Harmonic Analysis and Analytic Number Theory on $\mathbb{Y}_3$ Number Systems

Pu Justin Scarfy Yang

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1 Introduction

This document explores harmonic analysis and analytic number theory within the framework of $\mathbb{Y}_3$ number systems. We aim to generalize classical results, including the Riemann zeta function, to this non-associative setting.

2 Definition and Properties of $\mathbb{Y}_3$

2.1 Definition

Let $\mathbb{Y}_3$ be a non-associative number system with a binary operation $*$ satisfying the following properties:

- **Non-associativity**: For some $x, y, z \in \mathbb{Y}_3$, $(x * y) * z \neq x * (y * z)$.
- **Other Axioms**: Define additional axioms that characterize $\mathbb{Y}_3$.

Definition 2.1. *A $\mathbb{Y}_3$-algebra is a vector space with a bilinear product $*$ that satisfies the properties defined above.*

2.2 Defining $s \in \mathbb{Y}_3$

To handle $s \in \mathbb{Y}_3$ in the context of $\zeta_{\mathbb{Y}_3}(s)$, we need to extend the concept of exponentiation in a non-associative setting. Define the exponentiation x^s where $x \in \mathbb{Y}_3$ and $s \in \mathbb{Y}_3$ as follows:

Definition 2.2. *For $x \in \mathbb{Y}_3$ and $s \in \mathbb{Y}_3$, the exponentiation x^s is defined through a suitable extension of the usual exponential function. If $\mathbb{Y}_3$ has a structure allowing for logarithms and exponentials, define x^s using a logarithmic function:*

$$x^s = \exp(s \log x),$$

where $\log x$ and $\exp$ are appropriately defined for $\mathbb{Y}_3$.

3 Harmonic Analysis on $\mathbb{Y}_3$

3.1 Generalizing Fourier Analysis

Define the Fourier transform for $\mathbb{Y}_3$:

Definition 3.1. *Let $f : \mathbb{Y}_3 \to \mathbb{C}$ be a function. The $\mathbb{Y}_3$-Fourier transform is given by:*

$$\mathcal{F}_{\mathbb{Y}_3}(u) = \sum_{x \in \mathbb{Y}_3} f(x)\phi_u(x),$$

where ϕ_u is a character of $\mathbb{Y}_3$, if such characters exist.

3.2 Parseval's Identity

Theorem 3.2. *Let $f : \mathbb{Y}_3 \to \mathbb{C}$. The $\mathbb{Y}_3$-Fourier transform satisfies Parseval's identity:*

$$\|f\|^2 = \|\mathcal{F}_{\mathbb{Y}_3}(f)\|^2,$$

where the norms are defined as:

$$\|f\|^2 = \sum_{x \in \mathbb{Y}_3} |f(x)|^2 \quad \textit{and} \quad \|\mathcal{F}_{\mathbb{Y}_3}(f)\|^2 = \sum_{u \in \mathbb{Y}_3} |\mathcal{F}_{\mathbb{Y}_3}(u)|^2.$$

4 Analytic Number Theory with $\mathbb{Y}_3$

4.1 Generalized Zeta Function

Define the $\mathbb{Y}_3$-zeta function for $s \in \mathbb{Y}_3$:

Definition 4.1. *The $\mathbb{Y}_3$-zeta function is defined by:*

$$\zeta_{\mathbb{Y}_3}(s) = \sum_{x \in \mathbb{Y}_3} \frac{1}{x^s},$$

where $s \in \mathbb{Y}_3$ and x^s is defined as in the previous section.

4.2 Properties and Analytic Continuation

Investigate the properties of $\zeta_{\mathbb{Y}_3}$:

Theorem 4.2. *$\zeta_{\mathbb{Y}_3}$ satisfies a functional equation of the form:*

$$\zeta_{\mathbb{Y}_3}(s) = \Phi(s)\zeta_{\mathbb{Y}_3}(1-s),$$

where $\Phi(s)$ is a function related to $\mathbb{Y}_3$-algebra properties.

Proof. Provide detailed proof of the functional equation, utilizing properties of $\mathbb{Y}_3$ and $\mathbb{Y}_3$-Fourier analysis. □

5 Implications for the Riemann Hypothesis

5.1 Generalized Riemann Hypothesis

Define the $\mathbb{Y}_3$-Riemann Hypothesis:

Definition 5.1. *The $\mathbb{Y}_3$-Riemann Hypothesis posits that all non-trivial zeros of $\zeta_{\mathbb{Y}_3}(s)$ lie on the line $\Re(s) = \frac{1}{2}$.*

5.2 Comparative Analysis

Compare the $\mathbb{Y}_3$-zeta function with the classical Riemann zeta function. Discuss potential similarities and differences:

Theorem 5.2. *If $\zeta_{\mathbb{Y}_3}(s)$ has non-trivial zeros on $\Re(s) = \frac{1}{2}$, then similar structures or results might emerge as in the classical case.*

Proof. Provide a detailed analysis, including possible numerical experiments and theoretical insights. □

6 Conclusion

Summarize the results of the study, including any new insights into harmonic analysis and analytic number theory with $\mathbb{Y}_3$. Discuss the implications for the Riemann Hypothesis and future research directions.

7 References

List any references used throughout the document, formatted according to your preferred style.