EXISTENTIAL HOMOTOPY THEORY: FOUNDATIONS OF ONTOLOGICAL PATHS AND HIGHER EXISTENTIAL STRUCTURES

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Abstract. We initiate the development of *Existential Homotopy Theory* (EHT), a generalization of classical homotopy theory grounded in Ontology-Based Type Theory (OBTT) and Existential Moduli Spaces. In this framework, existential structures are not static entities but unfold dynamically through differentiation, symbolization, relation, fusion, and unfolding operations. EHT formalizes existential "paths," "homotopies," and "higher homotopies" between existential structures, providing a systematic model for the flows, deformations, and meta-evolutions across existential spaces and multiversal structures. This paper provides the foundational definitions, basic properties, and future directions of EHT.

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Date: May 10, 2025.

1. Introduction

Classical Homotopy Theory studies continuous deformations between topological spaces, capturing the essential shape and connectivity properties of spaces through paths and higher homotopies.

Existential Homotopy Theory (EHT) extends this idea into the realm of ontological structures, modeling the dynamic flows and transformations of existential structures themselves.

Grounded in:

- Ontology-Based Type Theory (OBTT),
- Existential Moduli Space ($\mathcal{E}$),
- Higher Topos Structures,

EHT introduces:

- Existential points,
- Existential paths (first-order unfoldings),
- Existential homotopies (deformations of unfoldings),
- Higher existential homotopies (meta-deformations).

The goal is to create a full existential analog of classical homotopy theory, but adapted to the dynamic, layered, multiversal context revealed by OBTT and its extensions.

This paper lays the groundwork for this new field.

2. Foundational Definitions of Existential Homotopy Theory

We now establish the basic definitions underlying Existential Homotopy Theory (EHT).

2.1. **Existential Points.**

Definition 2.1 (Existential Point)**.** *An* existential point *is an equivalence class $[e]_{\mathcal{N}} \in \mathcal{E}$ in the Existential Moduli Space.*

These represent "existential seeds" or "differentiated structures" arising from existential unfolding.

2.2. **Existential Paths.**

Definition 2.2 (Existential Path)**.** *Given two existential points* $[e_0]_{\mathcal{N}}, [e_1]_{\mathcal{N}} \in \mathcal{E}$*, an* existential path *from* $[e_0]_{\mathcal{N}}$ *to* $[e_1]_{\mathcal{N}}$ *is a sequence of existential operations:*

$$\gamma : [e_0]_{\mathcal{N}} \xrightarrow{\mathscr{D},\mathscr{S},\mathscr{R},\mathscr{F},\mathscr{U}} [e_1]_{\mathcal{N}}$$

modulo existential equivalences.

Intuitively, existential paths describe coherent unfoldings, symbolizations, relations, and fusions that deform one existential structure into another.

2.3. **Existential Homotopies.**

Definition 2.3 (Existential Homotopy)**.** *Two existential paths* γ_0, γ_1 *from* $[e_0]_{\mathcal{N}}$ *to* $[e_1]_{\mathcal{N}}$ *are* existentially homotopic *if there exists a higher existential deformation* H *interpolating between them:*

$$H : \gamma_0 \simeq \gamma_1 \quad \textit{through coherent unfolding deformations.}$$

H itself can be viewed as a second-order unfolding structure.

2.4. **Higher Existential Homotopies.**

Definition 2.4 (Higher Existential Homotopy)**.** *Given two existential homotopies* H_0, H_1 *between* γ_0 *and* γ_1*, a* higher existential homotopy *is a third-level deformation:*

$$K : H_0 \simeq H_1$$

and so on recursively for higher levels.

Thus, EHT naturally organizes existential structures into an ∞-groupoid-like hierarchy.

2.5. **Existential Homotopy Types.**

Definition 2.5 (Existential Homotopy Type)**.** *The* existential homotopy type *of an existential point* $[e]_{\mathcal{N}}$ *is the full* ∞*-structure of existential paths, homotopies, and higher homotopies emanating from it.*

This captures the "ontological connectedness" and deformation theory of existential structures.

3. Basic Properties and Constructions in EHT

We now explore some fundamental properties and constructions of Existential Homotopy Theory.

3.1. **Property 1: Identity Paths.**

Proposition 3.1. *For every existential point $[e]_{\mathcal{N}} \in \mathcal{E}$, there exists an identity existential path:*

$$id_{[e]_{\mathcal{N}}} : [e]_{\mathcal{N}} \to [e]_{\mathcal{N}}$$

corresponding to trivial existential unfolding (no effective transformation).

3.2. **Property 2: Composition of Existential Paths.**

Proposition 3.2. *Existential paths can be composed:*

$$\gamma_1 : [e_0]_{\mathcal{N}} \to [e_1]_{\mathcal{N}}, \quad \gamma_2 : [e_1]_{\mathcal{N}} \to [e_2]_{\mathcal{N}} \quad \Rightarrow \quad \gamma_2 \circ \gamma_1 : [e_0]_{\mathcal{N}} \to [e_2]_{\mathcal{N}}$$

by concatenating sequences of existential operations.

3.3. **Property 3: Inverses up to Homotopy.**

Proposition 3.3. *Existential paths admit inverses up to existential homotopy:*

$$\gamma : [e_0]_{\mathcal{N}} \to [e_1]_{\mathcal{N}} \quad \Rightarrow \quad \gamma^{-1} : [e_1]_{\mathcal{N}} \to [e_0]_{\mathcal{N}} \quad \textit{such that} \quad \gamma \circ \gamma^{-1} \simeq id_{[e_1]_{\mathcal{N}}}, \quad \gamma^{-1} \circ \gamma \simeq id_{[e_0]_{\mathcal{N}}}.$$

Thus, existential spaces behave like ∞-groupoids rather than strict categories.

3.4. **Construction: Existential Path Spaces.** Given two points $[e_0]_{\mathcal{N}}$ and $[e_1]_{\mathcal{N}}$, define the *existential path space*:

$$\mathrm{Path}([e_0]_{\mathcal{N}}, [e_1]_{\mathcal{N}})$$

as the higher structure consisting of all existential paths, homotopies, and higher homotopies between them.

3.5. **Construction: Loop Spaces.** Define the *existential loop space* at a point:

$$\Omega([e]_{\mathcal{N}}) := \mathrm{Path}([e]_{\mathcal{N}}, [e]_{\mathcal{N}})$$

capturing self-transformations, symmetries, and reflexive unfoldings of an existential structure.

4. Existential Homotopy Invariants and Applications

Existential Homotopy Theory (EHT) allows the definition of novel invariants capturing the fundamental existential structure of phenomena.

4.1. **Existential Homotopy Groups.**

Definition 4.1 (Existential Homotopy Groups). *Given an existential point $[e]_{\mathcal{N}} \in \mathcal{E}$, define the n-th existential homotopy group:*

$$\pi_n^{exist}([e]_{\mathcal{N}})$$

as the set of equivalence classes of n-fold higher existential loops at $[e]_{\mathcal{N}}$, modulo higher existential homotopies.

These groups capture the higher-order "existential holes," "twists," and "symmetries" in the unfolding structures around $[e]_{\mathcal{N}}$.

4.2. **Existential Homotopy Equivalence.**

Definition 4.2 (Existential Homotopy Equivalence). *Two existential points $[e_0]_{\mathcal{N}}, [e_1]_{\mathcal{N}}$ are* existentially homotopy equivalent *if there exist existential paths:*

$$\gamma : [e_0]_{\mathcal{N}} \to [e_1]_{\mathcal{N}}, \quad \delta : [e_1]_{\mathcal{N}} \to [e_0]_{\mathcal{N}}$$

such that:

$$\delta \circ \gamma \simeq id_{[e_0]_{\mathcal{N}}}, \quad \gamma \circ \delta \simeq id_{[e_1]_{\mathcal{N}}}.$$

Thus, existential structures can be grouped into homotopy classes under existential deformations.

4.3. **Applications.**

Potential applications of EHT include:

- Classification of existential seeds across multiversal structures,
- Detection of latent existential symmetries and anomalies,
- Formal modeling of meta-ontological feedback and reflexive emergence,
- Reconstruction of multiversal projections via existential homotopy invariants,
- New foundations for existential cohomology theories.

EHT thus opens a vast landscape of new research directions bridging ontology, mathematics, and metaphysical foundations.

5. Conclusion and Future Directions

In this paper, we initiated the foundational development of *Existential Homotopy Theory* (EHT), a higher-order framework generalizing classical homotopy theory to the domain of existential structures.

We have:

- Defined existential points, paths, homotopies, and higher homotopies,

- Established the basic properties of existential path spaces and loop spaces,
- Introduced existential homotopy groups as higher-order invariants,
- Outlined initial applications to multiversal classification and meta-ontological dynamics.

5.1. **Future Research Directions.**

- Develop existential homology and cohomology theories,
- Formalize ∞-groupoid models for existential unfolding systems,
- Investigate existential Whitehead theorems, Hurewicz theorems, and spectral sequences,
- Explore categorical and higher-topos models capturing full EHT dynamics,
- Apply EHT to reconstruct unexplained mathematical structures as existential homotopy artifacts across multiversal projections.

Existential Homotopy Theory opens an entirely new mathematical landscape where existence, transformation, and genesis are systematically modeled through paths, deformations, and higher-order relational dynamics.

References

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