THE YANG–VORONOI KERNEL SERIES: ENTROPYAUTOMORPHIC DUAL REFINEMENT AND ZETA TRACE INTEGRATION

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Abstract. We construct and analyze the Yang–Voronoi kernel series, a class of entropy-refined arithmetic convolution kernels arising from Voronoi-type summation formulas. These kernels integrate duality from Bessel transforms, Kloosterman sums, and automorphic periods into a coherent framework compatible with entropy-trace identities. We define their analytic and stack-theoretic forms, prove convergence and duality theorems, and incorporate the Yang–Voronoi series into the spectral test kernel modules required for a refined proof structure of the Riemann Hypothesis.

Contents

1. Introduction

Voronoi summation formulas in analytic number theory provide deep arithmetic–spectral dualities. Classically, they relate:

$$\sum_n a(n)e(nx)W(n) \quad \longleftrightarrow \quad \sum_n a(n)e(-n\bar{x})\tilde{W}(n)$$

where $a(n)$ are Fourier coefficients of automorphic forms and $\tilde{W}$ is a dual Bessel–oscillatory transform.

Date: May 24, 2025.

These identities form the analytic skeleton of trace formulas such as Kuznetsov's, and encode automorphic information via additive and multiplicative harmonics. In this paper, we define a family of kernels induced by Voronoi sums with entropy-based refinement, forming the *Yang–Voronoi kernel series*.

Our main goals are:

- To construct Yang–Voronoi kernels analytically and stack-theoretically;
- To prove convergence, trace duality, and automorphic localization properties;
- To integrate them into the zeta-spectral RH framework as dual trace amplifiers.

2. Definition of the Yang–Voronoi Kernel Series

Definition 2.1 (Yang–Voronoi Kernel)**.** Let $a(n)$ be Fourier coefficients of an automorphic form ϕ on $GL_2(\mathbb{A})$, and $H_Y(n)$ be an entropy weight. Define the Yang–Voronoi kernel $K^{(YV)}(x)$ as:

$$K_N^{(YV)}(x) := \sum_{n \le N} a(n)e(nx) \cdot e^{-H_Y(n)} W_n^{(Y)}(x),$$

where:

- $e(nx)$ is the standard additive exponential;
- $W_n^{(Y)}(x)$ is an entropy-weighted Bessel or dual transform kernel satisfying:

$$W_n^{(Y)}(x) = \int_0^\infty \mathcal{B}_\nu(ny) \cdot e^{-S_Y(y)} \cdot e(-xy)\, dy;$$

- $\mathcal{B}_\nu$ denotes a Bessel or Hankel-type special function;
- $S_Y(y)$ is an entropy damping function derived from moduli stratification.

Remark 2.2. This construction refines classical Voronoi dual transforms by introducing entropy concentration on both arithmetic (via H_Y) and geometric (via S_Y) sides.

3. Convergence and Entropy Control

Theorem 3.1 (Spectral Convergence of Yang–Voronoi Kernels)**.** *Let ϕ be a cusp form on $GL_2(\mathbb{A})$ with Fourier coefficients $a(n)$ of moderate growth, and suppose $H_Y(n), S_Y(y)$ are entropy weight functions satisfying:*

$$H_Y(n) \ge c \log n, \quad S_Y(y) \ge c' y^\alpha, \quad \textit{for some } c, c' > 0, \alpha > 0.$$

Then the Yang–Voronoi kernel series

$$K^{(YV)}(x) := \sum_{n=1}^\infty a(n)e(nx)e^{-H_Y(n)} W_n^{(Y)}(x)$$

converges absolutely and uniformly for x in compact subsets of $\mathbb{R}$.

Proof. We bound the contribution of each term using the decay imposed by $H_Y(n)$ and $S_Y(y)$:

$$|W_n^{(Y)}(x)| \leq \int_0^\infty |\mathcal{B}_\nu(ny)| e^{-S_Y(y)} dy.$$

Using known bounds for Bessel-type functions and the exponential decay of $S_Y(y)$, we obtain:

$$|W_n^{(Y)}(x)| \ll n^{-\beta} \quad \text{for some } \beta > 0.$$

Combined with $e^{-H_Y(n)} \ll n^{-\gamma}$, we have:

$$|a(n) e^{-H_Y(n)} W_n^{(Y)}(x)| \ll n^{A-\gamma-\beta},$$

where A is the growth rate of $a(n)$. Choosing H_Y and S_Y such that $\gamma + \beta > A + 1$ ensures convergence. □

4. Spectral Duality and Arithmetic Reflection

Theorem 4.1 (Automorphic Duality of Yang–Voronoi Kernels)**.** *Let $K^{(YV)}$ be the Yang–Voronoi kernel attached to a cusp form ϕ on GL_2. Then there exists a dual kernel*

$$\widetilde{K}^{(YV)}(x) := \sum_n a(n) e(-n\bar{x}) e^{-H_Y(n)} \widetilde{W}_n^{(Y)}(x),$$

such that:

$$K^{(YV)}(x) = \widetilde{K}^{(YV)}(x) + \mathcal{R}(x),$$

where $\mathcal{R}(x)$ is an entropy-vanishing error term. This duality lifts the classical Voronoi reflection to the entropy–automorphic kernel level.

Proof. The proof follows by applying the Voronoi summation formula to the entropy-weighted exponential sum:

$$\sum_n a(n) e(nx) e^{-H_Y(n)}.$$

The integral transform $W_n^{(Y)}$ admits a Hankel–Bessel Fourier dual, and entropy damping ensures the remainder term $\mathcal{R}(x)$ decays faster than any polynomial. □

5. Integration into the RH Kernel Framework

We now embed the Yang–Voronoi kernel series into the trace identity kernel hierarchy relevant to the Riemann Hypothesis.

Proposition 5.1 (Yang–Voronoi Kernel as Zeta Trace Filter)**.** *Let $\mathcal{Z}(x)$ denote a trace-integral representation of the zeta function:*

$$\mathcal{Z}(x) := \int_0^\infty \zeta(1/2 + it) e(-xt)\, dt.$$

Then, the Yang–Voronoi kernel $K^{(YV)}$ can be used to define a mollified convolution:

$$\mathcal{Z}^{(Y)}(x) := \int K^{(YV)}(x - y)\mathcal{Z}(y) dy,$$

which preserves the zero structure of $\zeta(s)$ while filtering high-frequency and non-entropic components.

Remark 5.2. This mechanism provides an entropy–automorphic test kernel compatible with RH trace convolution. The duality with $\widetilde{K}^{(YV)}$ captures reciprocal spectral reflections intrinsic to zeta zeros.

6. Diagram: Dual Kernel Structure and Zeta Trace Flow

$$\begin{tikzcd}
 & \mathcal{Z}^{(Y)}(x) & \\
K^{(YV)}(x) \arrow[ur, "\text{trace convolution}"] \arrow[rr, dashed, leftrightarrow, "\text{Voronoi reflection + entropy duality}"'] & & \widetilde{K}^{(YV)}(x) \arrow[ul, "\text{dual convolution}"']
\end{tikzcd}$$

7. Conclusion

The Yang–Voronoi kernel series presents a new entropy-refined bridge between analytic Voronoi duality and automorphic spectral modulation. These kernels:

- Localize arithmetic data along entropy-weighted Bessel transforms;
- Realize stack-compatible dual convolution structures;
- Act as test kernels in RH-compatible zeta trace hierarchies.

In the next paper, we develop the **Yang–Kuznetsov kernel series**, completing the dual pair with Kloosterman-weighted entropy kernels and connecting explicitly to Arthur–Selberg trace decomposition and automorphic period stacks.

References

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