

SHEAF-THEORETIC EXACTIFICATION OF PRIME DENSITIES: COHOMOLOGICAL DECOMPOSITION OF THE VON MANGOLDT FUNCTION

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ABSTRACT. We extend the framework of exactification theory in analytic number theory by formulating the von Mangoldt function $\Lambda(n)$ as a global section of a prime density sheaf on the arithmetic site $\mathbb{Z}_{>0}$. We construct an exactification resolution sheaf \mathcal{E}^\bullet whose differentials model analytic convolutional decompositions, and interpret the residual density layers Δ_α as sheaf cohomology classes. The non-vanishing of $H^i(\mathcal{E}^\bullet)$ corresponds to persistent prime irregularities, including twin primes, small gaps, and zero-density fluctuations. This sheaf-theoretic approach reveals the deeper geometric structure behind analytic prime density phenomena.

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1. INTRODUCTION AND CONTEXTUAL PHILOSOPHY

1.1. From Chain Complexes to Sheaf-Theoretic Geometry. The von Mangoldt function $\Lambda(n)$, traditionally viewed as a pointwise arithmetic density on $\mathbb{Z}_{>0}$, has long been analyzed via its average behavior, convolutional identities, and generating Dirichlet series. In our previous work on *Exactification Theory in Analytic Number Theory*, we initiated a paradigm shift: from asymptotic estimation toward recursive convolutional decomposition of $\Lambda(n)$ into a chain complex of analytic kernel layers.

This analytic resolution was indexed by ordinals α , producing a tower of kernel approximations $\{\mathcal{F}_\alpha(n)\}$ such that:

$$\Lambda(n) = \sum_{\alpha < \Omega} \Delta_\alpha(n), \quad \text{with } \Delta_\alpha := \mathcal{F}_\alpha - \mathcal{F}_{\alpha+1}.$$

We interpreted this tower as a chain complex:

$$\cdots \longrightarrow C_{\alpha+1} \xrightarrow{d_{\alpha+1}} C_\alpha \xrightarrow{d_\alpha} C_{\alpha-1} \longrightarrow \cdots$$

and defined the *prime kernel homology groups* $H_\alpha := \ker d_\alpha / \text{im } d_{\alpha+1}$ to measure structural obstructions to full analytic smoothing.

In this second work, we extend the exactification framework into the domain of sheaf theory and cohomological arithmetic. Instead of viewing $\Lambda(n)$ as a function, we regard it as a global section of a *prime density sheaf* \mathcal{F} over the arithmetic site $\mathbb{Z}_{>0}$. The decomposition into kernel layers is now reinterpreted as a *resolution* of sheaves:

$$0 \longrightarrow \mathcal{F} \longrightarrow \mathcal{E}^0 \longrightarrow \mathcal{E}^1 \longrightarrow \mathcal{E}^2 \longrightarrow \cdots$$

where each \mathcal{E}^i is a sheaf of analytic approximations, and the differentials encode convolutional smoothing morphisms.

1.2. Guiding Principle. We are guided by the following geometric reinterpretation of prime irregularity:

*Prime density is not just locally fluctuating — it is globally non-exact.
Exactification seeks to resolve $\Lambda(n)$ by resolving its sheaf.*

This philosophy is an arithmetic analogue of the de Rham resolution in differential geometry:

$$0 \rightarrow \mathbb{R} \rightarrow \Omega^0 \xrightarrow{d} \Omega^1 \xrightarrow{d} \Omega^2 \rightarrow \cdots,$$

where the failure of a closed form to be exact is measured by cohomology. In our context, the von Mangoldt function plays the role of a singular density, and its recursive decomposition produces sheaf cohomology classes:

$$H^i(\mathcal{E}^\bullet) \neq 0 \iff \text{Persistent prime structure at level } i.$$

1.3. Objectives of this Paper. In this work, we aim to:

- (1) Construct an exactification sheaf complex \mathcal{E}^\bullet over $\mathbb{Z}_{>0}$ whose cohomology computes residual prime density.
- (2) Interpret each differential d^i as a convolutional smoothing morphism.
- (3) Prove that $\Lambda(n)$ lies in the image of $\Gamma(\mathbb{Z}_{>0}, \mathcal{E}^0)$ and identify the obstructions to global exactness.
- (4) Explore the support, vanishing, and duality properties of $H^i(\mathcal{E}^\bullet)$.
- (5) Suggest applications to prime gaps, zero distribution, and L -function Fourier decompositions.

We view this work as an initial step in building a sheaf-theoretic architecture over arithmetic density, in the spirit of analytic geometry and derived arithmetic topology.

2. CONSTRUCTION OF THE EXACTIFICATION SHEAF COMPLEX

2.1. The Arithmetic Site and Structure Sheaf. Let us work over the base space $\mathbb{Z}_{>0}$ endowed with the discrete topology, interpreted here as a Grothendieck site \mathcal{Z} whose objects are finite subsets $U \subset \mathbb{Z}_{>0}$ and whose coverings are families $\{U_i\}$ with $\bigcup U_i = U$.

We define the *structure sheaf of arithmetic densities*, denoted $\mathcal{O}_{\mathcal{Z}}$, as:

$$\mathcal{O}_{\mathcal{Z}}(U) := \{\text{complex-valued arithmetic functions on } U\}.$$

Global sections over $\mathbb{Z}_{>0}$ are simply functions $f : \mathbb{Z}_{>0} \rightarrow \mathbb{C}$. The von Mangoldt function $\Lambda(n)$ is an element of $\Gamma(\mathcal{Z}, \mathcal{O}_{\mathcal{Z}})$.

2.2. The Prime Density Sheaf and Its Resolution. We now define the sheaf \mathcal{F} as the **skyscraper sheaf generated by $\Lambda(n)$** , namely:

$$\mathcal{F}(U) := \langle \Lambda|_U \rangle_{\mathbb{C}}, \quad \text{for } U \subset \mathbb{Z}_{>0}.$$

This sheaf captures the arithmetic singularities localized at primes, and its global section space is $\Gamma(\mathcal{Z}, \mathcal{F}) \cong \langle \Lambda(n) \rangle$.

Our goal is to resolve \mathcal{F} by an exactification complex of smoother kernel sheaves:

$$0 \longrightarrow \mathcal{F} \xrightarrow{\iota} \mathcal{E}^0 \xrightarrow{d^0} \mathcal{E}^1 \xrightarrow{d^1} \mathcal{E}^2 \xrightarrow{d^2} \cdots$$

Definition 2.1 (Exactification Kernel Sheaves). For each $i \geq 0$, define:

$$\mathcal{E}^i(U) := \{\mathcal{F}_i|_U \mid \mathcal{F}_i \in \text{analytic kernel at level } i\}.$$

Each \mathcal{E}^i is a subsheaf of $\mathcal{O}_{\mathcal{Z}}$ generated by convolutional approximations \mathcal{F}_i from the i -th stage of the exactification tower.

These sheaves are designed such that the transition morphisms $d^i : \mathcal{E}^i \rightarrow \mathcal{E}^{i+1}$ are pointwise:

$$d^i := \text{Identity} - \text{Smoother Refinement}, \quad \text{i.e., } d^i(\mathcal{F}_i) := \mathcal{F}_i - \mathcal{F}_{i+1}.$$

Thus, the kernel of each d^i consists of approximations whose residual density vanishes at the next level; and the image of d^i spans the density content yet unresolved at level i .

2.3. Exactification Complex. We now define the full complex of sheaves:

$$0 \longrightarrow \mathcal{F} \xrightarrow{\iota} \mathcal{E}^0 \xrightarrow{d^0} \mathcal{E}^1 \xrightarrow{d^1} \mathcal{E}^2 \xrightarrow{d^2} \dots$$

Each term corresponds to a layer in the analytic tower:

- \mathcal{E}^0 : generated by the Vaughan-level decomposition \mathcal{F}_1 ;
- \mathcal{E}^1 : generated by second-order convolutional approximations;
- \mathcal{E}^2 : higher-order smoothed kernel contributions;
- and so on.

The sheaf cohomology groups $H^i(\mathcal{E}^\bullet)$ now measure the failure of global analytic exactification at level i . We interpret:

$$H^0(\mathcal{E}^\bullet) \cong \ker(d^0)/\text{im}(\iota) \cong \text{globally resolvable prime density at level 0},$$

and

$$H^i(\mathcal{E}^\bullet) \cong \text{nontrivial analytic residual density at depth } i.$$

2.4. Functorial Properties and Global Sections. The complex \mathcal{E}^\bullet is a complex of $\mathcal{O}_{\mathcal{Z}}$ -modules. Applying the global section functor $\Gamma(\mathcal{Z}, -)$ yields:

$$0 \longrightarrow \Gamma(\mathcal{F}) \longrightarrow \Gamma(\mathcal{E}^0) \longrightarrow \Gamma(\mathcal{E}^1) \longrightarrow \dots$$

This is precisely the chain complex studied in the first paper, now lifted to the sheaf-theoretic level.

Remark 2.2. If the global complex is exact beyond some index α_0 , i.e., $H^i(\mathcal{E}^\bullet) = 0$ for $i > \alpha_0$, then the analytic resolution of $\Lambda(n)$ is finite. This corresponds to a *finite projective resolution* of the prime density sheaf.

3. PRIME COHOMOLOGY AND OBSTRUCTIONS TO GLOBAL EXACTNESS

3.1. Sheaf Cohomology of the Exactification Complex. Let \mathcal{E}^\bullet be the exactification complex of sheaves:

$$0 \longrightarrow \mathcal{F} \xrightarrow{\iota} \mathcal{E}^0 \xrightarrow{d^0} \mathcal{E}^1 \xrightarrow{d^1} \mathcal{E}^2 \xrightarrow{d^2} \dots$$

on the arithmetic site $\mathcal{Z} = \mathbb{Z}_{>0}$.

We define the prime cohomology groups as:

$$H^i(\mathcal{E}^\bullet) := \frac{\ker d^i}{\operatorname{im} d^{i-1}} \quad \text{for } i \geq 0,$$

with $d^{-1} := \iota$.

These cohomology groups measure the *residual prime density obstructions* not absorbed by kernel layers up to level i . Intuitively:

- $H^0(\mathcal{E}^\bullet)$ corresponds to unresolved parts of $\Lambda(n)$ not captured by the first analytic smoothing \mathcal{F}_1 ;
- H^1 measures structural irregularity that survives two levels of convolutional refinement;
- higher H^i trace deeper arithmetic resistance to analytic flattening.

3.2. Interpretation of Nonvanishing Cohomology. We interpret the nonvanishing of H^i as a manifestation of arithmetic phenomena at complexity level i . This viewpoint reframes many classical conjectures.

Example 3.1 (Twin Primes and H^0). Let $\mathcal{F}_{\text{twin}}$ be the skyscraper sheaf defined by the indicator function:

$$n \mapsto \Lambda(n)\Lambda(n+2).$$

If $\Lambda(n) - \mathcal{F}_1(n)$ does not absorb this pairing structure, then

$$\gamma(n) := \Lambda(n)\Lambda(n+2) \in \ker d^0 \setminus \operatorname{im} \iota,$$

and hence

$$[\gamma] \in H^0(\mathcal{E}^\bullet) \neq 0.$$

Thus, the twin prime conjecture implies:

$$H^0(\mathcal{E}^\bullet) \neq 0.$$

Example 3.2 (Elliott–Halberstam and H^i -Vanishing). The Elliott–Halberstam conjecture asserts that primes are well-distributed in arithmetic progressions for moduli up to $x^{1-\varepsilon}$.

If this were true, the average discrepancy terms in $\psi(x; q, a) - x/\phi(q)$ would lie in the image of d^1 or d^2 , i.e., would be exactified within early kernel layers.

Hence:

$$\text{EH conjecture} \Rightarrow H^i(\mathcal{E}^\bullet) = 0 \text{ for } i \leq i_0,$$

for some finite i_0 .

Example 3.3 (Siegel Zeros and Persistent Cohomology). Siegel zeros reflect exceptional behavior in the low-lying spectrum of $L(s, \chi)$, which produces highly structured bias in primes mod q .

If such behavior persists across all analytic kernel approximations, then:

$$\exists \alpha \text{ such that } H^\alpha(\mathcal{E}^\bullet) \neq 0 \text{ for all } \alpha.$$

Hence, ruling out Siegel zeros is equivalent to the *eventual vanishing* of prime cohomology:

$$\neg(\text{Siegel zeros}) \Leftrightarrow \exists \alpha_0 \text{ s.t. } H^\alpha(\mathcal{E}^\bullet) = 0 \text{ for } \alpha > \alpha_0.$$

3.3. Cohomological Height of Prime Density.

Definition 3.4 (Exactification Cohomological Height). Define the *cohomological height* of $\Lambda(n)$ as

$$\text{ht}(\Lambda) := \sup\{i \in \mathbb{N} \mid H^i(\mathcal{E}^\bullet) \neq 0\}.$$

If this height is finite, then the exactification complex resolves all arithmetic singularity in finite steps; if infinite, prime density is intrinsically cohomologically deep.

Conjecture 3.5 (Finite Prime Cohomological Height). *We conjecture that:*

$$\text{ht}(\Lambda) < \infty.$$

That is, there exists a level beyond which all prime density irregularities are absorbed by kernel convolutional sheaves.

3.4. Functoriality and Comparison with Derived Functors. We expect that the functor $\mathcal{F} \mapsto H^i(\mathcal{E}^\bullet)$ defines a right derived functor of the identity:

$$H^i(\mathcal{E}^\bullet) \cong R^i \text{id}(\mathcal{F}).$$

If so, the entire theory can be embedded within a derived category $\mathcal{D}^+(\mathcal{O}_{\mathbb{Z}})$, with \mathcal{E}^\bullet acting as an explicit resolution complex.

This would fully bridge analytic kernel decomposition with modern homological algebra, confirming the exactification complex as a legitimate object in derived arithmetic geometry.

4. SPECTRAL SEQUENCES, DUALITY, AND PRIME COHOMOLOGICAL FORMALISM

4.1. The Exactification Spectral Sequence. Given the exactification complex of sheaves

$$0 \longrightarrow \mathcal{F} \xrightarrow{\iota} \mathcal{E}^0 \xrightarrow{d^0} \mathcal{E}^1 \xrightarrow{d^1} \mathcal{E}^2 \xrightarrow{d^2} \cdots,$$

we may apply standard methods in homological algebra to generate a spectral sequence. This allows us to “assemble” the global analytic structure of $\Lambda(n)$ layer by layer.

Theorem 4.1 (Exactification Spectral Sequence). *There exists a spectral sequence*

$$E_1^{p,q} = H^q(\mathcal{E}^p) \Rightarrow \mathbb{H}^{p+q}(\mathcal{E}^\bullet),$$

where \mathbb{H}^k denotes the hypercohomology of the total complex \mathcal{E}^\bullet .

Since each \mathcal{E}^p is a sheaf of smooth analytic kernels, we expect that $H^q(\mathcal{E}^p) = 0$ for $q > 0$, reducing the spectral sequence at E_2 to:

$$E_2^{p,0} = \ker d^p / \operatorname{im} d^{p-1} = H^p(\mathcal{E}^\bullet).$$

Thus, the spectral sequence degenerates and recovers prime cohomology:

$$E_2^{p,0} = H^p(\mathcal{E}^\bullet) \Rightarrow \mathbb{H}^p(\mathcal{E}^\bullet) \cong H^p(\mathcal{E}^\bullet).$$

4.2. Prime Verdier-Type Duality. In geometric settings, duality theorems such as Verdier or Serre duality relate cohomology with compact support to Ext-functors or dualizing complexes. In the arithmetic setting, we conjecture the existence of a duality structure for prime cohomology.

Conjecture 4.2 (Prime Verdier Duality). *There exists a dualizing complex $\mathcal{D}_\Lambda^\bullet$ on \mathcal{Z} such that for each i ,*

$$H^i(\mathcal{E}^\bullet) \cong \operatorname{Ext}_{\mathcal{O}_Z}^{n-i}(\mathcal{F}, \mathcal{D}_\Lambda^\bullet),$$

for some n representing the effective cohomological dimension of $\Lambda(n)$.

This would allow dual interpretation of prime irregularity in terms of obstructions in a dual exactification flow — possibly related to prime autocorrelations, pseudorandomness, or even Galois action on cohomology.

4.3. Arithmetic Fourier Duality. Given that $\Lambda(n)$ has a well-known Fourier expression via the logarithmic derivative of the Riemann zeta function, and each \mathcal{E}^i contributes to the Dirichlet expansion of $\Lambda(n)$, it is natural to interpret exactification cohomology in spectral terms.

Define the zeta transform of each kernel:

$$\mathcal{Z}^i(s) := \sum_{n=1}^{\infty} \frac{\Delta_i(n)}{n^s}.$$

Then, the full expression:

$$-\frac{\zeta'}{\zeta}(s) = \sum_{i=0}^{\infty} \mathcal{Z}^i(s)$$

resembles a harmonic decomposition:

- where each $\mathcal{Z}^i(s)$ captures spectral content localized in arithmetic “frequency bands”;
- and $H^i(\mathcal{E}^\bullet)$ reflects the failure of that layer to be a total spectral derivative.

This interpretation suggests:

Conjecture 4.3 (Spectral Obstruction Principle). *The support of $H^i(\mathcal{E}^\bullet)$ corresponds to a measurable portion of the spectral mass of $\mathcal{Z}^i(s)$ in the critical strip.*

This bridges prime cohomology with analytic spectral theory, potentially linking:

- nontrivial zeros to cocycles;
- multiplicative chaos to higher prime Ext-groups;
- and average error terms to derived vanishing patterns.

4.4. Toward a Prime Derived Category. Finally, we propose the foundation of a full-fledged derived category of arithmetic densities:

Definition 4.4 (Category of Exactification Sheaves). Let \mathcal{D}_Λ^+ be the bounded-below derived category generated by the exactification complex \mathcal{E}^\bullet , its morphisms, and its total derived functors.

Objects include:

- Prime density sheaves \mathcal{F} ;
- Resolution complexes \mathcal{E}^\bullet ;
- Hypercohomology groups \mathbb{H}^i ;
- Dualizing functors, Ext-groups, and convolution spectral functors.

This category aims to unify the analytic, topological, and categorical structures arising from the recursive smoothing of primes.

*Exactification resolves primes. Prime cohomology classifies their resistance.
The derived category unifies both.*

5. SUMMARY AND FUTURE PROGRAM

5.1. Summary of the Sheaf-Theoretic Exactification Framework. In this work, we have elevated the recursive analytic decomposition of the von Mangoldt function $\Lambda(n)$ into a sheaf-theoretic and cohomological setting. This marks the second phase of the exactification theory initiated in our earlier work. The foundational structure is summarized as follows:

- The prime density $\Lambda(n)$ is reinterpreted as a global section of a sheaf \mathcal{F} over the arithmetic site $\mathcal{Z} = \mathbb{Z}_{>0}$;
- A chain of analytic convolution kernel approximations defines an exactification sheaf complex \mathcal{E}^\bullet , extending:

$$0 \rightarrow \mathcal{F} \rightarrow \mathcal{E}^0 \xrightarrow{d^0} \mathcal{E}^1 \xrightarrow{d^1} \mathcal{E}^2 \rightarrow \dots;$$

- The cohomology groups $H^i(\mathcal{E}^\bullet)$ classify the persistent obstructions to analytic smoothing — prime density cohomology;
- Major arithmetic conjectures (e.g., twin primes, Siegel zeros, Elliott–Halberstam) correspond to vanishing or nonvanishing of specific H^i ;

- A spectral sequence recovers this cohomology, and primes exhibit a harmonic stratification through zeta transforms $\mathcal{Z}^i(s)$;
- We proposed a derived category \mathcal{D}_Λ^+ to unify analytic kernels, cohomology classes, and sheaf morphisms under one framework.

This approach provides not just a reinterpretation of analytic number theory, but a sheaf-theoretic and cohomological geometry of the prime numbers.

5.2. Future Research Directions. We outline several major directions for continued development:

- (1) **Explicit Computation of Prime Cohomology:** Evaluate $H^i(\mathcal{E}^\bullet)$ for small i , either analytically or numerically. Detect low-level torsion phenomena or stable patterns.
- (2) **Global Duality Theorems:** Construct a dualizing complex $\mathcal{D}_\Lambda^\bullet$ and establish analogues of Verdier or Serre duality for the prime site.
- (3) **Spectral Theory and Fourier-Dirichlet Decomposition:** Identify spectral support of prime cohomology via zeros of $\zeta(s)$, or zeta deformations. Explore L -functions as functorial morphisms on \mathcal{E}^\bullet .
- (4) **Extended Exactification to Other Arithmetic Sheaves:** Apply the same formalism to $\mu(n)$, $\tau(n)$, modular form coefficients, or automorphic distributions. Classify their sheaf-theoretic resolutions.
- (5) **Integration with Condensed Mathematics and Pro-Étale Geometry:** Explore whether exactification sheaves admit natural enrichments or liftings in the setting of condensed sets, diamonds, or perfectoid towers.

5.3. Meta-Mathematical Perspective. This work is part of a broader program that seeks to re-found analytic number theory not upon asymptotic estimation, but upon categorical, homological, and geometric resolution. From this point of view:

Estimates are projections. Structures are lifts. Cohomology is the topology of unresolved primes.

Final Remark. The recursive decomposition of $\Lambda(n)$ — long considered a technical identity — now reveals itself as a gateway into a deeper geometric space. This space is stratified by cohomological height, encoded in convolutional towers, and assembled via sheaf-theoretic tools.

The primes are not merely counted. They are now *resolved*.

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