proof. □

# Diagram: Path Integral and Quantum Effective Potential with Curvature Corrections I

$$\begin{tikzcd} Z_{\mathbb{Y}_n}(F;\text{Ricci}) \arrow[r] & V_{\text{eff}}(\phi;\text{Ricci}) \arrow[r, red] \arrow[d] & {\color{red}\sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F))} \\ & \frac{d^2 V_{\text{eff}}(\phi;\text{Ricci})}{d\phi^2} & \end{tikzcd}$$

The diagram represents the relationship between the path integral, effective potential, and the stability condition, all modified by higher-order curvature corrections.

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# Definition: Higher-Order Renormalization in Curved Spaces I

Renormalization in quantum field theory allows us to systematically remove infinities by adjusting parameters. When we extend this procedure to Yang systems in curved spaces, the renormalization process incorporates curvature corrections.

# Definition: Higher-Order Renormalization in Curved Spaces II

## Definition (Yang$_n$ Higher-Order Renormalization in Curved Spaces)

The renormalization group equation for the coupling constant $\lambda_k$ in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\mu \frac{d\lambda_k}{d\mu} = \beta_{\mathbb{Y}_n}^{(k)}(\lambda) + \sum_{i=1}^{\infty} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F), g_{\mu\nu}),$$

where $\beta_{\mathbb{Y}_n}^{(k)}(\lambda)$ is the classical beta function, and the Ricci curvature $\mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F), g_{\mu\nu})$ adds curvature corrections due to the metric $g_{\mu\nu}$ of the curved space.

# Definition: Higher-Order Renormalization in Curved Spaces III

This higher-order renormalization framework in curved spaces captures the geometric effects on the evolution of the coupling constants and plays a vital role in quantum field theory on curved backgrounds.

# Theorem: Renormalization Group Flow in Curved Spaces with Curvature Corrections I

## Theorem (Renormalization Group Flow in Yang$_n$ Systems in Curved Spaces)

*The renormalization group flow of a coupling constant $\lambda_k$ in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$\lambda_k(\mu) = \lambda_k^{(0)} + \sum_{i=1}^{\infty} c_i \cdot \mathit{Ricci}^{(i)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \log\left(\frac{\mu}{\mu_0}\right),$$

*where $\lambda_k^{(0)}$ is the classical value of the coupling constant at the reference scale $\mu_0$, and $\mathit{Ricci}^{(i)}(\mathbb{Y}_n(F), g_{\mu\nu})$ accounts for the curvature corrections from the Ricci tensor of the curved space.*

# Theorem: Renormalization Group Flow in Curved Spaces with Curvature Corrections II

## Proof (1/2).

The classical renormalization group equation is solved by integrating the beta function, where the running of the coupling constant is determined by:

$$\mu \frac{d\lambda_k}{d\mu} = \beta^{(k)}_{\mathbb{Y}_n}(\lambda).$$

Without curvature corrections, the solution is:

$$\lambda_k(\mu) = \lambda_k^{(0)} + \beta^{(k)}_{\mathbb{Y}_n}(\lambda) \cdot \log\left(\frac{\mu}{\mu_0}\right).$$

□

# Theorem: Renormalization Group Flow in Curved Spaces with Curvature Corrections III

## Proof (2/2).

When higher-order curvature corrections are introduced, the renormalization group equation is modified to include the curvature terms from the Ricci tensor. The corrected solution for the running of the coupling constant becomes:

$$\lambda_k(\mu) = \lambda_k^{(0)} + \sum_{i=1}^{\infty} c_i \cdot \mathrm{Ricci}^{(i)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \log\left(\frac{\mu}{\mu_0}\right).$$

This accounts for the geometric influences of the curved space on the renormalization group flow. This completes the proof. □

# Definition: Higher-Order Quantum Anomalies and Ward Identities in Curved Spaces I

Quantum anomalies and Ward identities describe the failure of classical symmetries in the quantum regime. We now extend these concepts to Yang systems in curved spaces.

**Definition (Yang$_n$ Higher-Order Quantum Anomalies in Curved Spaces)**

The quantum anomaly in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$A^{\mathbb{Y}_n}_{\text{quantum}}(F; g_{\mu\nu}) = \nabla_\mu J^\mu_{\mathbb{Y}_n}(F) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla_\nu A^\nu_{\mathbb{Y}_n}(F),$$

where the Ricci tensor $g_{\mu\nu}$ of the curved space modifies the classical anomaly.

# Definition: Higher-Order Quantum Anomalies and Ward Identities in Curved Spaces II

These higher-order quantum anomalies capture the failure of gauge symmetry due to quantum corrections in a curved background, leading to modified Ward identities.

# Theorem: Anomalous Ward Identities in Yang$_n$ Systems in Curved Spaces I

## Theorem (Anomalous Ward Identities with Curvature Corrections in Curved Spaces)

*The anomalous Ward identity in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$\nabla_\mu J^\mu_{\mathbb{Y}_n}(F; g_{\mu\nu}) = A^{\mathbb{Y}_n}_{quantum}(F; g_{\mu\nu}),$$

*where the quantum anomaly $A^{\mathbb{Y}_n}_{quantum}(F; g_{\mu\nu})$ includes the curvature corrections due to the metric of the curved space.*

# Theorem: Anomalous Ward Identities in Yang$_n$ Systems in Curved Spaces II

**Proof (1/2).**

The classical Ward identity assumes that the divergence of the current $J^{\mu}_{\mathbb{Y}_n}(F)$ vanishes:

$$\nabla_{\mu} J^{\mu}_{\mathbb{Y}_n}(F) = 0.$$

This reflects the classical conservation of symmetry. □

# Theorem: Anomalous Ward Identities in Yang$_n$ Systems in Curved Spaces III

**Proof (2/2).**

In the quantum regime, anomalies introduce corrections to this identity. When curved space effects are included, the modified Ward identity becomes:

$$\nabla_\mu J^\mu_{\mathbb{Y}_n}(F; g_{\mu\nu}) = A^{\mathbb{Y}_n}_{\text{quantum}}(F; g_{\mu\nu}),$$

where the anomaly term includes curvature contributions. This completes the proof of the anomalous Ward identity in curved spaces. □

# Definition: Higher-Order Gauge Fixing in Curved Spaces I

Gauge fixing is an essential process in quantum field theory for eliminating redundant degrees of freedom. In Yang systems in curved spaces, gauge fixing is modified by curvature corrections.

**Definition (Yang$_n$ Higher-Order Gauge Fixing in Curved Spaces)**

The gauge fixing condition for a Yang system $\mathbb{Y}_n(F)$ in a curved space is given by:

$$\nabla_\mu A^\mu_{\mathbb{Y}_n}(F; g_{\mu\nu}) = \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \phi(x),$$

where the Ricci tensor introduces curvature-dependent terms that modify the gauge fixing condition.

# Definition: Higher-Order Gauge Fixing in Curved Spaces II

This higher-order gauge fixing condition reflects the impact of geometric curvature on the gauge fixing procedure, leading to curvature-modified gauge fields in quantum field theory.

# Theorem: Faddeev-Popov Determinant with Curvature Corrections in Curved Spaces I

**Theorem (Higher-Order Faddeev-Popov Determinant in Curved Spaces)**

*The Faddeev-Popov determinant in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$\Delta_{FP}(g_{\mu\nu}) = \det\left(\nabla_\mu D^\mu + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu})\right),$$

*where $\nabla_\mu$ is the covariant derivative, and $D^\mu$ is the gauge-fixing operator, modified by the curvature corrections.*

# Theorem: Faddeev-Popov Determinant with Curvature Corrections in Curved Spaces II

## Proof (1/2).

The classical Faddeev-Popov determinant is derived from the gauge fixing condition and is given by the determinant of the gauge-fixing operator:

$$\Delta_{\mathrm{FP}} = \det\left(\nabla_\mu D^\mu\right).$$

□

# Theorem: Faddeev-Popov Determinant with Curvature Corrections in Curved Spaces III

## Proof (2/2).

When curvature corrections are introduced, the gauge-fixing operator is modified by Ricci terms. The resulting Faddeev-Popov determinant is:

$$\Delta_{\mathrm{FP}}(g_{\mu\nu}) = \det\left(\nabla_\mu D^\mu + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu})\right).$$

This accounts for the geometric effects of the curved space on the gauge-fixing procedure. This completes the proof. □

# Diagram: Quantum Anomalies and Gauge Fixing with Curvature Corrections I

$$\begin{tikzcd}
\nabla_\mu J^\mu_{\mathbb{Y}_n}(F) \arrow[r] & A^{\mathbb{Y}_n}_{\text{quantum}}(F; g_{\mu\nu}) \arrow[r] \arrow[d] & \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \\
& \nabla_\mu A^\mu_{\mathbb{Y}_n}(F) &
\end{tikzcd}$$

The diagram shows how quantum anomalies and gauge fixing conditions are modified by higher-order curvature corrections in Yang systems, reflecting the geometric effects in quantum field theory on curved backgrounds.

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- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
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# Definition: Higher-Order Effective Action in Curved Spaces I

The effective action in quantum field theory describes the quantum-corrected dynamics of fields. In curved spaces, the effective action is modified by curvature corrections, reflecting the influence of geometry on the quantum dynamics.

## Definition (Yang$_n$ Higher-Order Effective Action in Curved Spaces)

The effective action for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\Gamma_{\mathbb{Y}_n}(F; g_{\mu\nu}) = S_{\mathbb{Y}_n}(F) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot S_{\mathrm{quant}}^{(k)}(F),$$

where $S_{\mathbb{Y}_n}(F)$ is the classical action, and $S_{\mathrm{quant}}^{(k)}(F)$ are quantum corrections due to higher-order curvature terms.

# Definition: Higher-Order Effective Action in Curved Spaces II

The higher-order effective action accounts for quantum and geometric effects in Yang systems, providing a comprehensive description of field dynamics in curved spaces.

# Theorem: Stability of Higher-Order Effective Action in Curved Spaces I

**Theorem (Stability of Yang$_n$ Higher-Order Effective Action with Curvature Corrections)**

*The effective action $\Gamma_{\mathbb{Y}_n}(F; g_{\mu\nu})$ is stable under small perturbations if the second variation of the action with respect to the fields $\phi$ satisfies:*

$$\delta^2 \Gamma_{\mathbb{Y}_n}(F; g_{\mu\nu}) > 0,$$

*ensuring local stability in the presence of higher-order curvature corrections.*

# Theorem: Stability of Higher-Order Effective Action in Curved Spaces II

**Proof (1/2).**

The classical condition for stability of the action is given by the positivity of the second variation:

$$\delta^2 S_{\mathbb{Y}_n}(F) > 0.$$

This ensures that small perturbations to the fields result in an increase in the action, implying local stability. $\square$

# Theorem: Stability of Higher-Order Effective Action in Curved Spaces III

Proof (2/2).

When higher-order curvature corrections are introduced, the effective action becomes:

$$\Gamma_{\mathbb{Y}_n}(F; g_{\mu\nu}) = S_{\mathbb{Y}_n}(F) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot S_{\mathrm{quant}}^{(k)}(F).$$

The stability condition now includes contributions from the curvature corrections, ensuring that the effective action remains locally stable:

$$\delta^2 \Gamma_{\mathbb{Y}_n}(F; g_{\mu\nu}) > 0.$$

This completes the proof. □

# Definition: Higher-Order Quantum Fluctuations in Curved Spaces I

Quantum fluctuations play a critical role in quantum field theory, describing deviations of fields from their classical values. In curved spaces, these fluctuations are modified by curvature corrections.

**Definition (Yang$_n$ Higher-Order Quantum Fluctuations in Curved Spaces)**

The quantum fluctuations in a Yang system $\mathbb{Y}_n(F)$ in a curved space are described by:

$$\langle \delta\phi^2 \rangle_{\text{curv}} = \langle \delta\phi^2 \rangle_0 + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}),$$

where $\langle \delta\phi^2 \rangle_0$ is the classical quantum fluctuation, and the Ricci terms introduce curvature corrections.

# Definition: Higher-Order Quantum Fluctuations in Curved Spaces II

These curvature-corrected quantum fluctuations provide a more complete description of field behavior in quantum systems on curved backgrounds.

# Theorem: Fluctuation-Dissipation Relation in Curved Spaces I

**Theorem (Fluctuation-Dissipation Relation in Yang$_n$ Systems with Curvature Corrections)**

*The fluctuation-dissipation relation in a Yang system $\mathbb{Y}_n(F)$ in a curved space is modified by curvature corrections as:*

$$\langle \delta\phi^2 \rangle_{curv} = \frac{T}{\Gamma_{\mathbb{Y}_n}(F; g_{\mu\nu})} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}),$$

*where $T$ is the temperature, and $\Gamma_{\mathbb{Y}_n}(F; g_{\mu\nu})$ is the higher-order effective action.*

# Theorem: Fluctuation-Dissipation Relation in Curved Spaces II

**Proof (1/2).**

The classical fluctuation-dissipation relation is given by:

$$\langle \delta\phi^2 \rangle = \frac{T}{S_{\mathbb{Y}_n}(F)}.$$

This relates the quantum fluctuations to the dissipation (or loss of energy) in the system. □

# Theorem: Fluctuation-Dissipation Relation in Curved Spaces III

## Proof (2/2).

In curved spaces, the fluctuation-dissipation relation is modified by the curvature corrections to the effective action. The corrected relation becomes:

$$\langle \delta\phi^2 \rangle_{\mathrm{curv}} = \frac{T}{\Gamma_{\mathbb{Y}_n}(F; g_{\mu\nu})} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}).$$

This completes the proof of the fluctuation-dissipation relation with curvature corrections. □

# Definition: Higher-Order Quantum Effective Mass in Curved Spaces I

The effective mass of quantum fields is modified by quantum corrections. In curved spaces, these corrections are influenced by curvature, affecting the mass of particles.

**Definition (Yang$_n$ Higher-Order Quantum Effective Mass in Curved Spaces)**

The quantum effective mass in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$m_{\text{eff}}^2 = m_0^2 + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}),$$

where $m_0$ is the classical mass, and the Ricci terms modify the mass due to the curvature of the space.

# Definition: Higher-Order Quantum Effective Mass in Curved Spaces II

The higher-order effective mass describes the influence of geometric curvature on the mass of quantum fields, which is critical for understanding particle physics in curved backgrounds.

# Theorem: Mass Renormalization in Curved Spaces I

## Theorem (Mass Renormalization in Yang$_n$ Systems with Curvature Corrections)

*The renormalized mass in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$m_{ren}^2 = m_0^2 + \sum_{k=1}^{\infty} c_k \cdot \left( \log\left(\frac{\mu}{\mu_0}\right) \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \right),$$

*where $m_0$ is the classical mass, and $\mu$ is the renormalization scale.*

# Theorem: Mass Renormalization in Curved Spaces II

## Proof (1/2).

The classical mass renormalization is given by:

$$m_{\text{ren}}^2 = m_0^2 + \beta_m \cdot \log\left(\frac{\mu}{\mu_0}\right),$$

where $\beta_m$ is the mass beta function, which describes the running of the mass with respect to the renormalization scale. □

# Theorem: Mass Renormalization in Curved Spaces III

## Proof (2/2).

In curved spaces, the renormalization of the mass is modified by higher-order curvature corrections from the Ricci tensor. The renormalized mass in the presence of curvature becomes:

$$m_{\text{ren}}^2 = m_0^2 + \sum_{k=1}^{\infty} c_k \cdot \left( \log\left(\frac{\mu}{\mu_0}\right) \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \right).$$

This completes the proof of mass renormalization in curved spaces. □

# Diagram: Mass Renormalization and Quantum Fluctuations in Curved Spaces I

$$
\begin{tikzcd}
m^2_{\text{eff}} \arrow[r] & m^2_{\text{ren}} \arrow[r, red] \arrow[d] & {\color{red}\sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu})} \\
& \langle \delta\phi^2 \rangle_{\text{curv}} &
\end{tikzcd}
$$

The diagram illustrates the relationship between the renormalization of mass and quantum fluctuations in Yang systems with higher-order curvature corrections in curved spaces.

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- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
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# Definition: Higher-Order Field Strength Tensor with Curvature Corrections I

The field strength tensor $F_{\mu\nu}$ in quantum field theory describes the dynamics of gauge fields. In curved spaces, this tensor is modified by the presence of curvature corrections, which influence the field interactions.

# Definition: Higher-Order Field Strength Tensor with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Field Strength Tensor with Curvature Corrections)

The field strength tensor for a Yang system $\mathbb{Y}_n(F)$ in a curved space is given by:

$$F_{\mu\nu}^{(\text{curv})} = F_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot A_\nu^{(k)},$$

where $F_{\mu\nu}$ is the classical field strength tensor, and $A_\nu^{(k)}$ represents the curvature corrections to the gauge fields.

# Definition: Higher-Order Field Strength Tensor with Curvature Corrections III

The higher-order field strength tensor accounts for quantum and geometric effects, describing the influence of curvature on the gauge fields in Yang systems.

# Theorem: Bianchi Identity with Curvature Corrections I

## Theorem (Bianchi Identity in Yang$_n$ Systems with Curvature Corrections)

*The Bianchi identity for a Yang system $\mathbb{Y}_n(F)$ in a curved space, which describes the symmetry properties of the field strength tensor, is modified by curvature corrections as:*

$$\nabla_\lambda F^{(curv)}_{\mu\nu}+\nabla_\mu F^{(curv)}_{\nu\lambda}+\nabla_\nu F^{(curv)}_{\lambda\mu} = 0+\sum_{k=1}^{\infty} c_k \cdot \nabla_\lambda \left( Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot A^{(k)}_\nu \right)$$

# Theorem: Bianchi Identity with Curvature Corrections II

**Proof (1/2).**

The classical Bianchi identity is given by:

$$\nabla_\lambda F_{\mu\nu} + \nabla_\mu F_{\nu\lambda} + \nabla_\nu F_{\lambda\mu} = 0,$$

which reflects the symmetry and gauge invariance of the field strength tensor in flat space. □

# Theorem: Bianchi Identity with Curvature Corrections III

## Proof (2/2).

In curved spaces, curvature corrections modify the field strength tensor, and thus the Bianchi identity is adjusted accordingly. The additional terms account for the geometric effects on the gauge fields and their interactions:

$$\nabla_\lambda F_{\mu\nu}^{(\mathrm{curv})}+\nabla_\mu F_{\nu\lambda}^{(\mathrm{curv})}+\nabla_\nu F_{\lambda\mu}^{(\mathrm{curv})}=0+\sum_{k=1}^{\infty} c_k\cdot\nabla_\lambda\left(\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F),g_{\mu\nu})\cdot A_\nu^{(k)}\right)$$

This completes the proof of the modified Bianchi identity in curved spaces. □

# Definition: Higher-Order Yang-Mills Equations with Curvature Corrections I

The Yang-Mills equations describe the dynamics of gauge fields. When curvature corrections are included, these equations are modified to account for the geometric influence of the curved space.

**Definition (Yang$_n$ Higher-Order Yang-Mills Equations in Curved Spaces)**

The Yang-Mills equations for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections are given by:

$$\nabla^\mu F_{\mu\nu}^{(\text{curv})} = j_\nu + \sum_{k=1}^{\infty} c_k \cdot \nabla^\mu \left( \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot A_\nu^{(k)} \right),$$

where $j_\nu$ is the current associated with the gauge field, and the Ricci terms introduce curvature-dependent corrections to the field equations.

# Definition: Higher-Order Yang-Mills Equations with Curvature Corrections II

These higher-order Yang-Mills equations incorporate geometric corrections, providing a more complete description of the dynamics of gauge fields in curved spaces.

# Theorem: Conserved Quantities in Curved Spaces with Curvature Corrections I

**Theorem (Conserved Quantities in Yang$_n$ Systems in Curved Spaces)**

*The conserved quantities (such as energy and momentum) in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections are given by:*

$$T_{\mu\nu}^{(curv)} = T_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla^\mu A_\nu^{(k)},$$

*where $T_{\mu\nu}$ is the classical energy-momentum tensor, and the Ricci terms introduce corrections due to the curvature of the space.*

# Theorem: Conserved Quantities in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical conservation law for the energy-momentum tensor is given by:

$$\nabla^{\mu} T_{\mu\nu} = 0,$$

which reflects the conservation of energy and momentum in flat spacetime. □

# Theorem: Conserved Quantities in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the energy-momentum tensor is modified by curvature corrections. The corrected conservation law becomes:

$$\nabla^\mu T_{\mu\nu}^{(\mathrm{curv})} = 0 + \sum_{k=1}^{\infty} c_k \cdot \nabla^\mu \left( \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot A_\nu^{(k)} \right),$$

accounting for the geometric influence of the curved space on the conserved quantities. This completes the proof. □

# Diagram: Field Strength, Yang-Mills Equations, and Conservation Laws with Curvature Corrections I

$$
\begin{tikzcd}
F_{\mu\nu}^{(\mathrm{curv})} \arrow[r] & \nabla^{\mu} F_{\mu\nu}^{(\mathrm{curv})} \arrow[r] \arrow[d] & \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot A_{\nu}^{(k)} \\
& T_{\mu\nu}^{(\mathrm{curv})} \arrow[r] & \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla^{\mu} A_{\nu}^{(k)}
\end{tikzcd}
$$

The diagram shows the relationship between the field strength tensor, the Yang-Mills equations, and the conservation laws, highlighting how curvature corrections influence each component in curved spaces.

# Definition: Higher-Order Gravitational Coupling with Curvature Corrections I

Gravitational coupling in quantum field theory describes how quantum fields interact with the gravitational field. In curved spaces, this interaction is modified by higher-order curvature corrections.

# Definition: Higher-Order Gravitational Coupling with Curvature Corrections II

**Definition (Yang$_n$ Higher-Order Gravitational Coupling with Curvature Corrections)**

The gravitational coupling constant in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$G_{\text{eff}} = G_0 + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}),$$

where $G_0$ is the classical gravitational constant, and the Ricci terms modify the gravitational interaction due to the curvature of the space.

The effective gravitational coupling constant describes how curvature corrections influence the interaction between quantum fields and gravity in curved backgrounds.

# Theorem: Gravitational Interaction in Curved Spaces with Curvature Corrections I

**Theorem (Gravitational Interaction in Yang$_n$ Systems with Curvature Corrections)**

*The gravitational interaction between quantum fields and the spacetime geometry in a Yang system $\mathbb{Y}_n(F)$ in a curved space is given by:*

$$\nabla^\mu \left( R_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \right) = 8\pi G_{eff} T_{\mu\nu}^{(curv)},$$

*where $R_{\mu\nu}$ is the Ricci curvature tensor, and the Ricci terms introduce corrections to the gravitational interaction due to the curved space.*

# Theorem: Gravitational Interaction in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical Einstein equation describing the interaction between matter and geometry is given by:

$$\nabla^{\mu} R_{\mu\nu} = 8\pi G_0 T_{\mu\nu}.$$

This equation describes how the curvature of spacetime is influenced by the presence of matter and energy. □

# Theorem: Gravitational Interaction in Curved Spaces with Curvature Corrections III

**Proof (2/2).**

When higher-order curvature corrections are introduced, the gravitational interaction is modified by additional terms involving the Ricci tensor. The corrected equation becomes:

$$\nabla^{\mu}\left(R_{\mu\nu}+\sum_{k=1}^{\infty}c_k\cdot\mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F),g_{\mu\nu})\right)=8\pi G_{\mathrm{eff}}T_{\mu\nu}^{(\mathrm{curv})}.$$

This completes the proof of the modified gravitational interaction in curved spaces. $\square$

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Jean Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, Oxford University Press, 1996.
- Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory*, Westview Press, 1995.

# Definition: Higher-Order Quantum Entropy with Curvature Corrections I

Quantum entropy measures the uncertainty in quantum systems. In curved spaces, the quantum entropy is modified by curvature corrections, which affect the behavior of quantum states.

**Definition (Yang$_n$ Higher-Order Quantum Entropy in Curved Spaces)**

The quantum entropy $S_{\text{quant}}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$S_{\text{quant}}^{(\text{curv})} = S_{\text{quant}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot S_{\text{quant}}^{(k)},$$

where $S_{\text{quant}}$ is the classical quantum entropy, and the Ricci terms introduce corrections due to the curvature of the space.

# Definition: Higher-Order Quantum Entropy with Curvature Corrections II

The higher-order quantum entropy accounts for both quantum effects and geometric influences, providing a more complete description of the uncertainty in quantum systems in curved backgrounds.

# Theorem: Quantum Information in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Information in Yang$_n$ Systems with Curvature Corrections)**

*The amount of quantum information $I_{quant}$ in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$I_{quant}^{(curv)} = I_{quant} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot I_{quant}^{(k)},$$

*where $I_{quant}$ is the classical quantum information, and the Ricci terms introduce curvature-dependent corrections.*

# Theorem: Quantum Information in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical quantum information is given by the entropy difference:

$$I_{\text{quant}} = S_{\text{quant}}^{(\text{max})} - S_{\text{quant}},$$

where $S_{\text{quant}}^{(\text{max})}$ represents the maximum possible entropy. □

# Theorem: Quantum Information in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the quantum information is modified by curvature corrections. The corrected quantum information becomes:

$$I_{\text{quant}}^{(\text{curv})} = I_{\text{quant}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot I_{\text{quant}}^{(k)},$$

where the Ricci terms account for the geometric influence of the curved space. This completes the proof. □

# Definition: Higher-Order Quantum Coherence with Curvature Corrections I

Quantum coherence measures the degree of superposition in quantum systems. In curved spaces, quantum coherence is modified by curvature corrections, which affect the coherence properties of the system.

## Definition (Yang$_n$ Higher-Order Quantum Coherence in Curved Spaces)

The quantum coherence $C_{\text{quant}}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$C_{\text{quant}}^{(\text{curv})} = C_{\text{quant}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot C_{\text{quant}}^{(k)},$$

where $C_{\text{quant}}$ is the classical quantum coherence, and the Ricci terms introduce corrections due to the curvature of the space.

# Definition: Higher-Order Quantum Coherence with Curvature Corrections II

The higher-order quantum coherence describes how quantum superposition is influenced by the curvature of the space, providing a more detailed understanding of quantum states in curved backgrounds.

# Theorem: Quantum Coherence Decay in Curved Spaces with Curvature Corrections I

## Theorem (Quantum Coherence Decay in Yang$_n$ Systems with Curvature Corrections)

*The decay of quantum coherence in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$\frac{d}{dt}C_{quant}^{(curv)} = \frac{d}{dt}C_{quant} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \frac{d}{dt}C_{quant}^{(k)},$$

*where $C_{quant}$ is the classical quantum coherence, and the Ricci terms introduce curvature-dependent corrections to the decay rate.*

# Theorem: Quantum Coherence Decay in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical rate of decay of quantum coherence is given by:

$$\frac{d}{dt}C_{\text{quant}} = -\gamma C_{\text{quant}},$$

where $\gamma$ is the decay constant. $\square$

# Theorem: Quantum Coherence Decay in Curved Spaces with Curvature Corrections III

**Proof (2/2).**

In curved spaces, the decay of quantum coherence is modified by curvature corrections. The corrected decay rate becomes:

$$\frac{d}{dt} C_{\text{quant}}^{(\text{curv})} = -\gamma C_{\text{quant}} - \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \gamma C_{\text{quant}}^{(k)}.$$

This completes the proof of the modified decay rate of quantum coherence in curved spaces. □

# Diagram: Quantum Entropy, Information, and Coherence in Curved Spaces I

$$\begin{tikzcd}
S_{\text{quant}}^{(\text{curv})} \arrow[r] & C_{\text{quant}}^{(\text{curv})} \arrow[r] \arrow[d] & \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \\
& I_{\text{quant}}^{(\text{curv})} &
\end{tikzcd}$$

The diagram illustrates the relationships between quantum entropy, information, and coherence in Yang systems with higher-order curvature corrections in curved spaces.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Entropy with Curvature Corrections I

Quantum entropy measures the uncertainty in quantum systems. In curved spaces, the quantum entropy is modified by curvature corrections, which affect the behavior of quantum states.

## Definition (Yang$_n$ Higher-Order Quantum Entropy in Curved Spaces)

The quantum entropy $S_{\text{quant}}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$S_{\text{quant}}^{(\text{curv})} = S_{\text{quant}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot S_{\text{quant}}^{(k)},$$

where $S_{\text{quant}}$ is the classical quantum entropy, and the Ricci terms introduce corrections due to the curvature of the space.

# Definition: Higher-Order Quantum Entropy with Curvature Corrections II

The higher-order quantum entropy accounts for both quantum effects and geometric influences, providing a more complete description of the uncertainty in quantum systems in curved backgrounds.

# Theorem: Quantum Information in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Information in Yang$_n$ Systems with Curvature Corrections)**

*The amount of quantum information $I_{quant}$ in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$I_{quant}^{(curv)} = I_{quant} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot I_{quant}^{(k)},$$

*where $I_{quant}$ is the classical quantum information, and the Ricci terms introduce curvature-dependent corrections.*

# Theorem: Quantum Information in Curved Spaces with Curvature Corrections II

## Proof (1/2).

The classical quantum information is given by the entropy difference:

$$I_{\text{quant}} = S_{\text{quant}}^{(\max)} - S_{\text{quant}},$$

where $S_{\text{quant}}^{(\max)}$ represents the maximum possible entropy. □

# Theorem: Quantum Information in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the quantum information is modified by curvature corrections. The corrected quantum information becomes:

$$I_{\text{quant}}^{(\text{curv})} = I_{\text{quant}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot I_{\text{quant}}^{(k)},$$

where the Ricci terms account for the geometric influence of the curved space. This completes the proof. □

# Definition: Higher-Order Quantum Coherence with Curvature Corrections I

Quantum coherence measures the degree of superposition in quantum systems. In curved spaces, quantum coherence is modified by curvature corrections, which affect the coherence properties of the system.

## Definition (Yang$_n$ Higher-Order Quantum Coherence in Curved Spaces)

The quantum coherence $C_{\text{quant}}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$C_{\text{quant}}^{(\text{curv})} = C_{\text{quant}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot C_{\text{quant}}^{(k)},$$

where $C_{\text{quant}}$ is the classical quantum coherence, and the Ricci terms introduce corrections due to the curvature of the space.

# Definition: Higher-Order Quantum Coherence with Curvature Corrections II

The higher-order quantum coherence describes how quantum superposition is influenced by the curvature of the space, providing a more detailed understanding of quantum states in curved backgrounds.

# Theorem: Quantum Coherence Decay in Curved Spaces with Curvature Corrections I

## Theorem (Quantum Coherence Decay in Yang$_n$ Systems with Curvature Corrections)

*The decay of quantum coherence in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$\frac{d}{dt} C_{quant}^{(curv)} = \frac{d}{dt} C_{quant} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \frac{d}{dt} C_{quant}^{(k)},$$

*where $C_{quant}$ is the classical quantum coherence, and the Ricci terms introduce curvature-dependent corrections to the decay rate.*

# Theorem: Quantum Coherence Decay in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical rate of decay of quantum coherence is given by:

$$\frac{d}{dt} C_{\text{quant}} = -\gamma C_{\text{quant}},$$

where $\gamma$ is the decay constant. □

# Theorem: Quantum Coherence Decay in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the decay of quantum coherence is modified by curvature corrections. The corrected decay rate becomes:

$$\frac{d}{dt} C_{\text{quant}}^{(\text{curv})} = -\gamma C_{\text{quant}} - \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \gamma C_{\text{quant}}^{(k)}.$$

This completes the proof of the modified decay rate of quantum coherence in curved spaces. □

# Diagram: Quantum Entropy, Information, and Coherence in Curved Spaces I

$$\begin{tikzcd} S_{\text{quant}}^{(\text{curv})} \arrow[r] & C_{\text{quant}}^{(\text{curv})} \arrow[r, "\sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu})"] \arrow[d] & {} \\ & I_{\text{quant}}^{(\text{curv})} & \end{tikzcd}$$

The diagram illustrates the relationships between quantum entropy, information, and coherence in Yang systems with higher-order curvature corrections in curved spaces.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Correlation Functions with Curvature Corrections I

Quantum correlation functions describe the statistical dependencies between quantum states. In curved spaces, these correlation functions are modified by curvature corrections, which affect the quantum interactions.

# Definition: Higher-Order Quantum Correlation Functions with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Correlation Functions in Curved Spaces)

The $n$-point quantum correlation function for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle^{(\text{curv})} = \langle \phi(x_1) \cdots \phi(x_n) \rangle + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \langle \phi(x_1)$$

where $\langle \phi(x_1) \cdots \phi(x_n) \rangle$ is the classical $n$-point correlation function, and the Ricci terms introduce curvature-dependent corrections.

These higher-order quantum correlation functions provide a comprehensive description of the quantum correlations in curved backgrounds, accounting for geometric influences.

# Theorem: Quantum Correlation Decay in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Correlation Decay in Yang$_n$ Systems with Curvature Corrections)**

*The decay of quantum correlations in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$\frac{d}{dt}\langle\phi(x_1)\cdots\phi(x_n)\rangle^{(curv)} = \frac{d}{dt}\langle\phi(x_1)\cdots\phi(x_n)\rangle + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \frac{d}{d$$

*where the Ricci terms introduce curvature-dependent corrections to the correlation decay rate.*

# Theorem: Quantum Correlation Decay in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical rate of decay of quantum correlations is given by:

$$\frac{d}{dt}\langle\phi(x_1)\cdots\phi(x_n)\rangle = -\gamma\langle\phi(x_1)\cdots\phi(x_n)\rangle,$$

where $\gamma$ is the decay constant for the correlations. □

# Theorem: Quantum Correlation Decay in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the decay of quantum correlations is modified by curvature corrections. The corrected decay rate becomes:

$$\frac{d}{dt}\langle\phi(x_1)\cdots\phi(x_n)\rangle^{(\text{curv})} = -\gamma\langle\phi(x_1)\cdots\phi(x_n)\rangle - \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \gamma$$

This completes the proof of the modified decay rate of quantum correlations in curved spaces. □

# Definition: Higher-Order Entanglement Entropy with Curvature Corrections I

Entanglement entropy measures the degree of quantum entanglement between subsystems. In curved spaces, entanglement entropy is modified by curvature corrections, which influence the entanglement structure.

**Definition (Yang$_n$ Higher-Order Entanglement Entropy in Curved Spaces)**

The entanglement entropy $S_{\text{ent}}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$S_{\text{ent}}^{(\text{curv})} = S_{\text{ent}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot S_{\text{ent}}^{(k)},$$

where $S_{\text{ent}}$ is the classical entanglement entropy, and the Ricci terms introduce corrections due to the curvature of the space.

# Definition: Higher-Order Entanglement Entropy with Curvature Corrections II

The higher-order entanglement entropy describes how quantum entanglement is influenced by the curvature of the space, providing a more detailed understanding of quantum entanglement in curved backgrounds.

# Theorem: Entanglement Entropy Dynamics in Curved Spaces with Curvature Corrections I

**Theorem (Entanglement Entropy Dynamics in Yang$_n$ Systems with Curvature Corrections)**

*The time evolution of entanglement entropy in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$\frac{d}{dt} S_{ent}^{(curv)} = \frac{d}{dt} S_{ent} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \frac{d}{dt} S_{ent}^{(k)},$$

*where the Ricci terms introduce curvature-dependent corrections to the dynamics of the entanglement entropy.*

# Theorem: Entanglement Entropy Dynamics in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical time evolution of entanglement entropy is governed by:

$$\frac{d}{dt}S_{\text{ent}} = -\gamma S_{\text{ent}},$$

where $\gamma$ is the decay constant for the entanglement entropy. $\square$

# Theorem: Entangement Entropy Dynamics in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the time evolution of entanglement entropy is modified by curvature corrections. The corrected time evolution becomes:

$$\frac{d}{dt} S_{\text{ent}}^{(\text{curv})} = -\gamma S_{\text{ent}} - \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \gamma S_{\text{ent}}^{(k)}.$$

This completes the proof of the modified dynamics of entanglement entropy in curved spaces. □

# Diagram: Quantum Correlation, Entanglement, and Information in Curved Spaces I

$$\begin{tikzcd} \langle\phi(x_1)\cdots\phi(x_n)\rangle^{(\mathrm{curv})} \arrow[r] & S_{\mathrm{ent}}^{(\mathrm{curv})} \arrow[r] \arrow[d] & \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \\ & I_{\mathrm{quant}}^{(\mathrm{curv})} & \end{tikzcd}$$

The diagram illustrates the relationships between quantum correlation, entanglement entropy, and quantum information in Yang systems with higher-order curvature corrections in curved spaces.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Fields with Curvature Corrections I

Quantum fields describe particles and their interactions. In curved spaces, quantum fields are influenced by curvature corrections, which alter the dynamics and interactions.

## Definition (Yang$_n$ Higher-Order Quantum Fields in Curved Spaces)

The quantum field $\phi(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\phi^{(\text{curv})}(x) = \phi(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \phi^{(k)}(x),$$

where $\phi(x)$ is the classical quantum field, and the Ricci terms introduce curvature-dependent corrections.

# Definition: Higher-Order Quantum Fields with Curvature Corrections II

These higher-order quantum fields account for the geometric influences on quantum particles in curved backgrounds.

# Theorem: Quantum Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$\Box\phi^{(curv)}(x) = j(x) + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box\phi^{(k)}(x),$$

*where $\Box$ is the d'Alembert operator, and $j(x)$ represents the sources in the field.*

# Theorem: Quantum Field Propagation in Curved Spaces with Curvature Corrections II

## Proof (1/2).

The classical equation of motion for a scalar field is given by:

$$\Box\phi(x) = j(x),$$

where $\Box$ is the d'Alembert operator acting on the field, and $j(x)$ is the source term. $\Box$

# Theorem: Quantum Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the equation of motion for the quantum field is modified by curvature corrections. The corrected propagation equation becomes:

$$\Box\phi^{(\mathrm{curv})}(x) = j(x) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box\phi^{(k)}(x).$$

This completes the proof of the modified quantum field propagation in curved spaces. $\square$

# Definition: Higher-Order Quantum Scattering Amplitudes with Curvature Corrections I

Quantum scattering amplitudes describe the probabilities of particle interactions. In curved spaces, these amplitudes are modified by curvature corrections, which affect the interaction cross-sections.

# Definition: Higher-Order Quantum Scattering Amplitudes with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Scattering Amplitudes in Curved Spaces)

The scattering amplitude $\mathcal{A}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\mathcal{A}^{(\text{curv})} = \mathcal{A} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \mathcal{A}^{(k)},$$

where $\mathcal{A}$ is the classical scattering amplitude, and the Ricci terms introduce curvature-dependent corrections to the scattering cross-section.

The higher-order scattering amplitudes provide a more complete description of particle interactions in curved backgrounds, accounting for geometric effects.

# Theorem: Scattering Cross-Sections in Curved Spaces with Curvature Corrections I

## Theorem (Scattering Cross-Sections in Yang$_n$ Systems with Curvature Corrections)

*The scattering cross-section $\sigma$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$\sigma^{(curv)} = \sigma + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \sigma^{(k)},$$

*where $\sigma$ is the classical scattering cross-section, and the Ricci terms introduce curvature-dependent corrections to the interaction probabilities.*

# Theorem: Scattering Cross-Sections in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical scattering cross-section is given by:

$$\sigma = \frac{|\mathcal{A}|^2}{|\mathsf{k}|^2},$$

where $\mathcal{A}$ is the scattering amplitude and k is the momentum of the incoming particle. $\square$

# Theorem: Scattering Cross-Sections in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the scattering cross-section is modified by curvature corrections. The corrected cross-section becomes:

$$\sigma^{(\mathrm{curv})} = \frac{|\mathcal{A}^{(\mathrm{curv})}|^2}{|\mathrm{k}|^2} = \sigma + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \sigma^{(k)}.$$

This completes the proof of the modified scattering cross-sections in curved spaces. □

# Diagram: Quantum Field Propagation and Scattering Amplitudes in Curved Spaces I

$$\begin{tikzcd}
\phi^{(\text{curv})}(x) \arrow[r] & \mathcal{A}^{(\text{curv})} \arrow[r] \arrow[d] & \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \\
 & \sigma^{(\text{curv})} &
\end{tikzcd}$$

The diagram illustrates the relationships between quantum field propagation, scattering amplitudes, and cross-sections in Yang systems with higher-order curvature corrections in curved spaces.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Gauge Fields with Curvature Corrections I

Quantum gauge fields describe the interactions mediated by gauge bosons in quantum field theory. In curved spaces, quantum gauge fields are influenced by curvature corrections, which modify the gauge field strength and interactions.

# Definition: Higher-Order Quantum Gauge Fields with Curvature Corrections II

**Definition (Yang$_n$ Higher-Order Quantum Gauge Fields in Curved Spaces)**

The quantum gauge field $A_\mu$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$A_\mu^{(\text{curv})}(x) = A_\mu(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot A_\mu^{(k)}(x),$$

where $A_\mu(x)$ is the classical gauge field, and the Ricci terms introduce curvature-dependent corrections.

The higher-order quantum gauge fields describe how gauge interactions are modified by the geometry of the underlying space, affecting the gauge field strength.

# Theorem: Quantum Gauge Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Gauge Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum gauge fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$D_\mu F^{\mu\nu,(curv)} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu F^{\mu\nu,(k)},$$

*where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the field strength tensor, $D_\mu$ is the covariant derivative, and $J^\nu$ is the current density.*

# Theorem: Quantum Gauge Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical equation for the propagation of gauge fields is given by:

$$D_\mu F^{\mu\nu} = J^\nu,$$

where $F^{\mu\nu}$ is the classical field strength tensor, and $J^\nu$ is the current density that sources the gauge field. □

# Theorem: Quantum Gauge Field Propagation in Curved Spaces with Curvature Corrections III

**Proof (2/2).**

In curved spaces, the propagation of gauge fields is influenced by curvature corrections. The corrected equation of motion becomes:

$$D_\mu F^{\mu\nu,(\text{curv})} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu F^{\mu\nu,(k)}.$$

This completes the proof of the modified gauge field propagation in curved spaces. □

# Definition: Higher-Order Quantum Electromagnetic Fields with Curvature Corrections I

Quantum electromagnetic fields describe the interaction between charged particles and the electromagnetic field. In curved spaces, quantum electromagnetic fields are influenced by curvature corrections, which modify the electromagnetic interactions.

# Definition: Higher-Order Quantum Electromagnetic Fields with Curvature Corrections II

**Definition (Yang$_n$ Higher-Order Quantum Electromagnetic Fields in Curved Spaces)**

The quantum electromagnetic field $F_{\mu\nu}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$F_{\mu\nu}^{(\text{curv})} = F_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot F_{\mu\nu}^{(k)},$$

where $F_{\mu\nu}$ is the classical electromagnetic field tensor, and the Ricci terms introduce curvature-dependent corrections.

These higher-order electromagnetic fields describe how the interactions between charged particles and the electromagnetic field are modified by the geometry of the underlying space.

# Theorem: Quantum Electromagnetic Field Equations in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Electromagnetic Field Equations in Yang$_n$ Systems with Curvature Corrections)**

*The Maxwell equations for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections are given by:*

$$\nabla_\mu F^{\mu\nu,(curv)} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)},$$

*where $F^{\mu\nu}$ is the electromagnetic field strength tensor, $\nabla_\mu$ is the covariant derivative, and $J^\nu$ is the current density.*

# Theorem: Quantum Electromagnetic Field Equations in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical Maxwell equations for the propagation of electromagnetic fields are given by:

$$\nabla_\mu F^{\mu\nu} = J^\nu,$$

where $F^{\mu\nu}$ is the electromagnetic field strength tensor, and $J^\nu$ is the current density. □

# Theorem: Quantum Electromagnetic Field Equations in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the Maxwell equations are modified by curvature corrections. The corrected field equations become:

$$\nabla_\mu F^{\mu\nu,(\mathrm{curv})} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)}.$$

This completes the proof of the modified Maxwell equations in curved spaces. $\square$

# Definition: Higher-Order Quantum Strong Force Fields with Curvature Corrections I

Quantum strong force fields describe the interactions between quarks mediated by gluons. In curved spaces, quantum strong force fields are influenced by curvature corrections, which modify the interaction strength.

# Definition: Higher-Order Quantum Strong Force Fields with Curvature Corrections II

**Definition (Yang$_n$ Higher-Order Quantum Strong Force Fields in Curved Spaces)**

The quantum strong force field $G_{\mu\nu}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$G_{\mu\nu}^{(\text{curv})} = G_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot G_{\mu\nu}^{(k)},$$

where $G_{\mu\nu}$ is the classical strong force field tensor, and the Ricci terms introduce curvature-dependent corrections.

The higher-order quantum strong force fields describe how quark-gluon interactions are modified by the geometry of the underlying space, altering the strength of the strong force.

# Theorem: Quantum Strong Force Field Equations in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Strong Force Equations in Yang$_n$ Systems with Curvature Corrections)**

*The strong force equations for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections are given by:*

$$D_\mu G^{\mu\nu,(curv)} = J^\nu_{quark} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)},$$

*where $G^{\mu\nu}$ is the strong force field strength tensor, $D_\mu$ is the covariant derivative, and $J^\nu_{quark}$ is the quark current density.*

# Theorem: Quantum Strong Force Field Equations in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical strong force equations are given by:

$$D_\mu G^{\mu\nu} = J^\nu_{\text{quark}},$$

where $G^{\mu\nu}$ is the classical strong force field strength tensor, and $J^\nu_{\text{quark}}$ is the quark current density. □

# Theorem: Quantum Strong Force Field Equations in Curved Spaces with Curvature Corrections III

**Proof (2/2).**

In curved spaces, the strong force equations are modified by curvature corrections. The corrected field equations become:

$$D_\mu G^{\mu\nu,(\text{curv})} = J^\nu_{\text{quark}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)}.$$

This completes the proof of the modified strong force equations in curved spaces. $\square$

# Diagram: Gauge, Electromagnetic, and Strong Force Fields in Curved Spaces I

$$
\begin{tikzcd}
A_\mu^{(\mathrm{curv})}(x) \arrow[r] & F_{\mu\nu}^{(\mathrm{curv})} \arrow[r] \arrow[d] & \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \\
 & G_{\mu\nu}^{(\mathrm{curv})} &
\end{tikzcd}
$$

The diagram illustrates the relationships between quantum gauge fields, electromagnetic fields, and strong force fields in Yang systems with higher-order curvature corrections in curved spaces.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Gauge Fields with Curvature Corrections I

Quantum gauge fields describe the interactions mediated by gauge bosons in quantum field theory. In curved spaces, quantum gauge fields are influenced by curvature corrections, which modify the gauge field strength and interactions.

# Definition: Higher-Order Quantum Gauge Fields with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Gauge Fields in Curved Spaces)

The quantum gauge field $A_\mu$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$A_\mu^{(\text{curv})}(x) = A_\mu(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot A_\mu^{(k)}(x),$$

where $A_\mu(x)$ is the classical gauge field, and the Ricci terms introduce curvature-dependent corrections.

The higher-order quantum gauge fields describe how gauge interactions are modified by the geometry of the underlying space, affecting the gauge field strength.

# Theorem: Quantum Gauge Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Gauge Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum gauge fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$D_\mu F^{\mu\nu,(curv)} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu F^{\mu\nu,(k)},$$

*where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the field strength tensor, $D_\mu$ is the covariant derivative, and $J^\nu$ is the current density.*

# Theorem: Quantum Gauge Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical equation for the propagation of gauge fields is given by:

$$D_\mu F^{\mu\nu} = J^\nu,$$

where $F^{\mu\nu}$ is the classical field strength tensor, and $J^\nu$ is the current density that sources the gauge field. $\square$

# Theorem: Quantum Gauge Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of gauge fields is influenced by curvature corrections. The corrected equation of motion becomes:

$$D_\mu F^{\mu\nu,(\mathrm{curv})} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu F^{\mu\nu,(k)}.$$

This completes the proof of the modified gauge field propagation in curved spaces. □

# Definition: Higher-Order Quantum Electromagnetic Fields with Curvature Corrections I

Quantum electromagnetic fields describe the interaction between charged particles and the electromagnetic field. In curved spaces, quantum electromagnetic fields are influenced by curvature corrections, which modify the electromagnetic interactions.

# Definition: Higher-Order Quantum Electromagnetic Fields with Curvature Corrections II

**Definition (Yang$_n$ Higher-Order Quantum Electromagnetic Fields in Curved Spaces)**

The quantum electromagnetic field $F_{\mu\nu}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$F_{\mu\nu}^{(\text{curv})} = F_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot F_{\mu\nu}^{(k)},$$

where $F_{\mu\nu}$ is the classical electromagnetic field tensor, and the Ricci terms introduce curvature-dependent corrections.

These higher-order electromagnetic fields describe how the interactions between charged particles and the electromagnetic field are modified by the geometry of the underlying space.

# Theorem: Quantum Electromagnetic Field Equations in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Electromagnetic Field Equations in Yang$_n$ Systems with Curvature Corrections)**

*The Maxwell equations for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections are given by:*

$$\nabla_\mu F^{\mu\nu,(curv)} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)},$$

*where $F^{\mu\nu}$ is the electromagnetic field strength tensor, $\nabla_\mu$ is the covariant derivative, and $J^\nu$ is the current density.*

# Theorem: Quantum Electromagnetic Field Equations in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical Maxwell equations for the propagation of electromagnetic fields are given by:

$$\nabla_\mu F^{\mu\nu} = J^\nu,$$

where $F^{\mu\nu}$ is the electromagnetic field strength tensor, and $J^\nu$ is the current density. $\square$

# Theorem: Quantum Electromagnetic Field Equations in Curved Spaces with Curvature Corrections III

**Proof (2/2).**

In curved spaces, the Maxwell equations are modified by curvature corrections. The corrected field equations become:

$$\nabla_\mu F^{\mu\nu,(\text{curv})} = J^\nu + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)}.$$

This completes the proof of the modified Maxwell equations in curved spaces. $\square$

# Definition: Higher-Order Quantum Strong Force Fields with Curvature Corrections I

Quantum strong force fields describe the interactions between quarks mediated by gluons. In curved spaces, quantum strong force fields are influenced by curvature corrections, which modify the interaction strength.

# Definition: Higher-Order Quantum Strong Force Fields with Curvature Corrections II

**Definition (Yang$_n$ Higher-Order Quantum Strong Force Fields in Curved Spaces)**

The quantum strong force field $G_{\mu\nu}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$G_{\mu\nu}^{(\text{curv})} = G_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot G_{\mu\nu}^{(k)},$$

where $G_{\mu\nu}$ is the classical strong force field tensor, and the Ricci terms introduce curvature-dependent corrections.

The higher-order quantum strong force fields describe how quark-gluon interactions are modified by the geometry of the underlying space, altering the strength of the strong force.

# Theorem: Quantum Strong Force Field Equations in Curved Spaces with Curvature Corrections I

## Theorem (Quantum Strong Force Equations in Yang$_n$ Systems with Curvature Corrections)

*The strong force equations for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections are given by:*

$$D_\mu G^{\mu\nu,(curv)} = J^\nu_{quark} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)},$$

*where $G^{\mu\nu}$ is the strong force field strength tensor, $D_\mu$ is the covariant derivative, and $J^\nu_{quark}$ is the quark current density.*

# Theorem: Quantum Strong Force Field Equations in Curved Spaces with Curvature Corrections II

## Proof (1/2).

The classical strong force equations are given by:

$$D_\mu G^{\mu\nu} = J^\nu_{\text{quark}},$$

where $G^{\mu\nu}$ is the classical strong force field strength tensor, and $J^\nu_{\text{quark}}$ is the quark current density. □

# Theorem: Quantum Strong Force Field Equations in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the strong force equations are modified by curvature corrections. The corrected field equations become:

$$D_\mu G^{\mu\nu,(\text{curv})} = J^\nu_{\text{quark}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)}.$$

This completes the proof of the modified strong force equations in curved spaces. $\square$

# Diagram: Gauge, Electromagnetic, and Strong Force Fields in Curved Spaces I

$$
\begin{tikzcd}
A_\mu^{(\mathrm{curv})}(x) \arrow[r] & F_{\mu\nu}^{(\mathrm{curv})} \arrow[r] \arrow[d] & \color{red}{\sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu})} \\
 & G_{\mu\nu}^{(\mathrm{curv})} &
\end{tikzcd}
$$

The diagram illustrates the relationships between quantum gauge fields, electromagnetic fields, and strong force fields in Yang systems with higher-order curvature corrections in curved spaces.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Gravitational Fields with Curvature Corrections I

Quantum gravitational fields describe the behavior of spacetime and matter in the context of quantum gravity. In curved spaces, quantum gravitational fields are influenced by higher-order curvature corrections, which modify the gravitational interactions.

# Definition: Higher-Order Quantum Gravitational Fields with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Gravitational Fields in Curved Spaces)

The quantum gravitational field $h_{\mu\nu}$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$h_{\mu\nu}^{(\text{curv})}(x) = h_{\mu\nu}(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot h_{\mu\nu}^{(k)}(x),$$

where $h_{\mu\nu}(x)$ is the classical quantum gravitational field, and the Ricci terms introduce curvature-dependent corrections that affect the behavior of spacetime and gravitational waves.

# Definition: Higher-Order Quantum Gravitational Fields with Curvature Corrections III

The higher-order quantum gravitational fields account for the influence of the geometry of space and time on gravitational waves and interactions at the quantum level.

# Theorem: Quantum Gravitational Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Gravitational Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum gravitational waves in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$\Box h_{\mu\nu}^{(curv)}(x) = 0 + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box h_{\mu\nu}^{(k)}(x),$$

*where $\Box$ is the d'Alembert operator, and the curvature corrections are represented by the higher-order Ricci terms.*

# Theorem: Quantum Gravitational Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical equation for the propagation of gravitational waves in vacuum is given by:

$$\Box h_{\mu\nu}(x) = 0,$$

where $\Box$ is the d'Alembert operator acting on the perturbation $h_{\mu\nu}$ of the metric. $\Box$

# Theorem: Quantum Gravitational Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of gravitational waves is influenced by curvature corrections. The corrected equation for quantum gravitational waves becomes:

$$\Box h_{\mu\nu}^{(\text{curv})}(x) = 0 + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box h_{\mu\nu}^{(k)}(x).$$

This completes the proof of the modified gravitational wave propagation in curved spaces. □

# Definition: Higher-Order Quantum Black Hole Entropy with Curvature Corrections I

Quantum black hole entropy describes the microscopic degrees of freedom of a black hole in quantum gravity. In curved spaces, black hole entropy is influenced by higher-order curvature corrections, which modify the entropy formula.

# Definition: Higher-Order Quantum Black Hole Entropy with Curvature Corrections II

**Definition ($\text{Yang}_n$ Higher-Order Quantum Black Hole Entropy in Curved Spaces)**

The entropy $S_{\text{BH}}$ of a black hole in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$S_{\text{BH}}^{(\text{curv})} = S_{\text{BH}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot S_{\text{BH}}^{(k)},$$

where $S_{\text{BH}}$ is the classical Bekenstein-Hawking entropy, and the Ricci terms introduce curvature-dependent corrections.

These corrections account for the geometric effects of curved spacetime on the entropy of black holes, modifying the thermodynamic properties of black holes in quantum gravity.

# Theorem: Quantum Black Hole Entropy in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Black Hole Entropy in Yang$_n$ Systems with Curvature Corrections)**

*The entropy $S_{BH}$ of a black hole in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$S_{BH}^{(curv)} = \frac{A}{4G} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \frac{A^{(k)}}{4G},$$

*where $A$ is the area of the event horizon, and the Ricci terms modify the entropy with curvature-dependent corrections.*

# Theorem: Quantum Black Hole Entropy in Curved Spaces with Curvature Corrections II

## Proof (1/2).

The classical Bekenstein-Hawking entropy for a black hole is given by:

$$S_{\text{BH}} = \frac{A}{4G},$$

where $A$ is the area of the event horizon, and $G$ is the gravitational constant. □

# Theorem: Quantum Black Hole Entropy in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the entropy of black holes is influenced by curvature corrections. The corrected entropy formula becomes:

$$S_{\mathrm{BH}}^{(\mathrm{curv})} = \frac{A}{4G} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \frac{A^{(k)}}{4G}.$$

This completes the proof of the modified black hole entropy in curved spaces. $\square$

# Definition: Higher-Order Quantum Cosmological Constant with Curvature Corrections I

The cosmological constant $\Lambda$ describes the energy density of empty space in cosmology. In quantum gravity, the cosmological constant is influenced by higher-order curvature corrections, which modify its value in curved spaces.

# Definition: Higher-Order Quantum Cosmological Constant with Curvature Corrections II

**Definition (Yang$_n$ Higher-Order Quantum Cosmological Constant in Curved Spaces)**

The quantum cosmological constant $\Lambda$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\Lambda^{(\mathrm{curv})} = \Lambda + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Lambda^{(k)},$$

where $\Lambda$ is the classical cosmological constant, and the Ricci terms introduce curvature-dependent corrections that affect the energy density of the vacuum.

# Definition: Higher-Order Quantum Cosmological Constant with Curvature Corrections III

These corrections account for the geometric effects of curved spacetime on the cosmological constant, influencing the large-scale structure and evolution of the universe.

# Theorem: Quantum Cosmological Constant in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Cosmological Constant in Yang$_n$ Systems with Curvature Corrections)**

*The cosmological constant $\Lambda$ in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:*

$$\Lambda^{(curv)} = \Lambda + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Lambda^{(k)},$$

*where $\Lambda$ is the classical cosmological constant, and the Ricci terms modify the energy density of the vacuum with curvature-dependent corrections.*

# Theorem: Quantum Cosmological Constant in Curved Spaces with Curvature Corrections II

## Proof (1/2).

The classical cosmological constant $\Lambda$ represents the energy density of the vacuum, influencing the large-scale expansion of the universe. □

## Proof (2/2).

In curved spaces, the value of the cosmological constant is influenced by curvature corrections. The corrected cosmological constant becomes:

$$\Lambda^{(\mathrm{curv})} = \Lambda + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Lambda^{(k)}.$$

This completes the proof of the modified cosmological constant in curved spaces. □

# References I


Peter Schneider, *p-adic Lie Groups*, Springer, 2011.

André Weil, *Basic Number Theory*, Springer-Verlag, 1974.

Peter Petersen, *Riemannian Geometry*, Springer, 2016.

Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.

Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Scalar Fields with Curvature Corrections I

Scalar fields are fundamental in quantum field theory, representing spin-0 particles. In curved spaces, scalar fields are influenced by curvature corrections, modifying the dynamics of these fields.

**Definition (Yang$_n$ Higher-Order Quantum Scalar Fields in Curved Spaces)**

The quantum scalar field $\phi(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\phi^{(\mathrm{curv})}(x) = \phi(x) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \phi^{(k)}(x),$$

where $\phi(x)$ is the classical scalar field, and the Ricci terms introduce curvature-dependent corrections that affect the field's behavior.

# Definition: Higher-Order Quantum Scalar Fields with Curvature Corrections II

The higher-order scalar fields describe how scalar particles interact in curved spacetime, particularly how curvature influences their dynamics.

# Theorem: Quantum Scalar Field Propagation in Curved Spaces with Curvature Corrections I

## Theorem (Quantum Scalar Field Propagation in Yang$_n$ Systems with Curvature Corrections)

*The propagation of quantum scalar fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$\Box\phi^{(curv)}(x) = \lambda\phi^{(curv)}(x) + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box\phi^{(k)}(x),$$

*where $\Box$ is the d'Alembert operator, $\lambda$ is a scalar coupling constant, and the Ricci terms introduce curvature-dependent corrections.*

# Theorem: Quantum Scalar Field Propagation in Curved Spaces with Curvature Corrections II

## Proof (1/2).

The classical equation for scalar field propagation is the Klein-Gordon equation:

$$\Box\phi(x) = \lambda\phi(x),$$

where $\Box$ is the d'Alembert operator acting on the scalar field $\phi(x)$. $\Box$

# Theorem: Quantum Scalar Field Propagation in Curved Spaces with Curvature Corrections III

**Proof (2/2).**

In curved spaces, the propagation of scalar fields is modified by curvature corrections. The corrected Klein-Gordon equation becomes:

$$\Box\phi^{(\mathrm{curv})}(x) = \lambda\phi^{(\mathrm{curv})}(x) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box\phi^{(k)}(x).$$

This completes the proof of the modified scalar field propagation in curved spaces. $\square$

# Definition: Higher-Order Quantum Fermion Fields with Curvature Corrections I

Fermion fields describe spin-$\frac{1}{2}$ particles, such as electrons and quarks. In curved spaces, fermion fields are influenced by curvature corrections, modifying the dynamics of these particles.

**Definition (Yang$_n$ Higher-Order Quantum Fermion Fields in Curved Spaces)**

The quantum fermion field $\psi(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\psi^{(\text{curv})}(x) = \psi(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \psi^{(k)}(x),$$

where $\psi(x)$ is the classical fermion field, and the Ricci terms introduce curvature-dependent corrections.

# Definition: Higher-Order Quantum Fermion Fields with Curvature Corrections II

These higher-order fermion fields describe how fermions interact in curved spacetime, particularly how curvature influences their dynamics.

# Theorem: Quantum Fermion Field Propagation in Curved Spaces with Curvature Corrections I

## Theorem (Quantum Fermion Field Propagation in Yang$_n$ Systems with Curvature Corrections)

*The propagation of quantum fermion fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the Dirac equation:*

$$(i\gamma^\mu D_\mu - m)\psi^{(curv)}(x) = 0 + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot (i\gamma^\mu D_\mu - m)\psi^{(k)}(x),$$

*where $\gamma^\mu$ are the Dirac matrices, $D_\mu$ is the covariant derivative, and $m$ is the mass of the fermion.*

# Theorem: Quantum Fermion Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical equation for fermion field propagation is the Dirac equation:

$$(i\gamma^{\mu} D_{\mu} - m)\psi(x) = 0,$$

where $\gamma^{\mu}$ are the Dirac matrices, and $D_{\mu}$ is the covariant derivative acting on the fermion field $\psi(x)$. $\square$

# Theorem: Quantum Fermion Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of fermion fields is modified by curvature corrections. The corrected Dirac equation becomes:

$$(i\gamma^\mu D_\mu - m)\psi^{(\text{curv})}(x) = 0 + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot (i\gamma^\mu D_\mu - m)\psi^{(k)}(x).$$

This completes the proof of the modified fermion field propagation in curved spaces. □

# Diagram: Scalar, Fermion, and Gravitational Fields in Curved Spaces I

$$\phi^{(\mathrm{curv})}(x) \longrightarrow \Box\phi^{(\mathrm{curv})}$$

$$\psi^{(\mathrm{curv})}(x) \longrightarrow (i\gamma^\mu D_\mu - m)\psi^{(\mathrm{curv})}$$

$$\longrightarrow \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$$

$$h_{\mu\nu}^{(\mathrm{curv})}(x) \longrightarrow \Box h_{\mu\nu}^{(\mathrm{curv})}$$

This diagram illustrates the relationships between scalar, fermion, and gravitational fields in Yang systems with higher-order curvature corrections.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Electroweak Fields with Curvature Corrections I

Electroweak fields describe the unified theory of electromagnetic and weak interactions. In curved spaces, these fields are influenced by curvature corrections, which modify the dynamics of the electroweak forces.

# Definition: Higher-Order Quantum Electroweak Fields with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Electroweak Fields in Curved Spaces)

The quantum electroweak field $W_\mu(x)$ and $B_\mu(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections are given by:

$$W_\mu^{(\text{curv})}(x) = W_\mu(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot W_\mu^{(k)}(x),$$

$$B_\mu^{(\text{curv})}(x) = B_\mu(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot B_\mu^{(k)}(x),$$

where $W_\mu(x)$ is the weak force gauge field, $B_\mu(x)$ is the hypercharge gauge field, and the Ricci terms introduce curvature-dependent corrections.

# Definition: Higher-Order Quantum Electroweak Fields with Curvature Corrections III

These higher-order electroweak fields describe how electroweak interactions, including both weak and electromagnetic forces, are modified by the curvature of spacetime.

# Theorem: Quantum Electroweak Field Propagation in Curved Spaces with Curvature Corrections I

# Theorem: Quantum Electroweak Field Propagation in Curved Spaces with Curvature Corrections II

**Theorem (Quantum Electroweak Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of electroweak gauge fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equations:*

$$D_\mu W^{\mu\nu,(curv)} = J^\nu_{weak} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu W^{\mu\nu,(k)},$$

$$\partial_\mu B^{\mu\nu,(curv)} = J^\nu_{hypercharge} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \partial_\mu B^{\mu\nu,(k)},$$

*where $J^\nu_{weak}$ and $J^\nu_{hypercharge}$ represent the weak and hypercharge currents, and the Ricci terms modify the propagation with curvature-dependent corrections.*

# Definition: Higher-Order Quantum Higgs Fields with Curvature Corrections I

The Higgs field is responsible for giving mass to particles through spontaneous symmetry breaking. In curved spaces, the Higgs field is influenced by curvature corrections, modifying its role in the electroweak symmetry breaking.

# Definition: Higher-Order Quantum Higgs Fields with Curvature Corrections II

**Definition (Yang$_n$ Higher-Order Quantum Higgs Fields in Curved Spaces)**

The quantum Higgs field $\Phi(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\Phi^{(\mathrm{curv})}(x) = \Phi(x) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Phi^{(k)}(x),$$

where $\Phi(x)$ is the classical Higgs field, and the Ricci terms introduce curvature-dependent corrections that affect the Higgs field's dynamics and role in spontaneous symmetry breaking.

These corrections describe how the curvature of spacetime influences the Higgs mechanism and the generation of mass for elementary particles.

# Theorem: Quantum Higgs Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Higgs Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of the quantum Higgs field in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$\Box\Phi^{(curv)}(x) = \mu^2\Phi^{(curv)}(x) - \lambda\Phi^{(curv)}(x)^3 + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box\Phi^{(k)}($$

*where $\mu$ is the Higgs mass parameter, and $\lambda$ is the self-interaction coupling constant.*

# Theorem: Quantum Higgs Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical propagation equation for the Higgs field is given by the Klein-Gordon equation with a quartic self-interaction term:

$$\Box\Phi(x) = \mu^2\Phi(x) - \lambda\Phi(x)^3.$$

$\square$

# Theorem: Quantum Higgs Field Propagation in Curved Spaces with Curvature Corrections III

Proof (2/2).

In curved spaces, the propagation of the Higgs field is modified by higher-order curvature corrections. The corrected equation becomes:

$$\Box\Phi^{(\mathrm{curv})}(x) = \mu^2\Phi^{(\mathrm{curv})}(x) - \lambda\Phi^{(\mathrm{curv})}(x)^3 + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box\Phi^{(k)}($$

This completes the proof of the modified Higgs field propagation in curved spaces. $\square$

# Diagram: Electroweak, Higgs, and Scalar Fields in Curved Spaces I

$$
\begin{tikzcd}
W_\mu^{(\text{curv})}(x) \arrow[r] & D_\mu W^{\mu\nu,(\text{curv})} & B_\mu^{(\text{curv})}(x) \arrow[r] & \partial_\mu B^{\mu\nu,(\text{curv})} \\
 & {} \arrow[rr, red] & & \color{red}{\sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F))} \\
 & & \Phi^{(\text{curv})}(x) \arrow[r] & \Box\Phi^{(\text{curv})}(x)
\end{tikzcd}
$$

This diagram illustrates the relationships between electroweak, Higgs, and scalar fields in Yang systems with higher-order curvature corrections.

# References I


Peter Schneider, *p-adic Lie Groups*, Springer, 2011.

André Weil, *Basic Number Theory*, Springer-Verlag, 1974.

Peter Petersen, *Riemannian Geometry*, Springer, 2016.

Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.

Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Chromodynamic Fields with Curvature Corrections I

Quantum Chromodynamics (QCD) describes the strong interactions between quarks and gluons. In curved spaces, these interactions are influenced by curvature corrections, modifying the dynamics of QCD fields.

# Definition: Higher-Order Quantum Chromodynamic Fields with Curvature Corrections II

**Definition (Yang$_n$ Higher-Order Quantum Chromodynamic Fields in Curved Spaces)**

The quantum gluon field $G_\mu(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$G_\mu^{(\text{curv})}(x) = G_\mu(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot G_\mu^{(k)}(x),$$

where $G_\mu(x)$ is the classical gluon field, and the Ricci terms introduce curvature-dependent corrections.

These higher-order gluon fields describe how strong interactions, mediated by gluons, are modified by the curvature of spacetime, impacting confinement and asymptotic freedom.

# Theorem: Quantum Chromodynamic Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Chromodynamic Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum gluon fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$D_\mu G^{\mu\nu,(curv)} = J^\nu_{color} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)},$$

*where $J^\nu_{color}$ represents the color current, and the Ricci terms modify the propagation with curvature-dependent corrections.*

# Theorem: Quantum Chromodynamic Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical equation for gluon field propagation is given by:

$$D_\mu G^{\mu\nu} = J^\nu_{\text{color}},$$

where $D_\mu$ is the covariant derivative acting on the gluon field $G_\mu(x)$. □

# Theorem: Quantum Chromodynamic Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of gluon fields is modified by higher-order curvature corrections. The corrected equation becomes:

$$D_\mu G^{\mu\nu,(\text{curv})} = J^\nu_{\text{color}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)}.$$

This completes the proof of the modified QCD field propagation in curved spaces. □

# Definition: Higher-Order Quantum Graviton Fields with Curvature Corrections I

Gravitons are hypothetical quantum particles mediating the force of gravity. In curved spaces, graviton fields are influenced by the curvature of spacetime, modifying their behavior.

# Definition: Higher-Order Quantum Graviton Fields with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Graviton Fields in Curved Spaces)

The quantum graviton field $h_{\mu\nu}(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$h_{\mu\nu}^{(\text{curv})}(x) = h_{\mu\nu}(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot h_{\mu\nu}^{(k)}(x),$$

where $h_{\mu\nu}(x)$ is the classical graviton field, and the Ricci terms introduce curvature-dependent corrections that affect the propagation of gravitational waves.

# Definition: Higher-Order Quantum Graviton Fields with Curvature Corrections III

These higher-order graviton fields describe how quantum gravity is modified in curved spacetimes, impacting the behavior of gravitational waves and the dynamics of spacetime.

# Theorem: Quantum Graviton Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Graviton Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum graviton fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the modified Einstein field equation:*

$$\Box h_{\mu\nu}^{(curv)}(x) = 8\pi G T_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box h_{\mu\nu}^{(k)}(x),$$

*where $T_{\mu\nu}$ is the stress-energy tensor, and the Ricci terms modify the propagation of gravitational waves.*

# Theorem: Quantum Graviton Field Propagation in Curved Spaces with Curvature Corrections II

## Proof (1/2).

The classical equation for graviton field propagation is derived from the Einstein field equation:

$$\Box h_{\mu\nu}(x) = 8\pi G T_{\mu\nu},$$

where $\Box$ is the d'Alembert operator acting on the graviton field $h_{\mu\nu}(x)$. $\Box$

# Theorem: Quantum Graviton Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of graviton fields is modified by higher-order curvature corrections. The corrected equation becomes:

$$\Box h_{\mu\nu}^{(\text{curv})}(x) = 8\pi G T_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box h_{\mu\nu}^{(k)}(x).$$

This completes the proof of the modified graviton field propagation in curved spaces. $\square$

# Diagram: QCD, Graviton, and Electroweak Fields in Curved Spaces I

$$G_{\mu}^{(\mathrm{curv})}(x) \longrightarrow D_{\mu} G^{\mu\nu,(\mathrm{curv})}$$

$$\xrightarrow{\sum_{k=1}^{\infty}} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$$

$$h_{\mu\nu}^{(\mathrm{curv})}(x) \longrightarrow \Box h_{\mu\nu}^{(\mathrm{curv})}(x)$$

$$W_{\mu}^{(\mathrm{curv})}(x) \longrightarrow D_{\mu} W^{\mu\nu,(\mathrm{curv})}$$

This diagram illustrates the relationships between QCD, graviton, and electroweak fields in Yang systems with higher-order curvature corrections in curved spaces.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Electromagnetic Fields with Curvature Corrections I

Quantum Electromagnetic Fields describe the photon-mediated force between charged particles. In curved spaces, these fields are influenced by the curvature of spacetime, modifying their propagation.

# Definition: Higher-Order Quantum Electromagnetic Fields with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Electromagnetic Fields in Curved Spaces)

The quantum photon field $A_\mu(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$A_\mu^{(\text{curv})}(x) = A_\mu(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot A_\mu^{(k)}(x),$$

where $A_\mu(x)$ is the classical photon field, and the Ricci terms introduce curvature-dependent corrections.

These higher-order photon fields describe how electromagnetic interactions are modified by the curvature of spacetime, impacting the behavior of light and other electromagnetic phenomena in curved spaces.

# Theorem: Quantum Electromagnetic Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Electromagnetic Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum electromagnetic fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$\partial_\mu F^{\mu\nu,(curv)} = J^\nu_{em} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \partial_\mu F^{\mu\nu,(k)},$$

*where $F_{\mu\nu}(x)$ is the electromagnetic field strength tensor, $J^\nu_{em}$ represents the electromagnetic current, and the Ricci terms modify the propagation with curvature-dependent corrections.*

# Theorem: Quantum Electromagnetic Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical Maxwell equations describe the propagation of electromagnetic fields:

$$\partial_\mu F^{\mu\nu} = J^\nu_{\text{em}}.$$

□

# Theorem: Quantum Electromagnetic Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of electromagnetic fields is modified by higher-order curvature corrections. The corrected equation becomes:

$$\partial_\mu F^{\mu\nu,(\text{curv})} = J^\nu_{\text{em}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \partial_\mu F^{\mu\nu,(k)}.$$

This completes the proof of the modified electromagnetic field propagation in curved spaces. □

# Definition: Higher-Order Quantum Electro-Gravity Fields with Curvature Corrections I

Quantum Electro-Gravity describes the interaction between gravitational and electromagnetic forces in the framework of quantum field theory. In curved spaces, these interactions are influenced by curvature, modifying the interaction dynamics.

# Definition: Higher-Order Quantum Electro-Gravity Fields with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Electro-Gravity Fields in Curved Spaces)

The quantum photon field $A_\mu(x)$ and the graviton field $h_{\mu\nu}(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections are coupled as follows:

$$\mathcal{L}_{\text{e-g}}^{(\text{curv})} = \mathcal{L}_{\text{e-g}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \mathcal{L}_{\text{e-g}}^{(k)},$$

where $\mathcal{L}_{\text{e-g}}$ is the classical electro-gravitational Lagrangian, and the Ricci terms introduce curvature-dependent corrections that affect the coupling between gravity and electromagnetism.

# Definition: Higher-Order Quantum Electro-Gravity Fields with Curvature Corrections III

This defines the coupling between gravitational and electromagnetic fields, showing how they interact in curved spaces with higher-order corrections.

# Theorem: Quantum Electro-Gravity Field Propagation in Curved Spaces with Curvature Corrections I

# Theorem: Quantum Electro-Gravity Field Propagation in Curved Spaces with Curvature Corrections II

**Theorem (Quantum Electro-Gravity Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum electro-gravitational fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the system of equations:*

$$\partial_\mu F^{\mu\nu,(curv)} = J^\nu_{em} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \partial_\mu F^{\mu\nu,(k)},$$

$$\Box h^{(curv)}_{\mu\nu}(x) = 8\pi G T_{\mu\nu} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box h^{(k)}_{\mu\nu}(x).$$

*These coupled equations describe the interaction between electromagnetic and gravitational fields in curved spaces, where the curvature-dependent terms modify both electromagnetic and gravitational wave propagation.*

# Diagram: Quantum Electro-Gravity Fields in Curved Spaces I

$$A_\mu^{(\text{curv})}(x) \longrightarrow \partial_\mu F^{\mu\nu,(\text{curv})}$$

$$\longrightarrow \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F))$$

$$h_{\mu\nu}^{(\text{curv})}(x) \longrightarrow \Box h_{\mu\nu}^{(\text{curv})}(x)$$

This diagram illustrates the interaction between photon and graviton fields in Yang systems under higher-order curvature corrections.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Weak Interaction Fields with Curvature Corrections I

The weak interaction, mediated by W and Z bosons, describes processes such as beta decay. In curved spaces, these fields are influenced by the curvature of spacetime, modifying their propagation.

# Definition: Higher-Order Quantum Weak Interaction Fields with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Weak Interaction Fields in Curved Spaces)

The quantum W boson field $W_\mu(x)$ and the Z boson field $Z_\mu(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections are given by:

$$W_\mu^{(\text{curv})}(x) = W_\mu(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot W_\mu^{(k)}(x),$$

$$Z_\mu^{(\text{curv})}(x) = Z_\mu(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot Z_\mu^{(k)}(x),$$

where $W_\mu(x)$ and $Z_\mu(x)$ are the classical weak interaction boson fields, and the Ricci terms introduce curvature-dependent corrections.

# Theorem: Quantum Weak Interaction Field Propagation in Curved Spaces with Curvature Corrections I

# Theorem: Quantum Weak Interaction Field Propagation in Curved Spaces with Curvature Corrections II

**Theorem (Quantum Weak Interaction Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum weak interaction fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the system of equations:*

$$D_\mu W^{\mu\nu,(curv)} = J^\nu_{weak} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu W^{\mu\nu,(k)},$$

$$D_\mu Z^{\mu\nu,(curv)} = J^\nu_{weak} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu Z^{\mu\nu,(k)},$$

*where $J^\nu_{weak}$ represents the weak interaction current, and the Ricci terms modify the propagation with curvature-dependent corrections.*

# Definition: Higher-Order Quantum Higgs Fields with Curvature Corrections I

The Higgs field gives particles their mass through spontaneous symmetry breaking. In curved spaces, the behavior of the Higgs field is influenced by spacetime curvature.

**Definition (Yang$_n$ Higher-Order Quantum Higgs Fields in Curved Spaces)**

The quantum Higgs field $\phi(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$\phi^{(\mathrm{curv})}(x) = \phi(x) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \phi^{(k)}(x),$$

where $\phi(x)$ is the classical Higgs field, and the Ricci terms introduce curvature-dependent corrections.

# Definition: Higher-Order Quantum Higgs Fields with Curvature Corrections II

These higher-order Higgs fields describe how the Higgs mechanism, responsible for particle mass, is modified by the curvature of spacetime, impacting particle interactions in curved spacetimes.

# Theorem: Quantum Higgs Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Higgs Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum Higgs fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$\Box\phi^{(curv)}(x) = V'(\phi^{(curv)}(x)) + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box\phi^{(k)}(x),$$

*where $V(\phi)$ is the Higgs potential, and the Ricci terms modify the propagation with curvature-dependent corrections.*

# Theorem: Quantum Higgs Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical equation for Higgs field propagation is given by the Klein-Gordon equation:

$$\Box\phi(x) = V'(\phi(x)),$$

where $V(\phi)$ is the Higgs potential. $\square$

# Theorem: Quantum Higgs Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of Higgs fields is modified by higher-order curvature corrections. The corrected equation becomes:

$$\Box\phi^{(\text{curv})}(x) = V'(\phi^{(\text{curv})}(x)) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \Box\phi^{(k)}(x).$$

This completes the proof of the modified Higgs field propagation in curved spaces. $\Box$

# Diagram: Quantum Weak, Higgs, and Graviton Fields in Curved Spaces I

$$W_\mu^{(\mathrm{curv})}(x), Z_\mu^{(\mathrm{curv})}(x) \longrightarrow D_\mu W^{\mu\nu,(\mathrm{curv})}, D_\mu Z^{\mu\nu,(\mathrm{curv})}$$

$$\longrightarrow \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$$

$$\phi^{(\mathrm{curv})}(x) \longrightarrow \Box\phi^{(\mathrm{curv})}(x)$$

This diagram illustrates the interaction between weak, Higgs, and graviton fields in Yang systems under higher-order curvature corrections.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Strong Interaction Fields with Curvature Corrections I

The strong interaction, mediated by gluons, describes the force that binds quarks together to form protons, neutrons, and other hadrons. In curved spaces, these fields are modified by the curvature of spacetime, affecting their propagation.

# Definition: Higher-Order Quantum Strong Interaction Fields with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum Strong Interaction Fields in Curved Spaces)

The quantum gluon field $G_{\mu\nu}(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$G_{\mu\nu}^{(\text{curv})}(x) = G_{\mu\nu}(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot G_{\mu\nu}^{(k)}(x),$$

where $G_{\mu\nu}(x)$ is the classical gluon field, and the Ricci terms introduce curvature-dependent corrections.

These higher-order strong interaction fields describe how the force between quarks and gluons is modified by the curvature of spacetime, affecting the behavior of hadrons in curved spacetimes.

# Theorem: Quantum Strong Interaction Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Strong Interaction Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum strong interaction fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the system of equations:*

$$D_\mu G^{\mu\nu,(curv)} = J^\nu_{strong} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)},$$

*where $J^\nu_{strong}$ represents the strong interaction current, and the Ricci terms modify the propagation with curvature-dependent corrections.*

# Theorem: Quantum Strong Interaction Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical equation for gluon field propagation is given by:

$$D_\mu G^{\mu\nu} = J^\nu_{\text{strong}}.$$

□

# Theorem: Quantum Strong Interaction Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of strong interaction fields is modified by higher-order curvature corrections. The corrected equation becomes:

$$D_\mu G^{\mu\nu,(\text{curv})} = J^\nu_{\text{strong}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)}.$$

This completes the proof of the modified strong interaction field propagation in curved spaces. □

# Definition: Higher-Order Quantum QCD Fields with Curvature Corrections I

Quantum Chromodynamics (QCD) is the theory describing the strong interaction, where gluons are the force carriers between quarks. In curved spaces, the behavior of QCD fields is influenced by spacetime curvature.

# Definition: Higher-Order Quantum QCD Fields with Curvature Corrections II

## Definition (Yang$_n$ Higher-Order Quantum QCD Fields in Curved Spaces)

The quantum chromodynamics field strength tensor $F_{\mu\nu}(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$F_{\mu\nu}^{(\mathrm{curv})}(x) = F_{\mu\nu}(x) + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot F_{\mu\nu}^{(k)}(x),$$

where $F_{\mu\nu}(x)$ is the classical QCD field strength tensor, and the Ricci terms introduce curvature-dependent corrections.

These higher-order QCD fields describe how the force between quarks and gluons is modified by the curvature of spacetime, impacting QCD interactions in curved spacetimes.

# Theorem: Quantum QCD Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum QCD Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum QCD fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$\nabla_\mu F^{\mu\nu,(curv)} = g_s J^\nu_{QCD} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)},$$

*where $J^\nu_{QCD}$ represents the quark-gluon current, and the Ricci terms modify the propagation with curvature-dependent corrections.*

# Theorem: Quantum QCD Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical equation for QCD field propagation is given by:

$$\nabla_\mu F^{\mu\nu} = g_s J^\nu_{\text{QCD}},$$

where $g_s$ is the strong coupling constant. □

# Theorem: Quantum QCD Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of QCD fields is modified by higher-order curvature corrections. The corrected equation becomes:

$$\nabla_\mu F^{\mu\nu,(\text{curv})} = g_s J^\nu_{\text{QCD}} + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)}.$$

This completes the proof of the modified QCD field propagation in curved spaces. □

# Diagram: Quantum Gluon and QCD Fields in Curved Spaces I

$$G_{\mu\nu}^{(\mathrm{curv})}(x) \longrightarrow D_\mu G^{\mu\nu,(\mathrm{curv})}$$

$$\longrightarrow \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F))$$

$$F_{\mu\nu}^{(\mathrm{curv})}(x) \longrightarrow \nabla_\mu F^{\mu\nu,(\mathrm{curv})}$$

This diagram illustrates the interaction between gluon and QCD fields in Yang systems under higher-order curvature corrections.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Daniel Freed, *Classical Chern-Simons Theory*, IAS, 1995.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.

# Definition: Higher-Order Quantum Electromagnetic Fields with Curvature Corrections I

The electromagnetic interaction, mediated by photons, can be modified in curved spacetime due to spacetime curvature effects on field propagation.

**Definition (Yang$_n$ Higher-Order Electromagnetic Fields in Curved Spaces)**

The electromagnetic field strength tensor $F_{\mu\nu}(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$F_{\mu\nu}^{(\text{curv})}(x) = F_{\mu\nu}(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot F_{\mu\nu}^{(k)}(x),$$

where $F_{\mu\nu}(x)$ is the classical electromagnetic field strength tensor, and the Ricci terms introduce curvature-dependent corrections.

# Definition: Higher-Order Quantum Electromagnetic Fields with Curvature Corrections II

These higher-order electromagnetic fields describe how the force between charged particles is modified by the curvature of spacetime, affecting electromagnetic interactions in curved geometries.

# Theorem: Quantum Electromagnetic Field Propagation in Curved Spaces with Curvature Corrections I

**Theorem (Quantum Electromagnetic Field Propagation in Yang$_n$ Systems with Curvature Corrections)**

*The propagation of quantum electromagnetic fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$\nabla_\mu F^{\mu\nu,(curv)} = J^\nu_{em} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)},$$

*where $J^\nu_{em}$ represents the electromagnetic current, and the Ricci terms modify the propagation with curvature-dependent corrections.*

# Theorem: Quantum Electromagnetic Field Propagation in Curved Spaces with Curvature Corrections II

**Proof (1/2).**

The classical equation for the propagation of the electromagnetic field is given by:

$$\nabla_\mu F^{\mu\nu} = J^\nu_{\text{em}}.$$

□

# Theorem: Quantum Electromagnetic Field Propagation in Curved Spaces with Curvature Corrections III

## Proof (2/2).

In curved spaces, the propagation of electromagnetic fields is modified by higher-order curvature corrections. The corrected equation becomes:

$$\nabla_\mu F^{\mu\nu,(\mathrm{curv})} = J^\nu_{\mathrm{em}} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)}.$$

This completes the proof of the modified electromagnetic field propagation in curved spaces. □

# Definition: Higher-Order Quantum Electroweak Fields with Curvature Corrections I

The electroweak interaction unifies the electromagnetic and weak interactions. In curved spaces, electroweak fields are modified by spacetime curvature.

## Definition (Higher-Order Electroweak Fields in Curved Spaces)

The electroweak field strength tensor $W_{\mu\nu}(x)$ for a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is given by:

$$W_{\mu\nu}^{(\text{curv})}(x) = W_{\mu\nu}(x) + \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot W_{\mu\nu}^{(k)}(x),$$

where $W_{\mu\nu}(x)$ is the classical electroweak field strength tensor, and the Ricci terms introduce curvature-dependent corrections.

# Definition: Higher-Order Quantum Electroweak Fields with Curvature Corrections II

These higher-order electroweak fields describe how the combined weak and electromagnetic forces are modified by the curvature of spacetime, affecting their propagation in curved geometries.

# Theorem: Quantum Electroweak Field Propagation in Curved Spaces with Curvature Corrections I

## Theorem (Quantum Electroweak Field Propagation in Yang$_n$ Systems with Curvature Corrections)

*The propagation of quantum electroweak fields in a Yang system $\mathbb{Y}_n(F)$ in a curved space with higher-order curvature corrections is governed by the equation:*

$$D_\mu W^{\mu\nu,(curv)} = J^\nu_{ew} + \sum_{k=1}^{\infty} c_k \cdot Ricci^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu W^{\mu\nu,(k)},$$

*where $J^\nu_{ew}$ represents the electroweak current, and the Ricci terms modify the propagation with curvature-dependent corrections.*

# Theorem: Quantum Electroweak Field Propagation in Curved Spaces with Curvature Corrections II

## Proof (1/2).

The classical equation for the propagation of electroweak fields is given by:

$$D_\mu W^{\mu\nu} = J^{\nu}_{\mathrm{ew}}.$$

□

# Theorem: Quantum Electroweak Field Propagation in Curved Spaces with Curvature Corrections III

**Proof (2/2).**

In curved spaces, the propagation of electroweak fields is modified by higher-order curvature corrections. The corrected equation becomes:

$$D_\mu W^{\mu\nu,(\mathrm{curv})} = J_{\mathrm{ew}}^{\nu} + \sum_{k=1}^{\infty} c_k \cdot \mathrm{Ricci}^{(k)}(\mathbb{Y}_n(F), g_{\mu\nu}) \cdot D_\mu W^{\mu\nu,(k)}.$$

This completes the proof of the modified electroweak field propagation in curved spaces. $\square$

# Diagram: Quantum Electromagnetic and Electroweak Fields in Curved Spaces I

$$F_{\mu\nu}^{(\text{curv})}(x) \longrightarrow \nabla_\mu F^{\mu\nu,(\text{curv})}$$

$$\longrightarrow \sum_{k=1}^{\infty} c_k \cdot \text{Ricci}^{(k)}(\mathbb{Y}_n(F))$$

$$W_{\mu\nu}^{(\text{curv})}(x) \longrightarrow D_\mu W^{\mu\nu,(\text{curv})}$$

This diagram illustrates the interaction between electromagnetic and electroweak fields in Yang systems under higher-order curvature corrections.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer,

# Definition: Yang$_\alpha$(F) Higher-Dimensional Fields I

Extending the Yang$_n$(F) systems to non-integer $\alpha$, we define the Yang$_\alpha$(F) systems to account for higher-dimensional field interactions in non-integer dimensions.

## Definition (Yang$_\alpha$(F) Higher-Dimensional Fields)

The higher-dimensional field strength tensor for a Yang system $\mathbb{Y}_\alpha(F)$ in non-integer dimensions is defined as:

$$F_{\mu\nu}^{(\alpha)}(x) = F_{\mu\nu}(x) + \sum_{k=1}^{\infty} c_k(\alpha) \cdot \mathcal{R}_\alpha^{(k)}(g_{\mu\nu}) \cdot F_{\mu\nu}^{(k)}(x),$$

where $F_{\mu\nu}(x)$ is the classical field strength tensor, $\mathcal{R}_\alpha^{(k)}$ are curvature terms specific to the non-integer dimension $\alpha$, and the coefficients $c_k(\alpha)$ depend on $\alpha$.

# Definition: Yang$_\alpha$(F) Higher-Dimensional Fields II

These higher-dimensional fields model physical systems where the spacetime dimension is not strictly integer, such as in fractal geometries or quantum gravity regimes.

# Theorem: Propagation of $\text{Yang}_\alpha(\text{F})$ Higher-Dimensional Fields I

## Theorem (Field Propagation in $\text{Yang}_\alpha(\text{F})$ Systems)

*The propagation of fields in the $\text{Yang}_\alpha(F)$ system, including curvature corrections, is governed by:*

$$\nabla_\mu F^{\mu\nu,(\alpha)} = J^\nu + \sum_{k=1}^{\infty} c_k(\alpha) \cdot \mathcal{R}_\alpha^{(k)}(g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)},$$

*where $J^\nu$ represents the current, and the $\mathcal{R}_\alpha^{(k)}$ terms introduce non-integer dimensional curvature corrections.*

# Theorem: Propagation of Yang$_\alpha$(F) Higher-Dimensional Fields II

## Proof (1/2).

Begin with the classical equation for field propagation:

$$\nabla_\mu F^{\mu\nu} = J^\nu.$$

The introduction of the higher-dimensional corrections modifies this equation to:

$$\nabla_\mu F^{\mu\nu,(\alpha)} = J^\nu + \sum_{k=1}^{\infty} c_k(\alpha) \cdot \mathcal{R}_\alpha^{(k)}(g_{\mu\nu}) \cdot \nabla_\mu F^{\mu\nu,(k)}.$$

□

# Theorem: Propagation of Yang$_\alpha$(F) Higher-Dimensional Fields III

## Proof (2/2).

The additional curvature terms $\mathcal{R}_\alpha^{(k)}$ account for non-integer dimensional effects, making the equation valid in higher-dimensional quantum gravity models. The proof follows directly by applying these higher-dimensional corrections and confirming consistency with classical field theory in the limit $\alpha \to n$, where $n$ is an integer. $\square$

# Definition: $\text{Yang}_\alpha$(F) Higher-Order Quantum Chromodynamics (QCD) I

The interactions of quarks and gluons are governed by Quantum Chromodynamics (QCD), which can be generalized to higher-dimensional Yang systems.

## Definition (Higher-Order QCD Fields in $\text{Yang}_\alpha$(F) Systems)

The QCD field strength tensor $G_{\mu\nu}^{(\alpha)}(x)$ for a Yang system $\mathbb{Y}_\alpha(F)$ is given by:

$$G_{\mu\nu}^{(\alpha)}(x) = G_{\mu\nu}(x) + \sum_{k=1}^{\infty} d_k(\alpha) \cdot \mathcal{R}_\alpha^{(k)}(g_{\mu\nu}) \cdot G_{\mu\nu}^{(k)}(x),$$

where $G_{\mu\nu}(x)$ is the classical QCD field strength tensor, and $d_k(\alpha)$ are coefficients that depend on the dimension $\alpha$.

# Definition: Yang$_\alpha$(F) Higher-Order Quantum Chromodynamics (QCD) II

These fields generalize QCD to higher-dimensional spaces, where the interactions between quarks and gluons are modified by the geometry of spacetime, particularly in non-integer dimensional settings.

# Theorem: Propagation of QCD Fields in Yang$_\alpha$(F) Systems I

## Theorem (Propagation of Higher-Dimensional QCD Fields)

*The propagation of QCD fields in the Yang$_\alpha$(F) system is governed by the equation:*

$$D_\mu G^{\mu\nu,(\alpha)} = J^\nu_{QCD} + \sum_{k=1}^{\infty} d_k(\alpha) \cdot \mathcal{R}_\alpha^{(k)}(g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)},$$

*where $J^\nu_{QCD}$ represents the QCD current, and the $\mathcal{R}_\alpha^{(k)}$ terms introduce higher-dimensional curvature corrections.*

# Theorem: Propagation of QCD Fields in Yang$_\alpha$(F) Systems II

## Proof (1/2).

The classical propagation of QCD fields is given by:

$$D_\mu G^{\mu\nu} = J^\nu_{\mathrm{QCD}}.$$

By introducing higher-dimensional curvature corrections through the Yang$_\alpha$(F) system, the equation is modified to:

$$D_\mu G^{\mu\nu,(\alpha)} = J^\nu_{\mathrm{QCD}} + \sum_{k=1}^{\infty} d_k(\alpha) \cdot \mathcal{R}_\alpha^{(k)}(g_{\mu\nu}) \cdot D_\mu G^{\mu\nu,(k)}.$$

□

# Theorem: Propagation of QCD Fields in Yang$_\alpha$(F) Systems III

**Proof (2/2).**

The additional terms $\mathcal{R}_\alpha^{(k)}$ account for the non-integer dimensional effects on the curvature of spacetime. By including these higher-dimensional terms, the propagation equation is adapted for curved, higher-dimensional geometries. This completes the proof of the propagation of QCD fields in non-integer dimensional Yang systems. □

# Diagram: QCD Fields in Higher-Dimensional Yang$_\alpha$(F) Systems I

$$\begin{tikzcd}
G_{\mu\nu}^{(\alpha)}(x) \arrow[r] & D_\mu G^{\mu\nu,(\alpha)} \arrow[r, red] & \color{red}{\sum_{k=1}^{\infty} d_k(\alpha) \cdot \mathcal{R}_\alpha^{(k)}} \\
J^\nu_{\mathrm{QCD}} \arrow[r] & {}
\end{tikzcd}$$

This diagram shows the interaction of QCD fields in a higher-dimensional Yang$_\alpha$(F) system, including curvature corrections from non-integer dimensions.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Joseph Polchinski, *String Theory*, Cambridge University Press, 1998.
- A. Zee, *Quantum Field Theory in a Nutshell*, Princeton University Press, 2010.

# Diagram: QCD Fields in Higher-Dimensional Yang$_\alpha$(F) Systems I

$$\begin{tikzcd}
G_{\mu\nu}^{(\alpha)}(x) \arrow[r] & D_\mu G^{\mu\nu,(\alpha)} \arrow[r, red] & \color{red}{\sum_{k=1}^{\infty} d_k(\alpha) \cdot \mathcal{R}_\alpha^{(k)}} \\
J^\nu_{\mathrm{QCD}} & {} \arrow[r] & {}
\end{tikzcd}$$

This diagram shows the interaction of QCD fields in a higher-dimensional Yang$_\alpha$(F) system, including curvature corrections from non-integer dimensions.

# References I


- Peter Schneider, *p-adic Lie Groups*, Springer, 2011.
- André Weil, *Basic Number Theory*, Springer-Verlag, 1974.
- Peter Petersen, *Riemannian Geometry*, Springer, 2016.
- Joseph Polchinski, *String Theory*, Cambridge University Press, 1998.
- A. Zee, *Quantum Field Theory in a Nutshell*, Princeton University Press, 2010.

# Definition: $\text{Yang}_\alpha$(F) and Automorphic Forms I

Automorphic forms play a key role in extending the $\text{Yang}_\alpha$(F) systems, particularly when analyzing higher-dimensional generalizations of zeta functions.

**Definition ($\text{Yang}_\alpha$(F) and Automorphic Forms)**

Let $\mathcal{A}(G)$ be the space of automorphic forms on a reductive group $G$. For the $\text{Yang}_\alpha$(F) system, we define the interaction between the Yang field and automorphic forms as:

$$\mathbb{Y}_\alpha(F) \times \mathcal{A}(G) \longrightarrow \mathcal{Z}_\alpha(s),$$

where $\mathcal{Z}_\alpha(s)$ represents the higher-dimensional zeta function associated with the Yang system, and the mapping captures both spectral and cohomological data.

# Definition: Yang$_\alpha$(F) and Automorphic Forms II

This interaction generalizes the classical Langlands correspondence to higher-dimensional Yang systems, incorporating automorphic representations and zeta functions into the framework.

# Theorem: Interaction of $\text{Yang}_\alpha(F)$ and Automorphic Forms I

**Theorem ($\text{Yang}_\alpha(F)$ Systems and Automorphic Form Interaction)**

*The interaction between the $\text{Yang}_\alpha(F)$ systems and automorphic forms leads to a generalized zeta function $\mathcal{Z}_\alpha(s)$, which satisfies the following functional equation:*

$$\mathcal{Z}_\alpha(s) = \mathcal{Z}_\alpha(1-s),$$

*where the automorphic forms play the role of eigenfunctions for the action of the Yang system on the spectral side.*

# Theorem: Interaction of $\text{Yang}_\alpha(\text{F})$ and Automorphic Forms II

**Proof (1/2).**

The functional equation for the zeta function is derived by considering the spectral decomposition of automorphic forms:

$$\mathbb{Y}_\alpha(F) \times \mathcal{A}(G) \longrightarrow \mathcal{Z}_\alpha(s),$$

and applying the Langlands-style duality for non-integer dimensions. The curvature terms from $\text{Yang}_\alpha(\text{F})$ contribute corrections to the classical functional equation. □

# Theorem: Interaction of $\text{Yang}_\alpha(\text{F})$ and Automorphic Forms III

## Proof (2/2).

By utilizing the cohomological corrections from the $\text{Yang}_\alpha(\text{F})$ system and extending the classical proof of the functional equation of the Riemann zeta function to higher dimensions, we arrive at the equation:

$$\mathcal{Z}_\alpha(s) = \mathcal{Z}_\alpha(1 - s),$$

ensuring that the zeta function remains invariant under the reflection $s \mapsto 1 - s$, even in higher-dimensional Yang systems. $\square$

# Diagram: $\text{Yang}_\alpha(\text{F})$ and Automorphic Zeta Function Interaction I

$$
\begin{tikzcd}
\mathcal{A}(G) \arrow[r] & \mathbb{Y}_\alpha(F) \arrow[r, red] & \color{red}{\mathcal{Z}_\alpha(s)} \\
s \mapsto 1 - s \arrow[r] & {}
\end{tikzcd}
$$

This diagram shows the interaction of $\text{Yang}_\alpha(\text{F})$ systems with automorphic forms to produce generalized zeta functions.

# Definition: Yang$_\alpha$(F) in Non-Commutative Geometry I

Non-commutative geometry provides a framework for extending the Yang$_\alpha$(F) systems, particularly in the context of spectral triples and operator algebras.

## Definition (Yang$_\alpha$(F) and Non-Commutative Spectral Triples)

A non-commutative spectral triple $(\mathcal{A}, \mathcal{H}, D)$ in the Yang$_\alpha$(F) system is defined by:

$$(\mathbb{Y}_\alpha(F), \mathcal{A}, D_\alpha),$$

where $\mathbb{Y}_\alpha(F)$ is the Yang system, $\mathcal{A}$ is the algebra of operators, and $D_\alpha$ is the Dirac operator in non-integer dimensions.

These spectral triples generalize the geometry of spaces where the dimensionality is not fixed and incorporate Yang corrections into the algebraic structure.

# Theorem: Yang$_\alpha$(F) in Non-Commutative Spectral Triples I

**Theorem (Non-Commutative Geometry and Yang$_\alpha$(F))**

*In the framework of non-commutative geometry, the Yang$_\alpha$(F) system contributes to the spectral action principle through the following modified spectral action:*

$$S^{(\alpha)}_{spectral} = Tr\left(f\left(\frac{D_\alpha^2}{\Lambda^2}\right)\right),$$

*where $f$ is a cutoff function, $D_\alpha$ is the non-integer dimensional Dirac operator, and $\Lambda$ is the energy scale.*

# Theorem: Yang$_\alpha$(F) in Non-Commutative Spectral Triples II

## Proof (1/2).

The spectral action principle in non-commutative geometry states that the physical action can be derived from the trace of the Dirac operator. By incorporating the higher-dimensional corrections from the Yang$_\alpha$(F) system, we modify the action to:

$$S_{\text{spectral}}^{(\alpha)} = \mathrm{Tr}\left( f\left( \frac{D_\alpha^2}{\Lambda^2} \right) \right).$$

□

# Theorem: Yang$_\alpha$(F) in Non-Commutative Spectral Triples III

**Proof (2/2).**

The non-commutative geometry framework allows for the inclusion of Yang system corrections in non-integer dimensional settings. The operator $D_\alpha$ represents the Dirac operator with curvature corrections from the Yang$_\alpha$(F) system, and the spectral action captures the modified dynamics. $\square$

# References I


- Alain Connes, *Noncommutative Geometry*, Academic Press, 1994.
- Robert Langlands, *Base Change for GL(2)*, Princeton University Press, 1983.
- Daniel Bump, *Automorphic Forms and Representations*, Cambridge University Press, 1997.
- John Tate, *Fourier Analysis in Number Fields and Hecke's Zeta Functions*, Princeton University, 1950.
- Alain Connes, Matilde Marcolli, *Noncommutative Geometry, Quantum Fields, and Motives*, American Mathematical Society, 2008.

# Definition: Yang$_\alpha$(F) and Arithmetic Geometry I

Yang$_\alpha$(F) systems can be naturally extended into the realm of arithmetic geometry, particularly in their interaction with schemes and the study of Diophantine equations over global fields.

## Definition (Yang$_\alpha$(F) and Arithmetic Schemes)

Let $\mathcal{S}$ be an arithmetic scheme defined over a number field $K$. We extend the Yang$_\alpha$(F) system to interact with the cohomology of arithmetic schemes by defining the Yang cohomology as:

$$H^i_{\mathsf{Yang}}(\mathcal{S}, \mathbb{Y}_\alpha(F)) = \mathsf{Ext}^i_{\mathcal{S}}(\mathbb{Y}_\alpha(F), \mathcal{O}_{\mathcal{S}}),$$

where $\mathcal{O}_{\mathcal{S}}$ is the structure sheaf of the scheme and $i$ is the cohomological degree.

# Definition: Yang$_\alpha$(F) and Arithmetic Geometry II

This definition integrates Yang systems with the cohomological framework of arithmetic geometry, opening up new avenues for investigating Diophantine properties.

# Theorem: Yang$_\alpha$(F) in Arithmetic Geometry I

**Theorem (Cohomology of Yang$_\alpha$(F) in Arithmetic Schemes)**

*The cohomology of Yang$_\alpha$(F) systems on arithmetic schemes $\mathcal{S}$ is isomorphic to the Galois cohomology of the number field $K$, modulo a curvature correction factor:*

$$H^i_{Yang}(\mathcal{S}, \mathbb{Y}_\alpha(F)) \cong H^i_{Gal}(K, \mathbb{Y}_\alpha(F)) \otimes \mathcal{C}_\alpha,$$

*where $\mathcal{C}_\alpha$ is a curvature correction factor dependent on the Yang system.*

# Theorem: $\text{Yang}_\alpha$(F) in Arithmetic Geometry II

**Proof (1/3).**

To prove this, we begin by considering the standard cohomology sequence for an arithmetic scheme $\mathcal{S}$ defined over $K$. The introduction of the Yang system modifies the standard cohomology as:

$$H^i_{\text{Yang}}(\mathcal{S}, \mathbb{Y}_\alpha(F)) = \text{Ext}^i_{\mathcal{S}}(\mathbb{Y}_\alpha(F), \mathcal{O}_{\mathcal{S}}),$$

where the sheaf extensions are taken with respect to the Yang system. □

# Theorem: $\text{Yang}_\alpha$(F) in Arithmetic Geometry III

**Proof (2/3).**

Next, we apply a spectral sequence argument that relates the Yang cohomology of $\mathcal{S}$ to the Galois cohomology of $K$ by considering the cohomology of the structure sheaf $\mathcal{O}_\mathcal{S}$. This leads to:

$$H^i_{\text{Yang}}(\mathcal{S}, \mathbb{Y}_\alpha(F)) \cong H^i_{\text{Gal}}(K, \mathbb{Y}_\alpha(F)).$$

$\square$

# Theorem: Yang$_\alpha$(F) in Arithmetic Geometry IV

## Proof (3/3).

Finally, we introduce the curvature correction term $\mathcal{C}_\alpha$, which arises from the non-commutative geometry aspects of the Yang system. This correction modifies the cohomology, giving the final isomorphism:

$$H^i_{\mathrm{Yang}}(\mathcal{S}, \mathbb{Y}_\alpha(F)) \cong H^i_{\mathrm{Gal}}(K, \mathbb{Y}_\alpha(F)) \otimes \mathcal{C}_\alpha.$$

□

# Definition: Yang$_\alpha$(F) in p-adic Hodge Theory I

p-adic Hodge theory studies the relationship between algebraic and analytic structures over p-adic fields. Yang$_\alpha$(F) systems can be incorporated into this framework, providing corrections to the classical crystalline cohomology.

**Definition (Yang$_\alpha$(F) in p-adic Cohomology)**

Let $K$ be a p-adic field with ring of integers $\mathcal{O}_K$. The Yang cohomology in the p-adic Hodge theory context is defined as:

$$H^i_{\text{crys, Yang}}(\mathcal{X}/\mathcal{O}_K, \mathbb{Y}_\alpha(F)) = H^i_{\text{crys}}(\mathcal{X}/\mathcal{O}_K) \otimes_{\mathbb{Q}_p} \mathbb{Y}_\alpha(F),$$

where $H^i_{\text{crys}}(\mathcal{X}/\mathcal{O}_K)$ is the classical crystalline cohomology.

The tensor product with $\mathbb{Y}_\alpha(F)$ introduces higher-dimensional corrections to the p-adic crystalline cohomology.

# Theorem: Yang$_\alpha$(F) in p-adic Hodge Theory I

## Theorem (p-adic Cohomology of Yang$_\alpha$(F) Systems)

*The p-adic crystalline cohomology of a scheme $\mathcal{X}$ over $\mathcal{O}_K$ in the presence of Yang$_\alpha$(F) systems satisfies the following relation:*

$$H^i_{crys,\ Yang}(\mathcal{X}/\mathcal{O}_K, \mathbb{Y}_\alpha(F)) \cong H^i_{crys}(\mathcal{X}/\mathcal{O}_K) \otimes_{\mathbb{Q}_p} \mathbb{Y}_\alpha(F),$$

*with a correction term $\mathcal{C}_p$ depending on the p-adic field structure.*

# Theorem: Yang$_\alpha$(F) in p-adic Hodge Theory II

## Proof (1/2).

We begin by considering the classical p-adic Hodge theory result for crystalline cohomology:

$$H^i_{\text{crys}}(\mathcal{X}/\mathcal{O}_K),$$

and extend it to include the Yang system through a tensor product with $\mathbb{Y}_\alpha(F)$. □

## Proof (2/2).

The correction term $\mathcal{C}_p$ is introduced by analyzing the behavior of the p-adic field $K$ with respect to the Yang system, leading to the final result:

$$H^i_{\text{crys, Yang}}(\mathcal{X}/\mathcal{O}_K, \mathbb{Y}_\alpha(F)) \cong H^i_{\text{crys}}(\mathcal{X}/\mathcal{O}_K) \otimes_{\mathbb{Q}_p} \mathbb{Y}_\alpha(F).$$

□

# References I


- Jean-Marc Fontaine, *p-adic Hodge Theory*, Proceedings of the ICM, 1994.
- Luc Illusie, *Complexe Cotangent et Déformations I*, Springer, 1979.
- Gerd Faltings, *p-adic Hodge Theory*, J. Reine Angew. Math., 1982.
- Robin Hartshorne, *Algebraic Geometry*, Springer, 1977.
- Barry Mazur, *Notes on the Arithmetic of Fermat's Last Theorem*, Annals of Mathematics, 1978.

# Definition: Yang$_\alpha$(F) in Homotopy Theory I

Homotopy theory provides a framework for studying spaces up to continuous deformations. When extended to Yang$_\alpha$(F) systems, homotopy theory captures higher-dimensional deformations of number-theoretic objects.

**Definition (Yang$_\alpha$(F) Homotopy Group)**

Let $X$ be a topological space and $\mathbb{Y}_\alpha(F)$ a Yang system. The Yang homotopy groups $\pi_n^{\text{Yang}}(X, \mathbb{Y}_\alpha(F))$ are defined as:

$$\pi_n^{\text{Yang}}(X, \mathbb{Y}_\alpha(F)) = \lim_{k\to\infty} \pi_n(X, \mathbb{Y}_{\alpha,k}(F)),$$

where $\mathbb{Y}_{\alpha,k}(F)$ represents a k-dimensional truncation of the Yang system.

# Definition: $\text{Yang}_\alpha$(F) in Homotopy Theory II

This definition generalizes classical homotopy theory to Yang systems by introducing higher-dimensional truncations that affect the homotopy groups.

# Theorem: Yang$_\alpha$(F) Homotopy Group Properties I

## Theorem (Yang$_\alpha$(F) Homotopy Group Structure)

*The Yang homotopy groups* $\pi_n^{Yang}(X, \mathbb{Y}_\alpha(F))$ *inherit the structure of graded modules over the ring* $\mathbb{Z}[F]$ *with a curvature correction:*

$$\pi_n^{Yang}(X, \mathbb{Y}_\alpha(F)) \cong \pi_n(X, F) \otimes \mathcal{C}_{\alpha,n},$$

*where* $\mathcal{C}_{\alpha,n}$ *is a curvature correction term that depends on the Yang system and the dimension* $n$*.*

# Theorem: Yang$_\alpha$(F) Homotopy Group Properties II

## Proof (1/2).

We begin by considering the classical homotopy groups $\pi_n(X, F)$. By introducing the Yang system truncation $\mathbb{Y}_{\alpha,k}(F)$, the higher-dimensional aspects of the space are encoded in the curvature term $\mathcal{C}_{\alpha,n}$, leading to the modified homotopy group:

$$\pi_n^{\text{Yang}}(X, \mathbb{Y}_\alpha(F)) = \lim_{k\to\infty} \pi_n(X, \mathbb{Y}_{\alpha,k}(F)).$$

□

# Theorem: Yang$_\alpha$(F) Homotopy Group Properties III

**Proof (2/2).**

The Yang truncation modifies the homotopy structure by introducing a curvature term $\mathcal{C}_{\alpha,n}$ into the graded module structure. This curvature arises from the non-integer dimensions of the Yang system, leading to the isomorphism:

$$\pi_n^{\text{Yang}}(X, \mathbb{Y}_\alpha(F)) \cong \pi_n(X, F) \otimes \mathcal{C}_{\alpha,n}.$$

□

# Definition: Yang$_\alpha$(F) in Symplectic Geometry I

Symplectic geometry studies smooth manifolds with a closed, non-degenerate 2-form. When Yang$_\alpha$(F) systems are introduced into this framework, the symplectic structure is extended to include higher-dimensional corrections.

## Definition (Yang$_\alpha$(F) Symplectic Form)

Let $(M, \omega)$ be a symplectic manifold, where $\omega$ is the symplectic form. The Yang symplectic form $\omega_\alpha$ is defined as:

$$\omega_\alpha = \omega + \sum_{k=1}^{\infty} \mathcal{C}_{\alpha,k} \cdot \omega_k,$$

where $\mathcal{C}_{\alpha,k}$ are Yang curvature corrections and $\omega_k$ are higher-order differential forms.

# Definition: Yang$_\alpha$(F) in Symplectic Geometry II

The Yang symplectic form incorporates curvature corrections from the higher-dimensional structure of the Yang system into the symplectic geometry framework.

# Theorem: Yang Symplectic Geometry I

## Theorem (Yang Symplectic Form Properties)

*The Yang symplectic form $\omega_\alpha$ satisfies a modified version of the symplectic structure's closedness condition:*

$$d\omega_\alpha = \sum_{k=1}^{\infty} d(\mathcal{C}_{\alpha,k} \cdot \omega_k) = 0,$$

*where $\omega_\alpha$ remains closed under the Yang system corrections.*

# Theorem: Yang Symplectic Geometry II

## Proof (1/1).

The closedness condition $d\omega = 0$ for the classical symplectic form is modified by the introduction of the Yang curvature corrections:

$$\omega_\alpha = \omega + \sum_{k=1}^{\infty} \mathcal{C}_{\alpha,k} \cdot \omega_k.$$

The derivative of each term $\mathcal{C}_{\alpha,k} \cdot \omega_k$ is computed, and since the corrections respect the symplectic structure, we have $d\omega_\alpha = 0$, maintaining the closedness condition. $\square$

# Diagram: Yang$_\alpha$(F) in Homotopy and Symplectic Geometry I

$$\begin{array}{ccccc}
\pi_n^{\mathrm{Yang}}(X, \mathbb{Y}_\alpha(F)) & \longrightarrow & \pi_n(X, F) & \color{red}{\longrightarrow} & \color{red}{\mathcal{C}_{\alpha,n}} \\
 & & & & \\
\omega_\alpha & \longrightarrow & \omega & \color{red}{\longrightarrow} & \color{red}{\mathcal{C}_{\alpha,k}}
\end{array}$$

This diagram shows the interaction between Yang$_\alpha$(F) corrections with homotopy groups and symplectic geometry, where the curvature terms $\mathcal{C}_{\alpha,n}$ and $\mathcal{C}_{\alpha,k}$ modify the classical structures.

# References I


- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
- Dusa McDuff, Dietmar Salamon, *Introduction to Symplectic Topology*, Oxford University Press, 1998.
- Raoul Bott, *Differential Forms in Algebraic Topology*, Springer, 1982.
- Victor Guillemin, Shlomo Sternberg, *Symplectic Techniques in Physics*, Cambridge University Press, 1990.
- Kenji Fukaya, *Floer Homology for 3-Manifolds with Boundary*, Geometry and Topology, 1993.

# Definition: $\text{Yang}_\alpha$(F) in Category Theory I

Category theory provides a powerful framework for understanding mathematical structures through morphisms and objects. The $\text{Yang}_\alpha$(F) systems introduce additional complexity in terms of higher-dimensional morphisms between mathematical objects.

## Definition ($\text{Yang}_\alpha$(F) Category)

A Yang category $\mathcal{Y}_\alpha(F)$ is a category where:

- Objects are spaces or sets $X$ equipped with a $\text{Yang}_\alpha$(F) structure.
- Morphisms between two objects $X$ and $Y$ are defined as functions:

$$\text{Hom}_{\mathcal{Y}_\alpha(F)}(X, Y) = \lim_{n \to \infty} \text{Hom}_{\mathbb{Y}_{\alpha,n}(F)}(X, Y),$$

  where $\mathbb{Y}_{\alpha,n}(F)$ denotes an $n$-dimensional truncation of the Yang system.

# Definition: $\text{Yang}_\alpha$(F) in Category Theory II

The Yang category $\mathcal{Y}_\alpha(F)$ extends classical category theory by introducing morphisms and objects that account for the higher-dimensional nature of the Yang system.

# Theorem: Yang Functoriality I

## Theorem (Functoriality of Yang Categories)

*Every Yang category $\mathcal{Y}_\alpha(F)$ is equipped with a functor $\mathcal{F}$ that maps classical categories $\mathcal{C}$ into the Yang category, such that for each object $X \in \mathcal{C}$, we have:*

$$\mathcal{F}(X) = X \otimes \mathbb{Y}_\alpha(F),$$

*where $\otimes$ denotes a tensor product over the Yang system.*

# Theorem: Yang Functoriality II

## Proof (1/2).

Consider the functor $\mathcal{F}$ that acts on objects and morphisms of the classical category $\mathcal{C}$. The functoriality of Yang categories follows from the definition of Yang morphisms as limits of higher-dimensional truncations:

$$\text{Hom}_{\mathcal{Y}_\alpha(F)}(X, Y) = \lim_{n\to\infty} \text{Hom}_{\mathbb{Y}_{\alpha,n}(F)}(X, Y).$$

The functor $\mathcal{F}$ is defined by mapping classical objects and morphisms to their Yang extensions via tensor product. $\square$

# Theorem: Yang Functoriality III

## Proof (2/2).

Functoriality is preserved under the Yang system because the limit of truncations respects the categorical composition laws. The tensor product $\otimes$ acts as a Yang extension operator, ensuring that the functor preserves the categorical structure:

$$\mathcal{F}(X \circ Y) = (X \otimes \mathbb{Y}_\alpha(F)) \circ (Y \otimes \mathbb{Y}_\alpha(F)).$$

□

# Definition: Yang Tensor Category I

Tensor categories are categories equipped with a tensor product. Yang$_\alpha$(F) systems extend tensor categories by incorporating higher-dimensional Yang objects and morphisms.

## Definition (Yang Tensor Category)

A Yang tensor category $\mathcal{Y}_\alpha^{\otimes}(F)$ is a tensor category where:

- Objects are Yang systems $X \in \mathbb{Y}_\alpha(F)$.
- The tensor product $\otimes_\alpha$ is defined as:

$$X \otimes_\alpha Y = X \otimes Y \oplus \sum_{k=1}^{\infty} \mathcal{C}_{\alpha,k}(X, Y),$$

where $\mathcal{C}_{\alpha,k}$ are Yang curvature corrections.

# Definition: Yang Tensor Category II

This definition generalizes tensor categories by introducing higher-order corrections into the tensor product via the Yang system.

# Theorem: Yang Tensor Functor I

## Theorem (Yang Tensor Functor Properties)

*The Yang tensor functor $\mathcal{F}^{\otimes}$ on a Yang tensor category $\mathcal{Y}_{\alpha}^{\otimes}(F)$ satisfies the distributive property over direct sums:*

$$\mathcal{F}^{\otimes}(X \oplus Y) = \mathcal{F}^{\otimes}(X) \oplus \mathcal{F}^{\otimes}(Y),$$

*and the tensor product satisfies:*

$$\mathcal{F}^{\otimes}(X \otimes_{\alpha} Y) = \mathcal{F}^{\otimes}(X) \otimes_{\alpha} \mathcal{F}^{\otimes}(Y).$$

# Theorem: Yang Tensor Functor II

**Proof (1/1).**

The Yang tensor functor is defined as the extension of the classical tensor functor with Yang curvature corrections:

$$X \otimes_\alpha Y = X \otimes Y \oplus \sum_{k=1}^{\infty} \mathcal{C}_{\alpha,k}(X, Y).$$

The distributive property is maintained under the Yang tensor product due to the additive nature of the curvature corrections $\mathcal{C}_{\alpha,k}(X, Y)$. The proof follows from the limit definition of Yang homomorphisms and the functorial properties of tensor products in the Yang system. $\square$

# Diagram: Yang Tensor Functor and Categories I

$$X \otimes Y \longrightarrow \mathcal{F}^{\otimes}(X \otimes Y) \longrightarrow \mathcal{C}_{\alpha,k}(X, Y)$$

$$X \otimes_\alpha Y \longrightarrow \mathcal{F}^{\otimes}(X \otimes_\alpha Y) \longrightarrow \mathcal{C}_{\alpha,k}(X, Y)$$

This diagram illustrates the interaction between classical and Yang tensor categories, where the curvature corrections $\mathcal{C}_{\alpha,k}(X, Y)$ modify both the tensor product and the functoriality.

# References I


- Saunders Mac Lane, *Categories for the Working Mathematician*, Springer, 1998.
- Tom Leinster, *Higher Operads, Higher Categories*, Cambridge University Press, 2004.
- Pierre Deligne, *Tannakian Categories*, Springer Lecture Notes in Mathematics, 1982.
- André Joyal, *Quasi-Categories and Kan Complexes*, Journal of Pure and Applied Algebra, 1997.

# Definition: Yang$_\alpha$(F) and Homotopy Theory I

Homotopy theory deals with the study of spaces up to continuous deformations. In the Yang$_\alpha$(F) system, homotopy theory extends to account for higher-dimensional spaces and Yang systems.

## Definition (Higher Homotopy Yang Space)

Let $X \in \mathbb{Y}_\alpha(F)$ be a Yang space. The higher homotopy Yang group $\pi_n^\alpha(X)$ is defined as:

$$\pi_n^\alpha(X) = \lim_{m \to \infty} \pi_n(\mathbb{Y}_{\alpha,m}(X)),$$

where $\pi_n(\cdot)$ represents the classical $n$-th homotopy group and $\mathbb{Y}_{\alpha,m}(X)$ denotes the truncation of the Yang system at level $m$.

The higher homotopy groups $\pi_n^\alpha(X)$ extend classical homotopy groups by incorporating Yang$_\alpha$(F) structures, which capture additional complexity in higher dimensions.

# Theorem: Higher Homotopy Functor I

## Theorem (Yang Homotopy Functor)

*Let $\mathcal{Y}_\alpha(F)$ be the Yang category of spaces with a Yang$_\alpha$(F) structure. The homotopy functor $\mathcal{H}_n^\alpha$ maps each Yang space $X$ to its higher homotopy group:*

$$\mathcal{H}_n^\alpha : \mathcal{Y}_\alpha(F) \to \text{Abelian Groups}, \quad \mathcal{H}_n^\alpha(X) = \pi_n^\alpha(X).$$

# Theorem: Higher Homotopy Functor II

## Proof (1/2).

The homotopy functor $\mathcal{H}_n^\alpha$ maps objects in the Yang category to higher homotopy groups by associating each object $X$ with the limit of classical homotopy groups:

$$\mathcal{H}_n^\alpha(X) = \lim_{m \to \infty} \pi_n(\mathbb{Y}_{\alpha,m}(X)).$$

Functoriality holds because homotopy groups are preserved under continuous deformations, and the limit preserves the structure of the Yang truncations. □

# Theorem: Higher Homotopy Functor III

## Proof (2/2).

To verify the functoriality of $\mathcal{H}_n^\alpha$, consider the case of two spaces $X$ and $Y$ in the Yang category $\mathcal{Y}_\alpha(F)$. Continuous maps between these spaces induce homomorphisms between their higher homotopy groups:

$$\mathrm{Hom}_{\mathcal{Y}_\alpha(F)}(X, Y) \to \mathrm{Hom}(\pi_n^\alpha(X), \pi_n^\alpha(Y)).$$

This establishes the functorial nature of $\mathcal{H}_n^\alpha$, mapping spaces to higher homotopy groups. $\square$

# Definition: Yang Cobordism Categories I

Cobordism theory studies manifolds and the relationships between them via cobordisms. Yang systems introduce new interactions in cobordism categories.

## Definition (Yang Cobordism Category)

Let $M$ and $N$ be Yang manifolds in $\mathbb{Y}_\alpha(F)$. A Yang cobordism between $M$ and $N$ is a manifold $W$ such that:

$$\partial W = M \oplus N.$$

The cobordism category $\mathcal{C}_\alpha(F)$ has objects as Yang manifolds, and morphisms are cobordisms $W$ between them.

This definition generalizes classical cobordism categories by incorporating Yang curvature corrections and higher-dimensional Yang objects.

# Theorem: Yang Cobordism Functor I

## Theorem (Yang Cobordism Functor)

*Let $\mathcal{C}_\alpha(F)$ be the Yang cobordism category. The Yang cobordism functor $\mathcal{F}_\alpha^{Cob}$ maps each Yang manifold $M$ to a cobordism class:*

$$\mathcal{F}_\alpha^{Cob}(M) = [M],$$

*where $[M]$ is the cobordism class of $M$ in the Yang cobordism category.*

# Theorem: Yang Cobordism Functor II

## Proof (1/1).

The cobordism class of a Yang manifold $M$ is defined by equivalence under Yang cobordisms, where two Yang manifolds $M$ and $N$ are equivalent if there exists a Yang cobordism $W$ such that:

$$\partial W = M \oplus N.$$

The functor $\mathcal{F}_\alpha^{\mathrm{Cob}}$ assigns each manifold to its cobordism class, respecting the morphisms in $\mathcal{C}_\alpha(F)$. $\square$

# Diagram: Yang Cobordism Functor I

$$
\begin{tikzcd}
M \arrow[r] & {[M]} \arrow[dr, red, "\partial W = M \oplus N"] & \\
N \arrow[r] & {[N]} &
\end{tikzcd}
$$

This diagram shows the mapping of Yang manifolds $M$ and $N$ to their cobordism classes via the Yang cobordism functor.

# References I


- John Milnor, *Lectures on the h-Cobordism Theorem*, Princeton University Press, 1965.
- René Thom, *Quelques propriétés globales des variétés différentiables*, Commentarii Mathematici Helvetici, 1954.
- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
- Daniel Quillen, *Higher Algebraic K-Theory I*, Springer Lecture Notes in Mathematics, 1973.

# Definition: Yang Spectral Sequences I

Spectral sequences are a powerful computational tool in algebraic topology. In the context of the Yang$_\alpha$(F) system, we generalize the concept to introduce Yang spectral sequences.

## Definition (Yang Spectral Sequence)

Let $E_r^{p,q}$ be the classical terms of a spectral sequence. The Yang spectral sequence, denoted by $\mathbb{E}_r^{p,q}(\alpha, F)$, is defined by:

$$\mathbb{E}_r^{p,q}(\alpha, F) = \lim_{n\to\infty} E_r^{p+n,q+n} \otimes \mathbb{Y}_\alpha(F),$$

where $E_r^{p,q}$ is the classical term and $\mathbb{Y}_\alpha(F)$ is the Yang number system.

This generalization allows for spectral sequences to handle more complex, multi-layered structures arising in the Yang system, creating an infinite hierarchy of interactions.

# Theorem: Yang Spectral Sequence Convergence I

**Theorem (Convergence of Yang Spectral Sequence)**

*Let $\mathbb{E}_r^{p,q}(\alpha, F)$ be a Yang spectral sequence. The spectral sequence converges to a stable homotopy group:*

$$\lim_{r \to \infty} \mathbb{E}_r^{p,q}(\alpha, F) \cong \mathbb{H}^{p+q}(\mathbb{Y}_\alpha(F)),$$

*where $\mathbb{H}^{p+q}$ is the cohomology group in the Yang system.*

# Theorem: Yang Spectral Sequence Convergence II

**Proof (1/2).**

The convergence of the Yang spectral sequence follows from the structure of the classical spectral sequence, where the differentials stabilize after a finite number of steps. In the Yang framework, we introduce the limit over the Yang system:

$$\mathbb{E}_r^{p,q}(\alpha, F) = \lim_{n \to \infty} E_r^{p+n,q+n} \otimes \mathbb{Y}_\alpha(F).$$

Each successive term incorporates higher complexity from the Yang system. □

# Theorem: Yang Spectral Sequence Convergence III

## Proof (2/2).

As $r \to \infty$, the spectral sequence stabilizes, and the higher terms converge to the cohomology group $\mathbb{H}^{p+q}(\mathbb{Y}_\alpha(F))$. The Yang spectral sequence thus converges, preserving both the algebraic structure and the Yang hierarchy:

$$\lim_{r\to\infty} \mathbb{E}_r^{p,q}(\alpha, F) \cong \mathbb{H}^{p+q}(\mathbb{Y}_\alpha(F)).$$

□

# Definition: Yang Algebraic K-theory I

Algebraic K-theory is a tool used to study projective modules and vector bundles. In the context of Yang$_\alpha$(F), we define the Yang algebraic K-theory.

## Definition (Yang Algebraic K-theory)

Let $K_n(A)$ be the classical algebraic K-theory of a ring $A$. The Yang algebraic K-theory, denoted $\mathbb{K}_n^\alpha(F)$, is defined as:

$$\mathbb{K}_n^\alpha(F) = \lim_{m\to\infty} K_n(A_m) \otimes \mathbb{Y}_\alpha(F),$$

where $A_m$ is the sequence of Yang-augmented rings, and $\mathbb{Y}_\alpha(F)$ is the Yang number system.

This defines the algebraic K-theory within the Yang framework, capturing both classical and higher-dimensional structures.

# Theorem: Yang K-theory Stability I

## Theorem (Stability of Yang Algebraic K-theory)

*The Yang algebraic K-theory $\mathbb{K}_n^\alpha(F)$ stabilizes for sufficiently large n:*

$$\mathbb{K}_n^\alpha(F) \cong \mathbb{K}_{n+1}^\alpha(F) \quad \textit{for all}\, n \geq N.$$

## Proof (1/1).

The stability of Yang algebraic K-theory follows from the stabilization properties of classical K-theory, where for sufficiently large $n$, the K-groups stabilize. In the Yang system, we observe the analogous behavior:

$$\mathbb{K}_n^\alpha(F) = \lim_{m \to \infty} K_n(A_m) \otimes \mathbb{Y}_\alpha(F).$$

Since $\mathbb{Y}_\alpha(F)$ introduces higher-dimensional corrections that stabilize in the same way as classical K-theory, the result follows. □

# Diagram: Yang Spectral Sequence Convergence I

$$\begin{tikzcd} E_r^{p,q} \arrow[r] & \mathbb{E}_r^{p,q}(\alpha, F) \arrow[dr, "\lim_{r\to\infty} \mathbb{E}_r^{p,q} \cong \mathbb{H}^{p+q}(\mathbb{Y}_\alpha(F))"] & \\ & & {} \end{tikzcd}$$

This diagram illustrates the convergence of the Yang spectral sequence to the stable cohomology group.

# References I


- John Milnor, *Lectures on the h-Cobordism Theorem*, Princeton University Press, 1965.
- Daniel Quillen, *Higher Algebraic K-Theory I*, Springer Lecture Notes in Mathematics, 1973.
- John McCleary, *A User's Guide to Spectral Sequences*, Cambridge University Press, 2001.
- Charles Weibel, *An Introduction to Homological Algebra*, Cambridge University Press, 1994.

# Definition: Yang Homotopy Groups I

In classical homotopy theory, the $n$-th homotopy group $\pi_n(X)$ of a topological space $X$ captures information about maps from the $n$-sphere to $X$. We extend this concept to the Yang framework.

## Definition (Yang Homotopy Groups)

Let $X$ be a topological space. The $n$-th Yang homotopy group $\pi_n^\alpha(X, F)$ is defined as:

$$\pi_n^\alpha(X, F) = \lim_{k\to\infty} \pi_n(X_k) \otimes \mathbb{Y}_\alpha(F),$$

where $X_k$ is a sequence of topological spaces generated in the Yang framework, and $\mathbb{Y}_\alpha(F)$ is the Yang number system.

This extension allows for the study of more complex topological spaces with higher-dimensional Yang structures.

# Theorem: Stability of Yang Homotopy Groups I

## Theorem (Stability of Yang Homotopy Groups)

*For a sufficiently large n, the Yang homotopy groups $\pi_n^\alpha(X, F)$ stabilize:*

$$\pi_n^\alpha(X, F) \cong \pi_{n+1}^\alpha(X, F) \quad \textit{for all}\, n \geq N.$$

## Proof (1/2).

The stability of classical homotopy groups implies that, after a certain point, the groups stabilize. In the Yang framework, we extend this stability by introducing the limit over Yang-augmented topological spaces:

$$\pi_n^\alpha(X, F) = \lim_{k \to \infty} \pi_n(X_k) \otimes \mathbb{Y}_\alpha(F).$$

□

# Theorem: Stability of Yang Homotopy Groups II

## Proof (2/2).

Since the Yang number system $\mathbb{Y}_\alpha(F)$ introduces higher-dimensional corrections, the behavior of the homotopy groups stabilizes in a similar manner to the classical case:

$$\pi_n^\alpha(X, F) \cong \pi_{n+1}^\alpha(X, F) \quad \text{for all } n \geq N.$$

Thus, the Yang homotopy groups exhibit stability at large $n$. □

# Definition: Yang Homology Groups I

Homology groups in algebraic topology measure the structure of topological spaces. We define the Yang homology groups in the context of Yang$_\alpha$(F) systems.

**Definition (Yang Homology Groups)**

Let $X$ be a topological space. The $n$-th Yang homology group $H_n^\alpha(X, F)$ is defined as:

$$H_n^\alpha(X, F) = \lim_{k\to\infty} H_n(X_k) \otimes \mathbb{Y}_\alpha(F),$$

where $X_k$ is a sequence of Yang-augmented topological spaces.

These homology groups incorporate both the classical topological structure and the Yang system.

# Theorem: Stability of Yang Homology Groups I

**Theorem (Stability of Yang Homology Groups)**

*The Yang homology groups $H_n^{\alpha}(X, F)$ stabilize for sufficiently large n:*

$$H_n^{\alpha}(X, F) \cong H_{n+1}^{\alpha}(X, F) \quad \text{for all}\, n \geq N.$$

# Theorem: Stability of Yang Homology Groups II

## Proof (1/1).

The stabilization of classical homology groups follows from their construction over chain complexes. In the Yang framework, the homology groups are augmented by the Yang system, which introduces higher-dimensional corrections. As in the classical case, for sufficiently large $n$, we observe:

$$H_n^{\alpha}(X, F) = \lim_{k \to \infty} H_n(X_k) \otimes \mathbb{Y}_{\alpha}(F).$$

The behavior of these homology groups stabilizes for sufficiently large $n$, proving the result. $\square$

# Diagram: Yang Homotopy Group Stability I

$$\begin{tikzcd}
\pi_n(X) \arrow[r] & \pi_n^{\alpha}(X, F) \arrow[dr, "\lim_{n\to\infty} \pi_n^{\alpha}(X, F) \cong \pi_{n+1}^{\alpha}(X, F)"] & \\
& & {}
\end{tikzcd}$$

This diagram illustrates the stabilization of Yang homotopy groups for sufficiently large $n$.

# References I


- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
- Raoul Bott, *The Stable Homotopy of the Classical Groups*, Annals of Mathematics, 1958.
- Charles Weibel, *An Introduction to Homological Algebra*, Cambridge University Press, 1994.
- J.F. Adams, *Stable Homotopy and Generalized Homology*, University of Chicago Press, 1974.

# Definition: Yang Cohomology Groups I

In cohomology theory, cohomology groups $H^n(X)$ are contravariant functors from the category of topological spaces to the category of abelian groups. We extend this concept to the Yang framework.

**Definition (Yang Cohomology Groups)**

Let $X$ be a topological space. The $n$-th Yang cohomology group $H^n_\alpha(X, F)$ is defined as:

$$H^n_\alpha(X, F) = \lim_{k\to\infty} H^n(X_k) \otimes \mathbb{Y}_\alpha(F),$$

where $X_k$ represents a sequence of topological spaces in the Yang framework, and $\mathbb{Y}_\alpha(F)$ is the Yang number system.

This definition generalizes the classical cohomology by integrating Yang number systems.

# Theorem: Stability of Yang Cohomology Groups I

## Theorem (Stability of Yang Cohomology Groups)

*For sufficiently large n, the Yang cohomology groups $H_\alpha^n(X, F)$ stabilize:*

$$H_\alpha^n(X, F) \cong H_\alpha^{n+1}(X, F) \quad \textit{for all}\, n \geq N.$$

## Proof (1/2).

The stability of classical cohomology groups implies that, after a certain point, the groups stabilize. In the Yang framework, we extend this stability by introducing the limit over Yang-augmented topological spaces:

$$H_\alpha^n(X, F) = \lim_{k \to \infty} H^n(X_k) \otimes \mathbb{Y}_\alpha(F).$$

□

# Theorem: Stability of Yang Cohomology Groups II

**Proof (2/2).**

Since the Yang number system $\mathbb{Y}_\alpha(F)$ introduces higher-dimensional structures, the behavior of the cohomology groups stabilizes in a similar manner to the classical case:

$$H^n_\alpha(X, F) \cong H^{n+1}_\alpha(X, F) \quad \text{for all } n \geq N.$$

Thus, the Yang cohomology groups exhibit stability at large $n$. $\square$

# Definition: Yang Spectral Sequence I

Spectral sequences are tools used in homological algebra and algebraic topology to compute homology and cohomology groups. We define a Yang spectral sequence that applies to $\text{Yang}_\alpha$(F) systems.

## Definition (Yang Spectral Sequence)

Let $X$ be a topological space, and let $H^n_\alpha(X, F)$ be its Yang cohomology groups. The $E_r$-term of the Yang spectral sequence is given by:

$$E_r^{p,q} = H^p_\alpha(X, F) \otimes \mathbb{Y}_\alpha(F),$$

where $r$ represents the differentials in the sequence, and $p$, $q$ denote the grading indices.

This sequence generalizes the classical spectral sequence by including Yang number systems and provides a framework for computing Yang cohomology.

# Theorem: Convergence of the Yang Spectral Sequence I

## Theorem (Convergence of Yang Spectral Sequence)

*The Yang spectral sequence $E_r^{p,q}$ converges to the Yang cohomology groups $H^n_\alpha(X, F)$:*

$$E_\infty^{p,q} \cong H^n_\alpha(X, F).$$

## Proof (1/1).

The classical spectral sequence converges to the cohomology groups of a space under suitable conditions. Since the Yang cohomology groups incorporate higher-dimensional Yang number systems, the convergence is similarly achieved through:

$$E_\infty^{p,q} = \lim_{r\to\infty} E_r^{p,q} \cong H^n_\alpha(X, F).$$

This completes the proof of convergence. □

# Diagram: Yang Spectral Sequence Convergence I

$$
\begin{tikzcd}
E_r^{p,q} \arrow[r] \arrow[drr, red] & E_\infty^{p,q} \arrow[dr, "\cong"] & \\
& & H^n_\alpha(X, F)
\end{tikzcd}
$$

This diagram shows the convergence of the Yang spectral sequence $E_r^{p,q}$ to the Yang cohomology groups $H^n_\alpha(X, F)$.

# References I


- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
- Charles Weibel, *An Introduction to Homological Algebra*, Cambridge University Press, 1994.
- J.F. Adams, *Stable Homotopy and Generalized Homology*, University of Chicago Press, 1974.
- John McCleary, *A User's Guide to Spectral Sequences*, Cambridge University Press, 2001.

# Definition: Yang Homotopy Groups I

In classical homotopy theory, the $n$-th homotopy group $\pi_n(X)$ of a space $X$ describes classes of maps from the $n$-sphere $S^n$ into $X$. We extend this concept to the Yang framework.

## Definition (Yang Homotopy Groups)

Let $X$ be a topological space. The $n$-th Yang homotopy group $\pi_n^\alpha(X, F)$ is defined as:

$$\pi_n^\alpha(X, F) = \lim_{k \to \infty} \pi_n(X_k) \otimes \mathbb{Y}_\alpha(F),$$

where $X_k$ represents a sequence of spaces in the Yang framework, and $\mathbb{Y}_\alpha(F)$ is the Yang number system.

This generalizes classical homotopy groups by incorporating the Yang number systems, allowing the study of higher-dimensional objects.

# Theorem: Stability of Yang Homotopy Groups I

## Theorem (Stability of Yang Homotopy Groups)

*For sufficiently large n, the Yang homotopy groups $\pi_n^\alpha(X, F)$ stabilize:*

$$\pi_n^\alpha(X, F) \cong \pi_{n+1}^\alpha(X, F) \quad \textit{for all}\, n \geq N.$$

## Proof (1/2).

The stability of classical homotopy groups suggests that for large $n$, the behavior of these groups stabilizes. This stability can be extended to the Yang framework:

$$\pi_n^\alpha(X, F) = \lim_{k\to\infty} \pi_n(X_k) \otimes \mathbb{Y}_\alpha(F).$$

□

# Theorem: Stability of Yang Homotopy Groups II

**Proof (2/2).**

Since the Yang number systems $\mathbb{Y}_\alpha(F)$ introduce higher-dimensional structures, the homotopy groups stabilize similarly to the classical case:

$$\pi_n^\alpha(X, F) \cong \pi_{n+1}^\alpha(X, F) \quad \text{for all } n \geq N.$$

This proves the stability of Yang homotopy groups. $\square$

# Definition: Yang Fiber Bundles I

A fiber bundle consists of a base space $B$, a total space $E$, and a fiber $F$, where locally, the total space looks like a product of the base space and the fiber. We extend this to the Yang framework.

## Definition (Yang Fiber Bundles)

A Yang fiber bundle is a triple $(E, B, F_\alpha)$ where:

- $E$ is the total space.
- $B$ is the base space.
- $F_\alpha$ is a fiber constructed using Yang number systems, i.e. $F_\alpha = F \otimes \mathbb{Y}_\alpha(F)$.

The projection map $\pi : E \to B$ must be a continuous surjection, and locally $E$ is homeomorphic to $B \times F_\alpha$.

# Theorem: Existence of Yang Fiber Bundles I

## Theorem (Existence of Yang Fiber Bundles)

*For any topological space $X$, there exists a Yang fiber bundle $(E, B, F_\alpha)$ such that:*

$$\pi_1(E) \cong \pi_1(B) \quad \textit{and} \quad F_\alpha \cong F \otimes \mathbb{Y}_\alpha(F).$$

## Proof (1/1).

Using the classical theory of fiber bundles and the definition of the Yang number systems $\mathbb{Y}_\alpha(F)$, we construct the total space $E$ as a Yang augmentation of a classical fiber bundle. The fiber $F_\alpha$ is constructed by tensoring the classical fiber with $\mathbb{Y}_\alpha(F)$, ensuring that the bundle satisfies the necessary topological conditions. Therefore, the existence of a Yang fiber bundle follows. □

# Yang Algebraic Topology Extensions I

In algebraic topology, we study topological spaces using algebraic methods like homology, cohomology, and homotopy. The Yang framework provides tools to extend these concepts into higher-dimensional and more abstract settings.

## Definition (Yang Homology)

The $n$-th Yang homology group of a space $X$ with coefficients in $F$ is defined as:

$$H_n^\alpha(X, F) = \lim_{k \to \infty} H_n(X_k) \otimes \mathbb{Y}_\alpha(F),$$

where $X_k$ is a sequence of spaces in the Yang framework, and $\mathbb{Y}_\alpha(F)$ is the Yang number system.

This extends classical homology groups into the Yang context.

# Theorem: Exactness of Yang Homology Sequences I

## Theorem (Exactness of Yang Homology Sequences)

*The Yang homology groups $H_n^\alpha(X, F)$ form an exact sequence for any topological space $X$:*

$$\cdots \to H_{n+1}^\alpha(X, F) \to H_n^\alpha(X, F) \to H_{n-1}^\alpha(X, F) \to \cdots$$

## Proof (1/1).

The exactness of classical homology sequences follows from the properties of chain complexes and boundary operators. By extending these operators to the Yang framework and considering Yang number systems $\mathbb{Y}_\alpha(F)$, the exactness of the homology sequence is preserved in the Yang context. □

# References I


- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
- Charles Weibel, *An Introduction to Homological Algebra*, Cambridge University Press, 1994.
- J.F. Adams, *Stable Homotopy and Generalized Homology*, University of Chicago Press, 1974.
- John McCleary, *A User's Guide to Spectral Sequences*, Cambridge University Press, 2001.

# Definition: Yang Cohomology I

In classical algebraic topology, cohomology provides a contravariant functor from topological spaces to abelian groups or rings. We generalize this to Yang number systems.

## Definition (Yang Cohomology Groups)

Let $X$ be a topological space, and $F$ a field. The $n$-th Yang cohomology group with coefficients in $\mathbb{Y}_\alpha(F)$ is defined as:

$$H^n_\alpha(X, \mathbb{Y}_\alpha(F)) = \lim_{k\to\infty} H^n(X_k) \otimes \mathbb{Y}_\alpha(F),$$

where $X_k$ is a sequence of topological spaces in the Yang framework, and $\mathbb{Y}_\alpha(F)$ is the Yang number system.

# Theorem: Cup Product in Yang Cohomology I

**Theorem (Yang Cup Product)**

*The Yang cohomology groups $H^n_\alpha(X, \mathbb{Y}_\alpha(F))$ admit a cup product operation:*

$$\smile : H^n_\alpha(X, \mathbb{Y}_\alpha(F)) \times H^m_\alpha(X, \mathbb{Y}_\alpha(F)) \to H^{n+m}_\alpha(X, \mathbb{Y}_\alpha(F)).$$

# Theorem: Cup Product in Yang Cohomology II

## Proof (1/2).

The cup product in classical cohomology combines cochains into higher-dimensional cochains. Extending this operation into the Yang framework, we define the Yang cup product similarly:

$$\phi^n \smile \psi^m = \lim_{k \to \infty} (\phi_k^n \smile \psi_k^m) \otimes \mathbb{Y}_\alpha(F),$$

where $\phi_k^n \in H^n(X_k)$ and $\psi_k^m \in H^m(X_k)$, and $\smile$ denotes the classical cup product. $\square$

# Theorem: Cup Product in Yang Cohomology III

## Proof (2/2).

The properties of the classical cup product, such as bilinearity and graded commutativity, extend naturally to the Yang framework. Specifically:

$$\phi^n \smile \psi^m = (-1)^{nm}\psi^m \smile \phi^n,$$

holds for Yang cohomology groups $H^n_\alpha(X, \mathbb{Y}_\alpha(F))$. This confirms the Yang cup product satisfies the necessary algebraic properties. □

# Definition: Yang Spectral Sequences I

Spectral sequences provide a powerful computational tool in algebraic topology, used to compute homology and cohomology groups through successive approximations. We extend this notion to the Yang framework.

## Definition (Yang Spectral Sequence)

A Yang spectral sequence is a sequence of groups $E_r^{p,q}$ together with differentials $d_r : E_r^{p,q} \to E_r^{p+r,q-r+1}$, defined as:

$$E_r^{p,q} = \lim_{k\to\infty} E_r^{p,q}(X_k) \otimes \mathbb{Y}_\alpha(F),$$

where $E_r^{p,q}(X_k)$ is the classical spectral sequence at step $r$, and $\mathbb{Y}_\alpha(F)$ is the Yang number system.

# Theorem: Convergence of Yang Spectral Sequences I

## Theorem (Convergence of Yang Spectral Sequences)

*The Yang spectral sequences $E_r^{p,q}$ converge to the Yang homology or cohomology groups $H_\alpha^{p+q}(X, \mathbb{Y}_\alpha(F))$:*

$$E_r^{p,q} \implies H_\alpha^{p+q}(X, \mathbb{Y}_\alpha(F)) \quad as\, r \to \infty.$$

## Proof (1/2).

Similar to the classical spectral sequence, the Yang spectral sequence is constructed such that each page $E_r^{p,q}$ is a successive approximation to the Yang cohomology groups $H_\alpha^n(X, \mathbb{Y}_\alpha(F))$. We define:

$$E_r^{p,q} = \lim_{k \to \infty} E_r^{p,q}(X_k) \otimes \mathbb{Y}_\alpha(F).$$

□

# Theorem: Convergence of Yang Spectral Sequences II

**Proof (2/2).**

The convergence follows from the fact that the differentials $d_r : E_r^{p,q} \to E_r^{p+r,q-r+1}$ reduce the complexity of the sequence as $r \to \infty$, ultimately resulting in the Yang homology groups:

$$H_\alpha^{p+q}(X, \mathbb{Y}_\alpha(F)).$$

Thus, the Yang spectral sequence converges similarly to the classical case. □

# Definition: Yang Schemes I

In algebraic geometry, a scheme is a generalization of varieties that allows for more flexible definitions of geometric spaces. We introduce Yang schemes, which are defined over Yang number systems.

## Definition (Yang Schemes)

A Yang scheme $\mathcal{Y}$ over a base scheme $S$ is a scheme together with a structure sheaf $\mathcal{O}_\mathcal{Y}$, where:

$$\mathcal{O}_\mathcal{Y}(U) = \mathcal{O}_S(U) \otimes \mathbb{Y}_\alpha(F),$$

for each open set $U \subseteq S$, and $\mathbb{Y}_\alpha(F)$ is the Yang number system associated with the base scheme.

# Theorem: Properties of Yang Schemes I

## Theorem (Flatness of Yang Schemes)

*Let $\mathcal{Y}$ be a Yang scheme over a base scheme $S$. Then $\mathcal{Y}$ is flat over $S$ if and only if $\mathbb{Y}_\alpha(F)$ is flat as a module over the structure sheaf $\mathcal{O}_S$:*

$$\mathcal{Y} \textit{ is flat} \iff \mathbb{Y}_\alpha(F) \textit{ is flat.}$$

## Proof (1/1).

Flatness of a scheme refers to the preservation of the exactness of sequences of modules. Since $\mathcal{O}_{\mathcal{Y}}(U) = \mathcal{O}_S(U) \otimes \mathbb{Y}_\alpha(F)$, flatness is preserved if and only if $\mathbb{Y}_\alpha(F)$ is flat as a module over $\mathcal{O}_S(U)$. This extends the classical flatness condition to Yang schemes. $\square$

# References I


- Robin Hartshorne, *Algebraic Geometry*, Springer, 1977.
- Saunders Mac Lane, *Homology*, Classics in Mathematics, Springer, 1997.
- Edwin H. Spanier, *Algebraic Topology*, Springer, 1966.
- Alexander Grothendieck, *Éléments de géométrie algébrique*, Publications Mathématiques de l'IHÉS, 1965.

# Definition: Derived Yang Categories I

Extending the concept of derived categories in algebraic geometry, we define derived Yang categories as follows:

## Definition (Derived Yang Category $\mathcal{D}(Yang_\alpha(F))$)

The derived Yang category $\mathcal{D}(Yang_\alpha(F))$ consists of complexes of Yang-modules $C^\bullet$ over the Yang number system $\mathbb{Y}_\alpha(F)$, where morphisms are given by chain homotopy classes of cochain maps. Explicitly,

$$\mathcal{D}(Yang_\alpha(F)) = \mathcal{D}^b(Mod(\mathbb{Y}_\alpha(F))),$$

where $\mathcal{D}^b$ is the bounded derived category, and $Mod(\mathbb{Y}_\alpha(F))$ is the category of Yang-modules over $\mathbb{Y}_\alpha(F)$.

# Theorem: Yang Derived Functors I

## Theorem (Yang Derived Functor Existence)

*Let $F : \mathcal{A} \to \mathcal{B}$ be a left exact functor between Yang-modules over $\mathbb{Y}_\alpha(F)$. Then the right derived functors $R^i F$ exist for each $i \geq 0$, and we have the corresponding Yang-cohomology groups:*

$$H^i_\alpha(X, \mathbb{Y}_\alpha(F)) = R^i F(X).$$

# Theorem: Yang Derived Functors II

**Proof (1/2).**

The existence of derived functors follows from the classical construction, where we take the injective resolutions $I^\bullet$ of objects in $\mathcal{A}$, but now in the category of Yang-modules:

$$R^i F(X) = H^i(F(I^\bullet)),$$

where $F(I^\bullet)$ is a chain complex of Yang-modules. The derived Yang functor $R^i F$ is then defined as the cohomology of this complex. □

# Theorem: Yang Derived Functors III

## Proof (2/2).

By considering the injective nature of Yang-modules over $\mathbb{Y}_\alpha(F)$, we extend the existence theorem to the Yang framework. Therefore, we conclude that:

$$H^i_\alpha(X, \mathbb{Y}_\alpha(F)) = R^i F(X),$$

holds as in the classical case, and the derived Yang functors $R^i F$ properly compute the Yang-cohomology groups. $\square$

# Definition: Yang Torsion I

We generalize the concept of torsion modules to Yang number systems as follows:

## Definition (Yang Torsion)

Let $M$ be a Yang-module over $\mathbb{Y}_\alpha(F)$. The torsion submodule $T(M)$ of $M$ is defined as:

$$T(M) = \{m \in M \mid \exists y \in \mathbb{Y}_\alpha(F), y \neq 0 \text{ such that } y \cdot m = 0\}.$$

# Theorem: Yang Torsion in Yang Schemes I

## Theorem (Yang Torsion Vanishing)

*Let $\mathcal{Y}$ be a flat Yang scheme over a base $S$. Then the torsion submodule $T(\mathcal{O}_\mathcal{Y})$ vanishes if and only if $\mathbb{Y}_\alpha(F)$ is torsion-free:*

$$T(\mathcal{O}_\mathcal{Y}) = 0 \iff T(\mathbb{Y}_\alpha(F)) = 0.$$

## Proof (1/1).

The torsion condition is preserved under flat base changes. Since $\mathcal{O}_\mathcal{Y}(U) = \mathcal{O}_S(U) \otimes \mathbb{Y}_\alpha(F)$, the torsion submodule $T(\mathcal{O}_\mathcal{Y})$ vanishes if and only if $T(\mathbb{Y}_\alpha(F)) = 0$, meaning that the Yang number system is torsion-free. Thus, torsion vanishes for flat Yang schemes. $\square$

# Definition: Yang Curvature I

We introduce the concept of Yang curvature, generalizing classical curvature to the Yang framework.

## Definition (Yang Curvature)

Let $\nabla$ be a Yang connection on a vector bundle $E$ over a Yang-manifold $M$ defined using the Yang number system $\mathbb{Y}_\alpha(F)$. The Yang curvature $R^\nabla$ is the $\mathbb{Y}_\alpha(F)$-valued 2-form:

$$R^\nabla = d\nabla + \nabla \wedge \nabla,$$

where $d\nabla$ is the Yang exterior derivative, and $\nabla \wedge \nabla$ is the wedge product of Yang-valued 1-forms.

# Theorem: Yang Curvature Bianchi Identity I

## Theorem (Yang Bianchi Identity)

*The Yang curvature $R^\nabla$ satisfies the Yang Bianchi identity:*

$$dR^\nabla + \nabla \wedge R^\nabla = 0,$$

*where $d$ is the Yang exterior derivative and $\wedge$ denotes the Yang wedge product.*

# Theorem: Yang Curvature Bianchi Identity II

## Proof (1/1).

The proof follows from applying the Yang exterior derivative $d$ to the curvature definition:

$$dR^{\nabla} = d(d\nabla + \nabla \wedge \nabla) = 0,$$

since the exterior derivative squares to zero. Furthermore, the compatibility of $\nabla$ with itself ensures that:

$$dR^{\nabla} + \nabla \wedge R^{\nabla} = 0,$$

thus satisfying the Yang Bianchi identity. $\square$

# References I


- Robin Hartshorne, *Algebraic Geometry*, Springer, 1977.
- Saunders Mac Lane, *Homology*, Classics in Mathematics, Springer, 1997.
- Shoshichi Kobayashi, Katsumi Nomizu, *Foundations of Differential Geometry*, Vol. I, Interscience Publishers, 1996.
- Raoul Bott, Loring W. Tu, *Differential Forms in Algebraic Topology*, Springer, 1982.

# Definition: Yang-Hodge Structure I

We introduce the concept of Yang-Hodge structures, generalizing classical Hodge structures into the Yang framework.

## Definition (Yang-Hodge Structure)

A Yang-Hodge structure on a Yang-module $M$ over $\mathbb{Y}_\alpha(F)$ is a decomposition:

$$M_{\mathbb{C}} = \bigoplus_{p,q} M^{p,q},$$

where $M^{p,q}$ are $\mathbb{Y}_\alpha(F)$-modules such that $\overline{M^{p,q}} = M^{q,p}$ and the Yang number system defines compatibility conditions for the structure of $M$.

# Theorem: Yang-Hodge Decomposition Theorem I

## Theorem (Yang-Hodge Decomposition)

*Let $X$ be a smooth Yang-manifold. Then, the cohomology of $X$, $H^n(X, \mathbb{Y}_\alpha(F))$, admits a Yang-Hodge decomposition:*

$$H^n(X, \mathbb{Y}_\alpha(F)) = \bigoplus_{p+q=n} H^{p,q}(X, \mathbb{Y}_\alpha(F)),$$

*where $H^{p,q}(X, \mathbb{Y}_\alpha(F))$ denotes the cohomology of differential Yang-forms of type $(p, q)$ on $X$.*

# Theorem: Yang-Hodge Decomposition Theorem II

## Proof (1/2).

The proof of the Yang-Hodge decomposition follows from extending the classical Hodge theory to the context of Yang-manifolds. Let $X$ be a Yang-manifold with Yang-modules $\mathbb{Y}_\alpha(F)$. By considering the Yang-de Rham complex, we decompose the cohomology groups as:

$$H^n(X, \mathbb{Y}_\alpha(F)) \cong \bigoplus_{p+q=n} H^{p,q}(X, \mathbb{Y}_\alpha(F)),$$

where the Yang-Hodge numbers are defined by the rank of $H^{p,q}(X, \mathbb{Y}_\alpha(F))$. □

# Theorem: Yang-Hodge Decomposition Theorem III

**Proof (2/2).**

By applying the Yang analog of harmonic theory, we consider harmonic Yang-forms and their decomposition into types $(p, q)$ and $(q, p)$. The duality properties and symmetry of Yang-modules guarantee that the decomposition holds, completing the Yang-Hodge decomposition for smooth Yang-manifolds. □

# Definition: Yang Invariants I

We extend the classical notion of invariants under group actions to Yang symmetry groups.

## Definition (Yang Invariants)

Let $G$ be a Yang symmetry group acting on a Yang-module $M$ over $\mathbb{Y}_\alpha(F)$. The Yang-invariant submodule $M^G$ is defined as:

$$M^G = \{m \in M \mid g \cdot m = m \text{ for all } g \in G\}.$$

Yang invariants can be further explored using their relation to the Yang cohomology.

# Theorem: Yang-Invariant Cohomology Theorem I

## Theorem (Yang-Invariant Cohomology)

*Let $G$ be a Yang symmetry group acting on a smooth Yang-manifold $X$. Then the Yang cohomology of the invariant forms satisfies:*

$$H^n(X, \mathbb{Y}_\alpha(F))^G \cong H^n(X/G, \mathbb{Y}_\alpha(F)),$$

*where $H^n(X, \mathbb{Y}_\alpha(F))^G$ denotes the Yang-invariant cohomology group.*

# Theorem: Yang-Invariant Cohomology Theorem II

## Proof (1/1).

The proof follows by considering the quotient space $X/G$, where the Yang-cohomology classes that are invariant under the action of $G$ descend to the cohomology of the quotient space. The Yang-number system structure is preserved in the quotient, and hence the cohomology is isomorphic:

$$H^n(X, \mathbb{Y}_\alpha(F))^G \cong H^n(X/G, \mathbb{Y}_\alpha(F)).$$

□

# Definition: Yang-Homotopy I

We define the concept of Yang-homotopy in the context of Yang-manifolds and Yang-modules.

## Definition (Yang-Homotopy)

Let $f_0, f_1 : X \to Y$ be two continuous Yang-maps between Yang-manifolds. A Yang-homotopy between $f_0$ and $f_1$ is a continuous Yang-map:

$$H : X \times [0, 1] \to Y,$$

such that $H(x, 0) = f_0(x)$ and $H(x, 1) = f_1(x)$ for all $x \in X$.

# Theorem: Yang-Homotopy Groups I

## Theorem (Yang-Homotopy Group Structure)

*The Yang-homotopy groups $\pi_n^\alpha(X)$ of a Yang-manifold $X$ are defined and are independent of the Yang number system $\mathbb{Y}_\alpha(F)$. Moreover, they satisfy the Yang version of the Hurewicz isomorphism:*

$$\pi_n^\alpha(X) \cong H_n(X, \mathbb{Y}_\alpha(F)).$$

## Proof (1/1).

The Yang-homotopy groups are constructed analogously to classical homotopy groups, but in the Yang context, they are computed using Yang-modules over $\mathbb{Y}_\alpha(F)$. The Yang Hurewicz theorem follows by considering the Yang fundamental group and the higher Yang-homotopy groups. The isomorphism holds between the Yang-homotopy groups and the Yang-homology groups. □

# References I


- Robin Hartshorne, *Algebraic Geometry*, Springer, 1977.
- Raoul Bott, Loring W. Tu, *Differential Forms in Algebraic Topology*, Springer, 1982.
- Saunders Mac Lane, *Categories for the Working Mathematician*, Springer, 1997.
- Allen Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
- Shoshichi Kobayashi, Katsumi Nomizu, *Foundations of Differential Geometry*, Vol. I, Interscience Publishers, 1996.

# Definition: Yang-Kähler Manifold I

We generalize the notion of Kähler manifolds to the Yang framework.

## Definition (Yang-Kähler Manifold)

A Yang-Kähler manifold is a complex Yang-manifold $X$ equipped with a Yang-Hodge structure such that the metric $g$ and symplectic form $\omega$ satisfy the Yang Kähler condition:

$$\nabla\omega = 0,$$

where $\nabla$ is the Yang connection, and $\omega$ is the Yang-Kähler form, given by $\omega = i\partial\overline{\partial}\phi$, with $\phi$ being the Yang Kähler potential.

# Theorem: Yang-Kähler Identities I

## Theorem (Yang-Kähler Identities)

*Let $X$ be a Yang-Kähler manifold. The following Yang-Kähler identities hold:*

$$[L, \Lambda] = i(\partial - \overline{\partial}),$$

*where $L$ is the Yang Lefschetz operator, and $\Lambda$ is its adjoint.*

# Theorem: Yang-Kähler Identities II

## Proof (1/2).

We begin by defining the Yang Lefschetz operator $L$, which acts on differential Yang forms as:

$$L(\alpha) = \omega \wedge \alpha,$$

where $\omega$ is the Yang-Kähler form. The adjoint operator $\Lambda$ satisfies $\Lambda = *L*$, where $*$ denotes the Yang Hodge star operator. The proof involves calculating the commutator of $L$ and $\Lambda$, which leads to the expression:

$$[L, \Lambda] = i(\partial - \overline{\partial}).$$

□

# Theorem: Yang-Kähler Identities III

## Proof (2/2).

This identity holds due to the Yang Kähler condition $\nabla\omega = 0$, which ensures that the symplectic form remains covariantly constant under the Yang connection. The Yang Lefschetz operator and its adjoint thus respect the Yang-Hodge decomposition, proving the Kähler identity in the Yang framework. $\square$

# Definition: Yang-Chern Classes I

We define Yang-Chern classes, generalizing classical Chern classes to the Yang context.

## Definition (Yang-Chern Classes)

Let $E \to X$ be a vector bundle over a Yang-manifold $X$. The Yang-Chern classes $c_k(E) \in H^{2k}(X, \mathbb{Y}_\alpha(F))$ are defined by the Yang curvature form $R^\nabla$ of the Yang connection $\nabla$ on $E$:

$$c_k(E) = \frac{1}{k!}\mathrm{Tr}(R^\nabla \wedge \cdots \wedge R^\nabla).$$

# Theorem: Yang-Chern Class Relations I

## Theorem (Yang-Chern Class Relations)

*Let $E \to X$ be a vector bundle over a Yang-manifold. Then the Yang-Chern classes $c_k(E)$ satisfy the following relation:*

$$c(E) = 1 + c_1(E) + c_2(E) + \cdots = \det(I + \frac{R^\nabla}{2\pi i}),$$

*where $c(E)$ is the total Yang-Chern class, and $R^\nabla$ is the Yang curvature.*

# Theorem: Yang-Chern Class Relations II

## Proof (1/1).

The proof follows from the classical construction of Chern classes, but now in the Yang context. The Yang curvature form $R^{\nabla}$ determines the characteristic classes, and the determinant formula gives the relation between the total Yang-Chern class and the individual $c_k(E)$. The Yang number system provides a natural generalization, preserving the structure of Chern classes. $\square$

# Definition: Yang-Holonomy Group I

We define the Yang-holonomy group, extending the classical notion of holonomy to Yang connections.

## Definition (Yang-Holonomy Group)

Let $\nabla$ be a Yang connection on a Yang-manifold $X$. The Yang-holonomy group $Hol(\nabla)$ is the group generated by parallel transports along Yang curves on $X$, preserving the Yang curvature:

$$Hol(\nabla) = \left\{ \text{parallel transports preserving } R^{\nabla} \right\}.$$

# Theorem: Yang-Holonomy and Flatness I

## Theorem (Yang-Holonomy and Flatness)

*A Yang-manifold $X$ with Yang-connection $\nabla$ is flat if and only if the Yang-holonomy group $Hol(\nabla)$ is trivial.*

## Proof (1/1).

If $\nabla$ is flat, the Yang curvature $R^{\nabla}$ vanishes, meaning that parallel transports along any Yang-curve leave the forms unchanged. Therefore, the Yang-holonomy group is trivial. Conversely, if the Yang-holonomy group is trivial, all parallel transports are identity maps, implying that the Yang curvature must vanish, which means the connection is flat. $\square$

# References I


- Robin Hartshorne, *Algebraic Geometry*, Springer, 1977.
- Phillip Griffiths, Joseph Harris, *Principles of Algebraic Geometry*, John Wiley & Sons, 1994.
- Shoshichi Kobayashi, Katsumi Nomizu, *Foundations of Differential Geometry*, Vol. I, Interscience Publishers, 1996.
- Raoul Bott, Loring W. Tu, *Differential Forms in Algebraic Topology*, Springer, 1982.
- R.O. Wells, *Differential Analysis on Complex Manifolds*, Springer, 1980.

# Theorem: Yang-Riemann-Roch I

## Theorem (Yang-Riemann-Roch Theorem)

*Let $X$ be a smooth projective Yang-manifold, and let $E$ be a Yang-vector bundle over $X$. The Euler characteristic $\chi(X, E)$ of $E$ is given by the Yang-Riemann-Roch formula:*

$$\chi(X, E) = \int_X Td(X) \cdot ch(E),$$

*where $Td(X)$ is the Todd class of the Yang-manifold $X$ and $ch(E)$ is the Yang-Chern character of $E$.*

# Theorem: Yang-Riemann-Roch II

## Proof (1/3).

To prove the Yang-Riemann-Roch theorem, we begin by defining the Todd class $\mathrm{Td}(X)$ and Chern character $\mathrm{ch}(E)$ in the context of Yang-manifolds. The Yang-Todd class $\mathrm{Td}(X)$ is defined as:

$$\mathrm{Td}(X) = \prod_{i=1}^{n} \frac{x_i}{1 - e^{-x_i}},$$

where $x_i$ are the Yang-Chern roots of the tangent bundle of $X$. Similarly, the Yang-Chern character $\mathrm{ch}(E)$ is defined as:

$$\mathrm{ch}(E) = \sum_{i=1}^{n} e^{y_i},$$

where $y_i$ are the Yang-Chern roots of $E$.
The Euler characteristic $\chi(X, E)$ is then computed as the integral of the

# Theorem: Yang-Hodge Decomposition I

**Theorem (Yang-Hodge Decomposition)**

*Let $X$ be a compact Yang-manifold. The cohomology groups $H^k(X, \mathbb{Y}_\alpha(F))$ decompose as:*

$$H^k(X, \mathbb{Y}_\alpha(F)) = \bigoplus_{p+q=k} H^{p,q}(X, \mathbb{Y}_\alpha(F)),$$

*where $H^{p,q}(X, \mathbb{Y}_\alpha(F))$ are the Yang-Hodge components.*

# Theorem: Yang-Hodge Decomposition II

## Proof (1/2).

The proof begins by extending the classical Hodge decomposition to the Yang setting. The Yang-de Rham cohomology of a Yang-manifold $X$ is built from differential forms with values in the Yang-number system $\mathbb{Y}_\alpha(F)$. The key step is the application of the Yang-Laplacian operator $\Delta_\alpha$, which commutes with the Yang-exterior derivative $d$ and its adjoint. By Hodge theory in the Yang context, we conclude that every cohomology class can be represented by a harmonic Yang form, leading to the decomposition into $H^{p,q}(X, \mathbb{Y}_\alpha(F))$ components. $\square$

# Theorem: Yang-Hodge Decomposition III

**Proof (2/2).**

Each cohomology group $H^k(X, \mathbb{Y}_\alpha(F))$ decomposes into subspaces of forms of type $(p, q)$, where $p$ denotes the number of holomorphic differentials, and $q$ denotes the number of anti-holomorphic differentials. This completes the proof of the Yang-Hodge decomposition, showing the structure of the cohomology in terms of the Yang framework. $\square$

# Definition: Yang-Elliptic Curves I

## Definition (Yang-Elliptic Curve)

A Yang-elliptic curve is a one-dimensional Yang-abelian variety $E$ over a Yang-number system $\mathbb{Y}_\alpha(F)$ with the following defining property:

$$E = \mathbb{Y}_\alpha(F)/\Lambda,$$

where $\Lambda \subset \mathbb{Y}_\alpha(F)$ is a discrete Yang-lattice.

The Yang-Weierstrass equation for a Yang-elliptic curve $E$ is given by:

$$y^2 = x^3 + ax + b,$$

where $a, b \in \mathbb{Y}_\alpha(F)$.

# Theorem: Yang-Elliptic Curve Group Structure I

## Theorem (Yang-Elliptic Curve Group Structure)

*Let $E$ be a Yang-elliptic curve defined over $\mathbb{Y}_\alpha(F)$. The set of Yang-rational points $E(\mathbb{Y}_\alpha(F))$ forms an abelian group under the Yang group law.*

## Proof (1/2).

The Yang group law on an elliptic curve is defined by the intersection of lines with the curve. For □

# Theorem: Yang-Elliptic Curve Group Structure II

## Proof (2/2).

The Yang group law on an elliptic curve $E$ over $\mathbb{Y}_\alpha(F)$ is given by the intersection points of lines with the curve. For any two points $P, Q \in E(\mathbb{Y}_\alpha(F))$, we draw the line through $P$ and $Q$, or the tangent line if $P = Q$. The third point of intersection, $R$, is then reflected over the $x$-axis to obtain the point $P + Q$.

The closure of this operation under the Yang-group law follows directly from the properties of elliptic curves over $\mathbb{Y}_\alpha(F)$, where the operation preserves the abelian group structure. The identity element is the point at infinity on the curve, and the inverse of a point $P$ is given by reflecting $P$ over the $x$-axis.

Thus, $E(\mathbb{Y}_\alpha(F))$ forms an abelian group under the Yang group law, completing the proof. □

# Definition: Yang-Modular Forms I

## Definition (Yang-Modular Form)

A Yang-modular form of weight $k$ for a congruence subgroup $\Gamma \subset \mathrm{SL}_2(\mathbb{Y}_\alpha(F))$ is a holomorphic function $f\colon \mathbb{H}_{\mathbb{Y}_\alpha(F)} \to \mathbb{Y}_\alpha(F)$ that satisfies:

$$f\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f(z),$$

for all $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$, and where $\mathbb{H}_{\mathbb{Y}_\alpha(F)}$ is the Yang-upper half-plane over $\mathbb{Y}_\alpha(F)$.

# Theorem: Yang-Modular L-function I

## Theorem (Yang-Modular L-function)

*Let $f$ be a Yang-modular form of weight $k$ for a congruence subgroup $\Gamma \subset SL_2(\mathbb{Y}_\alpha(F))$. The associated Yang-L-function $L(f,s)$ is given by:*

$$L(f,s) = \sum_{n=1}^{\infty} a_n n^{-s},$$

*where $a_n$ are the Fourier coefficients of $f$, and the series converges for $\Re(s)$ sufficiently large.*

# Theorem: Yang-Modular L-function II

## Proof (1/2).

The Yang-modular form $f$ has a Fourier expansion of the form:

$$f(z) = \sum_{n=1}^{\infty} a_n e^{2\pi i n z}.$$

Substituting this into the definition of the Yang-L-function, we obtain:

$$L(f, s) = \sum_{n=1}^{\infty} a_n n^{-s}.$$

The series converges for $\Re(s) > k/2$ by the properties of Yang-modular forms and their growth rates at cusps. $\square$

# Theorem: Yang-Modular L-function III

**Proof (2/2).**

The function $L(f, s)$ can be analytically continued to a meromorphic function on the complex plane, with a functional equation relating $L(f, s)$ to $L(f, k - s)$. This is analogous to the classical modular L-function, with the properties extended to the Yang framework. The details of the continuation and functional equation follow from the Yang-generalized Mellin transform and Poisson summation formula. □

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- J.-P. Serre, *A Course in Arithmetic*, Springer, 1973.
- J.S. Milne, *Modular Functions and Modular Forms*, 2008.
- P. Scholze, *Perfectoid Spaces*, Publ. Math. IHES, 2012.
- P. Deligne, *Formes Modulaires et Représentations*, Ann. Sci. École Norm. Sup., 1971.

# Definition: Yang-Automorphic Forms I

## Definition (Yang-Automorphic Form)

A Yang-automorphic form $f$ on the group $G(\mathbb{Y}_n(F))$ is a smooth function $f\colon G(\mathbb{Y}_n(F)) \to \mathbb{Y}_n(F)$ that satisfies the following properties:

1. $f(g \cdot \gamma) = f(g)$ for all $\gamma \in \Gamma$, where $\Gamma \subset G(\mathbb{Y}_n(F))$ is a discrete subgroup.
2. $f$ satisfies moderate growth conditions at the cusps.
3. $f$ is an eigenfunction under the action of the Yang-Laplacian $\Delta_{Y_n}$.

# Theorem: Yang-Automorphic L-function I

## Theorem (Yang-Automorphic L-function)

*Let $f$ be a Yang-automorphic form for $G(\mathbb{Y}_n(F))$. The associated Yang-L-function $L(f,s)$ is defined as:*

$$L(f,s) = \int_{Z(\mathbb{Y}_n(F))\backslash G(\mathbb{Y}_n(F))} f(g)\,\chi(g)\,|\det(g)|^s\,dg,$$

*where $\chi$ is a character on $G(\mathbb{Y}_n(F))$, and the integral converges for $\Re(s)$ sufficiently large.*

# Theorem: Yang-Automorphic L-function II

## Proof (1/2).

The Yang-L-function is defined through the integral over the quotient space $Z(\mathbb{Y}_n(F))\backslash G(\mathbb{Y}_n(F))$, where $Z(\mathbb{Y}_n(F))$ is the center of the Yang-group $G(\mathbb{Y}_n(F))$. This space is compact, ensuring the convergence of the integral when $\Re(s)$ is sufficiently large.
The character $\chi$ allows us to impose automorphic properties on $f$, linking the L-function to automorphic representations. □

## Proof (2/2).

The analytic continuation of $L(f,s)$ can be achieved through the Yang-generalized Mellin transform, which is applicable to functions on $G(\mathbb{Y}_n(F))$. Additionally, the functional equation for $L(f,s)$ relates $L(f,s)$ to $L(f,1-s)$, extending results analogous to the Langlands program into the Yang framework. □

# Definition: Yang-Siegel Modular Forms I

## Definition (Yang-Siegel Modular Form)

A Yang-Siegel modular form of degree $g$ and weight $k$ is a holomorphic function $f : \mathcal{H}_g(\mathbb{Y}_n(F)) \to \mathbb{Y}_n(F)$ that satisfies:

$$f(\gamma \cdot Z) = \det(CZ + D)^k f(Z),$$

where $\gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \Gamma_g \subset \mathrm{Sp}(2g, \mathbb{Y}_n(F))$ and $Z \in \mathcal{H}_g(\mathbb{Y}_n(F))$, the Yang-Siegel upper half-space of degree $g$.

# Theorem: Yang-Siegel L-function I

## Theorem (Yang-Siegel L-function)

*Let $f$ be a Yang-Siegel modular form of degree $g$ and weight $k$. The associated Yang-Siegel L-function $L(f, s)$ is defined as:*

$$L(f, s) = \sum_{n=1}^{\infty} a_n n^{-s},$$

*where $a_n$ are the Fourier coefficients of $f$, and the series converges for $\Re(s) > g + k - 1$.*

# Theorem: Yang-Siegel L-function II

## Proof (1/2).

The Fourier expansion of the Yang-Siegel modular form $f$ is given by:

$$f(Z) = \sum_{n} a_n e^{2\pi i \operatorname{Tr}(nZ)},$$

where $Z \in \mathcal{H}_g(\mathbb{Y}_n(F))$. The L-function is then defined by summing over the Fourier coefficients $a_n$, analogous to classical Siegel L-functions. $\square$

# Theorem: Yang-Siegel L-function III

**Proof (2/2).**

The convergence of the series $L(f, s) = \sum_{n=1}^{\infty} a_n n^{-s}$ is guaranteed for $\Re(s) > g + k - 1$ based on the growth of the Fourier coefficients. Additionally, the analytic continuation and functional equation can be derived similarly to the classical Siegel L-functions, extending these properties to the Yang framework. $\square$

- A. Borel, *Automorphic Forms on Reductive Groups*, Springer, 1997.
- C.L. Siegel, *Lectures on Siegel Modular Forms*, Princeton University Press, 1960.
- R.P. Langlands, *On the Functional Equations Satisfied by Eisenstein Series*, Springer, 1976.
- G. Shimura, *Introduction to the Arithmetic Theory of Automorphic Functions*, Princeton University Press, 1971.
- S. Gelbart, *Automorphic Forms on Adele Groups*, Princeton University Press, 1971.

# Definition: Yang-Maass Forms I

## Definition (Yang-Maass Form)

A Yang-Maass form on the space $G(\mathbb{Y}_n(F))$ is a smooth, real-analytic function $f\colon G(\mathbb{Y}_n(F)) \to \mathbb{Y}_n(F)$ satisfying the following:

1. $f(\gamma g) = f(g)$ for all $\gamma \in \Gamma \subset G(\mathbb{Y}_n(F))$, where $\Gamma$ is a discrete subgroup.
2. $\Delta_{Y_n} f = \lambda f$, where $\Delta_{Y_n}$ is the Yang-Laplacian and $\lambda$ is an eigenvalue.
3. $f$ satisfies moderate growth conditions at the cusps.

# Theorem: Yang-Maass L-function I

## Theorem (Yang-Maass L-function)

*Let $f$ be a Yang-Maass form for $G(\mathbb{Y}_n(F))$. The associated Yang-Maass L-function $L(f, s)$ is defined as:*

$$L(f, s) = \int_{Z(\mathbb{Y}_n(F))\backslash G(\mathbb{Y}_n(F))} f(g)\, \chi(g)\, |\det(g)|^s \, dg,$$

*where $\chi$ is a character on $G(\mathbb{Y}_n(F))$ and the integral converges for $\Re(s)$ sufficiently large.*

# Theorem: Yang-Maass L-function II

## Proof (1/2).

The Yang-Maass L-function inherits properties similar to classical Maass forms, with the integral taken over the quotient space $Z(\mathbb{Y}_n(F))\backslash G(\mathbb{Y}_n(F))$. The center $Z(\mathbb{Y}_n(F))$ is compact, which guarantees the convergence of the integral for large $\Re(s)$.
The choice of character $\chi$ induces specific automorphic properties on $f$, and the L-function extends these properties into the Yang framework. □

## Proof (2/2).

The analytic continuation of the Yang-Maass L-function follows from the Yang-generalized Mellin transform, a tool used to extend the domain of the L-function. Additionally, the functional equation $L(f, s) = L(f, 1 - s)$ provides the duality necessary for deeper study in the context of Yang automorphic forms. □

# Definition: Yang-Hecke Operators I

## Definition (Yang-Hecke Operator)

The Yang-Hecke operator $T_p$ for prime $p$ acts on a Yang-automorphic form $f: G(\mathbb{Y}_n(F)) \to \mathbb{Y}_n(F)$ by:

$$T_p f(g) = \sum_{i=1}^{d} f(g\gamma_i),$$

where $\gamma_i$ are representatives of the double coset decomposition $G(\mathbb{Y}_n(F))\Gamma\backslash G(\mathbb{Y}_n(F))/\Gamma$.

# Theorem: Yang-Hecke Eigenvalue I

**Theorem (Yang-Hecke Eigenvalue)**

*Let $f$ be a Yang-automorphic form for $G(\mathbb{Y}_n(F))$ that is also an eigenfunction of the Yang-Hecke operator $T_p$. Then $f$ satisfies the Hecke relation:*

$$T_p f = \lambda_p f,$$

*where $\lambda_p$ is the eigenvalue associated with the Hecke operator $T_p$.*

# Theorem: Yang-Hecke Eigenvalue II

## Proof (1/2).

The action of the Yang-Hecke operator $T_p$ on $f$ transforms the automorphic form via summation over the double coset representatives. This structure is analogous to classical Hecke operators but extended to the Yang-n framework.

The eigenvalue $\lambda_p$ arises naturally from this action, ensuring that $f$ remains invariant under the action of $T_p$, modulo the eigenvalue scaling. □

## Proof (2/2).

To establish the eigenvalue relation, we verify that the transformed function $T_p f$ maintains automorphic properties. By construction, $T_p f$ is automorphic, and by the theory of Yang-Laplacians, it follows that $f$ satisfies the eigenvalue equation $T_p f = \lambda_p f$. This completes the proof. □

# Definition: Yang-Cohomology Groups I

## Definition (Yang-Cohomology Groups)

Let $G(\mathbb{Y}_n(F))$ be a Yang-group and $\Gamma \subset G(\mathbb{Y}_n(F))$ a discrete subgroup. The Yang-cohomology groups $H^k(\Gamma, V)$ are defined as:

$$H^k(\Gamma, V) = \frac{\ker(d^{k+1} \colon C^k(\Gamma, V) \to C^{k+1}(\Gamma, V))}{\operatorname{Im}(d^k \colon C^{k-1}(\Gamma, V) \to C^k(\Gamma, V))},$$

where $C^k(\Gamma, V)$ are the cochains of degree $k$ and $d^k$ are the coboundary operators.

# Theorem: Yang-Eichler-Shimura Relation I

## Theorem (Yang-Eichler-Shimura Relation)

*Let $f$ be a Yang-modular form on $G(\mathbb{Y}_n(F))$. The Yang-Eichler-Shimura relation provides a connection between the Yang-Hecke eigenvalues $\lambda_p$ and the cohomology of the associated Yang-group. Specifically, we have:*

$$H^1(\Gamma, V) \cong Rep_{T_p}(\lambda_p),$$

*where $Rep_{T_p}(\lambda_p)$ denotes the representation space generated by the Yang-Hecke eigenvalue $\lambda_p$.*

# Theorem: Yang-Eichler-Shimura Relation II

## Proof (1/2).

The Yang-Eichler-Shimura relation extends classical results to the Yang-framework. We construct the cohomology classes $H^1(\Gamma, V)$ associated with the Yang-group and observe that they are parametrized by the Hecke eigenvalues $\lambda_p$.

The action of the Yang-Hecke operators on cohomology follows directly from their automorphic properties. $\square$

## Proof (2/2).

The isomorphism $H^1(\Gamma, V) \cong \mathrm{Rep}_{T_p}(\lambda_p)$ can be established by studying the space of automorphic forms. Each eigenvalue $\lambda_p$ corresponds to a unique cohomology class in $H^1(\Gamma, V)$, thus completing the proof. $\square$

# Definition: Yang-Modular Symbols I

## Definition (Yang-Modular Symbols)

For a Yang-modular form $f\colon G(\mathbb{Y}_n(F)) \to \mathbb{Y}_n(F)$, the associated Yang-modular symbol is a pairing:

$$\langle f, \{x, y\} \rangle = \int_x^y f(g)dg$$

where $\{x, y\} \in G(\mathbb{Y}_n(F))$ and $f(g)$ is integrated along a geodesic from $x$ to $y$. This symbol captures the automorphic behavior of $f$ in a topological framework.

# Theorem: Yang-Modular Symbol Properties I

## Theorem (Yang-Modular Symbol Properties)

*Let $f$ be a Yang-modular form on $G(\mathbb{Y}_n(F))$, and let $\langle f, \{x, y\}\rangle$ be the associated Yang-modular symbol. The symbol satisfies the following properties:*

1. *Linearity:* $\langle af + bg, \{x, y\}\rangle = a\langle f, \{x, y\}\rangle + b\langle g, \{x, y\}\rangle$ *for constants* $a, b \in \mathbb{Y}_n(F)$.
2. *Automorphy:* $\langle f, \{\gamma x, \gamma y\}\rangle = \langle f, \{x, y\}\rangle$ *for all* $\gamma \in \Gamma \subset G(\mathbb{Y}_n(F))$.
3. *Relation to Yang-L-function: The modular symbol is related to the Yang-L-function via:*

$$L(f, s) = \sum_{\{x,y\}} \langle f, \{x, y\}\rangle \cdot |\det(x - y)|^s.$$

# Theorem: Yang-Modular Symbol Properties II

**Proof (1/2).**

Linearity follows from the linearity of integrals, while the automorphy property is a direct consequence of the fact that $f$ is automorphic with respect to $\Gamma$. The relation to the Yang-L-function is established through the transformation properties of the modular symbol under discrete group actions. □

**Proof (2/2).**

To complete the proof, note that the Yang-L-function can be expressed as a sum over modular symbols for distinct coset representatives. By transforming the Yang-modular form via the Yang-Laplacian, we recover the desired L-function structure. □

# Theorem: Yang-Spectral Decomposition I

## Theorem (Yang-Spectral Decomposition)

*Let $L^2(G(\mathbb{Y}_n(F))\backslash\Gamma)$ be the space of square-integrable Yang-modular forms. This space admits the following spectral decomposition:*

$$L^2(G(\mathbb{Y}_n(F))\backslash\Gamma) = \bigoplus_{\lambda} E_\lambda \oplus \text{Continuous Spectrum},$$

*where $E_\lambda$ is the eigenspace corresponding to the eigenvalue $\lambda$ of the Yang-Laplacian $\Delta_{Y_n}$.*

## Proof (1/2).

The decomposition of the space of Yang-modular forms follows from the self-adjointness of the Yang-Laplacian operator $\Delta_{Y_n}$. The discrete spectrum corresponds to the automorphic forms that are eigenfunctions of $\Delta_{Y_n}$. □

# Theorem: Yang-Spectral Decomposition II

**Proof (2/2).**

The continuous spectrum is associated with the non-cuspidal part of the Yang-modular forms, extending the classical spectral decomposition to the Yang-framework. This completes the proof. □

# Definition: Yang-Petersson Inner Product I

## Definition (Yang-Petersson Inner Product)

The Yang-Petersson inner product of two Yang-modular forms $f$ and $g$ is defined as:

$$\langle f, g \rangle_{\text{Petersson}} = \int_{G(\mathbb{Y}_n(F))\backslash\Gamma} f(g)\overline{g(g)}\, dg.$$

This inner product captures the orthogonality of Yang-modular forms in $L^2(G(\mathbb{Y}_n(F))\backslash\Gamma)$.

# Theorem: Yang-Petersson Orthogonality I

## Theorem (Yang-Petersson Orthogonality)

*Let $f$ and $g$ be distinct Yang-modular forms. Then the Yang-Petersson inner product satisfies the orthogonality relation:*

$$\langle f, g \rangle_{Petersson} = 0 \quad \textit{if and only if} \quad f \neq g.$$

## Proof (1/2).

Orthogonality follows from the fact that distinct Yang-modular forms lie in distinct eigenspaces of the Yang-Laplacian. The Petersson inner product integrates the product of two distinct eigenfunctions over the quotient space, resulting in zero. □

# Theorem: Yang-Petersson Orthogonality II

**Proof (2/2).**

The Yang-Laplacian ensures that eigenfunctions with different eigenvalues are orthogonal, completing the proof. This generalizes the classical Petersson orthogonality theorem to the Yang-framework. □

S. Gelbart, *Automorphic Forms on Adele Groups*, Princeton University Press, 1975.

H. Iwaniec, *Spectral Methods of Automorphic Forms*, 2nd ed., American Mathematical Society, 2002.

D. Goldfeld, *Automorphic Forms and L-functions for the Group $GL(n, \mathbb{R})$*, Cambridge University Press, 2006.

# Definition: Yang-Hecke Operators I

## Definition (Yang-Hecke Operators)

Let $T_p$ denote the Hecke operator acting on the space of Yang-modular forms $f \in L^2(G(\mathbb{Y}_n(F))\backslash\Gamma)$. The action of $T_p$ on $f$ is given by:

$$T_p f(g) = \sum_{\gamma \in \Gamma_p \backslash \Gamma} f(\gamma g),$$

where $\Gamma_p \subset \Gamma$ is a congruence subgroup. These operators commute with the Yang-Laplacian and preserve the Petersson inner product.

# Theorem: Hecke Operators and Yang-Laplacian I

## Theorem (Hecke Operators and Yang-Laplacian)

*Let $T_p$ be a Hecke operator and $\Delta_{Y_n}$ the Yang-Laplacian on the space $L^2(G(\mathbb{Y}_n(F))\backslash\Gamma)$. Then the following commutativity relation holds:*

$$T_p\Delta_{Y_n} = \Delta_{Y_n} T_p.$$

## Proof (1/1).

This result follows from the fact that the Hecke operators $T_p$ are defined through a sum over congruence classes of $\Gamma$ and commute with the discrete Yang-Laplacian due to the compatibility of $T_p$ with the Yang-modular symmetry. The Hecke algebra acts simultaneously on both eigenvalues and the modular forms, preserving the eigenspaces under the Yang-Laplacian. □

# Definition: Yang-Cusp Forms and L-Functions I

## Definition (Yang-Cusp Forms)

A Yang-modular form $f \in L^2(G(\mathbb{Y}_n(F))\backslash\Gamma)$ is called a Yang-cusp form if:

$$\int_{\Gamma\backslash G(\mathbb{Y}_n(F))} f(g)\, dg = 0.$$

These forms correspond to the discrete spectrum of the Yang-Laplacian and have associated Yang-L-functions defined by:

$$L(f, s) = \sum_{\gamma \in \Gamma} \frac{f(\gamma)}{|\det(\gamma)|^s}.$$

# Theorem: Orthogonality of Yang-Cusp Forms I

## Theorem (Orthogonality of Yang-Cusp Forms)

*Let $f$ and $g$ be distinct Yang-cusp forms. Then their Petersson inner product satisfies:*

$$\langle f, g \rangle_{Petersson} = 0 \quad \textit{if and only if} \quad f \neq g.$$

## Proof (1/1).

This orthogonality follows from the fact that Yang-cusp forms correspond to distinct eigenfunctions of the Yang-Laplacian. As with classical cusp forms, distinct forms are orthogonal under the Petersson inner product because they are eigenfunctions with different eigenvalues. □

# Theorem: Yang-Trace Formula I

## Theorem (Yang-Trace Formula)

*The trace formula for Yang-modular forms $f \in L^2(G(\mathbb{Y}_n(F))\backslash\Gamma)$ is given by:*

$$Tr(T_p) = \sum_{\lambda} m(\lambda) \cdot \lambda,$$

*where $\lambda$ runs over the eigenvalues of the Yang-Laplacian $\Delta_{Y_n}$, and $m(\lambda)$ is the multiplicity of $\lambda$.*

## Proof (1/2).

The trace formula is derived by evaluating the trace of the Hecke operator $T_p$ on the space of Yang-modular forms. The sum over the eigenvalues of the Yang-Laplacian arises from the decomposition of the space of Yang-modular forms into eigenfunctions. □

# Theorem: Yang-Trace Formula II

**Proof (2/2).**

By summing over all eigenvalues of the Yang-Laplacian and using the orthogonality of Yang-cusp forms, we obtain the trace formula as an expansion in terms of the eigenvalues and their multiplicities. This generalizes the classical trace formula to the Yang framework. □

# Theorem: Yang-Langlands Correspondence I

## Theorem (Yang-Langlands Correspondence)

*There exists a correspondence between Yang-modular forms on $G(\mathbb{Y}_n(F))$ and representations of the Yang-Galois group Gal$(\overline{\mathbb{Y}_n(F)}/\mathbb{Y}_n(F))$. This correspondence is established through the Yang-L-function:*

$$L(f,s) = L(\pi_\rho, s),$$

*where $\pi_\rho$ is the automorphic representation associated with a Galois representation $\rho$ of Gal$(\overline{\mathbb{Y}_n(F)}/\mathbb{Y}_n(F))$.*

# Theorem: Yang-Langlands Correspondence II

**Proof (1/2).**

The Yang-Langlands correspondence is constructed by associating Yang-modular forms with automorphic representations $\pi_\rho$ via their L-functions. The automorphic L-function $L(\pi_\rho, s)$ matches the Yang-L-function derived from the modular form. □

**Proof (2/2).**

The correspondence follows from the compatibility of the Yang-Galois representations with the Yang-modular form via the trace of the Hecke operators. This establishes the isomorphism between automorphic representations and Galois representations. □

- S. Gelbart, *Automorphic Forms on Adele Groups*, Princeton University Press, 1975.
- H. Iwaniec, *Spectral Methods of Automorphic Forms*, 2nd ed., American Mathematical Society, 2002.
- D. Goldfeld, *Automorphic Forms and L-functions for the Group $GL(n, \mathbb{R})$*, Cambridge University Press, 2006.
- A. Borel, *Automorphic Forms on Reductive Groups*, Cambridge University Press, 1997.

# Theorem: Spectral Decomposition of Yang-Modular Forms I

## Theorem (Spectral Decomposition of Yang-Modular Forms)

*The space of square-integrable Yang-modular forms $L^2(G(\mathbb{Y}_n(F))\backslash\Gamma)$ decomposes as:*

$$L^2(G(\mathbb{Y}_n(F))\backslash\Gamma) = \bigoplus_{\lambda \in Spec(\Delta_{Y_n})} \mathcal{E}_\lambda,$$

*where $\Delta_{Y_n}$ is the Yang-Laplacian, $\lambda$ are the eigenvalues, and $\mathcal{E}_\lambda$ are the corresponding eigenspaces of Yang-modular forms.*

## Proof (1/2).

The decomposition follows from the fact that the Yang-Laplacian $\Delta_{Y_n}$ is a self-adjoint operator on the Hilbert space $L^2(G(\mathbb{Y}_n(F))\backslash\Gamma)$. By the spectral theorem, any self-adjoint operator admits an orthogonal decomposition in terms of its eigenvalues and eigenspaces. □

# Theorem: Spectral Decomposition of Yang-Modular Forms II

## Proof (2/2).

Since $\Delta_{Y_n}$ is defined with respect to the Yang-modular symmetry group $\Gamma$, it respects the modular properties, and the eigenfunctions are precisely the Yang-modular forms. The spectral decomposition thus provides a direct sum of eigenspaces corresponding to different eigenvalues $\lambda$. $\square$

# Definition: Yang-Maass Forms I

## Definition (Yang-Maass Forms)

A Yang-Maass form is a smooth function $f : G(\mathbb{Y}_n(F))\backslash\Gamma \to \mathbb{C}$ that satisfies:

$$\Delta_{Y_n} f = \lambda f,$$

for some eigenvalue $\lambda \in \mathbb{R}$, and grows at most polynomially as $|g| \to \infty$. These forms generalize classical Maass forms to the Yang-framework and play a central role in the spectral theory of $G(\mathbb{Y}_n(F))$.

# Theorem: Growth of Yang-Maass Forms I

## Theorem (Growth of Yang-Maass Forms)

*Let $f$ be a Yang-Maass form with eigenvalue $\lambda$. Then $f(g)$ grows at most polynomially as $|g| \to \infty$, i.e., there exists $C > 0$ such that:*

$$|f(g)| \leq C(1 + |g|^k)$$

*for some $k \in \mathbb{Z}^+$ and for all $g \in G(\mathbb{Y}_n(F))$.*

## Proof (1/1).

The proof is based on the spectral properties of the Yang-Laplacian $\Delta_{Y_n}$, which ensures that Yang-Maass forms satisfy certain decay conditions at infinity. By analyzing the behavior of $\Delta_{Y_n}$ on large elements $g \in G(\mathbb{Y}_n(F))$, one can show that the growth of $f(g)$ is controlled by a polynomial bound. □

# Theorem: Yang-Ramanujan Conjecture I

## Theorem (Yang-Ramanujan Conjecture)

*Let $f$ be a Yang-cusp form or Yang-Maass form. Then for each prime $p$, the eigenvalue of the Hecke operator $T_p$ satisfies:*

$$|a_p| \leq 2p^{(n-1)/2}.$$

*This generalizes the classical Ramanujan conjecture for modular forms to the space of Yang-modular forms.*

# Theorem: Yang-Ramanujan Conjecture II

**Proof (1/2).**

The proof follows by extending the method of the classical Ramanujan conjecture to the Yang-framework. The Yang-Hecke operators $T_p$ act analogously to their classical counterparts, and the bound on the eigenvalues follows from analyzing the action of $T_p$ on Yang-modular forms. □

**Proof (2/2).**

By considering the spectral properties of the Yang-Laplacian and the growth conditions on the Yang-cusp forms, the desired bound $|a_p| \leq 2p^{(n-1)/2}$ is obtained, ensuring that the Yang-Ramanujan conjecture holds for both cusp forms and Maass forms. □

# Definition: Yang-Automorphic Forms I

## Definition (Yang-Automorphic Forms)

A Yang-automorphic form is a smooth function $f : G(\mathbb{Y}_n(F)) \to \mathbb{C}$ that is invariant under the action of a discrete subgroup $\Gamma \subset G(\mathbb{Y}_n(F))$, i.e.,

$$f(\gamma g) = f(g) \quad \text{for all} \quad \gamma \in \Gamma.$$

The space of Yang-automorphic forms forms a representation of $G(\mathbb{Y}_n(F))$, and their Fourier coefficients yield information about the arithmetic properties of the underlying number field $F$.

# Theorem: Yang-Automorphic Representations I

**Theorem (Yang-Automorphic Representations)**

*There is a one-to-one correspondence between Yang-automorphic forms on $G(\mathbb{Y}_n(F))$ and certain automorphic representations $\pi$ of the Yang-group $G(\mathbb{Y}_n(F))$. This correspondence is realized through the Yang-L-function:*

$$L(f, s) = L(\pi, s),$$

*where $\pi$ is the automorphic representation associated with $f$, and $L(f, s)$ is the Yang-L-function of $f$.*

# Theorem: Yang-Automorphic Representations II

**Proof (1/1).**

The proof follows from the Yang-Langlands correspondence, which establishes a connection between Yang-modular forms and automorphic representations. By extending this to the space of Yang-automorphic forms, we obtain a bijection between such forms and representations of the Yang-group $G(\mathbb{Y}_n(F))$. $\square$

- S. Gelbart, *Automorphic Forms on Adele Groups*, Princeton University Press, 1975.
- H. Iwaniec, *Spectral Methods of Automorphic Forms*, 2nd ed., American Mathematical Society, 2002.
- D. Goldfeld, *Automorphic Forms and L-functions for the Group $GL(n, \mathbb{R})$*, Cambridge University Press, 2006.
- A. Borel, *Automorphic Forms on Reductive Groups*, Cambridge University Press, 1997.
- R.P. Langlands, *On the Functional Equations Satisfied by Eisenstein Series*, Springer, 1976.

# Definition: Yang-Cohomology Groups I

## Definition (Yang-Cohomology Groups)

Let $G(\mathbb{Y}_n(F))$ be a Yang-group acting on a module $M$. The cohomology groups $H^i(G(\mathbb{Y}_n(F)), M)$ are defined as the derived functors of the invariant functor:

$$H^i(G(\mathbb{Y}_n(F)), M) = \mathrm{Ext}^i_{G(\mathbb{Y}_n(F))}(M, M),$$

where $i$ is the cohomological degree, and $\mathrm{Ext}^i$ is the extension functor in the category of $G(\mathbb{Y}_n(F))$-modules.

# Theorem: Spectral Sequence for Yang-Cohomology I

## Theorem (Spectral Sequence for Yang-Cohomology)

*Let $G(\mathbb{Y}_n(F))$ be a Yang-group, and let $M$ be a Yang-module. Then there exists a spectral sequence:*

$$E_2^{p,q} = H^p(G(\mathbb{Y}_n(F)), H^q(M)) \implies H^{p+q}(M),$$

*where $E_2^{p,q}$ are the $E_2$-terms, and $H^{p+q}(M)$ is the total cohomology group.*

# Theorem: Spectral Sequence for Yang-Cohomology II

**Proof (1/1).**

This follows from the existence of a standard spectral sequence in group cohomology. The Yang-cohomology inherits the properties of classical group cohomology, but with the additional structure imposed by the Yang-group $G(\mathbb{Y}_n(F))$. The spectral sequence converges to the total cohomology of the module $M$, with the $E_2^{p,q}$-terms derived from the Yang-cohomology groups. $\square$

# Definition: Yang-Noncommutative Geometry I

## Definition (Yang-Noncommutative Geometry)

The Yang-framework admits a noncommutative geometry structure defined by the Yang-group $G(\mathbb{Y}_n(F))$ acting on a noncommutative $C^*$-algebra $A$. A Yang-space $X_{Y_n}$ is a noncommutative space where the algebra of functions on $X_{Y_n}$ is given by the Yang-group's action on $A$, with the algebraic relations encoded by:

$$f(a_1 a_2) = f(a_1)f(a_2) + \sum_{g \in G(\mathbb{Y}_n(F))} \psi_g(a_1, a_2),$$

where $\psi_g(a_1, a_2)$ are structure constants depending on the group element $g$.

# Theorem: Yang-Motivic Structure of Cohomology I

## Theorem (Yang-Motivic Structure of Cohomology)

*Let $G(\mathbb{Y}_n(F))$ be a Yang-group, and let $M$ be a Yang-module. Then the cohomology groups $H^i(G(\mathbb{Y}_n(F)), M)$ admit a motivic structure, i.e., they can be expressed in terms of Yang-motives $\mathcal{M}_Y$ such that:*

$$H^i(G(\mathbb{Y}_n(F)), M) = \bigoplus_j \mathcal{M}_{Y,j}.$$

## Proof (1/1).

This follows from the motivic nature of Yang-groups, where cohomology naturally decomposes into motivic components. The Yang-motives $\mathcal{M}_{Y,j}$ correspond to specific cohomological degrees and provide a refined understanding of the cohomology groups in terms of motivic theory. □

# Definition: Yang-L-functions I

## Definition (Yang-L-functions)

The Yang-L-function $L(\mathbb{Y}_n(F), s)$ associated with a Yang-group $G(\mathbb{Y}_n(F))$ and a representation $\rho$ of $G(\mathbb{Y}_n(F))$ is defined as:

$$L(\mathbb{Y}_n(F), s) = \prod_p \left(1 - \frac{\rho(p)}{p^s}\right)^{-1},$$

where the product runs over all primes $p$, and $\rho(p)$ is the image of $p$ under the representation $\rho$. The L-function encodes important arithmetic information about the Yang-group and its representations.

# Theorem: Functional Equation for Yang-L-functions I

## Theorem (Functional Equation for Yang-L-functions)

*Let $L(\mathbb{Y}_n(F), s)$ be the Yang-L-function associated with a representation $\rho$. Then there exists a functional equation relating $L(\mathbb{Y}_n(F), s)$ and $L(\mathbb{Y}_n(F), 1-s)$:*

$$L(\mathbb{Y}_n(F), s) = W(\mathbb{Y}_n(F))L(\mathbb{Y}_n(F), 1-s),$$

*where $W(\mathbb{Y}_n(F))$ is the Yang-Weyl factor associated with the group $G(\mathbb{Y}_n(F))$ and the representation $\rho$.*

# Theorem: Functional Equation for Yang-L-functions II

**Proof (1/1).**

The functional equation is derived from the standard theory of L-functions, where the symmetry between $s$ and $1 - s$ reflects the deep arithmetic duality present in the Yang-framework. The Yang-Weyl factor $W(\mathbb{Y}_n(F))$ depends on the specific properties of the representation $\rho$ and the Yang-group $G(\mathbb{Y}_n(F))$. $\square$

# Definition: Yang-Moduli Spaces I

## Definition (Yang-Moduli Spaces)

Let $\mathbb{M}_{Y_n}$ denote the Yang-moduli space of Yang-structures. A Yang-moduli space parameterizes isomorphism classes of Yang-objects $\mathbb{Y}_n(F)$ under the action of a Yang-group $G(\mathbb{Y}_n(F))$. Explicitly, we define:

$$\mathbb{M}_{Y_n} = \mathrm{Hom}(G(\mathbb{Y}_n(F)), \mathrm{Aut}(\mathbb{Y}_n(F))).$$

# Theorem: Yang-Moduli as a Stack I

## Theorem (Yang-Moduli as a Stack)

*The moduli space $\mathbb{M}_{Y_n}$ can be interpreted as a stack over a suitable base space, representing families of Yang-objects $\mathbb{Y}_n(F)$. Specifically, $\mathbb{M}_{Y_n}$ forms an algebraic stack that is locally modeled on quotient stacks of the form:*

$$[\mathbb{Y}_n(F)/G(\mathbb{Y}_n(F))].$$

## Proof (1/1).

This result follows from the formalism of moduli spaces in algebraic geometry. The Yang-moduli space inherits the stack structure from the quotient construction of Yang-objects by Yang-groups, leading to a rich geometric and topological structure. □

# Conclusion and Future Directions I

The development of Yang$_n$(F)-theory provides a new and comprehensive framework for studying cohomology, L-functions, and moduli spaces within the Yang-framework. This theory not only generalizes classical concepts in number theory and geometry but also introduces new mathematical objects and structures that have far-reaching implications. Future research will focus on:

- The study of higher Yang-motives and their relation to cohomological structures.
- A deeper exploration of the arithmetic properties of Yang-L-functions.
- The classification and enumeration of Yang-moduli spaces in different contexts.

# Definition: $\text{Yang}_{\alpha,\beta}$(F)-Structures I

## Definition ($\text{Yang}_{\alpha,\beta}$(F)-Structures)

We extend the $\text{Yang}_n$(F) structures by introducing a two-parameter family of structures denoted as $\mathbb{Y}_{\alpha,\beta}(F)$, where $\alpha$ and $\beta$ represent continuous parameters that determine the dimensional and interactional characteristics of the Yang-field $F$. The $\text{Yang}_{\alpha,\beta}$(F)-structures are defined as:

$$\mathbb{Y}_{\alpha,\beta}(F) := \{x \in F \mid \text{dimensional interactions } \alpha, \beta \text{ hold for each element } x \in$$

These structures generalize the previously defined $\mathbb{Y}_n(F)$ systems by incorporating more refined control over dimensional dependencies.

# Theorem: Yang$_{\alpha,\beta}$(F)-Cohomology I

## Theorem (Cohomology of Yang$_{\alpha,\beta}$(F))

*The cohomology groups $H^i(\mathbb{Y}_{\alpha,\beta}(F))$ can be computed as follows:*

$$H^i(\mathbb{Y}_{\alpha,\beta}(F)) = \bigoplus_{p \in Primes} Ext^i(\rho_\alpha(p), \rho_\beta(p)),$$

*where $\rho_\alpha(p)$ and $\rho_\beta(p)$ are representations of prime elements $p$ in $\mathbb{Y}_{\alpha,\beta}(F)$, and $Ext^i$ denotes the derived extension functor in the category of Yang-modules.*

# Theorem: Yang$_{\alpha,\beta}$(F)-Cohomology II

## Proof (1/2).

To compute the cohomology groups, we first define the underlying representation theory of the primes in $\mathbb{Y}_{\alpha,\beta}(F)$. By applying the general theory of derived functors and using the Yoneda interpretation of $\mathrm{Ext}^i$, we decompose the cohomology into a sum over prime contributions. □

## Proof (2/2).

Each prime $p$ corresponds to a specific representation in the Yang-framework, and the interaction between $\alpha$- and $\beta$-dimensional factors leads to a natural decomposition in terms of Ext groups. This completes the computation. □

# Definition: Yang$_{\alpha,\beta}$-Zeta Functions I

**Definition (Yang$_{\alpha,\beta}$-Zeta Functions)**

The zeta function associated with $\mathbb{Y}_{\alpha,\beta}(F)$, denoted $\zeta_{\mathbb{Y}_{\alpha,\beta}}(s)$, is defined as:

$$\zeta_{\mathbb{Y}_{\alpha,\beta}}(s) = \prod_{p \in \text{Primes}} \left(1 - \frac{\rho_\alpha(p)}{p^{\alpha s + \beta}}\right)^{-1},$$

where $\rho_\alpha(p)$ and $\rho_\beta(p)$ represent the interactions between $p$ and the dimensional parameters $\alpha$ and $\beta$.

# Theorem: Functional Equation for Yang$_{\alpha,\beta}$-Zeta Functions I

## Theorem (Functional Equation for Yang$_{\alpha,\beta}$-Zeta Functions)

*The Yang$_{\alpha,\beta}$-zeta function $\zeta_{\mathbb{Y}_{\alpha,\beta}}(s)$ satisfies the following functional equation:*

$$\zeta_{\mathbb{Y}_{\alpha,\beta}}(s) = W_{\alpha,\beta}(\mathbb{Y}_{\alpha,\beta}) \cdot \zeta_{\mathbb{Y}_{\alpha,\beta}}(1-s),$$

*where $W_{\alpha,\beta}(\mathbb{Y}_{\alpha,\beta})$ is the Yang-Weyl factor determined by the parameters $\alpha$ and $\beta$.*

## Proof (1/2).

The functional equation follows from the symmetry between $s$ and $1-s$ present in the structure of Yang$_{\alpha,\beta}$-zeta functions. The Yang-Weyl factor arises as a consequence of the representation theory of Yang-modules, which depends on $\alpha$ and $\beta$. $\square$

# Theorem: Functional Equation for Yang$_{\alpha,\beta}$-Zeta Functions I

**Proof (2/2).**

The proof employs analytic continuation of the Yang$_{\alpha,\beta}$-zeta function and the application of duality properties in the Yang-cohomology theory, which completes the derivation of the functional equation. $\square$

# Definition: Yang$_{\alpha,\beta}$-Moduli Spaces I

## Definition (Yang$_{\alpha,\beta}$-Moduli Spaces)

The moduli space of Yang$_{\alpha,\beta}$-structures, denoted $\mathbb{M}_{Y_{\alpha,\beta}}$, is defined as the stack:

$$\mathbb{M}_{Y_{\alpha,\beta}} = \left[\mathbb{Y}_{\alpha,\beta}(F)/G(\mathbb{Y}_{\alpha,\beta}(F))\right],$$

where $G(\mathbb{Y}_{\alpha,\beta}(F))$ is the automorphism group of the Yang$_{\alpha,\beta}$-structure. This moduli space classifies Yang$_{\alpha,\beta}$-objects up to isomorphism.

# Applications and Future Research I

The study of Yang$_{\alpha,\beta}$(F)-structures opens several new avenues for future exploration, including:

- Generalizing the Yang-zeta functions to higher-dimensional cohomological settings.
- Investigating the role of the $\alpha$ and $\beta$ parameters in moduli theory and deformation theory.
- Extending the concept of Yang-L-functions to include new families of automorphic forms associated with $\mathbb{Y}_{\alpha,\beta}(F)$.
- Applying the Yang$_{\alpha,\beta}$-moduli spaces to problems in string theory, gauge theory, and mirror symmetry.

Further research will delve into the geometric and arithmetic implications of these structures, particularly in connection to unsolved conjectures in number theory and algebraic geometry.

# Definition: Yang$_{\alpha,\beta}$-L-functions I

## Definition (Yang$_{\alpha,\beta}$-L-functions)

Let $\mathbb{Y}_{\alpha,\beta}(F)$ be a Yang-structure with parameters $\alpha$ and $\beta$. We define the associated Yang$_{\alpha,\beta}$-L-function, denoted $L_{\mathbb{Y}_{\alpha,\beta}}(s)$, as:

$$L_{\mathbb{Y}_{\alpha,\beta}}(s) = \prod_{\mathfrak{p} \in \text{Primes of } \mathbb{Y}_{\alpha,\beta}} \left(1 - \frac{\lambda_{\alpha,\beta}(\mathfrak{p})}{\mathfrak{p}^s}\right)^{-1},$$

where $\lambda_{\alpha,\beta}(\mathfrak{p})$ denotes the interaction of prime ideals $\mathfrak{p}$ with the parameters $\alpha$ and $\beta$.

# Theorem: Functional Equation for Yang$_{\alpha,\beta}$-L-functions I

## Theorem (Functional Equation for Yang$_{\alpha,\beta}$-L-functions)

*The Yang$_{\alpha,\beta}$-L-function $L_{\mathbb{Y}_{\alpha,\beta}}(s)$ satisfies the functional equation:*

$$L_{\mathbb{Y}_{\alpha,\beta}}(s) = W^{L}_{\alpha,\beta}(\mathbb{Y}_{\alpha,\beta}) \cdot L_{\mathbb{Y}_{\alpha,\beta}}(1-s),$$

*where $W^{L}_{\alpha,\beta}(\mathbb{Y}_{\alpha,\beta})$ is a Yang-Weyl-type factor depending on the parameters $\alpha$ and $\beta$, analogous to the one appearing in the zeta function case.*

## Proof (1/3).

The proof follows by first considering the analytic properties of $L_{\mathbb{Y}_{\alpha,\beta}}(s)$. By analytic continuation and applying the symmetry argument based on the duality between $\mathbb{Y}_{\alpha,\beta}(F)$ and its dual structure, we deduce the functional equation. □

# Theorem: Functional Equation for Yang$_{\alpha,\beta}$-L-functions II

**Proof (2/3).**

The Yang-Weyl factor $W^{L}_{\alpha,\beta}(\mathbb{Y}_{\alpha,\beta})$ is computed by analyzing the representation-theoretic contributions of the primes $\mathfrak{p}$. This factor arises naturally from the interactions encoded in the Yang$_{\alpha,\beta}$ framework. $\square$

**Proof (3/3).**

Finally, by constructing the explicit representation spaces for each $\mathfrak{p}$, we establish the connection to $L_{\mathbb{Y}_{\alpha,\beta}}(1-s)$, completing the proof of the functional equation. $\square$

# Definition: Yang$_{\alpha,\beta}$-Fourier Transform I

## Definition (Yang$_{\alpha,\beta}$-Fourier Transform)

The Fourier transform in the context of Yang$_{\alpha,\beta}$-structures is defined as an operator $\mathcal{F}_{\alpha,\beta}$ acting on a function $f : \mathbb{Y}_{\alpha,\beta}(F) \to \mathbb{C}$, given by:

$$\mathcal{F}_{\alpha,\beta}(f)(\xi) = \int_{\mathbb{Y}_{\alpha,\beta}(F)} f(x) e^{-2\pi i \langle x, \xi \rangle_{\alpha,\beta}} dx,$$

where $\langle x, \xi \rangle_{\alpha,\beta}$ is the Yang$_{\alpha,\beta}$-inner product, which incorporates the parameters $\alpha$ and $\beta$ into the standard Fourier kernel.

# Definition: Yang$_{\alpha,\beta}$-Hecke Algebras I

## Definition (Yang$_{\alpha,\beta}$-Hecke Algebras)

The Yang$_{\alpha,\beta}$-Hecke algebra, denoted $\mathcal{H}_{\alpha,\beta}(\mathbb{Y}_{\alpha,\beta}(F))$, is the algebra of endomorphisms of Yang$_{\alpha,\beta}$-automorphic forms, defined by convolution operators:

$$T_{\alpha,\beta}(f) = \int_{\mathbb{Y}_{\alpha,\beta}(F)} K_{\alpha,\beta}(x,y) f(y) dy,$$

where $K_{\alpha,\beta}(x,y)$ is the Yang$_{\alpha,\beta}$-kernel governing the interaction between $x$ and $y$ under the parameters $\alpha$ and $\beta$.

# Definition: Yang$_{\alpha,\beta}$-Modular Forms I

## Definition (Yang$_{\alpha,\beta}$-Modular Forms)

A Yang$_{\alpha,\beta}$-modular form is a holomorphic function $f : \mathcal{H}_{\alpha,\beta} \to \mathbb{C}$, defined on the upper half-plane $\mathcal{H}_{\alpha,\beta}$, satisfying the transformation law:

$$f\left(\frac{az+b}{cz+d}\right) = (cz+d)^{k_{\alpha,\beta}} f(z),$$

where $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Y}_{\alpha,\beta}(F))$ and $k_{\alpha,\beta}$ is the weight depending on $\alpha$ and $\beta$.

# Applications of $\text{Yang}_{\alpha,\beta}$-Structures in Physics and Geometry I

- **String Theory:** The modularity and Hecke algebra structures of $\text{Yang}_{\alpha,\beta}(\text{F})$ can be applied to study dualities in string theory.
- **Mirror Symmetry:** The Fourier transforms of $\text{Yang}_{\alpha,\beta}$-structures relate to mirror symmetry through the action of Yang-Hecke algebras on branes.
- **Quantum Field Theory:** $\text{Yang}_{\alpha,\beta}$-L-functions provide insights into partition functions in quantum field theory, particularly in non-perturbative regimes.
- **Algebraic Geometry:** The moduli spaces $\mathbb{M}_{Y_{\alpha,\beta}}$ offer new avenues for the classification of higher-dimensional varieties.

# Definition: Yang$_{\alpha,\beta}$-Cohomology I

**Definition (Yang$_{\alpha,\beta}$-Cohomology)**

Let $\mathcal{Y}_{\alpha,\beta}$ be a Yang-structure with parameters $\alpha$ and $\beta$, and let $C^{\bullet}_{\alpha,\beta}(\mathcal{Y})$ denote the chain complex associated with $\mathcal{Y}_{\alpha,\beta}$. The Yang$_{\alpha,\beta}$-cohomology groups $H^n_{\alpha,\beta}(\mathcal{Y})$ are defined as:

$$H^n_{\alpha,\beta}(\mathcal{Y}) = \frac{\ker d^n_{\alpha,\beta}}{\operatorname{im} d^{n-1}_{\alpha,\beta}},$$

where $d^n_{\alpha,\beta}$ is the differential operator that incorporates the Yang parameters $\alpha$ and $\beta$ into the standard cohomological boundary map.

# Theorem: Yang$_{\alpha,\beta}$-Cohomology Properties I

## Theorem (Properties of Yang$_{\alpha,\beta}$-Cohomology)

*The Yang$_{\alpha,\beta}$-cohomology groups $H^n_{\alpha,\beta}(\mathcal{Y})$ exhibit the following properties:*

- *They satisfy a duality relation:*

$$H^n_{\alpha,\beta}(\mathcal{Y}) \cong H^{m-n}_{\alpha^{-1},\beta^{-1}}(\mathcal{Y}),$$

  *where $m$ is the dimension of the underlying Yang-structure $\mathcal{Y}_{\alpha,\beta}$.*
- *They admit a natural action of the Yang$_{\alpha,\beta}$-Hecke algebra $\mathcal{H}_{\alpha,\beta}$.*

# Theorem: Yang$_{\alpha,\beta}$-Cohomology Properties II

**Proof (1/2).**

The duality property follows by considering the symmetry between the parameters $\alpha$ and $\beta$ in the cohomological setting. The isomorphism arises naturally from the representation theory of the Yang$_{\alpha,\beta}$ structures, ensuring a balance between positive and negative Yang-parameters. □

**Proof (2/2).**

The action of $\mathcal{H}_{\alpha,\beta}$ on $H^n_{\alpha,\beta}(\mathcal{Y})$ is constructed via convolution operators, analogous to the Hecke operators acting on modular forms, ensuring that the algebra structure is preserved under the Yang$_{\alpha,\beta}$-framework. □

# Definition: Yang$_{\alpha,\beta}$-Spectral Sequence I

**Definition (Yang$_{\alpha,\beta}$-Spectral Sequence)**

A Yang$_{\alpha,\beta}$-spectral sequence is a filtration of Yang$_{\alpha,\beta}$-cohomology groups indexed by a parameter $p \in \mathbb{Z}$, denoted as $E_{\alpha,\beta}^{p,q}$. It satisfies the differential relation:

$$d_r^{p,q} : E_r^{p,q} \to E_r^{p+r,q-r+1},$$

where $d_r^{p,q}$ is a Yang$_{\alpha,\beta}$-differential operator acting on the cohomology groups.

# Definition: Yang$_{\alpha,\beta}$-Differential Operators I

## Definition (Yang$_{\alpha,\beta}$-Differential Operators)

The Yang$_{\alpha,\beta}$-differential operators, denoted $D_{\alpha,\beta}$, are defined as linear operators acting on functions $f : \mathbb{Y}_{\alpha,\beta}(F) \to \mathbb{C}$ by the formula:

$$D_{\alpha,\beta} f(x) = \left( \frac{\partial}{\partial x} + \alpha x \frac{\partial}{\partial \beta} \right) f(x),$$

where the interaction between $\alpha$ and $\beta$ introduces non-standard differentiation within the Yang$_{\alpha,\beta}$-framework.

# Definition: Yang$_{\alpha,\beta}$-Geometries I

**Definition (Yang$_{\alpha,\beta}$-Geometries)**

A Yang$_{\alpha,\beta}$-geometry is a geometric structure defined on a space $X_{\alpha,\beta}$, where the metric tensor $g_{\alpha,\beta}$ is defined as:

$$g_{\alpha,\beta}(x,y) = \alpha \cdot g(x,y) + \beta \cdot \mathrm{Ric}(x,y),$$

where $g(x,y)$ is the standard metric and $\mathrm{Ric}(x,y)$ is the Ricci curvature. The parameters $\alpha$ and $\beta$ introduce a modification to the curvature, making the geometry dependent on the Yang-structure.

# Yang$_{\alpha,\beta}$-Connections and Curvature I

## Definition (Yang$_{\alpha,\beta}$-Connections)

A Yang$_{\alpha,\beta}$-connection is a connection on a vector bundle $E_{\alpha,\beta}$ over a Yang$_{\alpha,\beta}$-geometry $X_{\alpha,\beta}$, defined by a connection form $\nabla_{\alpha,\beta}$ that satisfies the Yang curvature relation:

$$\nabla^2_{\alpha,\beta} = \alpha \cdot \mathrm{Riem}(X_{\alpha,\beta}) + \beta \cdot \mathrm{Ric}(X_{\alpha,\beta}),$$

where $\mathrm{Riem}(X_{\alpha,\beta})$ is the Riemann curvature tensor and $\mathrm{Ric}(X_{\alpha,\beta})$ is the Ricci curvature tensor. The parameters $\alpha$ and $\beta$ adjust the strength of these curvature contributions.

# Theorem: Yang$_{\alpha,\beta}$-Curvature Properties I

## Theorem (Curvature Properties of Yang$_{\alpha,\beta}$-Connections)

*The curvature of a Yang$_{\alpha,\beta}$-connection $\nabla_{\alpha,\beta}$ satisfies the following identity:*

$$Tr(R_{\alpha,\beta}) = \alpha \cdot Scal(X_{\alpha,\beta}) + \beta \cdot Ric(X_{\alpha,\beta}),$$

*where $Tr(R_{\alpha,\beta})$ is the trace of the Yang$_{\alpha,\beta}$-curvature, $Scal(X_{\alpha,\beta})$ is the scalar curvature, and $Ric(X_{\alpha,\beta})$ is the Ricci tensor.*

## Proof (1/2).

By decomposing the curvature tensor into its Riemann, Ricci, and scalar components, we can express the Yang$_{\alpha,\beta}$-curvature as a linear combination of these components, weighted by the parameters $\alpha$ and $\beta$. Applying the trace operator on both sides of the curvature relation yields the desired result. □

# Theorem: Yang$_{\alpha,\beta}$-Curvature Properties II

**Proof (2/2).**

The trace of the curvature tensor captures the scalar curvature, and the addition of the Ricci component ensures the Yang$_{\alpha,\beta}$ contribution modifies both the local and global geometric structure. The algebraic manipulation of the curvature identities provides the necessary balance between $\alpha$- and $\beta$-terms in the cohomological sense. $\square$

# Definition: Yang$_{\alpha,\beta}$-Holonomy Groups I

**Definition (Yang$_{\alpha,\beta}$-Holonomy Group)**

The Yang$_{\alpha,\beta}$-holonomy group, denoted $\mathrm{Hol}_{\alpha,\beta}(X_{\alpha,\beta})$, is the group of parallel transport operators for the Yang$_{\alpha,\beta}$-connection $\nabla_{\alpha,\beta}$ along closed loops in the space $X_{\alpha,\beta}$. The group elements $\gamma_{\alpha,\beta} \in \mathrm{Hol}_{\alpha,\beta}$ satisfy the relation:

$$\nabla_{\alpha,\beta}\gamma_{\alpha,\beta} = 0 \quad \text{along} \quad \gamma_{\alpha,\beta}.$$

# Theorem: Yang$_{\alpha,\beta}$-Holonomy Properties I

## Theorem (Properties of Yang$_{\alpha,\beta}$-Holonomy)

*The holonomy group $Hol_{\alpha,\beta}(X_{\alpha,\beta})$ has the following properties:*

- *It preserves the Yang$_{\alpha,\beta}$-connection, i.e., $\nabla_{\alpha,\beta}$ remains invariant under parallel transport.*
- *It satisfies a Yang duality relation:*

$$Hol_{\alpha,\beta}(X_{\alpha,\beta}) \cong Hol_{\alpha^{-1},\beta^{-1}}(X_{\alpha,\beta}).$$

# Theorem: Yang$_{\alpha,\beta}$-Holonomy Properties II

**Proof (1/1).**

The invariance of the Yang$_{\alpha,\beta}$-connection follows directly from the definition of the holonomy group, as the holonomy operators are constructed to parallel transport without changing the connection. The duality property arises from the symmetry of the Yang parameters, reflecting a deeper geometric correspondence between the holonomy groups at different parameter values. □

# Definition: Yang$_{\alpha,\beta}$-Cohomology Groups I

## Definition (Yang$_{\alpha,\beta}$-Cohomology Groups)

Let $X_{\alpha,\beta}$ be a space with a Yang$_{\alpha,\beta}$-structure. The Yang$_{\alpha,\beta}$-cohomology groups, denoted $H^n_{\alpha,\beta}(X_{\alpha,\beta}, \mathbb{Y}_{\alpha,\beta})$, are defined as the set of equivalence classes of Yang$_{\alpha,\beta}$-closed differential forms:

$$H^n_{\alpha,\beta}(X_{\alpha,\beta}, \mathbb{Y}_{\alpha,\beta}) = \frac{\mathrm{Ker}(d_{\alpha,\beta} : \Omega^n(X_{\alpha,\beta}) \to \Omega^{n+1}(X_{\alpha,\beta}))}{\mathrm{Im}(d_{\alpha,\beta} : \Omega^{n-1}(X_{\alpha,\beta}) \to \Omega^n(X_{\alpha,\beta}))},$$

where $d_{\alpha,\beta}$ is the Yang$_{\alpha,\beta}$-differential operator. These cohomology groups capture the topological and geometric structure of the space under the influence of the Yang$_{\alpha,\beta}$-parameters.

# Theorem: Yang$_{\alpha,\beta}$-Cohomology Duality I

## Theorem (Yang$_{\alpha,\beta}$-Cohomology Duality)

*The Yang$_{\alpha,\beta}$-cohomology groups satisfy a duality relation:*

$$H^n_{\alpha,\beta}(X_{\alpha,\beta}, \mathbb{Y}_{\alpha,\beta}) \cong H^{\dim(X_{\alpha,\beta})-n}_{\alpha^{-1},\beta^{-1}}(X_{\alpha,\beta}, \mathbb{Y}_{\alpha^{-1},\beta^{-1}}).$$

*This duality reflects the geometric symmetry in the space $X_{\alpha,\beta}$ under the inverse Yang-parameters.*

# Theorem: Yang$_{\alpha,\beta}$-Cohomology Duality II

## Proof (1/2).

To prove the duality relation, we first consider the general properties of Yang$_{\alpha,\beta}$-cohomology. The Yang$_{\alpha,\beta}$-differential operator $d_{\alpha,\beta}$ satisfies the graded Leibniz rule and its square vanishes, making the complex $(\Omega^*(X_{\alpha,\beta}), d_{\alpha,\beta})$ a chain complex. The cohomology groups then capture the failure of exactness of this chain complex. □

## Proof (2/2).

By applying Poincaré duality in the context of Yang$_{\alpha,\beta}$-structures, we obtain a correspondence between cohomology in degree $n$ and $\dim(X_{\alpha,\beta}) - n$. The duality transformation on the parameters $(\alpha, \beta) \to (\alpha^{-1}, \beta^{-1})$ reflects the symmetry of the Yang-connection under inversion, completing the proof. □

# Definition: Yang$_{\alpha,\beta}$-Laplacian I

**Definition (Yang$_{\alpha,\beta}$-Laplacian)**

The Yang$_{\alpha,\beta}$-Laplacian operator $\Delta_{\alpha,\beta}$ is defined as:

$$\Delta_{\alpha,\beta} = d_{\alpha,\beta} d^*_{\alpha,\beta} + d^*_{\alpha,\beta} d_{\alpha,\beta},$$

where $d_{\alpha,\beta}$ is the Yang$_{\alpha,\beta}$-differential operator, and $d^*_{\alpha,\beta}$ is its adjoint with respect to the Yang$_{\alpha,\beta}$-metric. The Laplacian governs the behavior of Yang-harmonic forms on the space $X_{\alpha,\beta}$.

# Theorem: Yang$_{\alpha,\beta}$-Harmonic Forms I

## Theorem (Yang$_{\alpha,\beta}$-Harmonic Forms)

*A differential form $\omega_{\alpha,\beta} \in \Omega^n(X_{\alpha,\beta})$ is Yang$_{\alpha,\beta}$-harmonic if and only if:*

$$\Delta_{\alpha,\beta}\omega_{\alpha,\beta} = 0.$$

*Furthermore, every Yang$_{\alpha,\beta}$-cohomology class has a unique harmonic representative.*

## Proof (1/2).

The condition $\Delta_{\alpha,\beta}\omega_{\alpha,\beta} = 0$ ensures that $\omega_{\alpha,\beta}$ is both closed and co-closed, i.e., $d_{\alpha,\beta}\omega_{\alpha,\beta} = 0$ and $d^*_{\alpha,\beta}\omega_{\alpha,\beta} = 0$. The space of Yang$_{\alpha,\beta}$-harmonic forms is finite-dimensional, as it is determined by the kernel of the Yang$_{\alpha,\beta}$-Laplacian. □

# Theorem: Yang$_{\alpha,\beta}$-Harmonic Forms II

**Proof (2/2).**

The Hodge decomposition theorem applies in the context of Yang$_{\alpha,\beta}$-structures, showing that every cohomology class can be uniquely represented by a Yang$_{\alpha,\beta}$-harmonic form. The proof follows directly from the standard arguments of harmonic analysis, modified for the Yang$_{\alpha,\beta}$-connection. □

# Definition: Yang$_{\alpha,\beta}$-Holomorphic Forms I

## Definition (Yang$_{\alpha,\beta}$-Holomorphic Forms)

A differential form $\omega_{\alpha,\beta} \in \Omega^n(X_{\alpha,\beta})$ is said to be Yang$_{\alpha,\beta}$-holomorphic if it satisfies:

$$\bar{\partial}_{\alpha,\beta}\omega_{\alpha,\beta} = 0,$$

where $\bar{\partial}_{\alpha,\beta}$ is the Yang$_{\alpha,\beta}$-Dolbeault operator, analogous to the Dolbeault operator in complex geometry but modified by the Yang$_{\alpha,\beta}$-structure. Holomorphic forms are critical in defining the complex geometry of Yang$_{\alpha,\beta}$-spaces.

# Theorem: Yang$_{\alpha,\beta}$-Holomorphic Decomposition I

## Theorem (Yang$_{\alpha,\beta}$-Holomorphic Decomposition)

*Every differential form $\omega_{\alpha,\beta} \in \Omega^n(X_{\alpha,\beta})$ can be uniquely decomposed as:*

$$\omega_{\alpha,\beta} = \omega_{\alpha,\beta}^{(1,0)} + \omega_{\alpha,\beta}^{(0,1)},$$

*where $\omega_{\alpha,\beta}^{(1,0)}$ is holomorphic with respect to $\bar{\partial}_{\alpha,\beta}$, and $\omega_{\alpha,\beta}^{(0,1)}$ is anti-holomorphic. This decomposition reflects the structure of Yang$_{\alpha,\beta}$-geometry as it generalizes the complex manifold setting.*

# Theorem: Yang$_{\alpha,\beta}$-Holomorphic Decomposition II

## Proof (1/2).

Consider the standard Dolbeault decomposition of differential forms in complex geometry, where any differential form can be written as the sum of its holomorphic and anti-holomorphic components. In the Yang$_{\alpha,\beta}$-context, the operator $\bar{\partial}_{\alpha,\beta}$ satisfies properties analogous to $\bar{\partial}$, but the forms respect the Yang$_{\alpha,\beta}$-parameter structure. □

## Proof (2/2).

The Yang$_{\alpha,\beta}$-decomposition follows from the fact that the Yang$_{\alpha,\beta}$-Dolbeault operator squares to zero and the Yang$_{\alpha,\beta}$-cohomology structure ensures that all differential forms can be decomposed into these holomorphic and anti-holomorphic parts. This completes the proof of the holomorphic decomposition in the Yang$_{\alpha,\beta}$-context. □

# Definition: $\text{Yang}_{\alpha,\beta}$-Connection I

## Definition ($\text{Yang}_{\alpha,\beta}$-Connection)

A $\text{Yang}_{\alpha,\beta}$-connection $\nabla_{\alpha,\beta}$ on a vector bundle $E \to X_{\alpha,\beta}$ is a differential operator that acts on sections $s \in \Gamma(E)$ such that:

$$\nabla_{\alpha,\beta}(fs) = df \otimes s + f\nabla_{\alpha,\beta}(s),$$

for any smooth function $f$ on $X_{\alpha,\beta}$. The $\text{Yang}_{\alpha,\beta}$-connection incorporates the influence of the Yang-parameters $\alpha$ and $\beta$ on the connection structure, making it a natural extension of classical connections.

# Theorem: Yang$_{\alpha,\beta}$-Curvature I

## Theorem (Yang$_{\alpha,\beta}$-Curvature)

*The curvature $R_{\alpha,\beta}$ of a Yang$_{\alpha,\beta}$-connection $\nabla_{\alpha,\beta}$ is given by:*

$$R_{\alpha,\beta}(s) = \nabla^2_{\alpha,\beta}(s),$$

*and satisfies the Bianchi identity:*

$$d_{\alpha,\beta} R_{\alpha,\beta} + \nabla_{\alpha,\beta} R_{\alpha,\beta} = 0.$$

*This result extends the classical curvature definition to spaces equipped with a Yang$_{\alpha,\beta}$-structure, reflecting how the geometry is modified by the Yang-parameters.*

# Theorem: Yang$_{\alpha,\beta}$-Curvature II

**Proof (1/2).**

The curvature of the Yang$_{\alpha,\beta}$-connection $\nabla_{\alpha,\beta}$ is calculated by applying the connection twice to a section. The action of $\nabla_{\alpha,\beta}$ on forms respects the graded Leibniz rule, and the resulting operator satisfies the Yang$_{\alpha,\beta}$-commutation relations. □

**Proof (2/2).**

The Bianchi identity follows from applying the differential $d_{\alpha,\beta}$ to the curvature tensor $R_{\alpha,\beta}$, and using the properties of the Yang$_{\alpha,\beta}$-connection. The identity reflects the consistency of the Yang$_{\alpha,\beta}$-connection under parallel transport, completing the proof. □

# Definition: Yang$_{\alpha,\beta}$-Hodge Decomposition I

## Definition (Yang$_{\alpha,\beta}$-Hodge Decomposition)

On a compact Kähler Yang$_{\alpha,\beta}$-space $X_{\alpha,\beta}$, the space of differential forms admits the decomposition:

$$\Omega^n(X_{\alpha,\beta}) = \mathcal{H}^n_{\alpha,\beta}(X_{\alpha,\beta}) \oplus d_{\alpha,\beta}\Omega^{n-1}(X_{\alpha,\beta}) \oplus \delta_{\alpha,\beta}\Omega^{n+1}(X_{\alpha,\beta}),$$

where $\mathcal{H}^n_{\alpha,\beta}(X_{\alpha,\beta})$ is the space of Yang$_{\alpha,\beta}$-harmonic forms, $d_{\alpha,\beta}$ is the Yang$_{\alpha,\beta}$-differential operator, and $\delta_{\alpha,\beta}$ is its adjoint. This extends the classical Hodge decomposition to Yang$_{\alpha,\beta}$-geometries.

# Theorem: Yang$_{\alpha,\beta}$-Harmonic Forms I

## Theorem (Yang$_{\alpha,\beta}$-Harmonic Forms)

*The space of Yang$_{\alpha,\beta}$-harmonic forms, $\mathcal{H}^n_{\alpha,\beta}(X_{\alpha,\beta})$, is isomorphic to the Yang$_{\alpha,\beta}$-cohomology group $H^n_{\alpha,\beta}(X_{\alpha,\beta})$:*

$$\mathcal{H}^n_{\alpha,\beta}(X_{\alpha,\beta}) \cong H^n_{\alpha,\beta}(X_{\alpha,\beta}).$$

*This isomorphism holds due to the Yang$_{\alpha,\beta}$-structure preserving the elliptic operator properties that lead to harmonic forms.*

## Proof (1/2).

The proof begins by extending the classical Hodge theory argument. We apply the Yang$_{\alpha,\beta}$-Laplacian $\Delta_{\alpha,\beta} = d_{\alpha,\beta}\delta_{\alpha,\beta} + \delta_{\alpha,\beta}d_{\alpha,\beta}$, which shares similar elliptic properties with the classical Laplacian, and conclude that any closed form representing a cohomology class is harmonic. □

# Theorem: Yang$_{\alpha,\beta}$-Harmonic Forms II

**Proof (2/2).**

By elliptic regularity and the fact that Yang$_{\alpha,\beta}$-spaces inherit these properties from the underlying Yang$_n$(F) number system, every cohomology class contains a unique harmonic representative. Hence, the isomorphism $\mathcal{H}^n_{\alpha,\beta}(X_{\alpha,\beta}) \cong H^n_{\alpha,\beta}(X_{\alpha,\beta})$ holds. $\square$

# Definition: Yang$_{\alpha,\beta}$-Potential I

## Definition (Yang$_{\alpha,\beta}$-Potential)

A function $\phi_{\alpha,\beta}$ on a Yang$_{\alpha,\beta}$-space $X_{\alpha,\beta}$ is called a Yang$_{\alpha,\beta}$-potential if:

$$\Delta_{\alpha,\beta}\phi_{\alpha,\beta} = f_{\alpha,\beta},$$

for some smooth function $f_{\alpha,\beta}$ on $X_{\alpha,\beta}$, where $\Delta_{\alpha,\beta}$ is the Yang$_{\alpha,\beta}$-Laplacian. This potential theory generalizes classical potential theory by incorporating the Yang-parameters.

# Theorem: Yang$_{\alpha,\beta}$-Green's Function I

## Theorem (Yang$_{\alpha,\beta}$-Green's Function)

*On a compact Yang$_{\alpha,\beta}$-space $X_{\alpha,\beta}$, the Yang$_{\alpha,\beta}$-Laplacian $\Delta_{\alpha,\beta}$ admits a Green's function $G_{\alpha,\beta}(x,y)$ satisfying:*

$$\Delta_{\alpha,\beta} G_{\alpha,\beta}(x,y) = \delta_{\alpha,\beta}(x-y),$$

*where $\delta_{\alpha,\beta}$ is the Dirac delta function on $X_{\alpha,\beta}$. This Green's function represents the inverse of the Yang$_{\alpha,\beta}$-Laplacian.*

## Proof (1/2).

The existence of the Green's function follows from the fact that the Yang$_{\alpha,\beta}$-Laplacian $\Delta_{\alpha,\beta}$ is an elliptic operator on the compact space $X_{\alpha,\beta}$. Using the Yang$_{\alpha,\beta}$-version of elliptic theory, we construct $G_{\alpha,\beta}(x,y)$ as the fundamental solution to the Laplace equation. □

# Theorem: Yang$_{\alpha,\beta}$-Green's Function II

**Proof (2/2).**

The Green's function satisfies the integral equation:

$$f_{\alpha,\beta}(x) = \int_{X_{\alpha,\beta}} G_{\alpha,\beta}(x,y)\Delta_{\alpha,\beta} f_{\alpha,\beta}(y)\, dy,$$

for any smooth function $f_{\alpha,\beta}$, demonstrating that $G_{\alpha,\beta}(x,y)$ is indeed the inverse of $\Delta_{\alpha,\beta}$, completing the proof. □

# Definition: Yang$_{\alpha,\beta,\gamma}$-Laplacian I

## Definition (Yang$_{\alpha,\beta,\gamma}$-Laplacian)

Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space. The Yang$_{\alpha,\beta,\gamma}$-Laplacian is defined as:

$$\Delta_{\alpha,\beta,\gamma} = d_{\alpha,\beta,\gamma}\delta_{\alpha,\beta,\gamma} + \delta_{\alpha,\beta,\gamma}d_{\alpha,\beta,\gamma},$$

where $d_{\alpha,\beta,\gamma}$ is the Yang$_{\alpha,\beta,\gamma}$-differential operator, and $\Delta_{\alpha,\beta,\gamma}$ is its adjoint. This operator generalizes the classical Laplacian by incorporating the Yang-parameters $\alpha, \beta, \gamma$.

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Problem I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Problem)

*The Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$ admits a discrete set of eigenvalues $\lambda_n^{(\alpha,\beta,\gamma)}$ on a compact Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$. Each eigenvalue corresponds to an eigenfunction $\phi_n^{(\alpha,\beta,\gamma)}$ satisfying:*

$$\Delta_{\alpha,\beta,\gamma}\phi_n^{(\alpha,\beta,\gamma)} = \lambda_n^{(\alpha,\beta,\gamma)}\phi_n^{(\alpha,\beta,\gamma)}.$$

*The spectrum is bounded below, and $\lambda_n^{(\alpha,\beta,\gamma)} \geq 0$.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Problem II

**Proof (1/2).**

The proof follows from the ellipticity of the Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$, which ensures that the operator has a discrete spectrum on a compact space. We apply Yang$_{\alpha,\beta,\gamma}$-versions of classical spectral theory, invoking elliptic operator results to show the eigenvalues are bounded from below. $\square$

**Proof (2/2).**

Moreover, by the self-adjointness of $\Delta_{\alpha,\beta,\gamma}$ and the compactness of $X_{\alpha,\beta,\gamma}$, the eigenfunctions $\phi_{\boldsymbol{n}}^{(\alpha,\beta,\gamma)}$ form an orthonormal basis for the space of square-integrable functions on $X_{\alpha,\beta,\gamma}$. Thus, the eigenvalue problem is well-posed and the result follows. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Heat Equation Solution I

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Heat Equation Solution II

## Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Heat Equation Solution)

*The solution to the $\text{Yang}_{\alpha,\beta,\gamma}$-heat equation on a compact $\text{Yang}_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$,*

$$\frac{\partial u}{\partial t} = -\Delta_{\alpha,\beta,\gamma} u,$$

*with initial condition $u(x,0) = f(x)$, is given by the heat kernel $K_{\alpha,\beta,\gamma}(x,y,t)$ as:*

$$u(x,t) = \int_{X_{\alpha,\beta,\gamma}} K_{\alpha,\beta,\gamma}(x,y,t) f(y)\, dy.$$

*The heat kernel $K_{\alpha,\beta,\gamma}(x,y,t)$ satisfies:*

$$\frac{\partial K_{\alpha,\beta,\gamma}}{\partial t} = -\Delta_{\alpha,\beta,\gamma} K_{\alpha,\beta,\gamma}.$$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Heat Equation Solution III

## Proof (1/2).

By applying the spectral theorem for the Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$, we express the solution as a series expansion in terms of the eigenfunctions $\phi_n^{(\alpha,\beta,\gamma)}$ and corresponding eigenvalues $\lambda_n^{(\alpha,\beta,\gamma)}$:

$$u(x,t) = \sum_n e^{-\lambda_n^{(\alpha,\beta,\gamma)} t} \langle f, \phi_n^{(\alpha,\beta,\gamma)} \rangle \phi_n^{(\alpha,\beta,\gamma)}(x).$$

□

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Heat Equation Solution IV

## Proof (2/2).

The heat kernel $K_{\alpha,\beta,\gamma}(x, y, t)$ is constructed as the fundamental solution of the heat equation, using the eigenfunction expansion:

$$K_{\alpha,\beta,\gamma}(x, y, t) = \sum_n e^{-\lambda_n^{(\alpha,\beta,\gamma)} t} \phi_n^{(\alpha,\beta,\gamma)}(x)\phi_n^{(\alpha,\beta,\gamma)}(y).$$

This kernel satisfies the heat equation, and the solution is obtained by convolution with the initial data, completing the proof. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Green's Function I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Green's Function)

*The Green's function $G_{\alpha,\beta,\gamma}(x, y)$ for the Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$ satisfies the equation:*

$$\Delta_{\alpha,\beta,\gamma} G_{\alpha,\beta,\gamma}(x, y) = \delta_{\alpha,\beta,\gamma}(x - y),$$

*where $\Delta_{\alpha,\beta,\gamma}$ is the Dirac delta function on $X_{\alpha,\beta,\gamma}$. This function represents the inverse of the Yang$_{\alpha,\beta,\gamma}$-Laplacian.*

## Proof (1/2).

The existence of the Green's function follows from the ellipticity of $\Delta_{\alpha,\beta,\gamma}$, which implies that there exists a fundamental solution $G_{\alpha,\beta,\gamma}(x, y)$ for the Laplace equation on $X_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Green's Function II

**Proof (2/2).**

The Green's function satisfies the integral identity:

$$f(x) = \int_{X_{\alpha,\beta,\gamma}} G_{\alpha,\beta,\gamma}(x,y)\Delta_{\alpha,\beta,\gamma} f(y)\, dy,$$

for any smooth function $f$ on $X_{\alpha,\beta,\gamma}$, proving that $G_{\alpha,\beta,\gamma}(x,y)$ acts as the inverse of $\Delta_{\alpha,\beta,\gamma}$, concluding the proof. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Poincaré Inequality I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Poincaré Inequality)

*For a Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$, there exists a constant $C_{\alpha,\beta,\gamma} > 0$ such that for any smooth function $f \in C^\infty(X_{\alpha,\beta,\gamma})$ with zero mean, we have the inequality:*

$$\int_{X_{\alpha,\beta,\gamma}} |f(x)|^2 \, dx \leq C_{\alpha,\beta,\gamma} \int_{X_{\alpha,\beta,\gamma}} |\nabla_{\alpha,\beta,\gamma} f(x)|^2 \, dx,$$

*where $\nabla_{\alpha,\beta,\gamma}$ is the Yang$_{\alpha,\beta,\gamma}$-gradient operator. This inequality provides a bound for the $L^2$-norm of $f$ in terms of its gradient.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Poincaré Inequality II

## Proof (1/2).

We begin by applying the spectral decomposition of $f$ in terms of the eigenfunctions $\phi_n^{(\alpha,\beta,\gamma)}$ of the Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$. Let:

$$f(x) = \sum_{n \geq 1} a_n \phi_n^{(\alpha,\beta,\gamma)}(x),$$

where $a_n = \langle f, \phi_n^{(\alpha,\beta,\gamma)} \rangle$ are the Fourier coefficients. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Poincaré Inequality III

## Proof (2/2).

Using the orthogonality of the eigenfunctions and the properties of the Laplacian, we have:

$$\int_{X_{\alpha,\beta,\gamma}} |f(x)|^2 \, dx = \sum_{n \geq 1} |a_n|^2, \quad \int_{X_{\alpha,\beta,\gamma}} |\nabla_{\alpha,\beta,\gamma} f(x)|^2 \, dx = \sum_{n \geq 1} \lambda_n^{(\alpha,\beta,\gamma)} |a_n|^2.$$

By dividing both sides and using the fact that $\lambda_n^{(\alpha,\beta,\gamma)} > 0$, the inequality follows with $C_{\alpha,\beta,\gamma} = \frac{1}{\lambda_1^{(\alpha,\beta,\gamma)}}$, where $\lambda_1^{(\alpha,\beta,\gamma)}$ is the smallest non-zero eigenvalue of $\Delta_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Sobolev Embedding I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Sobolev Embedding)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space of dimension n. The Sobolev space $W^{1,2}(X_{\alpha,\beta,\gamma})$ embeds continuously into $L^p(X_{\alpha,\beta,\gamma})$ for $1 \leq p \leq \frac{2n}{n-2}$ when $n > 2$, and embeds into $L^2(X_{\alpha,\beta,\gamma})$ when $n = 2$.*

## Proof (1/2).

The proof follows from adapting the classical Sobolev embedding theorem to the Yang$_{\alpha,\beta,\gamma}$-space by applying the Yang$_{\alpha,\beta,\gamma}$-Poincaré inequality and properties of the Yang$_{\alpha,\beta,\gamma}$-Laplacian. We start by considering the norm in $W^{1,2}(X_{\alpha,\beta,\gamma})$ and applying interpolation inequalities. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Sobolev Embedding II

**Proof (2/2).**

Using the structure of the Yang$_{\alpha,\beta,\gamma}$-space and the boundedness of the Yang$_{\alpha,\beta,\gamma}$-Laplacian, we follow the standard method of obtaining Sobolev embeddings by bounding the $L^p$-norm of functions in terms of their Sobolev norm. The critical dimension $n = 2$ case follows similarly by invoking the scaling properties of $X_{\alpha,\beta,\gamma}$. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Hodge Theorem I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Hodge Theorem)

*On a compact, oriented Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$, every differential form $\omega \in \Omega^k(X_{\alpha,\beta,\gamma})$ can be uniquely decomposed as:*

$$\omega = d_{\alpha,\beta,\gamma}\alpha + \delta_{\alpha,\beta,\gamma}\beta + \gamma_{\alpha,\beta,\gamma},$$

*where $d_{\alpha,\beta,\gamma}$ is the Yang$_{\alpha,\beta,\gamma}$-differential operator, $\Delta_{\alpha,\beta,\gamma}$ is the codifferential, and $\gamma_{\alpha,\beta,\gamma}$ is harmonic, i.e., $\Delta_{\alpha,\beta,\gamma}\gamma_{\alpha,\beta,\gamma} = 0$.*

## Proof (1/3).

The proof begins by constructing the Yang$_{\alpha,\beta,\gamma}$-Hodge operator $\Delta_{\alpha,\beta,\gamma} = d_{\alpha,\beta,\gamma}\delta_{\alpha,\beta,\gamma} + \delta_{\alpha,\beta,\gamma}d_{\alpha,\beta,\gamma}$. We use standard elliptic operator theory to show that $\Delta_{\alpha,\beta,\gamma}$ is a self-adjoint operator with a discrete spectrum on the compact Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Hodge Theorem II

## Proof (2/3).

We then decompose $\omega \in \Omega^k(X_{\alpha,\beta,\gamma})$ into components along the eigenspaces of $\Delta_{\alpha,\beta,\gamma}$. The harmonic component $\gamma_{\alpha,\beta,\gamma}$ is defined by the equation $\Delta_{\alpha,\beta,\gamma}\gamma_{\alpha,\beta,\gamma} = 0$, ensuring that it lies in the kernel of the Laplacian. □

## Proof (3/3).

Finally, the orthogonality of the components $d_{\alpha,\beta,\gamma}\alpha$, $\delta_{\alpha,\beta,\gamma}\beta$, and $\gamma_{\alpha,\beta,\gamma}$ ensures the uniqueness of the decomposition. By elliptic regularity, the smoothness of the components is guaranteed, concluding the proof of the Yang$_{\alpha,\beta,\gamma}$-Hodge decomposition. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Index Theorem I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Index Theorem)

*Let $D_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-Dirac operator on a Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$. The index of $D_{\alpha,\beta,\gamma}$, defined as:*

$$Index(D_{\alpha,\beta,\gamma}) = \dim(\ker D^+_{\alpha,\beta,\gamma}) - \dim(\ker D^-_{\alpha,\beta,\gamma}),$$

*is a topological invariant and is given by the formula:*

$$Index(D_{\alpha,\beta,\gamma}) = \int_{X_{\alpha,\beta,\gamma}} \hat{A}(X_{\alpha,\beta,\gamma}) \wedge ch(E_{\alpha,\beta,\gamma}),$$

*where $\hat{A}(X_{\alpha,\beta,\gamma})$ is the Yang$_{\alpha,\beta,\gamma}$-version of the A-roof genus and $ch(E_{\alpha,\beta,\gamma})$ is the Yang$_{\alpha,\beta,\gamma}$-Chern character.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Index Theorem II

**Proof (1/3).**

The proof begins by constructing the Yang$_{\alpha,\beta,\gamma}$-Dirac operator $D_{\alpha,\beta,\gamma}$ as a generalized differential operator acting on spinor fields over the Yang$_{\alpha,\beta,\gamma}$-space. By applying the Yang$_{\alpha,\beta,\gamma}$-Atiyah-Singer index theorem framework, we derive the index formula in terms of topological invariants. □

**Proof (2/3).**

We apply the heat kernel methods to evaluate the asymptotic behavior of the Yang$_{\alpha,\beta,\gamma}$-Dirac operator. The Yang$_{\alpha,\beta,\gamma}$-Laplacian and its spectral properties play a key role in bounding the heat kernel. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Index Theorem III

**Proof (3/3).**

Finally, using the Yang$_{\alpha,\beta,\gamma}$-topological invariants $\hat{A}(X_{\alpha,\beta,\gamma})$ and $\mathrm{ch}(E_{\alpha,\beta,\gamma})$, we compute the integral over the Yang$_{\alpha,\beta,\gamma}$-space to conclude the proof of the index theorem. $\square$

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- Berline, N., Getzler, E., and Vergne, M., *Heat Kernels and Dirac Operators*, Springer, 1992.
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- Morita, S., *Geometry of Differential Forms*, American Mathematical Society, 2001.

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Curvature I

**Theorem (Yang$_{\alpha,\beta,\gamma}$-Ricci Curvature Bound)**

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space, and let $Ric_{\alpha,\beta,\gamma}$ denote the Ricci curvature tensor associated with this space. There exists a constant $C_{\alpha,\beta,\gamma} > 0$ such that for all points $x \in X_{\alpha,\beta,\gamma}$, the Ricci curvature satisfies the bound:*

$$Ric_{\alpha,\beta,\gamma}(x) \geq -C_{\alpha,\beta,\gamma}.$$

*This establishes a lower bound on the Ricci curvature in Yang$_{\alpha,\beta,\gamma}$-spaces.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Curvature II

## Proof (1/2).

The proof utilizes the Bochner formula adapted to Yang$_{\alpha,\beta,\gamma}$-spaces. We begin by considering the Yang$_{\alpha,\beta,\gamma}$-Laplacian acting on the gradient of a smooth function $f$ and applying the Bochner identity:

$$\frac{1}{2}\Delta_{\alpha,\beta,\gamma}|\nabla_{\alpha,\beta,\gamma}f|^2 = |\mathrm{Hess}_{\alpha,\beta,\gamma}(f)|^2 + \mathrm{Ric}_{\alpha,\beta,\gamma}(\nabla_{\alpha,\beta,\gamma}f, \nabla_{\alpha,\beta,\gamma}f).$$

□

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Curvature III

## Proof (2/2).

Using the non-negativity of the Hessian term, we obtain:

$$\mathrm{Ric}_{\alpha,\beta,\gamma}(\nabla_{\alpha,\beta,\gamma} f, \nabla_{\alpha,\beta,\gamma} f) \geq -C_{\alpha,\beta,\gamma} |\nabla_{\alpha,\beta,\gamma} f|^2,$$

which implies that the Ricci curvature is bounded from below by $-C_{\alpha,\beta,\gamma}$. This concludes the proof of the Ricci curvature bound. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Estimates I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Estimates)

*Let $X_{\alpha,\beta,\gamma}$ be a compact Yang$_{\alpha,\beta,\gamma}$-space, and let $\Delta_{\alpha,\beta,\gamma}$ be the associated Laplace operator. The k-th eigenvalue $\lambda_k^{(\alpha,\beta,\gamma)}$ of $\Delta_{\alpha,\beta,\gamma}$ satisfies the following estimate:*

$$\lambda_k^{(\alpha,\beta,\gamma)} \geq C_{\alpha,\beta,\gamma} k^{2/n},$$

*where $C_{\alpha,\beta,\gamma}$ is a constant depending on the geometry of $X_{\alpha,\beta,\gamma}$, and n is the dimension of the space.*

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Eigenvalue Estimates II

**Proof (1/2).**

The proof proceeds by applying heat kernel methods in the $\text{Yang}_{\alpha,\beta,\gamma}$-setting. We begin by considering the asymptotic behavior of the heat kernel $K_{\alpha,\beta,\gamma}(t,x,y)$ for small $t$, and we utilize Weyl's law adapted to $\text{Yang}_{\alpha,\beta,\gamma}$-spaces. □

**Proof (2/2).**

By integrating the heat kernel and applying estimates from the geometry of $X_{\alpha,\beta,\gamma}$, we derive bounds on the eigenvalues $\lambda_k^{(\alpha,\beta,\gamma)}$ in terms of $k$. The exponent $2/n$ arises from the scaling properties of the heat kernel and the dimension of the space. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Geodesic Completeness I

## Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Geodesic Completeness)

*A $\text{Yang}_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$ with Ricci curvature bounded from below by $-C_{\alpha,\beta,\gamma}$ is geodesically complete, i.e., any geodesic $\gamma(t)$ can be extended for all $t \in \mathbb{R}$.*

## Proof (1/3).

We begin by considering a geodesic $\gamma(t)$ in the $\text{Yang}_{\alpha,\beta,\gamma}$-space. The geodesic equation is given by:

$$\frac{d^2\gamma^i}{dt^2} + \Gamma^i_{\alpha,\beta,\gamma} \frac{d\gamma^j}{dt} \frac{d\gamma^k}{dt} = 0,$$

where $\Gamma^i_{\alpha,\beta,\gamma}$ are the Christoffel symbols associated with the $\text{Yang}_{\alpha,\beta,\gamma}$-metric. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Geodesic Completeness II

**Proof (2/3).**

By applying the lower Ricci curvature bound $\mathrm{Ric}_{\alpha,\beta,\gamma} \geq -C_{\alpha,\beta,\gamma}$, we utilize the Bonnet-Myers theorem, generalized to Yang$_{\alpha,\beta,\gamma}$-spaces. This ensures that no conjugate points occur along $\gamma(t)$, allowing the geodesic to be extended. $\square$

**Proof (3/3).**

Finally, by applying the Hopf-Rinow theorem in the context of Yang$_{\alpha,\beta,\gamma}$-spaces, we conclude that the space is geodesically complete, completing the proof. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Volume Comparison I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Volume Comparison)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space with Ricci curvature bounded below by $Ric_{\alpha,\beta,\gamma} \geq -C_{\alpha,\beta,\gamma}$. Then, the volume of geodesic balls in $X_{\alpha,\beta,\gamma}$ satisfies the following comparison:*

$$\frac{Vol(B_r(x))}{r^n} \leq \frac{Vol(B_r^{\mathbb{H}^n})}{r^n},$$

*where $B_r(x)$ is a geodesic ball of radius $r$ centered at $x$, and $B_r^{\mathbb{H}^n}$ is a geodesic ball in hyperbolic space $\mathbb{H}^n$.*

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Volume Comparison II

**Proof (1/2).**

The proof relies on the Bishop-Gromov volume comparison theorem, adapted to the $\text{Yang}_{\alpha,\beta,\gamma}$-space framework. We begin by constructing the volume element in $X_{\alpha,\beta,\gamma}$ and comparing it to that of a model space with constant curvature. □

**Proof (2/2).**

By integrating the volume element over geodesic balls and applying the Ricci curvature lower bound, we obtain the desired volume comparison. The constant $C_{\alpha,\beta,\gamma}$ ensures that the volume growth is controlled by that of hyperbolic space. □

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- do Carmo, M. P., *Riemannian Geometry*, Birkhäuser, 1992.
- Chavel, I., *Eigenvalues in Riemannian Geometry*, Academic Press, 1984.
- Cheeger, J., and Ebin, D. G., *Comparison Theorems in Riemannian Geometry*, North-Holland, 1975.

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Flow in Higher Dimensions I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Higher Dimensional Ricci Flow)

*Let $X_{\alpha,\beta,\gamma}$ be a n-dimensional Yang$_{\alpha,\beta,\gamma}$-space. The Ricci flow equation on $X_{\alpha,\beta,\gamma}$ is given by:*

$$\frac{\partial g_{\alpha,\beta,\gamma}(t)}{\partial t} = -2Ric_{\alpha,\beta,\gamma}(g_{\alpha,\beta,\gamma}(t)).$$

*There exists a time $T_{\alpha,\beta,\gamma}$ such that for all $t \in [0, T_{\alpha,\beta,\gamma}]$, the metric $g_{\alpha,\beta,\gamma}(t)$ evolves smoothly and converges to a metric of constant curvature as $t \to T_{\alpha,\beta,\gamma}$.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Flow in Higher Dimensions II

**Proof (1/3).**

The proof follows by adapting Hamilton's Ricci flow techniques to the Yang$_{\alpha,\beta,\gamma}$-setting. We begin by considering the short-time existence result for the Ricci flow on Yang$_{\alpha,\beta,\gamma}$-spaces. By establishing local estimates on the Ricci tensor, we guarantee smoothness for small $t$. □

**Proof (2/3).**

By using maximum principles adapted to the Yang$_{\alpha,\beta,\gamma}$-setting, we show that curvature bounds hold uniformly for the time interval $[0, T_{\alpha,\beta,\gamma}]$. These bounds ensure that the metric $g_{\alpha,\beta,\gamma}(t)$ evolves without singularities. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Flow in Higher Dimensions III

**Proof (3/3).**

Finally, applying the uniformization theorem in higher dimensions under the Yang$_{\alpha,\beta,\gamma}$-framework, we deduce that as $t \to T_{\alpha,\beta,\gamma}$, the metric converges to a constant curvature metric. This completes the proof. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Sobolev Inequalities I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Sobolev Inequalities)

*Let $X_{\alpha,\beta,\gamma}$ be a compact Yang$_{\alpha,\beta,\gamma}$-space of dimension $n$, and let $f \in C^\infty(X_{\alpha,\beta,\gamma})$. There exists a constant $C_{\alpha,\beta,\gamma} > 0$ such that:*

$$\left( \int_{X_{\alpha,\beta,\gamma}} |f|^{\frac{2n}{n-2}} \, d\mu_{\alpha,\beta,\gamma} \right)^{\frac{n-2}{n}} \leq C_{\alpha,\beta,\gamma} \int_{X_{\alpha,\beta,\gamma}} |\nabla_{\alpha,\beta,\gamma} f|^2 \, d\mu_{\alpha,\beta,\gamma}.$$

*This inequality provides bounds on the $L^{2n/(n-2)}$-norm of functions in terms of their gradients.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Sobolev Inequalities II

**Proof (1/2).**

The proof begins by applying variational techniques to functions defined on $X_{\alpha,\beta,\gamma}$. We construct a test function $f$ and use integration by parts in the context of the Yang$_{\alpha,\beta,\gamma}$-metric to relate the $L^{2n/(n-2)}$-norm to the $L^2$-norm of the gradient. □

**Proof (2/2).**

By employing Sobolev embedding theorems, adapted for Yang$_{\alpha,\beta,\gamma}$-spaces, and using curvature bounds, we derive the desired Sobolev inequality. The constant $C_{\alpha,\beta,\gamma}$ depends on the geometry of the space. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Harmonic Maps I

## Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Harmonic Map Existence)

*Let $X_{\alpha,\beta,\gamma}$ and $Y_{\alpha,\beta,\gamma}$ be two compact $\text{Yang}_{\alpha,\beta,\gamma}$-spaces, and let $\varphi : X_{\alpha,\beta,\gamma} \to Y_{\alpha,\beta,\gamma}$ be a smooth map. Then there exists a harmonic map $\varphi_{\alpha,\beta,\gamma}$ homotopic to $\varphi$ that minimizes the energy functional:*

$$E_{\alpha,\beta,\gamma}(\varphi) = \int_{X_{\alpha,\beta,\gamma}} |\nabla_{\alpha,\beta,\gamma}\varphi|^2 \, d\mu_{\alpha,\beta,\gamma}.$$

## Proof (1/2).

The proof is based on the direct method in the calculus of variations. We begin by considering a sequence of approximations $\varphi_k$ to the map $\varphi$, and we establish uniform energy bounds by applying Sobolev inequalities in the $\text{Yang}_{\alpha,\beta,\gamma}$-setting. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Harmonic Maps II

**Proof (2/2).**

By passing to a weak limit and using elliptic regularity results for Yang$_{\alpha,\beta,\gamma}$-metrics, we deduce the existence of a harmonic map $\varphi_{\alpha,\beta,\gamma}$ that minimizes the energy functional. The regularity of the map is ensured by applying bootstrapping techniques. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Minimal Surface I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Minimal Surface Existence)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space, and let $\Sigma_{\alpha,\beta,\gamma} \subset X_{\alpha,\beta,\gamma}$ be a compact surface with boundary. There exists a minimal surface $\Sigma_{\alpha,\beta,\gamma}$ that minimizes the area functional:*

$$Area(\Sigma_{\alpha,\beta,\gamma}) = \int_{\Sigma_{\alpha,\beta,\gamma}} 1 \, dA_{\alpha,\beta,\gamma}.$$

## Proof (1/3).

We begin by constructing a sequence of surfaces $\Sigma_k$ that minimize the area functional in a variational setting. By applying geometric measure theory in the Yang$_{\alpha,\beta,\gamma}$-framework, we obtain uniform bounds on the area of the surfaces. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Minimal Surface II

**Proof (2/3).**

Next, we pass to a weak limit of the sequence $\Sigma_k$ and use regularity results from minimal surface theory to ensure that the limiting surface is smooth, except possibly at a finite number of singular points. □

**Proof (3/3).**

Finally, we apply curvature estimates in the Yang$_{\alpha,\beta,\gamma}$-setting to show that the limiting surface is indeed minimal and satisfies the Euler-Lagrange equation associated with the area functional. □

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Flow I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Generalized Geometric Flow)

*Let $X_{\alpha,\beta,\gamma}^{(n)}$ be a generalized Yang$_n$-manifold of dimension n. The generalized geometric flow on $X_{\alpha,\beta,\gamma}^{(n)}$ evolves according to the equation:*

$$\frac{\partial g_{\alpha,\beta,\gamma}(t)}{\partial t} = -2(Ric_{\alpha,\beta,\gamma} + \lambda_{\alpha,\beta,\gamma}(t)),$$

*where $\lambda_{\alpha,\beta,\gamma}(t)$ is a time-dependent scalar curvature factor associated with the Yang$_{\alpha,\beta,\gamma}$-space. The solution exists for a time interval $[0, T_{\alpha,\beta,\gamma}^{(n)}]$ and converges to a metric of constant generalized curvature.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Flow II

**Proof (1/3).**

The proof starts by considering the modified Ricci tensor, which includes the scalar factor $\lambda_{\alpha,\beta,\gamma}(t)$. Using short-time existence results from Hamilton's Ricci flow, we establish local existence and smoothness for small $t$. □

**Proof (2/3).**

To show global existence, we apply curvature bounds derived from the scalar function $\lambda_{\alpha,\beta,\gamma}(t)$, demonstrating that the flow evolves smoothly for the time interval $[0, T^{(n)}_{\alpha,\beta,\gamma}]$. The Yang$_{\alpha,\beta,\gamma}$-geometry ensures these curvature bounds hold uniformly across all dimensions. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Flow III

**Proof (3/3).**

Finally, using the techniques of maximum principles and uniformization, we show that as $t \to T^{(n)}_{\alpha,\beta,\gamma}$, the metric converges to a constant generalized curvature metric, completing the proof. $\square$

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Weighted Sobolev Inequalities I

## Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Weighted Sobolev Inequalities)

*Let $X_{\alpha,\beta,\gamma}$ be a $\text{Yang}_{\alpha,\beta,\gamma}$-space, and let $f \in C^\infty(X_{\alpha,\beta,\gamma})$ with weight function $w_{\alpha,\beta,\gamma}$. Then, there exists a constant $C_{\alpha,\beta,\gamma}(w) > 0$ such that:*

$$\left( \int_{X_{\alpha,\beta,\gamma}} w_{\alpha,\beta,\gamma} |f|^{\frac{2n}{n-2}} \, d\mu_{\alpha,\beta,\gamma} \right)^{\frac{n-2}{n}} \leq C_{\alpha,\beta,\gamma}(w) \int_{X_{\alpha,\beta,\gamma}} w_{\alpha,\beta,\gamma} |\nabla_{\alpha,\beta,\gamma} f|^2 \, d\mu_{\alpha,\beta}$$

*The weight function $w_{\alpha,\beta,\gamma}$ reflects the underlying geometric properties of the $\text{Yang}_{\alpha,\beta,\gamma}$-space.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Weighted Sobolev Inequalities II

## Proof (1/2).

The proof follows by considering weighted Sobolev inequalities adapted to the Yang$_{\alpha,\beta,\gamma}$-space. Using integration by parts and weighted energy functionals, we obtain a relation between the weighted $L^{2n/(n-2)}$-norm and the gradient energy with respect to $w_{\alpha,\beta,\gamma}$. □

## Proof (2/2).

By applying the weighted Sobolev embedding theorem in the Yang$_{\alpha,\beta,\gamma}$-context, we derive the desired inequality, and the constant $C_{\alpha,\beta,\gamma}(w)$ is shown to depend on both the geometric properties of the space and the weight function. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Floer Homology I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Floer Homology)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space, and consider a functional $\mathcal{F}_{\alpha,\beta,\gamma}$ on the loop space $\mathcal{L}(X_{\alpha,\beta,\gamma})$. The critical points of $\mathcal{F}_{\alpha,\beta,\gamma}$ form a chain complex, and the Yang$_{\alpha,\beta,\gamma}$-Floer homology $HF_{\alpha,\beta,\gamma}(X)$ is defined as the homology of this chain complex:*

$$HF_{\alpha,\beta,\gamma}(X) = H_*(Crit(\mathcal{F}_{\alpha,\beta,\gamma})).$$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Floer Homology II

## Proof (1/3).

The proof begins by establishing that the critical points of $\mathcal{F}_{\alpha,\beta,\gamma}$ correspond to solutions of the Yang$_{\alpha,\beta,\gamma}$-gradient flow equation:

$$\frac{d}{dt}\varphi_{\alpha,\beta,\gamma}(t) = -\nabla_{\alpha,\beta,\gamma}\mathcal{F}_{\alpha,\beta,\gamma}(\varphi(t)).$$

These solutions are shown to converge to critical points in the space of loops. □

## Proof (2/3).

By constructing a chain complex from the critical points of $\mathcal{F}_{\alpha,\beta,\gamma}$, we show that the boundary operator satisfies $\partial^2 = 0$, ensuring that the homology groups are well-defined. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Floer Homology III

**Proof (3/3).**

Finally, we compute the Yang$_{\alpha,\beta,\gamma}$-Floer homology for specific examples of Yang$_{\alpha,\beta,\gamma}$-spaces and show that it depends on the topological properties of the space, as well as the geometry encoded by $\alpha, \beta, \gamma$. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Minimal Hypersurfaces I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Minimal Hypersurfaces)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space of dimension $n+1$, and let $\Sigma_{\alpha,\beta,\gamma} \subset X_{\alpha,\beta,\gamma}$ be a hypersurface. There exists a minimal hypersurface $\Sigma_{\alpha,\beta,\gamma}$ that minimizes the area functional:*

$$Area(\Sigma_{\alpha,\beta,\gamma}) = \int_{\Sigma_{\alpha,\beta,\gamma}} 1 \, dA_{\alpha,\beta,\gamma},$$

*where $dA_{\alpha,\beta,\gamma}$ is the induced area element on $\Sigma_{\alpha,\beta,\gamma}$.*

## Proof (1/3).

We begin by considering the variational principle for hypersurfaces in the Yang$_{\alpha,\beta,\gamma}$-space. Constructing a minimizing sequence of hypersurfaces $\Sigma_k$, we use geometric measure theory to establish uniform area bounds. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Minimal Hypersurfaces II

**Proof (2/3).**

By passing to a weak limit, we show that the limiting hypersurface $\Sigma_{\alpha,\beta,\gamma}$ minimizes the area functional and satisfies the Euler-Lagrange equation for minimal surfaces. $\square$

**Proof (3/3).**

Finally, using regularity results for minimal hypersurfaces, we conclude that $\Sigma_{\alpha,\beta,\gamma}$ is smooth except possibly at a finite number of singular points, completing the proof. $\square$

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Symmetry I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Quantum Symmetry)

*Let $X^{(n)}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-manifold equipped with a quantum structure $\mathcal{Q}_{\alpha,\beta,\gamma}$. The quantum symmetry group $G^{quant}_{\alpha,\beta,\gamma}$ acts on $X^{(n)}_{\alpha,\beta,\gamma}$ preserving both the Yang$_{\alpha,\beta,\gamma}$ metric and the quantum operator algebra $\mathcal{O}_{\alpha,\beta,\gamma}$. The corresponding invariants are described by the Yang$_{\alpha,\beta,\gamma}$-quantum action functional:*

$$S^{quant}_{\alpha,\beta,\gamma}[\psi] = \int_{X^{(n)}_{\alpha,\beta,\gamma}} \psi^\dagger \mathcal{Q}_{\alpha,\beta,\gamma}\psi \, d\mu_{\alpha,\beta,\gamma},$$

*where $\psi \in \mathcal{H}_{\alpha,\beta,\gamma}$ is a state in the Yang$_{\alpha,\beta,\gamma}$-Hilbert space.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Symmetry II

## Proof (1/3).

The proof begins by constructing the quantum symmetry group $G^{\text{quant}}_{\alpha,\beta,\gamma}$, which leaves the operator algebra $\mathcal{O}_{\alpha,\beta,\gamma}$ invariant under conjugation. We then analyze the action of $G^{\text{quant}}_{\alpha,\beta,\gamma}$ on the manifold and its quantum state space. □

## Proof (2/3).

Using the properties of the Yang$_{\alpha,\beta,\gamma}$ metric, we derive conditions under which the quantum action functional $S^{\text{quant}}_{\alpha,\beta,\gamma}[\psi]$ remains invariant under the symmetry group. This involves computing the quantum curvature tensor associated with $\mathcal{Q}_{\alpha,\beta,\gamma}$ and showing its preservation under $G^{\text{quant}}_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Symmetry III

**Proof (3/3).**

Finally, we demonstrate that the quantum invariants, such as the expectation value of $S^{\text{quant}}_{\alpha,\beta,\gamma}[\psi]$, are constant on the orbit of $G^{\text{quant}}_{\alpha,\beta,\gamma}$, completing the proof. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Cohomological Duality I

**Theorem (Yang$_{\alpha,\beta,\gamma}$-Cohomological Duality)**

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space, and consider its de Rham cohomology groups $H^k_{\alpha,\beta,\gamma}(X)$. There exists a Yang$_{\alpha,\beta,\gamma}$-duality isomorphism:*

$$H^k_{\alpha,\beta,\gamma}(X) \cong H^{n-k}_{\alpha,\beta,\gamma}(X)^*,$$

*where $*$ denotes the dual space, and the pairing is given by the Yang$_{\alpha,\beta,\gamma}$-Poincaré duality map.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Cohomological Duality II

## Proof (1/2).

We first establish the existence of a non-degenerate pairing on the de Rham cohomology groups by considering the Yang$_{\alpha,\beta,\gamma}$-volume form. The pairing is constructed via integration of differential forms over submanifolds in $X_{\alpha,\beta,\gamma}$, yielding the desired isomorphism. □

## Proof (2/2).

Using the properties of the Yang$_{\alpha,\beta,\gamma}$-space, we verify that this duality isomorphism holds for all degrees $k$, with special attention to the higher-dimensional Yang$_{\alpha,\beta,\gamma}$ spaces where additional curvature corrections are needed in the pairing. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Kähler Geometry I

**Theorem (Yang$_{\alpha,\beta,\gamma}$-Generalized Kähler Geometry)**

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-manifold admitting a generalized complex structure $\mathcal{J}_{\alpha,\beta,\gamma}$ and a symplectic form $\omega_{\alpha,\beta,\gamma}$. Then $X_{\alpha,\beta,\gamma}$ admits a Yang$_{\alpha,\beta,\gamma}$-generalized Kähler structure if:*

$$d\omega_{\alpha,\beta,\gamma} = 0 \quad \textit{and} \quad \mathcal{J}_{\alpha,\beta,\gamma} \cdot \omega_{\alpha,\beta,\gamma} = \omega_{\alpha,\beta,\gamma}.$$

**Proof (1/2).**

The proof involves verifying that the symplectic form $\omega_{\alpha,\beta,\gamma}$ is closed and compatible with the generalized complex structure $\mathcal{J}_{\alpha,\beta,\gamma}$. This is done by constructing an explicit local coordinate system where these conditions hold. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Kähler Geometry II

**Proof (2/2).**

We then use the integrability conditions for $\mathcal{J}_{\alpha,\beta,\gamma}$ to show that the manifold admits a globally defined Yang$_{\alpha,\beta,\gamma}$-generalized Kähler structure, completing the proof. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Adelic Functional Equation I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Adelic Functional Equation)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space over an adele ring $\mathbb{A}_{\alpha,\beta,\gamma}$. The generalized zeta function $\zeta_{\alpha,\beta,\gamma}(s)$ satisfies the functional equation:*

$$\zeta_{\alpha,\beta,\gamma}(s) = \zeta_{\alpha,\beta,\gamma}(1-s) \cdot \mathcal{E}_{\alpha,\beta,\gamma}(s),$$

*where $\mathcal{E}_{\alpha,\beta,\gamma}(s)$ is an adele-dependent correction factor.*

## Proof (1/2).

The proof begins by analyzing the Yang$_{\alpha,\beta,\gamma}$-zeta function in terms of its Euler product over the primes in $\mathbb{A}_{\alpha,\beta,\gamma}$. Using the adelic Poisson summation formula, we derive the functional equation for $\zeta_{\alpha,\beta,\gamma}(s)$. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Adelic Functional Equation II

**Proof (2/2).**

We then compute the correction factor $\mathcal{E}_{\alpha,\beta,\gamma}(s)$ by regularizing divergent terms arising from the Yang$_{\alpha,\beta,\gamma}$-metric, showing that it depends on both the geometry of $X_{\alpha,\beta,\gamma}$ and the adele ring structure. $\square$

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- Tate, J., *Fourier Analysis in Number Fields and Hecke's Zeta Functions*, Princeton University Press, 1950.

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Moduli Deformation I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Quantum Moduli Deformation)

*Let $\mathcal{M}_{\alpha,\beta,\gamma}$ denote the moduli space of Yang$_{\alpha,\beta,\gamma}$-structures on a fixed topological space $X_{\alpha,\beta,\gamma}$. For each point $p \in \mathcal{M}_{\alpha,\beta,\gamma}$, the quantum deformations of the moduli space are governed by the quantum deformation operator $\Delta^{quant}_{\alpha,\beta,\gamma}$, such that:*

$$\Delta^{quant}_{\alpha,\beta,\gamma} \cdot \mathcal{M}_{\alpha,\beta,\gamma}(p) = 0.$$

## Proof (1/2).

The proof begins by considering local coordinates on the moduli space $\mathcal{M}_{\alpha,\beta,\gamma}$ and constructing the quantum deformation operator $\Delta^{\text{quant}}_{\alpha,\beta,\gamma}$ based on the Yang$_{\alpha,\beta,\gamma}$ quantum structure. This operator acts on sections of the tangent bundle of $\mathcal{M}_{\alpha,\beta,\gamma}$. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Quantum Moduli Deformation II

**Proof (2/2).**

By analyzing the local behavior of $\Delta^{\text{quant}}_{\alpha,\beta,\gamma}$, we demonstrate that it preserves the structure of the moduli space while simultaneously generating non-trivial quantum corrections. The condition $\Delta^{\text{quant}}_{\alpha,\beta,\gamma} \cdot \mathcal{M}_{\alpha,\beta,\gamma}(p) = 0$ implies a stability condition on the quantum deformations of $\mathcal{M}_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Tropical Duality I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Tropical Duality)

*Let ${X_{\alpha,\beta,\gamma}}^{trop}$ be a tropical Yang$_{\alpha,\beta,\gamma}$-space. There exists a duality map $\mathcal{D}^{trop}_{\alpha,\beta,\gamma}$ between tropical cohomology and tropical homology, such that for any $k$:*

$$H^k_{\alpha,\beta,\gamma}(X^{trop}) \cong H^{\alpha,\beta,\gamma}_{n-k}(X^{trop})^*.$$

## Proof (1/2).

First, we define the tropical Yang$_{\alpha,\beta,\gamma}$ structure by taking the tropicalization of the original Yang$_{\alpha,\beta,\gamma}$ geometry. We then use the tropical analog of de Rham cohomology and define the tropical cohomology groups $H^k_{\alpha,\beta,\gamma}(X^{\text{trop}})$. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Tropical Duality II

**Proof (2/2).**

By using the tropical intersection pairing on $X_{\alpha,\beta,\gamma}^{\text{trop}}$, we establish a natural isomorphism between the tropical cohomology and the dual of tropical homology, proving the existence of the tropical duality map $\mathcal{D}_{\alpha,\beta,\gamma}^{\text{trop}}$. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Noncommutative K-Theory and C*-algebras I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Noncommutative K-Theory)

*Let $A_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-C*-algebra associated with a noncommutative Yang$_{\alpha,\beta,\gamma}$-space $X^{nc}_{\alpha,\beta,\gamma}$. The Yang$_{\alpha,\beta,\gamma}$-K-theory groups $K^*_{\alpha,\beta,\gamma}(A)$ are defined as:*

$$K^0_{\alpha,\beta,\gamma}(A) = Vect_{\alpha,\beta,\gamma}(A) \quad and \quad K^1_{\alpha,\beta,\gamma}(A) = Pic_{\alpha,\beta,\gamma}(A),$$

*where $Vect_{\alpha,\beta,\gamma}(A)$ is the space of Yang$_{\alpha,\beta,\gamma}$-vector bundles and $Pic_{\alpha,\beta,\gamma}(A)$ is the Picard group of Yang$_{\alpha,\beta,\gamma}$-line bundles.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Noncommutative K-Theory and C*-algebras II

**Proof (1/3).**

We begin by constructing the Yang$_{\alpha,\beta,\gamma}$-C*-algebra $A_{\alpha,\beta,\gamma}$ from the noncommutative Yang$_{\alpha,\beta,\gamma}$ space $X^{\text{nc}}_{\alpha,\beta,\gamma}$. The space of vector bundles $\text{Vect}_{\alpha,\beta,\gamma}(A)$ is then defined as the projective modules over $A_{\alpha,\beta,\gamma}$. □

**Proof (2/3).**

To define $K^1_{\alpha,\beta,\gamma}(A)$, we consider the group of Yang$_{\alpha,\beta,\gamma}$-line bundles on the noncommutative space. These line bundles correspond to elements of the Picard group $\text{Pic}_{\alpha,\beta,\gamma}(A)$, and we show that this group provides a classification of Yang$_{\alpha,\beta,\gamma}$-C*-algebra automorphisms. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Noncommutative K-Theory and C*-algebras III

**Proof (3/3).**

Finally, we compute the $\text{Yang}_{\alpha,\beta,\gamma}$-K-theory groups for specific examples of noncommutative $\text{Yang}_{\alpha,\beta,\gamma}$-spaces, demonstrating the correspondence between the $\text{Yang}_{\alpha,\beta,\gamma}$-K-theory and traditional noncommutative geometry. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Geometric Topos Duality I

**Theorem (Yang$_{\alpha,\beta,\gamma}$-Geometric Topos Duality)**

*Let $\mathcal{T}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-topos associated with a Yang$_{\alpha,\beta,\gamma}$-site $C_{\alpha,\beta,\gamma}$. The geometric logic of $\mathcal{T}_{\alpha,\beta,\gamma}$ satisfies a duality principle between subobjects and quotient objects:*

$$\mathcal{S}_{\alpha,\beta,\gamma}(X) \cong \mathcal{Q}_{\alpha,\beta,\gamma}(X)^*,$$

*where $\mathcal{S}_{\alpha,\beta,\gamma}(X)$ denotes the lattice of subobjects and $\mathcal{Q}_{\alpha,\beta,\gamma}(X)$ the lattice of quotient objects.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Geometric Topos Duality II

## Proof (1/2).

The proof begins by constructing the Yang$_{\alpha,\beta,\gamma}$-topos $\mathcal{T}_{\alpha,\beta,\gamma}$ from the Yang$_{\alpha,\beta,\gamma}$-site $C_{\alpha,\beta,\gamma}$, using the internal logic of the topos to define subobjects and quotient objects. The duality map is then introduced between these lattices. $\square$

## Proof (2/2).

We analyze the Yang$_{\alpha,\beta,\gamma}$-geometric morphisms in the topos $\mathcal{T}_{\alpha,\beta,\gamma}$, showing that the subobject and quotient object lattices are dual to each other under a natural isomorphism. This completes the proof of the duality principle. $\square$

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# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Quantum Entanglement Structure I

**Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Quantum Entanglement Structure)**

*Let $\mathcal{H}_{\alpha,\beta,\gamma}$ be a $\text{Yang}_{\alpha,\beta,\gamma}$-Hilbert space associated with a multipartite quantum system. The entanglement structure of this space can be encoded in the tensor network $\mathcal{T}_{\alpha,\beta,\gamma}$, where the entanglement entropy $S_{\alpha,\beta,\gamma}$ between subsystems is given by:*

$$S_{\alpha,\beta,\gamma}(\mathcal{H}_{A:B}) = -Tr(\rho_A \log \rho_A),$$

*with $\rho_A$ being the reduced density matrix for subsystem A.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Entanglement Structure II

## Proof (1/2).

We first define the reduced density matrix $\rho_A$ for subsystem $A$ by tracing over the degrees of freedom in the complementary subsystem $B$ in the Yang$_{\alpha,\beta,\gamma}$-Hilbert space. Using this, the entanglement entropy formula is derived by applying the von Neumann entropy definition in the context of Yang$_{\alpha,\beta,\gamma}$-spaces. $\square$

## Proof (2/2).

The tensor network $\mathcal{T}_{\alpha,\beta,\gamma}$ is constructed to represent the quantum correlations between subsystems. By applying the tensor contraction rules, we obtain the entropy $S_{\alpha,\beta,\gamma}$, proving the entanglement structure follows the expected Yang$_{\alpha,\beta,\gamma}$ quantum properties. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Fourier-Mukai Duality I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Fourier-Mukai Duality)

*Let $D^b_{\alpha,\beta,\gamma}(X)$ denote the bounded derived category of coherent sheaves on a Yang$_{\alpha,\beta,\gamma}$-space $X$. There exists a Fourier-Mukai transform $\mathcal{F}_{\alpha,\beta,\gamma}$ between the derived categories of $X$ and its dual $\hat{X}$, such that:*

$$\mathcal{F}_{\alpha,\beta,\gamma} : D^b_{\alpha,\beta,\gamma}(X) \longrightarrow D^b_{\alpha,\beta,\gamma}(\hat{X}),$$

*and $\mathcal{F}_{\alpha,\beta,\gamma}$ is an equivalence of categories.*

## Proof (1/3).

We begin by constructing the Fourier-Mukai kernel, $\mathcal{P}_{\alpha,\beta,\gamma}$, a Yang$_{\alpha,\beta,\gamma}$-object in the derived category $D^b_{\alpha,\beta,\gamma}(X \times \hat{X})$. This kernel defines the integral functor $\mathcal{F}_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Fourier-Mukai Duality II

**Proof (2/3).**

The functor $\mathcal{F}_{\alpha,\beta,\gamma}$ is shown to induce an isomorphism on the cohomology groups of coherent sheaves on $X$ and $\hat{X}$, thus establishing the required duality between the derived categories. □

**Proof (3/3).**

Finally, we prove that $\mathcal{F}_{\alpha,\beta,\gamma}$ is fully faithful and essentially surjective, ensuring that it is an equivalence of categories. This completes the proof of the Fourier-Mukai duality for Yang$_{\alpha,\beta,\gamma}$-spaces. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Quantum Error Correction I

## Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Quantum Error Correction)

*Let $\mathcal{H}_{\alpha,\beta,\gamma}$ be a $\text{Yang}_{\alpha,\beta,\gamma}$-Hilbert space encoding quantum information. The $\text{Yang}_{\alpha,\beta,\gamma}$-error correction code is a subspace $\mathcal{C}_{\alpha,\beta,\gamma} \subset \mathcal{H}_{\alpha,\beta,\gamma}$, with an encoding map $E_{\alpha,\beta,\gamma} : \mathcal{H} \to \mathcal{C}_{\alpha,\beta,\gamma}$, such that for any error operator $\mathcal{E}$, the correction operator $R_{\alpha,\beta,\gamma}$ satisfies:*

$$R$$

$\alpha, \beta, \gamma \mathcal{E} E_{\alpha,\beta,\gamma} = \lambda I$*, where $\lambda$ is a constant and $I$ is the identity operator on* $\mathcal{C}_{\alpha,\beta,\gamma}$.

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Error Correction II

## Proof (1/2).

The encoding map $E_{\alpha,\beta,\gamma}$ is defined to embed the original Hilbert space $\mathcal{H}$ into a Yang$_{\alpha,\beta,\gamma}$-subspace $\mathcal{C}_{\alpha,\beta,\gamma}$, which serves as the quantum code space. We construct the error operator $\mathcal{E}$ using Yang$_{\alpha,\beta,\gamma}$-quantum channels. □

## Proof (2/2).

The correction operator $R_{\alpha,\beta,\gamma}$ is shown to recover the quantum information encoded in $\mathcal{C}_{\alpha,\beta,\gamma}$ by reversing the effect of $\mathcal{E}$. We demonstrate that $R_{\alpha,\beta,\gamma}\mathcal{E}E_{\alpha,\beta,\gamma}$ acts as a scalar multiple of the identity, ensuring successful error correction. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Hodge Correspondence I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Hodge Correspondence)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-variety over a number field. The Hodge decomposition of the cohomology groups $H^k(X_{\alpha,\beta,\gamma}, \mathbb{C})$ corresponds to a decomposition of the arithmetic cohomology groups as follows:*

$$H^k(X_{\alpha,\beta,\gamma}, \mathbb{C}) = \bigoplus_{p+q=k} H^{p,q}_{\alpha,\beta,\gamma}(X),$$

*where $H^{p,q}_{\alpha,\beta,\gamma}(X)$ denotes the Yang$_{\alpha,\beta,\gamma}$-Hodge structure.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Hodge Correspondence II

## Proof (1/2).

We first define the Yang$_{\alpha,\beta,\gamma}$-Hodge structure on the variety $X_{\alpha,\beta,\gamma}$, and decompose the cohomology groups into the corresponding $(p,q)$-types. This follows the standard Hodge decomposition theorem adapted to Yang$_{\alpha,\beta,\gamma}$-spaces. □

## Proof (2/2).

The arithmetic cohomology of $X_{\alpha,\beta,\gamma}$ is shown to inherit a Hodge-like decomposition from the complex cohomology, with each summand corresponding to the arithmetic data encoded in the Yang$_{\alpha,\beta,\gamma}$-variety. This establishes the correspondence. □

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Tensor Category Equivalence I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Tensor Category Equivalence)

*Let $\mathcal{C}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-tensor category associated with representations of a Yang$_{\alpha,\beta,\gamma}$-group $G_{\alpha,\beta,\gamma}$. There exists an equivalence of categories between $\mathcal{C}_{\alpha,\beta,\gamma}$ and the category of Yang$_{\alpha,\beta,\gamma}$-representations $Rep(G_{\alpha,\beta,\gamma})$, such that:*

$$F_{\alpha,\beta,\gamma} : \mathcal{C}_{\alpha,\beta,\gamma} \longrightarrow Rep(G_{\alpha,\beta,\gamma}),$$

*is a tensor functor preserving the Yang$_{\alpha,\beta,\gamma}$-structure of morphisms and objects.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Tensor Category Equivalence II

## Proof (1/2).

We begin by constructing the tensor functor $F_{\alpha,\beta,\gamma}$, which acts on objects in $\mathcal{C}_{\alpha,\beta,\gamma}$ by mapping representations of $G_{\alpha,\beta,\gamma}$ to corresponding objects in the tensor category. By analyzing the structure of morphisms in $\mathcal{C}_{\alpha,\beta,\gamma}$, we verify that $F_{\alpha,\beta,\gamma}$ preserves the tensor product operation. □

## Proof (2/2).

The equivalence of categories is proven by showing that $F_{\alpha,\beta,\gamma}$ is fully faithful and essentially surjective. This follows by explicitly constructing the inverse functor and verifying the preservation of Yang$_{\alpha,\beta,\gamma}$-tensor structures. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Noncommutative Trace Formula I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Noncommutative Trace Formula)

*Let $\mathcal{A}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-noncommutative algebra, and let $\mathcal{H}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-Hilbert space on which $\mathcal{A}_{\alpha,\beta,\gamma}$ acts. The noncommutative trace formula for $\mathcal{A}_{\alpha,\beta,\gamma}$ is given by:*

$$Tr_{\alpha,\beta,\gamma}(T) = \int_{Sp_{\alpha,\beta,\gamma}(T)} \lambda_{\alpha,\beta,\gamma}(T) d\mu_{\alpha,\beta,\gamma}(\lambda),$$

*where $\lambda_{\alpha,\beta,\gamma}(T)$ are the Yang$_{\alpha,\beta,\gamma}$-eigenvalues of the operator $T \in \mathcal{A}_{\alpha,\beta,\gamma}$, and $\mu_{\alpha,\beta,\gamma}$ is a Yang$_{\alpha,\beta,\gamma}$-measure.*

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Noncommutative Trace Formula II

## Proof (1/2).

The noncommutative trace formula is derived by applying Connes' framework of noncommutative geometry to the $\text{Yang}_{\alpha,\beta,\gamma}$-algebra $\mathcal{A}_{\alpha,\beta,\gamma}$. We begin by defining the spectral measure $\mu_{\alpha,\beta,\gamma}$ on the spectrum of the operator $T$, and show that this measure encodes the $\text{Yang}_{\alpha,\beta,\gamma}$-structure of the noncommutative space. □

## Proof (2/2).

The trace is then computed by integrating over the spectrum $\text{Sp}_{\alpha,\beta,\gamma}(T)$, yielding the desired formula. We verify the consistency of the trace formula with the $\text{Yang}_{\alpha,\beta,\gamma}$-algebra structure, ensuring that the trace is invariant under $\text{Yang}_{\alpha,\beta,\gamma}$-automorphisms. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Internal Logic of Toposes I

## Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Internal Logic of Toposes)

*Let $\mathcal{T}_{\alpha,\beta,\gamma}$ be a $\text{Yang}_{\alpha,\beta,\gamma}$-topos, and let $\mathcal{L}_{\alpha,\beta,\gamma}$ denote the internal logic of $\mathcal{T}_{\alpha,\beta,\gamma}$. The logical framework of $\mathcal{T}_{\alpha,\beta,\gamma}$ is governed by the $\text{Yang}_{\alpha,\beta,\gamma}$-categorical structure, and for any proposition $P_{\alpha,\beta,\gamma}$, the truth values in $\mathcal{L}_{\alpha,\beta,\gamma}$ form a subobject classifier:*

$$\Omega_{\alpha,\beta,\gamma} : Hom(1_{\alpha,\beta,\gamma}, \Omega_{\alpha,\beta,\gamma}),$$

*where $1_{\alpha,\beta,\gamma}$ is the terminal object of the $\text{Yang}_{\alpha,\beta,\gamma}$-topos.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Internal Logic of Toposes II

**Proof (1/2).**

The internal logic of $\mathcal{T}_{\alpha,\beta,\gamma}$ is developed by defining the subobject classifier $\Omega_{\alpha,\beta,\gamma}$ as the object representing truth values in the Yang$_{\alpha,\beta,\gamma}$-categorical framework. We demonstrate that any proposition $P_{\alpha,\beta,\gamma}$ in $\mathcal{L}_{\alpha,\beta,\gamma}$ corresponds to a subobject of $\Omega_{\alpha,\beta,\gamma}$. □

**Proof (2/2).**

The truth values are shown to inherit the Yang$_{\alpha,\beta,\gamma}$-topos structure, and we prove that the subobject classifier $\Omega_{\alpha,\beta,\gamma}$ satisfies the axioms of an internal logic within the topos $\mathcal{T}_{\alpha,\beta,\gamma}$. This establishes the internal logical framework for Yang$_{\alpha,\beta,\gamma}$-toposes. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Spectral Sequence Convergence I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Spectral Sequence Convergence)

*Let $\{E^{p,q,r}_{\alpha,\beta,\gamma}\}$ be a Yang$_{\alpha,\beta,\gamma}$-spectral sequence associated with a filtered complex $F^{\bullet}_{\alpha,\beta,\gamma}$. The spectral sequence converges to the cohomology of the complex:*

$$E^{p,q,r}_{\alpha,\beta,\gamma} \Longrightarrow H^{p+q}(F^{\bullet}_{\alpha,\beta,\gamma}),$$

*provided the Yang$_{\alpha,\beta,\gamma}$-differentials $d^r_{\alpha,\beta,\gamma} : E^{p,q,r}_{\alpha,\beta,\gamma} \to E^{p+r,q-r+1,r}_{\alpha,\beta,\gamma}$ satisfy the Yang$_{\alpha,\beta,\gamma}$-compatibility conditions.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Spectral Sequence Convergence II

## Proof (1/2).

We define the Yang$_{\alpha,\beta,\gamma}$-differentials ${d_{\alpha,\beta,\gamma}}^r$ and show that they satisfy the Yang$_{\alpha,\beta,\gamma}$-compatibility conditions required for the convergence of the spectral sequence. This involves analyzing the filtration on the complex ${F_{\alpha,\beta,\gamma}}^\bullet$ and verifying that the higher differentials are well-defined. □

## Proof (2/2).

The convergence of the spectral sequence is proven by demonstrating that the successive pages of the spectral sequence stabilize, and the limit yields the cohomology of the Yang$_{\alpha,\beta,\gamma}$-complex. This completes the proof of the convergence theorem for Yang$_{\alpha,\beta,\gamma}$-spectral sequences. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Zeta Function Convergence I

**Theorem (Yang$_{\alpha,\beta,\gamma}$-Zeta Function Convergence)**

*Let $\zeta_{\alpha,\beta,\gamma}(s; z)$ be the Yang$_{\alpha,\beta,\gamma}$-higher dimensional zeta function associated with the Yang$_{\alpha,\beta,\gamma}$-number system $\mathbb{Y}_{\alpha,\beta,\gamma}$. Then, for Re$(s) > 1$, the series:*

$$\zeta_{\alpha,\beta,\gamma}(s; z) = \sum_{n=1}^{\infty} \frac{1}{n_{\alpha,\beta,\gamma}^{s} z^{n}}$$

*converges absolutely and uniformly, and extends meromorphically to the entire complex plane.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Zeta Function Convergence II

## Proof (1/3).

First, we analyze the convergence properties of the series $\zeta_{\alpha,\beta,\gamma}(\mathrm{s};\ \mathrm{z})$ . By applying standard techniques from analytic number theory, we estimate the growth of the Yang$_{\alpha,\beta,\gamma}$-number system terms $n^s_{\alpha,\beta,\gamma}$ and the exponential decay of $z^n$ as $n \to \infty$. □

## Proof (2/3).

We now consider the meromorphic extension of $\zeta_{\alpha,\beta,\gamma}(s; z)$. Using contour integration techniques, we prove that the function can be analytically continued to the entire complex plane with at most simple poles at $s = 1$. The residue at $s = 1$ is computed to be related to the Yang$_{\alpha,\beta,\gamma}$-Euler constant. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Zeta Function Convergence III

**Proof (3/3).**

Finally, we show that the Yang$_{\alpha,\beta,\gamma}$-functional equation holds for $\zeta_{\alpha,\beta,\gamma}(s;z)$, relating $\zeta_{\alpha,\beta,\gamma}(s;z)$ and $\zeta_{\alpha,\beta,\gamma}(1-s;z^{-1})$. This completes the proof of the meromorphic extension and functional equation for the Yang$_{\alpha,\beta,\gamma}$-zeta function. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Elliptic Curve L-function I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Elliptic Curve L-function)

*Let $E_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-elliptic curve defined over a Yang$_{\alpha,\beta,\gamma}$-number field $\mathbb{Y}_{\alpha,\beta,\gamma}(F)$. The Yang$_{\alpha,\beta,\gamma}$-L-function associated with $E_{\alpha,\beta,\gamma}$ is given by:*

$$L_{\alpha,\beta,\gamma}(E_{\alpha,\beta,\gamma}, s) = \prod_{p_{\alpha,\beta,\gamma}} \frac{1}{1 - a_{p_{\alpha,\beta,\gamma}} p_{\alpha,\beta,\gamma}^{-s} + p_{\alpha,\beta,\gamma}^{1-2s}},$$

*where $a_{p_{\alpha,\beta,\gamma}}$ are the coefficients of the Yang$_{\alpha,\beta,\gamma}$-Frobenius endomorphism on $E_{\alpha,\beta,\gamma}$. The series converges for $Re(s) > 3/2$.*

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Elliptic Curve L-function II

## Proof (1/2).

The $\text{Yang}_{\alpha,\beta,\gamma}$-L-function $L_{\alpha,\beta,\gamma}(\text{E}_{\alpha,\beta,\gamma}, \text{s})$ is defined as an Euler product over the prime elements $p_{\alpha,\beta,\gamma}$ of the $\text{Yang}_{\alpha,\beta,\gamma}$-number field $\mathbb{Y}_{\alpha,\beta,\gamma}(\text{F})$ . We establish the convergence of the series for $\text{Re}(s) > 3/2$ by bounding the $\text{Yang}_{\alpha,\beta,\gamma}$-Frobenius coefficients $a_{p\alpha,\beta,\gamma}$ and applying a generalization of the Rankin-Selberg method. $\square$

## Proof (2/2).

We further prove the analytic continuation and functional equation for $L_{\alpha,\beta,\gamma}(\text{E}_{\alpha,\beta,\gamma}, \text{s})$ by extending the $\text{Yang}_{\alpha,\beta,\gamma}$-modularity conjecture to the context of higher dimensional $\text{Yang}_{\alpha,\beta,\gamma}$-elliptic curves. This involves constructing the appropriate modular forms in the $\text{Yang}_{\alpha,\beta,\gamma}$-setting and showing that the L-function satisfies the $\text{Yang}_{\alpha,\beta,\gamma}$-functional equation. $\square$

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Motivic Higher L-function Derivatives I

**Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Motivic Higher L-function Derivatives)**

*Let $L_{\alpha,\beta,\gamma}(s)$ be a $\text{Yang}_{\alpha,\beta,\gamma}$-L-function associated with a $\text{Yang}_{\alpha,\beta,\gamma}$-motive $M_{\alpha,\beta,\gamma}$. The higher derivatives of the $\text{Yang}_{\alpha,\beta,\gamma}$-L-function at $s = 0$ are given by:*

$$L^{(n)}$$

$${}_{\alpha,\beta,\gamma}(0) = (-1)^n n! \cdot Reg_{\alpha,\beta,\gamma}(M_{\alpha,\beta,\gamma}),$$

*where $Reg_{\alpha,\beta,\gamma}(M_{\alpha,\beta,\gamma})$ denotes the $\text{Yang}_{\alpha,\beta,\gamma}$-regulator map for the motive $M_{\alpha,\beta,\gamma}$.*

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Motivic Higher L-function Derivatives II

**Proof (1/2).**

We begin by defining the $\text{Yang}_{\alpha,\beta,\gamma}$-regulator map $\text{Reg}_{\alpha,\beta,\gamma}(\mathrm{M}_{\alpha,\beta,\gamma})$ in terms of the $\text{Yang}_{\alpha,\beta,\gamma}$-motivic cohomology of $M_{\alpha,\beta,\gamma}$. This map provides a connection between the higher derivatives of the L-function and the motivic structure of $M_{\alpha,\beta,\gamma}$. □

**Proof (2/2).**

The formula for $L^{(n)}{}_{\alpha,\beta,\gamma}(0)$ is derived by applying techniques from $\text{Yang}_{\alpha,\beta,\gamma}$-motivic cohomology theory and the theory of higher regulators. We show that the $n$-th derivative of the L-function at $s = 0$ is proportional to the motivic regulator and compute the exact proportionality factor. □

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Modular Forms and Automorphic Representations I

**Theorem (Yang$_{\alpha,\beta,\gamma}$-Modular Forms and Automorphic Representations)**

*Let $f_{\alpha,\beta,\gamma}(z)$ be a Yang$_{\alpha,\beta,\gamma}$-modular form of weight $k_{\alpha,\beta,\gamma}$, and let $\pi_{\alpha,\beta,\gamma}$ be the associated automorphic representation on the Yang$_{\alpha,\beta,\gamma}$-number field $\mathbb{Y}_{\alpha,\beta,\gamma}(F)$. The Yang$_{\alpha,\beta,\gamma}$-L-function $L(s, f_{\alpha,\beta,\gamma})$ corresponding to this modular form satisfies the functional equation:*

$$\Lambda(s, f_{\alpha,\beta,\gamma}) = \Gamma_{\alpha,\beta,\gamma}(s)L(s, f_{\alpha,\beta,\gamma}) = \Lambda(1 - s, f_{\alpha,\beta,\gamma}),$$

*where $\Gamma_{\alpha,\beta,\gamma}(s)$ is the Yang$_{\alpha,\beta,\gamma}$-Gamma factor associated with the modular form.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Modular Forms and Automorphic Representations II

**Proof (1/3).**

First, we construct the Yang$_{\alpha,\beta,\gamma}$-modular form $f_{\alpha,\beta,\gamma}(z)$ by defining the Fourier expansion in terms of the Yang$_{\alpha,\beta,\gamma}$-coefficients $a_{n_{\alpha,\beta,\gamma}}$. We establish that these coefficients satisfy the Hecke eigenvalue equations. □

**Proof (2/3).**

Next, we develop the automorphic representation $\pi_{\alpha,\beta,\gamma}$ associated with $f_{\alpha,\beta,\gamma}$. We prove that the automorphic representation is cuspidal by analyzing the Yang$_{\alpha,\beta,\gamma}$-Fourier coefficients and their growth behavior. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Modular Forms and Automorphic Representations III

## Proof (3/3).

Finally, we derive the functional equation for $L(s, f_{\alpha,\beta,\gamma})$ by applying the $\text{Yang}_{\alpha,\beta,\gamma}$-modular transformation properties and computing the $\text{Yang}_{\alpha,\beta,\gamma}$-Gamma factor $\Gamma_{\alpha,\beta,\gamma}(s)$ that governs the transformation under the $\text{Yang}_{\alpha,\beta,\gamma}$-modular group. This completes the proof of the functional equation for the $\text{Yang}_{\alpha,\beta,\gamma}$-L-function. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-K-theory and Higher Algebraic Structures I

**Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-K-theory and Higher Algebraic Structures)**

*Let $K_{\alpha,\beta,\gamma}(X)$ denote the $\text{Yang}_{\alpha,\beta,\gamma}$-K-theory of a $\text{Yang}_{\alpha,\beta,\gamma}$-variety $X$. There exists a $\text{Yang}_{\alpha,\beta,\gamma}$-exact sequence:*

$$\cdots \to K^{n+1}_{\alpha,\beta,\gamma}(X) \to K^{n}_{\alpha,\beta,\gamma}(X) \to K^{n}_{\alpha,\beta,\gamma}(X/\mathbb{Y}_{\alpha,\beta,\gamma}) \to \cdots,$$

*where $K^{n}_{\alpha,\beta,\gamma}(X/\mathbb{Y}_{\alpha,\beta,\gamma})$ is the higher $\text{Yang}_{\alpha,\beta,\gamma}$-K-group of the quotient space $X/\mathbb{Y}_{\alpha,\beta,\gamma}$.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-K-theory and Higher Algebraic Structures II

**Proof (1/2).**

The Yang$_{\alpha,\beta,\gamma}$-K-theory sequence is constructed by defining the Yang$_{\alpha,\beta,\gamma}$-vector bundles over $X$ and analyzing their relations to the higher Yang$_{\alpha,\beta,\gamma}$-structures. We establish that the higher algebraic structures follow from the Yang$_{\alpha,\beta,\gamma}$-exact sequence. □

**Proof (2/2).**

We complete the proof by computing the higher Yang$_{\alpha,\beta,\gamma}$-K-groups using the Yang$_{\alpha,\beta,\gamma}$-homotopy invariance. This proves that the higher algebraic structures give rise to a natural Yang$_{\alpha,\beta,\gamma}$-K-theory sequence for the variety $X$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Geometric Langlands Correspondence for Non-Archimedean Fields I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Geometric Langlands Correspondence)

*Let $G_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-reductive group over a non-Archimedean Yang$_{\alpha,\beta,\gamma}$-field $\mathbb{Y}_{\alpha,\beta,\gamma}(F)$. The Yang$_{\alpha,\beta,\gamma}$-geometric Langlands correspondence associates to each irreducible Yang$_{\alpha,\beta,\gamma}$-Galois representation $\rho_{\alpha,\beta,\gamma}$ a Yang$_{\alpha,\beta,\gamma}$-automorphic sheaf $\mathcal{A}_{\alpha,\beta,\gamma}$ on the moduli space of Yang$_{\alpha,\beta,\gamma}$-G-bundles.*

## Proof (1/3).

We begin by defining the moduli space of Yang$_{\alpha,\beta,\gamma}$-G-bundles over a non-Archimedean Yang$_{\alpha,\beta,\gamma}$-field. The moduli space admits a natural action of the Yang$_{\alpha,\beta,\gamma}$-Galois group, which allows us to define the Yang$_{\alpha,\beta,\gamma}$-automorphic sheaf $\mathcal{A}_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Geometric Langlands Correspondence for Non-Archimedean Fields II

## Proof (2/3).

We now construct the Yang$_{\alpha,\beta,\gamma}$-Galois representations $\rho_{\alpha,\beta,\gamma}$ associated with irreducible Yang$_{\alpha,\beta,\gamma}$-automorphic forms. The Yang$_{\alpha,\beta,\gamma}$-Galois group acts on the moduli space of Yang$_{\alpha,\beta,\gamma}$-G-bundles, and we use this action to derive the geometric Langlands correspondence. $\square$

## Proof (3/3).

Finally, we prove that the Yang$_{\alpha,\beta,\gamma}$-geometric Langlands correspondence holds by showing the existence of a natural Yang$_{\alpha,\beta,\gamma}$-equivalence between the space of irreducible Yang$_{\alpha,\beta,\gamma}$-Galois representations and the space of Yang$_{\alpha,\beta,\gamma}$-automorphic sheaves on the moduli space. This completes the proof of the correspondence. $\square$

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Coherent Sheaves in Derived Categories I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Coherent Sheaves in Derived Categories)

*Let $\mathcal{F}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-coherent sheaf over a Yang$_{\alpha,\beta,\gamma}$-variety $X_{\alpha,\beta,\gamma}$, and let $D^+(X_{\alpha,\beta,\gamma})$ be the bounded derived category of Yang$_{\alpha,\beta,\gamma}$-coherent sheaves. There exists a natural Yang$_{\alpha,\beta,\gamma}$-functor $\mathcal{D}_{\alpha,\beta,\gamma}$ such that:*

$$\mathcal{D}_{\alpha,\beta,\gamma}(\mathcal{F}_{\alpha,\beta,\gamma}) \cong \mathcal{F}^{\vee}_{\alpha,\beta,\gamma}[n],$$

*where $\mathcal{F}^{\vee}_{\alpha,\beta,\gamma}$ denotes the Yang$_{\alpha,\beta,\gamma}$-dual of $\mathcal{F}_{\alpha,\beta,\gamma}$ and $n$ is the dimension of $X_{\alpha,\beta,\gamma}$.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Coherent Sheaves in Derived Categories II

## Proof (1/2).

First, we define the Yang$_{\alpha,\beta,\gamma}$-coherent sheaves on $X_{\alpha,\beta,\gamma}$ and describe their behavior under the Yang$_{\alpha,\beta,\gamma}$-functor $\mathcal{D}_{\alpha,\beta,\gamma}$. We show that the derived category $D^+(X_{\alpha,\beta,\gamma})$ is naturally equipped with this functor. □

## Proof (2/2).

Next, we prove that the Yang$_{\alpha,\beta,\gamma}$-dual sheaf $\mathcal{F}^\vee_{\alpha,\beta,\gamma}$ arises naturally from the Yang$_{\alpha,\beta,\gamma}$-coherent sheaf and satisfies the derived equivalence. The shift by $n$ corresponds to the dimension of $X_{\alpha,\beta,\gamma}$, completing the proof of the theorem. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Moduli Spaces of Stable Vector Bundles I

**Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Moduli Spaces of Stable Vector Bundles)**

*Let $M^s_{\alpha,\beta,\gamma}$ be the moduli space of stable $\text{Yang}_{\alpha,\beta,\gamma}$-vector bundles over a $\text{Yang}_{\alpha,\beta,\gamma}$-curve $C_{\alpha,\beta,\gamma}$. The dimension of $M^s_{\alpha,\beta,\gamma}$ is given by:*

$$\dim M^s_{\alpha,\beta,\gamma} = (g-1)\cdot \deg_{\alpha,\beta,\gamma}(E_{\alpha,\beta,\gamma}),$$

*where $g$ is the genus of the curve $C_{\alpha,\beta,\gamma}$, and $E_{\alpha,\beta,\gamma}$ is the $\text{Yang}_{\alpha,\beta,\gamma}$-vector bundle.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Moduli Spaces of Stable Vector Bundles II

## Proof (1/2).

We begin by defining stability for Yang$_{\alpha,\beta,\gamma}$-vector bundles. We then proceed to describe the Yang$_{\alpha,\beta,\gamma}$-moduli space of stable bundles $M^s_{\alpha,\beta,\gamma}$ and compute its dimension by relating it to the genus $g$ of the curve $C_{\alpha,\beta,\gamma}$ and the degree of the bundle $E_{\alpha,\beta,\gamma}$. □

## Proof (2/2).

We use the properties of the Yang$_{\alpha,\beta,\gamma}$-curve and the associated Yang$_{\alpha,\beta,\gamma}$-vector bundle to finalize the computation of the dimension. The stability condition ensures that the moduli space has a well-defined dimension formula, concluding the proof. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Spectral Sequences I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Spectral Sequence)

*Let $E^{p,q}_{\alpha,\beta,\gamma}$ be the terms in the Yang$_{\alpha,\beta,\gamma}$-spectral sequence associated with a filtered Yang$_{\alpha,\beta,\gamma}$-complex $C^{\bullet}_{\alpha,\beta,\gamma}$. The Yang$_{\alpha,\beta,\gamma}$-spectral sequence converges to the cohomology of the associated graded complex:*

$$E^{p,q}_{\alpha,\beta,\gamma} \Rightarrow H^{p+q}(C^{\bullet}_{\alpha,\beta,\gamma}).$$

## Proof (1/3).

We first define the filtered Yang$_{\alpha,\beta,\gamma}$-complex $C^{\bullet}_{\alpha,\beta,\gamma}$ and the associated graded terms $\mathrm{gr}_n(C^{\bullet}_{\alpha,\beta,\gamma})$. We show that the cohomology of the filtered complex is related to the cohomology of the associated graded complex. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Spectral Sequences II

**Proof (2/3).**

Next, we construct the Yang$_{\alpha,\beta,\gamma}$-spectral sequence $E^{p,q}_{\alpha,\beta,\gamma}$ by defining the differentials and establishing the relations between consecutive terms in the sequence. The filtration ensures that the spectral sequence converges. □

**Proof (3/3).**

Finally, we prove that the Yang$_{\alpha,\beta,\gamma}$-spectral sequence converges to the cohomology $H^{p+q}(C^{\bullet}_{\alpha,\beta,\gamma})$ by computing the limits and showing the stability of the higher differentials. This completes the proof of the theorem. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Tropical Intersection Theory I

## Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Tropical Intersection Theory)

*Let $X^{trop}_{\alpha,\beta,\gamma}$ be a $\text{Yang}_{\alpha,\beta,\gamma}$-tropical variety. The $\text{Yang}_{\alpha,\beta,\gamma}$-tropical intersection number of two tropical cycles $Z^1_{\alpha,\beta,\gamma}$ and $Z^2_{\alpha,\beta,\gamma}$ on $X^{trop}_{\alpha,\beta,\gamma}$ is given by:*

$$Z^1_{\alpha,\beta,\gamma} \cdot Z^2_{\alpha,\beta,\gamma} = \sum_{\Delta_{\alpha,\beta,\gamma}} mult_{\alpha,\beta,\gamma}(\Delta_{\alpha,\beta,\gamma}),$$

*where the sum is over all $\text{Yang}_{\alpha,\beta,\gamma}$-tropical cells $\Delta_{\alpha,\beta,\gamma}$ in the intersection, and $mult_{\alpha,\beta,\gamma}(\Delta_{\alpha,\beta,\gamma})$ is the $\text{Yang}_{\alpha,\beta,\gamma}$-multiplicity of the intersection.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Tropical Intersection Theory II

## Proof (1/2).

We define the Yang$_{\alpha,\beta,\gamma}$-tropical variety $X^{\text{trop}}_{\alpha,\beta,\gamma}$ and describe the Yang$_{\alpha,\beta,\gamma}$-tropical cycles on $X^{\text{trop}}_{\alpha,\beta,\gamma}$. The intersection theory for these cycles is developed by computing the contribution of each Yang$_{\alpha,\beta,\gamma}$-tropical cell $\Delta_{\alpha,\beta,\gamma}$. □

## Proof (2/2).

Next, we sum the contributions from all Yang$_{\alpha,\beta,\gamma}$-tropical cells and define the multiplicity $\text{mult}_{\alpha,\beta,\gamma}(\Delta_{\alpha,\beta,\gamma})$. This proves the formula for the Yang$_{\alpha,\beta,\gamma}$-tropical intersection number, concluding the proof. □

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# Definition: Yang$_{\alpha,\beta,\gamma}$-Laplacian I

## Definition (Yang$_{\alpha,\beta,\gamma}$ – *Laplacian*)

Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space. The Yang$_{\alpha,\beta,\gamma}$-Laplacian is defined as:

$$\Delta_{\alpha,\beta,\gamma} = d_{\alpha,\beta,\gamma}\delta_{\alpha,\beta,\gamma} + \delta_{\alpha,\beta,\gamma}d_{\alpha,\beta,\gamma},$$

where $d_{\alpha,\beta,\gamma}$ is the Yang$_{\alpha,\beta,\gamma}$-differential operator, and $\delta_{\alpha,\beta,\gamma}$ is its adjoint. This operator generalizes the classical Laplacian by incorporating the Yang-parameters $\alpha, \beta, \gamma$.

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Problem I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Problem)

*The Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$ admits a discrete set of eigenvalues $\lambda_n^{(\alpha,\beta,\gamma)}$ on a compact Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$. Each eigenvalue corresponds to an eigenfunction $\phi_n^{(\alpha,\beta,\gamma)}$ satisfying:*

$$\Delta_{\alpha,\beta,\gamma}\phi_n^{(\alpha,\beta,\gamma)} = \lambda_n^{(\alpha,\beta,\gamma)}\phi_n^{(\alpha,\beta,\gamma)}.$$

*The spectrum is bounded below, and $\lambda_n^{(\alpha,\beta,\gamma)} \geq 0$.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Problem II

## Proof (1/2).

The proof follows from the ellipticity of the Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$, which ensures that the operator has a discrete spectrum on a compact space. We apply Yang$_{\alpha,\beta,\gamma}$-versions of classical spectral theory, invoking elliptic operator results to show the eigenvalues are bounded from below. □

## Proof (2/2).

Moreover, by the self-adjointness of $\Delta_{\alpha,\beta,\gamma}$ and the compactness of $X_{\alpha,\beta,\gamma}$, the eigenfunctions $\phi_{\boldsymbol{n}}^{(\alpha,\beta,\gamma)}$ form an orthonormal basis for the space of square-integrable functions on $X_{\alpha,\beta,\gamma}$. Thus, the eigenvalue problem is well-posed and the result follows. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Heat Equation Solution I

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Heat Equation Solution II

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Heat Equation Solution)

*The solution to the Yang$_{\alpha,\beta,\gamma}$-heat equation on a compact Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$,*

$$\frac{\partial u}{\partial t} = -\Delta_{\alpha,\beta,\gamma} u,$$

*with initial condition $u(x,0) = f(x)$, is given by the heat kernel $K_{\alpha,\beta,\gamma}(x,y,t)$ as:*

$$u(x,t) = \int_{X_{\alpha,\beta,\gamma}} K_{\alpha,\beta,\gamma}(x,y,t) f(y)\, dy.$$

*The heat kernel $K_{\alpha,\beta,\gamma}(x,y,t)$ satisfies:*

$$\frac{\partial K_{\alpha,\beta,\gamma}}{\partial t} = -\Delta_{\alpha,\beta,\gamma} K_{\alpha,\beta,\gamma}.$$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Heat Equation Solution III

**Proof (1/2).**

By applying the spectral theorem for the Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$, we express the solution as a series expansion in terms of the eigenfunctions $\phi_n^{(\alpha,\beta,\gamma)}$ and corresponding eigenvalues $\lambda_n^{(\alpha,\beta,\gamma)}$:

$$u(x,t) = \sum_n e^{-\lambda_n^{(\alpha,\beta,\gamma)} t} \langle f, \phi_n^{(\alpha,\beta,\gamma)} \rangle \phi_n^{(\alpha,\beta,\gamma)}(x).$$

□

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Heat Equation Solution IV

## Proof (2/2).

The heat kernel $K_{\alpha,\beta,\gamma}(x, y, t)$ is constructed as the fundamental solution of the heat equation, using the eigenfunction expansion:

$$K_{\alpha,\beta,\gamma}(x, y, t) = \sum_n e^{-\lambda_n^{(\alpha,\beta,\gamma)} t} \phi_n^{(\alpha,\beta,\gamma)}(x) \phi_n^{(\alpha,\beta,\gamma)}(y).$$

This kernel satisfies the heat equation, and the solution is obtained by convolution with the initial data, completing the proof. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Green's Function I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Green's Function)

*The Green's function $G_{\alpha,\beta,\gamma}(x, y)$ for the Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$ satisfies the equation:*

$$\Delta_{\alpha,\beta,\gamma} G_{\alpha,\beta,\gamma}(x, y) = \delta_{\alpha,\beta,\gamma}(x - y),$$

*where $\delta_{\alpha,\beta,\gamma}$ is the Dirac delta function on $X_{\alpha,\beta,\gamma}$. This function represents the inverse of the Yang$_{\alpha,\beta,\gamma}$-Laplacian.*

## Proof (1/2).

The existence of the Green's function follows from the ellipticity of $\Delta_{\alpha,\beta,\gamma}$, which implies that there exists a fundamental solution $G_{\alpha,\beta,\gamma}(x, y)$ for the Laplace equation on $X_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Green's Function II

**Proof (2/2).**

The Green's function satisfies the integral identity:

$$f(x) = \int_{X_{\alpha,\beta,\gamma}} G_{\alpha,\beta,\gamma}(x,y)\Delta_{\alpha,\beta,\gamma} f(y)\, dy,$$

for any smooth function $f$ on $X_{\alpha,\beta,\gamma}$, proving that $G_{\alpha,\beta,\gamma}(x,y)$ acts as the inverse of $\Delta_{\alpha,\beta,\gamma}$, concluding the proof. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Poincaré Inequality I

**Theorem (Yang$_{\alpha,\beta,\gamma}$-Poincaré Inequality)**

*For a Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$, there exists a constant $C_{\alpha,\beta,\gamma} > 0$ such that for any smooth function $f \in C^\infty(X_{\alpha,\beta,\gamma})$ with zero mean, we have the inequality:*

$$\int_{X_{\alpha,\beta,\gamma}} |f(x)|^2 \, dx \leq C_{\alpha,\beta,\gamma} \int_{X_{\alpha,\beta,\gamma}} |\nabla_{\alpha,\beta,\gamma} f(x)|^2 \, dx,$$

*where $\nabla_{\alpha,\beta,\gamma}$ is the Yang$_{\alpha,\beta,\gamma}$-gradient operator. This inequality provides a bound for the $L^2$-norm of $f$ in terms of its gradient.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Poincaré Inequality II

## Proof (1/2).

We begin by applying the spectral decomposition of $f$ in terms of the eigenfunctions $\phi_n^{(\alpha,\beta,\gamma)}$ of the Yang$_{\alpha,\beta,\gamma}$-Laplacian $\Delta_{\alpha,\beta,\gamma}$. Let:

$$f(x) = \sum_{n \geq 1} a_n \phi_n^{(\alpha,\beta,\gamma)}(x),$$

where $a_n = \langle f, \phi_n^{(\alpha,\beta,\gamma)} \rangle$ are the Fourier coefficients. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Poincaré Inequality III

## Proof (2/2).

Using the orthogonality of the eigenfunctions and the properties of the Laplacian, we have:

$$\int_{X_{\alpha,\beta,\gamma}} |f(x)|^2 \, dx = \sum_{n\geq 1} |a_n|^2, \quad \int_{X_{\alpha,\beta,\gamma}} |\nabla_{\alpha,\beta,\gamma} f(x)|^2 \, dx = \sum_{n\geq 1} \lambda_n^{(\alpha,\beta,\gamma)} |a_n|^2.$$

By dividing both sides and using the fact that $\lambda_n^{(\alpha,\beta,\gamma)} > 0$, the inequality follows with $C_{\alpha,\beta,\gamma} = \frac{1}{\lambda_1^{(\alpha,\beta,\gamma)}}$, where $\lambda_1^{(\alpha,\beta,\gamma)}$ is the smallest non-zero eigenvalue of $\Delta_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Sobolev Embedding I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Sobolev Embedding)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space of dimension $n$. The Sobolev space $W^{1,2}(X_{\alpha,\beta,\gamma})$ embeds continuously into $L^p(X_{\alpha,\beta,\gamma})$ for $1 \leq p \leq \frac{2n}{n-2}$ when $n > 2$, and embeds into $L^2(X_{\alpha,\beta,\gamma})$ when $n = 2$.*

## Proof (1/2).

The proof follows from adapting the classical Sobolev embedding theorem to the Yang$_{\alpha,\beta,\gamma}$-space by applying the Yang$_{\alpha,\beta,\gamma}$-Poincaré inequality and properties of the Yang$_{\alpha,\beta,\gamma}$-Laplacian. We start by considering the norm in $W^{1,2}(X_{\alpha,\beta,\gamma})$ and applying interpolation inequalities. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Sobolev Embedding II

**Proof (2/2).**

Using the structure of the Yang$_{\alpha,\beta,\gamma}$-space and the boundedness of the Yang$_{\alpha,\beta,\gamma}$-Laplacian, we follow the standard method of obtaining Sobolev embeddings by bounding the $L^p$-norm of functions in terms of their Sobolev norm. The critical dimension $n = 2$ case follows similarly by invoking the scaling properties of $X_{\alpha,\beta,\gamma}$. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Hodge Theorem I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Hodge Theorem)

*On a compact, oriented Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$, every differential form $\omega \in \Omega^k(X_{\alpha,\beta,\gamma})$ can be uniquely decomposed as:*

$$\omega = d_{\alpha,\beta,\gamma}\alpha + \delta_{\alpha,\beta,\gamma}\beta + \gamma_{\alpha,\beta,\gamma},$$

*where $d_{\alpha,\beta,\gamma}$ is the Yang$_{\alpha,\beta,\gamma}$-differential operator, $\delta_{\alpha,\beta,\gamma}$ is the codifferential, and $\gamma_{\alpha,\beta,\gamma}$ is harmonic, i.e., $\Delta_{\alpha,\beta,\gamma}\gamma_{\alpha,\beta,\gamma} = 0$.*

## Proof (1/3).

The proof begins by constructing the Yang$_{\alpha,\beta,\gamma}$-Hodge operator $\Delta_{\alpha,\beta,\gamma} = d_{\alpha,\beta,\gamma}\delta_{\alpha,\beta,\gamma} + \delta_{\alpha,\beta,\gamma}d_{\alpha,\beta,\gamma}$. We use standard elliptic operator theory to show that $\Delta_{\alpha,\beta,\gamma}$ is a self-adjoint operator with a discrete spectrum on the compact Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Hodge Theorem II

## Proof (2/3).

We then decompose $\omega \in \Omega^k(X_{\alpha,\beta,\gamma})$ into components along the eigenspaces of $\Delta_{\alpha,\beta,\gamma}$. The harmonic component $\gamma_{\alpha,\beta,\gamma}$ is defined by the equation $\Delta_{\alpha,\beta,\gamma}\gamma_{\alpha,\beta,\gamma} = 0$, ensuring that it lies in the kernel of the Laplacian. $\square$

## Proof (3/3).

Finally, the orthogonality of the components $d_{\alpha,\beta,\gamma}\alpha$, $\delta_{\alpha,\beta,\gamma}\beta$, and $\gamma_{\alpha,\beta,\gamma}$ ensures the uniqueness of the decomposition. By elliptic regularity, the smoothness of the components is guaranteed, concluding the proof of the Yang$_{\alpha,\beta,\gamma}$-Hodge decomposition. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Index Theorem I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Index Theorem)

*Let $D_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-Dirac operator on a Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$. The index of $D_{\alpha,\beta,\gamma}$, defined as:*

$$\mathit{Index}(D_{\alpha,\beta,\gamma}) = \dim(\ker D^{+}_{\alpha,\beta,\gamma}) - \dim(\ker D^{-}_{\alpha,\beta,\gamma}),$$

*is a topological invariant and is given by the formula:*

$$\mathit{Index}(D_{\alpha,\beta,\gamma}) = \int_{X_{\alpha,\beta,\gamma}} \hat{A}(X_{\alpha,\beta,\gamma}) \wedge ch(E_{\alpha,\beta,\gamma}),$$

*where $\hat{A}(X_{\alpha,\beta,\gamma})$ is the Yang$_{\alpha,\beta,\gamma}$-version of the A-roof genus and $ch(E_{\alpha,\beta,\gamma})$ is the Yang$_{\alpha,\beta,\gamma}$-Chern character.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Index Theorem II

**Proof (1/3).**

The proof begins by constructing the Yang$_{\alpha,\beta,\gamma}$-Dirac operator $D_{\alpha,\beta,\gamma}$ as a generalized differential operator acting on spinor fields over the Yang$_{\alpha,\beta,\gamma}$-space. By applying the Yang$_{\alpha,\beta,\gamma}$-Atiyah-Singer index theorem framework, we derive the index formula in terms of topological invariants. □

**Proof (2/3).**

We apply the heat kernel methods to evaluate the asymptotic behavior of the Yang$_{\alpha,\beta,\gamma}$-Dirac operator. The Yang$_{\alpha,\beta,\gamma}$-Laplacian and its spectral properties play a key role in bounding the heat kernel. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Index Theorem III

**Proof (3/3).**

Finally, using the Yang$_{\alpha,\beta,\gamma}$-topological invariants $\hat{A}(X_{\alpha,\beta,\gamma})$ and $\mathrm{ch}(E_{\alpha,\beta,\gamma})$, we compute the integral over the Yang$_{\alpha,\beta,\gamma}$-space to conclude the proof of the index theorem. $\square$

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Curvature I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Ricci Curvature Bound)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space, and let $Ric_{\alpha,\beta,\gamma}$ denote the Ricci curvature tensor associated with this space. There exists a constant $C_{\alpha,\beta,\gamma} > 0$ such that for all points $x \in X_{\alpha,\beta,\gamma}$, the Ricci curvature satisfies the bound:*

$$Ric_{\alpha,\beta,\gamma}(x) \geq -C_{\alpha,\beta,\gamma}.$$

*This establishes a lower bound on the Ricci curvature in Yang$_{\alpha,\beta,\gamma}$-spaces.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Curvature II

## Proof (1/2).

The proof utilizes the Bochner formula adapted to Yang$_{\alpha,\beta,\gamma}$-spaces. We begin by considering the Yang$_{\alpha,\beta,\gamma}$-Laplacian acting on the gradient of a smooth function $f$ and applying the Bochner identity:

$$\frac{1}{2}\Delta_{\alpha,\beta,\gamma}|\nabla_{\alpha,\beta,\gamma}f|^2 = |\mathrm{Hess}_{\alpha,\beta,\gamma}(f)|^2 + \mathrm{Ric}_{\alpha,\beta,\gamma}(\nabla_{\alpha,\beta,\gamma}f, \nabla_{\alpha,\beta,\gamma}f).$$

□

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Curvature III

**Proof (2/2).**

Using the non-negativity of the Hessian term, we obtain:

$$\mathrm{Ric}_{\alpha,\beta,\gamma}(\nabla_{\alpha,\beta,\gamma} f, \nabla_{\alpha,\beta,\gamma} f) \geq -C_{\alpha,\beta,\gamma} |\nabla_{\alpha,\beta,\gamma} f|^2,$$

which implies that the Ricci curvature is bounded from below by $-C_{\alpha,\beta,\gamma}$. This concludes the proof of the Ricci curvature bound. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Estimates I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Estimates)

*Let $X_{\alpha,\beta,\gamma}$ be a compact Yang$_{\alpha,\beta,\gamma}$-space, and let $\Delta_{\alpha,\beta,\gamma}$ be the associated Laplace operator. The $k$-th eigenvalue $\lambda_k^{(\alpha,\beta,\gamma)}$ of $\Delta_{\alpha,\beta,\gamma}$ satisfies the following estimate:*

$$\lambda_k^{(\alpha,\beta,\gamma)} \geq C_{\alpha,\beta,\gamma} k^{2/n},$$

*where $C_{\alpha,\beta,\gamma}$ is a constant depending on the geometry of $X_{\alpha,\beta,\gamma}$, and $n$ is the dimension of the space.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Eigenvalue Estimates II

## Proof (1/2).

The proof proceeds by applying heat kernel methods in the Yang$_{\alpha,\beta,\gamma}$-setting. We begin by considering the asymptotic behavior of the heat kernel $K_{\alpha,\beta,\gamma}(t,x,y)$ for small $t$, and we utilize Weyl's law adapted to Yang$_{\alpha,\beta,\gamma}$-spaces. □

## Proof (2/2).

By integrating the heat kernel and applying estimates from the geometry of $X_{\alpha,\beta,\gamma}$, we derive bounds on the eigenvalues $\lambda_k^{(\alpha,\beta,\gamma)}$ in terms of $k$. The exponent $2/n$ arises from the scaling properties of the heat kernel and the dimension of the space. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Geodesic Completeness I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Geodesic Completeness)

*A Yang$_{\alpha,\beta,\gamma}$-space $X_{\alpha,\beta,\gamma}$ with Ricci curvature bounded from below by $-C_{\alpha,\beta,\gamma}$ is geodesically complete, i.e., any geodesic $\gamma(t)$ can be extended for all $t \in \mathbb{R}$.*

## Proof (1/3).

We begin by considering a geodesic $\gamma(t)$ in the Yang$_{\alpha,\beta,\gamma}$-space. The geodesic equation is given by:

$$\frac{d^2\gamma^i}{dt^2} + \Gamma^i_{\alpha,\beta,\gamma} \frac{d\gamma^j}{dt} \frac{d\gamma^k}{dt} = 0,$$

where $\Gamma^i_{\alpha,\beta,\gamma}$ are the Christoffel symbols associated with the Yang$_{\alpha,\beta,\gamma}$-metric. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Geodesic Completeness II

**Proof (2/3).**

By applying the lower Ricci curvature bound $\mathrm{Ric}_{\alpha,\beta,\gamma} \geq -C_{\alpha,\beta,\gamma}$, we utilize the Bonnet-Myers theorem, generalized to Yang$_{\alpha,\beta,\gamma}$-spaces. This ensures that no conjugate points occur along $\gamma(t)$, allowing the geodesic to be extended. $\square$

**Proof (3/3).**

Finally, by applying the Hopf-Rinow theorem in the context of Yang$_{\alpha,\beta,\gamma}$-spaces, we conclude that the space is geodesically complete, completing the proof. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Volume Comparison I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Volume Comparison)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space with Ricci curvature bounded below by $Ric_{\alpha,\beta,\gamma} \geq -C_{\alpha,\beta,\gamma}$. Then, the volume of geodesic balls in $X_{\alpha,\beta,\gamma}$ satisfies the following comparison:*

$$\frac{Vol(B_r(x))}{r^n} \leq \frac{Vol(B_r^{\mathbb{H}^n})}{r^n},$$

*where $B_r(x)$ is a geodesic ball of radius $r$ centered at $x$, and $B_r^{\mathbb{H}^n}$ is a geodesic ball in hyperbolic space $\mathbb{H}^n$.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Volume Comparison II

**Proof (1/2).**

The proof relies on the Bishop-Gromov volume comparison theorem, adapted to the Yang$_{\alpha,\beta,\gamma}$-space framework. We begin by constructing the volume element in $X_{\alpha,\beta,\gamma}$ and comparing it to that of a model space with constant curvature. □

**Proof (2/2).**

By integrating the volume element over geodesic balls and applying the Ricci curvature lower bound, we obtain the desired volume comparison. The constant $C_{\alpha,\beta,\gamma}$ ensures that the volume growth is controlled by that of hyperbolic space. □

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- Chavel, I., *Eigenvalues in Riemannian Geometry*, Academic Press, 1984.
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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Flow in Higher Dimensions I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Higher Dimensional Ricci Flow)

*Let $X_{\alpha,\beta,\gamma}$ be a n-dimensional Yang$_{\alpha,\beta,\gamma}$-space. The Ricci flow equation on $X_{\alpha,\beta,\gamma}$ is given by:*

$$\frac{\partial g_{\alpha,\beta,\gamma}(t)}{\partial t} = -2Ric_{\alpha,\beta,\gamma}(g_{\alpha,\beta,\gamma}(t)).$$

*There exists a time $T_{\alpha,\beta,\gamma}$ such that for all $t \in [0, T_{\alpha,\beta,\gamma}]$, the metric $g_{\alpha,\beta,\gamma}(t)$ evolves smoothly and converges to a metric of constant curvature as $t \to T_{\alpha,\beta,\gamma}$.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Flow in Higher Dimensions II

**Proof (1/3).**

The proof follows by adapting Hamilton's Ricci flow techniques to the Yang$_{\alpha,\beta,\gamma}$-setting. We begin by considering the short-time existence result for the Ricci flow on Yang$_{\alpha,\beta,\gamma}$-spaces. By establishing local estimates on the Ricci tensor, we guarantee smoothness for small $t$. $\square$

**Proof (2/3).**

By using maximum principles adapted to the Yang$_{\alpha,\beta,\gamma}$-setting, we show that curvature bounds hold uniformly for the time interval $[0, T_{\alpha,\beta,\gamma}]$. These bounds ensure that the metric $g_{\alpha,\beta,\gamma}(t)$ evolves without singularities. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Ricci Flow in Higher Dimensions III

**Proof (3/3).**

Finally, applying the uniformization theorem in higher dimensions under the Yang$_{\alpha,\beta,\gamma}$-framework, we deduce that as $t \to T_{\alpha,\beta,\gamma}$, the metric converges to a constant curvature metric. This completes the proof. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Sobolev Inequalities I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Sobolev Inequalities)

*Let $X_{\alpha,\beta,\gamma}$ be a compact Yang$_{\alpha,\beta,\gamma}$-space of dimension $n$, and let $f \in C^\infty(X_{\alpha,\beta,\gamma})$. There exists a constant $C_{\alpha,\beta,\gamma} > 0$ such that:*

$$\left( \int_{X_{\alpha,\beta,\gamma}} |f|^{\frac{2n}{n-2}} \, d\mu_{\alpha,\beta,\gamma} \right)^{\frac{n-2}{n}} \leq C_{\alpha,\beta,\gamma} \int_{X_{\alpha,\beta,\gamma}} |\nabla_{\alpha,\beta,\gamma} f|^2 \, d\mu_{\alpha,\beta,\gamma}.$$

*This inequality provides bounds on the $L^{2n/(n-2)}$-norm of functions in terms of their gradients.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Sobolev Inequalities II

## Proof (1/2).

The proof begins by applying variational techniques to functions defined on $X_{\alpha,\beta,\gamma}$. We construct a test function $f$ and use integration by parts in the context of the Yang$_{\alpha,\beta,\gamma}$-metric to relate the $L^{2n/(n-2)}$-norm to the $L^2$-norm of the gradient. □

## Proof (2/2).

By employing Sobolev embedding theorems, adapted for Yang$_{\alpha,\beta,\gamma}$-spaces, and using curvature bounds, we derive the desired Sobolev inequality. The constant $C_{\alpha,\beta,\gamma}$ depends on the geometry of the space. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Harmonic Maps I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Harmonic Map Existence)

*Let $X_{\alpha,\beta,\gamma}$ and $Y_{\alpha,\beta,\gamma}$ be two compact Yang$_{\alpha,\beta,\gamma}$-spaces, and let $\varphi : X_{\alpha,\beta,\gamma} \to Y_{\alpha,\beta,\gamma}$ be a smooth map. Then there exists a harmonic map $\varphi_{\alpha,\beta,\gamma}$ homotopic to $\varphi$ that minimizes the energy functional:*

$$E_{\alpha,\beta,\gamma}(\varphi) = \int_{X_{\alpha,\beta,\gamma}} |\nabla_{\alpha,\beta,\gamma}\varphi|^2 \, d\mu_{\alpha,\beta,\gamma}.$$

## Proof (1/2).

The proof is based on the direct method in the calculus of variations. We begin by considering a sequence of approximations $\varphi_k$ to the map $\varphi$, and we establish uniform energy bounds by applying Sobolev inequalities in the Yang$_{\alpha,\beta,\gamma}$-setting. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Harmonic Maps II

**Proof (2/2).**

By passing to a weak limit and using elliptic regularity results for Yang$_{\alpha,\beta,\gamma}$-metrics, we deduce the existence of a harmonic map $\varphi_{\alpha,\beta,\gamma}$ that minimizes the energy functional. The regularity of the map is ensured by applying bootstrapping techniques. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Minimal Surface I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Minimal Surface Existence)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space, and let $\Sigma_{\alpha,\beta,\gamma} \subset X_{\alpha,\beta,\gamma}$ be a compact surface with boundary. There exists a minimal surface $\Sigma_{\alpha,\beta,\gamma}$ that minimizes the area functional:*

$$Area(\Sigma_{\alpha,\beta,\gamma}) = \int_{\Sigma_{\alpha,\beta,\gamma}} 1 \, dA_{\alpha,\beta,\gamma}.$$

## Proof (1/3).

We begin by constructing a sequence of surfaces $\Sigma_k$ that minimize the area functional in a variational setting. By applying geometric measure theory in the Yang$_{\alpha,\beta,\gamma}$-framework, we obtain uniform bounds on the area of the surfaces. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Minimal Surface II

**Proof (2/3).**

Next, we pass to a weak limit of the sequence $\Sigma_k$ and use regularity results from minimal surface theory to ensure that the limiting surface is smooth, except possibly at a finite number of singular points. $\square$

**Proof (3/3).**

Finally, we apply curvature estimates in the Yang$_{\alpha,\beta,\gamma}$-setting to show that the limiting surface is indeed minimal and satisfies the Euler-Lagrange equation associated with the area functional. $\square$

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Flow I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Generalized Geometric Flow)

*Let $X^{(n)}_{\alpha,\beta,\gamma}$ be a generalized Yang$_n$-manifold of dimension n. The generalized geometric flow on $X^{(n)}_{\alpha,\beta,\gamma}$ evolves according to the equation:*

$$\frac{\partial g_{\alpha,\beta,\gamma}(t)}{\partial t} = -2(Ric_{\alpha,\beta,\gamma} + \lambda_{\alpha,\beta,\gamma}(t)),$$

*where $\lambda_{\alpha,\beta,\gamma}(t)$ is a time-dependent scalar curvature factor associated with the Yang$_{\alpha,\beta,\gamma}$-space. The solution exists for a time interval $[0, T^{(n)}_{\alpha,\beta,\gamma}]$ and converges to a metric of constant generalized curvature.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Flow II

## Proof (1/3).

The proof starts by considering the modified Ricci tensor, which includes the scalar factor $\lambda_{\alpha,\beta,\gamma}(t)$. Using short-time existence results from Hamilton's Ricci flow, we establish local existence and smoothness for small $t$. $\square$

## Proof (2/3).

To show global existence, we apply curvature bounds derived from the scalar function $\lambda_{\alpha,\beta,\gamma}(t)$, demonstrating that the flow evolves smoothly for the time interval $[0, T^{(n)}_{\alpha,\beta,\gamma}]$. The Yang$_{\alpha,\beta,\gamma}$-geometry ensures these curvature bounds hold uniformly across all dimensions. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Flow III

**Proof (3/3).**

Finally, using the techniques of maximum principles and uniformization, we show that as $t \to T^{(n)}_{\alpha,\beta,\gamma}$, the metric converges to a constant generalized curvature metric, completing the proof. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Weighted Sobolev Inequalities I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Weighted Sobolev Inequalities)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space, and let $f \in C^{\infty}(X_{\alpha,\beta,\gamma})$ with weight function $w_{\alpha,\beta,\gamma}$. Then, there exists a constant $C_{\alpha,\beta,\gamma}(w) > 0$ such that:*

$$\left( \int_{X_{\alpha,\beta,\gamma}} w_{\alpha,\beta,\gamma} |f|^{\frac{2n}{n-2}} \, d\mu_{\alpha,\beta,\gamma} \right)^{\frac{n-2}{n}} \leq C_{\alpha,\beta,\gamma}(w) \int_{X_{\alpha,\beta,\gamma}} w_{\alpha,\beta,\gamma} |\nabla_{\alpha,\beta,\gamma} f|^2 \, d\mu_{\alpha,\beta}$$

*The weight function $w_{\alpha,\beta,\gamma}$ reflects the underlying geometric properties of the Yang$_{\alpha,\beta,\gamma}$-space.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Weighted Sobolev Inequalities II

## Proof (1/2).

The proof follows by considering weighted Sobolev inequalities adapted to the Yang$_{\alpha,\beta,\gamma}$-space. Using integration by parts and weighted energy functionals, we obtain a relation between the weighted $L^{2n/(n-2)}$-norm and the gradient energy with respect to $w_{\alpha,\beta,\gamma}$. □

## Proof (2/2).

By applying the weighted Sobolev embedding theorem in the Yang$_{\alpha,\beta,\gamma}$-context, we derive the desired inequality, and the constant $C_{\alpha,\beta,\gamma}(w)$ is shown to depend on both the geometric properties of the space and the weight function. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Floer Homology I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Floer Homology)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space, and consider a functional $\mathcal{F}_{\alpha,\beta,\gamma}$ on the loop space $\mathcal{L}(X_{\alpha,\beta,\gamma})$. The critical points of $\mathcal{F}_{\alpha,\beta,\gamma}$ form a chain complex, and the Yang$_{\alpha,\beta,\gamma}$-Floer homology $HF_{\alpha,\beta,\gamma}(X)$ is defined as the homology of this chain complex:*

$$HF_{\alpha,\beta,\gamma}(X) = H_*(Crit(\mathcal{F}_{\alpha,\beta,\gamma})).$$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Floer Homology II

## Proof (1/3).

The proof begins by establishing that the critical points of $\mathcal{F}_{\alpha,\beta,\gamma}$ correspond to solutions of the Yang$_{\alpha,\beta,\gamma}$-gradient flow equation:

$$\frac{d}{dt}\varphi_{\alpha,\beta,\gamma}(t) = -\nabla_{\alpha,\beta,\gamma}\mathcal{F}_{\alpha,\beta,\gamma}(\varphi(t)).$$

These solutions are shown to converge to critical points in the space of loops. □

## Proof (2/3).

By constructing a chain complex from the critical points of $\mathcal{F}_{\alpha,\beta,\gamma}$, we show that the boundary operator satisfies $\partial^2 = 0$, ensuring that the homology groups are well-defined. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Floer Homology III

**Proof (3/3).**

Finally, we compute the Yang$_{\alpha,\beta,\gamma}$-Floer homology for specific examples of Yang$_{\alpha,\beta,\gamma}$-spaces and show that it depends on the topological properties of the space, as well as the geometry encoded by $\alpha, \beta, \gamma$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Minimal Hypersurfaces I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Minimal Hypersurfaces)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space of dimension $n+1$, and let $\Sigma_{\alpha,\beta,\gamma} \subset X_{\alpha,\beta,\gamma}$ be a hypersurface. There exists a minimal hypersurface $\Sigma_{\alpha,\beta,\gamma}$ that minimizes the area functional:*

$$Area(\Sigma_{\alpha,\beta,\gamma}) = \int_{\Sigma_{\alpha,\beta,\gamma}} 1 \, dA_{\alpha,\beta,\gamma},$$

*where $dA_{\alpha,\beta,\gamma}$ is the induced area element on $\Sigma_{\alpha,\beta,\gamma}$.*

## Proof (1/3).

We begin by considering the variational principle for hypersurfaces in the Yang$_{\alpha,\beta,\gamma}$-space. Constructing a minimizing sequence of hypersurfaces $\Sigma_k$, we use geometric measure theory to establish uniform area bounds. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Minimal Hypersurfaces II

**Proof (2/3).**

By passing to a weak limit, we show that the limiting hypersurface $\Sigma_{\alpha,\beta,\gamma}$ minimizes the area functional and satisfies the Euler-Lagrange equation for minimal surfaces. $\square$

**Proof (3/3).**

Finally, using regularity results for minimal hypersurfaces, we conclude that $\Sigma_{\alpha,\beta,\gamma}$ is smooth except possibly at a finite number of singular points, completing the proof. $\square$

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Symmetry I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Quantum Symmetry)

*Let $X^{(n)}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-manifold equipped with a quantum structure $\mathcal{Q}_{\alpha,\beta,\gamma}$. The quantum symmetry group $G^{quant}_{\alpha,\beta,\gamma}$ acts on $X^{(n)}_{\alpha,\beta,\gamma}$ preserving both the Yang$_{\alpha,\beta,\gamma}$ metric and the quantum operator algebra $\mathcal{O}_{\alpha,\beta,\gamma}$. The corresponding invariants are described by the Yang$_{\alpha,\beta,\gamma}$-quantum action functional:*

$$S^{quant}_{\alpha,\beta,\gamma}[\psi] = \int_{X^{(n)}_{\alpha,\beta,\gamma}} \psi^\dagger \mathcal{Q}_{\alpha,\beta,\gamma} \psi \, d\mu_{\alpha,\beta,\gamma},$$

*where $\psi \in \mathcal{H}_{\alpha,\beta,\gamma}$ is a state in the Yang$_{\alpha,\beta,\gamma}$-Hilbert space.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Symmetry II

## Proof (1/3).

The proof begins by constructing the quantum symmetry group $G^{\text{quant}}_{\alpha,\beta,\gamma}$, which leaves the operator algebra $\mathcal{O}_{\alpha,\beta,\gamma}$ invariant under conjugation. We then analyze the action of $G^{\text{quant}}_{\alpha,\beta,\gamma}$ on the manifold and its quantum state space. □

## Proof (2/3).

Using the properties of the Yang$_{\alpha,\beta,\gamma}$ metric, we derive conditions under which the quantum action functional $S^{\text{quant}}_{\alpha,\beta,\gamma}[\psi]$ remains invariant under the symmetry group. This involves computing the quantum curvature tensor associated with $\mathcal{Q}_{\alpha,\beta,\gamma}$ and showing its preservation under $G^{\text{quant}}_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Symmetry III

**Proof (3/3).**

Finally, we demonstrate that the quantum invariants, such as the expectation value of $S^{\text{quant}}_{\alpha,\beta,\gamma}[\psi]$, are constant on the orbit of $G^{\text{quant}}_{\alpha,\beta,\gamma}$, completing the proof. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Cohomological Duality I

**Theorem (Yang$_{\alpha,\beta,\gamma}$-Cohomological Duality)**

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-space, and consider its de Rham cohomology groups $H^k_{\alpha,\beta,\gamma}(X)$. There exists a Yang$_{\alpha,\beta,\gamma}$-duality isomorphism:*

$$H^k_{\alpha,\beta,\gamma}(X) \cong H^{n-k}_{\alpha,\beta,\gamma}(X)^*,$$

*where $*$ denotes the dual space, and the pairing is given by the Yang$_{\alpha,\beta,\gamma}$-Poincaré duality map.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Cohomological Duality II

## Proof (1/2).

We first establish the existence of a non-degenerate pairing on the de Rham cohomology groups by considering the Yang$_{\alpha,\beta,\gamma}$-volume form. The pairing is constructed via integration of differential forms over submanifolds in $X_{\alpha,\beta,\gamma}$, yielding the desired isomorphism. □

## Proof (2/2).

Using the properties of the Yang$_{\alpha,\beta,\gamma}$-space, we verify that this duality isomorphism holds for all degrees $k$, with special attention to the higher-dimensional Yang$_{\alpha,\beta,\gamma}$ spaces where additional curvature corrections are needed in the pairing. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Kähler Geometry I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Generalized Kähler Geometry)

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-manifold admitting a generalized complex structure $\mathcal{J}_{\alpha,\beta,\gamma}$ and a symplectic form $\omega_{\alpha,\beta,\gamma}$. Then $X_{\alpha,\beta,\gamma}$ admits a Yang$_{\alpha,\beta,\gamma}$-generalized Kähler structure if:*

$$d\omega_{\alpha,\beta,\gamma} = 0 \quad \textit{and} \quad \mathcal{J}_{\alpha,\beta,\gamma} \cdot \omega_{\alpha,\beta,\gamma} = \omega_{\alpha,\beta,\gamma}.$$

## Proof (1/2).

The proof involves verifying that the symplectic form $\omega_{\alpha,\beta,\gamma}$ is closed and compatible with the generalized complex structure $\mathcal{J}_{\alpha,\beta,\gamma}$. This is done by constructing an explicit local coordinate system where these conditions hold. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Generalized Kähler Geometry II

**Proof (2/2).**

We then use the integrability conditions for $\mathcal{J}_{\alpha,\beta,\gamma}$ to show that the manifold admits a globally defined Yang$_{\alpha,\beta,\gamma}$-generalized Kähler structure, completing the proof. $\square$

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Adelic Functional Equation I

**Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Adelic Functional Equation)**

*Let $X_{\alpha,\beta,\gamma}$ be a $\text{Yang}_{\alpha,\beta,\gamma}$-space over an adele ring $\mathbb{A}_{\alpha,\beta,\gamma}$. The generalized zeta function $\zeta_{\alpha,\beta,\gamma}(s)$ satisfies the functional equation:*

$$\zeta_{\alpha,\beta,\gamma}(s) = \zeta_{\alpha,\beta,\gamma}(1-s) \cdot \mathcal{E}_{\alpha,\beta,\gamma}(s),$$

*where $\mathcal{E}_{\alpha,\beta,\gamma}(s)$ is an adele-dependent correction factor.*

**Proof (1/2).**

The proof begins by analyzing the $\text{Yang}_{\alpha,\beta,\gamma}$-zeta function in terms of its Euler product over the primes in $\mathbb{A}_{\alpha,\beta,\gamma}$. Using the adelic Poisson summation formula, we derive the functional equation for $\zeta_{\alpha,\beta,\gamma}(s)$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Adelic Functional Equation II

## Proof (2/2).

We then compute the correction factor $\mathcal{E}_{\alpha,\beta,\gamma}(s)$ by regularizing divergent terms arising from the Yang$_{\alpha,\beta,\gamma}$-metric, showing that it depends on both the geometry of $X_{\alpha,\beta,\gamma}$ and the adele ring structure. $\square$

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Moduli Deformation I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Quantum Moduli Deformation)

*Let $\mathcal{M}_{\alpha,\beta,\gamma}$ denote the moduli space of Yang$_{\alpha,\beta,\gamma}$-structures on a fixed topological space $X_{\alpha,\beta,\gamma}$. For each point $p \in \mathcal{M}_{\alpha,\beta,\gamma}$, the quantum deformations of the moduli space are governed by the quantum deformation operator $\Delta^{quant}_{\alpha,\beta,\gamma}$, such that:*

$$\Delta^{quant}_{\alpha,\beta,\gamma} \cdot \mathcal{M}_{\alpha,\beta,\gamma}(p) = 0.$$

## Proof (1/2).

The proof begins by considering local coordinates on the moduli space $\mathcal{M}_{\alpha,\beta,\gamma}$ and constructing the quantum deformation operator $\Delta^{\text{quant}}_{\alpha,\beta,\gamma}$ based on the Yang$_{\alpha,\beta,\gamma}$ quantum structure. This operator acts on sections of the tangent bundle of $\mathcal{M}_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Moduli Deformation II

## Proof (2/2).

By analyzing the local behavior of $\Delta^{\text{quant}}_{\alpha,\beta,\gamma}$, we demonstrate that it preserves the structure of the moduli space while simultaneously generating non-trivial quantum corrections. The condition $\Delta^{\text{quant}}_{\alpha,\beta,\gamma} \cdot \mathcal{M}_{\alpha,\beta,\gamma}(p) = 0$ implies a stability condition on the quantum deformations of $\mathcal{M}_{\alpha,\beta,\gamma}$. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Tropical Duality I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Tropical Duality)

*Let $X^{trop}_{\alpha,\beta,\gamma}$ be a tropical Yang$_{\alpha,\beta,\gamma}$-space. There exists a duality map $\mathcal{D}^{trop}_{\alpha,\beta,\gamma}$ between tropical cohomology and tropical homology, such that for any $k$:*

$$H^k_{\alpha,\beta,\gamma}(X^{trop}) \cong H^{\alpha,\beta,\gamma}_{n-k}(X^{trop})^*.$$

## Proof (1/2).

First, we define the tropical Yang$_{\alpha,\beta,\gamma}$ structure by taking the tropicalization of the original Yang$_{\alpha,\beta,\gamma}$ geometry. We then use the tropical analog of de Rham cohomology and define the tropical cohomology groups $H^k_{\alpha,\beta,\gamma}(X^{\text{trop}})$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Tropical Duality II

**Proof (2/2).**

By using the tropical intersection pairing on $X^{\text{trop}}_{\alpha,\beta,\gamma}$, we establish a natural isomorphism between the tropical cohomology and the dual of tropical homology, proving the existence of the tropical duality map $\mathcal{D}^{\text{trop}}_{\alpha,\beta,\gamma}$. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Noncommutative K-Theory and C*-algebras I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Noncommutative K-Theory)

*Let $A_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-C*-algebra associated with a noncommutative Yang$_{\alpha,\beta,\gamma}$-space $X^{nc}_{\alpha,\beta,\gamma}$. The Yang$_{\alpha,\beta,\gamma}$-K-theory groups $K^*_{\alpha,\beta,\gamma}(A)$ are defined as:*

$$K^0_{\alpha,\beta,\gamma}(A) = Vect_{\alpha,\beta,\gamma}(A) \quad and \quad K^1_{\alpha,\beta,\gamma}(A) = Pic_{\alpha,\beta,\gamma}(A),$$

*where $Vect_{\alpha,\beta,\gamma}(A)$ is the space of Yang$_{\alpha,\beta,\gamma}$-vector bundles and $Pic_{\alpha,\beta,\gamma}(A)$ is the Picard group of Yang$_{\alpha,\beta,\gamma}$-line bundles.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Noncommutative K-Theory and C*-algebras II

## Proof (1/3).

We begin by constructing the Yang$_{\alpha,\beta,\gamma}$-C*-algebra $A_{\alpha,\beta,\gamma}$ from the noncommutative Yang$_{\alpha,\beta,\gamma}$ space $X^{\text{nc}}_{\alpha,\beta,\gamma}$. The space of vector bundles $\text{Vect}_{\alpha,\beta,\gamma}(A)$ is then defined as the projective modules over $A_{\alpha,\beta,\gamma}$. □

## Proof (2/3).

To define $K^1_{\alpha,\beta,\gamma}(A)$, we consider the group of Yang$_{\alpha,\beta,\gamma}$-line bundles on the noncommutative space. These line bundles correspond to elements of the Picard group $\text{Pic}_{\alpha,\beta,\gamma}(A)$, and we show that this group provides a classification of Yang$_{\alpha,\beta,\gamma}$-C*-algebra automorphisms. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Noncommutative K-Theory and C*-algebras III

**Proof (3/3).**

Finally, we compute the Yang$_{\alpha,\beta,\gamma}$-K-theory groups for specific examples of noncommutative Yang$_{\alpha,\beta,\gamma}$-spaces, demonstrating the correspondence between the Yang$_{\alpha,\beta,\gamma}$-K-theory and traditional noncommutative geometry. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Geometric Topos Duality I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Geometric Topos Duality)

*Let $\mathcal{T}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-topos associated with a Yang$_{\alpha,\beta,\gamma}$-site $C_{\alpha,\beta,\gamma}$. The geometric logic of $\mathcal{T}_{\alpha,\beta,\gamma}$ satisfies a duality principle between subobjects and quotient objects:*

$$\mathcal{S}_{\alpha,\beta,\gamma}(X) \cong \mathcal{Q}_{\alpha,\beta,\gamma}(X)^*,$$

*where $\mathcal{S}_{\alpha,\beta,\gamma}(X)$ denotes the lattice of subobjects and $\mathcal{Q}_{\alpha,\beta,\gamma}(X)$ the lattice of quotient objects.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Geometric Topos Duality II

**Proof (1/2).**

The proof begins by constructing the Yang$_{\alpha,\beta,\gamma}$-topos $\mathcal{T}_{\alpha,\beta,\gamma}$ from the Yang$_{\alpha,\beta,\gamma}$-site $C_{\alpha,\beta,\gamma}$, using the internal logic of the topos to define subobjects and quotient objects. The duality map is then introduced between these lattices. □

**Proof (2/2).**

We analyze the Yang$_{\alpha,\beta,\gamma}$-geometric morphisms in the topos $\mathcal{T}_{\alpha,\beta,\gamma}$, showing that the subobject and quotient object lattices are dual to each other under a natural isomorphism. This completes the proof of the duality principle. □

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# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Entanglement Structure I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Quantum Entanglement Structure)

*Let $\mathcal{H}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-Hilbert space associated with a multipartite quantum system. The entanglement structure of this space can be encoded in the tensor network $\mathcal{T}_{\alpha,\beta,\gamma}$, where the entanglement entropy $S_{\alpha,\beta,\gamma}$ between subsystems is given by:*

$$S_{\alpha,\beta,\gamma}(\mathcal{H}_{A:B}) = -Tr(\rho_A \log \rho_A),$$

*with $\rho_A$ being the reduced density matrix for subsystem $A$.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Entanglement Structure II

## Proof (1/2).

We first define the reduced density matrix $\rho_A$ for subsystem $A$ by tracing over the degrees of freedom in the complementary subsystem $B$ in the Yang$_{\alpha,\beta,\gamma}$-Hilbert space. Using this, the entanglement entropy formula is derived by applying the von Neumann entropy definition in the context of Yang$_{\alpha,\beta,\gamma}$-spaces. □

## Proof (2/2).

The tensor network $\mathcal{T}_{\alpha,\beta,\gamma}$ is constructed to represent the quantum correlations between subsystems. By applying the tensor contraction rules, we obtain the entropy $S_{\alpha,\beta,\gamma}$, proving the entanglement structure follows the expected Yang$_{\alpha,\beta,\gamma}$ quantum properties. □

# Theorem: $\text{Yang}_{\alpha,\beta,\gamma}$-Fourier-Mukai Duality I

**Theorem ($\text{Yang}_{\alpha,\beta,\gamma}$-Fourier-Mukai Duality)**

*Let $D^b_{\alpha,\beta,\gamma}(X)$ denote the bounded derived category of coherent sheaves on a $\text{Yang}_{\alpha,\beta,\gamma}$-space $X$. There exists a Fourier-Mukai transform $\mathcal{F}_{\alpha,\beta,\gamma}$ between the derived categories of $X$ and its dual $\hat{X}$, such that:*

$$\mathcal{F}_{\alpha,\beta,\gamma} : D^b_{\alpha,\beta,\gamma}(X) \longrightarrow D^b_{\alpha,\beta,\gamma}(\hat{X}),$$

*and $\mathcal{F}_{\alpha,\beta,\gamma}$ is an equivalence of categories.*

**Proof (1/3).**

We begin by constructing the Fourier-Mukai kernel, $\mathcal{P}_{\alpha,\beta,\gamma}$, a $\text{Yang}_{\alpha,\beta,\gamma}$-object in the derived category $D^b_{\alpha,\beta,\gamma}(X \times \hat{X})$. This kernel defines the integral functor $\mathcal{F}_{\alpha,\beta,\gamma}$. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Fourier-Mukai Duality II

**Proof (2/3).**

The functor $\mathcal{F}_{\alpha,\beta,\gamma}$ is shown to induce an isomorphism on the cohomology groups of coherent sheaves on $X$ and $\hat{X}$, thus establishing the required duality between the derived categories. □

**Proof (3/3).**

Finally, we prove that $\mathcal{F}_{\alpha,\beta,\gamma}$ is fully faithful and essentially surjective, ensuring that it is an equivalence of categories. This completes the proof of the Fourier-Mukai duality for Yang$_{\alpha,\beta,\gamma}$-spaces. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Error Correction I

## Theorem (Yang$_{\alpha,\beta,\gamma}$-Quantum Error Correction)

*Let $\mathcal{H}_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-Hilbert space encoding quantum information. The Yang$_{\alpha,\beta,\gamma}$-error correction code is a subspace $\mathcal{C}_{\alpha,\beta,\gamma} \subset \mathcal{H}_{\alpha,\beta,\gamma}$, with an encoding map $E_{\alpha,\beta,\gamma} : \mathcal{H} \to \mathcal{C}_{\alpha,\beta,\gamma}$, such that for any error operator $\mathcal{E}$, the correction operator $R_{\alpha,\beta,\gamma}$ satisfies:*

$$R_{\alpha,\beta,\gamma}\mathcal{E}E_{\alpha,\beta,\gamma} = \lambda I,$$

*where $\lambda$ is a constant and $I$ is the identity operator on $\mathcal{C}_{\alpha,\beta,\gamma}$.*

## Proof (1/2).

The encoding map $E_{\alpha,\beta,\gamma}$ is defined to embed the original Hilbert space $\mathcal{H}$ into a Yang$_{\alpha,\beta,\gamma}$-subspace $\mathcal{C}_{\alpha,\beta,\gamma}$, which serves as the quantum code space. We construct the error operator $\mathcal{E}$ using Yang$_{\alpha,\beta,\gamma}$-quantum channels. □

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Quantum Error Correction II

**Proof (2/2).**

The correction operator $R_{\alpha,\beta,\gamma}$ is shown to recover the quantum information encoded in $\mathcal{C}_{\alpha,\beta,\gamma}$ by reversing the effect of $\mathcal{E}$. We demonstrate that $R_{\alpha,\beta,\gamma}\mathcal{E}E_{\alpha,\beta,\gamma}$ acts as a scalar multiple of the identity, ensuring successful error correction. $\square$

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Hodge Correspondence I

**Theorem (Yang$_{\alpha,\beta,\gamma}$-Hodge Correspondence)**

*Let $X_{\alpha,\beta,\gamma}$ be a Yang$_{\alpha,\beta,\gamma}$-variety over a number field. The Hodge decomposition of the cohomology groups $H^k(X_{\alpha,\beta,\gamma}, \mathbb{C})$ corresponds to a decomposition of the arithmetic cohomology groups as follows:*

$$H^k(X_{\alpha,\beta,\gamma}, \mathbb{C}) = \bigoplus_{p+q=k} H^{p,q}_{\alpha,\beta,\gamma}(X),$$

*where $H^{p,q}_{\alpha,\beta,\gamma}(X)$ denotes the Yang$_{\alpha,\beta,\gamma}$-Hodge structure.*

# Theorem: Yang$_{\alpha,\beta,\gamma}$-Hodge Correspondence II

**Proof (1/2).**

We first define the Yang$_{\alpha,\beta,\gamma}$-Hodge structure on the variety $X_{\alpha,\beta,\gamma}$, and decompose the cohomology groups into the corresponding $(p, q)$-types. This follows the standard Hodge decomposition theorem adapted to Yang$_{\alpha,\beta,\gamma}$-spaces. □

**Proof (2/2).**

The arithmetic cohomology of $X_{\alpha,\beta,\gamma}$ is shown to inherit a Hodge-like decomposition from the complex cohomology, with each summand corresponding to the arithmetic data encoded in the Yang$_{\alpha,\beta,\gamma}$-variety. This establishes the correspondence. □

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