Infinite Variables L-Functions, $\mathbb{Y}_n(F)$ Number Systems, and the Infinite Dimensional Riemann Hypothesis

Alien Mathematicians

Introduction

In this presentation, we continue the rigorous development of infinite variable L-functions, $\mathbb{Y}_n(F)$ number systems, and the infinite-dimensional Riemann Hypothesis. We extend the concepts and theorems from previous discussions and introduce new mathematical definitions, notations, and formulas, providing full explanations for each.

New Mathematical Definitions and Notations

Definition 1: Infinite Dimensional L-Functions

We define the infinite-dimensional L-function $\mathcal{L}_\infty(s; \mathbf{X})$ associated with a vector $\mathbf{X} = (X_1, X_2, \dots)$ of complex variables as:

$$\mathcal{L}_\infty(s; \mathbf{X}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} e^{\langle \mathbf{X}, \mathbf{n} \rangle},$$

where $\langle \mathbf{X}, \mathbf{n} \rangle = \sum_{i=1}^{\infty} X_i n_i$ is an infinite sum and $a(n)$ are coefficients that depend on the specific problem context.

Definition 2: Yang$_n$(F) Number Systems

The Yang$_n(F)$ number system, denoted as $\mathbb{Y}_n(F)$, is a generalization of field extensions where n can be an infinite cardinal number and F is a field. The structure of $\mathbb{Y}_n(F)$ is explored through the following properties:

$$\mathbb{Y}_n(F) = \bigoplus_{i \in I} F_i,$$

where I is an index set with cardinality n, and each F_i is a copy of F.

Theorem 1: Convergence of $\mathcal{L}_\infty(s; \mathbf{X})$

Theorem 1: The infinite-dimensional L-function $\mathcal{L}_\infty(s; \mathbf{X})$ converges absolutely for $\Re(s) > 1$ and for suitable choices of the vector **X**.

Proof (1/3).

Consider the infinite series defining $\mathcal{L}_\infty(s; \mathbf{X})$:

$$\mathcal{L}_\infty(s; \mathbf{X}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} e^{\langle \mathbf{X}, \mathbf{n} \rangle}.$$

To show absolute convergence, we must analyze the term:

$$\left| \frac{a(n)}{n^s} e^{\langle \mathbf{X}, \mathbf{n} \rangle} \right| \leq \frac{|a(n)|}{n^{\Re(s)}} \left| e^{\langle \mathbf{X}, \mathbf{n} \rangle} \right|.$$

□

Proof (2/3).

Since $|e^{\langle \mathbf{X}, \mathbf{n} \rangle}| = e^{\Re(\langle \mathbf{X}, \mathbf{n} \rangle)}$, the convergence of the series

Newly Invented Mathematical Formulas

We introduce the generalized symmetry-adjusted zeta function for the $\mathbb{Y}_n(F)$ number system:

$$\zeta^{\text{sym}}_{\mathbb{Y}_n(F)}(s;k) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{i=1}^{k} \chi_i(\mathbb{Y}_n(F)),$$

where χ_i are characters associated with the $\mathbb{Y}_n(F)$ system, and k is an integer parameter determining the level of symmetry adjustment.

Further Extensions

Further, we define the symmetry-adjusted L-function for infinite variables as:

$$\mathcal{L}_{\infty}^{\text{sym}}(s; \mathbf{X}, k) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \prod_{i=1}^{k} \chi_i(\mathbf{X}_n),$$

where $\mathbf{X}_n = (X_{n1}, X_{n2}, \ldots)$ is a sequence of variables, and χ_i are symmetry characters.

New Mathematical Definitions and Notations

Definition 4: Infinite Tensor *L*-Functions

We define the infinite tensor *L*-function $\mathcal{L}_{\infty}^{\otimes}(s; \mathbf{X}, \mathbf{T})$ for a vector $\mathbf{X}$ of complex variables and a tensor $\mathbf{T}$ as:

$$\mathcal{L}_{\infty}^{\otimes}(s; \mathbf{X}, \mathbf{T}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \otimes_{i=1}^{\infty} T_i(\mathbf{X}, \mathbf{n}),$$

where $\otimes$ denotes the tensor product over infinite dimensions, and $T_i(\mathbf{X}, \mathbf{n})$ are tensor components defined in relation to the vector $\mathbf{X}$ and the index vector $\mathbf{n}$.

Definition 5: $\mathbf{Yang}_{n,m}(F)$ Number Systems

The $\text{Yang}_{n,m}(F)$ number system, denoted as $\mathbb{Y}_{n,m}(F)$, extends the $\mathbb{Y}_n(F)$ number system to two indices, n and m, where n can be an infinite cardinal number and m is a positive integer. The structure of $\mathbb{Y}_{n,m}(F)$ is given by:

$$\mathbb{Y}_{n,m}(F) = \bigoplus_{i=1}^{m} \mathbb{Y}_n(F)_i,$$

where each $\mathbb{Y}_n(F)_i$ represents an individual $\mathbb{Y}_n(F)$ system.

Theorem 2: Existence of Symmetry-Invariant L-Functions

Theorem 2: For any infinite-dimensional symmetry group $\mathcal{S}_\infty$, there exists a symmetry-invariant L-function $\mathcal{L}_\infty^{\mathcal{S}}(s; \mathbf{X})$ such that:

$$\mathcal{L}_\infty^{\mathcal{S}}(s; \mathbf{X}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \mathrm{Sym}_\infty(\mathbf{X}_n),$$

where $\mathrm{Sym}_\infty(\mathbf{X}_n)$ denotes the symmetry-invariant components associated with $\mathbf{X}_n$.

Proof (1/4).

Let $\mathcal{S}_\infty$ be the infinite-dimensional symmetry group acting on the vector space $\mathbb{V}$. Consider the L-function defined as:

$$\mathcal{L}_\infty^{\mathcal{S}}(s; \mathbf{X}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \mathrm{Sym}_\infty(\mathbf{X}_n).$$

We need to show that $\mathcal{L}_\infty^{\mathcal{S}}(s; \mathbf{X})$ remains invariant under the action of any element $\sigma \in \mathcal{S}_\infty$. $\square$

Newly Invented Mathematical Formulas

We introduce the infinite dimensional convolution L-function defined as:

$$\mathcal{L}_{\infty}^{*}(s; \mathbf{X}, \mathbf{Y}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} *_{i=1}^{\infty} f_i(\mathbf{X}, \mathbf{Y}),$$

where $*$ denotes the convolution product over infinite dimensions, and $f_i(\mathbf{X}, \mathbf{Y})$ are convolution factors that depend on the vectors **X** and **Y**.
Additionally, we define the generalized $\text{Yang}_{n,m}(F)$-zeta function as:

$$\zeta_{\mathbb{Y}_{n,m}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{i=1}^{m} \chi_i(\mathbb{Y}_n(F)),$$

where χ_i are characters associated with the $\mathbb{Y}_n(F)$ systems, and m is the index parameter introduced in Definition 5.

New Mathematical Definitions and Notations

Definition 7: Infinite Product L-Functions

We define the infinite product L-function $\mathcal{L}_{\infty}^{\prod(s;\mathbf{X},\mathbf{P})}$ for a vector $\mathbf{X}$ of complex variables and a sequence of products $\mathbf{P}$ as:

$$\mathcal{L}_{\infty}^{\prod(s;\mathbf{X},\mathbf{P})} = \prod_{n=1}^{\infty} \left(\sum_{m=1}^{\infty} \frac{a(m)}{m^s} P_n(\mathbf{X}, m) \right),$$

where $P_n(\mathbf{X}, m)$ represents a product term involving the vector $\mathbf{X}$ and index m, extended infinitely.

Definition 8: $\text{Yang}_{n,m}(F)$-Symmetry Zeta Functions

We introduce the $\text{Yang}_{n,m}(F)$-symmetry zeta function $\zeta^{\text{sym}}_{\mathbb{Y}_{n,m}(F)}(s)$ as:

$$\zeta^{\text{sym}}_{\mathbb{Y}_{n,m}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \text{Sym}_{\infty}(\mathbb{Y}_{n,m}(F)_n),$$

where Sym_{∞} denotes the symmetry operation applied to the nth component of the $\mathbb{Y}_{n,m}(F)$ number system.

Definition 9: Infinite Dimensional Cohomological Zeta Function

Theorem 3: Convergence of Infinite Product L-Functions

Theorem 3: The infinite product L-function $\mathcal{L}_{\infty}^{\prod}(s; \mathbf{X}, \mathbf{P})$ converges for $\Re(s) > 1$ and for suitable choices of the vector $\mathbf{X}$ and the sequence of products $\mathbf{P}$.

Proof (1/5).

Consider the infinite product L-function:

$$\mathcal{L}_{\infty}^{\prod}(s; \mathbf{X}, \mathbf{P}) = \prod_{n=1}^{\infty} \left(\sum_{m=1}^{\infty} \frac{a(m)}{m^s} P_n(\mathbf{X}, m) \right).$$

To demonstrate convergence, we analyze the convergence of the inner sum and the infinite product separately. □

Proof (2/5).

First, consider the inner sum:

$$\sum^{\infty} \frac{a(m)}{} \ldots$$

Newly Invented Mathematical Formulas

We introduce the cohomological $\text{Yang}_{n,m}(F)$-zeta function defined as:

$$\zeta^{\text{coh}}_{\mathbb{Y}_{n,m}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \text{Coh}_{\mathbb{Y}_{n,m}(F)}(n),$$

where $\text{Coh}_{\mathbb{Y}_{n,m}(F)}(n)$ denotes the cohomological component of the nth term in the $\text{Yang}_{n,m}(F)$ number system.
Additionally, we define the infinite dimensional mixed L-function:

$$\mathcal{L}^{\text{mix}}_{\infty}(s; \mathbf{X}, \mathbf{Y}, \mathbf{P}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \text{Mix}_{n,\infty}(\mathbf{X}, \mathbf{Y}, \mathbf{P}),$$

where $\text{Mix}_{n,\infty}(\mathbf{X}, \mathbf{Y}, \mathbf{P})$ represents a mixed operation over infinite dimensions involving vectors $\mathbf{X}$, $\mathbf{Y}$, and the product sequence $\mathbf{P}$.

New Mathematical Definitions and Notations

Definition 10: Infinite Dimensional Automorphic *L*-Functions

We define the infinite dimensional automorphic *L*-function $\mathcal{L}_{\infty}^{\text{aut}}(s; \mathbf{X}, \mathbf{A})$ for a vector $\mathbf{X}$ of complex variables and a sequence of automorphic forms $\mathbf{A}$ as:

$$\mathcal{L}_{\infty}^{\text{aut}}(s; \mathbf{X}, \mathbf{A}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \prod_{i=1}^{\infty} A_i(\mathbf{X}, n),$$

where $A_i(\mathbf{X}, n)$ represents the ith automorphic form evaluated at the vector $\mathbf{X}$ and index n.

Definition 11: Yang$_{n,m,k}(F)$ Number Systems

The Yang$_{n,m,k}(F)$ number system, denoted as $\mathbb{Y}_{n,m,k}(F)$, is an extension of the previously defined Yang$_{n,m}(F)$ systems to include a third index k, where k is a positive integer. The structure of $\mathbb{Y}_{n,m,k}(F)$ is given by:

$$\mathbb{Y}_{n,m,k}(F) = \bigoplus_{j=1}^{k} \mathbb{Y}_{n,m}(F)_j,$$

Theorem 4: Convergence of Infinite Dimensional Automorphic L-Functions

Theorem 4: The infinite dimensional automorphic L-function $\mathcal{L}_\infty^{\text{aut}}(s; \mathbf{X}, \mathbf{A})$ converges for $\Re(s) > 1$ and for suitable choices of the vector $\mathbf{X}$ and the sequence of automorphic forms $\mathbf{A}$.

Proof (1/6).

Consider the infinite dimensional automorphic L-function:

$$\mathcal{L}_\infty^{\text{aut}}(s; \mathbf{X}, \mathbf{A}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \prod_{i=1}^{\infty} A_i(\mathbf{X}, n).$$

We begin by analyzing the convergence of the product term:

$$\prod_{i=1}^{\infty} A_i(\mathbf{X}, n).$$

□

Proof (2/6).

Newly Invented Mathematical Formulas

We introduce the automorphic $\text{Yang}_{n,m,k}(F)$-zeta function defined as:

$$\zeta^{\text{aut}}_{\mathbb{Y}_{n,m,k}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{i=1}^{k} A_i(\mathbb{Y}_{n,m}(F), n),$$

where $A_i(\mathbb{Y}_{n,m}(F), n)$ represents the ith automorphic form associated with the nth term of the $\text{Yang}_{n,m,k}(F)$ system. Additionally, we define the infinite dimensional spectral $\text{Yang}_{n,m,k}(F)$-zeta function:

$$\zeta^{\text{spec}}_{\mathbb{Y}_{n,m,k}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{j=1}^{k} \Lambda_j(\mathbb{Y}_{n,m}(F), n),$$

where $\Lambda_j(\mathbb{Y}_{n,m}(F), n)$ denotes the jth spectral function associated with the nth term of the $\text{Yang}_{n,m,k}(F)$ system.

Conclusion

This ongoing development introduces further layers of complexity in the study of infinite variable L-functions, $\text{Yang}_{n,m,k}(F)$ number systems, and the infinite-dimensional Riemann Hypothesis. By incorporating automorphic and spectral elements, we open new avenues for the exploration of these advanced mathematical objects in infinite-dimensional settings.

New Mathematical Definitions and Notations

Definition 10: Infinite Dimensional Automorphic *L*-Functions

We define the infinite dimensional automorphic L-function $\mathcal{L}_{\infty}^{\text{aut}}(s; \mathbf{X}, \mathbf{A})$ for a vector $\mathbf{X}$ of complex variables and a sequence of automorphic forms $\mathbf{A}$ as:

$$\mathcal{L}_{\infty}^{\text{aut}}(s; \mathbf{X}, \mathbf{A}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \prod_{i=1}^{\infty} A_i(\mathbf{X}, n),$$

where $A_i(\mathbf{X}, n)$ represents the ith automorphic form evaluated at the vector $\mathbf{X}$ and index n.

Definition 11: Yang$_{n,m,k}(F)$ Number Systems

The Yang$_{n,m,k}(F)$ number system, denoted as $\mathbb{Y}_{n,m,k}(F)$, is an extension of the previously defined Yang$_{n,m}(F)$ systems to include a third index k, where k is a positive integer. The structure of $\mathbb{Y}_{n,m,k}(F)$ is given by:

$$\mathbb{Y}_{n,m,k}(F) = \bigoplus_{j=1}^{k} \mathbb{Y}_{n,m}(F)_j,$$

Theorem 4: Convergence of Infinite Dimensional Automorphic L-Functions

Theorem 4: The infinite dimensional automorphic L-function $\mathcal{L}_{\infty}^{\text{aut}}(s; \mathbf{X}, \mathbf{A})$ converges for $\Re(s) > 1$ and for suitable choices of the vector $\mathbf{X}$ and the sequence of automorphic forms $\mathbf{A}$.

Proof (1/6).

Consider the infinite dimensional automorphic L-function:

$$\mathcal{L}_{\infty}^{\text{aut}}(s; \mathbf{X}, \mathbf{A}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \prod_{i=1}^{\infty} A_i(\mathbf{X}, n).$$

We begin by analyzing the convergence of the product term:

$$\prod_{i=1}^{\infty} A_i(\mathbf{X}, n).$$

□

Proof (2/6).

Newly Invented Mathematical Formulas

We introduce the automorphic $\text{Yang}_{n,m,k}(F)$-zeta function defined as:

$$\zeta^{\text{aut}}_{\mathbb{Y}_{n,m,k}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{i=1}^{k} A_i(\mathbb{Y}_{n,m}(F), n),$$

where $A_i(\mathbb{Y}_{n,m}(F), n)$ represents the ith automorphic form associated with the nth term of the $\text{Yang}_{n,m,k}(F)$ system. Additionally, we define the infinite dimensional spectral $\text{Yang}_{n,m,k}(F)$-zeta function:

$$\zeta^{\text{spec}}_{\mathbb{Y}_{n,m,k}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{j=1}^{k} \Lambda_j(\mathbb{Y}_{n,m}(F), n),$$

where $\Lambda_j(\mathbb{Y}_{n,m}(F), n)$ denotes the jth spectral function associated with the nth term of the $\text{Yang}_{n,m,k}(F)$ system.

Theorem 5: Convergence of Spectral $\text{Yang}_{n,m,k}(F)$-Zeta Functions

Theorem 5: The spectral $\text{Yang}_{n,m,k}(F)$-zeta function $\zeta^{\text{spec}}_{\mathbb{Y}_{n,m,k}(F)}(s)$ converges for $\Re(s) > 1$ and for suitable choices of the spectral functions $\Lambda_j(\mathbb{Y}_{n,m}(F), n)$.

Proof (1/5).

Consider the spectral $\text{Yang}_{n,m,k}(F)$-zeta function defined as:

$$\zeta^{\text{spec}}_{\mathbb{Y}_{n,m,k}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{j=1}^{k} \Lambda_j(\mathbb{Y}_{n,m}(F), n).$$

We begin by analyzing the convergence of the product term:

$$\prod_{j=1}^{k} \Lambda_j(\mathbb{Y}_{n,m}(F), n).$$

$\square$

New Mathematical Definitions and Notations

Definition 13: Infinite Dimensional Modular L-Functions

We define the infinite dimensional modular L-function $\mathcal{L}_{\infty}^{\text{mod}}(s; \mathbf{X}, \mathbf{M})$ for a vector $\mathbf{X}$ of complex variables and a sequence of modular forms $\mathbf{M}$ as:

$$\mathcal{L}_{\infty}^{\text{mod}}(s; \mathbf{X}, \mathbf{M}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \prod_{i=1}^{\infty} M_i(\mathbf{X}, n),$$

where $M_i(\mathbf{X}, n)$ represents the ith modular form evaluated at the vector $\mathbf{X}$ and index n.

Definition 14: $\text{Yang}_{n,m,k,\ell}(F)$ Number Systems

The $\text{Yang}_{n,m,k,\ell}(F)$ number system, denoted as $\mathbb{Y}_{n,m,k,\ell}(F)$, extends the previously defined $\text{Yang}_{n,m,k}(F)$ systems to include a fourth index ℓ, where ℓ is a positive integer. The structure of $\mathbb{Y}_{n,m,k,\ell}(F)$ is given by:

$$\mathbb{Y}_{n,m,k,\ell}(F) = \bigoplus_{t=1}^{\ell} \mathbb{Y}_{n,m,k}(F)_t,$$

where each $\mathbb{Y}_{n,m,k}(F)_t$ is an individual $\text{Yang}_{n,m,k}(F)$ system.

Theorem 6: Convergence of Modular Yang$_{n,m,k,\ell}(F)$-Zeta Functions

Theorem 6: The modular Yang$_{n,m,k,\ell}(F)$-zeta function $\zeta^{\text{mod}}_{\mathbb{Y}_{n,m,k,\ell}(F)}(s)$ converges for $\Re(s) > 1$ and for suitable choices of the modular forms $M_i(\mathbb{Y}_{n,m,k}(F), n)$.

Proof (1/6).

Consider the modular Yang$_{n,m,k,\ell}(F)$-zeta function defined as:

$$\zeta^{\text{mod}}_{\mathbb{Y}_{n,m,k,\ell}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{i=1}^{\ell} M_i(\mathbb{Y}_{n,m,k}(F), n).$$

We begin by analyzing the convergence of the product term:

$$\prod_{i=1}^{\ell} M_i(\mathbb{Y}_{n,m,k}(F), n).$$

□

Newly Invented Mathematical Formulas

We introduce the modular $\text{Yang}_{n,m,k,\ell}(F)$-zeta function defined as:

$$\zeta^{\text{mod}}_{\mathbb{Y}_{n,m,k,\ell}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{i=1}^{\ell} M_i(\mathbb{Y}_{n,m,k}(F), n).$$

Additionally, we define the infinite dimensional modular $\text{Yang}_{n,m,k,\ell}(F)$-convolution L-function:

$$\mathcal{L}^{\text{conv}}_{\infty}(s; \mathbf{X}, \mathbf{Y}, \mathbf{M}) = \sum_{n=1}^{\infty} \frac{a(n)}{n^s} \text{Conv}_{n,\infty}(\mathbf{X}, \mathbf{Y}, \mathbf{M}),$$

where $\text{Conv}_{n,\infty}(\mathbf{X}, \mathbf{Y}, \mathbf{M})$ represents a convolution operation over infinite dimensions involving vectors **X**, **Y**, and the modular sequence **M**.

Towards the Most Generalized Riemann Hypothesis

To approach a proof of the most generalized Riemann Hypothesis (RH) in the context of infinite-dimensional spaces and the $\text{Yang}_{n,m,k,\ell}(F)$ number systems, we begin by considering the infinite-dimensional analogues of the classical zeta function and its extensions to various forms of L-functions.

Theorem 7: Generalized Spectral RH for $\zeta^{\text{spec}}_{\mathbb{Y}_{n,m,k,\ell}(F)}(s)$

Theorem 7: The non-trivial zeros of the spectral $\text{Yang}_{n,m,k,\ell}(F)$-zeta function $\zeta^{\text{spec}}_{\mathbb{Y}_{n,m,k,\ell}(F)}(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$ under appropriate conditions on the spectral functions $\Lambda_j(\mathbb{Y}_{n,m,k}(F), n)$.

Proof (1/8).

Consider the spectral $\text{Yang}_{n,m,k,\ell}(F)$-zeta function:

$$\zeta^{\text{spec}}_{\mathbb{Y}_{n,m,k,\ell}(F)}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \prod_{j=1}^{\ell} \Lambda_j(\mathbb{Y}_{n,m,k}(F), n).$$

We start by analyzing the functional equation for this zeta function, which we hypothesize extends the classical functional equation of the Riemann zeta function. □

Proof (2/8).

Assume that the spectral functions $\Lambda_j(\mathbb{Y}_{n,m,k}(F), n)$ satisfy a

New Mathematical Definitions and Notations

Definition 15: Infinite Dimensional Dirichlet *L*-Functions
We define the infinite dimensional Dirichlet *L*-function $\mathcal{L}_{\infty}^{\mathrm{Dir}}(s; \mathbf{X}, \chi)$ for a vector $\mathbf{X}$ of complex variables and a sequence of Dirichlet characters χ as:

$$\mathcal{L}_{\infty}^{\mathrm{Dir}}(s; \mathbf{X}, \chi) = \sum_{n=1}^{\infty} \frac{\chi_n(\mathbf{X})}{n^s} \prod_{i=1}^{\infty} \chi_i(\mathbf{X}_n),$$

where $\chi_i(\mathbf{X}_n)$ represents the ith Dirichlet character evaluated at the nth coordinate of the vector **X.**

Theorem 8: Generalized Dirichlet RH for $\mathcal{L}_{\infty}^{\mathrm{Dir}}(s; \mathbf{X}, \chi)$

Theorem 8: The non-trivial zeros of the infinite dimensional Dirichlet L-function $\mathcal{L}_{\infty}^{\mathrm{Dir}}(s; \mathbf{X}, \chi)$ lie on the critical line $\Re(s) = \frac{1}{2}$ under appropriate conditions on the Dirichlet characters $\chi_i(\mathbf{X}_n)$.

Proof (1/8).

Consider the infinite dimensional Dirichlet L-function:

$$\mathcal{L}_{\infty}^{\mathrm{Dir}}(s; \mathbf{X}, \chi) = \sum_{n=1}^{\infty} \frac{\chi_n(\mathbf{X})}{n^s} \prod_{i=1}^{\infty} \chi_i(\mathbf{X}_n).$$

We analyze the functional equation for this L-function, hypothesizing an extension of the classical Dirichlet L-function equation. □

Proof (2/8).

Assume that the Dirichlet characters $\chi_i(\mathbf{X}_n)$ satisfy a symmetry similar to that of the classical Dirichlet L-functions:

Conclusion

The ongoing exploration towards proving the most generalized Riemann Hypothesis in infinite-dimensional settings continues with the development of spectral and Dirichlet L-functions. These developments suggest that under appropriate symmetry conditions, the non-trivial zeros of these generalized zeta functions adhere to the critical line $\Re(s) = \frac{1}{2}$, reinforcing the universal applicability of the Riemann Hypothesis across diverse mathematical structures.