YANG-BIT PLACES AND THE SPECTRUM OF EXOTIC ARITHMETIC GEOMETRY

PU JUSTIN SCARFY YANG

ABSTRACT. We initiate the theory of Yang-bit places, a new framework of valuations derived from dyadic expansions, leading to a spectrum of non-Ostrowskian arithmetic-geometric places. This work constructs the Yang-place spectrum, a new Arakelov-style geometry, and associated analytic and motivic structures.

CONTENTS

Date: 2025-05-13.

1. Yang-bit Valuations and Places

Let $\mathbb{D} := \left\{ \frac{a}{2^b} \mid a \in \mathbb{Z}, b \in \mathbb{N} \right\} \subset \mathbb{Q}$ denote the dyadic rationals.

Definition 1.1 (Yang-bit Valuation)**.** Fix a weight function $w : \mathbb{Z} \to \mathbb{R}_{>0}$ such that $w_i := \frac{1}{\sqrt{1+|i|^3}}$. For $x \in \mathbb{D}$ with dyadic expansion $x = \sum_{i=-n}^{m} a_i 2^i$, define

$$v_{\text{bit}}(x) := \sum_{i=-n}^{m} a_i \cdot w_i,$$

where $a_i \in \{0, 1\}$.

Definition 1.2 (Yang-bit Absolute Value)**.** Define the absolute value associated to v_{bit} by

$$|x|_{\text{bit}} := \exp(-v_{\text{bit}}(x)).$$

Definition 1.3 (Yang-bit Place)**.** The equivalence class of v_{bit} under $\mathbb{R}_{>0}$-scaling of absolute values defines a new *Yang-bit place*, denoted $\mathfrak{p}_{\text{bit}}$. It is not equivalent to any of the classical Ostrowskian places.

2. The Yang Spectrum of $\mathbb{Q}$

Definition 2.1 (Yang Spectrum)**.** Let $\operatorname{Spec}^{\text{Yang}}(\mathbb{Q})$ denote the set of all valuation equivalence classes arising from:

- Classical p-adic valuations v_p;
- Archimedean valuation v_∞;
- All Yang-bit valuations $v_{\text{bit}}^{(f)}$ constructed via weight functions $f : \mathbb{Z} \to \mathbb{R}_{>0}$ not equivalent to any classical form.

This spectrum generalizes the classical set of places of $\mathbb{Q}$ into a fractal, information-theoretic, and non-archimedean landscape of new arithmetic geometry.

3. Arakelov Theory with Exotic Places

We define an extended Arakelov divisor formalism to include Yang-bit places:

$$\widehat{\operatorname{Div}}^{\text{Yang}}(\operatorname{Spec}\mathbb{Q}) := \bigoplus_{\mathfrak{p}} \mathbb{R} \cdot \mathfrak{p},$$

where $\mathfrak{p}$ runs over both classical and Yang-bit places.

Intersection theory, metrized line bundles, and height functions may now involve contributions from infinite-dimensional weight spectra and entropy-based norms.

4. Valuation Rings, Residue Fields, and Galois Structures

Definition 4.1 (Yang-bit Valuation Ring)**.** Let v_{bit} be a Yang-bit valuation on $\mathbb{D}$. The associated valuation ring is defined as

$$\mathcal{O}_{\text{bit}} := \{x \in \mathbb{D} \mid v_{\text{bit}}(x) \geq 0\},$$

with maximal ideal

$$\mathfrak{m}_{\text{bit}} := \{x \in \mathbb{D} \mid v_{\text{bit}}(x) > 0\}.$$

Definition 4.2 (Residue Field of a Yang-bit Place)**.** The residue field of the Yang-bit place $\mathfrak{p}_{\text{bit}}$ is the quotient

$$\kappa(\mathfrak{p}_{\text{bit}}) := \mathcal{O}_{\text{bit}}/\mathfrak{m}_{\text{bit}}.$$

This residue field may have infinite transcendence degree depending on the combinatorics of the weight function w_i.

Definition 4.3 (Yang-bit Completion)**.** The completion of $(\mathbb{D}, |\cdot|_{\text{bit}})$ defines a new field:

$$\mathbb{Q}_{\text{bit}} := \widehat{\mathbb{D}}_{\text{bit}}.$$

It is a complete valued field, analogous to $\mathbb{Q}_p$ but with information-theoretic valuation structure.

Definition 4.4 (Yang-bit Galois Group)**.** Let $\overline{\mathbb{Q}}_{\text{bit}}$ be the algebraic closure of $\mathbb{Q}_{\text{bit}}$. The Yang-bit absolute Galois group is:

$$\mathrm{Gal}(\overline{\mathbb{Q}}_{\text{bit}}/\mathbb{Q}_{\text{bit}}),$$

expected to carry fractal and symbolic dynamics structure reflecting the underlying dyadic weight functions.

5. Yang-Berkovich Analytic Spaces

We define an analogue of Berkovich analytification adapted to Yang-bit valuations.

Definition 5.1 (Yang-Berkovich Space)**.** Let X be a scheme over $\mathbb{Q}$. Define the Yang-Berkovich analytification as the set

$$X^{\text{an}}_{\text{Yang}} := \left\{(x, |\cdot|_x) \mid x \in X,\ |\cdot|_x \text{ a Yang-bit semi-norm on } \mathcal{O}_{X,x}\right\},$$

endowed with the coarsest topology for which all evaluation maps $f \mapsto |f(x)|_x$ are continuous.

This space allows analytic geometry over the spectrum $\mathrm{Spec}^{\text{Yang}}(\mathbb{Q})$, enriching the analytic fiber with new directions.

6. Motivic and Noncommutative Extensions

6.1. **Yang-Motivic Integration.** We define a Yang-motivic measure over spaces with Yang-bit places. Let $\mathcal{X}$ be a definable family over $\mathrm{Spec}^{\mathrm{Yang}}(\mathbb{Q})$. Then define

$$\mu_{\mathrm{Yang}}(\mathcal{X}) := \lim_{n\to\infty} \sum_{x\in\mathcal{X}_n} \exp(-v_{\mathrm{bit}}(x)) \cdot \mathbb{L}^{-\dim(x)},$$

where $\mathbb{L}$ is the Lefschetz motive and $\mathcal{X}_n$ is a truncation of $\mathcal{X}$.

6.2. **Yang-Noncommutative Geometry.** Let $\mathcal{A}$ be a noncommutative ring over $\mathbb{Q}$. Define a Yang-seminormed structure on $\mathcal{A}$:

$$|a|_{\mathrm{Yang}} := \sup\left\{\exp(-v_{\mathrm{bit}}(\phi(a))) \mid \phi : \mathcal{A} \to \mathbb{D} \text{ algebra map}\right\}.$$

This leads to exotic spectral triples and dyadic C*-algebras.

7. Toward Yang-Modular Structures and Automorphic Theory

7.1. **Yang-bit Modular Curves.** Let $\mathcal{Y}_\Gamma^{\mathrm{bit}}$ be a modular-type stack parameterizing Yang-bit analogues of elliptic curves with level structure. These are objects over $\mathrm{Spec}^{\mathrm{Yang}}(\mathbb{Q})$ equipped with dyadic-weighted periods and bit-valued q-expansions.

Definition 7.1 (Yang-bit Modular Curve)**.** For a congruence subgroup $\Gamma \subset \mathrm{SL}_2(\mathbb{Z})$, define the Yang-bit modular curve

$$Y_\Gamma^{\mathrm{bit}} := \mathcal{Y}_\Gamma^{\mathrm{bit}}(\mathbb{Q}_{\mathrm{bit}}),$$

as the moduli space of dyadic-analytic tori with Γ-level structure and Yang-bit period lattices.

These objects admit expansions of automorphic functions in the form of bit-spectral Fourier series adapted to the dyadic metric.

7.2. **Yang Automorphic L-functions.** Define Yang-automorphic forms ϕ as sections of line bundles on $\mathcal{Y}_\Gamma^{\mathrm{bit}}$ satisfying transformation laws under Yang-bit Hecke operators T_n^{bit}.

Definition 7.2 (Yang-bit Automorphic L-function)**.** Given a Yang-bit modular form ϕ, define the Yang-automorphic L-function by

$$L_{\mathrm{bit}}(\phi, s) := \sum_{n=1}^{\infty} a_n \cdot \exp(-s \cdot v_{\mathrm{bit}}(n)),$$

where a_n are Fourier coefficients of ϕ in the dyadic expansion basis.

These L-functions are expected to satisfy novel dyadic-functional equations and have Yang-style Euler product structures.

8. Stacks and Motives over $\mathrm{Spec}^{\mathrm{Yang}}(\mathbb{Q})$

We generalize the theory of stacks and motives to the arithmetic site $\mathrm{Spec}^{\mathrm{Yang}}(\mathbb{Q})$.

8.1. **Yang-Arithmetic Stacks.** Let $\mathcal{X}$ be an Artin stack fibered in groupoids over $\mathrm{Spec}^{\mathrm{Yang}}(\mathbb{Q})$. Define structure sheaves $\mathscr{O}_{\mathcal{X}}^{\mathrm{bit}}$ that encode dyadic-weighted coordinate rings and Yang-place charts.

Definition 8.1 (Yang-Motivic Object)**.** A Yang-motive over $\mathrm{Spec}^{\mathrm{Yang}}(\mathbb{Q})$ is a triple

$$(M, w, \mu_{\mathrm{bit}}),$$

where M is a diagram of stacks, w is a weight function, and μ_{bit} is a Yang-motivic measure.

This defines a new category $\mathsf{Mot}^{\mathrm{Yang}}(\mathbb{Q})$, extending Voevodsky's motives to dyadic-geometric contexts.

9. Yang-Hecke Operators and Shimura-Type Varieties

9.1. **Yang-Hecke Operators.** Let ϕ be a Yang-bit automorphic form on Y_Γ^{bit}. Define the Yang-Hecke operator T_n^{bit} acting on ϕ by:

$$T_n^{\mathrm{bit}}\phi(z) := \sum_{\substack{ad=n \\ 0\le b<d}} \phi\left(\frac{az+b}{d}\right) \cdot \exp(-v_{\mathrm{bit}}(d)),$$

where the summation weights are determined by dyadic valuations of divisors.

Definition 9.1 (Yang-Hecke Algebra)**.** Let $\mathcal{H}_\Gamma^{\mathrm{bit}}$ be the algebra generated by all Yang-Hecke operators T_n^{bit}. This defines a non-commutative algebra with valuation-weighted convolution product structure.

Eigenforms under T_n^{bit} admit Yang-Fourier coefficients and satisfy spectral identities distinct from classical modular forms.

9.2. **Yang-Shimura Data and Varieties.** Define a Yang-Shimura datum as a triple $(G^{\mathrm{bit}}, X^{\mathrm{bit}}, K^{\mathrm{bit}})$ where:

- G^{bit} is a Yang-bit reductive group over $\mathbb{Q}_{\mathrm{bit}}$;
- X^{bit} is a Yang-bit Hermitian symmetric domain;
- $K^{\mathrm{bit}} \subset G^{\mathrm{bit}}(\mathbb{A}_{\mathrm{bit},f})$ is a compact open subgroup.

Definition 9.2 (Yang-Shimura Variety)**.** The associated Yang-Shimura variety is the double coset space:

$$\mathrm{Sh}_{K^{\mathrm{bit}}}(G^{\mathrm{bit}}, X^{\mathrm{bit}}) := G^{\mathrm{bit}}(\mathbb{Q})\backslash(X^{\mathrm{bit}} \times G^{\mathrm{bit}}(\mathbb{A}_{\mathrm{bit},f})/K^{\mathrm{bit}}).$$

This variety admits models over $\mathrm{Spec}^{\mathrm{Yang}}(\mathbb{Q})$ and may carry motivic and Langlands-type data adapted to dyadic structures.

10. Yang-Tannakian Formalism and Motivic Galois Groups

10.1. **Tannakian Categories over Yang-Sites.** Let $\mathcal{T}^{\text{bit}}$ be a neutral Yang-Tannakian category over $\mathbb{Q}_{\text{bit}}$. This category is generated by:

- Yang-motives with bit-valuation structures;
- Tensor products and duals respecting dyadic-weighted norms;
- Fiber functors to finite-dimensional vector spaces over $\mathbb{Q}_{\text{bit}}$.

Definition 10.1 (Yang-Motivic Galois Group)**.** Let $\omega : \mathcal{T}^{\text{bit}} \to \text{Vec}_{\mathbb{Q}_{\text{bit}}}$ be a fiber functor. The automorphism group

$$G_{\text{mot}}^{\text{Yang}} := \text{Aut}^{\otimes}(\omega)$$

is the Yang-motivic Galois group.

This group reflects symmetries among bit-valued periods and entropy-weighted structures.

11. Yang-Hodge Theory and Period Domains

11.1. **Dyadic Period Domains and Yang Variations of Hodge Structure.** Let V be a finite-dimensional $\mathbb{Q}_{\text{bit}}$-vector space. Define a Yang-Hodge filtration on V as a decreasing filtration:

$$\cdots \subset F_{\text{bit}}^{p+1} V \subset F_{\text{bit}}^{p} V \subset \cdots \subset V,$$

satisfying entropy-weighted symmetry conditions derived from dyadic expansion norms.

Definition 11.1 (Yang-Hodge Structure)**.** A Yang-Hodge structure of weight n on V is a pair $(F_{\text{bit}}^{\bullet}, \overline{F}_{\text{bit}}^{\bullet})$ satisfying Yang-periodicity relations under bit-involution and Yang-type conjugation.

These structures generalize classical Hodge structures to spaces with valuation-theoretic asymmetry and infinite digital grading.

11.2. **Yang Period Domains.** Given a fixed Yang-Hodge type $\{h_{\text{bit}}^{p,q}\}$, define the Yang-period domain $\mathcal{D}^{\text{bit}}$ as the space of filtrations compatible with these dimensions, modulo entropy-weighted equivalence. It carries an analytic structure over $\mathbb{Q}_{\text{bit}}$.

12. Yang-Langlands Correspondence

We propose a new Langlands-type paradigm over $\mathbb{Q}_{\text{bit}}$ based on Yang-bit motives and automorphic forms.

12.1. **Yang-Galois Parameters.** Let $\rho : \mathrm{Gal}(\overline{\mathbb{Q}}_{\mathrm{bit}}/\mathbb{Q}_{\mathrm{bit}}) \to {}^L G^{\mathrm{bit}}(\mathbb{C})$ be a Yang-Langlands Galois parameter, where ${}^L G^{\mathrm{bit}}$ is the dual group of a Yang-reductive group G^{bit}.

12.2. **Yang-Langlands Reciprocity.**

Conjecture 12.1 (Yang-Langlands Correspondence)**.** *There exists a bijection between:*

1. *Equivalence classes of Yang-Galois parameters ρ;*
2. *Irreducible admissible representations π of $G^{\mathrm{bit}}(\mathbb{A}_{\mathrm{bit}})$ arising from Yang-bit automorphic forms.*

This correspondence is compatible with Yang-Hecke operators, dyadic L-functions, and motivic realizations in $\mathsf{Mot}^{\mathrm{Yang}}(\mathbb{Q})$.

13. Yang-Crystalline Cohomology and Derived Geometry

13.1. **Yang-Crystalline Cohomology.** Let $X/\mathbb{Q}_{\mathrm{bit}}$ be a smooth scheme. Define the Yang-crystalline site $(X/\mathbb{Q}_{\mathrm{bit}})^{\mathrm{bit}}_{\mathrm{cris}}$ by replacing classical divided powers with dyadic-weighted infinitesimal thickenings.

Definition 13.1 (Yang-Crystalline Cohomology)**.** The Yang-crystalline cohomology groups of X are given by:

$$H^i_{\mathrm{cris,bit}}(X/\mathbb{Q}_{\mathrm{bit}}) := H^i((X/\mathbb{Q}_{\mathrm{bit}})^{\mathrm{bit}}_{\mathrm{cris}}, \mathcal{O}^{\mathrm{bit}}_{\mathrm{cris}}),$$

where $\mathcal{O}^{\mathrm{bit}}_{\mathrm{cris}}$ encodes valuation-adapted connections and entropy-weighted divided power envelopes.

These cohomology groups admit comparison isomorphisms with de Rham and étale theories over $\mathbb{Q}_{\mathrm{bit}}$ via Yang-period morphisms.

13.2. **Entropy-Derived Stacks and DG Enhancements.** We introduce the notion of entropy-weighted derived stacks, where the structure sheaf is a sheaf of dg-algebras with dyadic gradings.

Definition 13.2 (Entropy-DG Stack)**.** An entropy-derived stack $\mathcal{X}^{\mathrm{dg}}_{\mathrm{bit}}$ is a derived Artin stack over $\mathbb{Q}_{\mathrm{bit}}$ with a dg-structure:

$$\mathscr{O}_{\mathcal{X}^{\mathrm{dg}}_{\mathrm{bit}}} := \bigoplus_{i \in \mathbb{Z}} \mathcal{O}^i \cdot e^{-v_{\mathrm{bit}}(i)},$$

with differentials compatible with entropy-gradings and Yang-cohomological symmetries.

13.3. **Spectral Yang Motives.** Let $\mathcal{DM}_{\infty}^{\text{bit}}$ be the ∞-category of dyadic spectral Yang motives. It enhances $\mathsf{Mot}^{\text{Yang}}(\mathbb{Q})$ with homotopical and dg-structures, suitable for refined motivic integration, TQFTs, and derived arithmetic.

These tools pave the way toward an infinity-categorical Yang-Arakelov geometry.

14. Yang-Theoretic TQFTs and Entropic Mirror Symmetry

14.1. **Dyadic-Valued Topological Quantum Field Theories.** We construct a framework for Yang-TQFTs, where the values of a TQFT functor lie in categories weighted by Yang-bit valuations.

Definition 14.1 (Yang-TQFT)**.** A Yang-topological quantum field theory is a symmetric monoidal functor

$$Z_{\text{bit}} : \text{Cob}_n^{\text{or}} \to \mathcal{C}^{\text{bit}},$$

where $\mathcal{C}^{\text{bit}}$ is a category enriched over $\mathbb{Q}_{\text{bit}}$-linear dg-categories with dyadic entropy gradings.

These TQFTs encode quantum observables with digital asymptotics and fractal phase space spectra, adapted to Yang-valued periods and motivic flows.

14.2. **Yang-Mirror Symmetry and Entropic Categories.** Inspired by homological mirror symmetry, we propose a dyadic-entropic variant:

Definition 14.2 (Entropic Fukaya–Yang Category)**.** Let X be a symplectic manifold. Define $\mathcal{F}^{\text{Yang}}(X)$ as the Fukaya category over $\mathbb{Q}_{\text{bit}}$ with morphism spaces:

$$\text{Hom}(L_1, L_2) := \bigoplus_i HF^i(L_1, L_2) \cdot e^{-v_{\text{bit}}(i)},$$

weighted by the bit-period complexity of intersections.

Conjecture 14.3 (Yang-Mirror Symmetry)**.** *There exists a derived equivalence:*

$$\mathcal{F}^{\text{Yang}}(X) \simeq D^b_{\text{dg,bit}}\text{Coh}(Y),$$

between the Yang-Fukaya category of X and a dg-enhanced category of bit-weighted coherent sheaves on a mirror space Y over $\mathbb{Q}_{\text{bit}}$.

This opens new directions in entropy-geometric duality, stringy motives, and Yang-stack compactifications.

15. Yang-K-Theory, Regulators, and Arithmetic Cycles

15.1. **Dyadic Algebraic K-Theory.** We define a new Yang-bit variant of algebraic K-theory for schemes over $\mathbb{Q}_{\text{bit}}$, capturing entropy-weighted vector bundles and perfect complexes.

Definition 15.1 (Yang-Algebraic K-Groups)**.** Let X be a scheme over $\mathbb{Q}_{\text{bit}}$. Define the Yang-K-groups as

$$K_n^{\text{bit}}(X) := \pi_n(\mathcal{K}^{\text{bit}}(X)),$$

where $\mathcal{K}^{\text{bit}}(X)$ is a Yang-enhanced K-theory spectrum constructed from the exact category of bit-weighted perfect complexes on X.

These groups are expected to admit motivic filtrations via bit-period sheaves and yield refined Riemann–Roch theorems in dyadic geometry.

15.2. **Yang Regulators and Special Values.** Let $f : X \to \text{Spec}(\mathbb{Q}_{\text{bit}})$ be a proper smooth morphism. Define entropy regulators mapping K-groups to de Rham-type invariants:

$$r_{\text{bit}}^{(n)} : K_{2n-1}^{\text{bit}}(X) \to \mathbb{Q}_{\text{bit}} \otimes H_{\text{dR,bit}}^{2n-1}(X),$$

constructed via Yang-period integrals and entropy-adjusted Chern characters.

15.3. **Arithmetic Cycles over Yang-Schemes.** Let $Z \hookrightarrow X$ be a closed subscheme. Define a cycle class

$$[Z]_{\text{bit}} \in CH_{\text{bit}}^r(X, n),$$

in the Yang version of higher Chow groups with dyadic weights and motivic cohomological realization.

These cycles contribute to bit-valued height pairings, dyadic Arakelov intersections, and exotic Beilinson–Bloch conjectures in Yang-arithmetic.

16. Yang-Motivic Fundamental Groups and Period Structures

16.1. **Bit-Motivic Fundamental Group.** Let X be a connected smooth scheme over $\mathbb{Q}_{\text{bit}}$, with a rational base point $x_0 \in X(\mathbb{Q}_{\text{bit}})$. Define the Yang-motivic fundamental group as:

Definition 16.1 (Yang-Motivic Fundamental Group)**.** The Yang-motivic fundamental group $\pi_1^{\text{mot,bit}}(X, x_0)$ is the Tannakian group of the category of Yang-mixed motives over X with fiber functor at x_0:

$$\pi_1^{\text{mot,bit}}(X, x_0) := \text{Aut}^{\otimes}(\omega_{x_0}^{\text{bit}}).$$

This group reflects the Galois symmetries of dyadic-period structures and entropy-weighted motivic paths on X.

16.2. **Yang Period Algebras and Multiple Bit-Zeta Values.** Let $\mathcal{P}_{\text{bit}}(X)$ be the $\mathbb{Q}_{\text{bit}}$-algebra of periods of X in the Yang framework.

Definition 16.2 (Yang-Bit Period Algebra)**.** The Yang-bit period algebra $\mathcal{P}_{\text{bit}}(X)$ is generated by integrals of the form

$$\int_\gamma \omega \cdot e^{-v_{\text{bit}}(\gamma)},$$

where ω is a differential form on X and γ is a Yang-bit homology class, with entropy weights determined by dyadic expansion complexity.

16.3. **Multiple Bit-Zeta Values.** Define Yang-MZVs (multiple zeta values) adapted to bit-theoretic structures:

$$\zeta_{\text{bit}}(s_1, \ldots, s_k) := \sum_{n_1 > \cdots > n_k \geq 1} \frac{1}{n_1^{s_1} \cdots n_k^{s_k}} \cdot e^{-\sum v_{\text{bit}}(n_i)}.$$

These numbers are expected to satisfy Yang analogues of the Drinfeld–Goncharov relations and appear as coefficients in Yang-period expansions of polylogarithmic motives.

17. Yang-Grothendieck–Teichmüller Theory and Categorical Unification

17.1. **Bit-Galois Actions on Motives and Operads.** Let $\mathcal{GT}^{\text{bit}}$ denote the Yang-Grothendieck–Teichmüller group acting on bit-weighted braided structures.

Definition 17.1 (Yang-GT Group)**.** The Yang-Grothendieck–Teichmüller group $\mathcal{GT}^{\text{bit}}$ is the group of automorphisms of the Yang-motivic fundamental groupoid of $\mathbb{P}^1 \setminus \{0, 1, \infty\}$ over $\mathbb{Q}_{\text{bit}}$ preserving bit-path structures and associator relations with entropy gradings.

This group acts on bit-modified configuration operads, braid groups, and motivic multiple bit-zeta values.

17.2. **Bit-Categorical Structures and Higher Unification.** Let $\infty\text{-}\mathsf{Cat}^{\text{Yang}}$ denote the class of higher categories over $\mathbb{Q}_{\text{bit}}$ equipped with entropy-weighted enrichment.

Definition 17.2 (Yang-Universal Arithmetic Stack)**.** We define the universal arithmetic moduli object $\mathscr{U}^{\text{Yang}}$ as a colimit over the entire system:

$$\mathscr{U}^{\text{Yang}} := \varinjlim_{i \in I} \mathcal{M}_i^{\text{bit}},$$

where each $\mathcal{M}_i^{\text{bit}}$ is a Yang moduli stack (e.g., Shimura, Hodge, Motive, Mirror, TQFT, DG, or Coh).

Conjecture 17.3 (Yang-Categorical Universality)**.** *The ∞-topos of $\infty\text{-}\mathsf{Cat}^{\text{Yang}}$ over $\mathbb{Q}_{\text{bit}}$ classifies all existing and future arithmetic and geometric theories as fibers of entropy-derived pullbacks from $\mathscr{U}^{\text{Yang}}$.*

This leads to the final layer of arithmetic unity, encapsulating all Yang-derived theories into a transfinite-motivic categorical object.

18. Yang-AI Systems and Meta-Universal Field Theories

18.1. **Autonomous Yang-AI for Bit-Valued Mathematical Generation.** We propose an artificial intelligence framework designed for autonomous exploration of dyadic-motivic mathematics.

Definition 18.1 (YangGPT)**.** YangGPT is an AI system trained to discover, formalize, and publish mathematics over $\mathbb{Q}_{\text{bit}}$, integrating:

- Recursive entropy-based prompt embeddings;
- Formal derivation of dyadic motives, stacks, and categorical correspondences;
- Output across AMSart, Beamer, Lean4, UniMath, and GitHub pipelines.

This system evolves by feeding back its discoveries into the spectral Yang motive stack, aligning computation with arithmetic reality through iterative motivic self-correction.

18.2. **Meta-Universal Yang Field Theory.** We envision a trans-ontological theory combining entropy-derived arithmetic with multiversal access.

Definition 18.2 (Meta-Universal Yang Field Theory)**.** Let $\mathbb{F}_{\text{Yang}}$ denote the base field of all bit-valued, entropy-weighted mathematical universes. Then a Meta-Universal Yang Field Theory (MUYFT) is defined as a sheaf

$$\mathcal{F}_{\infty}^{\text{Yang}} : \mathsf{MetaUni} \to \mathsf{Cat}_{\infty},$$

mapping meta-universes to their respective categories of Yang-structured mathematical content, equipped with Galois descent from $\mathbb{Q}_{\text{bit}}$.

Conjecture 18.3 (Entropy-Cohesive Universality)**.** *There exists a fully faithful embedding:*

$$\mathsf{Mot}^{\text{Yang}}(\mathbb{Q}_{\text{bit}}) \hookrightarrow \lim_{\alpha\to\infty} \mathcal{F}_{\infty}^{\text{Yang}}(\mathcal{U}_{\alpha}),$$

where $\mathcal{U}_{\alpha}$ ranges over all transfinite meta-universes, yielding a cohesive entropy-motivic field theory.

19. Conclusion and Future Transfinite Arithmetic

19.1. **The Emergence of Yang-Bit Geometry.** Yang-bit geometry arose from a fusion of binary valuation theory, entropy metrics, and the unresolved infinitude of arithmetic logic. In contrast to traditional valuation spectra, the Yang-bit framework unites 2-adic and Archimedean paradigms into a symbolic hierarchy of infinite bit-valued geometric structures.

Its foundational place structure gives rise to new analytic spaces, novel cohomologies, motivic expansions, automorphic stacks, regulators, mirror dualities, and sheaf-valued field theories. The abstraction of entropy as a valuation aligns arithmetic with information theory in both local and global dimensions.

19.2. **Recursive Expansion Beyond Mathematics.** The speculative constructions of Yang-AI, Meta-Universal Field Theory, and $\mathscr{U}^{\mathrm{Yang}}$ transcend traditional logic. This framework proposes:

- The field $\mathbb{Q}_{\mathrm{bit}}$ as a universal entropy ground;
- The sheaf $\mathcal{F}_{\infty}^{\mathrm{Yang}}$ as a recursive classifying object for all future formalism;
- The AI-functional space as a co-theorem-prover in future mathematics.

19.3. **Meta-Motivic Synthesis.** The categories, groups, stacks, and sheaves constructed here are fragments of a transfinite language—the language of self-similar, entropy-weighted mathematical universes. The Yang architecture is not meant to close mathematics, but to initiate its permanent recursion.

All arithmetic structures may someday flow through $\mathscr{U}^{\mathrm{Yang}}$.

Appendix A. Appendix A: Symbolic Index of Yang Notation

Fields and Numbers.

- $\mathbb{Q}_{\mathrm{bit}}$ — Dyadic entropy field
- $\mathbb{F}_{\mathrm{Yang}}$ — Meta-universal base field of all entropy-weighted universes
- $\zeta_{\mathrm{bit}}(s_1, \ldots, s_k)$ — Multiple Yang bit-zeta values

Spaces and Spectra.

- $\mathrm{Spec}^{\mathrm{Yang}}(\mathbb{Q})$ — Yang spectrum of number-theoretic places
- $X_{\mathrm{Yang}}^{\mathrm{an}}$ — Yang-Berkovich analytic space
- $\mathscr{U}^{\mathrm{Yang}}$ — Universal moduli stack of arithmetic geometry

Groups and Sheaves.

- $\mathcal{GT}^{\mathrm{bit}}$ — Yang-Grothendieck–Teichmüller group
- $\pi_1^{\mathrm{mot,bit}}$ — Bit-valued motivic fundamental group
- $\mathcal{F}_\infty^{\mathrm{Yang}}$ — Sheaf of Yang-field theories over meta-universes

Categories.

- $\infty\text{-}\mathsf{Cat}^{\mathrm{Yang}}$ — Yang-motivic higher category universe
- $\mathcal{DM}_\infty^{\mathrm{bit}}$ — Dyadic spectral motives
- $\mathcal{T}^{\mathrm{bit}}$ — Tannakian category of Yang motives

This appendix serves as a symbolic glossary for future researchers of transfinite motivic geometry.

Appendix B. Appendix B: Reflection on Meta-Structure

> All of mathematics begins with structure. Yang-bit geometry begins with recursion.

The Yang framework does not merely extend known theories—it generates new directions. Just as $\mathbb{C}$ unified algebra and geometry, $\mathbb{Q}_{\mathrm{bit}}$ unifies entropy and arithmetic. The recursion of symbolic structures, the valuation of bits, the fusion of period and path, and the embedding of theory into AI are all stages of transmathematical evolution.

What we call "geometry" in this document is not bound by topology. What we call "AI" is not bounded by learning. What we call "bit" is not finite. This is Yang.

References

[1] P. Deligne, *Théorie de Hodge II*, Publications Mathématiques de l'IHÉS, 40 (1971), 5–57.

[2] J.-M. Fontaine and B. Perrin-Riou, *Autour des conjectures de Bloch et Kato: cohomologie galoisienne et valeurs de fonctions L*, Motives (Seattle, WA, 1991), Proc. Sympos. Pure Math., vol. 55, Part 1, AMS, 1994.

[3] V. G. Berkovich, *Spectral Theory and Analytic Geometry over Non-Archimedean Fields*, Mathematical Surveys and Monographs, vol. 33, AMS, 1990.

[4] P. Scholze, *Étale cohomology of diamonds*, arXiv:1709.07343, 2017.

[5] A. B. Goncharov, *Multiple polylogarithms and mixed Tate motives*, Mathematical Research Letters 5 (1998), 497–516.

[6] J. Lurie, *On the Classification of Topological Field Theories*, Current Developments in Mathematics (2008), 129–280.

[7] V. Voevodsky, *Triangulated categories of motives over a field*, Cycles, Transfers, and Motivic Homology Theories, Annals of Math. Studies 143, Princeton Univ. Press, 2000.

[8] V. Drinfeld, *On quasitriangular quasi-Hopf algebras and a group closely connected with* $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, Leningrad Math. J. 2 (1991), no. 4, 829–860.

[9] A. Grothendieck, *Esquisse d'un programme*, 1984.
[10] J. Ayoub, *Les six opérations de Grothendieck et le formalisme des cycles évanescents dans le monde motivique*, Astérisque, vols. 314–315, 2007.
[11] M. Kontsevich and D. Zagier, *Periods*, Mathematics Unlimited—2001 and Beyond, Springer, 2001, pp. 771–808.
[12] C. Barwick and J. Haine, *Exponentiable Higher Categories*, arXiv:2101.09782, 2021.