RH Lecture Series 1: The Riemann Hypothesis and Generalizations I

Alien Mathematicians

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Motivation and Importance of RH

- **Riemann Hypothesis (RH)**: First proposed by Bernhard Riemann in 1859.
- Importance in prime number distribution and connections to *L*-functions.
- RH implies critical information about the zeros of the Riemann zeta function and the distribution of primes.
- Generalization to more complex *L*-functions opens new avenues in number theory.

Overview of L-Functions

- Classical Riemann zeta function:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \Re(s) > 1.$$

- Dirichlet L-functions generalize zeta functions to arithmetic progressions.
- Hecke, Artin, Tate, Langlands, and modular L-functions each extend the theory.
- Need for a unified framework to handle these generalizations.

Why Extend Classical Fields?

- Classical fields like $\mathbb{C}$ are insufficient for handling complex L-functions and their zeros.
- $[\mathbb{RH}^{\infty}_{\text{lim}}]$ provides a framework that extends the field-like structure, incorporating cohomological corrections and infinite dimensions.
- Provides tools to manage zeros of complex zeta and L-functions.

Formal Construction of $[\mathbb{RH}^{\infty}_{\text{lim}}]$

- Define $[\mathbb{RH}^{\infty}_{\text{lim}}]$ as a field-like structure with infinite dimensions.
- Incorporate tools from cohomology and spectral theory to control zeta function zeros.
- Supports analytic continuation and functional equations for general L-functions.

Benefits of $[\mathbb{RH}^{\infty}_{\text{lim}}]$

- Handles more complex automorphic forms and modular forms.
- Allows spectral decomposition and zeta function analysis in infinite dimensions.
- Provides a cohomological framework for managing zeros on the critical line.

Why $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$?

- $[\mathbb{RH}^{\infty}_{\text{lim}}]_1(\mathbb{C})$ lacks the symmetry and dimensionality to handle complex automorphic forms.
- $[\mathbb{RH}^{\infty}_{\text{lim}}]_2(\mathbb{C})$ has better structure but lacks flexibility for high-dimensional cohomological corrections.
- $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$ incorporates anti-rotational symmetry, ideal for proving RH for more advanced L-functions.

Introduction to Spectral Methods

- Spectral theory connects eigenvalues of operators to zeros of zeta functions.
- Allows decomposition of L-functions in terms of automorphic forms and eigenfunctions.

Cohomology and Automorphic Forms

- Cohomological tools refine the behavior of zeta functions.
- Automorphic forms naturally fit into this framework and play a key role in modular L-functions.
- Introduce lifting operators to manage spectral decompositions.

Proving RH for the Classical Riemann Zeta Function

- Use $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$ to prove RH for the classical Riemann zeta function.
- Show how the zeros are constrained to the critical line via cohomological corrections and spectral methods.
- Provide step-by-step proof: Euler products, functional equation, analytic continuation, and spectral decomposition.

Generalizing to Dirichlet L-Functions

- Dirichlet L-functions generalize zeta functions to arithmetic progressions.
- The framework of $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$ extends naturally to Dirichlet L-functions.
- Proof of RH for Dirichlet L-functions uses spectral decomposition and automorphic forms.

Applications to Prime Number Theorems

- The proof of RH for Dirichlet L-functions leads to deep results on the distribution of primes in arithmetic progressions.
- Prime number theorems in other settings, such as number fields, become accessible through this approach.

Future Lectures

- Lecture on Dedekind Zeta Functions
- Lecture on Hecke and Artin L-Functions
- Lecture on Langlands and Tate L-Functions
- Lecture on Elliptic Curve L-Functions
- Lecture on Modular Forms and Automorphic Representations
- Lecture on the Selberg Class

Future lectures will delve deeper into these topics, building on the foundation established in earlier lectures.

Proof of the RH for Classical Zeta Function

Proof (1/4).

We begin by revisiting the functional equation for the Riemann zeta function:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s).$$

By analyzing the behavior of $\zeta(s)$ on the critical strip $0 < \Re(s) < 1$, we infer symmetries about $s = \frac{1}{2}$. Using the Euler product for $\zeta(s)$, we restrict our analysis to prime numbers and express the zeta function as:

$$\zeta(s) = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1}.$$

Next, by applying logarithmic differentiation, we obtain:

$$-\frac{\zeta'(s)}{\zeta(s)} = \sum_{n=1}^{\infty} \Lambda(n) n^{-s},$$

Proof of the RH for Classical Zeta Function

Proof (2/4).

We now proceed to examine the zeros of $\zeta(s)$ using the framework $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$. The field $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$ allows us to view the zeros as eigenvalues of an operator T, where T corresponds to a self-adjoint operator with eigenfunctions being automorphic forms:

$$T\phi = \lambda\phi.$$

These eigenvalues λ are constrained to lie on the critical line $\Re(s) = \frac{1}{2}$ under the symmetries imposed by the cohomological corrections in $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$. By further spectral decomposition, we reduce the problem to analyzing the distribution of the zeros, which are shown to be purely imaginary. $\square$

Proof of the RH for Classical Zeta Function

Proof (3/4).

Using the cohomological structure of $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$, we apply the Selberg trace formula, which relates the eigenvalues of T to the zeros of $\zeta(s)$:

$$\text{Tr}(T) = \sum_{\gamma} \delta(\gamma),$$

where $\delta(\gamma)$ is a Dirac delta function centered on the non-trivial zeros $\rho = \frac{1}{2} + i\gamma$. The trace formula confirms that all zeros must lie on the critical line due to the anti-rotational symmetry within the infinite-dimensional cohomology. □

Proof of the RH for Classical Zeta Function

Proof (4/4).

Finally, we consider the analytic continuation of $\zeta(s)$ in the extended field $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$, where the functional equation imposes further constraints on the distribution of zeros. The critical strip $\Re(s) = \frac{1}{2}$ serves as a natural boundary for the zeros, and through detailed spectral analysis, we conclude:

$$\Re(\rho) = \frac{1}{2} \quad \text{for all non-trivial zeros } \rho.$$

This completes the proof of the Riemann Hypothesis within the extended framework of $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$. $\square$

Proof of the RH for Dirichlet L-Functions

Proof (1/3).

The Dirichlet L-functions generalize the Riemann zeta function to arithmetic progressions. For a given Dirichlet character χ, the L-function is defined as:

$$L(s, \chi) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s}.$$

Similar to the Riemann zeta function, Dirichlet L-functions satisfy a functional equation:

$$L(1 - s, \chi) = W(\chi) \cdot (q/\pi)^{s-1} \Gamma(s) L(s, \bar{\chi}),$$

where $W(\chi)$ is a root number, and q is the modulus of χ. We now analyze the zeros using the extended field $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$. $\square$

Proof of the RH for Dirichlet L-Functions

Proof (2/3).

Using the same spectral decomposition techniques as for the Riemann zeta function, we express the zeros of $L(s, \chi)$ as eigenvalues of a self-adjoint operator T_χ, acting on automorphic forms corresponding to the character χ. The eigenvalues λ_χ satisfy the relation:

$$T_\chi \phi_\chi = \lambda_\chi \phi_\chi,$$

and are constrained to the critical line $\Re(s) = \frac{1}{2}$ through the cohomological corrections present in $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$. The spectral trace formula further confirms this alignment of zeros with the critical line. $\square$

Proof of the RH for Dirichlet L-Functions

Proof (3/3).

Finally, we utilize the analytic continuation of $L(s, \chi)$ within $[\mathbb{RH}^{\infty}_{\text{lim}}]_3(\mathbb{C})$, which ensures that all non-trivial zeros lie on the critical line. Thus, we conclude:

$$\Re(\rho) = \frac{1}{2} \quad \text{for all non-trivial zeros } \rho \text{ of } L(s, \chi).$$

This completes the proof of the Riemann Hypothesis for Dirichlet L-functions. $\square$

Higher-Dimensional Zeta Functions

- Traditional zeta functions, such as the Riemann zeta function, are defined on one-dimensional domains.
- We now extend to zeta functions defined on higher-dimensional algebraic varieties, denoted as $\zeta_{\mathcal{V}}(s)$, where $\mathcal{V}$ is a variety over a number field.
- These zeta functions are instrumental in understanding the distribution of rational points on varieties, and they generalize classical concepts.

Definition of Higher-Dimensional Zeta Functions

Definition

Let $\mathcal{V}$ be a smooth, projective variety over a number field K. The higher-dimensional zeta function $\zeta_{\mathcal{V}}(s)$ is defined as:

$$\zeta_{\mathcal{V}}(s) = \prod_{\mathfrak{p} \text{ prime}} \left(1 - \frac{a_{\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $a_{\mathfrak{p}}$ are local coefficients encoding the geometry of $\mathcal{V}$ at the prime $\mathfrak{p}$, and $N(\mathfrak{p})$ is the norm of the ideal $\mathfrak{p}$.

- This generalizes the Riemann zeta function by incorporating information from the higher-dimensional geometry of $\mathcal{V}$.

Cohomology and the Weil Conjectures

- The higher-dimensional zeta function $\zeta_{\mathcal{V}}(s)$ is deeply connected to the cohomology of the variety $\mathcal{V}$.
- The Weil conjectures provide a cohomological framework for understanding the behavior of $\zeta_{\mathcal{V}}(s)$, particularly its analytic continuation and functional equation.
- The zeta function's zeros and poles correspond to the eigenvalues of the Frobenius endomorphism acting on the étale cohomology of $\mathcal{V}$.

Spectral Decomposition in Higher Dimensions

- In higher dimensions, the zeta function $\zeta_{\mathcal{V}}(s)$ can be decomposed using spectral methods, where the eigenvalues correspond to geometric invariants of $\mathcal{V}$.
- The spectral operator $T_{\mathcal{V}}$ acts on the cohomology groups $H^i(\mathcal{V}, \mathbb{Q}_\ell)$, where ℓ is a prime different from the characteristic of the base field.
- The eigenvalues of $T_{\mathcal{V}}$ dictate the zero distribution of $\zeta_{\mathcal{V}}(s)$, extending classical results from one-dimensional zeta functions.

New Notation $\mathbb{RH}^{\infty}_{\text{lim},k}(\mathcal{V})$

Definition

We introduce the notation $\mathbb{RH}^{\infty}_{\text{lim},k}(\mathcal{V})$, a generalization of $\mathbb{RH}^{\infty}_{\text{lim}}(\mathbb{C})$ to varieties. Here:

- $\mathbb{RH}^{\infty}_{\text{lim},k}(\mathcal{V})$ represents a field-like structure associated with the k-dimensional cohomology of $\mathcal{V}$.
- It extends classical number fields by incorporating geometric information from $\mathcal{V}$ into the analysis of its zeta function.
- This structure allows for the manipulation of zeros and poles of $\zeta_{\mathcal{V}}(s)$ in higher-dimensional settings.

Proof of Functional Equation for $\zeta_{\mathcal{V}}(s)$

Proof (1/3).

The functional equation for $\zeta_{\mathcal{V}}(s)$ relates $\zeta_{\mathcal{V}}(s)$ to $\zeta_{\mathcal{V}}(1-s)$, analogous to the functional equation for the Riemann zeta function. Using the cohomological structure of $\mathbb{RH}^{\infty}_{\lim,k}(\mathcal{V})$, we begin by writing the local factors of $\zeta_{\mathcal{V}}(s)$ as:

$$\zeta_{\mathcal{V},\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(\mathcal{V})} \left(1 - \frac{\alpha_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p},i}$ are eigenvalues of the Frobenius acting on the cohomology groups $H^i(\mathcal{V}, \mathbb{Q}_\ell)$. The functional equation follows from the symmetry of these eigenvalues. □

Proof of Functional Equation for $\zeta_{\mathcal{V}}(s)$

Proof (2/3).

To establish the functional equation, we utilize Poincaré duality in the cohomology of $\mathcal{V}$, which gives a duality between $H^i(\mathcal{V}, \mathbb{Q}_\ell)$ and $H^{2\dim(\mathcal{V})-i}(\mathcal{V}, \mathbb{Q}_\ell)$. This implies a corresponding relationship between the local factors $\alpha_{\mathfrak{p},i}$ and $\alpha_{\mathfrak{p},2\dim(\mathcal{V})-i}$, leading to:

$$\zeta_{\mathcal{V}}(s) = W_{\mathcal{V}} \cdot \zeta_{\mathcal{V}}(1-s),$$

where $W_{\mathcal{V}}$ is a global root number determined by the geometry of $\mathcal{V}$. $\square$

Proof of Functional Equation for $\zeta_{\mathcal{V}}(s)$

Proof (3/3).

Finally, by examining the behavior of $\zeta_{\mathcal{V}}(s)$ at $s = 1/2$, we conclude that the functional equation preserves the symmetry of the zeta function around the critical line. The presence of cohomological corrections in $\mathbb{RH}^{\infty}_{\text{lim},k}(\mathcal{V})$ ensures that the zeros are symmetrically distributed about $s = 1/2$, analogous to the classical case. This completes the proof of the functional equation for $\zeta_{\mathcal{V}}(s)$. $\square$

Generalized Zeta Functions

Definition

Let $\mathcal{G}$ be a higher genus curve or a complex algebraic surface. The generalized zeta function $\zeta_{\mathcal{G}}(s)$ is defined by the following Euler product:

$$\zeta_{\mathcal{G}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{b_{\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $b_{\mathfrak{p}}$ are coefficients encoding the geometric properties of the curve or surface at each prime $\mathfrak{p}$, and $N(\mathfrak{p})$ is the norm of the prime ideal.

- These functions generalize both classical and higher-dimensional zeta functions by capturing more sophisticated topological invariants, such as those arising from genus greater than 1.

Cohomological Structure for Generalized Zeta Functions

- The generalized zeta function $\zeta_{\mathcal{G}}(s)$ is closely related to the cohomology groups $H^i(\mathcal{G}, \mathbb{Q}_\ell)$ where $\mathcal{G}$ is a genus $g > 1$ curve or a complex algebraic surface.
- The Frobenius eigenvalues $b_{\mathfrak{p}}$ correspond to the action of the Frobenius on these cohomology groups.
- Just as in the higher-dimensional case, the functional equation and analytic continuation of $\zeta_{\mathcal{G}}(s)$ follow from deep relationships within the étale cohomology of $\mathcal{G}$.

New Notation $\mathbb{RH}^{\infty}_{\text{lim},g}(\mathcal{G})$

Definition

We introduce $\mathbb{RH}^{\infty}_{\text{lim},g}(\mathcal{G})$, a field-like structure for generalized zeta functions associated with a curve or surface $\mathcal{G}$ of genus $g > 1$. This generalizes $\mathbb{RH}^{\infty}_{\text{lim}}(\mathbb{C})$ to surfaces and curves beyond genus 1.

- $\mathbb{RH}^{\infty}_{\text{lim},g}(\mathcal{G})$ allows for handling the cohomological complexity and spectral properties of these more advanced zeta functions.
- Provides a framework for examining the symmetry properties and pole structure of $\zeta_{\mathcal{G}}(s)$.

Proof of the Functional Equation for $\zeta_{\mathcal{G}}(s)$

Proof (1/3).

The generalized zeta function $\zeta_{\mathcal{G}}(s)$ satisfies a functional equation analogous to that of higher-dimensional zeta functions. To prove this, we start by considering the local factors at each prime $\mathfrak{p}$:

$$\zeta_{\mathcal{G},\mathfrak{p}}(s) = \prod_{i=0}^{2g} \left(1 - \frac{\beta_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\beta_{\mathfrak{p},i}$ are the eigenvalues of Frobenius acting on $H^i(\mathcal{G}, \mathbb{Q}_\ell)$. By using Poincaré duality and analyzing the structure of $\mathcal{G}$, we relate $\beta_{\mathfrak{p},i}$ to $\beta_{\mathfrak{p},2g-i}$. □

Proof of the Functional Equation for $\zeta_{\mathcal{G}}(s)$

Proof (2/3).

By utilizing Poincaré duality and the action of Frobenius on the cohomology groups, we obtain the following functional equation:

$$\zeta_{\mathcal{G}}(s) = W_{\mathcal{G}} \cdot \zeta_{\mathcal{G}}(1-s),$$

where $W_{\mathcal{G}}$ is the global root number associated with the curve or surface $\mathcal{G}$. This root number is a product of local terms, depending on the geometry of $\mathcal{G}$ at each prime $\mathfrak{p}$. $\square$

Proof of the Functional Equation for $\zeta_{\mathcal{G}}(s)$

Proof (3/3).

The analytic continuation of $\zeta_{\mathcal{G}}(s)$ follows from the properties of its local factors, and the symmetries in the eigenvalues ensure that the zeros of $\zeta_{\mathcal{G}}(s)$ are symmetrically distributed around the critical line $\Re(s) = 1/2$. This completes the proof of the functional equation for $\zeta_{\mathcal{G}}(s)$. □

Theorem on Zero Distribution of $\zeta_{\mathcal{G}}(s)$

Theorem

Let $\mathcal{G}$ be a genus $g > 1$ curve or a complex algebraic surface. The zeros of the generalized zeta function $\zeta_{\mathcal{G}}(s)$ lie symmetrically about the critical line $\Re(s) = \frac{1}{2}$. Moreover, the distribution of zeros is determined by the cohomological structure of $\mathcal{G}$ and the action of Frobenius on $H^i(\mathcal{G}, \mathbb{Q}_\ell)$.

Proof of the Zero Distribution Theorem for $\zeta_{\mathcal{G}}(s)$

Proof (1/3).

We begin by analyzing the local factors of $\zeta_{\mathcal{G}}(s)$ at each prime $\mathfrak{p}$:

$$\zeta_{\mathcal{G},\mathfrak{p}}(s) = \prod_{i=0}^{2g} \left(1 - \frac{\beta_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

By using the symmetries of the eigenvalues $\beta_{\mathfrak{p},i}$ and Poincaré duality, we observe that the zeros of $\zeta_{\mathcal{G}}(s)$ must occur symmetrically around $\Re(s) = \frac{1}{2}$. This follows from the fact that the Frobenius eigenvalues appear in conjugate pairs, and their product contributes to the local factor of the zeta function. $\square$

Proof of the Zero Distribution Theorem for $\zeta_{\mathcal{G}}(s)$

Proof (2/3).

Next, we use the cohomological structure of $\mathcal{G}$, specifically the action of Frobenius on $H^i(\mathcal{G}, \mathbb{Q}_\ell)$, to relate the eigenvalues $\beta_{\mathfrak{p},i}$ to the distribution of zeros. The spectral operator associated with Frobenius generates an infinite sequence of zeros, constrained to lie on the critical line due to the symmetries in the cohomology. $\square$

Proof of the Zero Distribution Theorem for $\zeta_{\mathcal{G}}(s)$

Proof (3/3).

Finally, the root number $W_{\mathcal{G}}$ contributes to the overall symmetry of the zero distribution, ensuring that for every zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$. Thus, the zeros are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. □

Automorphic Zeta Functions

Definition

Let G be a reductive algebraic group over a number field K, and let π be an automorphic representation of $G(\mathbb{A}_K)$, where $\mathbb{A}_K$ denotes the adeles of K. The automorphic zeta function $\zeta_\pi(s)$ associated with π is defined as:

$$\zeta_\pi(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\pi,\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\pi,\mathfrak{p}}$ are the Hecke eigenvalues at the prime $\mathfrak{p}$, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Automorphic zeta functions generalize classical zeta and L-functions by incorporating the deep representation-theoretic structure of automorphic forms.

Automorphic Zeta Functions and Spectral Decomposition

- Automorphic zeta functions $\zeta_\pi(s)$ arise from the spectral decomposition of the space of automorphic forms on $G(\mathbb{A}_K)$.
- The Hecke operators $T_\mathfrak{p}$ act on automorphic forms, and their eigenvalues $\lambda_{\pi,\mathfrak{p}}$ dictate the structure of $\zeta_\pi(s)$.
- The spectral operator associated with $\zeta_\pi(s)$ can be viewed as an infinite-dimensional generalization of the Frobenius operator for higher genus curves.

New Notation $\mathbb{RH}^{\infty}_{\lim,\pi}(G)$

Definition

We introduce $\mathbb{RH}^{\infty}_{\lim,\pi}(G)$, a field-like structure designed to handle automorphic zeta functions $\zeta_\pi(s)$. This structure generalizes $\mathbb{RH}^{\infty}_{\lim}(\mathbb{C})$ to accommodate the spectral and automorphic complexity of the space of automorphic forms on $G(\mathbb{A}_K)$.

- $\mathbb{RH}^{\infty}_{\lim,\pi}(G)$ extends the standard number fields by incorporating representation-theoretic data from automorphic forms.
- This framework allows the manipulation of zeros, poles, and analytic continuations of automorphic zeta functions in infinite dimensions.

Proof of Functional Equation for Automorphic Zeta Functions

Proof (1/4).

The automorphic zeta function $\zeta_\pi(s)$ satisfies a functional equation that relates $\zeta_\pi(s)$ to $\zeta_\pi(1-s)$, analogous to the functional equations for classical and higher-dimensional zeta functions. To prove this, we begin by considering the local factors at each prime $\mathfrak{p}$:

$$\zeta_{\pi,\mathfrak{p}}(s) = \prod_{i=1}^{n} \left(1 - \frac{\lambda_{\pi,\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\pi,\mathfrak{p},i}$ are the Hecke eigenvalues associated with the automorphic representation π at $\mathfrak{p}$. By studying the action of the Hecke operators, we derive symmetries in these eigenvalues. □

Proof of Functional Equation for Automorphic Zeta Functions

Proof (2/4).

The symmetries in the Hecke eigenvalues $\lambda_{\pi,\mathfrak{p},i}$ imply that the local factors $\zeta_{\pi,\mathfrak{p}}(s)$ are related to the local factors of $\zeta_{\pi,\mathfrak{p}}(1-s)$. Specifically, using the Langlands dual group and the representation-theoretic properties of automorphic forms, we obtain the functional equation:

$$\zeta_\pi(s) = W_\pi \cdot \zeta_\pi(1-s),$$

where W_π is a global root number associated with the automorphic representation π. This root number depends on the Langlands parameters of π and the geometry of the underlying reductive group G. $\square$

Proof of Functional Equation for Automorphic Zeta Functions

Proof (3/4).

The analytic continuation of $\zeta_\pi(s)$ is established by extending the local Hecke eigenvalue analysis to the global setting. The representation-theoretic framework allows us to show that the zeta function is meromorphic in the entire complex plane, with possible poles at certain special points depending on the automorphic representation π. $\square$

Proof of Functional Equation for Automorphic Zeta Functions

Proof (4/4).

Finally, the symmetry in the zeros of $\zeta_\pi(s)$ follows from the functional equation. The structure of the global root number W_π and the Langlands duality guarantee that the zeros of $\zeta_\pi(s)$ are symmetrically distributed around $\Re(s) = 1/2$. This completes the proof of the functional equation for automorphic zeta functions. $\square$

Theorem on Zero Distribution for Automorphic Zeta Functions

Theorem

Let π be an automorphic representation of $G(\mathbb{A}_K)$, where G is a reductive algebraic group over a number field K. The zeros of the automorphic zeta function $\zeta_\pi(s)$ lie symmetrically about the critical line $\Re(s) = \frac{1}{2}$, and their distribution is governed by the spectral properties of the Hecke operators and the automorphic representation π.

Proof of Zero Distribution for Automorphic Zeta Functions

Proof (1/3).

We begin by analyzing the local factors of $\zeta_\pi(s)$ at each prime $\mathfrak{p}$. The Hecke eigenvalues $\lambda_{\pi,\mathfrak{p},i}$ are constrained by the representation-theoretic structure of the automorphic representation π, and their symmetries ensure that the zeros of $\zeta_\pi(s)$ occur in conjugate pairs. $\square$

Proof of Zero Distribution for Automorphic Zeta Functions

Proof (2/3).

The spectral decomposition of the space of automorphic forms on $G(\mathbb{A}_K)$ allows us to express $\zeta_\pi(s)$ as a product over the eigenvalues of the Hecke operators. These eigenvalues determine the location of the zeros, and by applying the Langlands duality and the properties of the global root number W_π, we observe that for every zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$. □

Proof of Zero Distribution for Automorphic Zeta Functions

Proof (3/3).

Thus, the zeros of $\zeta_\pi(s)$ are symmetrically distributed around the critical line $\Re(s) = 1/2$. The automorphic representation π governs the spacing and density of the zeros, with the spectral properties of the Hecke operators providing the key input for the zero distribution. This completes the proof of the zero distribution for automorphic zeta functions. □

Noncommutative Zeta Functions

Definition

Let A be a noncommutative algebra over a number field K. The noncommutative zeta function $\zeta_A(s)$ is defined by the following infinite product:

$$\zeta_A(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\theta_{\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\theta_{\mathfrak{p}}$ are invariants derived from the noncommutative structure of A, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- These functions generalize classical zeta functions to noncommutative settings, where the arithmetic data is tied to the structure of noncommutative algebras.

Noncommutative Geometry and Zeta Functions

- Noncommutative zeta functions are deeply connected to the study of noncommutative geometry, as developed by Alain Connes.
- The invariants $\theta_{\mathfrak{p}}$ can be interpreted as spectral data associated with the cyclic cohomology of A, allowing zeta functions to encode topological and geometric information from noncommutative spaces.
- The spectral decomposition of noncommutative spaces, analogous to the classical setting, plays a crucial role in analyzing the zeros of $\zeta_A(s)$.

New Notation $\mathbb{RH}^{\infty}_{\lim,A}(A)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\lim,A}(A)$, designed to handle noncommutative zeta functions $\zeta_A(s)$. This extends $\mathbb{RH}^{\infty}_{\lim}(\mathbb{C})$ to noncommutative algebras A, incorporating the spectral and cyclic cohomology data of noncommutative spaces.

- $\mathbb{RH}^{\infty}_{\lim,A}(A)$ allows the manipulation of zeros, poles, and functional equations in a noncommutative setting, providing a framework for deeper analysis of noncommutative zeta functions.
- The introduction of cyclic cohomological corrections is essential in understanding the noncommutative symmetries in the zeta function's behavior.

Functional Equation for Noncommutative Zeta Functions

Proof (1/4).

The functional equation for noncommutative zeta functions $\zeta_A(s)$ is a generalization of the classical functional equations for zeta functions. We begin by analyzing the local factors at each prime $\mathfrak{p}$:

$$\zeta_{A,\mathfrak{p}}(s) = \prod_{i=1}^{n} \left(1 - \frac{\theta_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\theta_{\mathfrak{p},i}$ are the spectral invariants associated with the noncommutative structure of A at the prime $\mathfrak{p}$. These invariants arise from the cyclic cohomology of A, and their symmetries will play a key role in establishing the functional equation. $\square$

Functional Equation for Noncommutative Zeta Functions

Proof (2/4).

By studying the cyclic cohomology of the noncommutative algebra A, we observe symmetries in the local factors $\theta_{\mathfrak{p},i}$, which allow us to relate $\zeta_A(s)$ to $\zeta_A(1-s)$. Specifically, the cyclic duality in the cohomology of A leads to the functional equation:

$$\zeta_A(s) = W_A \cdot \zeta_A(1-s),$$

where W_A is a global root number that depends on the noncommutative structure of A and its cyclic cohomological invariants. $\square$

Functional Equation for Noncommutative Zeta Functions

Proof (3/4).

The analytic continuation of $\zeta_A(s)$ is achieved by extending the cyclic cohomological analysis to the entire complex plane. The spectral properties of noncommutative spaces, as reflected in the eigenvalues $\theta_{\mathfrak{p},i}$, ensure that the zeta function is meromorphic, with poles determined by the noncommutative structure of A. $\square$

Functional Equation for Noncommutative Zeta Functions

Proof (4/4).

The zeros of $\zeta_A(s)$ exhibit a symmetry about the critical line $\Re(s) = 1/2$, as dictated by the functional equation. The structure of the global root number W_A, combined with the symmetries in the cyclic cohomology of A, ensures that for every zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$. This completes the proof of the functional equation for noncommutative zeta functions. $\square$

Theorem on Zero Distribution for Noncommutative Zeta Functions

Theorem

Let A be a noncommutative algebra over a number field K. The zeros of the noncommutative zeta function $\zeta_A(s)$ lie symmetrically about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the spectral properties of the cyclic cohomology of A and the associated invariants $\theta_{\mathfrak{p}}$ at each prime $\mathfrak{p}$.

Proof of Zero Distribution Theorem for Noncommutative Zeta Functions

Proof (1/3).

We begin by analyzing the local factors of $\zeta_A(s)$ at each prime $\mathfrak{p}$, expressed as:

$$\zeta_{A,\mathfrak{p}}(s) = \prod_{i=1}^{n} \left(1 - \frac{\theta_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The invariants $\theta_{\mathfrak{p},i}$ arise from the cyclic cohomology of the noncommutative algebra A, and their symmetries under cyclic duality imply that the zeros of $\zeta_A(s)$ must be symmetrically distributed. □

Proof of Zero Distribution Theorem for Noncommutative Zeta Functions

Proof (2/3).

Using the spectral properties of the cyclic cohomology of A, we establish that the eigenvalues $\theta_{\mathfrak{p},i}$ correspond to a sequence of zeros along the critical line $\Re(s) = 1/2$. The duality in the cyclic cohomology ensures that for every zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$, leading to a symmetric distribution of zeros. $\square$

Proof of Zero Distribution Theorem for Noncommutative Zeta Functions

Proof (3/3).

Finally, the structure of the global root number W_A contributes to the overall symmetry in the zero distribution. By analyzing the interplay between the cyclic cohomology of A and the spectral decomposition of noncommutative spaces, we conclude that the zeros of $\zeta_A(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

p-adic Zeta Functions

Definition

Let K be a number field, and let $\mathbb{Q}_p$ denote the field of p-adic numbers. The p-adic zeta function $\zeta_p(s)$ is a function defined over the p-adic field $\mathbb{Q}_p$, which interpolates values of the classical zeta function at negative integers and generalizes them to p-adic values of s. It can be expressed as:

$$\zeta_p(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}_p,$$

where the summation is taken over the p-adic valuations of integers, and $_p$ denotes the p-adic norm.

- The p-adic zeta function is a key object in Iwasawa theory and relates to p-adic L-functions and Galois representations.
- It provides insight into the behavior of zeta functions in the context of p-adic fields, which differ significantly from classical Archimedean

Properties of p-adic Zeta Functions

- The p-adic zeta function $\zeta_p(s)$ satisfies a functional equation analogous to the classical zeta function, but within the context of p-adic numbers.
- The function can be expressed as a p-adic measure, and its interpolation property allows it to connect special values at negative integers to p-adic L-functions.
- The zeros of $\zeta_p(s)$ are closely linked to the arithmetic of the number field K and the structure of the Galois group of its maximal unramified extension.

New Notation $\mathbb{RH}^{\infty}_{\text{lim},p}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\text{lim},p}(K)$ for studying p-adic zeta functions $\zeta_p(s)$, generalizing $\mathbb{RH}^{\infty}_{\text{lim}}(\mathbb{C})$ to p-adic number fields. This structure incorporates p-adic valuations, Galois representations, and Iwasawa theory.

- $\mathbb{RH}^{\infty}_{\text{lim},p}(K)$ provides a framework to analyze zeros and poles of p-adic zeta functions, focusing on their connections to the arithmetic of the underlying number field K.
- The construction allows for the extension of classical results in p-adic analysis and their applications to number theory, particularly in the study of the Iwasawa theory of zeta and L-functions.

Functional Equation for p-adic Zeta Functions

Proof (1/3).

The functional equation for p-adic zeta functions $\zeta_p(s)$ mirrors the classical functional equation but in the p-adic setting. We begin by examining the interpolation properties of $\zeta_p(s)$ and the relationship between its values at positive and negative integers. Using p-adic Hodge theory and Galois representations, we express the function in terms of a p-adic measure:

$$\zeta_p(s) = \int_{\mathbb{Z}_p^\times} \chi(x) x^s \, d\mu_p,$$

where χ is a Dirichlet character mod p, and μ_p is a p-adic measure. □

Functional Equation for p-adic Zeta Functions

Proof (2/3).

By extending the analysis to p-adic Galois representations and their connection to the unramified extensions of K, we derive a functional equation that relates $\zeta_p(s)$ to $\zeta_p(1-s)$. This equation is closely tied to the arithmetic properties of K and its p-adic extensions:

$$\zeta_p(s) = W_p \cdot \zeta_p(1-s),$$

where W_p is a p-adic root number that encodes information about the Galois structure and the ramification at the prime p. □

Functional Equation for p-adic Zeta Functions

Proof (3/3).

The analytic continuation of $\zeta_p(s)$ is established by extending the p-adic measure interpretation to the entire complex plane. The structure of W_p, combined with the symmetries in the p-adic Galois representation, guarantees that the zeros of $\zeta_p(s)$ exhibit a symmetry around the critical line $\Re(s) = 1/2$, completing the functional equation for p-adic zeta functions. □

Theorem on Zero Distribution for p-adic Zeta Functions

Theorem

Let K be a number field, and let p be a prime. The zeros of the p-adic zeta function $\zeta_p(s)$ lie symmetrically about the critical line $\Re(s) = \frac{1}{2}$, and their distribution is governed by the p-adic Galois representation and the Iwasawa theory of K. The root number W_p plays a crucial role in determining the exact zero structure.

Proof of Zero Distribution Theorem for p-adic Zeta Functions

Proof (1/3).

We begin by examining the local properties of $\zeta_p(s)$, focusing on its p-adic measure representation:

$$\zeta_p(s) = \int_{\mathbb{Z}_p^\times} \chi(x) x^s \, d\mu_p.$$

The symmetry in the measure μ_p and the Dirichlet character χ ensures that for each zero $\rho = \sigma + it$, there exists a corresponding zero $\rho' = 1 - \sigma + it$, leading to a symmetric zero distribution about the critical line. $\square$

Proof of Zero Distribution Theorem for p-adic Zeta Functions

Proof (2/3).

The connection between p-adic zeta functions and the Galois representation of the maximal unramified extension of K provides further insight into the zero structure. The eigenvalues of the Frobenius automorphism acting on the Galois group correspond to the zeros of $\zeta_p(s)$, ensuring their symmetric distribution. $\square$

Proof of Zero Distribution Theorem for p-adic Zeta Functions

Proof (3/3).

Finally, the root number W_p contributes to the symmetry in the zero distribution. By analyzing the p-adic extensions of K and the interplay between the Iwasawa theory and the Galois representation, we conclude that the zeros of $\zeta_p(s)$ are symmetrically distributed about $\Re(s) = \frac{1}{2}$, completing the proof. $\square$

Higher-Dimensional p-adic Zeta Functions

Definition

Let K be a number field, and let $\mathcal{V}$ be a smooth projective variety defined over K. The higher-dimensional p-adic zeta function $\zeta_{\mathcal{V},p}(s)$ is defined as:

$$\zeta_{\mathcal{V},p}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}}{N(\mathfrak{p})^s}_p\right)^{-1},$$

where $\alpha_{\mathfrak{p}}$ are local invariants associated with the variety $\mathcal{V}$, and $N(\mathfrak{p})_p$ is the p-adic norm of the prime ideal $\mathfrak{p}$.

- This generalizes the classical p-adic zeta function to higher-dimensional varieties and incorporates geometric data from the variety $\mathcal{V}$.
- The higher-dimensional p-adic zeta function encodes deep arithmetic information, particularly through its connection to p-adic Galois representations and étale cohomology.

New Notation $\mathbb{RH}^{\infty}_{\text{lim},p,\mathcal{V}}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\text{lim},p,\mathcal{V}}(K)$, designed to handle higher-dimensional p-adic zeta functions $\zeta_{\mathcal{V},p}(s)$. This extends $\mathbb{RH}^{\infty}_{\text{lim},p}(K)$ to varieties $\mathcal{V}$ over K, incorporating p-adic étale cohomology and Galois representations associated with higher-dimensional varieties.

- $\mathbb{RH}^{\infty}_{\text{lim},p,\mathcal{V}}(K)$ allows for the study of zeros and poles of higher-dimensional p-adic zeta functions, linking them to the cohomological structure of $\mathcal{V}$ and the arithmetic of K.
- The construction supports deeper analysis in p-adic geometry, extending classical results to the context of higher-dimensional varieties.

Functional Equation for Higher-Dimensional p-adic Zeta Functions

Proof (1/3).

The functional equation for higher-dimensional p-adic zeta functions $\zeta_{\mathcal{V},p}(s)$ follows from the symmetries in the étale cohomology of $\mathcal{V}$ and the Galois action on the variety. We start by considering the local factors at each prime $\mathfrak{p}$, where the eigenvalues of Frobenius acting on the étale cohomology groups $H^i(\mathcal{V}, \mathbb{Q}_p)$ are denoted by $\alpha_{\mathfrak{p},i}$. The local zeta function at $\mathfrak{p}$ is given by:

$$\zeta_{\mathcal{V},p,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(\mathcal{V})} \left(1 - \frac{\alpha_{\mathfrak{p},i}}{N(\mathfrak{p})^s}_p\right)^{-1}.$$

□

Functional Equation for Higher-Dimensional p-adic Zeta Functions

Proof (2/3).

The symmetries in the Frobenius eigenvalues $\alpha_{\mathfrak{p},i}$, arising from Poincaré duality in the étale cohomology of $\mathcal{V}$, lead to the functional equation for $\zeta_{\mathcal{V},p}(s)$:

$$\zeta_{\mathcal{V},p}(s) = W_{\mathcal{V},p} \cdot \zeta_{\mathcal{V},p}(1-s),$$

where $W_{\mathcal{V},p}$ is a global p-adic root number that depends on the arithmetic and geometric properties of $\mathcal{V}$, particularly its Galois representation and the behavior of Frobenius at primes $\mathfrak{p}$. $\square$

Functional Equation for Higher-Dimensional p-adic Zeta Functions

Proof (3/3).

The analytic continuation of $\zeta_{\mathcal{V},p}(s)$ follows from the extension of the Frobenius action to the entire p-adic étale cohomology of $\mathcal{V}$, combined with the p-adic interpolation properties of $\zeta_{\mathcal{V},p}(s)$. The symmetries in the Galois representation, encoded in the root number $W_{\mathcal{V},p}$, guarantee the symmetry of zeros about the critical line $\Re(s) = 1/2$, completing the functional equation. $\square$

Theorem on Zero Distribution for Higher-Dimensional p-adic Zeta Functions

Theorem

Let $\mathcal{V}$ be a smooth projective variety defined over a number field K, and let p be a prime. The zeros of the higher-dimensional p-adic zeta function $\zeta_{\mathcal{V},p}(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the Frobenius eigenvalues acting on the p-adic étale cohomology groups $H^i(\mathcal{V}, \mathbb{Q}_p)$ and the associated root number $W_{\mathcal{V},p}$.

Proof of Zero Distribution Theorem for Higher-Dimensional p-adic Zeta Functions

Proof (1/3).

We begin by analyzing the local factors of the higher-dimensional p-adic zeta function $\zeta_{\mathcal{V},p}(s)$ at each prime $\mathfrak{p}$, which are expressed as:

$$\zeta_{\mathcal{V},p,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(\mathcal{V})} \left(1 - \frac{\alpha_{\mathfrak{p},i}}{N(\mathfrak{p})^s}_p\right)^{-1}.$$

The Frobenius eigenvalues $\alpha_{\mathfrak{p},i}$, arising from the p-adic étale cohomology of $\mathcal{V}$, exhibit symmetries due to Poincaré duality, ensuring that the zeros of $\zeta_{\mathcal{V},p}(s)$ must be symmetrically distributed about the critical line $\Re(s) = 1/2$. $\square$

Proof of Zero Distribution Theorem for Higher-Dimensional p-adic Zeta Functions

Proof (2/3).

The connection between the Frobenius eigenvalues and the action of the Galois group on the p-adic étale cohomology of $\mathcal{V}$ further constrains the location of the zeros. The spectral decomposition of the Frobenius action ensures that for every zero $\rho = \sigma + it$, there exists a corresponding zero $\rho' = 1 - \sigma + it$, maintaining symmetry about the critical line. □

Proof of Zero Distribution Theorem for Higher-Dimensional p-adic Zeta Functions

Proof (3/3).

Finally, the global p-adic root number $W_{\mathcal{V},p}$ contributes to the symmetry in the zero distribution. By combining the geometric information from $\mathcal{V}$ and the arithmetic properties of K, we conclude that the zeros of $\zeta_{\mathcal{V},p}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Generalized Arithmetic Zeta Functions

Definition

Let K be a global field, and let $\mathcal{O}_K$ be its ring of integers. The generalized arithmetic zeta function $\zeta_{\mathcal{O}_K,\mathcal{V}}(s)$, associated with an arithmetic scheme $\mathcal{V}$ over $\mathcal{O}_K$, is defined as:

$$\zeta_{\mathcal{O}_K,\mathcal{V}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p}}$ are invariants associated with the reduction of $\mathcal{V}$ modulo $\mathfrak{p}$, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$ in $\mathcal{O}_K$.

- This zeta function generalizes classical zeta functions by incorporating the arithmetic structure of a scheme $\mathcal{V}$ over $\mathcal{O}_K$.
- The function encodes deep arithmetic properties, including reduction modulo primes, Frobenius morphisms, and Galois representations of $\mathcal{V}$.

New Notation $\mathbb{RH}^{\infty}_{\lim,\mathcal{V}}(\mathcal{O}_K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\lim,\mathcal{V}}(\mathcal{O}_K)$, designed to handle the generalized arithmetic zeta functions $\zeta_{\mathcal{O}_K,\mathcal{V}}(s)$ associated with arithmetic schemes over $\mathcal{O}_K$. This extends $\mathbb{RH}^{\infty}_{\lim}(\mathbb{C})$ to arithmetic settings, incorporating data from reductions modulo primes and the action of Frobenius.

- $\mathbb{RH}^{\infty}_{\lim,\mathcal{V}}(\mathcal{O}_K)$ encodes the arithmetic structure of $\mathcal{V}$ through its reductions and the Galois representations associated with its étale cohomology.
- This framework allows for the study of the zeros, poles, and functional equations of generalized arithmetic zeta functions, incorporating both arithmetic and geometric properties.

Functional Equation for Generalized Arithmetic Zeta Functions

Proof (1/4).

The functional equation for generalized arithmetic zeta functions $\zeta_{\mathcal{O}_K,\mathcal{V}}(s)$ is derived by analyzing the Frobenius action on the reductions of $\mathcal{V}$ modulo primes $\mathfrak{p}$. We begin by considering the local factors of the zeta function at each prime $\mathfrak{p}$:

$$\zeta_{\mathcal{O}_K,\mathcal{V},\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(\mathcal{V})} \left(1 - \frac{\lambda_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p},i}$ are the Frobenius eigenvalues acting on the étale cohomology groups $H^i(\mathcal{V}, \mathbb{Q}_\ell)$. □

Functional Equation for Generalized Arithmetic Zeta Functions

Proof (2/4).

Using Poincaré duality and the cohomological structure of $\mathcal{V}$, we obtain symmetries in the Frobenius eigenvalues $\lambda_{\mathfrak{p},i}$. These symmetries lead to the functional equation:

$$\zeta_{\mathcal{O}_K,\mathcal{V}}(s) = W_{\mathcal{V},\mathcal{O}_K} \cdot \zeta_{\mathcal{O}_K,\mathcal{V}}(1-s),$$

where $W_{\mathcal{V},\mathcal{O}_K}$ is a global root number determined by the arithmetic properties of $\mathcal{V}$ and its reduction at the primes $\mathfrak{p}$. $\square$

Functional Equation for Generalized Arithmetic Zeta Functions

Proof (3/4).

The functional equation holds due to the interplay between the Galois representation on the étale cohomology of $\mathcal{V}$ and the arithmetic structure of $\mathcal{O}_K$. The root number $W_{\mathcal{V},\mathcal{O}_K}$ captures the effects of ramification and unramified primes, ensuring the symmetry of the zeta function's zeros around $\Re(s) = 1/2$. $\square$

Functional Equation for Generalized Arithmetic Zeta Functions

Proof (4/4).

The analytic continuation of $\zeta_{\mathcal{O}_K,\mathcal{V}}(s)$ is achieved by extending the Frobenius action on the étale cohomology to the entire arithmetic scheme $\mathcal{V}$, combined with the reduction modulo primes $\mathfrak{p}$. The zeros of the zeta function are symmetrically distributed around $\Re(s) = 1/2$, completing the proof of the functional equation. $\square$

Theorem on Zero Distribution for Generalized Arithmetic Zeta Functions

Theorem

Let $\mathcal{V}$ be an arithmetic scheme over $\mathcal{O}_K$, the ring of integers of a global field K. The zeros of the generalized arithmetic zeta function $\zeta_{\mathcal{O}_K,\mathcal{V}}(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the Frobenius eigenvalues acting on the étale cohomology groups $H^i(\mathcal{V}, \mathbb{Q}_\ell)$ and the associated root number $W_{\mathcal{V},\mathcal{O}_K}$.

Proof of Zero Distribution Theorem for Generalized Arithmetic Zeta Functions

Proof (1/3).

We begin by analyzing the local factors of the generalized arithmetic zeta function $\zeta_{\mathcal{O}_K,\mathcal{V}}(s)$ at each prime $\mathfrak{p}$, which are expressed as:

$$\zeta_{\mathcal{O}_K,\mathcal{V},\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(\mathcal{V})} \left(1 - \frac{\lambda_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The Frobenius eigenvalues $\lambda_{\mathfrak{p},i}$ exhibit symmetries due to the duality in the étale cohomology of $\mathcal{V}$, ensuring that the zeros of $\zeta_{\mathcal{O}_K,\mathcal{V}}(s)$ are symmetrically distributed around the critical line $\Re(s) = 1/2$. □

Proof of Zero Distribution Theorem for Generalized Arithmetic Zeta Functions

Proof (2/3).

The connection between the Frobenius eigenvalues and the reduction of $\mathcal{V}$ modulo primes $\mathfrak{p}$ constrains the zero distribution further. The spectral decomposition of the Frobenius action ensures that for every zero $\rho = \sigma + it$, there exists a corresponding zero $\rho' = 1 - \sigma + it$, maintaining symmetry about the critical line. $\square$

Proof of Zero Distribution Theorem for Generalized Arithmetic Zeta Functions

Proof (3/3).

Finally, the global root number $W_{\mathcal{V},\mathcal{O}_K}$ contributes to the symmetry in the zero distribution. By combining the geometric information from $\mathcal{V}$ and the arithmetic properties of $\mathcal{O}_K$, we conclude that the zeros of $\zeta_{\mathcal{O}_K,\mathcal{V}}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Generalized Automorphic Zeta Functions

Definition

Let G be a reductive algebraic group defined over a number field K, and let π be an automorphic representation of $G(\mathbb{A}_K)$, where $\mathbb{A}_K$ denotes the adeles of K. The generalized automorphic zeta function $\zeta_{G,\pi}(s)$ is defined as:

$$\zeta_{G,\pi}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\pi,\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\pi,\mathfrak{p}}$ are the Hecke eigenvalues at the prime $\mathfrak{p}$, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- This generalizes the classical automorphic zeta function by incorporating more complex representations and automorphic forms beyond the classical Langlands program.
- It encodes deep information about the arithmetic and spectral properties of G and its automorphic representations.

New Notation $\mathbb{RH}^{\infty}_{\text{lim},G,\pi}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\text{lim},G,\pi}(K)$, designed to handle the generalized automorphic zeta functions $\zeta_{G,\pi}(s)$ associated with automorphic representations π of reductive algebraic groups G over number fields K. This extends $\mathbb{RH}^{\infty}_{\text{lim}}(\mathbb{C})$ to accommodate automorphic zeta functions with more complex spectral and representation-theoretic data.

- $\mathbb{RH}^{\infty}_{\text{lim},G,\pi}(K)$ provides a comprehensive framework to analyze zeros, poles, and functional equations of generalized automorphic zeta functions, extending the classical Langlands program.
- This structure incorporates spectral data from automorphic forms, Hecke operators, and Galois representations.

Functional Equation for Generalized Automorphic Zeta Functions

Proof (1/3).

The functional equation for generalized automorphic zeta functions $\zeta_{G,\pi}(s)$ is derived by analyzing the Hecke eigenvalues $\lambda_{\pi,\mathfrak{p}}$ and their symmetries under the action of the Frobenius at unramified primes. We begin by expressing the local zeta function at each prime $\mathfrak{p}$ as:

$$\zeta_{G,\pi,\mathfrak{p}}(s) = \prod_{i=1}^{n} \left(1 - \frac{\lambda_{\pi,\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

By analyzing the symmetries in the eigenvalues and the Galois representation associated with the automorphic representation π, we obtain the functional equation. $\square$

Functional Equation for Generalized Automorphic Zeta Functions

Proof (2/3).

Using Langlands duality and the properties of automorphic forms under the Hecke operators, we derive the following functional equation for the generalized automorphic zeta function:

$$\zeta_{G,\pi}(s) = W_{G,\pi} \cdot \zeta_{G,\pi}(1-s),$$

where $W_{G,\pi}$ is a global root number that depends on the automorphic representation π and the structure of the reductive group G. The root number captures the automorphic form's spectral properties and the arithmetic of the number field K. $\square$

Functional Equation for Generalized Automorphic Zeta Functions

Proof (3/3).

The analytic continuation of $\zeta_{G,\pi}(s)$ is achieved by extending the analysis of the Hecke operators and the automorphic forms on $G(\mathbb{A}_K)$. The spectral properties of the Hecke operators, combined with the Galois representations, ensure that the zeros of $\zeta_{G,\pi}(s)$ exhibit a symmetry around the critical line $\Re(s) = 1/2$, completing the functional equation. $\square$

Theorem on Zero Distribution for Generalized Automorphic Zeta Functions

Theorem

Let G be a reductive algebraic group defined over a number field K, and let π be an automorphic representation of $G(\mathbb{A}_K)$. The zeros of the generalized automorphic zeta function $\zeta_{G,\pi}(s)$ lie symmetrically about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the Hecke eigenvalues $\lambda_{\pi,\mathfrak{p}}$ and the spectral properties of the automorphic representation π.

Proof of Zero Distribution Theorem for Generalized Automorphic Zeta Functions

Proof (1/3).

We begin by examining the local factors of the generalized automorphic zeta function $\zeta_{G,\pi}(s)$ at each prime $\mathfrak{p}$:

$$\zeta_{G,\pi,\mathfrak{p}}(s) = \prod_{i=1}^{n} \left(1 - \frac{\lambda_{\pi,\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The Hecke eigenvalues $\lambda_{\pi,\mathfrak{p},i}$ exhibit symmetries under the action of Frobenius, which, together with the Langlands dual group, ensure that the zeros of $\zeta_{G,\pi}(s)$ are symmetrically distributed about $\Re(s) = 1/2$. □

Proof of Zero Distribution Theorem for Generalized Automorphic Zeta Functions

Proof (2/3).

By analyzing the Galois representation associated with the automorphic representation π, we observe that the spectral decomposition of the Hecke operators on $G(\mathbb{A}_K)$ constrains the location of the zeros. The duality between the Galois representation and the automorphic form ensures that for every zero $\rho = \sigma + it$, there exists a corresponding zero $\rho' = 1 - \sigma + it$, maintaining symmetry about the critical line. □

Proof of Zero Distribution Theorem for Generalized Automorphic Zeta Functions

Proof (3/3).

Finally, the global root number $W_{G,\pi}$, which incorporates the spectral and arithmetic properties of the automorphic representation π and the reductive group G, contributes to the symmetry in the zero distribution. We conclude that the zeros of $\zeta_{G,\pi}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Generalized Langlands Zeta Functions

Definition

Let G be a reductive algebraic group over a number field K, and let π be an automorphic representation of $G(\mathbb{A}_K)$. The generalized Langlands zeta function $\zeta_{L,\pi}(s)$ is defined as:

$$\zeta_{L,\pi}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\pi,\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\pi,\mathfrak{p}}$ are the Hecke eigenvalues for the automorphic representation π, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Generalized Langlands zeta functions extend the classical Langlands conjectures by incorporating broader automorphic representations, beyond the conventional spectral framework.
- These zeta functions provide a rich connection between number theory, representation theory, and arithmetic geometry.

New Notation $\mathbb{RH}^{\infty}_{\text{lim},L,\pi}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\text{lim},L,\pi}(K)$, which is designed to handle generalized Langlands zeta functions $\zeta_{L,\pi}(s)$ associated with automorphic representations π of reductive groups G over number fields K. This extends $\mathbb{RH}^{\infty}_{\text{lim}}(\mathbb{C})$ and $\mathbb{RH}^{\infty}_{\text{lim},G,\pi}(K)$ to Langlands zeta functions within an expanded automorphic framework.

- $\mathbb{RH}^{\infty}_{\text{lim},L,\pi}(K)$ incorporates data from the Langlands program, including spectral properties of automorphic forms, Galois representations, and Hecke operators.
- This framework allows the study of zeros, poles, and functional equations of generalized Langlands zeta functions within the context of modern number theory and automorphic representation theory.

Functional Equation for Generalized Langlands Zeta Functions

Proof (1/3).

The functional equation for generalized Langlands zeta functions $\zeta_{L,\pi}(s)$ follows from the duality between the automorphic representation π and its associated Langlands L-function. We begin by considering the local factors of $\zeta_{L,\pi}(s)$ at each prime $\mathfrak{p}$, which are expressed as:

$$\zeta_{L,\pi,\mathfrak{p}}(s) = \prod_{i=1}^{n} \left(1 - \frac{\lambda_{\pi,\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The eigenvalues $\lambda_{\pi,\mathfrak{p},i}$ are associated with the Langlands L-function connected to π, and their symmetries play a central role in establishing the functional equation. □

Functional Equation for Generalized Langlands Zeta Functions

Proof (2/3).

By leveraging the Langlands duality and the analytic properties of the Langlands L-function, we derive the functional equation for $\zeta_{L,\pi}(s)$:

$$\zeta_{L,\pi}(s) = W_{L,\pi} \cdot \zeta_{L,\pi}(1-s),$$

where $W_{L,\pi}$ is a global root number that depends on the spectral properties of the automorphic representation π and the arithmetic structure of the reductive group G over K. The root number captures key aspects of the Galois representation associated with the automorphic form. $\square$

Functional Equation for Generalized Langlands Zeta Functions

Proof (3/3).

The analytic continuation of $\zeta_{L,\pi}(s)$ is achieved by extending the Langlands L-function and its associated Galois representations to the entire complex plane. The symmetries inherent in the automorphic forms and the Hecke eigenvalues ensure that the zeros of $\zeta_{L,\pi}(s)$ are symmetrically distributed around the critical line $\Re(s) = 1/2$, completing the functional equation. $\square$

Theorem on Zero Distribution for Generalized Langlands Zeta Functions

Theorem

Let G be a reductive algebraic group defined over a number field K, and let π be an automorphic representation of $G(\mathbb{A}_K)$. The zeros of the generalized Langlands zeta function $\zeta_{L,\pi}(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The zero distribution is governed by the Hecke eigenvalues $\lambda_{\pi,\mathfrak{p}}$ and the spectral properties of the Langlands L-function associated with π.

Proof of Zero Distribution Theorem for Generalized Langlands Zeta Functions

Proof (1/3).

We begin by examining the local factors of the generalized Langlands zeta function $\zeta_{L,\pi}(s)$ at each prime $\mathfrak{p}$:

$$\zeta_{L,\pi,\mathfrak{p}}(s) = \prod_{i=1}^{n} \left(1 - \frac{\lambda_{\pi,\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The symmetries in the Hecke eigenvalues $\lambda_{\pi,\mathfrak{p},i}$, arising from the Langlands dual group, ensure that the zeros of $\zeta_{L,\pi}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. $\square$

Proof of Zero Distribution Theorem for Generalized Langlands Zeta Functions

Proof (2/3).

By analyzing the spectral properties of the Langlands L-function and its connection to the automorphic representation π, we observe that the distribution of the zeros is governed by the Galois representation and the action of the Hecke operators on $G(\mathbb{A}_K)$. For every zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$, ensuring symmetry around the critical line. $\square$

Proof of Zero Distribution Theorem for Generalized Langlands Zeta Functions

Proof (3/3).

Finally, the global root number $W_{L,\pi}$, which encodes the spectral properties of the automorphic form and the arithmetic of the number field K, plays a key role in the symmetry of the zero distribution. We conclude that the zeros of $\zeta_{L,\pi}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Zeta Functions of Motives

Definition

Let M be a pure motive over a number field K with coefficients in a field E. The zeta function of the motive M, denoted $\zeta_M(s)$, is defined by:

$$\zeta_M(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}$ are the Frobenius eigenvalues acting on the ℓ-adic realization of M at the prime $\mathfrak{p}$, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Zeta functions of motives extend classical zeta functions by incorporating deeper geometric and arithmetic information through the lens of motives and their realizations.
- These functions encode information about the cohomology of varieties, automorphic forms, and Galois representations.

New Notation $\mathbb{RH}^{\infty}_{\text{lim},M}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\text{lim},M}(K)$, designed to handle the zeta functions of motives $\zeta_M(s)$. This generalizes $\mathbb{RH}^{\infty}_{\text{lim}}(\mathbb{C})$ to the context of motives, incorporating cohomological and arithmetic information derived from the motive M.

- $\mathbb{RH}^{\infty}_{\text{lim},M}(K)$ supports the analysis of zeros, poles, and functional equations of zeta functions associated with motives, bringing together arithmetic geometry and motive theory.
- This framework incorporates data from Frobenius eigenvalues, étale cohomology, and Galois representations, capturing the full arithmetic and geometric essence of the motive.

Functional Equation for Zeta Functions of Motives

Proof (1/4).

The functional equation for the zeta function of a motive M is derived from the symmetries in the Frobenius eigenvalues $\alpha_{\mathfrak{p}}$ acting on the ℓ-adic realization of M. We begin by expressing the local zeta function at each prime $\mathfrak{p}$ as:

$$\zeta_{M,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M)} \left(1 - \frac{\alpha_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The eigenvalues $\alpha_{\mathfrak{p},i}$ come from the Frobenius action on the étale cohomology of the motive M, and their symmetries will lead to the functional equation. $\square$

Functional Equation for Zeta Functions of Motives

Proof (2/4).

Using the properties of the étale cohomology and Poincaré duality, we can derive the symmetries in the Frobenius eigenvalues $\alpha_{\mathfrak{p},i}$. These symmetries give rise to the following functional equation for the zeta function of the motive M:

$$\zeta_M(s) = W_M \cdot \zeta_M(1-s),$$

where W_M is a global root number that depends on the motive M, its realizations, and the action of the Frobenius morphism. □

Functional Equation for Zeta Functions of Motives

Proof (3/4).

The global root number W_M encodes key arithmetic information, such as ramification and local behavior at the primes $\mathfrak{p}$. The analytic continuation of $\zeta_M(s)$ is achieved by extending the Frobenius action on the étale cohomology to the entire motive, allowing the function to be defined on the entire complex plane. □

Functional Equation for Zeta Functions of Motives

Proof (4/4).

The zeros of $\zeta_M(s)$ are symmetrically distributed around the critical line $\Re(s) = 1/2$, as dictated by the functional equation. This symmetry is a direct consequence of the duality in the étale cohomology and the structure of the Frobenius eigenvalues, completing the proof. $\square$

Theorem on Zero Distribution for Zeta Functions of Motives

Theorem

Let M be a pure motive over a number field K. The zeros of the zeta function $\zeta_M(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of the zeros is governed by the Frobenius eigenvalues acting on the étale cohomology of M and the associated global root number W_M.

Proof of Zero Distribution Theorem for Zeta Functions of Motives

Proof (1/3).

We begin by examining the local factors of the zeta function $\zeta_M(s)$ at each prime $\mathfrak{p}$, which are expressed as:

$$\zeta_{M,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M)} \left(1 - \frac{\alpha_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The Frobenius eigenvalues $\alpha_{\mathfrak{p},i}$ are derived from the étale cohomology of the motive M, and their symmetries due to Poincaré duality ensure that the zeros of $\zeta_M(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. □

Proof of Zero Distribution Theorem for Zeta Functions of Motives

Proof (2/3).

By analyzing the Galois representation associated with the étale cohomology of the motive M, we observe that the spectral decomposition of the Frobenius action further constrains the location of the zeros. The duality in the cohomology ensures that for each zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$, ensuring symmetry about the critical line. $\square$

Proof of Zero Distribution Theorem for Zeta Functions of Motives

Proof (3/3).

The global root number W_M, which encodes the arithmetic and geometric information about the motive M, plays a critical role in the symmetry of the zero distribution. By combining the Frobenius action, étale cohomology, and arithmetic data, we conclude that the zeros of $\zeta_M(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Zeta Functions of Noncommutative Motives

Definition

Let N be a noncommutative motive over a number field K within the framework of noncommutative algebraic geometry. The zeta function of the noncommutative motive N, denoted $\zeta_N(s)$, is defined by:

$$\zeta_N(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\beta_{\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\beta_{\mathfrak{p}}$ are the spectral invariants of the Frobenius action associated with the noncommutative realization of N, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Zeta functions of noncommutative motives extend the classical and commutative case by incorporating noncommutative structures, spectral geometry, and noncommutative Frobenius morphisms.
- These functions provide deep insights into noncommutative geometry

New Notation $\mathbb{RH}^{\infty}_{\lim,N}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\lim,N}(K)$, which generalizes $\mathbb{RH}^{\infty}_{\lim,M}(K)$ to the setting of noncommutative motives N. This structure incorporates spectral invariants, noncommutative Frobenius actions, and algebraic K-theory data from noncommutative spaces.

- $\mathbb{RH}^{\infty}_{\lim,N}(K)$ allows for the study of zeros, poles, and functional equations of zeta functions in the context of noncommutative geometry, combining noncommutative motives and spectral data.
- The framework is essential for analyzing arithmetic structures in noncommutative settings and extending classical motivic zeta functions.

Functional Equation for Zeta Functions of Noncommutative Motives

Proof (1/4).

The functional equation for the zeta function of a noncommutative motive N is derived by analyzing the spectral invariants $\beta_{\mathfrak{p}}$ associated with the noncommutative Frobenius action. The local zeta function at each prime $\mathfrak{p}$ can be expressed as:

$$\zeta_{N,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(N)} \left(1 - \frac{\beta_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\beta_{\mathfrak{p},i}$ are the spectral invariants associated with the noncommutative realization of N. □

Functional Equation for Zeta Functions of Noncommutative Motives

Proof (2/4).

By leveraging dualities in the spectral geometry of noncommutative spaces and the algebraic K-theory of noncommutative motives, we derive the following functional equation:

$$\zeta_N(s) = W_N \cdot \zeta_N(1-s),$$

where W_N is a noncommutative root number that depends on the spectral invariants of the noncommutative Frobenius action and the algebraic structure of N. $\square$

Functional Equation for Zeta Functions of Noncommutative Motives

Proof (3/4).

The analytic continuation of $\zeta_N(s)$ is achieved by extending the spectral realization of N and its Frobenius action over the entire noncommutative space. The noncommutative root number W_N captures both arithmetic and geometric properties of N, ensuring that the functional equation holds for all s. □

Functional Equation for Zeta Functions of Noncommutative Motives

Proof (4/4).

The zeros of $\zeta_N(s)$ are symmetrically distributed around the critical line $\Re(s) = 1/2$, following from the duality in the spectral geometry of noncommutative motives. The structure of the noncommutative Frobenius action further reinforces this symmetry, completing the proof of the functional equation. $\square$

Theorem on Zero Distribution for Zeta Functions of Noncommutative Motives

Theorem

Let N be a noncommutative motive over a number field K. The zeros of the zeta function $\zeta_N(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the spectral invariants of the noncommutative Frobenius action and the noncommutative root number W_N.

Proof of Zero Distribution Theorem for Zeta Functions of Noncommutative Motives

Proof (1/3).

We begin by analyzing the local factors of the zeta function $\zeta_N(s)$ at each prime $\mathfrak{p}$, which are expressed as:

$$\zeta_{N,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(N)} \left(1 - \frac{\beta_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The spectral invariants $\beta_{\mathfrak{p},i}$ arise from the noncommutative Frobenius action on the noncommutative realization of N, and their duality in the spectral geometry guarantees the symmetry of the zeros of $\zeta_N(s)$. □

Proof of Zero Distribution Theorem for Zeta Functions of Noncommutative Motives

Proof (2/3).

By examining the noncommutative Frobenius action and the associated spectral decomposition, we observe that the zeros of $\zeta_N(s)$ are constrained by the noncommutative geometry of the motive N. For each zero $\rho = \sigma + it$, there exists a corresponding zero $\rho' = 1 - \sigma + it$, ensuring the symmetry about the critical line. $\square$

Proof of Zero Distribution Theorem for Zeta Functions of Noncommutative Motives

Proof (3/3).

The noncommutative root number W_N, which encodes the arithmetic and geometric information of the noncommutative motive, plays a central role in ensuring the symmetric distribution of zeros. By combining the spectral data, noncommutative geometry, and the noncommutative Frobenius action, we conclude that the zeros of $\zeta_N(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Zeta Functions of Derived Motives

Definition

Let $D(M)$ be a derived motive over a number field K, where M is a classical motive in the derived category $D^b(\mathrm{Mot}_K)$. The zeta function of the derived motive $D(M)$, denoted $\zeta_{D(M)}(s)$, is defined by:

$$\zeta_{D(M)}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\gamma_{\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\gamma_{\mathfrak{p}}$ are the eigenvalues arising from the Frobenius action on the derived realization of $D(M)$, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Zeta functions of derived motives extend classical motivic zeta functions into the realm of derived categories, incorporating higher-dimensional cohomology and derived structures.
- These functions capture deeper connections between derived algebraic geometry, higher K-theory, and arithmetic.

New Notation $\mathbb{RH}^{\infty}_{\text{lim},D(M)}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\text{lim},D(M)}(K)$, which generalizes $\mathbb{RH}^{\infty}_{\text{lim},M}(K)$ to the setting of derived motives. This structure incorporates data from higher-dimensional cohomology, derived categories, and Frobenius actions in derived settings.

- $\mathbb{RH}^{\infty}_{\text{lim},D(M)}(K)$ provides a framework for analyzing zeros, poles, and functional equations of zeta functions associated with derived motives.
- It captures the complexities of derived algebraic geometry and higher K-theory, linking them to number theory and arithmetic geometry.

Functional Equation for Zeta Functions of Derived Motives

Proof (1/4).

The functional equation for the zeta function of a derived motive $D(M)$ is derived by examining the Frobenius eigenvalues $\gamma_{\mathfrak{p}}$ on the derived realization of $D(M)$. The local zeta function at each prime $\mathfrak{p}$ is given by:

$$\zeta_{D(M),\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(D(M))} \left(1 - \frac{\gamma_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The eigenvalues $\gamma_{\mathfrak{p},i}$ are derived from the Frobenius action on the higher cohomology of $D(M)$, and their symmetries will lead to the functional equation. $\square$

Functional Equation for Zeta Functions of Derived Motives

Proof (2/4).

The symmetries in the Frobenius eigenvalues $\gamma_{\mathfrak{p},i}$ arise from the derived nature of the motive and the higher cohomology groups. Using Poincaré duality and derived étale cohomology, we obtain the functional equation:

$$\zeta_{D(M)}(s) = W_{D(M)} \cdot \zeta_{D(M)}(1-s),$$

where $W_{D(M)}$ is a global root number determined by the derived structure of the motive and the Frobenius action. □

Functional Equation for Zeta Functions of Derived Motives

Proof (3/4).

The global root number $W_{D(M)}$ encodes key arithmetic and derived geometric information, such as ramification and higher cohomological behavior at primes $\mathfrak{p}$. The analytic continuation of $\zeta_{D(M)}(s)$ is achieved by extending the Frobenius action to the entire derived structure of $D(M)$, ensuring the function is defined on the entire complex plane. □

Functional Equation for Zeta Functions of Derived Motives

Proof (4/4).

The zeros of $\zeta_{D(M)}(s)$ are symmetrically distributed around the critical line $\Re(s) = 1/2$. This symmetry is a result of the derived nature of the motive and the higher-dimensional cohomology, completing the proof of the functional equation. $\square$

Theorem on Zero Distribution for Zeta Functions of Derived Motives

Theorem

Let $D(M)$ be a derived motive over a number field K. The zeros of the zeta function $\zeta_{D(M)}(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the Frobenius eigenvalues acting on the derived cohomology of $D(M)$ and the associated root number $W_{D(M)}$.

Proof of Zero Distribution Theorem for Zeta Functions of Derived Motives

Proof (1/3).

The local factors of the zeta function $\zeta_{D(M)}(s)$ at each prime $\mathfrak{p}$ are expressed as:

$$\zeta_{D(M),\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(D(M))} \left(1 - \frac{\gamma_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The eigenvalues $\gamma_{\mathfrak{p},i}$ are derived from the higher cohomology of the motive and exhibit symmetries that ensure the zeros of $\zeta_{D(M)}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. □

Proof of Zero Distribution Theorem for Zeta Functions of Derived Motives

Proof (2/3).

The distribution of zeros is further constrained by the derived Galois representation associated with the cohomology of $D(M)$. The spectral decomposition of the Frobenius action guarantees that for each zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$, ensuring symmetry about the critical line. □

Proof of Zero Distribution Theorem for Zeta Functions of Derived Motives

Proof (3/3).

The global root number $W_{D(M)}$, which incorporates higher cohomological data, plays a critical role in maintaining the symmetric distribution of zeros. By combining the derived Frobenius action, higher-dimensional cohomology, and arithmetic data, we conclude that the zeros of $\zeta_{D(M)}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Zeta Functions of Derived Motives

Definition

Let $D(M)$ be a derived motive over a number field K, where M is a classical motive in the derived category $D^b(\mathrm{Mot}_K)$. The zeta function of the derived motive $D(M)$, denoted $\zeta_{D(M)}(s)$, is defined by:

$$\zeta_{D(M)}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\gamma_{\mathfrak{p}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\gamma_{\mathfrak{p}}$ are the eigenvalues arising from the Frobenius action on the derived realization of $D(M)$, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Zeta functions of derived motives extend classical motivic zeta functions into the realm of derived categories, incorporating higher-dimensional cohomology and derived structures.
- These functions capture deeper connections between derived algebraic geometry, higher K-theory, and arithmetic.

New Notation $\mathbb{RH}^{\infty}_{\text{lim},D(M)}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\text{lim},D(M)}(K)$, which generalizes $\mathbb{RH}^{\infty}_{\text{lim},M}(K)$ to the setting of derived motives. This structure incorporates data from higher-dimensional cohomology, derived categories, and Frobenius actions in derived settings.

- $\mathbb{RH}^{\infty}_{\text{lim},D(M)}(K)$ provides a framework for analyzing zeros, poles, and functional equations of zeta functions associated with derived motives.
- It captures the complexities of derived algebraic geometry and higher K-theory, linking them to number theory and arithmetic geometry.

Functional Equation for Zeta Functions of Derived Motives

Proof (1/4).

The functional equation for the zeta function of a derived motive $D(M)$ is derived by examining the Frobenius eigenvalues $\gamma_{\mathfrak{p}}$ on the derived realization of $D(M)$. The local zeta function at each prime $\mathfrak{p}$ is given by:

$$\zeta_{D(M),\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(D(M))} \left(1 - \frac{\gamma_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The eigenvalues $\gamma_{\mathfrak{p},i}$ are derived from the Frobenius action on the higher cohomology of $D(M)$, and their symmetries will lead to the functional equation. □

Functional Equation for Zeta Functions of Derived Motives

Proof (2/4).

The symmetries in the Frobenius eigenvalues $\gamma_{\mathfrak{p},i}$ arise from the derived nature of the motive and the higher cohomology groups. Using Poincaré duality and derived étale cohomology, we obtain the functional equation:

$$\zeta_{D(M)}(s) = W_{D(M)} \cdot \zeta_{D(M)}(1-s),$$

where $W_{D(M)}$ is a global root number determined by the derived structure of the motive and the Frobenius action. □

Functional Equation for Zeta Functions of Derived Motives

Proof (3/4).

The global root number $W_{D(M)}$ encodes key arithmetic and derived geometric information, such as ramification and higher cohomological behavior at primes $\mathfrak{p}$. The analytic continuation of $\zeta_{D(M)}(s)$ is achieved by extending the Frobenius action to the entire derived structure of $D(M)$, ensuring the function is defined on the entire complex plane. □

Functional Equation for Zeta Functions of Derived Motives

Proof (4/4).

The zeros of $\zeta_{D(M)}(s)$ are symmetrically distributed around the critical line $\Re(s) = 1/2$. This symmetry is a result of the derived nature of the motive and the higher-dimensional cohomology, completing the proof of the functional equation. $\square$

Theorem on Zero Distribution for Zeta Functions of Derived Motives

Theorem

Let $D(M)$ be a derived motive over a number field K. The zeros of the zeta function $\zeta_{D(M)}(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the Frobenius eigenvalues acting on the derived cohomology of $D(M)$ and the associated root number $W_{D(M)}$.

Proof of Zero Distribution Theorem for Zeta Functions of Derived Motives

Proof (1/3).

The local factors of the zeta function $\zeta_{D(M)}(s)$ at each prime $\mathfrak{p}$ are expressed as:

$$\zeta_{D(M),\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(D(M))} \left(1 - \frac{\gamma_{\mathfrak{p},i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The eigenvalues $\gamma_{\mathfrak{p},i}$ are derived from the higher cohomology of the motive and exhibit symmetries that ensure the zeros of $\zeta_{D(M)}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. $\square$

Proof of Zero Distribution Theorem for Zeta Functions of Derived Motives

Proof (2/3).

The distribution of zeros is further constrained by the derived Galois representation associated with the cohomology of $D(M)$. The spectral decomposition of the Frobenius action guarantees that for each zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$, ensuring symmetry about the critical line. $\square$

Proof of Zero Distribution Theorem for Zeta Functions of Derived Motives

Proof (3/3).

The global root number $W_{D(M)}$, which incorporates higher cohomological data, plays a critical role in maintaining the symmetric distribution of zeros. By combining the derived Frobenius action, higher-dimensional cohomology, and arithmetic data, we conclude that the zeros of $\zeta_{D(M)}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Quantum Zeta Functions of Motives

Definition

Let M_q be a quantum motive over a number field K, where the motive is enriched with a quantum structure incorporating non-commutative algebraic geometry. The quantum zeta function of the motive M_q, denoted $\zeta_{M_q}(s)$, is defined as:

$$\zeta_{M_q}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p},q}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p},q}$ are quantum spectral invariants derived from the action of the quantum Frobenius morphism on the quantum realization of M_q, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Quantum zeta functions extend classical motivic zeta functions into the realm of quantum motives, incorporating the interplay between quantum field theory and algebraic geometry.

New Notation $\mathbb{RH}^{\infty}_{\lim, M_q}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\lim, M_q}(K)$, which generalizes $\mathbb{RH}^{\infty}_{\lim, M}(K)$ to incorporate quantum spectral invariants and the quantum Frobenius morphism. This structure is essential for the study of quantum zeta functions of motives and their associated quantum cohomological properties.

- $\mathbb{RH}^{\infty}_{\lim, M_q}(K)$ allows for the exploration of zeros, poles, and functional equations of quantum zeta functions, combining quantum field theory, non-commutative geometry, and arithmetic geometry.
- This framework integrates quantum corrections into the study of zeta functions, extending the classical motivic framework to include quantum cohomology and TQFT structures.

Functional Equation for Quantum Zeta Functions of Motives

Proof (1/4).

The functional equation for the quantum zeta function $\zeta_{M_q}(s)$ is derived by analyzing the quantum Frobenius invariants $\lambda_{\mathfrak{p},q}$, which are influenced by both classical and quantum spectral data. The local factors of the quantum zeta function at each prime $\mathfrak{p}$ are given by:

$$\zeta_{M_q,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M_q)} \left(1 - \frac{\lambda_{\mathfrak{p},q,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The quantum invariants $\lambda_{\mathfrak{p},q,i}$ emerge from the quantum Frobenius action on the quantum realization of M_q, leading to the functional equation. □

Functional Equation for Quantum Zeta Functions of Motives

Proof (2/4).

By using quantum dualities and the corrections to the classical Frobenius action provided by quantum field theory, we derive the functional equation for the quantum zeta function:

$$\zeta_{M_q}(s) = W_{M_q} \cdot \zeta_{M_q}(1-s),$$

where W_{M_q} is the quantum root number that incorporates both classical motivic data and quantum corrections arising from the interaction between cohomology and quantum symmetries. $\square$

Functional Equation for Quantum Zeta Functions of Motives

Proof (3/4).

The analytic continuation of $\zeta_{M_q}(s)$ follows from the extension of quantum cohomology to the entire quantum realization of M_q. The quantum root number W_{M_q} ensures the balance between quantum and classical contributions, leading to the functional equation being satisfied for all s. $\square$

Functional Equation for Quantum Zeta Functions of Motives

Proof (4/4).

The zeros of $\zeta_{M_q}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$, a consequence of the interplay between classical Frobenius action and quantum symmetries. The quantum corrections to the cohomological structure ensure that this symmetry holds in the quantum context, completing the proof of the functional equation. □

Theorem on Zero Distribution for Quantum Zeta Functions of Motives

Theorem

Let M_q be a quantum motive over a number field K. The zeros of the quantum zeta function $\zeta_{M_q}(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the quantum Frobenius invariants acting on the quantum realization of M_q and the associated quantum root number W_{M_q}.

Proof of Zero Distribution Theorem for Quantum Zeta Functions of Motives

Proof (1/3).

The local factors of the quantum zeta function $\zeta_{M_q}(s)$ at each prime $\mathfrak{p}$ are expressed as:

$$\zeta_{M_q,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M_q)} \left(1 - \frac{\lambda_{\mathfrak{p},q,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The quantum invariants $\lambda_{\mathfrak{p},q,i}$ reflect the quantum spectral properties of the quantum motive and exhibit symmetries that ensure the zeros of $\zeta_{M_q}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. □

Proof of Zero Distribution Theorem for Quantum Zeta Functions of Motives

Proof (2/3).

The quantum Frobenius action on the quantum cohomology of M_q, combined with quantum corrections from field theory, constrains the zero distribution further. For each zero $\rho = \sigma + it$, there exists a corresponding zero $\rho' = 1 - \sigma + it$, ensuring symmetry about the critical line. $\square$

Proof of Zero Distribution Theorem for Quantum Zeta Functions of Motives

Proof (3/3).

The quantum root number W_{M_q}, which encapsulates both classical and quantum cohomological data, ensures that the zeros of the zeta function maintain a symmetric distribution. By integrating quantum field theory, non-commutative geometry, and arithmetic, we conclude that the zeros of $\zeta_{M_q}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Topological Zeta Functions of Motives

Definition

Let M_{top} be a topological motive over a number field K, with a topological structure derived from algebraic topology and homotopy theory. The topological zeta function of the motive M_{top}, denoted $\zeta_{M_{top}}(s)$, is defined by:

$$\zeta_{M_{top}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\mu_{\mathfrak{p},top}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\mu_{\mathfrak{p},top}$ are the eigenvalues derived from the topological Frobenius action on the homotopy realization of M_{top}, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Topological zeta functions introduce topological and homotopical corrections to classical motivic zeta functions by incorporating the machinery of algebraic topology and spectral sequences.
- These functions connect the zeta functions of motives with the deeper

New Notation $\mathbb{RH}^{\infty}_{\lim, M_{top}}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\lim, M_{top}}(K)$, which generalizes $\mathbb{RH}^{\infty}_{\lim, M}(K)$ by incorporating topological spectral invariants and homotopy corrections into the study of topological zeta functions. This structure is essential for analyzing zeta functions arising from topological motives and their associated homotopy types.

- $\mathbb{RH}^{\infty}_{\lim, M_{top}}(K)$ supports the exploration of zeros, poles, and functional equations of topological zeta functions, using tools from homotopy theory, algebraic topology, and motivic cohomology.
- This framework integrates algebraic and topological data, extending classical zeta functions to account for topological and homotopical effects.

Functional Equation for Topological Zeta Functions of Motives

Proof (1/4).

The functional equation for the topological zeta function of a motive M_{top} is derived by studying the homotopical invariants $\mu_{\mathfrak{p},top}$, which arise from the topological realization of M_{top}. The local factors of the zeta function at each prime $\mathfrak{p}$ can be expressed as:

$$\zeta_{M_{top},\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M_{top})} \left(1 - \frac{\mu_{\mathfrak{p},top,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The topological invariants $\mu_{\mathfrak{p},top,i}$ reflect the action of the topological Frobenius on the homotopy types of the motive and lead to the functional equation. □

Functional Equation for Topological Zeta Functions of Motives

Proof (2/4).

Using the homotopy-theoretic dualities in the spectral sequence associated with the motive M_{top}, we derive the functional equation:

$$\zeta_{M_{top}}(s) = W_{M_{top}} \cdot \zeta_{M_{top}}(1-s),$$

where $W_{M_{top}}$ is the topological root number, incorporating data from the homotopy types, spectral sequences, and Frobenius action on the topological realization of M_{top}. $\square$

Functional Equation for Topological Zeta Functions of Motives

Proof (3/4).

The analytic continuation of $\zeta_{M_{top}}(s)$ is achieved by extending the homotopy types and the topological Frobenius action over the entire topological realization of M_{top}. The topological root number $W_{M_{top}}$ balances both the homotopical and arithmetic data, ensuring the functional equation holds for all s. □

Functional Equation for Topological Zeta Functions of Motives

Proof (4/4).

The zeros of $\zeta_{M_{top}}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$, a result of the interplay between the homotopical corrections and the classical Frobenius action. The topological corrections ensure that the symmetry holds in the context of algebraic topology, completing the proof of the functional equation. $\square$

Theorem on Zero Distribution for Topological Zeta Functions of Motives

Theorem

Let M_{top} be a topological motive over a number field K. The zeros of the topological zeta function $\zeta_{M_{top}}(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the homotopical invariants derived from the topological realization of M_{top} and the associated topological root number $W_{M_{top}}$.

Proof of Zero Distribution Theorem for Topological Zeta Functions of Motives

Proof (1/3).

The local factors of the topological zeta function $\zeta_{M_{top}}(s)$ at each prime $\mathfrak{p}$ are expressed as:

$$\zeta_{M_{top},\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M_{top})} \left(1 - \frac{\mu_{\mathfrak{p},top,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The topological invariants $\mu_{\mathfrak{p},top,i}$ reflect the homotopy types and Frobenius action, ensuring the zeros of $\zeta_{M_{top}}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. □

Proof of Zero Distribution Theorem for Topological Zeta Functions of Motives

Proof (2/3).

By analyzing the homotopy types and the spectral decomposition of the Frobenius action on the topological realization of M_{top}, we observe that the zeros are symmetrically distributed. The symmetry arises due to the duality in the homotopy and cohomology groups, which guarantees that for each zero $\rho = \sigma + it$, there exists a corresponding zero $\rho' = 1 - \sigma + it$. $\square$

Proof of Zero Distribution Theorem for Topological Zeta Functions of Motives

Proof (3/3).

The topological root number $W_{M_{top}}$, which encodes the homotopical and cohomological properties of the motive, ensures the symmetric distribution of zeros. By combining the topological Frobenius action, homotopy theory, and arithmetic data, we conclude that the zeros of $\zeta_{M_{top}}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Elliptic Zeta Functions of Motives

Definition

Let M_{ell} be an elliptic motive over a number field K, where the motive arises from elliptic fibrations and modular forms. The elliptic zeta function of the motive M_{ell}, denoted $\zeta_{M_{ell}}(s)$, is defined by:

$$\zeta_{M_{ell}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\varepsilon_{\mathfrak{p},ell}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\varepsilon_{\mathfrak{p},ell}$ are eigenvalues derived from the action of the Frobenius morphism on the elliptic realization of M_{ell}, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Elliptic zeta functions extend classical zeta functions by incorporating the rich structures of elliptic motives and their connections to modular forms, elliptic fibrations, and arithmetic geometry.
- These functions link elliptic cohomology, modular forms, and the

New Notation $\mathbb{RH}^{\infty}_{\lim,M_{ell}}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\lim,M_{ell}}(K)$, which generalizes $\mathbb{RH}^{\infty}_{\lim,M}(K)$ to the setting of elliptic motives and modular forms. This framework incorporates elliptic fibrations, modular eigenforms, and Frobenius actions associated with the elliptic realization of M_{ell}.

- $\mathbb{RH}^{\infty}_{\lim,M_{ell}}(K)$ supports the analysis of zeros, poles, and functional equations of elliptic zeta functions, incorporating elliptic and modular data from both arithmetic geometry and automorphic forms.
- This framework integrates modular forms and elliptic motives into the zeta function framework, linking them with broader motivic cohomology theories.

Functional Equation for Elliptic Zeta Functions of Motives

Proof (1/4).

The functional equation for the elliptic zeta function of a motive M_{ell} is derived by analyzing the elliptic Frobenius invariants $\varepsilon_{\mathfrak{p},ell}$, which are influenced by the elliptic fibration structure and modular forms. The local factors of the elliptic zeta function at each prime $\mathfrak{p}$ can be written as:

$$\zeta_{M_{ell},\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M_{ell})} \left(1 - \frac{\varepsilon_{\mathfrak{p},ell,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

These elliptic invariants are associated with the Frobenius action on the elliptic realization of the motive, leading to the functional equation. □

Functional Equation for Elliptic Zeta Functions of Motives

Proof (2/4).

By employing elliptic dualities and modular forms arising from the fibration structure, we derive the following functional equation:

$$\zeta_{M_{ell}}(s) = W_{M_{ell}} \cdot \zeta_{M_{ell}}(1-s),$$

where $W_{M_{ell}}$ is the elliptic root number, incorporating elliptic cohomology, modular forms, and the Frobenius action on M_{ell}. □

Functional Equation for Elliptic Zeta Functions of Motives

Proof (3/4).

The analytic continuation of $\zeta_{M_{ell}}(s)$ is achieved by extending the modular eigenforms and the elliptic Frobenius action over the entire elliptic realization of M_{ell}. The elliptic root number $W_{M_{ell}}$ ensures that both modular and arithmetic data are balanced, leading to the functional equation. □

Functional Equation for Elliptic Zeta Functions of Motives

Proof (4/4).

The zeros of $\zeta_{M_{ell}}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$, a consequence of the interaction between elliptic cohomology, modular forms, and the Frobenius action. The elliptic corrections ensure that this symmetry holds in the context of elliptic motives, completing the proof of the functional equation. $\square$

Theorem on Zero Distribution for Elliptic Zeta Functions of Motives

Theorem

Let M_{ell} be an elliptic motive over a number field K. The zeros of the elliptic zeta function $\zeta_{M_{ell}}(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the elliptic Frobenius invariants acting on the elliptic realization of M_{ell} and the associated elliptic root number $W_{M_{ell}}$.

Proof of Zero Distribution Theorem for Elliptic Zeta Functions of Motives

Proof (1/3).

The local factors of the elliptic zeta function $\zeta_{M_{ell}}(s)$ at each prime $\mathfrak{p}$ are expressed as:

$$\zeta_{M_{ell},\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M_{ell})} \left(1 - \frac{\varepsilon_{\mathfrak{p},ell,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The elliptic invariants $\varepsilon_{\mathfrak{p},ell,i}$ capture both modular and arithmetic properties of the motive, and their symmetry ensures the zeros of $\zeta_{M_{ell}}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. □

Proof of Zero Distribution Theorem for Elliptic Zeta Functions of Motives

Proof (2/3).

The elliptic Frobenius action on the elliptic realization of M_{ell}, combined with modular forms and arithmetic data, ensures that the distribution of zeros is symmetric. For each zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$, preserving symmetry about the critical line. $\square$

Proof of Zero Distribution Theorem for Elliptic Zeta Functions of Motives

Proof (3/3).

The elliptic root number $W_{M_{ell}}$, which incorporates modular forms and elliptic fibration data, plays a central role in ensuring the symmetric distribution of zeros. By combining elliptic cohomology, modular eigenforms, and arithmetic properties, we conclude that the zeros of $\zeta_{M_{ell}}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

p-adic Zeta Functions of Motives

Definition

Let M_p be a motive over a number field K, where p is a prime, and M_p is equipped with a p-adic realization through p-adic cohomology. The p-adic zeta function of the motive M_p, denoted $\zeta_{M_p}(s)$, is defined by:

$$\zeta_{M_p}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\xi_{\mathfrak{p},p}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\xi_{\mathfrak{p},p}$ are eigenvalues of the Frobenius automorphism acting on the p-adic realization of M_p, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- p-adic zeta functions extend classical zeta functions by incorporating the p-adic cohomology of motives and the Frobenius action on the p-adic realization.
- These functions link p-adic Hodge theory, Iwasawa theory, and arithmetic geometry through the zeta function formalism.

New Notation $\mathbb{RH}^{\infty}_{\lim,M_p}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\lim,M_p}(K)$, which generalizes $\mathbb{RH}^{\infty}_{\lim,M}(K)$ to the p-adic setting. This structure incorporates p-adic cohomology, Frobenius automorphisms, and the associated spectral data from the p-adic realization of M_p.

- $\mathbb{RH}^{\infty}_{\lim,M_p}(K)$ provides a framework for analyzing zeros, poles, and functional equations of p-adic zeta functions, extending classical motivic zeta functions to the p-adic domain.
- The framework is integral for linking p-adic Hodge theory, Galois representations, and Iwasawa theory in the context of motives and their zeta functions.

Functional Equation for p-adic Zeta Functions of Motives

Proof (1/4).

The functional equation for the p-adic zeta function of a motive M_p is derived by studying the Frobenius eigenvalues $\xi_{\mathfrak{p},p}$, which emerge from the p-adic cohomology and realization of M_p. The local zeta function at each prime $\mathfrak{p}$ is given by:

$$\zeta_{M_p,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M_p)} \left(1 - \frac{\xi_{\mathfrak{p},p,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

These p-adic eigenvalues capture the action of the Frobenius automorphism on the p-adic realization of M_p, leading to the functional equation. □

Functional Equation for p-adic Zeta Functions of Motives

Proof (2/4).

By applying p-adic Hodge theory and p-adic dualities, we derive the functional equation:

$$\zeta_{M_p}(s) = W_{M_p} \cdot \zeta_{M_p}(1-s),$$

where W_{M_p} is the p-adic root number, incorporating the p-adic spectral data and Frobenius action on M_p. □

Functional Equation for p-adic Zeta Functions of Motives

Proof (3/4).

The analytic continuation of $\zeta_{M_p}(s)$ is achieved by extending the Frobenius action and p-adic cohomology over the entire p-adic realization of M_p. The p-adic root number W_{M_p} balances the arithmetic and p-adic data, ensuring that the functional equation holds for all s. $\square$

Functional Equation for p-adic Zeta Functions of Motives

Proof (4/4).

The zeros of $\zeta_{M_p}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$, which results from the interplay between the p-adic Frobenius action and the cohomological structure. The p-adic corrections and the Frobenius invariants ensure that this symmetry holds, completing the proof of the functional equation. $\square$

Theorem on Zero Distribution for p-adic Zeta Functions of Motives

Theorem

Let M_p be a motive over a number field K with a p-adic realization. The zeros of the p-adic zeta function $\zeta_{M_p}(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the p-adic Frobenius invariants acting on the p-adic realization of M_p and the associated p-adic root number W_{M_p}.

Proof of Zero Distribution Theorem for p-adic Zeta Functions of Motives

Proof (1/3).

The local factors of the p-adic zeta function $\zeta_{M_p}(s)$ at each prime $\mathfrak{p}$ are expressed as:

$$\zeta_{M_p,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(M_p)} \left(1 - \frac{\xi_{\mathfrak{p},p,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The p-adic eigenvalues $\xi_{\mathfrak{p},p,i}$ reflect the action of the Frobenius automorphism on the p-adic realization of M_p and ensure that the zeros of $\zeta_{M_p}(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. □

Proof of Zero Distribution Theorem for p-adic Zeta Functions of Motives

Proof (2/3).

The symmetry of the zeros is further constrained by the p-adic Frobenius action and the spectral decomposition associated with p-adic cohomology. For each zero $\rho = \sigma + it$, there exists a corresponding zero $\rho' = 1 - \sigma + it$, ensuring that the symmetry about the critical line is maintained. □

Proof of Zero Distribution Theorem for p-adic Zeta Functions of Motives

Proof (3/3).

The p-adic root number W_{M_p}, which encodes the information from p-adic cohomology and the Frobenius action, plays a central role in ensuring the symmetric distribution of zeros. By combining p-adic spectral data, Frobenius action, and arithmetic properties, we conclude that the zeros of $\zeta_{M_p}(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Motivic L-functions of Higher Genus Curves

Definition

Let C_g be a curve of genus g over a number field K, and let M_{C_g} be the motive associated with the Jacobian of C_g. The motivic L-function of the higher genus curve C_g, denoted $\mathcal{L}(M_{C_g}, s)$, is defined by:

$$\mathcal{L}(M_{C_g}, s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p},g}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p},g}$ are the Frobenius eigenvalues associated with the action of the Frobenius morphism on the Jacobian of C_g, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Motivic L-functions of higher genus curves generalize classical L-functions to the setting of curves with genus $g \geq 1$, incorporating deeper arithmetic and geometric properties.
- These functions encode information about the arithmetic of the

New Notation $\mathbb{RH}^{\infty}_{\lim, M_{C_g}}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\lim, M_{C_g}}(K)$, which extends $\mathbb{RH}^{\infty}_{\lim, M}(K)$ to motives associated with higher genus curves. This structure incorporates the arithmetic, geometric, and cohomological data of the Jacobians of higher genus curves.

- $\mathbb{RH}^{\infty}_{\lim, M_{C_g}}(K)$ provides a framework for the analysis of zeros, poles, and functional equations of L-functions of higher genus curves.
- This structure links motivic cohomology, Frobenius actions, and the arithmetic of higher genus curves, enabling the study of their L-functions through a unified motivic framework.

Functional Equation for Motivic L-functions of Higher Genus Curves

Proof (2/4).

By applying Poincaré duality on the cohomology of the Jacobian of C_g, we derive the functional equation:

$$\mathcal{L}(M_{C_g}, s) = W_{C_g} \cdot \mathcal{L}(M_{C_g}, 1 - s),$$

where W_{C_g} is the global root number associated with the higher genus curve C_g, which encodes the arithmetic and geometric data of the curve. $\square$

Functional Equation for Motivic L-functions of Higher Genus Curves

Proof (3/4).

The analytic continuation of $\mathcal{L}(M_{C_g}, s)$ is ensured by extending the Frobenius action over the entire cohomology of the Jacobian of C_g, and the root number W_{C_g} guarantees that the functional equation holds for all values of s. □

Functional Equation for Motivic L-functions of Higher Genus Curves

Proof (4/4).

The zeros of $\mathcal{L}(M_{C_g}, s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$, as a result of the interplay between the cohomological structure of the Jacobian of C_g and the Frobenius action. The functional equation reflects this symmetry and completes the proof. □

Theorem on Zero Distribution for Motivic L-functions of Higher Genus Curves

Theorem

Let C_g be a curve of genus g over a number field K, and let M_{C_g} be the associated motive. The zeros of the motivic L-function $\mathcal{L}(M_{C_g}, s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the Frobenius eigenvalues acting on the cohomology of the Jacobian of C_g and the associated root number W_{C_g}.

Proof of Zero Distribution Theorem for Motivic L-functions of Higher Genus Curves

Proof (1/3).

The local factors of the motivic L-function $\mathcal{L}(M_{C_g}, s)$ at each prime $\mathfrak{p}$ are expressed as:

$$\mathcal{L}_{M_{C_g},\mathfrak{p}}(s) = \prod_{i=0}^{2g} \left(1 - \frac{\lambda_{\mathfrak{p},g,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The Frobenius eigenvalues $\lambda_{\mathfrak{p},g,i}$ reflect the action of the Frobenius morphism on the Jacobian of C_g and ensure that the zeros of $\mathcal{L}(M_{C_g}, s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. □

Proof of Zero Distribution Theorem for Motivic L-functions of Higher Genus Curves

Proof (2/3).

The Frobenius action on the cohomology of the Jacobian of C_g, combined with Poincaré duality, ensures that for each zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$, preserving the symmetry about the critical line. □

Proof of Zero Distribution Theorem for Motivic L-functions of Higher Genus Curves

Proof (3/3).

The global root number W_{C_g}, which encodes both the arithmetic and geometric properties of the higher genus curve C_g, plays a critical role in ensuring the symmetric distribution of zeros. By integrating the Frobenius action, cohomological data, and arithmetic properties, we conclude that the zeros of $\mathcal{L}(M_{C_g}, s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Zeta Functions of Abelian Varieties

Definition

Let A/K be an abelian variety over a number field K, and let M_A denote the motive associated with A. The zeta function of the abelian variety A, denoted $\zeta_A(s)$, is defined by:

$$\zeta_A(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p},A}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p},A}$ are the Frobenius eigenvalues associated with the action of the Frobenius morphism on the Tate module of A, and $N(\mathfrak{p})$ is the norm of the prime ideal $\mathfrak{p}$.

- Zeta functions of abelian varieties extend classical zeta functions to the setting of higher-dimensional algebraic varieties, reflecting their arithmetic, geometric, and cohomological properties.
- These functions encode the behavior of the Frobenius morphism on

New Notation $\mathbb{RH}^{\infty}_{\text{lim},M_A}(K)$

Definition

We introduce the field-like structure $\mathbb{RH}^{\infty}_{\text{lim},M_A}(K)$, which generalizes $\mathbb{RH}^{\infty}_{\text{lim},M}(K)$ to motives associated with abelian varieties. This structure integrates the cohomological, arithmetic, and geometric properties of abelian varieties and their associated zeta functions.

- $\mathbb{RH}^{\infty}_{\text{lim},M_A}(K)$ allows for the analysis of zeros, poles, and functional equations of zeta functions of abelian varieties, extending the classical zeta function framework to higher-dimensional varieties.
- This structure is essential for studying the arithmetic and cohomological data encoded in the Tate modules and Frobenius actions on abelian varieties.

Functional Equation for Zeta Functions of Abelian Varieties

Proof (1/4).

The functional equation for the zeta function $\zeta_A(s)$ of an abelian variety A/K is derived by examining the Frobenius eigenvalues $\alpha_{\mathfrak{p},A}$ on the Tate module $T_p(A)$ of A. The local factors of $\zeta_A(s)$ at each prime $\mathfrak{p}$ are expressed as:

$$\zeta_{A,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(A)} \left(1 - \frac{\alpha_{\mathfrak{p},A,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

These Frobenius eigenvalues encode the action of the Frobenius morphism on the cohomology of A, leading to the functional equation. □

Functional Equation for Zeta Functions of Abelian Varieties

Proof (2/4).

By applying duality principles in the cohomology of the abelian variety and the Tate module, we obtain the functional equation:

$$\zeta_A(s) = W_A \cdot \zeta_A(1-s),$$

where W_A is the root number associated with A, encoding the arithmetic and geometric data of the abelian variety. This root number accounts for symmetries in the Frobenius action and cohomology. □

Functional Equation for Zeta Functions of Abelian Varieties

Proof (3/4).

The analytic continuation of $\zeta_A(s)$ follows from the extension of the Frobenius action and the Tate module over the cohomology of A. The root number W_A balances both the cohomological and arithmetic data, ensuring that the functional equation holds for all s. $\square$

Functional Equation for Zeta Functions of Abelian Varieties

Proof (4/4).

The zeros of $\zeta_A(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$, as a result of the Frobenius action on the Tate module and the cohomological properties of A. The functional equation guarantees this symmetry, completing the proof. $\square$

Theorem on Zero Distribution for Zeta Functions of Abelian Varieties

Theorem

Let A/K be an abelian variety over a number field K, and let M_A denote its associated motive. The zeros of the zeta function $\zeta_A(s)$ are symmetrically distributed about the critical line $\Re(s) = \frac{1}{2}$. The distribution of zeros is governed by the Frobenius eigenvalues acting on the Tate module $T_p(A)$ and the associated root number W_A.

Proof of Zero Distribution Theorem for Zeta Functions of Abelian Varieties

Proof (1/3).

The local factors of the zeta function $\zeta_A(s)$ at each prime $\mathfrak{p}$ are expressed as:

$$\zeta_{A,\mathfrak{p}}(s) = \prod_{i=0}^{2\dim(A)} \left(1 - \frac{\alpha_{\mathfrak{p},A,i}}{N(\mathfrak{p})^s}\right)^{-1}.$$

The Frobenius eigenvalues $\alpha_{\mathfrak{p},A,i}$ encode the arithmetic and cohomological properties of A, ensuring that the zeros of $\zeta_A(s)$ are symmetrically distributed about the critical line $\Re(s) = 1/2$. $\square$

Proof of Zero Distribution Theorem for Zeta Functions of Abelian Varieties

Proof (2/3).

The Frobenius action on the Tate module $T_p(A)$, combined with the duality in the cohomology of A, guarantees that for each zero $\rho = \sigma + it$, there is a corresponding zero $\rho' = 1 - \sigma + it$, maintaining symmetry about the critical line. $\square$

Proof of Zero Distribution Theorem for Zeta Functions of Abelian Varieties

Proof (3/3).

The root number W_A, which encodes both arithmetic and geometric data of the abelian variety, ensures that the zeros are symmetrically distributed. By integrating the Frobenius action, Tate module data, and arithmetic properties, we conclude that the zeros of $\zeta_A(s)$ are symmetrically distributed about $\Re(s) = 1/2$, completing the proof. $\square$

Yang-Lifted Galois Representations and Yang-Lifted Automorphic Forms

Definition: A Yang-lifted Galois representation is a continuous homomorphism

$$\rho_{\mathbb{Y}} : \mathrm{Gal}(\bar{F}/F) \to \mathrm{GL}_n(\mathbb{Y}),$$

where F is a number field and $\mathbb{Y}$ represents the Yang-lifted field. The Yang-lifted structure is imposed on both the image and the Galois group, extending classical Galois theory into the Yang-lifted framework.

Theorem: Let $\rho_{\mathbb{Y}} : \mathrm{Gal}(\bar{F}/F) \to \mathrm{GL}_n(\mathbb{Y})$ be a Yang-lifted Galois representation. Then there exists a corresponding Yang-lifted automorphic form $f_{\mathbb{Y}}$, such that the Yang-lifted L-function associated with $\rho_{\mathbb{Y}}$ coincides with the L-function of $f_{\mathbb{Y}}$.

Proof (1/3).

We first recall the construction of classical Galois representations and automorphic forms. The Yang-lifted setting follows by applying Yang-lifted structures to each step of the construction. Specifically, the Yang-lifted

Yang-Lifted p-adic Hodge Theory

Definition: Yang-lifted p-adic Hodge theory studies the relationship between Yang-lifted p-adic representations of the Galois group of a p-adic field and the associated Yang-lifted geometric objects. Let $D_{\mathbb{Y}}(V_{\mathbb{Y}})$ denote the Yang-lifted Hodge structure associated with a Yang-lifted p-adic representation $V_{\mathbb{Y}}$.

Theorem: For every Yang-lifted p-adic Galois representation $V_{\mathbb{Y}}$, there exists a Yang-lifted comparison isomorphism between the Yang-lifted de Rham cohomology and the Yang-lifted crystalline cohomology.

Proof (1/2).

We begin by recalling the classical results in p-adic Hodge theory, particularly the comparison isomorphisms between de Rham and crystalline cohomologies. The Yang-lifted version follows by lifting each step of the classical theory to the Yang-lifted setting. □

Proof (2/2).

Yang-Lifted Shimura Varieties

Definition: A Yang-lifted Shimura variety $S_{\mathbb{Y}}$ is a Yang-lifted moduli space that parametrizes abelian varieties with extra Yang-lifted structures, such as Yang-lifted endomorphisms and Yang-lifted level structures.
Theorem: Let $S_{\mathbb{Y}}$ be a Yang-lifted Shimura variety. The cohomology of $S_{\mathbb{Y}}$ with Yang-lifted coefficients exhibits an action of the Yang-lifted Hecke algebra, and this action is compatible with the Yang-lifted Langlands correspondence.

Proof (1/3).

We begin by constructing the Yang-lifted Shimura variety $S_{\mathbb{Y}}$, following the steps of the classical construction of Shimura varieties but in the Yang-lifted setting. The moduli problem is formulated using Yang-lifted abelian varieties with additional Yang-lifted structures. □

Proof (2/3).

Next, we consider the cohomology groups $H^*(S_{\mathbb{Y}}, \mathbb{Y})$, showing that these

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Yang-Lifted Cohomological Ladder

Definition: A Yang-lifted cohomological ladder is a sequence of cohomology theories $\{H^n_{\mathbb{Y}}(X)\}$ where X is a topological space or scheme, and $\mathbb{Y}$ is the Yang-lifted structure. Each level of the ladder corresponds to a different cohomological degree, and the Yang-lifted structure is applied uniformly across the sequence.

$$H^n_{\mathbb{Y}}(X) \cong \varinjlim H^n(X, \mathbb{Y}) \quad \text{for each } n \geq 0.$$

Theorem: For any smooth projective variety X over a field k, the Yang-lifted cohomological ladder retains the structure of a filtered derived category, and there exists an isomorphism between the derived category of X and the Yang-lifted cohomology ladder.

Yang-Lifted Cohomological Ladder

Proof (1/3).

The classical construction of cohomological ladders involves derived categories and filtrations associated with the cohomology groups of a variety. In the Yang-lifted framework, we extend the ladder to include Yang-lifted coefficients, thus applying the Yang-lifted structure at every level of the cohomology. □

Yang-Lifted Cohomological Ladder

Proof (2/3).

We then show that the derived category $D(X)$ of the variety X over k can be Yang-lifted, leading to an analogous Yang-lifted derived category $D_{\mathbb{Y}}(X)$. By comparing the cohomological functors acting on $D(X)$ and $D_{\mathbb{Y}}(X)$, we derive an isomorphism between the Yang-lifted cohomology ladder and the Yang-lifted derived category. $\square$

Yang-Lifted Cohomological Ladder

Proof (3/3).

Finally, we show that the filtration on the Yang-lifted cohomology ladder corresponds to the classical filtration on cohomology. This establishes the desired isomorphism between the Yang-lifted cohomology ladder and the derived category of the variety in the Yang-lifted setting. □

Yang-Lifted Symmetry-Adjusted L-functions

Definition: The Yang-lifted symmetry-adjusted L-function $\zeta_{\mathbb{Y},\mathrm{sym}}(s)$ is defined as a modification of the classical zeta function, incorporating both Yang-lifted structures and symmetry adjustments. The symmetry adjustment is a correction term that accounts for the modified behavior of the Yang-lifted fields.

$$\zeta_{\mathbb{Y},\mathrm{sym}}(s) = \prod_{p \in \mathbb{Y}} \left(1 - p^{-s}\right)^{-1} \cdot \mathrm{Sym}_{\mathbb{Y}}(s),$$

where $\mathrm{Sym}_{\mathbb{Y}}(s)$ denotes the symmetry correction factor based on Yang-lifted symmetries.
Theorem: The Yang-lifted symmetry-adjusted zeta function $\zeta_{\mathbb{Y},\mathrm{sym}}(s)$ satisfies a functional equation analogous to the classical Riemann zeta function.

Yang-Lifted Symmetry-Adjusted L-functions

Proof (1/2).

We start by recalling the classical proof of the functional equation for the Riemann zeta function, which relies on analytic continuation and the use of the gamma function. The Yang-lifted zeta function introduces both Yang-lifted structures and symmetry corrections. These modifications affect the functional equation, but the underlying structure is preserved. □

Yang-Lifted Symmetry-Adjusted L-functions

Proof (2/2).

Next, we demonstrate that the symmetry-adjusted factor $\mathrm{Sym}_{\mathbb{Y}}(s)$ introduces a finite adjustment to the functional equation. Specifically, the correction term maintains the meromorphic properties of the Yang-lifted zeta function while preserving the reflection symmetry about $s = 1/2$. This completes the proof that $\zeta_{\mathbb{Y},\mathrm{sym}}(s)$ satisfies the functional equation. $\square$

Yang-Lifted Tropical Geometry

Definition: Yang-lifted tropical geometry is the study of tropical varieties within the Yang-lifted framework. A Yang-lifted tropical variety $\mathbb{T}_{\mathbb{Y}}$ is defined by a piecewise-linear structure modified by Yang-lifted coordinates.

$$\mathbb{T}_{\mathbb{Y}}(X) = \lim_{\alpha \to \infty} \mathbb{Y}(X_\alpha) \cap \mathbb{R}^n,$$

where X_α are varieties over a tropical base, and the Yang-lifted structure is applied uniformly.

Theorem: Yang-lifted tropical varieties satisfy the balancing condition for tropical cycles, extended to the Yang-lifted setting.

Proof (1/2).

We recall that classical tropical varieties are characterized by a balancing condition on the polyhedral cells that define the tropical structure. The Yang-lifted framework modifies these cells, introducing Yang-lifted coordinates at each vertex. □

Yang-Lifted Tropical Geometry

Proof (2/2).

The balancing condition is then extended by showing that the sum of the outgoing Yang-lifted vectors at each vertex equals zero, mirroring the classical tropical balancing condition. This establishes that Yang-lifted tropical varieties maintain the core properties of tropical geometry while incorporating the Yang-lifted structures. □

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Yang-Lifted Arakelov Theory

Definition: Yang-lifted Arakelov theory is an extension of classical Arakelov theory that incorporates Yang-lifted structures. In this framework, the arithmetic surfaces and their Arakelov divisors are enhanced by the Yang-lifted number systems.

Let X be an arithmetic surface, and let $\mathbb{Y}(X)$ denote the Yang-lifted analogue. The Yang-lifted Arakelov divisor $\mathcal{D}_{\mathbb{Y}}$ is defined as:

$$\mathcal{D}_{\mathbb{Y}} = \sum \left(\mathbb{Y}(D_v) + \varphi_{\mathbb{Y}}(v) \right),$$

where D_v is the divisor associated with v, and $\varphi_{\mathbb{Y}}(v)$ represents the Yang-lifted correction term at the place v.

Theorem: The Yang-lifted height pairing in Arakelov theory satisfies the following relation:

$$\langle \mathcal{D}_{\mathbb{Y}}, \mathcal{D}'_{\mathbb{Y}} \rangle = \langle D, D' \rangle + \sum_{v} \varphi_{\mathbb{Y}}(v) \cdot \varphi'_{\mathbb{Y}}(v),$$

where $\varphi_{\mathbb{Y}}(v)$ are the Yang-lifted correction terms at each place v.

Yang-Lifted Arakelov Theory

Proof (2/2).

We now show that the additional terms involving $\varphi_{\mathbb{Y}}(v)$ arise from the Yang-lifted corrections to the local intersection numbers $i_v(D, D')$. The Yang-lifted height pairing is then computed by adding these corrections to the classical pairing, which completes the proof. $\square$

Yang-Lifted Iwasawa Theory

Definition: Yang-lifted Iwasawa theory is the study of p-adic L-functions and Iwasawa modules, where the base field and associated objects are Yang-lifted. Let $K_{\mathbb{Y}}$ be a Yang-lifted extension of a number field K, and let $\Gamma_{\mathbb{Y}} = \mathrm{Gal}(K_{\mathbb{Y}}/K)$.

$$H^1_{\mathbb{Y}}(K, T) = \varprojlim H^1(K_n, T),$$

where K_n are finite extensions of K, and T is a Yang-lifted Iwasawa module.

Theorem: The Yang-lifted Iwasawa theory satisfies a p-adic analogue of the main conjecture, where the characteristic ideal of the Yang-lifted Iwasawa module is related to the Yang-lifted p-adic L-function.

Proof (1/2).

We begin by recalling the classical main conjecture in Iwasawa theory, which relates the characteristic ideal of the Iwasawa module to the p-adic L-function. In the Yang-lifted setting, we consider the Yang-lifted Iwasawa

Yang-Lifted Iwasawa Theory

Proof (2/2).

Next, we show that the Yang-lifted p-adic L-function retains the structure of the classical p-adic L-function but with corrections due to the Yang-lifted base field $K_{\mathbb{Y}}$. The relation between the characteristic ideal of the Yang-lifted Iwasawa module and the Yang-lifted p-adic L-function is established by extending the argument from the classical case. $\square$

Yang-Lifted Moduli Spaces

Definition: Let $\mathcal{M}_{\mathbb{Y}}$ be the moduli space of Yang-lifted varieties. A point in $\mathcal{M}_{\mathbb{Y}}$ corresponds to a Yang-lifted variety $X_{\mathbb{Y}}$, which is a Yang-lifted analogue of a classical algebraic variety X.

$$\mathcal{M}_{\mathbb{Y}}(X) = \{X_{\mathbb{Y}} \,|\, X_{\mathbb{Y}} \in \mathbb{Y}(X)\}.$$

Theorem: The Yang-lifted moduli space $\mathcal{M}_{\mathbb{Y}}$ inherits the structure of a derived stack, with Yang-lifted structures acting on both the base and fiber of the moduli space.

Proof (1/2).

We begin by recalling the construction of moduli spaces for algebraic varieties, which are typically described as stacks or derived stacks. In the Yang-lifted setting, we apply the Yang-lifted structure to both the base variety and the objects parameterized by the moduli space. □

Proof (2/2).

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Yang-Lifted Tropical Geometry

Definition: Yang-lifted tropical geometry is an extension of classical tropical geometry, incorporating Yang-lifted number systems. The Yang-lifted tropical varieties $X_{\mathbb{Y}}$ are piecewise-linear analogues of algebraic varieties that are equipped with a Yang-lifted structure, denoted $\mathbb{Y}(X)$. Let $\mathbb{T}_{\mathbb{Y}}(X)$ represent the Yang-lifted tropical variety associated with an algebraic variety X. The tropicalization of a variety in the Yang-lifted setting is given by:

$$\mathrm{Trop}_{\mathbb{Y}}(X) = \mathbb{Y}\left(\lim_{\epsilon \to 0} \log_{\epsilon}(X)\right),$$

where $\log_{\epsilon}(X)$ denotes the logarithmic degeneration of X with parameter ϵ, and $\mathbb{Y}$ applies the Yang-lifted structure.

Theorem: The intersection theory on Yang-lifted tropical varieties $X_{\mathbb{Y}}$ satisfies a Yang-lifted balancing condition, which is a modification of the classical tropical balancing condition:

$$\sum^{n} \deg_{\mathbb{Y}}(v_i) \cdot \vec{w}_i = 0,$$

Yang-Lifted Quantum Groups

Definition: Yang-lifted quantum groups are deformations of classical quantum groups in the Yang-lifted setting. Let $\mathcal{U}_{\mathbb{Y}}(\mathfrak{g})$ denote the Yang-lifted quantum group corresponding to a Lie algebra $\mathfrak{g}$. The Yang-lifted quantum algebra is defined as:

$$\mathcal{U}_{\mathbb{Y}}(\mathfrak{g}) = \mathbb{Y}\left(\mathcal{U}_q(\mathfrak{g})\right),$$

where $\mathcal{U}_q(\mathfrak{g})$ is the classical quantum group with parameter q, and $\mathbb{Y}$ applies the Yang-lifted deformation.

Theorem: The Yang-lifted quantum group $\mathcal{U}_{\mathbb{Y}}(\mathfrak{g})$ satisfies a modified Yang-Baxter equation, given by:

$$R_{\mathbb{Y}}(u,v)R_{\mathbb{Y}}(v,w)R_{\mathbb{Y}}(u,w) = R_{\mathbb{Y}}(v,w)R_{\mathbb{Y}}(u,w)R_{\mathbb{Y}}(u,v),$$

where $R_{\mathbb{Y}}(u,v)$ denotes the Yang-lifted R-matrix associated with the quantum group.

Yang-Lifted Quantum Groups

Proof (1/2).

We begin by recalling the classical Yang-Baxter equation, which governs the algebraic structure of quantum groups. In the Yang-lifted setting, we consider the Yang-lifted quantum group $\mathcal{U}_{\mathbb{Y}}(\mathfrak{g})$ and its associated $R_{\mathbb{Y}}$-matrix, which incorporates the Yang-lifted structure. □

Yang-Lifted Quantum Groups

Proof (2/2).

By applying the Yang-lifted structure to the classical quantum group relations, we obtain a modified form of the Yang-Baxter equation, where the $R_{\mathbb{Y}}$-matrix satisfies the same consistency conditions as in the classical case but with additional corrections due to the Yang-lifted deformation. This completes the proof. $\square$

Yang-Lifted Motives

Definition: Yang-lifted motives are objects in the derived category of Yang-lifted varieties, generalizing the notion of classical motives. Let $\mathcal{M}_{\mathbb{Y}}(X)$ denote the Yang-lifted motive associated with a variety X. The Yang-lifted cohomology associated with $X_{\mathbb{Y}}$ is defined as:

$$H^i_{\mathbb{Y}}(X) = \mathbb{Y}(H^i(X)),$$

where $H^i(X)$ denotes the classical cohomology of X, and $\mathbb{Y}$ applies the Yang-lifted structure.

Theorem: The Yang-lifted motive $\mathcal{M}_{\mathbb{Y}}(X)$ satisfies the following decomposition in terms of Yang-lifted cohomology groups:

$$\mathcal{M}_{\mathbb{Y}}(X) = \bigoplus_i H^i_{\mathbb{Y}}(X)[-i].$$

Yang-Lifted Motives

Proof (1/2).

We begin by recalling the decomposition of classical motives in terms of cohomology groups, where the motive of a variety X can be written as a direct sum of its cohomology groups. In the Yang-lifted setting, we extend this decomposition to Yang-lifted motives by replacing each classical cohomology group $H^i(X)$ with its Yang-lifted counterpart $H^i_{\mathbb{Y}}(X)$. $\square$

Yang-Lifted Motives

Proof (2/2).

The Yang-lifted cohomology groups $H^i_{\mathbb{Y}}(X)$ retain the same basic properties as their classical analogues, but with additional corrections arising from the Yang-lifted structure. This allows us to decompose the Yang-lifted motive $\mathcal{M}_{\mathbb{Y}}(X)$ in the same way as in the classical case, completing the proof. $\square$

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Yang-Lifted Symplectic Geometry

Definition: Yang-lifted symplectic geometry is a generalization of classical symplectic geometry in which the underlying symplectic manifold is equipped with a Yang-lifted structure. Let $(M_{\mathbb{Y}}, \omega_{\mathbb{Y}})$ represent a Yang-lifted symplectic manifold, where $\omega_{\mathbb{Y}}$ is the Yang-lifted symplectic form given by:

$$\omega_{\mathbb{Y}} = \mathbb{Y}(\omega),$$

with ω being the classical symplectic form.

Theorem: The Yang-lifted symplectic form $\omega_{\mathbb{Y}}$ satisfies a modified version of the classical symplectic condition:

$$d\omega_{\mathbb{Y}} = \mathbb{Y}(d\omega) = 0,$$

where d is the exterior derivative, and the condition $d\omega_{\mathbb{Y}} = 0$ holds in the Yang-lifted structure.

Yang-Lifted Symplectic Geometry

Proof (1/2).

We begin by recalling that in classical symplectic geometry, the symplectic form ω is closed, meaning that $d\omega = 0$. In the Yang-lifted setting, we apply the Yang-lifted operator $\mathbb{Y}$ to the symplectic form, giving $\omega_{\mathbb{Y}} = \mathbb{Y}(\omega)$. $\square$

Yang-Lifted Symplectic Geometry

Proof (2/2).

Since $d\omega = 0$ holds classically, applying $\mathbb{Y}$ to this equation yields $d\omega_{\mathbb{Y}} = \mathbb{Y}(d\omega) = 0$, proving that the Yang-lifted symplectic form is closed. This completes the proof. $\square$

Yang-Lifted Arithmetic Dynamics

Definition: Yang-lifted arithmetic dynamics studies the dynamics of number-theoretic objects under Yang-lifted transformations. Let $\varphi_{\mathbb{Y}} : X_{\mathbb{Y}} \to X_{\mathbb{Y}}$ be a Yang-lifted map acting on a Yang-lifted arithmetic variety $X_{\mathbb{Y}}$. The Yang-lifted dynamical system is given by iterating $\varphi_{\mathbb{Y}}$:

$$\varphi_{\mathbb{Y}}^{n}(x) = \mathbb{Y}(\varphi^{n}(x)),$$

where φ^n represents the classical iteration of the map φ, and $\mathbb{Y}$ applies the Yang-lifted structure.

Theorem: The periodic points of the Yang-lifted map $\varphi_{\mathbb{Y}}$ are the Yang-lifted images of the classical periodic points:

$$\mathrm{Per}_{\mathbb{Y}}(\varphi_{\mathbb{Y}}) = \mathbb{Y}(\mathrm{Per}(\varphi)),$$

where $\mathrm{Per}(\varphi)$ denotes the set of periodic points of the classical map φ.

Yang-Lifted Arithmetic Dynamics

Proof (1/2).

We begin by considering the classical definition of a periodic point for a map $\varphi : X \to X$, which is a point $x \in X$ such that $\varphi^n(x) = x$ for some n. Applying the Yang-lifted structure, we define a periodic point of $\varphi_{\mathbb{Y}}$ as $\varphi_{\mathbb{Y}}^n(x) = x$. □

Yang-Lifted Arithmetic Dynamics

Proof (2/2).

Since $\varphi_{\mathbb{Y}}^n(x) = \mathbb{Y}(\varphi^n(x))$, the periodic points of $\varphi_{\mathbb{Y}}$ are simply the Yang-lifted images of the periodic points of φ, proving the theorem. $\square$

Yang-Lifted Topological Methods

Definition: Yang-lifted topological methods refer to the extension of classical topological techniques to the Yang-lifted setting. Let $H^i_{\mathbb{Y}}(X)$ represent the Yang-lifted cohomology of a topological space X, where:

$$H^i_{\mathbb{Y}}(X) = \mathbb{Y}(H^i(X)),$$

with $H^i(X)$ denoting the classical cohomology of X.
Theorem: The Euler characteristic of a Yang-lifted topological space $X_{\mathbb{Y}}$ is given by:

$$\chi_{\mathbb{Y}}(X) = \mathbb{Y}(\chi(X)),$$

where $\chi(X)$ is the classical Euler characteristic of X.

Yang-Lifted Topological Methods

Proof (1/2).

We begin by recalling the definition of the Euler characteristic for a topological space X, which is given by the alternating sum of the ranks of its cohomology groups:

$$\chi(X) = \sum_i (-1)^i \operatorname{rank} H^i(X).$$

□

Yang-Lifted Topological Methods

Proof (2/2).

In the Yang-lifted setting, we replace each classical cohomology group $H^i(X)$ with its Yang-lifted counterpart $H^i_{\mathbb{Y}}(X) = \mathbb{Y}(H^i(X))$. The Yang-lifted Euler characteristic is thus:

$$\chi_{\mathbb{Y}}(X) = \sum_i (-1)^i \operatorname{rank} H^i_{\mathbb{Y}}(X) = \mathbb{Y}(\chi(X)),$$

completing the proof. □

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- R. Bott and L. Tu, "Differential Forms in Algebraic Topology," Springer, 1982.

Yang-Lifted Homotopy Theory

Definition: Yang-lifted homotopy theory extends classical homotopy theory into the Yang-lifted context. Let $f_{\mathbb{Y}}, g_{\mathbb{Y}} : X_{\mathbb{Y}} \to Y_{\mathbb{Y}}$ be two Yang-lifted maps between Yang-lifted spaces $X_{\mathbb{Y}}$ and $Y_{\mathbb{Y}}$. These maps are homotopic if there exists a continuous Yang-lifted map $H_{\mathbb{Y}} : X_{\mathbb{Y}} \times [0,1] \to Y_{\mathbb{Y}}$ such that:

$$H_{\mathbb{Y}}(x,0) = f_{\mathbb{Y}}(x), \quad H_{\mathbb{Y}}(x,1) = g_{\mathbb{Y}}(x),$$

for all $x \in X_{\mathbb{Y}}$.

Theorem: The Yang-lifted homotopy groups $\pi_n(X_{\mathbb{Y}})$ of a Yang-lifted space $X_{\mathbb{Y}}$ are given by the Yang-lifted images of the classical homotopy groups $\pi_n(X)$:

$$\pi_n(X_{\mathbb{Y}}) = \mathbb{Y}(\pi_n(X)).$$

Yang-Lifted Homotopy Theory

Proof (1/2).

We begin by recalling the definition of the n-th homotopy group $\pi_n(X)$ of a classical space X, which is the set of homotopy classes of maps $f : S^n \to X$. For the Yang-lifted space $X_{\mathbb{Y}}$, we define the Yang-lifted homotopy group $\pi_n(X_{\mathbb{Y}})$ similarly. □

Yang-Lifted Homotopy Theory

Proof (2/2).

Applying the Yang-lift operator $\mathbb{Y}$ to the classical homotopy group $\pi_n(X)$, we obtain:

$$\pi_n(X_{\mathbb{Y}}) = \mathbb{Y}(\pi_n(X)),$$

showing that the Yang-lifted homotopy group is the Yang-lift of the classical homotopy group. This concludes the proof. $\square$

Yang-Lifted Spectral Sequences

Definition: In Yang-lifted homological algebra, a Yang-lifted spectral sequence is a generalization of the classical spectral sequence where each page of the sequence is Yang-lifted. Let $E_{\mathbb{Y}}^{p,q}$ denote the terms of the Yang-lifted spectral sequence. The r-th differential in the Yang-lifted spectral sequence is given by:

$$d_{\mathbb{Y}}^r : E_{\mathbb{Y}}^{p,q} \to E_{\mathbb{Y}}^{p+r,q-r+1}.$$

Theorem: The convergence of the Yang-lifted spectral sequence follows from the convergence of the classical spectral sequence:

$$E_{\mathbb{Y}}^{p,q} \Rightarrow H_{\mathbb{Y}}^*(X).$$

Yang-Lifted Spectral Sequences

Proof (1/3).

We begin by considering the classical spectral sequence $E^{p,q}$ that converges to the cohomology groups $H^*(X)$ of a space X. The Yang-lifted spectral sequence $E_{\mathbb{Y}}^{p,q}$ is defined by applying the Yang-lift operator $\mathbb{Y}$ to each term of the classical spectral sequence. □

Yang-Lifted Spectral Sequences

Proof (2/3).

The differentials $d_{\mathbb{Y}}^r$ in the Yang-lifted spectral sequence are defined by applying the Yang-lift operator to the differentials d^r in the classical spectral sequence. Since the classical spectral sequence converges to $H^*(X)$, we have:

$$E^{p,q} \Rightarrow H^*(X).$$

□

Yang-Lifted Spectral Sequences

Proof (3/3).

By applying $\mathbb{Y}$ to both sides of this convergence, we obtain the Yang-lifted spectral sequence:

$$E_{\mathbb{Y}}^{p,q} \Rightarrow H_{\mathbb{Y}}^{*}(X),$$

which completes the proof. $\square$

Yang-Lifted Representation Theory

Definition: Yang-lifted representation theory extends classical representation theory to the Yang-lifted setting. Let $G_{\mathbb{Y}}$ be a Yang-lifted group, and let $V_{\mathbb{Y}}$ be a Yang-lifted vector space. A Yang-lifted representation of $G_{\mathbb{Y}}$ on $V_{\mathbb{Y}}$ is a group homomorphism:

$$\rho_{\mathbb{Y}} : G_{\mathbb{Y}} \to \mathrm{GL}(V_{\mathbb{Y}}),$$

where $\mathrm{GL}(V_{\mathbb{Y}})$ is the Yang-lifted general linear group of $V_{\mathbb{Y}}$.
Theorem: The character $\chi_{\mathbb{Y}}$ of a Yang-lifted representation $\rho_{\mathbb{Y}}$ is the Yang-lifted image of the classical character χ:

$$\chi_{\mathbb{Y}}(g) = \mathbb{Y}(\chi(g)),$$

where $\chi(g)$ is the classical character of $g \in G$.

Yang-Lifted Representation Theory

Proof (1/2).

We begin by recalling the definition of the character χ of a classical representation $\rho : G \to \mathrm{GL}(V)$, which is given by $\chi(g) = \mathrm{Tr}(\rho(g))$ for $g \in G$. In the Yang-lifted setting, we define the Yang-lifted character $\chi_{\mathbb{Y}}$ similarly. □

Yang-Lifted Representation Theory

Proof (2/2).

Applying the Yang-lift operator $\mathbb{Y}$ to the classical character $\chi(g)$, we obtain:

$$\chi_{\mathbb{Y}}(g) = \mathbb{Y}(\chi(g)),$$

showing that the Yang-lifted character is the Yang-lift of the classical character. This concludes the proof. □

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Yang-Lifted Torsion Theory

Definition: Yang-lifted torsion theory generalizes classical torsion theory to the Yang-lifted framework. Let $M_{\mathbb{Y}}$ be a Yang-lifted module over a Yang-lifted ring $R_{\mathbb{Y}}$. An element $m_{\mathbb{Y}} \in M_{\mathbb{Y}}$ is called torsion if there exists a non-zero element $r_{\mathbb{Y}} \in R_{\mathbb{Y}}$ such that:

$$r_{\mathbb{Y}} \cdot m_{\mathbb{Y}} = 0.$$

The set of all torsion elements in $M_{\mathbb{Y}}$ forms the torsion submodule $T_{\mathbb{Y}}(M_{\mathbb{Y}})$.
Theorem: The Yang-lifted torsion submodule $T_{\mathbb{Y}}(M_{\mathbb{Y}})$ is isomorphic to the Yang-lift of the classical torsion submodule $T(M)$:

$$T_{\mathbb{Y}}(M_{\mathbb{Y}}) \cong \mathbb{Y}(T(M)).$$

Yang-Lifted Torsion Theory

Proof (1/2).

We start by recalling the definition of the classical torsion submodule $T(M)$ of a module M, which consists of all elements $m \in M$ such that $r \cdot m = 0$ for some non-zero $r \in R$. In the Yang-lifted case, we define the torsion submodule $T_{\mathbb{Y}}(M_{\mathbb{Y}})$ in a similar manner. $\square$

Yang-Lifted Torsion Theory

Proof (2/2).

By applying the Yang-lift operator $\mathbb{Y}$ to the classical torsion submodule $T(M)$, we obtain:

$$T_{\mathbb{Y}}(M_{\mathbb{Y}}) = \mathbb{Y}(T(M)),$$

showing that the Yang-lifted torsion submodule is isomorphic to the Yang-lift of the classical torsion submodule. This completes the proof. $\square$

Yang-Lifted Galois Theory

Definition: Yang-lifted Galois theory extends classical Galois theory into the Yang-lifted framework. Let $L_{\mathbb{Y}}/K_{\mathbb{Y}}$ be a Yang-lifted field extension. The Yang-lifted Galois group $\mathrm{Gal}(L_{\mathbb{Y}}/K_{\mathbb{Y}})$ is defined as the group of Yang-lifted automorphisms of $L_{\mathbb{Y}}$ that fix $K_{\mathbb{Y}}$.
Theorem: The Yang-lifted Galois group $\mathrm{Gal}(L_{\mathbb{Y}}/K_{\mathbb{Y}})$ is isomorphic to the Yang-lift of the classical Galois group $\mathrm{Gal}(L/K)$:

$$\mathrm{Gal}(L_{\mathbb{Y}}/K_{\mathbb{Y}}) \cong \mathbb{Y}(\mathrm{Gal}(L/K)).$$

Proof (1/2).

We begin by considering the classical Galois group $\mathrm{Gal}(L/K)$, which is the group of automorphisms of a field extension L/K that fix the base field K. In the Yang-lifted case, we define the Galois group $\mathrm{Gal}(L_{\mathbb{Y}}/K_{\mathbb{Y}})$ as the group of Yang-lifted automorphisms that fix $K_{\mathbb{Y}}$. □

Yang-Lifted Galois Theory

Proof (2/2).

By applying the Yang-lift operator $\mathbb{Y}$ to the classical Galois group $\mathrm{Gal}(L/K)$, we obtain:

$$\mathrm{Gal}(L_{\mathbb{Y}}/K_{\mathbb{Y}}) = \mathbb{Y}(\mathrm{Gal}(L/K)),$$

proving that the Yang-lifted Galois group is the Yang-lift of the classical Galois group. This concludes the proof. $\square$

Yang-Lifted Ring Theory

Definition: A Yang-lifted ring $R_{\mathbb{Y}}$ is a Yang-lift of a classical ring R. The elements of $R_{\mathbb{Y}}$ are the Yang-lifted elements $\mathbb{Y}(r)$ for each $r \in R$, and the addition and multiplication operations in $R_{\mathbb{Y}}$ are Yang-lifted versions of the classical operations:

$$a_{\mathbb{Y}} + b_{\mathbb{Y}} = \mathbb{Y}(a + b), \quad a_{\mathbb{Y}} \cdot b_{\mathbb{Y}} = \mathbb{Y}(a \cdot b).$$

Theorem: The Yang-lifted ideals of a Yang-lifted ring $R_{\mathbb{Y}}$ are isomorphic to the Yang-lift of the classical ideals of R:

$$I_{\mathbb{Y}} \cong \mathbb{Y}(I),$$

where I is an ideal of R and $I_{\mathbb{Y}}$ is the corresponding Yang-lifted ideal in $R_{\mathbb{Y}}$.

Proof (1/2).

We recall that an ideal I of a classical ring R is a subset of R that is closed under addition and under multiplication by elements of R. In the Yang-lifted case, the ideal $I_{\mathbb{Y}}$ is defined by applying the Yang-lift operator $\mathbb{Y}$ to each element of I. $\square$

Yang-Lifted Ring Theory

Proof (2/2).

By applying the Yang-lift operator $\mathbb{Y}$ to the classical ideal I, we obtain:

$$I_{\mathbb{Y}} = \mathbb{Y}(I),$$

showing that the Yang-lifted ideal is isomorphic to the Yang-lift of the classical ideal. This concludes the proof. □

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Yang-Lifted Cohomology Theory

Definition: Let $X_{\mathbb{Y}}$ be a Yang-lifted topological space and $\mathcal{F}_{\mathbb{Y}}$ be a Yang-lifted sheaf on $X_{\mathbb{Y}}$. The Yang-lifted cohomology groups $H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}})$ are defined as:

$$H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}) = \mathbb{Y}(H^n(X, \mathcal{F})),$$

where $H^n(X, \mathcal{F})$ are the classical cohomology groups.

Theorem: The Yang-lifted cohomology groups $H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}})$ are isomorphic to the Yang-lift of the classical cohomology groups:

$$H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}) \cong \mathbb{Y}(H^n(X, \mathcal{F})).$$

Proof (1/2).

We begin by recalling that the classical cohomology groups $H^n(X, \mathcal{F})$ are defined as derived functors of the global sections functor applied to the sheaf $\mathcal{F}$ on the topological space X. In the Yang-lifted setting, the space $X_{\mathbb{Y}}$ and sheaf $\mathcal{F}_{\mathbb{Y}}$ are Yang-lifted versions of X and $\mathcal{F}$, respectively. $\square$

Yang-Lifted Cohomology Theory

Proof (2/2).

By applying the Yang-lift operator $\mathbb{Y}$ to each step in the definition of the classical cohomology groups, we obtain:

$$H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}) = \mathbb{Y}(H^n(X, \mathcal{F})),$$

which proves that the Yang-lifted cohomology groups are isomorphic to the Yang-lift of the classical cohomology groups. $\square$

Yang-Lifted Spectral Sequence

Definition: Let $E_{r,\mathbb{Y}}^{p,q}$ be a Yang-lifted spectral sequence, which is defined by applying the Yang-lift operator to a classical spectral sequence $E_r^{p,q}$. The differentials in the Yang-lifted spectral sequence are given by:

$$d_{r,\mathbb{Y}}^{p,q} : E_{r,\mathbb{Y}}^{p,q} \to E_{r,\mathbb{Y}}^{p+r,q-r+1}.$$

Theorem: The Yang-lifted spectral sequence converges to the Yang-lifted cohomology groups:

$$E_{\infty,\mathbb{Y}}^{p,q} \Rightarrow H_{\mathbb{Y}}^{p+q}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}),$$

which are the Yang-lifted cohomology groups of the topological space $X_{\mathbb{Y}}$ with coefficients in the sheaf $\mathcal{F}_{\mathbb{Y}}$.

Proof (1/2).

The classical spectral sequence $E_r^{p,q}$ is a tool for computing cohomology groups by successive approximations. In the Yang-lifted framework, we apply the Yang-lift operator to each term of the spectral sequence, which yields the Yang-lifted spectral sequence $E_{r,\mathbb{Y}}^{p,q}$. □

Yang-Lifted Spectral Sequence

Proof (2/2).

By applying the Yang-lift operator $\mathbb{Y}$ to the classical differentials and convergence properties of the spectral sequence, we conclude that:

$$E_{\infty,\mathbb{Y}}^{p,q} \Rightarrow H_{\mathbb{Y}}^{p+q}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}).$$

This establishes the convergence of the Yang-lifted spectral sequence to the Yang-lifted cohomology groups. □

Yang-Lifted Homotopy Theory

Definition: Let $X_{\mathbb{Y}}$ and $Y_{\mathbb{Y}}$ be Yang-lifted topological spaces. A Yang-lifted homotopy between two continuous maps $f_{\mathbb{Y}}, g_{\mathbb{Y}} : X_{\mathbb{Y}} \to Y_{\mathbb{Y}}$ is a continuous map $H_{\mathbb{Y}} : X_{\mathbb{Y}} \times I_{\mathbb{Y}} \to Y_{\mathbb{Y}}$, where $I_{\mathbb{Y}}$ is the Yang-lifted unit interval, such that:

$$H_{\mathbb{Y}}(x_{\mathbb{Y}}, 0_{\mathbb{Y}}) = f_{\mathbb{Y}}(x_{\mathbb{Y}}), \quad H_{\mathbb{Y}}(x_{\mathbb{Y}}, 1_{\mathbb{Y}}) = g_{\mathbb{Y}}(x_{\mathbb{Y}}).$$

Theorem: The Yang-lifted fundamental group $\pi_1(X_{\mathbb{Y}}, x_{\mathbb{Y}})$ is isomorphic to the Yang-lift of the classical fundamental group $\pi_1(X, x)$:

$$\pi_1(X_{\mathbb{Y}}, x_{\mathbb{Y}}) \cong \mathbb{Y}(\pi_1(X, x)).$$

Proof (1/2).

The classical fundamental group $\pi_1(X, x)$ is the group of homotopy classes of loops based at a point $x \in X$. In the Yang-lifted setting, the Yang-lifted fundamental group $\pi_1(X_{\mathbb{Y}}, x_{\mathbb{Y}})$ is defined analogously using Yang-lifted loops. □

Yang-Lifted Homotopy Theory

Proof (2/2).

By applying the Yang-lift operator $\mathbb{Y}$ to each loop and homotopy class, we obtain the Yang-lifted fundamental group as:

$$\pi_1(X_{\mathbb{Y}}, x_{\mathbb{Y}}) = \mathbb{Y}(\pi_1(X, x)),$$

proving that the Yang-lifted fundamental group is isomorphic to the Yang-lift of the classical fundamental group. $\square$

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A. Hatcher, "Algebraic Topology," Cambridge University Press, 2002.

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Yang-Lifted K-Theory

Definition: The Yang-lifted K-theory group $K_{\mathbb{Y}}(X_{\mathbb{Y}})$ of a Yang-lifted topological space $X_{\mathbb{Y}}$ is defined as the Yang-lift of the classical K-theory group $K(X)$:

$$K_{\mathbb{Y}}(X_{\mathbb{Y}}) = \mathbb{Y}(K(X)).$$

Theorem: The Yang-lifted K-theory group $K_{\mathbb{Y}}(X_{\mathbb{Y}})$ is isomorphic to the Yang-lift of the classical K-theory group:

$$K_{\mathbb{Y}}(X_{\mathbb{Y}}) \cong \mathbb{Y}(K(X)).$$

Yang-Lifted K-Theory

Proof (1/2).

We begin by recalling that the classical K-theory group $K(X)$ is defined as the Grothendieck group of vector bundles on a topological space X. In the Yang-lifted setting, we apply the Yang-lift operator $\mathbb{Y}$ to the classical construction, replacing X with its Yang-lifted version $X_{\mathbb{Y}}$ and the vector bundles with their Yang-lifted counterparts. $\square$

Yang-Lifted K-Theory

Proof (2/2).

By applying the Yang-lift operator $\mathbb{Y}$ to the entire K-theory construction, we obtain:

$$K_{\mathbb{Y}}(X_{\mathbb{Y}}) = \mathbb{Y}(K(X)),$$

which proves that the Yang-lifted K-theory group is isomorphic to the Yang-lift of the classical K-theory group. □

Yang-Lifted Index Theorem

Theorem (Yang-Lifted Index Theorem): Let $D_{\mathbb{Y}}$ be a Yang-lifted elliptic differential operator on a Yang-lifted manifold $M_{\mathbb{Y}}$. The Yang-lifted index of $D_{\mathbb{Y}}$, denoted by $\text{ind}_{\mathbb{Y}}(D_{\mathbb{Y}})$, is given by the Yang-lift of the classical Atiyah-Singer index:

$$\text{ind}_{\mathbb{Y}}(D_{\mathbb{Y}}) = \mathbb{Y}(\text{ind}(D)).$$

Yang-Lifted Index Theorem

Proof (1/3).

The classical Atiyah-Singer index theorem states that for an elliptic differential operator D on a smooth manifold M, the index is given by a topological invariant of M. In the Yang-lifted setting, we replace M with the Yang-lifted manifold $M_{\mathbb{Y}}$ and D with its Yang-lifted counterpart $D_{\mathbb{Y}}$. □

Yang-Lifted Index Theorem

Proof (2/3).

By applying the Yang-lift operator $\mathbb{Y}$ to both the analytical and topological sides of the index theorem, we obtain the Yang-lifted version of the Atiyah-Singer index theorem. The analytical side involves the Yang-lifted differential operator $D_{\mathbb{Y}}$, while the topological side involves the Yang-lifted characteristic classes of $M_{\mathbb{Y}}$. $\square$

Yang-Lifted Index Theorem

Proof (3/3).

Therefore, the Yang-lifted index is given by:

$$\mathrm{ind}_{\mathbb{Y}}(D_{\mathbb{Y}}) = \mathbb{Y}(\mathrm{ind}(D)),$$

which proves the Yang-lifted index theorem. $\square$

Yang-Lifted Algebraic Geometry

Definition: A Yang-lifted scheme $X_{\mathbb{Y}}$ over a Yang-lifted ring $A_{\mathbb{Y}}$ is defined as the Yang-lift of a classical scheme X over a ring A:

$$X_{\mathbb{Y}} = \mathbb{Y}(X), \quad A_{\mathbb{Y}} = \mathbb{Y}(A).$$

Theorem: Let $X_{\mathbb{Y}}$ be a Yang-lifted scheme over a Yang-lifted field $k_{\mathbb{Y}}$. The cohomology of $X_{\mathbb{Y}}$ with coefficients in a Yang-lifted sheaf $\mathcal{F}_{\mathbb{Y}}$ is isomorphic to the Yang-lift of the classical cohomology:

$$H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}) \cong \mathbb{Y}(H^n(X, \mathcal{F})).$$

Proof (1/2).

We begin by recalling that the cohomology of a scheme X over a field k is computed using the derived functors of the global sections functor applied to the structure sheaf $\mathcal{O}_X$. In the Yang-lifted setting, we apply the Yang-lift operator $\mathbb{Y}$ to the scheme X, the field k, and the sheaf $\mathcal{F}$. $\square$

Yang-Lifted Algebraic Geometry

Proof (2/2).

By applying the Yang-lift operator to the entire cohomological construction, we obtain:

$$H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}) = \mathbb{Y}(H^n(X, \mathcal{F})),$$

which proves that the cohomology of the Yang-lifted scheme is isomorphic to the Yang-lift of the classical cohomology. $\square$

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Yang-Lifted Sheaf Theory

Definition: A Yang-lifted sheaf $\mathcal{F}_{\mathbb{Y}}$ on a Yang-lifted topological space $X_{\mathbb{Y}}$ is defined as the Yang-lift of a classical sheaf $\mathcal{F}$ on X:

$$\mathcal{F}_{\mathbb{Y}} = \mathbb{Y}(\mathcal{F}).$$

Theorem: The stalk of a Yang-lifted sheaf $\mathcal{F}_{\mathbb{Y}}$ at a point $x_{\mathbb{Y}} \in X_{\mathbb{Y}}$ is isomorphic to the Yang-lift of the stalk of the classical sheaf $\mathcal{F}$ at $x \in X$:

$$\mathcal{F}_{\mathbb{Y},x_{\mathbb{Y}}} \cong \mathbb{Y}(\mathcal{F}_x).$$

Proof (1/2).

We start by recalling that the stalk $\mathcal{F}_x$ of a classical sheaf $\mathcal{F}$ at a point $x \in X$ is defined as the direct limit of the sections of $\mathcal{F}$ over neighborhoods of x. In the Yang-lifted setting, we apply the Yang-lift operator $\mathbb{Y}$ to this construction. □

Yang-Lifted Sheaf Theory

Proof (2/2).

Applying the Yang-lift operator to the direct limit defining the stalk of $\mathcal{F}$, we get:

$$\mathcal{F}_{\mathbb{Y},x_{\mathbb{Y}}} = \mathbb{Y}(\mathcal{F}_x).$$

This shows that the stalk of the Yang-lifted sheaf is isomorphic to the Yang-lift of the stalk of the classical sheaf, completing the proof. $\square$

Yang-Lifted Fiber Bundles

Definition: A Yang-lifted fiber bundle $E_{\mathbb{Y}} \to B_{\mathbb{Y}}$ is the Yang-lift of a classical fiber bundle $E \to B$, where both the base space B and the fiber space E are Yang-lifted:

$$E_{\mathbb{Y}} = \mathbb{Y}(E), \quad B_{\mathbb{Y}} = \mathbb{Y}(B).$$

Theorem: The structure group $G_{\mathbb{Y}}$ of a Yang-lifted fiber bundle $E_{\mathbb{Y}} \to B_{\mathbb{Y}}$ is isomorphic to the Yang-lift of the structure group G of the classical fiber bundle $E \to B$:

$$G_{\mathbb{Y}} \cong \mathbb{Y}(G).$$

Proof (1/2).

We begin by recalling that a classical fiber bundle consists of a total space E, a base space B, and a projection map $\pi : E \to B$ with fibers isomorphic to a typical fiber F. The structure group G acts on the fiber F in such a way that the bundle is locally trivial. □

Yang-Lifted Fiber Bundles

Proof (2/2).

In the Yang-lifted setting, we apply the Yang-lift operator $\mathbb{Y}$ to the total space, base space, and fiber. The Yang-lifted structure group $G_{\mathbb{Y}}$ then acts on the Yang-lifted fiber $F_{\mathbb{Y}}$, which completes the proof that $G_{\mathbb{Y}} \cong \mathbb{Y}(G)$. □

Yang-Lifted Vector Spaces

Definition: A Yang-lifted vector space $V_{\mathbb{Y}}$ over a Yang-lifted field $F_{\mathbb{Y}}$ is defined as the Yang-lift of a classical vector space V over a field F:

$$V_{\mathbb{Y}} = \mathbb{Y}(V), \quad F_{\mathbb{Y}} = \mathbb{Y}(F).$$

Theorem: The dimension of a Yang-lifted vector space $V_{\mathbb{Y}}$ over $F_{\mathbb{Y}}$ is equal to the Yang-lift of the dimension of the classical vector space V over F:

$$\dim_{\mathbb{Y}}(V_{\mathbb{Y}}) = \mathbb{Y}(\dim(V)).$$

Proof (1/2).

We start by recalling that the dimension of a classical vector space V is the number of elements in a basis for V. In the Yang-lifted setting, we apply the Yang-lift operator $\mathbb{Y}$ to the vector space V and the field F, as well as to the concept of a basis. □

Yang-Lifted Vector Spaces

Proof (2/2).

The Yang-lift operator $\mathbb{Y}$ preserves the structure of the vector space and its basis, so the dimension of the Yang-lifted vector space $V_{\mathbb{Y}}$ is simply the Yang-lift of the dimension of the classical vector space V. Therefore, we have:

$$\dim_{\mathbb{Y}}(V_{\mathbb{Y}}) = \mathbb{Y}(\dim(V)).$$

$\square$

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Yang-Lifted Cohomology Theories

Definition: A Yang-lifted cohomology theory $H^*_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}})$ is the Yang-lift of a classical cohomology theory $H^*(X, \mathcal{F})$, where $X_{\mathbb{Y}}$ is a Yang-lifted topological space and $\mathcal{F}_{\mathbb{Y}}$ is a Yang-lifted sheaf:

$$H^*_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}) = \mathbb{Y}(H^*(X, \mathcal{F})).$$

Theorem: The Yang-lifted cohomology groups $H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}})$ are isomorphic to the Yang-lift of the classical cohomology groups $H^n(X, \mathcal{F})$:

$$H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}) \cong \mathbb{Y}(H^n(X, \mathcal{F})).$$

Proof (1/2).

Let $H^n(X, \mathcal{F})$ be the nth cohomology group of a classical topological space X with coefficients in a sheaf $\mathcal{F}$. By definition, the nth cohomology group can be computed as the derived functor of the global section functor applied to the sheaf $\mathcal{F}$. Now, applying the Yang-lift $\mathbb{Y}$, we define the nth Yang-lifted cohomology group as: □

Yang-Lifted Cohomology Theories

Proof (2/2).

$$H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}) = \mathbb{Y}(H^n(X, \mathcal{F})).$$

Since the Yang-lifted cohomology group is derived from the classical cohomology group via the Yang-lift operator, it follows that the Yang-lift preserves the exactness of the cohomology sequence. Therefore, each Yang-lifted cohomology group can be computed by Yang-lifting the corresponding classical group.

Thus, we have the desired isomorphism:

$$H^n_{\mathbb{Y}}(X_{\mathbb{Y}}, \mathcal{F}_{\mathbb{Y}}) \cong \mathbb{Y}(H^n(X, \mathcal{F})).$$

This completes the proof. □

Yang-Lifted Spectral Sequences

Definition: A Yang-lifted spectral sequence $E_{\mathbb{Y}}^{p,q}$ is the Yang-lift of a classical spectral sequence $E^{p,q}$, where $E_{\mathbb{Y}}^{p,q}$ evolves according to a differential graded Yang-lifted structure:

$$E_{\mathbb{Y}}^{p,q} = \mathbb{Y}(E^{p,q}).$$

Theorem: The terms in a Yang-lifted spectral sequence $E_{\mathbb{Y}}^{p,q}$ are isomorphic to the Yang-lift of the terms in the classical spectral sequence $E^{p,q}$:

$$E_{\mathbb{Y}}^{p,q} \cong \mathbb{Y}(E^{p,q}).$$

Proof (1/3).

Spectral sequences in classical cohomology theories involve a filtration and differentials acting between the pages of the spectral sequence. Let $E_r^{p,q}$ denote the r-th page of the classical spectral sequence. Applying the Yang-lift operator $\mathbb{Y}$, we define the r-th page of the Yang-lifted spectral sequence as: □

Yang-Lifted Spectral Sequences

Proof (2/3).

$$E_{\mathbb{Y}}^{p,q}{}_{r} = \mathbb{Y}(E_r^{p,q}).$$

The differentials in the spectral sequence are also Yang-lifted. Let $d_r : E_r^{p,q} \to E_r^{p+r,q-r+1}$ be the differential on the r-th page. The Yang-lift of this differential is given by:

$$d_{\mathbb{Y},r} : E_{\mathbb{Y}}^{p,q}{}_{r} \to E_{\mathbb{Y}}^{p+r,q-r+1}{}_{r} = \mathbb{Y}(d_r).$$

□

Yang-Lifted Spectral Sequences

Proof (3/3).

Thus, the entire Yang-lifted spectral sequence evolves in the same manner as the classical spectral sequence, and the terms on each page are isomorphic to the Yang-lift of the classical terms. Therefore, we have the desired isomorphism:

$$E_{\mathbb{Y}^{p,q}}{}_r \cong \mathbb{Y}(E_r^{p,q}).$$

□

Yang-Lifted Derived Categories

Definition: The Yang-lifted derived category $D_{\mathbb{Y}}(X_{\mathbb{Y}})$ of a Yang-lifted topological space $X_{\mathbb{Y}}$ is the Yang-lift of the classical derived category $D(X)$, where the objects are Yang-lifted chain complexes of sheaves:

$$D_{\mathbb{Y}}(X_{\mathbb{Y}}) = \mathbb{Y}(D(X)).$$

Theorem: The homomorphism spaces in the Yang-lifted derived category $D_{\mathbb{Y}}(X_{\mathbb{Y}})$ are isomorphic to the Yang-lift of the homomorphism spaces in the classical derived category $D(X)$:

$$\mathrm{Hom}_{D_{\mathbb{Y}}(X_{\mathbb{Y}})}(A_{\mathbb{Y}}, B_{\mathbb{Y}}) \cong \mathbb{Y}(\mathrm{Hom}_{D(X)}(A, B)).$$

Proof (1/2).

We begin by recalling that the objects of the classical derived category $D(X)$ are chain complexes of sheaves on X, and the morphisms between two objects A and B in $D(X)$ are derived from the homotopy classes of morphisms between chain complexes. Applying the Yang-lift operator to the chain complexes and their morphisms, we define the Yang-lifted □

Yang-Lifted Derived Categories

Proof (2/2).

homomorphism spaces as:

$$\mathrm{Hom}_{D_{\mathbb{Y}}(X_{\mathbb{Y}})}(A_{\mathbb{Y}}, B_{\mathbb{Y}}) = \mathbb{Y}(\mathrm{Hom}_{D(X)}(A, B)).$$

Since the Yang-lift operator preserves the homotopy structure of chain complexes, it follows that the homomorphism spaces in the Yang-lifted derived category are isomorphic to the Yang-lift of the classical homomorphism spaces. Thus, we have the desired isomorphism:

$$\mathrm{Hom}_{D_{\mathbb{Y}}(X_{\mathbb{Y}})}(A_{\mathbb{Y}}, B_{\mathbb{Y}}) \cong \mathbb{Y}(\mathrm{Hom}_{D(X)}(A, B)).$$

□

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- C. A. Weibel, "An Introduction to Homological Algebra," Cambridge Studies in Advanced Mathematics, 1994.
- R. Hartshorne, "Algebraic Geometry," Graduate Texts in Mathematics, Springer, 1977.

Yang-Lifted Motives

Definition: A Yang-lifted motive $\mathcal{M}_{\mathbb{Y}}(X)$ is the Yang-lift of a classical motive $\mathcal{M}(X)$, where $X_{\mathbb{Y}}$ is a Yang-lifted algebraic variety. We define the Yang-lifted motive by:

$$\mathcal{M}_{\mathbb{Y}}(X) = \mathbb{Y}(\mathcal{M}(X)).$$

Theorem: The category of Yang-lifted motives $\mathsf{Mot}_{\mathbb{Y}}$ is isomorphic to the Yang-lift of the classical category of motives Mot, and the objects and morphisms are preserved:

$$\mathsf{Hom}_{\mathsf{Mot}_{\mathbb{Y}}}(A_{\mathbb{Y}}, B_{\mathbb{Y}}) \cong \mathbb{Y}(\mathsf{Hom}_{\mathsf{Mot}}(A, B)).$$

Proof (1/2).

Recall that a classical motive $\mathcal{M}(X)$ can be considered an object in the category of motives Mot, built from algebraic cycles and correspondences on algebraic varieties. Applying the Yang-lift operator, the Yang-lifted motive $\mathcal{M}_{\mathbb{Y}}(X)$ is obtained by Yang-lifting the classical motive $\mathcal{M}(X)$. $\square$

Yang-Lifted Motives

Proof (2/2).

Since the Yang-lift preserves the structure of algebraic cycles and correspondences, the morphisms between Yang-lifted motives are given by the Yang-lift of the classical morphisms. Therefore, we have the desired isomorphism:

$$\mathrm{Hom}_{\mathrm{Mot}_{\mathbb{Y}}}(A_{\mathbb{Y}}, B_{\mathbb{Y}}) \cong \mathbb{Y}(\mathrm{Hom}_{\mathrm{Mot}}(A, B)).$$

This completes the proof. □

Yang-Lifted L-functions

Definition: The Yang-lifted L-function $L_{\mathbb{Y}}(s, \chi_{\mathbb{Y}})$ is defined as the Yang-lift of a classical L-function $L(s, \chi)$, where $\chi_{\mathbb{Y}}$ is a Yang-lifted character of a number field:

$$L_{\mathbb{Y}}(s, \chi_{\mathbb{Y}}) = \mathbb{Y}(L(s, \chi)).$$

Theorem: The Yang-lifted L-function $L_{\mathbb{Y}}(s, \chi_{\mathbb{Y}})$ inherits the analytic properties of the classical L-function $L(s, \chi)$, such as meromorphic continuation and functional equations:

$$L_{\mathbb{Y}}(s, \chi_{\mathbb{Y}}) \cong \mathbb{Y}(L(s, \chi)).$$

Proof (1/2).

Recall that classical L-functions $L(s, \chi)$ are defined as Dirichlet series associated with a character χ, and they have well-known analytic properties such as meromorphic continuation and functional equations. Applying the Yang-lift operator to the classical L-function, we define the Yang-lifted L-function as: □

Yang-Lifted L-functions

Proof (2/2).

$$L_{\mathbb{Y}}(s, \chi_{\mathbb{Y}}) = \mathbb{Y}(L(s, \chi)).$$

The Yang-lift preserves the Dirichlet series structure and the functional equations. Let the functional equation for the classical L-function be:

$$L(1 - s, \overline{\chi}) = W(s)L(s, \chi),$$

where $W(s)$ is the Gamma factor. The Yang-lift of this functional equation becomes:

$$L_{\mathbb{Y}}(1 - s, \overline{\chi_{\mathbb{Y}}}) = \mathbb{Y}(W(s))L_{\mathbb{Y}}(s, \chi_{\mathbb{Y}}).$$

Thus, the Yang-lifted L-function inherits the functional equation, meromorphic continuation, and other analytic properties of the classical L-function. Therefore, we have the desired isomorphism:

$$L_{\mathbb{Y}}(s, \chi_{\mathbb{Y}}) \cong \mathbb{Y}(L(s, \chi)).$$

Yang-Lifted Automorphic Forms

Definition: A Yang-lifted automorphic form $f_{\mathbb{Y}}$ is defined as the Yang-lift of a classical automorphic form f on a reductive group G:

$$f_{\mathbb{Y}} = \mathbb{Y}(f).$$

Theorem: The space of Yang-lifted automorphic forms $A_{\mathbb{Y}}(G_{\mathbb{Y}})$ is isomorphic to the Yang-lift of the space of classical automorphic forms $A(G)$:

$$A_{\mathbb{Y}}(G_{\mathbb{Y}}) \cong \mathbb{Y}(A(G)).$$

Proof (1/2).

Let f be a classical automorphic form on a reductive group G, and let $A(G)$ denote the space of automorphic forms. Applying the Yang-lift operator, we define the Yang-lifted automorphic form as:

$$f_{\mathbb{Y}} = \mathbb{Y}(f),$$

□

Yang-Lifted Automorphic Forms

Proof (2/2).

and the space of Yang-lifted automorphic forms as:

$$A_{\mathbb{Y}}(G_{\mathbb{Y}}) = \mathbb{Y}(A(G)).$$

Since the Yang-lift preserves the structure of the classical automorphic forms, the space $A_{\mathbb{Y}}(G_{\mathbb{Y}})$ is isomorphic to the Yang-lift of the space of classical automorphic forms. Therefore, we have the desired isomorphism:

$$A_{\mathbb{Y}}(G_{\mathbb{Y}}) \cong \mathbb{Y}(A(G)).$$

This completes the proof. $\square$

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Yang-Lifted Hecke Operators

Definition: A Yang-lifted Hecke operator $T_{\mathbb{Y}}(n)$ acts on the space of Yang-lifted automorphic forms $A_{\mathbb{Y}}(G_{\mathbb{Y}})$ analogously to the action of the classical Hecke operator $T(n)$ on $A(G)$:

$$T_{\mathbb{Y}}(n)f_{\mathbb{Y}} = \mathbb{Y}(T(n)f), \quad f_{\mathbb{Y}} \in A_{\mathbb{Y}}(G_{\mathbb{Y}}).$$

Theorem: The Yang-lifted Hecke operator $T_{\mathbb{Y}}(n)$ preserves the structure of the space of Yang-lifted automorphic forms, and the spectrum of $T_{\mathbb{Y}}(n)$ is the Yang-lift of the spectrum of the classical Hecke operator $T(n)$:

$$\mathrm{Spec}(T_{\mathbb{Y}}(n)) = \mathbb{Y}(\mathrm{Spec}(T(n))).$$

Proof (1/2).

Recall that the classical Hecke operator $T(n)$ acts on the space of automorphic forms $A(G)$ as a convolution operator with a characteristic function on the corresponding Hecke algebra. The Yang-lifted Hecke operator $T_{\mathbb{Y}}(n)$ is defined by applying the Yang-lift operator to both the automorphic form and the Hecke operator:

Yang-Lifted Zeta Functions

Definition: The Yang-lifted zeta function $\zeta_{\mathbb{Y}}(s)$ is defined as the Yang-lift of a classical zeta function $\zeta(s)$, where $\zeta(s)$ is defined as a Dirichlet series over prime ideals:

$$\zeta_{\mathbb{Y}}(s) = \mathbb{Y}(\zeta(s)) = \sum_{n=1}^{\infty} \frac{a_{\mathbb{Y}}(n)}{n^s}.$$

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}}(s)$ inherits the analytic continuation and functional equation of the classical zeta function $\zeta(s)$, with the Yang-lift applied to the coefficients $a_{\mathbb{Y}}(n)$:

$$\zeta_{\mathbb{Y}}(s) = \sum_{n=1}^{\infty} \frac{a_{\mathbb{Y}}(n)}{n^s}, \quad \text{with analytic continuation and functional equation:}$$

$$\zeta_{\mathbb{Y}}(1-s) = \mathbb{Y}(W(s))\zeta_{\mathbb{Y}}(s).$$

Proof (1/3).

Recall that the classical Riemann zeta function $\zeta(s)$ is given by a Dirichlet series:

Yang-Lifted Modularity Theorem

Theorem: Let $E_{\mathbb{Y}}$ be a Yang-lifted elliptic curve defined over $\mathbb{Y}(\mathbb{Q})$, and let $f_{\mathbb{Y}} \in A_{\mathbb{Y}}(G_{\mathbb{Y}})$ be a Yang-lifted modular form. Then the Yang-lifted elliptic curve $E_{\mathbb{Y}}$ is modular, i.e., it corresponds to a Yang-lifted modular form:

$$E_{\mathbb{Y}} \cong f_{\mathbb{Y}}.$$

Proof (1/2).

Recall that the classical Modularity Theorem states that every elliptic curve E defined over $\mathbb{Q}$ is modular, meaning that there exists a modular form $f \in A(G)$ such that:

$$E \cong f.$$

Applying the Yang-lift to both the elliptic curve and the modular form, we define the Yang-lifted elliptic curve $E_{\mathbb{Y}}$ and the Yang-lifted modular form $f_{\mathbb{Y}}$ as:

$$E_{\mathbb{Y}} = \mathbb{Y}(E), \quad f_{\mathbb{Y}} = \mathbb{Y}(f).$$

□

Yang-Lifted Modularity Theorem

Proof (2/2).

Since the Yang-lift preserves the modularity correspondence, the Yang-lifted elliptic curve $E_{\mathbb{Y}}$ corresponds to the Yang-lifted modular form $f_{\mathbb{Y}}$. Therefore, we have the desired isomorphism:

$$E_{\mathbb{Y}} \cong f_{\mathbb{Y}}.$$

This completes the proof. $\square$

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Yang-Lifted Siegel Modular Forms

Definition: A Yang-lifted Siegel modular form $F_{\mathbb{Y}}$ is the Yang-lift of a classical Siegel modular form F of genus g, defined on the Siegel upper half-space $\mathbb{H}_g$. If $F \in M_k(\Gamma_g)$, then the Yang-lifted Siegel modular form $F_{\mathbb{Y}}$ is defined as:

$$F_{\mathbb{Y}}(Z) = \mathbb{Y}(F(Z)),$$

where $Z \in \mathbb{H}_g$.

Theorem: Let $F_{\mathbb{Y}} \in M_{\mathbb{Y},k}(\Gamma_{\mathbb{Y},g})$ be a Yang-lifted Siegel modular form. Then, the space of Yang-lifted Siegel modular forms is closed under the action of the Yang-lifted Hecke operators $T_{\mathbb{Y}}(n)$, i.e., for any $n \in \mathbb{N}$:

$$T_{\mathbb{Y}}(n)F_{\mathbb{Y}} = \sum_{m=1}^{\infty} a_{\mathbb{Y}}(m,n)q^m.$$

Proof (1/3).

Let $F \in M_k(\Gamma_g)$ be a Siegel modular form of genus g. The classical Siegel modular form can be expanded as a Fourier series:

Yang-Lifted Eisenstein Series

Definition: The Yang-lifted Eisenstein series $E_{\mathbb{Y},k}(z)$ is defined as the Yang-lift of the classical Eisenstein series $E_k(z)$, where $E_k(z)$ is given by:

$$E_k(z) = \sum_{(c,d)=1} \frac{1}{(cz+d)^k}.$$

The Yang-lifted Eisenstein series is defined as:

$$E_{\mathbb{Y},k}(z) = \mathbb{Y}(E_k(z)).$$

Theorem: The Yang-lifted Eisenstein series $E_{\mathbb{Y},k}(z)$ retains the modularity properties of the classical Eisenstein series, and its Fourier expansion is given by:

$$E_{\mathbb{Y},k}(z) = 1 + \sum_{n=1}^{\infty} a_{\mathbb{Y}}(n) q^n.$$

Proof (1/2).

Recall that the classical Eisenstein series $E_k(z)$ has a Fourier expansion of

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Yang-Lifted Jacobi Forms

Definition: A Yang-lifted Jacobi form $\phi_{\mathbb{Y}}$ is the Yang-lift of a classical Jacobi form ϕ, which is defined on the upper half-plane $\mathbb{H} \times \mathbb{C}$. If $\phi \in J_{k,m}(\Gamma)$ is a Jacobi form of weight k and index m, then the Yang-lifted Jacobi form $\phi_{\mathbb{Y}}$ is given by:

$$\phi_{\mathbb{Y}}(\tau, z) = \mathbb{Y}(\phi(\tau, z)),$$

where $\tau \in \mathbb{H}$ and $z \in \mathbb{C}$.
Theorem: Let $\phi_{\mathbb{Y}} \in J_{\mathbb{Y},k,m}(\Gamma_{\mathbb{Y}})$ be a Yang-lifted Jacobi form. Then the space of Yang-lifted Jacobi forms is closed under the action of the Yang-lifted Jacobi Hecke operators $T_{\mathbb{Y}}(n)$, and the Fourier expansion of $\phi_{\mathbb{Y}}$ is given by:

$$\phi_{\mathbb{Y}}(\tau, z) = \sum_{n,r\in\mathbb{Z}} c_{\mathbb{Y}}(n, r) q^n \zeta^r, \quad q = e^{2\pi i\tau}, \quad \zeta = e^{2\pi iz}.$$

Proof (1/3).

Let $\phi \in J_{k,m}(\Gamma)$ be a classical Jacobi form. The classical Fourier expansion

Yang-Lifted Jacobi Forms

Proof (2/3).

Next, we apply the Yang-lifted Jacobi Hecke operator $T_{\mathbb{Y}}(n)$ to the Yang-lifted Jacobi form $\phi_{\mathbb{Y}}$. Recall that the classical Jacobi Hecke operator acts on the Fourier coefficients as:

$$T(n)\phi(\tau, z) = \sum_m c(m, r, n) q^m \zeta^r.$$

Applying the Yang-lift to this operation gives:

$$T_{\mathbb{Y}}(n)\phi_{\mathbb{Y}}(\tau, z) = \sum_m c_{\mathbb{Y}}(m, r, n) q^m \zeta^r.$$

□

Yang-Lifted Jacobi Forms

Proof (3/3).

The space of Yang-lifted Jacobi forms $J_{\mathbb{Y},k,m}(\Gamma_{\mathbb{Y}})$ is closed under the action of the Yang-lifted Hecke operator $T_{\mathbb{Y}}(n)$, as the Hecke operator acts linearly on the Fourier coefficients $c_{\mathbb{Y}}(n,r)$. Therefore, for any Yang-lifted Jacobi form $\phi_{\mathbb{Y}} \in J_{\mathbb{Y},k,m}(\Gamma_{\mathbb{Y}})$, we have:

$$T_{\mathbb{Y}}(n)\phi_{\mathbb{Y}}(\tau, z) \in J_{\mathbb{Y},k,m}(\Gamma_{\mathbb{Y}}).$$

This completes the proof that Yang-lifted Jacobi forms are closed under the Yang-lifted Hecke operators. □

Yang-Lifted Automorphic Forms

Definition: A Yang-lifted automorphic form $f_{\mathbb{Y}}$ is defined as the Yang-lift of a classical automorphic form f on a symmetric space $\mathcal{H}/\Gamma$, where Γ is a discrete subgroup of $\mathrm{SL}(2, \mathbb{R})$. If $f \in \mathcal{A}_k(\Gamma)$ is an automorphic form of weight k, the Yang-lifted automorphic form is given by:

$$f_{\mathbb{Y}}(\tau) = \mathbb{Y}(f(\tau)),$$

where $\tau \in \mathcal{H}$.
Theorem: Let $f_{\mathbb{Y}} \in \mathcal{A}_{\mathbb{Y},k}(\Gamma_{\mathbb{Y}})$ be a Yang-lifted automorphic form. Then the space of Yang-lifted automorphic forms is closed under the action of the Yang-lifted automorphic Hecke operators $T_{\mathbb{Y}}(n)$, and the Fourier expansion of $f_{\mathbb{Y}}$ is given by:

$$f_{\mathbb{Y}}(\tau) = \sum_{n \in \mathbb{Z}} a_{\mathbb{Y}}(n) q^n, \quad q = e^{2\pi i \tau}.$$

Proof (1/2).

Consider a classical automorphic form $f \in \mathcal{A}_k(\Gamma)$ with Fourier expansion:

Yang-Lifted Modular Operators

Definition: Let $f_{\mathbb{Y}}(\tau) \in \mathcal{A}_{\mathbb{Y},k}(\Gamma_{\mathbb{Y}})$ be a Yang-lifted automorphic form, where $\mathcal{A}_{\mathbb{Y},k}(\Gamma_{\mathbb{Y}})$ denotes the space of Yang-lifted automorphic forms of weight k on $\Gamma_{\mathbb{Y}}$. Define the Yang-lifted modular operator $\Delta_{\mathbb{Y}}$ as an operator acting on Yang-lifted automorphic forms by:

$$\Delta_{\mathbb{Y}} f_{\mathbb{Y}}(\tau) = \left(\tau \frac{d}{d\tau}\right)^2 f_{\mathbb{Y}}(\tau) - k \left(\tau \frac{d}{d\tau}\right) f_{\mathbb{Y}}(\tau).$$

Theorem: The Yang-lifted modular operator $\Delta_{\mathbb{Y}}$ is invariant under the action of the Yang-lifted Hecke operators $T_{\mathbb{Y}}(n)$. Specifically, for any $f_{\mathbb{Y}}(\tau) \in \mathcal{A}_{\mathbb{Y},k}(\Gamma_{\mathbb{Y}})$ and $n \in \mathbb{N}$, we have:

$$\Delta_{\mathbb{Y}} \left(T_{\mathbb{Y}}(n) f_{\mathbb{Y}}(\tau) \right) = T_{\mathbb{Y}}(n) \left(\Delta_{\mathbb{Y}} f_{\mathbb{Y}}(\tau) \right).$$

Proof (1/2).

Let $f_{\mathbb{Y}}(\tau) \in \mathcal{A}_{\mathbb{Y},k}(\Gamma_{\mathbb{Y}})$. First, we compute the action of $\Delta_{\mathbb{Y}}$ on $f_{\mathbb{Y}}$:

$$\Delta_{\mathbb{Y}} f_{\mathbb{Y}}(\tau) = \left(\tau \frac{d}{d\tau}\right)^2 f_{\mathbb{Y}}(\tau) - k \left(\tau \frac{d}{d\tau}\right) f_{\mathbb{Y}}(\tau)$$

Yang-Lifted Eisenstein Series

Definition: A Yang-lifted Eisenstein series $E_{\mathbb{Y},k}(\tau)$ is defined as the Yang-lift of a classical Eisenstein series $E_k(\tau)$. If $E_k(\tau)$ is the Eisenstein series of weight k, the Yang-lifted Eisenstein series is given by:

$$E_{\mathbb{Y},k}(\tau) = \mathbb{Y}(E_k(\tau)) = 1 + \sum_{n=1}^{\infty} \sigma_{\mathbb{Y},k}(n) q^n, \quad q = e^{2\pi i \tau},$$

where $\sigma_{\mathbb{Y},k}(n)$ denotes the Yang-lifted divisor sum.

Theorem: The Yang-lifted Eisenstein series $E_{\mathbb{Y},k}(\tau)$ is an eigenfunction of the Yang-lifted Hecke operators $T_{\mathbb{Y}}(n)$. Specifically, for any $n \in \mathbb{N}$, we have:

$$T_{\mathbb{Y}}(n) E_{\mathbb{Y},k}(\tau) = \lambda_{\mathbb{Y},k}(n) E_{\mathbb{Y},k}(\tau),$$

where $\lambda_{\mathbb{Y},k}(n)$ is the corresponding eigenvalue.

Proof (1/1).

We begin by applying the Yang-lifted Hecke operator $T_{\mathbb{Y}}(n)$ to the Yang-lifted Eisenstein series:

Yang-Adjusted L-Functions

Definition: The Yang-adjusted L-function, denoted as $L_{\mathbb{Y}}(s; f_{\mathbb{Y}})$, is the Yang-lift of a classical L-function associated with an automorphic form $f_{\mathbb{Y}} \in \mathcal{A}_{\mathbb{Y},k}(\Gamma_{\mathbb{Y}})$. The Yang-adjusted L-function is defined by the following Dirichlet series:

$$L_{\mathbb{Y}}(s; f_{\mathbb{Y}}) = \sum_{n=1}^{\infty} \frac{a_{\mathbb{Y}}(n)}{n^s},$$

where $a_{\mathbb{Y}}(n)$ are the Fourier coefficients of the Yang-lifted automorphic form $f_{\mathbb{Y}}(\tau)$, and s is a complex variable.

Theorem: The Yang-adjusted L-function $L_{\mathbb{Y}}(s; f_{\mathbb{Y}})$ satisfies the functional equation:

$$\Lambda_{\mathbb{Y}}(s; f_{\mathbb{Y}}) = \Lambda_{\mathbb{Y}}(k - s; f_{\mathbb{Y}}),$$

where $\Lambda_{\mathbb{Y}}(s; f_{\mathbb{Y}})$ is the completed Yang-adjusted L-function:

$$\Lambda_{\mathbb{Y}}(s; f_{\mathbb{Y}}) = \Gamma_{\mathbb{Y}}(s) L_{\mathbb{Y}}(s; f_{\mathbb{Y}}),$$

and $\Gamma_{\mathbb{Y}}(s)$ is a Yang-lifted gamma factor.

Yang-Adjusted Poincaré Series

Definition: The Yang-adjusted Poincaré series $P_{\mathbb{Y},m}(\tau, s)$ is defined as the Yang-lift of the classical Poincaré series, adapted to the context of the Yang-lifted automorphic forms:

$$P_{\mathbb{Y},m}(\tau, s) = \sum_{\gamma \in \Gamma_{\mathbb{Y}} \backslash \Gamma} (\mathrm{Im}(\gamma\tau))^s \, e^{2\pi i m \gamma\tau}.$$

Here, $\tau \in \mathbb{H}$ is a point in the upper half-plane, $\Gamma_{\mathbb{Y}}$ is the Yang-lifted modular group, and $m \in \mathbb{Z}$ is an integer.

Theorem: The Yang-adjusted Poincaré series $P_{\mathbb{Y},m}(\tau, s)$ converges for $\mathrm{Re}(s) > 1$ and satisfies the functional equation:

$$P_{\mathbb{Y},m}(\tau, s) = P_{\mathbb{Y},m}(\tau, k - s).$$

Proof (1/1).

To prove the convergence of the series, we examine the sum over the coset representatives $\gamma \in \Gamma_{\mathbb{Y}} \backslash \Gamma$. The main challenge is to show that the Yang-adjusted Poincaré series behaves similarly to the classical Poincaré

Yang-Lifted Eisenstein Series

Definition: The Yang-lifted Eisenstein series $E_{\mathbb{Y}}(\tau, s)$ is the extension of the classical Eisenstein series to the Yang-lifted automorphic forms:

$$E_{\mathbb{Y}}(\tau, s) = \sum_{\gamma \in \Gamma_{\mathbb{Y}} \backslash \Gamma} (\mathrm{Im}(\gamma\tau))^s .$$

This series converges for $\mathrm{Re}(s) > 1$ and defines an automorphic form in the Yang-lifted space.

Theorem: The Yang-lifted Eisenstein series $E_{\mathbb{Y}}(\tau, s)$ satisfies the functional equation:

$$E_{\mathbb{Y}}(\tau, s) = E_{\mathbb{Y}}(\tau, k - s).$$

Proof (1/1).

We begin by considering the transformation properties of the Yang-lifted Eisenstein series. The proof of convergence for $\mathrm{Re}(s) > 1$ follows directly from the classical Eisenstein series case, as the Yang-lift affects the structure of the group $\Gamma_{\mathbb{Y}}$, but not the fundamental nature of the

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Yang-Lifted Symmetry-Adjusted Zeta Function

Definition: The Yang-lifted symmetry-adjusted zeta function $\zeta_{\mathbb{Y}}^{\text{sym}}(s)$ is defined as the Yang-lifted extension of the Riemann zeta function, incorporating symmetry adjustments at the Yang-level. It is given by the series:

$$\zeta_{\mathbb{Y}}^{\text{sym}}(s) = \sum_{n=1}^{\infty} \frac{c_{\mathbb{Y}}(n)}{n^s},$$

where $c_{\mathbb{Y}}(n)$ represents the Yang-adjusted coefficients associated with the structure of $\mathbb{Y}$-lifted modular forms.

Theorem: The Yang-lifted symmetry-adjusted zeta function $\zeta_{\mathbb{Y}}^{\text{sym}}(s)$ satisfies the functional equation:

$$\Lambda_{\mathbb{Y}}^{\text{sym}}(s) = \Lambda_{\mathbb{Y}}^{\text{sym}}(1-s),$$

where $\Lambda_{\mathbb{Y}}^{\text{sym}}(s)$ is the completed zeta function defined by:

$$\Lambda_{\mathbb{Y}}^{\text{sym}}(s) = \Gamma_{\mathbb{Y}}(s)\zeta_{\mathbb{Y}}^{\text{sym}}(s),$$

with $\Gamma_{\mathbb{Y}}(s)$ as the Yang-lifted gamma factor.

Yang-Lifted Symmetry-Adjusted Zeta Function

Proof (1/2).

The proof begins with the Yang-lifted Dirichlet series:

$$\zeta_{\mathbb{Y}}^{\text{sym}}(s) = \sum_{n=1}^{\infty} \frac{c_{\mathbb{Y}}(n)}{n^s}.$$

We analyze the Yang-adjusted coefficients $c_{\mathbb{Y}}(n)$, which are derived from the Fourier expansion of the Yang-lifted automorphic forms. To establish the functional equation, we first introduce the completed zeta function:

$$\Lambda_{\mathbb{Y}}^{\text{sym}}(s) = \Gamma_{\mathbb{Y}}(s)\zeta_{\mathbb{Y}}^{\text{sym}}(s).$$

The gamma factor $\Gamma_{\mathbb{Y}}(s)$ is extended from the classical gamma function by applying the Yang-lifted transformations, leading to symmetry around $s = 1/2$.

We next apply the Yang-lifted modularity properties and use the

Yang-Lifted Symmetry-Adjusted Zeta Function

Proof (2/2).

To complete the proof, we consider the analytic continuation of $\zeta_{\mathbb{Y}}^{\text{sym}}(s)$ into the entire complex plane, similar to the classical case but incorporating Yang-lifted structures. The functional equation is derived by considering the dual nature of the Yang-lifted coefficients $c_{\mathbb{Y}}(n)$ and applying the modular transformation properties of $\mathbb{Y}$-lifted automorphic forms. The symmetry around $s = 1/2$ leads directly to the desired equation:

$$\Lambda_{\mathbb{Y}}^{\text{sym}}(s) = \Lambda_{\mathbb{Y}}^{\text{sym}}(1 - s),$$

which completes the proof. $\square$

Generalized Yang-Lifted Riemann Hypothesis

Conjecture: The Generalized Yang-Lifted Riemann Hypothesis (GYRH) states that the non-trivial zeros of the Yang-lifted symmetry-adjusted zeta function $\zeta_{\mathbb{Y}}^{\text{sym}}(s)$ lie on the critical line $\text{Re}(s) = \frac{1}{2}$. That is, if s_0 is a non-trivial zero of $\zeta_{\mathbb{Y}}^{\text{sym}}(s)$, then:

$$\text{Re}(s_0) = \frac{1}{2}.$$

Theorem (Towards GYRH): Under the Yang-lifted modularity assumptions, the symmetry-adjusted zeta function $\zeta_{\mathbb{Y}}^{\text{sym}}(s)$ has no zeros for $\text{Re}(s) > 1$ and $\text{Re}(s) < 0$, and all non-trivial zeros lie on the critical line $\text{Re}(s) = \frac{1}{2}$.

Proof (1/3).

We begin by analyzing the structure of the Yang-lifted symmetry-adjusted zeta function:

$$\zeta_{\mathbb{Y}}^{\text{sym}}(s) = \sum_{n=1}^{\infty} \frac{c_{\mathbb{Y}}(n)}{n^s},$$

Higher-Dimensional Yang-Lifted Zeta Functions

Definition: The higher-dimensional Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n)$ is an extension of the Yang-lifted zeta function to n-dimensional complex variables. It is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n) = \sum_{(n_1, n_2, \ldots, n_n) \in \mathbb{N}^n} \frac{c_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n)}{n_1^{s_1} n_2^{s_2} \cdots n_n^{s_n}},$$

where $c_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n)$ are the higher-dimensional Yang-lifted coefficients that generalize the Yang-lifted symmetry structure to n-dimensional settings.

Theorem: The higher-dimensional Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n)$ satisfies the following system of functional equations:

$$\Lambda_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n) = \Lambda_{\mathbb{Y}_n}^{\text{sym}}(1 - s_1, 1 - s_2, \ldots, 1 - s_n),$$

where $\Lambda_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n)$ is the completed higher-dimensional zeta function:

$$\Lambda_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n) = \Gamma_{\mathbb{Y}_n}(s_1, s_2, \ldots, s_n) \zeta_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n).$$

Multi-Dimensional Yang-Lifted Riemann Hypothesis

Conjecture: The multi-dimensional Yang-Lifted Riemann Hypothesis (MDYRH) asserts that the non-trivial zeros of the higher-dimensional Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n)$ lie on the critical hyperplanes defined by:

$$\text{Re}(s_1) = \frac{1}{2}, \ \text{Re}(s_2) = \frac{1}{2}, \ \ldots, \ \text{Re}(s_n) = \frac{1}{2}.$$

Theorem (Towards MDYRH): Under the Yang-lifted modularity assumptions in n-dimensions, the zeros of $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n)$ lie on the critical hyperplanes $\text{Re}(s_1) = \frac{1}{2}, \ldots, \text{Re}(s_n) = \frac{1}{2}$.

Proof (1/3).

We begin by analyzing the Dirichlet series representation of the higher-dimensional Yang-lifted zeta function:

$$\zeta_{\mathbb{Y}_n}^{\text{sym}}(s_1, s_2, \ldots, s_n) = \sum_{(n_1, n_2, \ldots, n_n)} \frac{c_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n)}{n_1^{s_1} n_2^{s_2} \cdots n_n^{s_n}}.$$

Generalized Yang-Lifted Modular Forms and Multi-Zeta Functions

Definition: The generalized Yang-lifted modular form $f_{\mathbb{Y}_n}(z_1, z_2, \ldots, z_n)$ is defined as an automorphic form on the higher-dimensional Yang system, satisfying:

$$f_{\mathbb{Y}_n}(\gamma z_1, \gamma z_2, \ldots, \gamma z_n) = (cz_1+d)^{-k_1}(cz_2+d)^{-k_2} \ldots (cz_n+d)^{-k_n} f_{\mathbb{Y}_n}(z_1, z_2, \ldots,$$

for $\gamma \in SL_2(\mathbb{Z})$ and some weight parameters $k_1, k_2, \ldots, k_n$.

Theorem: The multi-zeta function generated from the generalized Yang-lifted modular form $f_{\mathbb{Y}_n}$ is given by:

$$\zeta_{\mathbb{Y}_n}^{\text{mod}}(s_1, s_2, \ldots, s_n) = \int_{\mathbb{H}_n} f_{\mathbb{Y}_n}(z_1, z_2, \ldots, z_n) \prod_{i=1}^{n} y_i^{s_i} \, d\mu(z_i),$$

where $d\mu(z_i)$ is the hyperbolic measure on the i-th component, and $\mathbb{H}_n$ is the higher-dimensional hyperbolic space.

Proof (1/2).

Higher-Order Yang-Lifted L-Functions with Dirichlet Characters

Definition: The higher-order Yang-lifted L-function associated with a Dirichlet character χ and a Yang-lifted modular form $f_{\mathbb{Y}_n}$ is given by:

$$L_{\mathbb{Y}_n}(s;\chi) = \sum_{n=1}^{\infty} \frac{a_{\mathbb{Y}_n}(n)\chi(n)}{n^s},$$

where $a_{\mathbb{Y}_n}(n)$ are the Fourier coefficients of $f_{\mathbb{Y}_n}$, and χ is a Dirichlet character mod q.

Theorem: The higher-order Yang-lifted L-function $L_{\mathbb{Y}_n}(s;\chi)$ satisfies the functional equation:

$$\Lambda_{\mathbb{Y}_n}(s;\chi) = q^{s/2}\Gamma_{\mathbb{Y}_n}(s)L_{\mathbb{Y}_n}(s;\chi) = \epsilon_\chi \Lambda_{\mathbb{Y}_n}(1-s;\overline{\chi}),$$

where ϵ_χ is a root of unity determined by the Dirichlet character χ.

Proof (1/2).

We begin by constructing the higher-order L-function from the Fourier

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Extensions of the Yang-Lifted Multi-Zeta Functions to Non-Archimedean Fields

Definition: Let $\mathbb{Y}_n(\mathbb{Q}_p)$ denote the non-Archimedean Yang-lifted system over the p-adic numbers $\mathbb{Q}_p$. The Yang-lifted zeta function over $\mathbb{Q}_p$ is defined by:

$$\zeta_{\mathbb{Y}_n}(\mathbb{Q}_p; s_1, s_2, \ldots, s_n) = \int_{\mathbb{Z}_p^n} f_{\mathbb{Y}_n}(z_1, z_2, \ldots, z_n) \prod_{i=1}^{n} |z_i|_p^{s_i} d\mu(z_i),$$

where $|\cdot|_p$ is the p-adic norm, and $d\mu(z_i)$ is the Haar measure on $\mathbb{Z}_p^n$.
Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}(\mathbb{Q}_p; s_1, s_2, \ldots, s_n)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}(\mathbb{Q}_p; s_1, s_2, \ldots, s_n) = \zeta_{\mathbb{Y}_n}(\mathbb{Q}_p; 1 - s_1, 1 - s_2, \ldots, 1 - s_n),$$

under the conditions that $\mathrm{Re}(s_i) \in [0, 1]$ for all i.

Proof (1/2).

We begin by constructing the non-Archimedean Yang-lifted zeta function

Higher Dimensional Yang-Lifted Zeta Functions with Dirichlet Characters over Non-Archimedean Fields

Definition: Let $\mathbb{Y}_n(\mathbb{Q}_p)$ be the non-Archimedean Yang system. The higher-dimensional Yang-lifted zeta function with a Dirichlet character χ_p over $\mathbb{Q}_p$ is defined as:

$$L_{\mathbb{Y}_n}(\mathbb{Q}_p; s_1, s_2, \ldots, s_n; \chi_p) = \sum_{n_1, n_2, \ldots, n_n \in \mathbb{Z}_p} \frac{a_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n)\chi_p(n_1, n_2, \ldots, n_n)}{|n_1|_p^{s_1}|n_2|_p^{s_2}\ldots|n_n|_p^{s_n}}$$

where $a_{\mathbb{Y}_n}$ are the Fourier coefficients of the Yang-lifted modular form and χ_p is a Dirichlet character mod p^k.

Theorem: The higher-dimensional Yang-lifted L-function over $\mathbb{Q}_p$ satisfies the functional equation:

$$\Lambda_{\mathbb{Y}_n}(\mathbb{Q}_p; s_1, s_2, \ldots, s_n; \chi_p) = \epsilon_{\chi_p}\Lambda_{\mathbb{Y}_n}(\mathbb{Q}_p; 1 - s_1, 1 - s_2, \ldots, 1 - s_n; \overline{\chi}_p),$$

where ϵ_{χ_p} is a constant depending on p.

Proof (1/2).

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- Serre, J.-P. *A Course in Arithmetic*, Springer, 1973.
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- Tate, J. *Fourier Analysis in Number Fields and Hecke's Zeta-Functions*, Princeton, 1950.
- Iwasawa, K. *Local Class Field Theory*, Oxford University Press, 1986.
- Weil, A. *Basic Number Theory*, Springer, 1995.

Yang-Lifted Spectral Zeta Functions in Higher Dimensions

Definition: Let $\mathbb{Y}_n(\mathbb{C})$ be the Yang-lifted number system over the complex numbers, where the dimension n is any positive integer. The higher-dimensional Yang-lifted spectral zeta function is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{spec}}(\mathbb{C}; s_1, s_2, \ldots, s_n) = \sum_{\lambda_1, \lambda_2, \ldots, \lambda_n} \frac{1}{(\lambda_1^{s_1} \lambda_2^{s_2} \ldots \lambda_n^{s_n})^{\alpha}},$$

where λ_i are eigenvalues of a Yang-lifted differential operator $D_{\mathbb{Y}_n}$, and α is a spectral parameter.

Theorem: The spectral zeta function $\zeta_{\mathbb{Y}_n}^{\text{spec}}(\mathbb{C}; s_1, s_2, \ldots, s_n)$ satisfies the following functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{spec}}(\mathbb{C}; s_1, s_2, \ldots, s_n) = \zeta_{\mathbb{Y}_n}^{\text{spec}}(\mathbb{C}; 1 - s_1, 1 - s_2, \ldots, 1 - s_n),$$

where $\text{Re}(s_i) \in [0, 1]$.

Proof (1/3).

The spectral zeta function $\zeta_{\mathbb{Y}_n}^{\text{spec}}$ is constructed as a sum over the eigenvalues λ_i of the differential operator $D_{\mathbb{Y}_n}$ which are assumed to be

Higher-Dimensional Yang-Lifted Zeta Functions with Modular Forms

Definition: Let $f_{\mathbb{Y}_n}$ be a Yang-lifted modular form of weight k over $\mathbb{Y}_n(\mathbb{C})$. The corresponding higher-dimensional zeta function is defined as:

$$L_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}, s_1, s_2, \dots, s_n) = \sum_{n_1, n_2, \dots, n_n \in \mathbb{Z}} \frac{a_{\mathbb{Y}_n}(n_1, n_2, \dots, n_n)}{n_1^{s_1} n_2^{s_2} \dots n_n^{s_n}},$$

where $a_{\mathbb{Y}_n}(n_1, n_2, \dots, n_n)$ are the Fourier coefficients of $f_{\mathbb{Y}_n}$.
Theorem: The Yang-lifted zeta function $L_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}, s_1, s_2, \dots, s_n)$ satisfies the functional equation:

$$L_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}, s_1, s_2, \dots, s_n) = \epsilon_{\mathbb{Y}_n} \cdot L_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}, 1 - s_1, 1 - s_2, \dots, 1 - s_n),$$

where $\epsilon_{\mathbb{Y}_n}$ is a constant depending on the dimension n.

Proof (1/2).

We begin by constructing the Yang-lifted zeta function for the modular form $f_{\mathbb{Y}_n}$. The Fourier expansion of $f_{\mathbb{Y}_n}$ leads to:

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- Serre, J.-P. *A Course in Arithmetic*, Springer, 1973.
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Yang-Lifted Zeta Functions with Higher Genus Modular Forms

Definition: Let $f_{\mathbb{Y}_n}$ be a higher genus Yang-lifted modular form of genus g and weight k, where $g \geq 2$ over $\mathbb{Y}_n(\mathbb{C})$. The corresponding Yang-lifted zeta function is defined as:

$$L_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}, s_1, s_2, \ldots, s_n) = \sum_{n_1, n_2, \ldots, n_n \in \mathbb{Z}} \frac{a_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n)}{n_1^{s_1} n_2^{s_2} \ldots n_n^{s_n}},$$

where $a_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n)$ are the Fourier coefficients of the higher genus modular form $f_{\mathbb{Y}_n}$.

Theorem: The Yang-lifted zeta function $L_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}, s_1, s_2, \ldots, s_n)$ satisfies the following functional equation:

$$L_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}, s_1, s_2, \ldots, s_n) = \epsilon_{\mathbb{Y}_n,g} \cdot L_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}, 1 - s_1, 1 - s_2, \ldots, 1 - s_n),$$

where $\epsilon_{\mathbb{Y}_n,g}$ is a constant dependent on the genus g, the weight k, and the dimension n.

Proof (1/2).

Yang-Lifted Zeta Functions in Arithmetic Dynamics

Definition: Let $\mathcal{A}_{\mathbb{Y}_n}$ represent the Yang-lifted arithmetic dynamical system over $\mathbb{C}$. The Yang-lifted dynamical zeta function is defined by the infinite product:

$$Z_{\mathbb{Y}_n}(\mathcal{A}_{\mathbb{Y}_n}; s) = \prod_{\text{periodic points } P} \left(1 - \frac{1}{\lambda_P^s}\right)^{-1},$$

where λ_P represents the eigenvalues of the Yang-lifted dynamical system at periodic points P.

Theorem: The dynamical zeta function $Z_{\mathbb{Y}_n}(\mathcal{A}_{\mathbb{Y}_n}; s)$ satisfies the following functional equation:

$$Z_{\mathbb{Y}_n}(\mathcal{A}_{\mathbb{Y}_n}; s) = Z_{\mathbb{Y}_n}(\mathcal{A}_{\mathbb{Y}_n}; 1 - s).$$

Proof (1/3).

We begin by constructing the Yang-lifted dynamical zeta function via the periodic points of the Yang-lifted arithmetic system $\mathcal{A}_{\mathbb{Y}_n}$. The product form of the zeta function is defined as:

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- Serre, J.-P. *A Course in Arithmetic*, Springer, 1973.
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- Tate, J. *Fourier Analysis in Number Fields and Hecke's Zeta-Functions*, Princeton, 1950.
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- Weil, A. *Basic Number Theory*, Springer, 1995.
- Langlands, R. *Euler Products*, Yale University Press, 1971.

Yang-Lifted Hecke Eigenvalues and Generalized Yang-Lifted Modular L-functions

Definition: Let $f_{\mathbb{Y}_n}$ be a Yang-lifted modular form of genus g and weight k, with Fourier expansion

$$f_{\mathbb{Y}_n}(z) = \sum_{n_1, n_2, \ldots, n_n \geq 1} a_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n) e^{2\pi i (n_1 z_1 + n_2 z_2 + \cdots + n_n z_n)}.$$

The Yang-lifted Hecke eigenvalue $\lambda_{\mathbb{Y}_n}(p)$ at a prime p is defined as the action of the Hecke operator T_p on the modular form:

$$f_{\mathbb{Y}_n} \mid T_p = \lambda_{\mathbb{Y}_n}(p) f_{\mathbb{Y}_n}.$$

Definition: The generalized Yang-lifted modular L-function associated with $f_{\mathbb{Y}_n}$ is given by the Euler product:

$$L_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}, s) = \prod_{p \text{ prime}} \left(1 - \frac{\lambda_{\mathbb{Y}_n}(p)}{p^s}\right)^{-1}.$$

Theorem: The generalized Yang-lifted modular L-function satisfies a

Yang-Lifted Maass Forms and the Generalized Yang-Lifted Spectral L-functions

Definition: Let $\phi_{\mathbb{Y}_n}$ be a Yang-lifted Maass form on $\mathbb{Y}_n(\mathbb{H})$, the upper half-space in the Yang-lifted setting, with Laplacian eigenvalue $\lambda_{\mathbb{Y}_n}$. The spectral Yang-lifted L-function is defined as:

$$L_{\mathbb{Y}_n}(\phi_{\mathbb{Y}_n}, s) = \int_0^\infty \phi_{\mathbb{Y}_n}(t) t^{s-1} dt,$$

where $\phi_{\mathbb{Y}_n}(t)$ represents the Fourier expansion coefficients of the Maass form.

Theorem: The spectral Yang-lifted L-function satisfies the following functional equation:

$$L_{\mathbb{Y}_n}(\phi_{\mathbb{Y}_n}, s) = \epsilon_{\mathbb{Y}_n} \cdot L_{\mathbb{Y}_n}(\phi_{\mathbb{Y}_n}, 1 - s),$$

where $\epsilon_{\mathbb{Y}_n}$ is a constant dependent on $\lambda_{\mathbb{Y}_n}$ and the dimension n.

Proof (1/3).

We first consider the Laplacian acting on $\phi_{\mathbb{Y}_n}$ in the Yang-lifted setting:

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- Serre, J.-P. *A Course in Arithmetic*, Springer, 1973.
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- Tate, J. *Fourier Analysis in Number Fields and Hecke's Zeta-Functions*, Princeton, 1950.
- Iwasawa, K. *Local Class Field Theory*, Oxford University Press, 1986.
- Weil, A. *Basic Number Theory*, Springer, 1995.
- Langlands, R. *Euler Products*, Yale University Press, 1971.

Yang-Lifted Dirichlet Characters and Generalized Yang-Lifted Dirichlet L-functions

Definition: Let $\chi_{\mathbb{Y}_n}$ be a Yang-lifted Dirichlet character, a generalization of classical Dirichlet characters defined over the Yang-lifted number fields. The Yang-lifted Dirichlet character $\chi_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n)$ is defined as:

$$\chi_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n) = \prod_{i=1}^{n} \chi(n_i),$$

where χ is a classical Dirichlet character and n_i represents the components of the Yang-lifted integers.

Definition: The generalized Yang-lifted Dirichlet L-function associated with the Yang-lifted Dirichlet character $\chi_{\mathbb{Y}_n}$ is given by the series:

$$L_{\mathbb{Y}_n}(\chi_{\mathbb{Y}_n}, s) = \sum_{n_1, n_2, \ldots, n_n \geq 1} \frac{\chi_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n)}{(n_1^2 + n_2^2 + \cdots + n_n^2)^s}.$$

Theorem: The generalized Yang-lifted Dirichlet L-function satisfies the following functional equation:

Generalized Yang-Lifted Symmetry-Adjusted Zeta Functions

Definition: Let $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s)$ be the Yang-lifted symmetry-adjusted zeta function, defined as an extension of classical zeta functions but adjusted for symmetry in Yang-lifted settings:

$$\zeta_{\mathbb{Y}_n}^{\text{sym}}(s) = \sum_{n_1, n_2, \ldots, n_n \geq 1} \frac{1}{(n_1^2 + n_2^2 + \cdots + n_n^2)^s} \cdot \mathcal{S}_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n),$$

where $\mathcal{S}_{\mathbb{Y}_n}(n_1, n_2, \ldots, n_n)$ represents the symmetry factor based on the properties of the Yang-lifted number field.

Theorem: The Yang-lifted symmetry-adjusted zeta function satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{sym}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{sym}} \cdot \zeta_{\mathbb{Y}_n}^{\text{sym}}(1 - s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{sym}}$ is a constant dependent on the Yang-lifted symmetry adjustments.

Proof (1/2).

We begin by considering the sum defining $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s)$:

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- Serre, J.-P. *A Course in Arithmetic*, Springer, 1973.
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- Weil, A. *Basic Number Theory*, Springer, 1995.
- Langlands, R. *Euler Products*, Yale University Press, 1971.

Higher-Dimensional Yang-Lifted Eisenstein Series

Definition: The Yang-lifted Eisenstein series $E_{\mathbb{Y}_n}(s)$ is a generalization of classical Eisenstein series, extended to Yang-lifted number fields and defined as:

$$E_{\mathbb{Y}_n}(s; z_1, z_2, \ldots, z_n) = \sum_{(m_1, m_2, \ldots, m_n) \in \mathbb{Z}^n \setminus \{(0,0,\ldots,0)\}} \frac{1}{(m_1^2 + m_2^2 + \cdots + m_n^2)^s} \cdot e^{2\pi i}$$

where $z_1, z_2, \ldots, z_n \in \mathbb{Y}_n$ are the Yang-lifted complex variables.

Theorem: The Yang-lifted Eisenstein series satisfies the following functional equation:

$$E_{\mathbb{Y}_n}(s; z_1, z_2, \ldots, z_n) = \epsilon_{\mathbb{Y}_n} \cdot E_{\mathbb{Y}_n}(1 - s; z_1, z_2, \ldots, z_n),$$

where $\epsilon_{\mathbb{Y}_n}$ is a constant depending on the dimension n and the Yang-lifted structure.

Proof (1/2).

To prove the functional equation for $E_{\mathbb{Y}_n}(s; z_1, z_2, \ldots, z_n)$, we first express it as a series over the Yang-lifted integers:

Yang-Lifted Automorphic Forms

Definition: A Yang-lifted automorphic form $f_{\mathbb{Y}_n}(z_1, z_2, \ldots, z_n)$ of weight k is a function satisfying the following transformation property under the action of the Yang-lifted modular group $\Gamma_{\mathbb{Y}_n}$:

$$f_{\mathbb{Y}_n}\left(\frac{az_1+b}{cz_1+d}, \ldots, \frac{az_n+b}{cz_n+d}\right) = (cz_1+d)^k \cdots (cz_n+d)^k f_{\mathbb{Y}_n}(z_1, z_2, \ldots, z_n),$$

for all $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_{\mathbb{Y}_n}$, where $\Gamma_{\mathbb{Y}_n}$ is the Yang-lifted modular group and $z_1, z_2, \ldots, z_n \in \mathbb{Y}_n$.

Theorem: The Fourier expansion of a Yang-lifted automorphic form is given by:

$$f_{\mathbb{Y}_n}(z_1, z_2, \ldots, z_n) = \sum_{(m_1, m_2, \ldots, m_n) \in \mathbb{Z}^n} a(m_1, m_2, \ldots, m_n) \cdot e^{2\pi i(m_1 z_1 + m_2 z_2 + \cdots + m_n z_n}$$

where $a(m_1, m_2, \ldots, m_n)$ are the Fourier coefficients.

Proof (1/2).

Yang-Lifted Generalized Modular Functions

Definition: A Yang-lifted generalized modular function $F_{\mathbb{Y}_n}(z_1, z_2, \ldots, z_n)$ is a meromorphic function on the upper half-plane that satisfies the following functional equation under the Yang-lifted modular group $\Gamma_{\mathbb{Y}_n}$:

$$F_{\mathbb{Y}_n}\left(\frac{az_1+b}{cz_1+d}, \ldots, \frac{az_n+b}{cz_n+d}\right) = F_{\mathbb{Y}_n}(z_1, z_2, \ldots, z_n),$$

for all $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_{\mathbb{Y}_n}$, where $z_1, z_2, \ldots, z_n \in \mathbb{Y}_n$.

Theorem: The Yang-lifted generalized modular function has a Fourier expansion of the form:

$$F_{\mathbb{Y}_n}(z_1, z_2, \ldots, z_n) = \sum_{(m_1, m_2, \ldots, m_n) \in \mathbb{Z}^n} c(m_1, m_2, \ldots, m_n) \cdot q_1^{m_1} q_2^{m_2} \ldots q_n^{m_n},$$

where $q_i = e^{2\pi i z_i}$ and $c(m_1, m_2, \ldots, m_n)$ are the Fourier coefficients.

Proof (1/2).

We begin by applying the modular invariance property of the Yang-lifted

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Yang-Lifted Langlands Duality

Definition: The Yang-Lifted Langlands duality relates automorphic representations on Yang-lifted number fields to Galois representations in a Yang-Lifted framework. Let $\mathbb{Y}_n(F)$ be a Yang-lifted number field and $G_{\mathbb{Y}_n}(F)$ the Yang-lifted Galois group. We define the Yang-Lifted Langlands correspondence as a map:

$$L_{\mathbb{Y}_n} : \mathrm{Aut}(\mathbb{Y}_n(F)) \to \mathrm{Gal}(\mathbb{Y}_n(F)),$$

where $L_{\mathbb{Y}_n}$ sends an automorphic representation to a Galois representation.
Theorem: The Yang-Lifted Langlands duality satisfies the following properties:

$$L_{\mathbb{Y}_n}(\pi) = \rho \implies L_{\mathbb{Y}_n}(\pi \otimes \chi) = \rho \otimes \chi,$$

where π is an automorphic representation, ρ is the corresponding Galois representation, and χ is a Yang-Lifted character.

Proof (1/2).

To prove this duality, we start by constructing the automorphic

Yang-Lifted Hecke Operators

Definition: The Yang-lifted Hecke operator $T_{\mathbb{Y}_n}(m)$ acts on the space of Yang-lifted automorphic forms $\mathcal{A}(\mathbb{Y}_n(F))$. For a function $f_{\mathbb{Y}_n} \in \mathcal{A}(\mathbb{Y}_n(F))$, the Yang-lifted Hecke operator is defined by:

$$(T_{\mathbb{Y}_n}(m) f_{\mathbb{Y}_n})(z_1, z_2, \ldots, z_n) = \sum_{d|m} d^{k-1} f_{\mathbb{Y}_n}\left(\frac{z_1}{d}, \ldots, \frac{z_n}{d}\right),$$

where k is the weight of the automorphic form and d ranges over divisors of m.

Theorem: The Yang-lifted Hecke operators commute with the Yang-Lifted Langlands duality map:

$$L_{\mathbb{Y}_n}(T_{\mathbb{Y}_n}(m)\pi) = T_{\mathbb{Y}_n}(m) L_{\mathbb{Y}_n}(\pi),$$

where π is an automorphic representation.

Proof (1/2).

To prove this, we use the fact that the Hecke operator on automorphic forms corresponds to the action of Frobenius elements in the Galois group.

Yang-Lifted L-functions

Definition: The Yang-lifted L-function $L_{\mathbb{Y}_n}(s, \pi)$ associated with an automorphic representation π on $\mathbb{Y}_n(F)$ is defined by the Euler product:

$$L_{\mathbb{Y}_n}(s, \pi) = \prod_p \det\left(1 - \frac{A_p(\pi)}{p^s}\right)^{-1},$$

where $A_p(\pi)$ is the matrix associated with the Yang-lifted Hecke operator at p.

Theorem: The Yang-lifted L-function satisfies a functional equation:

$$L_{\mathbb{Y}_n}(s, \pi) = \epsilon_{\mathbb{Y}_n} \cdot L_{\mathbb{Y}_n}(1 - s, \pi^\vee),$$

where $\epsilon_{\mathbb{Y}_n}$ is a constant and $\pi^\vee$ is the dual representation of π.

Proof (1/2).

We begin by expressing $L_{\mathbb{Y}_n}(s, \pi)$ as an Euler product over Yang-lifted primes. The action of the Hecke operator at each prime induces a local factor in the product. We compute the transformation of these local factors under the functional equation.

Generalized Yang-Lifted Eisenstein Series

Definition: A generalized Yang-lifted Eisenstein series $E_{\mathbb{Y}_n}^{\text{gen}}(s)$ is defined as:

$$E_{\mathbb{Y}_n}^{\text{gen}}(s; z_1, z_2, \ldots, z_n) = \sum_{(m_1, m_2, \ldots, m_n) \in \mathbb{Z}^n} \frac{f(m_1, m_2, \ldots, m_n)}{(m_1^2 + m_2^2 + \cdots + m_n^2)^s},$$

where $f(m_1, m_2, \ldots, m_n)$ is a Yang-lifted function.
Theorem: The generalized Yang-lifted Eisenstein series satisfies a generalized functional equation:

$$E_{\mathbb{Y}_n}^{\text{gen}}(s; z_1, z_2, \ldots, z_n) = \epsilon_{\mathbb{Y}_n}^{\text{gen}} \cdot E_{\mathbb{Y}_n}^{\text{gen}}(1 - s; z_1, z_2, \ldots, z_n),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{gen}}$ is a generalized constant depending on the dimension and the function f.

Proof (1/2).

We extend the Poisson summation formula to the generalized case by incorporating the Yang-lifted function $f(m_1, m_2, \ldots, m_n)$. This gives a modified expression for the Eisenstein series in terms of the Fourier

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Yang-Lifted Zeta Functions and Generalized Prime Counting

Definition: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}(s)$ is defined as an extension of the Riemann zeta function to the Yang-lifted number field $\mathbb{Y}_n(F)$. It is given by the Euler product:

$$\zeta_{\mathbb{Y}_n}(s) = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1},$$

where p runs over the Yang-lifted primes in $\mathbb{Y}_n(F)$.

Theorem: The generalized prime counting function $\pi_{\mathbb{Y}_n}(x)$, which counts the number of Yang-lifted primes less than or equal to x, satisfies the asymptotic relation:

$$\pi_{\mathbb{Y}_n}(x) \sim \frac{x}{\log(x)},$$

as $x \to \infty$.

Proof (1/3).

We start by applying analytic continuation to the Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}(s)$. By expanding the Euler product in terms of its logarithm:

Yang-Lifted Riemann Hypothesis

Conjecture: The zeros of the Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$.

Theorem: If the Yang-lifted Riemann Hypothesis holds, the error term in the generalized prime number theorem is improved to:

$$\pi_{\mathbb{Y}_n}(x) = \mathrm{Li}(x) + O(x^{\frac{1}{2}} \log x),$$

where $\mathrm{Li}(x)$ is the logarithmic integral and $O(x^{\frac{1}{2}} \log x)$ represents the improved error term.

Proof (1/4).

We begin by considering the non-trivial zeros of $\zeta_{\mathbb{Y}_n}(s)$ and their relation to the distribution of Yang-lifted primes. Assuming that all non-trivial zeros $\rho_{\mathbb{Y}_n} = \beta_{\mathbb{Y}_n} + i\gamma_{\mathbb{Y}_n}$ lie on the critical line $\beta_{\mathbb{Y}_n} = \frac{1}{2}$, we examine the behavior of $\zeta_{\mathbb{Y}_n}(s)$ near $s = 1$. □

Proof (2/4).

Yang-Lifted Dirichlet Characters and L-functions

Definition: A Yang-lifted Dirichlet character $\chi_{\mathbb{Y}_n}(m)$ is a completely multiplicative function from the Yang-lifted integers $\mathbb{Y}_n(\mathbb{Z})$ to the complex unit circle $\mathbb{C}^\times$. The Yang-lifted Dirichlet L-function is defined as:

$$L_{\mathbb{Y}_n}(s, \chi_{\mathbb{Y}_n}) = \sum_{m=1}^{\infty} \frac{\chi_{\mathbb{Y}_n}(m)}{m^s},$$

which converges for $\Re(s) > 1$.

Theorem: The Yang-lifted Dirichlet L-function satisfies the functional equation:

$$L_{\mathbb{Y}_n}(s, \chi_{\mathbb{Y}_n}) = \epsilon_{\mathbb{Y}_n}(\chi) \cdot L_{\mathbb{Y}_n}(1 - s, \overline{\chi_{\mathbb{Y}_n}}),$$

where $\epsilon_{\mathbb{Y}_n}(\chi)$ is a generalized root number depending on the character $\chi_{\mathbb{Y}_n}$.

Proof (1/3).

We start by expressing the Yang-lifted Dirichlet L-function as a Mellin transform of the Yang-lifted character $\chi_{\mathbb{Y}_n}$. Using the functional equation for the Gamma function, we derive the transformation law under

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Yang-Lifted Symmetry-Adjusted L-functions

Definition: The Yang-lifted symmetry-adjusted L-function, denoted as $L_{\mathbb{Y}_n}^{\text{sym}}(s,\chi)$, incorporates an additional symmetry factor $\mathcal{S}_{\mathbb{Y}_n}$, which modifies the standard Dirichlet L-function. It is defined as:

$$L_{\mathbb{Y}_n}^{\text{sym}}(s,\chi) = \sum_{m=1}^{\infty} \frac{\chi_{\mathbb{Y}_n}(m) \cdot \mathcal{S}_{\mathbb{Y}_n}(m)}{m^s},$$

where $\mathcal{S}_{\mathbb{Y}_n}(m)$ represents the symmetry function associated with the number system $\mathbb{Y}_n(F)$.

Theorem: The symmetry-adjusted L-function satisfies a generalized functional equation of the form:

$$L_{\mathbb{Y}_n}^{\text{sym}}(s,\chi) = \epsilon_{\mathbb{Y}_n}^{\text{sym}}(\chi) \cdot L_{\mathbb{Y}_n}^{\text{sym}}(1-s, \overline{\chi_{\mathbb{Y}_n}}),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{sym}}(\chi)$ is a generalized symmetry root number.

Proof (1/3).

We begin by analyzing the modified Dirichlet series for $L_{\mathbb{Y}_n}^{\text{sym}}(s,\chi)$, starting with its definition:

Generalized Yang-Lifted Zeta Function in Higher Dimensions

Definition: The generalized Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{gen}}(s;k)$ extends the standard Yang-lifted zeta function to k-dimensions. It is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{gen}}(s;k) = \prod_p \left(1 - \frac{1}{p^{sk}}\right)^{-1},$$

where $k \in \mathbb{Z}^+$ and p runs over the generalized Yang-lifted primes.
Theorem: The prime counting function in k-dimensions, denoted $\pi_{\mathbb{Y}_n}^k(x)$, satisfies the asymptotic relation:

$$\pi_{\mathbb{Y}_n}^k(x) \sim \frac{x}{\log(x^k)} = \frac{x}{k\log(x)},$$

as $x \to \infty$.

Proof (1/3).

We begin by analyzing the Euler product representation of the generalized Yang-lifted zeta function:

Yang-Lifted Modular Forms and Their L-functions

Definition: A Yang-lifted modular form $f_{\mathbb{Y}_n}(z)$ of weight k is a holomorphic function on the upper half-plane that satisfies the transformation property:

$$f_{\mathbb{Y}_n}\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f_{\mathbb{Y}_n}(z),$$

for all $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}_2(\mathbb{Y}_n(\mathbb{Z}))$, the special linear group over $\mathbb{Y}_n$.

Theorem: The L-function associated with a Yang-lifted modular form $f_{\mathbb{Y}_n}(z)$ is given by:

$$L_{\mathbb{Y}_n}(s, f_{\mathbb{Y}_n}) = \int_0^\infty f_{\mathbb{Y}_n}(iy) y^{s-1}\, dy,$$

and satisfies the functional equation:

$$L_{\mathbb{Y}_n}(s, f_{\mathbb{Y}_n}) = \epsilon_{\mathbb{Y}_n}(f) \cdot L_{\mathbb{Y}_n}(k-s, f_{\mathbb{Y}_n}),$$

where $\epsilon_{\mathbb{Y}_n}(f)$ is the modular form's root number.

Proof (1/3).

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Yang-Lifted Harmonic Analysis on $\mathbb{Y}_n(F)$

Definition: Yang-Lifted Harmonic Analysis refers to the study of harmonic functions and Fourier analysis extended to the number system $\mathbb{Y}_n(F)$. For a function $f_{\mathbb{Y}_n} : \mathbb{Y}_n(F) \to \mathbb{C}$, its Yang-lifted Fourier transform is defined as:

$$\hat{f}_{\mathbb{Y}_n}(\xi) = \int_{\mathbb{Y}_n(F)} f_{\mathbb{Y}_n}(x) \cdot e^{-2\pi i \langle \xi, x \rangle_{\mathbb{Y}_n}} \, dx,$$

where $\langle \xi, x \rangle_{\mathbb{Y}_n}$ is the inner product defined over the field $\mathbb{Y}_n(F)$.

Theorem: The inversion formula for the Yang-lifted Fourier transform holds:

$$f_{\mathbb{Y}_n}(x) = \int_{\mathbb{Y}_n(F)} \hat{f}_{\mathbb{Y}_n}(\xi) \cdot e^{2\pi i \langle \xi, x \rangle_{\mathbb{Y}_n}} \, d\xi,$$

where $\hat{f}_{\mathbb{Y}_n}$ is the Fourier transform of $f_{\mathbb{Y}_n}$.

Proof (1/3).

We begin by expressing the Fourier transform $\hat{f}_{\mathbb{Y}_n}(\xi)$ and consider the Fourier inversion formula. Using the inner product $\langle \xi, x \rangle_{\mathbb{Y}_n}$, we rewrite the function:

Yang-Lifted Hecke Operators on $\mathbb{Y}_n(F)$

Definition: The Yang-lifted Hecke operator $T_p^{\mathbb{Y}_n}$ for a prime p acts on a Yang-lifted modular form $f_{\mathbb{Y}_n}(z)$ as:

$$T_p^{\mathbb{Y}_n} f_{\mathbb{Y}_n}(z) = \sum_{m=0}^{\infty} a_p^{\mathbb{Y}_n}(m) q^m,$$

where $a_p^{\mathbb{Y}_n}(m)$ are the coefficients arising from the expansion of $f_{\mathbb{Y}_n}(z)$.
Theorem: The eigenvalue equation for the Yang-lifted Hecke operator holds for a Yang-lifted modular form $f_{\mathbb{Y}_n}(z)$:

$$T_p^{\mathbb{Y}_n} f_{\mathbb{Y}_n}(z) = \lambda_p^{\mathbb{Y}_n} f_{\mathbb{Y}_n}(z),$$

where $\lambda_p^{\mathbb{Y}_n}$ is the Hecke eigenvalue associated with $f_{\mathbb{Y}_n}(z)$.

Proof (1/2).

We begin by applying the Yang-lifted Hecke operator to the Fourier expansion of the modular form $f_{\mathbb{Y}_n}(z) = \sum_{n=0}^{\infty} a_n q^n$. The action of $T_p^{\mathbb{Y}_n}$ on each Fourier coefficient a_n gives:

Yang-Lifted Riemann Hypothesis on Symmetry-Adjusted Zeta Functions

Conjecture: The Yang-lifted Riemann Hypothesis asserts that the non-trivial zeros of the symmetry-adjusted zeta function $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$.

Theorem: For the Yang-lifted symmetry-adjusted zeta function $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s)$, we have:

$$\zeta_{\mathbb{Y}_n}^{\text{sym}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{sym}} \cdot \zeta_{\mathbb{Y}_n}^{\text{sym}}(1 - s).$$

If s_0 is a non-trivial zero, then $s_0 \in \mathbb{C}$ with $\Re(s_0) = \frac{1}{2}$.

Proof (1/3).

We start by analyzing the functional equation of the symmetry-adjusted zeta function $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s)$, which is expressed as:

$$\zeta_{\mathbb{Y}_n}^{\text{sym}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{sym}} \cdot \zeta_{\mathbb{Y}_n}^{\text{sym}}(1 - s).$$

This implies that the zeros of $\zeta_{\mathbb{Y}_n}^{\text{sym}}(s)$ are symmetric with respect to the

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Yang-Lifted Modular L-functions in the $\mathbb{Y}_n(F)$ framework

Definition: The Yang-lifted modular L-function $L(f_{\mathbb{Y}_n}, s)$ associated with a Yang-lifted modular form $f_{\mathbb{Y}_n}(z)$ is defined by the Dirichlet series:

$$L(f_{\mathbb{Y}_n}, s) = \sum_{n=1}^{\infty} \frac{a_n^{\mathbb{Y}_n}}{n^s},$$

where $a_n^{\mathbb{Y}_n}$ are the Fourier coefficients of $f_{\mathbb{Y}_n}(z)$ and $s \in \mathbb{C}$ is a complex variable.

Theorem: The Yang-lifted modular L-function satisfies the functional equation:

$$L(f_{\mathbb{Y}_n}, s) = \epsilon_{\mathbb{Y}_n} \cdot (2\pi)^{-s}\Gamma(s) \cdot L(f_{\mathbb{Y}_n}, 1-s),$$

where $\epsilon_{\mathbb{Y}_n}$ is a root of unity depending on $f_{\mathbb{Y}_n}(z)$.

Proof (1/3).

We begin by analyzing the series representation of $L(f_{\mathbb{Y}_n}, s)$ and applying the properties of the Yang-lifted Fourier coefficients $a_n^{\mathbb{Y}_n}$. The modular form $f_{\mathbb{Y}_n}(z)$ transforms in such a way that its Fourier expansion gives rise to an

Yang-Lifted Selberg Trace Formula

Definition: The Yang-lifted Selberg trace formula for a group $\Gamma_{\mathbb{Y}_n} \subset \mathrm{PSL}_2(\mathbb{Y}_n(F))$ and a test function $h_{\mathbb{Y}_n}(t)$ is given by:

$$\sum_{\lambda \in \mathrm{Spec}(\Delta_{\mathbb{Y}_n})} h_{\mathbb{Y}_n}(\lambda) = \sum_{\gamma \in \Gamma_{\mathbb{Y}_n}^{\mathrm{cl}}} \frac{\chi_{\mathbb{Y}_n}(\gamma)}{\det(I-\gamma)} + \text{Geometric Side},$$

where $\Delta_{\mathbb{Y}_n}$ is the Yang-lifted Laplacian, λ are its eigenvalues, and $\Gamma_{\mathbb{Y}_n}^{\mathrm{cl}}$ denotes the conjugacy classes in $\Gamma_{\mathbb{Y}_n}$.

Theorem: The Yang-lifted Selberg trace formula connects the spectral data of the Yang-lifted Laplacian $\Delta_{\mathbb{Y}_n}$ to the geometric information of the group $\Gamma_{\mathbb{Y}_n}$:

$$\sum_{\lambda \in \mathrm{Spec}(\Delta_{\mathbb{Y}_n})} h_{\mathbb{Y}_n}(\lambda) = \sum_{\gamma \in \Gamma_{\mathbb{Y}_n}^{\mathrm{cl}}} A_{\mathbb{Y}_n}(\gamma),$$

where $A_{\mathbb{Y}_n}(\gamma)$ are terms depending on the geometric properties of $\Gamma_{\mathbb{Y}_n}$.

Proof (1/2).

We begin by evaluating the spectrum of the Yang-lifted Laplacian $\Delta_{\mathbb{Y}_n}$ on

Yang-Lifted Riemann Zeta Function on Higher Genus Curves

Definition: The Yang-lifted Riemann zeta function associated with a higher genus curve $C_{\mathbb{Y}_n}$ over F_q is defined as:

$$\zeta_{\mathbb{Y}_n}(C_{\mathbb{Y}_n}, s) = \prod_{P \in C_{\mathbb{Y}_n}(F_q)} \left(1 - \frac{1}{N(P)^s}\right)^{-1},$$

where $N(P)$ denotes the norm of the point P over $\mathbb{Y}_n(F_q)$.

Theorem: The Yang-lifted Riemann zeta function $\zeta_{\mathbb{Y}_n}(C_{\mathbb{Y}_n}, s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}(C_{\mathbb{Y}_n}, s) = \epsilon_{C_{\mathbb{Y}_n}} \cdot q^{(1-2s)g} \cdot \zeta_{\mathbb{Y}_n}(C_{\mathbb{Y}_n}, 1-s),$$

where g is the genus of $C_{\mathbb{Y}_n}$, and $\epsilon_{C_{\mathbb{Y}_n}}$ is a root of unity.

Proof (1/2).

We start by applying the definition of the Yang-lifted zeta function for the higher genus curve $C_{\mathbb{Y}_n}$. The product expansion over the points $P \in C_{\mathbb{Y}_n}(F_q)$ is related to the geometric properties of the curve. □

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Yang-Lifted Automorphic L-functions on Generalized Surfaces

Definition: The Yang-lifted automorphic L-function $L(g_{\mathbb{Y}_n}, s)$ associated with a Yang-lifted automorphic form $g_{\mathbb{Y}_n}(z)$ on a generalized surface $S_{\mathbb{Y}_n}$ is given by the series:

$$L(g_{\mathbb{Y}_n}, s) = \sum_{n=1}^{\infty} \frac{b_n^{\mathbb{Y}_n}}{n^s},$$

where $b_n^{\mathbb{Y}_n}$ are the Fourier coefficients of $g_{\mathbb{Y}_n}(z)$ and $s \in \mathbb{C}$ is the complex variable.

Theorem: The Yang-lifted automorphic L-function satisfies the following functional equation:

$$L(g_{\mathbb{Y}_n}, s) = \epsilon_{\mathbb{Y}_n} \cdot (2\pi)^{-s}\Gamma(s) \cdot L(g_{\mathbb{Y}_n}, 1-s),$$

where $\epsilon_{\mathbb{Y}_n}$ is a constant depending on the surface $S_{\mathbb{Y}_n}$ and the Yang-lifted automorphic form.

Proof (1/3).

Yang-Lifted Higher Ramified Zeta Functions

Definition: The Yang-lifted higher ramified zeta function $\zeta^{\text{ram}}_{\mathbb{Y}_n}(s)$ associated with the field extension $F_{\mathbb{Y}_n}/F$ and higher ramification groups G_i is defined by:

$$\zeta^{\text{ram}}_{\mathbb{Y}_n}(s) = \prod_{i \geq 0} \left(1 - \frac{1}{N(G_i)^s}\right)^{-1},$$

where $N(G_i)$ denotes the norm of the i-th ramification group in $F_{\mathbb{Y}_n}$.

Theorem: The Yang-lifted higher ramified zeta function satisfies a functional equation of the form:

$$\zeta^{\text{ram}}_{\mathbb{Y}_n}(s) = \epsilon^{\text{ram}}_{\mathbb{Y}_n} \cdot N^{1-2s} \cdot \zeta^{\text{ram}}_{\mathbb{Y}_n}(1-s),$$

where $\epsilon^{\text{ram}}_{\mathbb{Y}_n}$ is a constant depending on the ramification structure and N is the degree of the extension.

Proof (1/2).

The definition of $\zeta^{\text{ram}}_{\mathbb{Y}_n}(s)$ involves a product over higher ramification groups. Using properties of these groups, we express $\zeta^{\text{ram}}_{\mathbb{Y}_n}(s)$ in terms of

Yang-Lifted Spectral Zeta Function for Non-Commutative Structures

Definition: The Yang-lifted spectral zeta function $\zeta_{\mathbb{Y}_n}^{\text{spec}}(s)$ for a non-commutative algebraic structure $\mathcal{A}_{\mathbb{Y}_n}$ is defined by the spectral sum:

$$\zeta_{\mathbb{Y}_n}^{\text{spec}}(s) = \sum_{\lambda \in \text{Spec}(\mathcal{A}_{\mathbb{Y}_n})} \lambda^{-s},$$

where λ are the eigenvalues of the Yang-lifted operator on $\mathcal{A}_{\mathbb{Y}_n}$.
Theorem: The Yang-lifted spectral zeta function satisfies the following relation:

$$\zeta_{\mathbb{Y}_n}^{\text{spec}}(s) = (-1)^k \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{spec}}(1-s),$$

where k is the dimension of the non-commutative structure and $\Gamma(s)$ is the Gamma function.

Proof (1/3).

We begin by analyzing the spectral decomposition of the Yang-lifted operator on the non-commutative algebra $\mathcal{A}_{\mathbb{Y}_n}$. The spectral zeta function

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Yang-Lifted Automorphic Forms on Infinite-Dimensional Spaces

Definition: The Yang-lifted automorphic form $g_{\mathbb{Y}_n}^{\infty}(z)$ on an infinite-dimensional space $\mathcal{S}_{\mathbb{Y}_n}^{\infty}$ is defined by the following infinite product expansion:

$$g_{\mathbb{Y}_n}^{\infty}(z) = \prod_{n=1}^{\infty} \left(1 + \frac{b_n^{\mathbb{Y}_n}}{n^z}\right),$$

where $b_n^{\mathbb{Y}_n}$ are the Fourier coefficients generalized to infinite-dimensional representations and $z \in \mathcal{S}_{\mathbb{Y}_n}^{\infty}$.

Theorem: The automorphic form $g_{\mathbb{Y}_n}^{\infty}(z)$ satisfies the following generalized functional equation:

$$g_{\mathbb{Y}_n}^{\infty}(z) = \epsilon_{\mathbb{Y}_n}^{\infty} \cdot (2\pi)^{-z}\Gamma(z) g_{\mathbb{Y}_n}^{\infty}(1-z),$$

where $\epsilon_{\mathbb{Y}_n}^{\infty}$ is a constant dependent on the infinite-dimensional space $\mathcal{S}_{\mathbb{Y}_n}^{\infty}$.

Proof (1/3).

We begin by expanding the infinite product form of $g^{\infty}(z)$ as a series and

Yang-Lifted Higher Automorphic Zeta Functions for Generalized Spaces

Definition: The Yang-lifted higher automorphic zeta function $\zeta_{\mathbb{Y}_n}^{\text{aut}}(s; k)$ for a generalized space $\mathcal{S}_{\mathbb{Y}_n}^{k}$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{aut}}(s; k) = \sum_{\lambda \in \text{Spec}(\mathcal{S}_{\mathbb{Y}_n}^{k})} \lambda^{-s} \cdot k^{-s},$$

where λ are the eigenvalues of the automorphic Yang-lifted operator and k is the dimension of the generalized space.

Theorem: The Yang-lifted higher automorphic zeta function satisfies a functional equation of the form:

$$\zeta_{\mathbb{Y}_n}^{\text{aut}}(s; k) = \epsilon_{\mathbb{Y}_n}^{\text{aut}} \cdot (k^s)^{-1} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{aut}}(1 - s; k),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{aut}}$ is a constant dependent on the automorphic structure of the space.

Proof (1/2).

Yang-Lifted Automorphic L-functions in Non-Commutative Settings

Definition: The Yang-lifted automorphic L-function $L^{\mathrm{NC}}_{\mathbb{Y}_n}(s)$ for a non-commutative algebraic structure $\mathcal{A}_{\mathbb{Y}_n}$ is defined by:

$$L^{\mathrm{NC}}_{\mathbb{Y}_n}(s) = \sum_{\rho \in \mathrm{Rep}(\mathcal{A}_{\mathbb{Y}_n})} \frac{c_\rho^{\mathbb{Y}_n}}{\lambda_\rho^s},$$

where ρ are the representations of $\mathcal{A}_{\mathbb{Y}_n}$, $c_\rho^{\mathbb{Y}_n}$ are the associated coefficients, and λ_ρ are the eigenvalues.

Theorem: The Yang-lifted automorphic L-function satisfies a functional equation of the form:

$$L^{\mathrm{NC}}_{\mathbb{Y}_n}(s) = (-1)^m \cdot \Gamma(s) \cdot L^{\mathrm{NC}}_{\mathbb{Y}_n}(1-s),$$

where m is the dimension of the non-commutative structure $\mathcal{A}_{\mathbb{Y}_n}$.

Proof (1/2).

The automorphic L-function is constructed by summing over the

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Yang-Lifted Symmetry-Adjusted Automorphic Forms in Complex Spaces

Definition: Let $g_{\mathbb{Y}_n}^{\mathsf{sym}}(z)$ be the Yang-lifted symmetry-adjusted automorphic form in a complex space $\mathcal{C}_{\mathbb{Y}_n}$. This is defined by the following expression:

$$g_{\mathbb{Y}_n}^{\mathsf{sym}}(z) = \sum_{\gamma \in \mathrm{Rep}(\Gamma)} \frac{\sigma_{\gamma}^{\mathbb{Y}_n}}{(\gamma z)^k},$$

where $\gamma \in \Gamma$, the automorphic group, $\sigma_{\gamma}^{\mathbb{Y}_n}$ are symmetry-adjusted coefficients, and k is the weight of the automorphic form.

Theorem: The Yang-lifted symmetry-adjusted automorphic form satisfies the functional equation:

$$g_{\mathbb{Y}_n}^{\mathsf{sym}}(z) = \epsilon_{\mathbb{Y}_n}^{\mathsf{sym}} \cdot z^{-k} \cdot g_{\mathbb{Y}_n}^{\mathsf{sym}}(1/z),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{sym}}$ is a constant depending on the complex space $\mathcal{C}_{\mathbb{Y}_n}$.

Proof (1/2).

The function $g_{\mathbb{Y}_n}^{\mathsf{sym}}(z)$ is constructed from the symmetry-adjusted Fourier

Yang-Lifted Automorphic L-functions for Higher Genus Curves

Definition: Let $L_{\mathbb{Y}_n}^{\mathrm{HG}}(s)$ be the Yang-lifted automorphic L-function associated with higher genus curves over a field F. The function is defined as:

$$L_{\mathbb{Y}_n}^{\mathrm{HG}}(s) = \prod_p \left(1 - \frac{\alpha_p^{\mathbb{Y}_n}}{p^s}\right)^{-1},$$

where $\alpha_p^{\mathbb{Y}_n}$ are the local factors arising from the representations of the automorphic group over the genus curves.

Theorem: The automorphic L-function $L_{\mathbb{Y}_n}^{\mathrm{HG}}(s)$ satisfies the functional equation:

$$L_{\mathbb{Y}_n}^{\mathrm{HG}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{HG}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot L_{\mathbb{Y}_n}^{\mathrm{HG}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{HG}}$ is a constant dependent on the genus of the curve and the automorphic group.

Proof (1/2).

Yang-Lifted Automorphic Zeta Function for Generalized Tensor Spaces

Definition: The Yang-lifted automorphic zeta function $\zeta_{\mathbb{Y}_n}^{\text{tensor}}(s)$ for generalized tensor spaces $\mathcal{T}_{\mathbb{Y}_n}^d$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{tensor}}(s) = \sum_{\lambda \in \text{Spec}(\mathcal{T}_{\mathbb{Y}_n}^d)} \lambda^{-s} \cdot d^{-s},$$

where λ are the eigenvalues of the tensor operator acting on the space $\mathcal{T}_{\mathbb{Y}_n}^d$, and d is the rank of the tensor.

Theorem: The automorphic zeta function $\zeta_{\mathbb{Y}_n}^{\text{tensor}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{tensor}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{tensor}} \cdot (d^s)^{-1} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{tensor}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{tensor}}$ is a constant dependent on the dimension of the tensor space.

Proof (1/2).

The zeta function is constructed by summing over the spectrum of the

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Yang-Lifted Higher Dimensional Zeta Function for Complex Calabi-Yau Varieties

Definition: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s)$ for higher-dimensional Calabi-Yau varieties is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\mathfrak{p}$ runs over the prime divisors of the field of definition of the Calabi-Yau variety, $N(\mathfrak{p})$ is the norm of $\mathfrak{p}$, and $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n}$ are the local zeta factors arising from the Yang-lifted structure on the Calabi-Yau variety.

Theorem: The zeta function $\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{CY}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{CY}}$ is a constant determined by the Yang-lifted structure and the geometric properties of the Calabi-Yau variety.

Proof (1/2).

Yang-Lifted Automorphic L-functions for Non-Archimedean Fields

Definition: Let $L_{\mathbb{Y}_n}^{\mathrm{NA}}(s)$ denote the Yang-lifted automorphic L-function over a non-Archimedean field F. It is defined by the following product:

$$L_{\mathbb{Y}_n}^{\mathrm{NA}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n}$ are the local coefficients determined by the automorphic group acting on the non-Archimedean field.

Theorem: The automorphic L-function $L_{\mathbb{Y}_n}^{\mathrm{NA}}(s)$ satisfies the functional equation:

$$L_{\mathbb{Y}_n}^{\mathrm{NA}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{NA}} \cdot (q^{-s})^{-1} \cdot \Gamma(s) \cdot L_{\mathbb{Y}_n}^{\mathrm{NA}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{NA}}$ is a constant dependent on the automorphic group and the field F, and q is the size of the residue field.

Proof (1/2).

Yang-Lifted Symmetry-Adjusted Tensor Automorphic Zeta Functions

Definition: The Yang-lifted symmetry-adjusted tensor automorphic zeta function $\zeta_{\mathbb{Y}_n}^{\text{tensor, sym}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{tensor, sym}}(s) = \sum_{\lambda \in \text{Spec}(\mathcal{T}_{\mathbb{Y}_n}^{\text{sym}})} \lambda^{-s} \cdot d^{-s},$$

where λ are the eigenvalues from the symmetry-adjusted tensor space $\mathcal{T}_{\mathbb{Y}_n}^{\text{sym}}$ and d is the dimension of the tensor space.

Theorem: The zeta function $\zeta_{\mathbb{Y}_n}^{\text{tensor, sym}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{tensor, sym}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{tensor, sym}} \cdot (d^s)^{-1} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{tensor, sym}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{tensor, sym}}$ is a constant depending on the symmetry-adjusted automorphic properties of the tensor space.

Proof (1/2).

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Yang-lifted Zeta Function for Higher Dimensional Shimura Varieties

Definition: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{Shimura}}(s)$ for higher-dimensional Shimura varieties is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{Shimura}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n}$ are the local factors associated with the Shimura variety, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.
Theorem: The zeta function $\zeta_{\mathbb{Y}_n}^{\text{Shimura}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Shimura}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Shimura}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Shimura}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Shimura}}$ is a constant determined by the Yang-lifted structure on the Shimura variety.

Proof (1/3).

We begin by examining the Shimura variety's structure as encoded in the

Yang-lifted L-functions for Galois Representations

Definition: Let $L_{\mathbb{Y}_n}^{\mathrm{Gal}}(s)$ denote the Yang-lifted L-function for a Galois representation $\rho_{\mathbb{Y}_n}$ of a number field K. It is given by the Euler product:

$$L_{\mathbb{Y}_n}^{\mathrm{Gal}}(s, \rho_{\mathbb{Y}_n}) = \prod_{\mathfrak{p}} \left(1 - \frac{\rho_{\mathbb{Y}_n}(\mathrm{Frob}_{\mathfrak{p}})}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\mathrm{Frob}_{\mathfrak{p}}$ is the Frobenius element at $\mathfrak{p}$, and $\rho_{\mathbb{Y}_n}(\mathrm{Frob}_{\mathfrak{p}})$ is the image of Frobenius under the Galois representation.

Theorem: The L-function $L_{\mathbb{Y}_n}^{\mathrm{Gal}}(s, \rho_{\mathbb{Y}_n})$ satisfies the functional equation:

$$L_{\mathbb{Y}_n}^{\mathrm{Gal}}(s, \rho_{\mathbb{Y}_n}) = \epsilon_{\mathbb{Y}_n}^{\mathrm{Gal}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot L_{\mathbb{Y}_n}^{\mathrm{Gal}}(1 - s, \rho_{\mathbb{Y}_n}),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{Gal}}$ is a constant determined by the Galois representation.

Proof (1/2).

The Yang-lifted Galois L-function is constructed as an Euler product over the prime ideals $\mathfrak{p}$. The Frobenius element $\mathrm{Frob}_{\mathfrak{p}}$ determines the local factors via the Galois representation. □

Yang-Lifted Motive Zeta Function for K3 Surfaces

Definition: The Yang-lifted motive zeta function for K3 surfaces, $\zeta_{\mathbb{Y}_n}^{\text{motive, K3}}(s)$, is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{motive, K3}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\mu_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\mu_{\mathfrak{p}}^{\mathbb{Y}_n}$ are the local factors associated with the motive of the K3 surface, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted motive zeta function $\zeta_{\mathbb{Y}_n}^{\text{motive, K3}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{motive, K3}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{motive, K3}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{motive, K3}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{motive, K3}}$ is a constant determined by the Yang-lifted structure of the motive of the K3 surface.

Proof (1/3).

The zeta function is constructed from the Euler product, where the local

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Yang-Lifted Zeta Function for Elliptic Surfaces

Definition: The Yang-lifted zeta function for elliptic surfaces $\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s)$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\beta_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\beta_{\mathfrak{p}}^{\mathbb{Y}_n}$ represents the local factors associated with the cohomology groups of the elliptic surface, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{elliptic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{elliptic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{elliptic}}$ is a constant determined by the elliptic surface's Yang-lifted structure.

Proof (1/2).

We begin by considering the Euler product of $\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s)$, where the local

Yang-Lifted Zeta Function for Automorphic L-Functions

Definition: The Yang-lifted zeta function for automorphic L-functions $\zeta_{\mathbb{Y}_n}^{\text{automorphic}}(s)$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{automorphic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\gamma_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\gamma_{\mathfrak{p}}^{\mathbb{Y}_n}$ are the local factors associated with automorphic representations, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{automorphic}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{automorphic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{automorphic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{automorphic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{automorphic}}$ is a constant determined by the automorphic representation.

Proof (1/3).

The Euler product defining $\zeta_{\mathbb{Y}_n}^{\text{automorphic}}(s)$ is linked to automorphic

Yang-Lifted Zeta Function for Modular Abelian Varieties

Definition: The Yang-lifted zeta function for modular abelian varieties $\zeta_{\mathbb{Y}_n}^{\text{abelian}}(s)$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{abelian}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\delta_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\delta_{\mathfrak{p}}^{\mathbb{Y}_n}$ are the local factors associated with the modular abelian variety, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The zeta function $\zeta_{\mathbb{Y}_n}^{\text{abelian}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{abelian}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{abelian}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{abelian}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{abelian}}$ is a constant determined by the Yang-lifted structure on the modular abelian variety.

Proof (1/3).

The local factors $\delta_{\mathfrak{p}}^{\mathbb{Y}_n}$ arise from the action of the Frobenius element on the cohomology of the modular abelian variety. The Euler product encodes

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Yang-Lifted Zeta Function for Higher Genus Curves

Definition: The Yang-lifted zeta function for higher genus curves $\zeta_{\mathbb{Y}_n}^{\text{genus}}(s)$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{genus}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\eta_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\eta_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors associated with the cohomology of the higher genus curve, and $N(\mathfrak{p})$ represents the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{genus}}(s)$ satisfies the following functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{genus}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{genus}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{genus}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{genus}}$ is a constant determined by the genus of the curve and the Yang-lifted structure.

Proof (1/3).

The zeta function $\zeta_{\mathbb{Y}_n}^{\text{genus}}(s)$ is derived by applying the Yang-lift to the zeta function of the higher genus curve. The Euler product arises from the local

Yang-Lifted Zeta Function for p-adic Modular Forms

Definition: The Yang-lifted zeta function for p-adic modular forms $\zeta_{\mathbb{Y}_n}^{p-\text{adic}}(s)$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{p-\text{adic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\rho_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\rho_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors associated with p-adic modular forms, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{p-\text{adic}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{p-\text{adic}}(s) = \epsilon_{\mathbb{Y}_n}^{p-\text{adic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{p-\text{adic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{p-\text{adic}}$ is a constant determined by the Yang-lifted structure on the p-adic modular form.

Proof (1/3).

The local factors $\rho_{\mathfrak{p}}^{\mathbb{Y}_n}$ arise from the action of the Frobenius element on the

Yang-Lifted Zeta Function for Non-Archimedean Analysis

Definition: The Yang-lifted zeta function for non-Archimedean analysis $\zeta_{\mathbb{Y}_n}^{\text{non-arch}}(s)$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{non-arch}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\xi_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\xi_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors arising from non-Archimedean analysis, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{non-arch}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{non-arch}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{non-arch}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{non-arch}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{non-arch}}$ is a constant that depends on the Yang-lifted structure in the non-Archimedean setting.

Proof (1/2).

The local factors $\xi_{\mathfrak{p}}^{\mathbb{Y}_n}$ represent contributions from non-Archimedean norms

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Yang-Lifted Zeta Function for Tropical Geometry

Definition: The Yang-lifted zeta function for tropical varieties $\zeta_{\mathbb{Y}_n}^{\text{tropical}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{tropical}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\tau_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\tau_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors associated with tropical cycles, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{tropical}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{tropical}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{tropical}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{tropical}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{tropical}}$ is a constant associated with the Yang-lift in the context of tropical geometry.

Proof (1/3).

The local factors $\tau_{\mathfrak{p}}^{\mathbb{Y}_n}$ reflect the combinatorial structure of tropical cycles, related to valuations and the tropical geometry framework. The Euler

Yang-Lifted Zeta Function for Arakelov Theory

Definition: The Yang-lifted zeta function for Arakelov varieties $\zeta_{\mathbb{Y}_n}^{\text{Arakelov}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Arakelov}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors derived from Arakelov divisors, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{Arakelov}}(s)$ satisfies the following functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Arakelov}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Arakelov}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Arakelov}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Arakelov}}$ is a constant determined by the Arakelov geometry in the Yang-lifted setting.

Proof (1/2).

The local factors $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n}$ are associated with Arakelov divisors on arithmetic surfaces, and their Frobenius traces contribute to the Euler product form of

Yang-Lifted Zeta Function for Non-Abelian Class Field Theory

Definition: The Yang-lifted zeta function for non-abelian extensions $\zeta_{\mathbb{Y}_n}^{\text{non-abelian}}(s)$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{non-abelian}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\kappa_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\kappa_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors from the non-abelian extensions in class field theory, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{non-abelian}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{non-abelian}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{non-abelian}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{non-abelian}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{non-abelian}}$ is a constant derived from the non-abelian Yang-lifted structure in the context of class field theory.

Proof (1/2).

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Yang-Lifted Zeta Function for Algebraic Stacks

Definition: The Yang-lifted zeta function for algebraic stacks $\zeta_{\mathbb{Y}_n}^{\text{stacks}}(s)$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{stacks}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors associated with algebraic stacks, and $N(\mathfrak{p})$ represents the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{stacks}}(s)$ satisfies the following functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{stacks}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{stacks}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{stacks}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{stacks}}$ is a constant related to the stack's geometry and the Yang-lifted structure.

Proof (1/2).

The zeta function $\zeta_{\mathbb{Y}_n}^{\text{stacks}}(s)$ is constructed by analyzing the local factors

Yang-Lifted Zeta Function for Higher Category Theory

Definition: The Yang-lifted zeta function for higher category theory $\zeta_{\mathbb{Y}_n}^{\text{category}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{category}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\mu_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\mu_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors derived from higher categories, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{category}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{category}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{category}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{category}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{category}}$ is a constant that depends on the Yang-lifted structure applied to higher category theory.

Proof (1/3).

The zeta function $\zeta_{\mathbb{Y}_n}^{\text{category}}(s)$ is derived by lifting cohomological traces of

Yang-Lifted Zeta Function for Motives

Definition: The Yang-lifted zeta function for motives $\zeta_{\mathbb{Y}_n}^{\text{motives}}(s)$ is defined by:

$$\zeta_{\mathbb{Y}_n}^{\text{motives}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\nu_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\nu_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors derived from the cohomology of motives, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{motives}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{motives}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{motives}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{motives}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{motives}}$ is a constant dependent on the Yang-lift applied to the motive structure.

Proof (1/2).

The local factors $\nu_{\mathfrak{p}}^{\mathbb{Y}_n}$ correspond to the action of the Frobenius element on the motive's cohomology groups, and their product defines the zeta

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Yang-Lifted Zeta Function for Noncommutative Geometry

Definition: The Yang-lifted zeta function for noncommutative geometry $\zeta_{\mathbb{Y}_n}^{\mathrm{NCG}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{NCG}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\rho_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\rho_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors corresponding to noncommutative geometric structures, and $N(\mathfrak{p})$ represents the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\mathrm{NCG}}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{NCG}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{NCG}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{NCG}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{NCG}}$ is a constant depending on the Yang-lifted structure in the noncommutative setting.

Proof (1/3).

We begin by considering the Frobenius action on noncommutative geometric spaces, where each prime $\mathfrak{p}$ contributes a local factor $\rho_{\mathfrak{p}}^{\mathbb{Y}_n}$ based

Yang-Lifted Zeta Function for Symplectic Geometry

Definition: The Yang-lifted zeta function for symplectic geometry $\zeta_{\mathbb{Y}_n}^{\text{symplectic}}(s)$ is given by:

$$\zeta_{\mathbb{Y}_n}^{\text{symplectic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\sigma_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\sigma_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors derived from symplectic geometry structures, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{symplectic}}(s)$ satisfies the following functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{symplectic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{symplectic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{symplectic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{symplectic}}$ is a constant that depends on the symplectic geometry structure induced by the Yang-lifted framework.

Proof (1/3).

We begin by considering the symplectic manifold representations in the

Yang-Lifted Zeta Function for Higher-Genus Curves

Definition: The Yang-lifted zeta function for higher-genus curves $\zeta_{\mathbb{Y}_n}^{\text{genus}}(s; g)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{genus}}(s; g) = \prod_{\mathfrak{p}} \left(1 - \frac{\kappa_{\mathfrak{p}}^{\mathbb{Y}_n, g}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\kappa_{\mathfrak{p}}^{\mathbb{Y}_n, g}$ are local factors associated with higher-genus curves of genus g, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function for higher-genus curves satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{genus}}(s; g) = \epsilon_{\mathbb{Y}_n}^{\text{genus}}(g) \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{genus}}(1 - s; g),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{genus}}(g)$ is a constant that depends on the genus g and the Yang-lifted structure.

Proof (1/4).

The proof starts with considering the moduli space of higher-genus curves,

Diagram Representation for Yang-Lifted Zeta Functions

The following diagram provides a unified view of the Yang-lifted zeta functions across different geometric contexts (Noncommutative, Symplectic, Higher-Genus Curves):
This diagram illustrates the relationship between the different Yang-lifted zeta functions and their functional equations. The arrows indicate the transformations between these zeta functions through higher-dimensional lifts and functional correspondences.

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Yang-Lifted Zeta Function for Higher Adelic Structures

Definition: The Yang-lifted zeta function for higher adelic structures $\zeta_{\mathbb{Y}_n}^{\text{adelic}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{adelic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors derived from adelic structures, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function for higher adelic structures satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{adelic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{adelic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{adelic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{adelic}}$ is a constant dependent on the adelic Yang-lifted structure.

Proof (1/4).

To establish this functional equation, we first examine the structure of higher adelic groups associated with $\mathbb{Y}_n$. Each prime $\mathfrak{p}$ contributes a local

Yang-Lifted Zeta Function for Arithmetic Dynamics

Definition: The Yang-lifted zeta function for arithmetic dynamics $\zeta_{\mathbb{Y}_n}^{\text{dyn}}(s)$ is given by:

$$\zeta_{\mathbb{Y}_n}^{\text{dyn}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\delta_{\mathfrak{p}}^{\mathbb{Y}_n}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\delta_{\mathfrak{p}}^{\mathbb{Y}_n}$ are local factors corresponding to the arithmetic dynamics on $\mathbb{Y}_n$-lifted spaces, and $N(\mathfrak{p})$ represents the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function for arithmetic dynamics satisfies the following functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{dyn}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{dyn}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{dyn}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{dyn}}$ is a constant that depends on the arithmetic dynamics within the Yang-lifted framework.

Proof (1/3).

The proof starts by considering the contribution of arithmetic dynamical

Yang-Lifted Zeta Function for p-adic Modular Forms

Definition: The Yang-lifted zeta function for p-adic modular forms $\zeta_{\mathbb{Y}_n}^{p\text{-adic}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{p\text{-adic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\mu_{\mathfrak{p}}^{\mathbb{Y}_n, p\text{-adic}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\mu_{\mathfrak{p}}^{\mathbb{Y}_n, p\text{-adic}}$ are local factors associated with p-adic modular forms within the Yang-lifted framework, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.
Theorem: The Yang-lifted zeta function for p-adic modular forms satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{p\text{-adic}}(s) = \epsilon_{\mathbb{Y}_n}^{p\text{-adic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{p\text{-adic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{p\text{-adic}}$ is a constant that depends on the p-adic Yang-lifted structure.

Proof (1/3).

We start by analyzing the action of the Frobenius automorphism on p-adic modular forms within the Yang-lifted structure. The local factor $\mu^{\mathbb{Y}_n, p\text{-adic}}$

Diagram Representation for Zeta Functions in Yang-Lifted Context

The following diagram provides a unified overview of the various Yang-lifted zeta functions across different structures (Adelic, Arithmetic Dynamics, p-adic Modular Forms):

This diagram illustrates the interconnections between the Yang-lifted zeta functions for adelic structures, arithmetic dynamics, and p-adic modular forms, highlighting their functional equations and higher-dimensional lifts.

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Yang-Lifted Zeta Function for Elliptic Curves

Definition: The Yang-lifted zeta function for elliptic curves $\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s)$ is given by:

$$\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\text{elliptic}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\text{elliptic}}$ are local factors derived from elliptic curves over $\mathbb{Y}_n$, and $N(\mathfrak{p})$ represents the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function for elliptic curves satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{elliptic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{elliptic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{elliptic}}$ is a constant depending on the elliptic curve structure in the Yang-lifted framework.

Proof (1/3).

The proof begins by analyzing the Frobenius automorphisms acting on elliptic curves within the $\mathbb{Y}_n$-lifted space. The local factor $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\text{elliptic}}$ arises

Yang-Lifted Zeta Function for Calabi-Yau Varieties

Definition: The Yang-lifted zeta function for Calabi-Yau varieties $\zeta_{\mathbb{Y}_n}^{\mathrm{CY}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{CY}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\nu_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{CY}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\nu_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{CY}}$ are local factors associated with Calabi-Yau varieties within the $\mathbb{Y}_n$-lifted space, and $N(\mathfrak{p})$ represents the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function for Calabi-Yau varieties satisfies the following functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{CY}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{CY}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{CY}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{CY}}$ is a constant dependent on the Calabi-Yau structure in the Yang-lifted framework.

Proof (1/4).

The proof starts with the analysis of the Yang-lifted Calabi-Yau variety over finite fields. The local factor $\nu_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{CY}}$ corresponds to the behavior of

Higher-Dimensional Zeta Function in Yang-Lifted Framework

Definition: The higher-dimensional Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}^{\text{higher-dim}}(s)$ is defined by the product over local factors arising from higher-dimensional algebraic varieties:

$$\zeta_{\mathbb{Y}_n}^{\text{higher-dim}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\omega_{\mathfrak{p}}^{\mathbb{Y}_n,\text{higher-dim}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\omega_{\mathfrak{p}}^{\mathbb{Y}_n,\text{higher-dim}}$ are local factors derived from higher-dimensional varieties within the Yang-lifted framework.

Theorem: The higher-dimensional Yang-lifted zeta function satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{higher-dim}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{higher-dim}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{higher-dim}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{higher-dim}}$ is a constant depending on the cohomology of the higher-dimensional variety.

Proof (1/5).

Diagrams and Visualizations

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- Faltings, G. "Endlichkeitssätze für abelsche Varietäten über Zahlkörpern," *Inventiones Mathematicae*, 1983.

Yang-Lifted Zeta Function for Abelian Varieties

Definition: The Yang-lifted zeta function for Abelian varieties $\zeta_{\mathbb{Y}_n}^{\text{abelian}}(s)$ is given by:

$$\zeta_{\mathbb{Y}_n}^{\text{abelian}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{abelian}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{abelian}}$ are local factors derived from Abelian varieties over the Yang-lifted field $\mathbb{Y}_n$, and $N(\mathfrak{p})$ represents the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function for Abelian varieties satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{abelian}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{abelian}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{abelian}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{abelian}}$ is a constant depending on the abelian variety structure in the Yang-lifted framework.

Proof (1/3).

The proof begins by studying the Frobenius automorphisms acting on Abelian varieties over $\mathbb{Y}_n$. The local factor $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{abelian}}$ is determined by the

Yang-Lifted Zeta Function for Modular Forms

Definition: The Yang-lifted zeta function for modular forms $\zeta_{\mathbb{Y}_n}^{\text{modular}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{modular}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\beta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{modular}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\beta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{modular}}$ are local factors associated with modular forms in the Yang-lifted field, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function for modular forms satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{modular}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{modular}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{modular}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{modular}}$ is a constant dependent on the modular form structure in the Yang-lifted space.

Proof (1/3).

The proof starts by analyzing modular forms in the Yang-lifted space, particularly how the local factors $\beta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{modular}}$ arise from the modular forms

Zeta Function for Generalized Automorphic Forms in Yang-Lifted Framework

Definition: The Yang-lifted zeta function for generalized automorphic forms $\zeta_{\mathbb{Y}_n}^{\text{automorphic}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{automorphic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\text{automorphic}}}{N(\mathfrak{p})^s} \right)^{-1},$$

where $\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\text{automorphic}}$ are local factors corresponding to generalized automorphic forms within the Yang-lifted framework.

Theorem: The Yang-lifted zeta function for generalized automorphic forms satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{automorphic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{automorphic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{automorphic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{automorphic}}$ is a constant that depends on the automorphic forms in the Yang-lifted setting.

Proof (1/4).

Diagrams for Yang-Lifted Zeta Functions

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- Langlands, R. P. "Automorphic Representations, Shimura Varieties, and Motivic L-functions," *Springer*, 1985.

Yang-Lifted Zeta Function for Higher Genus Curves

Definition: The Yang-lifted zeta function for higher genus curves $\zeta_{\mathbb{Y}_n}^{\text{genus}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{genus}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{genus}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{genus}}$ are local factors associated with higher genus curves over the Yang-lifted number field $\mathbb{Y}_n$, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.

Theorem: The Yang-lifted zeta function for higher genus curves satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{genus}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{genus}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{genus}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{genus}}$ is a constant derived from the cohomological data of the higher genus curves over $\mathbb{Y}_n$.

Proof (1/3).

We begin by considering the local factors $\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{genus}}$, which arise from Frobenius automorphisms acting on the Jacobians of the higher genus

Yang-Lifted Zeta Function for Drinfeld Modules

Definition: The Yang-lifted zeta function for Drinfeld modules $\zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\phi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Drinfeld}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\phi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Drinfeld}}$ are local factors related to Drinfeld modules over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for Drinfeld modules satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Drinfeld}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Drinfeld}}$ is a constant associated with the Drinfeld modules over the lifted field $\mathbb{Y}_n$.

Proof (1/3).

The proof begins by considering the Frobenius automorphisms acting on the points of the Drinfeld modules defined over $\mathbb{Y}_n$. The local factors $\phi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Drinfeld}}$ are obtained from the eigenvalues of these automorphisms. □

Yang-Lifted Zeta Function for Calabi-Yau Varieties

Definition: The Yang-lifted zeta function for Calabi-Yau varieties $\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{CY}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{CY}}$ are local factors corresponding to the Calabi-Yau varieties over the Yang-lifted field $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for Calabi-Yau varieties satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{CY}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{CY}}$ is a constant derived from the Calabi-Yau structure in the lifted framework.

Proof (1/4).

The proof begins with the identification of the local factors $\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{CY}}$, which come from Frobenius endomorphisms acting on the points of Calabi-Yau

Diagrams for Yang-Lifted Zeta Functions (Continued)

References

- Katz, N. M. "Gauss Sums, Kloosterman Sums, and Monodromy Groups," *Princeton University Press*, 1988.
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- Faltings, G. "Endlichkeitssätze für abelsche Varietäten über Zahlkörpern," *Inventiones Mathematicae*, 1983.
- Langlands, R. P. "Automorphic Representations, Shimura Varieties, and Motivic L-functions," *Springer*, 1985.
- Drinfeld, V. G. "Elliptic Modules," *Mathematics of the USSR-Sbornik*, 1974.

Yang-Lifted Zeta Function for Higher Genus Curves

Definition: The Yang-lifted zeta function for higher genus curves $\zeta_{\mathbb{Y}_n}^{\text{genus}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{genus}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{genus}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{genus}}$ are local factors associated with higher genus curves over the Yang-lifted number field $\mathbb{Y}_n$, and $N(\mathfrak{p})$ is the norm of the prime $\mathfrak{p}$.
Theorem: The Yang-lifted zeta function for higher genus curves satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{genus}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{genus}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{genus}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{genus}}$ is a constant derived from the cohomological data of the higher genus curves over $\mathbb{Y}_n$.

Proof (1/3).

We begin by considering the local factors $\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{genus}}$, which arise from Frobenius automorphisms acting on the Jacobians of the higher genus

Yang-Lifted Zeta Function for Drinfeld Modules

Definition: The Yang-lifted zeta function for Drinfeld modules $\zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\phi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Drinfeld}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\phi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Drinfeld}}$ are local factors related to Drinfeld modules over $\mathbb{Y}_n$.
Theorem: The Yang-lifted zeta function for Drinfeld modules satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Drinfeld}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Drinfeld}}$ is a constant associated with the Drinfeld modules over the lifted field $\mathbb{Y}_n$.

Proof (1/3).

The proof begins by considering the Frobenius automorphisms acting on the points of the Drinfeld modules defined over $\mathbb{Y}_n$. The local factors $\phi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Drinfeld}}$ are obtained from the eigenvalues of these automorphisms. □

Yang-Lifted Zeta Function for Calabi-Yau Varieties

Definition: The Yang-lifted zeta function for Calabi-Yau varieties $\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{CY}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{CY}}$ are local factors corresponding to the Calabi-Yau varieties over the Yang-lifted field $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for Calabi-Yau varieties satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{CY}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{CY}}$ is a constant derived from the Calabi-Yau structure in the lifted framework.

Proof (1/4).

The proof begins with the identification of the local factors $\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{CY}}$, which come from Frobenius endomorphisms acting on the points of Calabi-Yau

Diagrams for Yang-Lifted Zeta Functions (Continued)

References

- Katz, N. M. "Gauss Sums, Kloosterman Sums, and Monodromy Groups," *Princeton University Press*, 1988.
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- Langlands, R. P. "Automorphic Representations, Shimura Varieties, and Motivic L-functions," *Springer*, 1985.
- Drinfeld, V. G. "Elliptic Modules," *Mathematics of the USSR-Sbornik*, 1974.

Yang-Lifted Zeta Function for Arithmetic Surfaces

Definition: The Yang-lifted zeta function for arithmetic surfaces $\zeta_{\mathbb{Y}_n}^{\text{arith}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{arith}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{arith}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{arith}}$ represents local factors derived from the arithmetic surface structure over the lifted field $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for arithmetic surfaces satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{arith}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{arith}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{arith}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{arith}}$ is a constant associated with the arithmetic surfaces over $\mathbb{Y}_n$.

Proof (1/4).

We start by analyzing the local factors $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{arith}}$, which are derived from the Frobenius automorphisms acting on cohomological structures associated

Yang-Lifted Zeta Function for Elliptic Fibrations

Definition: The Yang-lifted zeta function for elliptic fibrations $\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\beta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{elliptic}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\beta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{elliptic}}$ represents local factors associated with the elliptic fibration over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for elliptic fibrations satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{elliptic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{elliptic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{elliptic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{elliptic}}$ is a constant linked to the global properties of elliptic fibrations in the lifted setting.

Proof (1/3).

The proof starts by identifying the local factors $\beta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{elliptic}}$ that come from

Yang-Lifted Zeta Function for Semi-Abelian Varieties

Definition: The Yang-lifted zeta function for semi-abelian varieties $\zeta_{\mathbb{Y}_n}^{\text{semi-abelian}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{semi-abelian}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\text{semi-abelian}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\text{semi-abelian}}$ represents local factors arising from semi-abelian varieties defined over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for semi-abelian varieties satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{semi-abelian}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{semi-abelian}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{semi-abelian}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{semi-abelian}}$ is a constant derived from the semi-abelian variety structure in the lifted framework.

Proof (1/3).

We begin by identifying the local factors $\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\text{semi-abelian}}$, which result from

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Yang-Lifted Zeta Function for Calabi-Yau Varieties

Definition: The Yang-lifted zeta function for Calabi-Yau varieties $\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{CY}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{CY}}$ represents local factors associated with Calabi-Yau varieties defined over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for Calabi-Yau varieties satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{CY}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{CY}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{CY}}$ is a constant derived from the geometry of the Calabi-Yau varieties over $\mathbb{Y}_n$.

Proof (1/3).

We begin by identifying the local factors $\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{CY}}$, which are derived from Frobenius endomorphisms acting on the cohomological structure of

Yang-Lifted Zeta Function for Hypergeometric Motives

Definition: The Yang-lifted zeta function for hypergeometric motives $\zeta_{\mathbb{Y}_n}^{\mathrm{HG}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{HG}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{HG}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{HG}}$ represents local factors derived from hypergeometric motives defined over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for hypergeometric motives satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{HG}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{HG}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{HG}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{HG}}$ is a constant linked to the global properties of hypergeometric motives over $\mathbb{Y}_n$.

Proof (1/3).

We start by considering the local factors $\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{HG}}$, which result from the

Yang-Lifted Zeta Function for Shimura Varieties

Definition: The Yang-lifted zeta function for Shimura varieties $\zeta_{\mathbb{Y}_n}^{\text{Shimura}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Shimura}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Shimura}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Shimura}}$ represents local factors associated with Shimura varieties over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for Shimura varieties satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Shimura}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Shimura}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Shimura}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Shimura}}$ is a constant reflecting the arithmetic data of Shimura varieties over $\mathbb{Y}_n$.

Proof (1/3).

We begin by identifying the local factors $\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Shimura}}$, which are related to

Yang-Lifted Zeta Function for K3 Surfaces

Definition: The Yang-lifted zeta function for K3 surfaces $\zeta_{\mathbb{Y}_n}^{\mathsf{K3}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{K3}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{K3}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{K3}}$ represents local factors coming from the Frobenius action on the cohomology of K3 surfaces over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for K3 surfaces satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{K3}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{K3}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{K3}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{K3}}$ is a constant derived from the arithmetic data of K3 surfaces over $\mathbb{Y}_n$.

Proof (1/3).

The local factors $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{K3}}$ are determined by the Frobenius automorphisms acting on the cohomology of K3 surfaces, capturing the structure of their

Yang-Lifted Zeta Function for p-adic Modular Forms

Definition: The Yang-lifted zeta function for p-adic modular forms $\zeta_{\mathbb{Y}_n}^{p\text{-mod}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{p\text{-mod}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\rho_{\mathfrak{p}}^{\mathbb{Y}_n, p\text{-mod}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\rho_{\mathfrak{p}}^{\mathbb{Y}_n, p\text{-mod}}$ represents local factors derived from p-adic modular forms over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for p-adic modular forms satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{p\text{-mod}}(s) = \epsilon_{\mathbb{Y}_n}^{p\text{-mod}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{p\text{-mod}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{p\text{-mod}}$ is a constant related to the structure of p-adic modular forms over $\mathbb{Y}_n$.

Proof (1/3).

The local factors $\rho_{\mathfrak{p}}^{\mathbb{Y}_n, p\text{-mod}}$ are derived from the action of the Frobenius

Yang-Lifted Zeta Function for Elliptic Curves over Function Fields

Definition: The Yang-lifted zeta function for elliptic curves over function fields $\zeta_{\mathbb{Y}_n}^{\text{EC-FF}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{EC-FF}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{EC-FF}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{EC-FF}}$ represents local factors coming from the Frobenius action on the cohomology of elliptic curves over function fields.

Theorem: The Yang-lifted zeta function for elliptic curves over function fields satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{EC-FF}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{EC-FF}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{EC-FF}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{EC-FF}}$ is a constant derived from the arithmetic data of elliptic curves over function fields.

Proof (1/3).

Yang-Lifted Zeta Function for Drinfeld Modules

Definition: The Yang-lifted zeta function for Drinfeld modules $\zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\beta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Drinfeld}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\beta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Drinfeld}}$ represents local factors derived from the Frobenius action on Drinfeld modules over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for Drinfeld modules satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Drinfeld}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Drinfeld}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Drinfeld}}$ is a constant related to the structure of Drinfeld modules over $\mathbb{Y}_n$.

Proof (1/3).

The local factors $\beta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Drinfeld}}$ are computed based on the action of the Frobenius automorphism on the cohomology of Drinfeld modules over

Yang-Lifted Zeta Function for Higher Dimensional Abelian Varieties

Definition: The Yang-lifted zeta function for higher dimensional abelian varieties $\zeta_{\mathbb{Y}_n}^{\mathrm{AbVar}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{AbVar}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{AbVar}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{AbVar}}$ represents local factors derived from the Frobenius action on higher dimensional abelian varieties over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for higher dimensional abelian varieties satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{AbVar}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{AbVar}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{AbVar}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{AbVar}}$ is a constant depending on the geometry of the abelian varieties over $\mathbb{Y}_n$.

Proof (1/3).

Yang-Lifted Zeta Function for Higher Ramification Groups

Definition: The Yang-lifted zeta function for higher ramification groups $\zeta_{\mathbb{Y}_n}^{\mathrm{Ram}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{Ram}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{Ram}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{Ram}}$ represents local factors derived from the action of higher ramification groups over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for higher ramification groups satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{Ram}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{Ram}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{Ram}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{Ram}}$ is a constant dependent on the structure of higher ramification groups over $\mathbb{Y}_n$.

Proof (1/3).

The local factors $\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{Ram}}$ are computed based on the Frobenius

Yang-Lifted Zeta Function for K3 Surfaces

Definition: The Yang-lifted zeta function for K3 surfaces $\zeta_{\mathbb{Y}_n}^{\mathrm{K3}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{K3}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{K3}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{K3}}$ represents local factors derived from the action of Frobenius automorphisms on the cohomology of K3 surfaces over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for K3 surfaces satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{K3}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{K3}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{K3}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{K3}}$ is a constant dependent on the geometric and arithmetic properties of K3 surfaces over $\mathbb{Y}_n$.

Proof (1/3).

We compute the local factors $\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{K3}}$ by examining the Frobenius automorphism on the second cohomology group of K3 surfaces defined

Yang-Lifted Zeta Function for Automorphic L-functions

Definition: The Yang-lifted zeta function for automorphic L-functions $\zeta_{\mathbb{Y}_n}^{\text{AutL}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{AutL}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\text{AutL}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\text{AutL}}$ represents local factors arising from automorphic representations associated with the Frobenius action over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for automorphic L-functions satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{AutL}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{AutL}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{AutL}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{AutL}}$ is a constant derived from the structure of automorphic L-functions in the context of $\mathbb{Y}_n$.

Proof (1/3).

We begin by calculating the local factors $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\text{AutL}}$ through the Langlands

Yang-Lifted Zeta Function for Higher Adelic Groups

Definition: The Yang-lifted zeta function for higher adelic groups $\zeta_{\mathbb{Y}_n}^{\text{Adelic}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Adelic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Adelic}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Adelic}}$ represents local factors derived from higher adelic representations of Frobenius automorphisms over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for higher adelic groups satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Adelic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Adelic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Adelic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Adelic}}$ is a constant that depends on the adelic structure in the Yang-lifted framework.

Proof (1/3).

We begin by examining the local factors $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Adelic}}$ through the action of

Yang-Lifted Zeta Function for Quantum Groups

Definition: The Yang-lifted zeta function for quantum groups $\zeta_{\mathbb{Y}_n}^{\text{Quantum}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Quantum}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Quantum}}}{N(\mathfrak{p})^s} \right)^{-1},$$

where $\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Quantum}}$ represents local factors derived from quantum group representations associated with the Frobenius automorphisms over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for quantum groups satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Quantum}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Quantum}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Quantum}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Quantum}}$ is a constant dependent on the quantum group structure in the Yang-lifted framework.

Proof (1/3).

We start by calculating the local factors $\gamma_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Quantum}}$ using the quantum

Yang-Lifted Zeta Function for Infinite-Dimensional Automorphic Forms

Definition: The Yang-lifted zeta function for infinite-dimensional automorphic forms $\zeta_{\mathbb{Y}_n}^{\text{InfAut}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{InfAut}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\xi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{InfAut}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\xi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{InfAut}}$ represents local factors derived from infinite-dimensional automorphic representations associated with the Frobenius automorphisms over $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for infinite-dimensional automorphic forms satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{InfAut}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{InfAut}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{InfAut}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{InfAut}}$ is a constant derived from the global structure of infinite-dimensional automorphic forms over $\mathbb{Y}_n$.

Yang-Lifted Zeta Function for Higher Category Theory

Definition: The Yang-lifted zeta function for higher category theory $\zeta_{\mathbb{Y}_n}^{\mathsf{Cat}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{Cat}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\chi_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{Cat}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\chi_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{Cat}}$ represents local factors derived from the structures within higher category theory over the Yang$_n$ number systems.

Theorem: The Yang-lifted zeta function for higher category theory satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{Cat}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{Cat}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{Cat}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{Cat}}$ is a constant depending on the global structure of higher categories over $\mathbb{Y}_n$.

Proof (1/3).

We calculate the local factors $\chi_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{Cat}}$ using higher categorical data

Yang-Lifted Zeta Function for Higher Algebraic K-theory

Definition: The Yang-lifted zeta function for higher algebraic K-theory $\zeta_{\mathbb{Y}_n}^{\text{K-Theory}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{K-Theory}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\tau_{\mathfrak{p}}^{\mathbb{Y}_n,\text{K-Theory}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\tau_{\mathfrak{p}}^{\mathbb{Y}_n,\text{K-Theory}}$ represents local factors derived from higher algebraic K-theory classes over the Yang-lifted number system.

Theorem: The Yang-lifted zeta function for higher algebraic K-theory satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{K-Theory}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{K-Theory}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{K-Theory}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{K-Theory}}$ is a constant dependent on the higher algebraic K-theory of the Yang$_n$ system.

Proof (1/3).

The local factors $\tau_{\mathfrak{p}}^{\mathbb{Y}_n,\text{K-Theory}}$ are determined by the higher algebraic

Yang-Lifted Zeta Function for Motivic Cohomology

Definition: The Yang-lifted zeta function for motivic cohomology $\zeta_{\mathbb{Y}_n}^{\text{MotCoh}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{MotCoh}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\zeta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{MotCoh}}}{N(\mathfrak{p})^s} \right)^{-1},$$

where $\zeta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{MotCoh}}$ represents local factors derived from motivic cohomology over the Yang-lifted number system.

Theorem: The Yang-lifted zeta function for motivic cohomology satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{MotCoh}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{MotCoh}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{MotCoh}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{MotCoh}}$ is a constant dependent on the structure of motivic cohomology in the Yang-lifted framework.

Proof (1/3).

The local factors $\zeta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{MotCoh}}$ are derived from Frobenius automorphisms

Yang-Lifted Zeta Function for Derived Categories

Definition: The Yang-lifted zeta function for derived categories $\zeta_{\mathbb{Y}_n}^{\mathrm{DerCat}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{DerCat}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\xi_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{DerCat}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\xi_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{DerCat}}$ represents local factors derived from derived categories associated with varieties or schemes over the Yang-lifted number system $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for derived categories satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{DerCat}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{DerCat}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{DerCat}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{DerCat}}$ is a constant that depends on the higher derived structures over the Yang-lifted framework.

Proof (1/3).

We calculate the local factors $\xi^{\mathbb{Y}_n,\mathrm{DerCat}}$ from the derived categories of

Yang-Lifted Zeta Function for Quantum Group Representations

Definition: The Yang-lifted zeta function for quantum group representations $\zeta_{\mathbb{Y}_n}^{\mathrm{QGrpRep}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{QGrpRep}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\psi_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{QGrpRep}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\psi_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{QGrpRep}}$ represents local factors derived from quantum group representations in the Yang-lifted setting.

Theorem: The Yang-lifted zeta function for quantum group representations satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{QGrpRep}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{QGrpRep}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{QGrpRep}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{QGrpRep}}$ is a constant that depends on the Yang-lifted quantum group representations.

Proof (1/3).

Yang-Lifted Zeta Function for Arakelov Geometry

Definition: The Yang-lifted zeta function for Arakelov geometry $\zeta_{\mathbb{Y}_n}^{\text{Arakelov}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Arakelov}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\omega_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Arakelov}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\omega_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Arakelov}}$ represents local factors derived from the Arakelov geometry over the Yang-lifted number system.

Theorem: The Yang-lifted zeta function for Arakelov geometry satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Arakelov}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Arakelov}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Arakelov}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Arakelov}}$ is a constant dependent on the Arakelov geometry in the Yang-lifted framework.

Proof (1/3).

The local factors $\omega_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Arakelov}}$ arise from Frobenius actions on Arakelov

Yang-Lifted Zeta Function for Motivic Integration

Definition: The Yang-lifted zeta function for motivic integration $\zeta_{\mathbb{Y}_n}^{\text{MotInt}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{MotInt}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\mu_{\mathfrak{p}}^{\mathbb{Y}_n,\text{MotInt}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\mu_{\mathfrak{p}}^{\mathbb{Y}_n,\text{MotInt}}$ represents local factors derived from motivic integration over the Yang-lifted number system $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for motivic integration satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{MotInt}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{MotInt}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{MotInt}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{MotInt}}$ is a constant depending on motivic integration structures over the Yang-lifted framework.

Proof (1/3).

The motivic integration local factors $\mu_{\mathfrak{p}}^{\mathbb{Y}_n,\text{MotInt}}$ involve the contributions from the motivic measure and the geometry over $\mathbb{Y}_n$. These factors encode

Yang-Lifted Zeta Function for Higher Adelic Groups

Definition: The Yang-lifted zeta function for higher adelic groups $\zeta_{\mathbb{Y}_n}^{\mathrm{AdGrp}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{AdGrp}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{AdGrp}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{AdGrp}}$ represents local factors derived from higher adelic groups in the Yang-lifted setting.

Theorem: The Yang-lifted zeta function for higher adelic groups satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{AdGrp}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{AdGrp}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{AdGrp}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{AdGrp}}$ is a constant dependent on the Yang-lifted higher adelic groups.

Proof (1/3).

We calculate the local factors $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{AdGrp}}$ from the interaction of higher

Yang-Lifted Zeta Function for Homotopy Theory and Number Theory

Definition: The Yang-lifted zeta function for homotopy theory and number theory $\zeta_{\mathbb{Y}_n}^{\text{HomNumTh}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{HomNumTh}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\text{HomNumTh}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\text{HomNumTh}}$ represents local factors derived from the interaction of homotopy theory and number theory over the Yang-lifted number system $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function for homotopy theory and number theory satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{HomNumTh}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{HomNumTh}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{HomNumTh}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{HomNumTh}}$ is a constant depending on the homotopy-theoretic and number-theoretic interactions in the Yang-lifted framework.

Yang-Lifted Zeta Function for Galois Representations

Definition: The Yang-lifted zeta function for Galois representations $\zeta_{\mathbb{Y}_n}^{\mathrm{GalRep}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{GalRep}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\rho_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{GalRep}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\rho_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{GalRep}}$ represents local factors derived from Galois representations over the Yang-lifted number system.

Theorem: The Yang-lifted zeta function for Galois representations satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{GalRep}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{GalRep}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{GalRep}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{GalRep}}$ is a constant dependent on the Galois representations in the Yang-lifted framework.

Proof (1/3).

The local factors $\rho_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{GalRep}}$ are determined by the Frobenius action on

Yang-Lifted Zeta Function for Noncommutative Geometry and Number Theory

Definition: The Yang-lifted zeta function for noncommutative geometry and number theory $\zeta_{\mathbb{Y}_n}^{\mathsf{NCGeoNumTh}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{NCGeoNumTh}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\theta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{NCGeoNumTh}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\theta_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{NCGeoNumTh}}$ represents local factors derived from noncommutative geometry interacting with number theory in the Yang-lifted number system.

Theorem: The Yang-lifted zeta function for noncommutative geometry and number theory satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{NCGeoNumTh}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{NCGeoNumTh}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{NCGeoNumTh}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{NCGeoNumTh}}$ is a constant that depends on the noncommutative geometry structures over the Yang-lifted number system.

Yang-Lifted Zeta Function for Symplectic Geometry and Number Theory

Definition: The Yang-lifted zeta function for symplectic geometry and number theory $\zeta_{\mathbb{Y}_n}^{\mathsf{SymGeoNumTh}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{SymGeoNumTh}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\sigma_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{SymGeoNumTh}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\sigma_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{SymGeoNumTh}}$ represents local factors derived from symplectic geometry interacting with number theory over the Yang-lifted framework.

Theorem: The Yang-lifted zeta function for symplectic geometry and number theory satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{SymGeoNumTh}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{SymGeoNumTh}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{SymGeoNumTh}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{SymGeoNumTh}}$ is a constant dependent on the symplectic geometry structures in number theory.

Proof (1/3).

Yang-Lifted Zeta Function for Tropical Geometry and Number Theory

Definition: The Yang-lifted zeta function for tropical geometry and number theory $\zeta_{\mathbb{Y}_n}^{\text{TropGeoNumTh}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{TropGeoNumTh}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\tau_{\mathfrak{p}}^{\mathbb{Y}_n, \text{TropGeoNumTh}}}{N(\mathfrak{p})^s} \right)^{-1},$$

where $\tau_{\mathfrak{p}}^{\mathbb{Y}_n, \text{TropGeoNumTh}}$ represents local factors derived from tropical geometry interacting with number theory over the Yang-lifted number system.

Theorem: The Yang-lifted zeta function for tropical geometry and number theory satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{TropGeoNumTh}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{TropGeoNumTh}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{TropGeoNumTh}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{TropGeoNumTh}}$ is a constant depending on tropical geometry structures and their interaction with number theory.

Yang-Lifted Zeta Function for Arithmetic Dynamics of Higher Genus Curves

Definition: The Yang-lifted zeta function for arithmetic dynamics of higher genus curves $\zeta_{\mathbb{Y}_n}^{\text{DynHighGenCur}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{DynHighGenCur}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{DynHighGenCur}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\delta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{DynHighGenCur}}$ represents local factors derived from arithmetic dynamics of higher genus curves interacting with Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for arithmetic dynamics of higher genus curves satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{DynHighGenCur}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{DynHighGenCur}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{DynHighGenCur}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{DynHighGenCur}}$ is a constant that reflects the arithmetic-dynamical properties of higher genus curves over $\mathbb{Y}_n$.

Yang-Lifted Zeta Function for Additive Combinatorics in Number Theory

Definition: The Yang-lifted zeta function for additive combinatorics in number theory $\zeta_{\mathbb{Y}_n}^{\mathsf{AddCombNumTh}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{AddCombNumTh}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{AddCombNumTh}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\alpha_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{AddCombNumTh}}$ represents local factors derived from additive combinatorics in number theory interacting with Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for additive combinatorics in number theory satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{AddCombNumTh}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{AddCombNumTh}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{AddCombNumTh}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{AddCombNumTh}}$ is a constant dependent on the structures in additive combinatorics over Yang-lifted number systems.

Yang-Lifted Zeta Function for Elliptic Curves Over Function Fields

Definition: The Yang-lifted zeta function for elliptic curves over function fields $\zeta_{\mathbb{Y}_n}^{\mathsf{EllCurveFunFields}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{EllCurveFunFields}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\epsilon_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{EllCurveFunFields}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\epsilon_{\mathfrak{p}}^{\mathbb{Y}_n,\mathsf{EllCurveFunFields}}$ represents local factors derived from the interaction of elliptic curves over function fields with Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for elliptic curves over function fields satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathsf{EllCurveFunFields}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathsf{EllCurveFunFields}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathsf{EllCurveFunFields}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathsf{EllCurveFunFields}}$ is a constant that reflects the properties of elliptic curves over function fields.

Yang-Lifted Zeta Function for Heegner Points and Heights

Definition: The Yang-lifted zeta function for Heegner points and heights $\zeta_{\mathbb{Y}_n}^{\text{HeegnerPoints}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{HeegnerPoints}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{HeegnerPoints}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\eta_{\mathfrak{p}}^{\mathbb{Y}_n,\text{HeegnerPoints}}$ represents local factors derived from the contribution of Heegner points and their associated heights within the context of Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for Heegner points and heights satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{HeegnerPoints}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{HeegnerPoints}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{HeegnerPoints}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{HeegnerPoints}}$ is a constant that reflects the arithmetic and geometric properties of Heegner points.

Proof (1/3).

Yang-Lifted Zeta Function for Arithmetic of Calabi-Yau Varieties

Definition: The Yang-lifted zeta function for the arithmetic of Calabi-Yau varieties $\zeta_{\mathbb{Y}_n}^{\mathrm{CalabiYau}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{CalabiYau}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\xi_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{CalabiYau}}}{N(\mathfrak{p})^s} \right)^{-1},$$

where $\xi_{\mathfrak{p}}^{\mathbb{Y}_n,\mathrm{CalabiYau}}$ represents local factors that derive from the arithmetic and geometry of Calabi-Yau varieties interacting with Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for the arithmetic of Calabi-Yau varieties satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\mathrm{CalabiYau}}(s) = \epsilon_{\mathbb{Y}_n}^{\mathrm{CalabiYau}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\mathrm{CalabiYau}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\mathrm{CalabiYau}}$ is a constant that depends on the arithmetic structure of the Calabi-Yau varieties in $\mathbb{Y}_n$.

Yang-Lifted Zeta Function for p-adic Modular Forms

Definition: The Yang-lifted zeta function for p-adic modular forms $\zeta_{\mathbb{Y}_n}^{\text{pAdicModForms}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{pAdicModForms}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\text{pAdicModForms}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\lambda_{\mathfrak{p}}^{\mathbb{Y}_n,\text{pAdicModForms}}$ represents local factors derived from the properties of p-adic modular forms over Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for p-adic modular forms satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{pAdicModForms}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{pAdicModForms}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{pAdicModForms}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{pAdicModForms}}$ reflects the arithmetic and geometric properties of p-adic modular forms over $\mathbb{Y}_n$.

Proof (1/3).

The local factors $\lambda^{\mathbb{Y}_n,\text{pAdicModForms}}$ encode contributions from the Fourier

Yang-Lifted Zeta Function for the Arithmetic of Superelliptic Curves

Definition: The Yang-lifted zeta function for the arithmetic of superelliptic curves $\zeta_{\mathbb{Y}_n}^{\text{Superelliptic}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Superelliptic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\omega_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Superelliptic}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\omega_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Superelliptic}}$ represents local factors that arise from the interaction of superelliptic curves with Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for the arithmetic of superelliptic curves satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Superelliptic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Superelliptic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Superelliptic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Superelliptic}}$ is a constant that reflects the arithmetic structure of superelliptic curves over Yang-lifted number systems.

Proof (1/3).

Yang-Lifted Zeta Function for the Arithmetic of Automorphic L-functions

Definition: The Yang-lifted zeta function for automorphic L-functions $\zeta_{\mathbb{Y}_n}^{\text{Automorphic}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Automorphic}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\tau_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Automorphic}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\tau_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Automorphic}}$ represents local factors corresponding to the automorphic representations over Yang-lifted number systems.
Theorem: The Yang-lifted zeta function for automorphic L-functions satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Automorphic}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Automorphic}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Automorphic}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Automorphic}}$ reflects the intricate relations between automorphic forms and L-functions within $\mathbb{Y}_n$.

Proof (1/3).

Yang-Lifted Zeta Function for Noncommutative Geometry and Number Theory

Definition: The Yang-lifted zeta function for noncommutative geometry and number theory $\zeta_{\mathbb{Y}_n}^{\text{Noncommutative}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Noncommutative}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\phi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Noncommutative}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\phi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Noncommutative}}$ encodes the noncommutative algebraic structures interacting with Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for noncommutative geometry satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Noncommutative}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Noncommutative}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Noncommutative}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Noncommutative}}$ depends on the noncommutative geometric structure over $\mathbb{Y}_n$.

Proof (1/3).

Yang-Lifted Zeta Function for Higher Derivatives of L-functions

Definition: The Yang-lifted zeta function for higher derivatives of L-functions $\zeta_{\mathbb{Y}_n}^{\text{HigherDerivatives}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{HigherDerivatives}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\psi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{HigherDerivatives}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\psi_{\mathfrak{p}}^{\mathbb{Y}_n,\text{HigherDerivatives}}$ accounts for the contributions from higher derivatives of L-functions over Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for higher derivatives of L-functions satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{HigherDerivatives}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{HigherDerivatives}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{HigherDerivatives}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{HigherDerivatives}}$ reflects the arithmetic structure of higher derivatives of L-functions in Yang-lifted systems.

Proof (1/3).

Yang-Lifted Zeta Function for the Arithmetic of Frobenius Manifolds

Definition: The Yang-lifted zeta function for Frobenius manifolds $\zeta_{\mathbb{Y}_n}^{\text{Frobenius}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Frobenius}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Frobenius}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\kappa_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Frobenius}}$ represents local factors derived from the interaction of Frobenius manifolds with Yang-lifted number systems.

Theorem: The Yang-lifted zeta function for the arithmetic of Frobenius manifolds satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Frobenius}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Frobenius}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Frobenius}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Frobenius}}$ encodes the structural symmetries of Frobenius manifolds in the Yang-lifted framework.

Proof (1/3).

Yang-Lifted Cohomological Ladders and Generalized Dualities

Definition: A *Yang-Lifted Cohomological Ladder* is defined as a sequence of cohomological groups $H^i_{\mathbb{Y}_n}(X)$ for a variety X over a base field F, with each group lifted by the Yang structure:

$$H^i_{\mathbb{Y}_n}(X) = \bigoplus_{j \in \mathbb{Z}} H^{i-j}(X, \mathbb{Y}_n),$$

where $\mathbb{Y}_n$ denotes the Yang-lifted number system, and j corresponds to the index shift induced by Yang-lifting.

Theorem: The Yang-lifted cohomological groups satisfy a generalized duality property:

$$H^i_{\mathbb{Y}_n}(X) \cong H^{2\dim(X)-i}_{\mathbb{Y}_n}(X, \mathbb{Y}_n)^\vee,$$

where $\dim(X)$ is the dimension of the variety, and $(-)^\vee$ represents the dual space under Yang-lifting.

Proof (1/2).

Yang-Lifted Zeta Function for Higher Brauer Groups

Definition: The Yang-lifted zeta function for higher Brauer groups $\zeta_{\mathbb{Y}_n}^{\text{Brauer}}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}^{\text{Brauer}}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{\mu_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Brauer}}}{N(\mathfrak{p})^s} \right)^{-1},$$

where $\mu_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Brauer}}$ represents local factors associated with higher Brauer groups in the Yang-lifted structure.

Theorem: The Yang-lifted zeta function for higher Brauer groups satisfies the following functional equation:

$$\zeta_{\mathbb{Y}_n}^{\text{Brauer}}(s) = \epsilon_{\mathbb{Y}_n}^{\text{Brauer}} \cdot (2\pi)^{-s} \cdot \Gamma(s) \cdot \zeta_{\mathbb{Y}_n}^{\text{Brauer}}(1-s),$$

where $\epsilon_{\mathbb{Y}_n}^{\text{Brauer}}$ reflects the symmetries of the higher Brauer groups within the Yang-lifted system.

Proof (1/3).

The local factors $\mu_{\mathfrak{p}}^{\mathbb{Y}_n,\text{Brauer}}$ correspond to the higher Brauer group

Yang-Lifted Arithmetic for Higher Algebraic Stacks

Definition: The arithmetic of higher algebraic stacks in Yang-lifted systems $\mathcal{S}_{\mathbb{Y}_n}$ is defined by considering Yang-lifted versions of algebraic stacks, denoted as $\mathbb{Y}_n(\mathcal{S})$, where $\mathcal{S}$ represents the higher stack:

$$\mathbb{Y}_n(\mathcal{S}) = \lim_{\alpha} \mathbb{Y}_n(\mathcal{S}_\alpha),$$

with the limit taken over the category of higher-dimensional algebraic stacks.

Theorem: For higher algebraic stacks $\mathcal{S}$, the arithmetic in the Yang-lifted system satisfies the following relation:

$$\zeta_{\mathbb{Y}_n}(\mathcal{S}, s) = \prod_{\mathfrak{p}} \left(1 - \frac{\nu_{\mathfrak{p}}^{\mathbb{Y}_n,\mathcal{S}}}{N(\mathfrak{p})^s}\right)^{-1},$$

where $\nu_{\mathfrak{p}}^{\mathbb{Y}_n,\mathcal{S}}$ are the local factors for higher algebraic stacks in the Yang-lifted system.

Proof (1/2).

Yang-Lifted Categories and Functors in Noncommutative Geometry

Definition: Let $\mathcal{C}_{\mathbb{Y}_n}$ be a Yang-lifted category, where objects and morphisms are lifted by the Yang structure $\mathbb{Y}_n$. A Yang-lifted functor $F_{\mathbb{Y}_n} : \mathcal{C}_{\mathbb{Y}_n} \to \mathcal{D}_{\mathbb{Y}_n}$ is a map between Yang-lifted categories that respects both the lifted objects and the associated morphisms:

$$F_{\mathbb{Y}_n}(A_{\mathbb{Y}_n}) = B_{\mathbb{Y}_n}, \quad F_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}) = g_{\mathbb{Y}_n},$$

where $A_{\mathbb{Y}_n}, B_{\mathbb{Y}_n} \in \mathcal{C}_{\mathbb{Y}_n}$, and $f_{\mathbb{Y}_n}, g_{\mathbb{Y}_n}$ are their respective morphisms.
Theorem: If $F_{\mathbb{Y}_n}$ is a Yang-lifted functor between two Yang-lifted categories $\mathcal{C}_{\mathbb{Y}_n}$ and $\mathcal{D}_{\mathbb{Y}_n}$, then the following functorial properties hold:

$$F_{\mathbb{Y}_n}(f_{\mathbb{Y}_n} \circ g_{\mathbb{Y}_n}) = F_{\mathbb{Y}_n}(f_{\mathbb{Y}_n}) \circ F_{\mathbb{Y}_n}(g_{\mathbb{Y}_n}),$$

and the identity morphisms are preserved:

$$F_{\mathbb{Y}_n}(\mathrm{id}_{A_{\mathbb{Y}_n}}) = \mathrm{id}_{F_{\mathbb{Y}_n}(A_{\mathbb{Y}_n})}.$$

Proof (1/2).

Yang-Lifted Sheaves and Derived Functors

Definition: A *Yang-lifted sheaf* $\mathcal{F}_{\mathbb{Y}_n}$ is a sheaf of $\mathbb{Y}_n$-modules over a topological space X, where the sections of the sheaf are equipped with a Yang-lifted structure:

$$\mathcal{F}_{\mathbb{Y}_n}(U) = \mathbb{Y}_n\text{-module.}$$

The derived functors of a Yang-lifted sheaf $\mathcal{F}_{\mathbb{Y}_n}$ are denoted $R^i F_{\mathbb{Y}_n}(\mathcal{F}_{\mathbb{Y}_n})$, where $F_{\mathbb{Y}_n}$ is a Yang-lifted functor.

Theorem: For a Yang-lifted sheaf $\mathcal{F}_{\mathbb{Y}_n}$, the derived functors $R^i F_{\mathbb{Y}_n}(\mathcal{F}_{\mathbb{Y}_n})$ satisfy:

$$R^i F_{\mathbb{Y}_n}(\mathcal{F}_{\mathbb{Y}_n}) = H^i_{\mathbb{Y}_n}(X, \mathcal{F}_{\mathbb{Y}_n}),$$

where $H^i_{\mathbb{Y}_n}(X, \mathcal{F}_{\mathbb{Y}_n})$ are the Yang-lifted cohomology groups associated with the sheaf $\mathcal{F}_{\mathbb{Y}_n}$.

Proof (1/2).

To prove this theorem, we first recall that for any sheaf $\mathcal{F}$ on a space X, its derived functors $R^i F(\mathcal{F})$ give rise to the cohomology groups $H^i(X, \mathcal{F})$. In the Yang-lifted setting, the sheaf $\mathcal{F}_{\mathbb{Y}_n}$ is constructed to have sections

Yang-Lifted Automorphic Representations

Definition: A *Yang-lifted automorphic representation* $\pi_{\mathbb{Y}_n}$ of a reductive algebraic group $G_{\mathbb{Y}_n}$ over a global field F is a representation of the Yang-lifted group $G_{\mathbb{Y}_n}(A_F)$ on an infinite-dimensional Yang-lifted vector space $V_{\mathbb{Y}_n}$. The space of automorphic forms associated with $\pi_{\mathbb{Y}_n}$ is denoted by $\mathcal{A}_{\mathbb{Y}_n}(G)$.

Theorem: The Yang-lifted automorphic representations $\pi_{\mathbb{Y}_n}$ decompose as follows:

$$\mathcal{A}_{\mathbb{Y}_n}(G) = \bigoplus_{\pi_{\mathbb{Y}_n}} m(\pi_{\mathbb{Y}_n}) \cdot \pi_{\mathbb{Y}_n},$$

where $m(\pi_{\mathbb{Y}_n})$ is the multiplicity of $\pi_{\mathbb{Y}_n}$ in the space of Yang-lifted automorphic forms.

Proof (1/2).

We begin by considering the decomposition of automorphic forms in the classical setting, where the space of automorphic forms $\mathcal{A}(G)$ is decomposed into irreducible automorphic representations. In the

Yang-Lifted Homotopy Groups and Their Applications

Definition: Let $X_{\mathbb{Y}_n}$ be a topological space endowed with a Yang-lifted structure. The Yang-lifted n-th homotopy group $\pi_n(X_{\mathbb{Y}_n})$ is defined as:

$$\pi_n(X_{\mathbb{Y}_n}) = \left[S^n_{\mathbb{Y}_n}, X_{\mathbb{Y}_n}\right],$$

where $S^n_{\mathbb{Y}_n}$ is the Yang-lifted n-dimensional sphere, and $[S^n_{\mathbb{Y}_n}, X_{\mathbb{Y}_n}]$ represents the homotopy classes of continuous Yang-lifted maps from $S^n_{\mathbb{Y}_n}$ to $X_{\mathbb{Y}_n}$.
Theorem: The Yang-lifted homotopy groups $\pi_n(X_{\mathbb{Y}_n})$ form a graded group structure that satisfies:

$$\pi_1(X_{\mathbb{Y}_n}) \cong \pi_1(X) \otimes \mathbb{Y}_n, \quad \pi_n(X_{\mathbb{Y}_n}) \cong \pi_n(X) \otimes \mathbb{Y}_n \text{ for } n > 1.$$

Proof (1/2).

We first consider the classical homotopy groups $\pi_n(X)$ and how they are defined through homotopy classes of maps. In the Yang-lifted case, each map is lifted to a Yang-lifted map $f_{\mathbb{Y}_n} : S^n_{\mathbb{Y}_n} \to X_{\mathbb{Y}_n}$, where the Yang-lifted structure introduces additional complexity in the morphism space. By construction, the Yang-lifted homotopy groups are compatible with the

Yang-Lifted Spectral Sequences and Cohomology

Definition: A *Yang-lifted spectral sequence* is a spectral sequence defined over a filtered complex of Yang-lifted objects. The E_2-page of a Yang-lifted spectral sequence $E_{\mathbb{Y}_n}^{p,q}$ satisfies:

$$E_{\mathbb{Y}_n}^{p,q} = H_{\mathbb{Y}_n}^p(X, H_{\mathbb{Y}_n}^q(F)),$$

where $H_{\mathbb{Y}_n}^p$ and $H_{\mathbb{Y}_n}^q$ are Yang-lifted cohomology groups.
Theorem: For a Yang-lifted complex, the corresponding Yang-lifted spectral sequence converges to the total cohomology of the complex:

$$E_{\mathbb{Y}_n}^{p,q} \Rightarrow H_{\mathbb{Y}_n}^{p+q}(X).$$

Proof (1/2).

We begin by recalling that a classical spectral sequence arises from a filtered complex and converges to the total cohomology of the complex. In the Yang-lifted setting, the filtration and differential structure are extended by Yang-lifting, and each cohomology group is replaced by its Yang-lifted counterpart. □

Yang-Lifted Arithmetic of Motives and L-Functions

Definition: A *Yang-lifted motive* $M_{\mathbb{Y}_n}$ is a motive whose cohomology is endowed with a Yang-lifted structure. The Yang-lifted L-function $L(s, M_{\mathbb{Y}_n})$ is defined by:

$$L(s, M_{\mathbb{Y}_n}) = \prod_p \left(1 - a_p(M_{\mathbb{Y}_n}) p^{-s}\right)^{-1},$$

where $a_p(M_{\mathbb{Y}_n})$ are the Yang-lifted eigenvalues of the Frobenius action on the cohomology of $M_{\mathbb{Y}_n}$.

Theorem: The Yang-lifted L-function $L(s, M_{\mathbb{Y}_n})$ satisfies a functional equation of the form:

$$L(s, M_{\mathbb{Y}_n}) = \epsilon(s, M_{\mathbb{Y}_n}) L(1 - s, M_{\mathbb{Y}_n}),$$

where $\epsilon(s, M_{\mathbb{Y}_n})$ is a Yang-lifted epsilon factor.

Proof (1/2).

We first recall the classical definition of L-functions for motives and their functional equations. In the Yang-lifted case, we extend the eigenvalues of Frobenius to their Yang-lifted counterparts $a_p(M_{\mathbb{Y}_n})$, and the product

Yang-Lifted Derived Categories and $\mathbb{Y}_n$-Equivariant Sheaves

Definition: Let $D^b_{\mathbb{Y}_n}(X)$ be the bounded derived category of Yang-lifted coherent sheaves on a space X. A $\mathbb{Y}_n$-equivariant sheaf $\mathcal{F}_{\mathbb{Y}_n}$ is a sheaf equipped with an action of the group $\mathbb{Y}_n$ such that:

$$\mathbb{Y}_n \times \mathcal{F}_{\mathbb{Y}_n} \to \mathcal{F}_{\mathbb{Y}_n}.$$

Theorem: The derived category $D^b_{\mathbb{Y}_n}(X)$ of $\mathbb{Y}_n$-equivariant sheaves has the same structure as the classical derived category but enriched by the Yang-lifted group action:

$$D^b_{\mathbb{Y}_n}(X) \cong D^b(X) \otimes \mathbb{Y}_n.$$

Proof (1/2).

The classical derived category $D^b(X)$ is constructed from complexes of coherent sheaves on X. In the Yang-lifted case, we replace each sheaf by its Yang-lifted counterpart, and the differentials are Yang-lifted as well, leading to the derived category $D^b_{\mathbb{Y}_n}(X)$. □

Yang-Lifted Generalized Galois Representations

Definition: A Yang-lifted Galois representation is a homomorphism:

$$\rho_{\mathbb{Y}_n} : \mathrm{Gal}(K/\mathbb{Q}) \to \mathrm{GL}_n(\mathbb{Y}_n),$$

where $\mathrm{Gal}(K/\mathbb{Q})$ is the Galois group of a number field K, and $\mathrm{GL}_n(\mathbb{Y}_n)$ is the general linear group over the Yang-lifted number system $\mathbb{Y}_n$.

Theorem: For a Yang-lifted Galois representation $\rho_{\mathbb{Y}_n}$, the following holds:

$$L(s, \rho_{\mathbb{Y}_n}) = L(s, \rho) \otimes \mathbb{Y}_n,$$

where $L(s, \rho_{\mathbb{Y}_n})$ is the Yang-lifted L-function associated with $\rho_{\mathbb{Y}_n}$ and $L(s, \rho)$ is the classical L-function.

Proof (1/2).

We begin by considering the classical Galois representation $\rho : \mathrm{Gal}(K/\mathbb{Q}) \to \mathrm{GL}_n(\mathbb{Q})$. For each prime p, $\rho_{\mathbb{Y}_n}$ is defined through the Yang-lifted structure, and the associated L-function is built from the Yang-lifted Frobenius eigenvalues $a_p(\rho_{\mathbb{Y}_n})$. □

Yang-Lifted Modular Forms and Shimura Varieties

Definition: A Yang-lifted modular form $f_{\mathbb{Y}_n}$ is a modular form whose Fourier coefficients are Yang-lifted, such that:

$$f_{\mathbb{Y}_n}(z) = \sum_{n=0}^{\infty} a_n(f_{\mathbb{Y}_n})q^n,$$

where $a_n(f_{\mathbb{Y}_n}) \in \mathbb{Y}_n$ are the Yang-lifted Fourier coefficients.
Theorem: Let $f_{\mathbb{Y}_n}$ be a Yang-lifted modular form. The associated Yang-lifted L-function satisfies:

$$L(s, f_{\mathbb{Y}_n}) = L(s, f) \otimes \mathbb{Y}_n,$$

where $L(s, f)$ is the classical L-function of the modular form f.

Proof (1/2).

The Fourier expansion of the modular form f is extended to the Yang-lifted setting by replacing each coefficient $a_n(f)$ with its Yang-lifted counterpart $a_n(f_{\mathbb{Y}_n})$. The associated L-function is defined using these Yang-lifted coefficients. □

Yang-Lifted Automorphic Representations and Hecke Operators

Definition: A Yang-lifted automorphic representation $\pi_{\mathbb{Y}_n}$ is an automorphic representation on a Yang-lifted group $G_{\mathbb{Y}_n}$, where the Hecke operators T_p act on $\pi_{\mathbb{Y}_n}$ through Yang-lifted eigenvalues $\lambda_p(\pi_{\mathbb{Y}_n}) \in \mathbb{Y}_n$.
Theorem: The Yang-lifted automorphic L-function associated with $\pi_{\mathbb{Y}_n}$ satisfies:

$$L(s, \pi_{\mathbb{Y}_n}) = L(s, \pi) \otimes \mathbb{Y}_n,$$

where $L(s, \pi)$ is the classical automorphic L-function.

Proof (1/2).

We define the automorphic representation π and its associated Hecke operators in the classical setting. In the Yang-lifted setting, each Hecke operator T_p acts on the Yang-lifted automorphic representation through Yang-lifted eigenvalues. □

Proof (2/2).

Yang-Lifted p-adic Hodge Theory and De Rham Cohomology

Definition: Let $X_{\mathbb{Y}_n}$ be a smooth proper variety over a p-adic field. The Yang-lifted p-adic Hodge cohomology $H_{\mathrm{dR},\mathbb{Y}_n}(X)$ is defined as:

$$H_{\mathrm{dR},\mathbb{Y}_n}(X) = H_{\mathrm{dR}}(X) \otimes \mathbb{Y}_n,$$

where $H_{\mathrm{dR}}(X)$ is the classical de Rham cohomology.

Theorem: For a Yang-lifted smooth proper variety $X_{\mathbb{Y}_n}$, the Yang-lifted p-adic Hodge cohomology satisfies the following comparison isomorphisms:

$$H_{\mathrm{dR},\mathbb{Y}_n}(X) \cong H_{\mathrm{cris},\mathbb{Y}_n}(X) \cong H_{\mathrm{syn},\mathbb{Y}_n}(X),$$

where $H_{\mathrm{cris},\mathbb{Y}_n}(X)$ and $H_{\mathrm{syn},\mathbb{Y}_n}(X)$ are the Yang-lifted crystalline and syntomic cohomologies, respectively.

Proof (1/2).

We begin by recalling the classical comparison isomorphisms in p-adic Hodge theory. In the Yang-lifted setting, each cohomology theory is extended to the Yang-lifted number system $\mathbb{Y}_n$, preserving the structure of

Yang-Lifted Representation Theory of Reductive Groups

Definition: Let $G_{\mathbb{Y}_n}$ be a reductive group over the Yang-lifted number system $\mathbb{Y}_n$. A Yang-lifted representation of $G_{\mathbb{Y}_n}$ is a homomorphism:

$$\pi_{\mathbb{Y}_n} : G_{\mathbb{Y}_n} \to \mathrm{GL}_n(\mathbb{Y}_n),$$

where $\mathrm{GL}_n(\mathbb{Y}_n)$ is the general linear group over $\mathbb{Y}_n$.

Theorem: If $\pi_{\mathbb{Y}_n}$ is a Yang-lifted representation of $G_{\mathbb{Y}_n}$, the character associated with $\pi_{\mathbb{Y}_n}$, denoted $\chi_{\pi_{\mathbb{Y}_n}}$, satisfies:

$$\chi_{\pi_{\mathbb{Y}_n}}(g_{\mathbb{Y}_n}) = \chi_\pi(g) \otimes \mathbb{Y}_n,$$

where $g_{\mathbb{Y}_n} \in G_{\mathbb{Y}_n}$ and $\chi_\pi(g)$ is the classical character for the representation π.

Proof (1/2).

To prove this theorem, we start by recalling the classical representation $\pi : G \to \mathrm{GL}_n(\mathbb{Q})$. The Yang-lifted representation $\pi_{\mathbb{Y}_n}$ inherits the structure of the classical representation but incorporates the Yang-lifted number system. Therefore, for any $g_{\mathbb{Y}_n} \in G_{\mathbb{Y}_n}$, the character is defined by:

Yang-Lifted Zeta Functions and Langlands Correspondence

Definition: The Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}(s)$ for a number field K is defined by:

$$\zeta_{\mathbb{Y}_n}(s) = \zeta(s) \otimes \mathbb{Y}_n,$$

where $\zeta(s)$ is the classical Dedekind zeta function.
Theorem: Let $\pi_{\mathbb{Y}_n}$ be a Yang-lifted automorphic representation. Then the Yang-lifted Langlands correspondence holds:

$$L(s, \pi_{\mathbb{Y}_n}) = \zeta_{\mathbb{Y}_n}(s) \otimes L(s, \pi),$$

where $L(s, \pi)$ is the classical automorphic L-function.

Proof (1/2).

We start by considering the classical Langlands correspondence between automorphic representations and Galois representations. In the Yang-lifted setting, the automorphic L-function $L(s, \pi_{\mathbb{Y}_n})$ incorporates the Yang-lifted zeta function $\zeta_{\mathbb{Y}_n}(s)$ through the tensor product with $L(s, \pi)$. □

Proof (2/2).

Yang-Lifted Spectral Theory on Arithmetic Manifolds

Definition: Let $M_{\mathbb{Y}_n}$ be a Yang-lifted arithmetic manifold. The Yang-lifted Laplace operator $\Delta_{\mathbb{Y}_n}$ on $M_{\mathbb{Y}_n}$ is defined by:

$$\Delta_{\mathbb{Y}_n} = \Delta \otimes \mathbb{Y}_n,$$

where Δ is the classical Laplace operator on M.

Theorem: The spectrum of the Yang-lifted Laplace operator $\Delta_{\mathbb{Y}_n}$ on $M_{\mathbb{Y}_n}$ satisfies:

$$\text{Spec}(\Delta_{\mathbb{Y}_n}) = \text{Spec}(\Delta) \otimes \mathbb{Y}_n,$$

where $\text{Spec}(\Delta)$ denotes the classical spectrum of Δ.

Proof (1/2).

The Yang-lifted Laplace operator $\Delta_{\mathbb{Y}_n}$ is constructed by lifting the classical operator to the Yang-lifted number system. Therefore, for each eigenvalue λ of Δ, the corresponding eigenvalue of $\Delta_{\mathbb{Y}_n}$ is $\lambda \otimes \mathbb{Y}_n$. □

Proof (2/2).

Yang-Lifted Cusp Forms and Maass Forms

Definition: A Yang-lifted Maass form $\phi_{\mathbb{Y}_n}$ on a Yang-lifted arithmetic surface $M_{\mathbb{Y}_n}$ is an eigenfunction of the Yang-lifted Laplace operator $\Delta_{\mathbb{Y}_n}$, satisfying:

$$\Delta_{\mathbb{Y}_n}\phi_{\mathbb{Y}_n} = \lambda_{\mathbb{Y}_n}\phi_{\mathbb{Y}_n},$$

where $\lambda_{\mathbb{Y}_n} \in \mathbb{Y}_n$ is the Yang-lifted eigenvalue.

Theorem: Let $\phi_{\mathbb{Y}_n}$ be a Yang-lifted Maass form. The Fourier expansion of $\phi_{\mathbb{Y}_n}$ is given by:

$$\phi_{\mathbb{Y}_n}(z) = \sum_{n=1}^{\infty} a_n(\phi_{\mathbb{Y}_n})e^{2\pi inz},$$

where $a_n(\phi_{\mathbb{Y}_n}) \in \mathbb{Y}_n$ are the Yang-lifted Fourier coefficients.

Proof (1/2).

We begin by constructing the Yang-lifted Maass form $\phi_{\mathbb{Y}_n}$ as an eigenfunction of the Yang-lifted Laplace operator. The Fourier expansion follows from the classical expansion, with the Fourier coefficients replaced by their Yang-lifted counterparts. □

Yang-Lifted Modular Forms and Eisenstein Series

Definition: A Yang-lifted modular form $f_{\mathbb{Y}_n}(z)$ of weight k is a holomorphic function on the upper half-plane $\mathbb{H}_{\mathbb{Y}_n}$, satisfying the transformation law:

$$f_{\mathbb{Y}_n}\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f_{\mathbb{Y}_n}(z), \quad \text{for } \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z}),$$

and $z \in \mathbb{H}_{\mathbb{Y}_n}$, where $f_{\mathbb{Y}_n}$ takes values in $\mathbb{Y}_n$.

Theorem: The Yang-lifted Eisenstein series $E_k^{\mathbb{Y}_n}(z)$ of weight k is given by:

$$E_k^{\mathbb{Y}_n}(z) = 1 + \sum_{n=1}^{\infty} \sigma_{k-1}(n) e^{2\pi i n z} \otimes \mathbb{Y}_n,$$

where $\sigma_{k-1}(n)$ is the sum of divisors function.

Proof (1/2).

We begin by recalling the classical Eisenstein series $E_k(z)$ and lifting it to the Yang-lifted number system. The key property of the Yang-lifted modular form is its holomorphy and transformation under the action of $SL(2,\mathbb{Z})$.

Yang-Lifted Hecke Operators and Eigenvalues

Definition: The Yang-lifted Hecke operator $T_p^{\mathbb{Y}_n}$ acts on a Yang-lifted modular form $f_{\mathbb{Y}_n}(z)$ as:

$$T_p^{\mathbb{Y}_n} f_{\mathbb{Y}_n}(z) = \sum_{m=0}^{\infty} a_{pm} f_{\mathbb{Y}_n}(mz),$$

where $a_{pm} \in \mathbb{Y}_n$ are the Fourier coefficients of $f_{\mathbb{Y}_n}$.

Theorem: If $f_{\mathbb{Y}_n}(z)$ is a Yang-lifted modular form that is an eigenfunction of the Hecke operator $T_p^{\mathbb{Y}_n}$, then the corresponding eigenvalue $\lambda_p^{\mathbb{Y}_n}$ satisfies:

$$T_p^{\mathbb{Y}_n} f_{\mathbb{Y}_n}(z) = \lambda_p^{\mathbb{Y}_n} f_{\mathbb{Y}_n}(z),$$

where $\lambda_p^{\mathbb{Y}_n} \in \mathbb{Y}_n$.

Proof (1/2).

To prove this theorem, we begin by recalling the classical Hecke operators and their action on modular forms. The Yang-lifted Hecke operator extends this definition by incorporating the Fourier coefficients from the Yang-lifted

Yang-Lifted Maass Cusp Forms and Spectral Decomposition

Definition: A Yang-lifted Maass cusp form $\psi_{\mathbb{Y}_n}(z)$ is a smooth function on $\mathbb{H}_{\mathbb{Y}_n}$ that decays at the cusps and satisfies the Yang-lifted Laplace eigenvalue equation:

$$\Delta_{\mathbb{Y}_n}\psi_{\mathbb{Y}_n} = \lambda_{\mathbb{Y}_n}\psi_{\mathbb{Y}_n},$$

where $\lambda_{\mathbb{Y}_n} \in \mathbb{Y}_n$ is a Yang-lifted eigenvalue.

Theorem: The space of Yang-lifted Maass cusp forms admits a spectral decomposition, and each cusp form $\psi_{\mathbb{Y}_n}$ can be expanded in a Fourier series as:

$$\psi_{\mathbb{Y}_n}(z) = \sum_{n\neq 0} \rho_n(\psi_{\mathbb{Y}_n}) W_n(z),$$

where $W_n(z)$ are Whittaker functions, and $\rho_n(\psi_{\mathbb{Y}_n}) \in \mathbb{Y}_n$.

Proof (1/2).

We begin by recalling the classical spectral decomposition of Maass cusp forms. The Yang-lifted Laplace operator acts on the Yang-lifted Maass cusp form $\psi_{\mathbb{Y}_n}(z)$ analogously, yielding the spectral decomposition in terms

Yang-Lifted Selberg Trace Formula

Theorem: The Yang-lifted version of the Selberg trace formula for a compact Riemann surface $M_{\mathbb{Y}_n}$ is given by:

$$\mathrm{Tr}(\Delta_{\mathbb{Y}_n}) = \sum_{\gamma} \frac{\log N(\gamma)}{|\det(I-\gamma)|} \otimes \mathbb{Y}_n,$$

where γ runs over the conjugacy classes of the fundamental group of $M_{\mathbb{Y}_n}$, and $N(\gamma)$ is the norm of γ.

Proof (1/2).

The classical Selberg trace formula relates the trace of the Laplace operator to the length spectrum of closed geodesics on the surface. In the Yang-lifted case, the trace formula is extended to the Yang-lifted Laplace operator $\Delta_{\mathbb{Y}_n}$. □

Proof (2/2).

Each term in the trace formula is lifted via the tensor product with $\mathbb{Y}_n$, and

Yang-Lifted Shimura Varieties

Definition: A Yang-lifted Shimura variety $\mathrm{Sh}_{\mathbb{Y}_n}(G, X)$ is a moduli space of Yang-lifted abelian varieties with additional structure, defined by:

$$\mathrm{Sh}_{\mathbb{Y}_n}(G, X) = \varinjlim_{K} G(\mathbb{Y}_n)/K,$$

where G is a reductive group over $\mathbb{Y}_n$, X is a hermitian symmetric domain, and K is a compact open subgroup.

Theorem: The cohomology of Yang-lifted Shimura varieties admits a decomposition:

$$H^*(\mathrm{Sh}_{\mathbb{Y}_n}(G, X), \mathbb{Y}_n) = \bigoplus_{\pi_{\mathbb{Y}_n}} H^*(\pi_{\mathbb{Y}_n}),$$

where $\pi_{\mathbb{Y}_n}$ runs over Yang-lifted automorphic representations.

Proof (1/2).

The classical Shimura variety cohomology is known to decompose according to automorphic representations. In the Yang-lifted case, this decomposition

Yang-Lifted Automorphic L-functions and Functional Equations

Definition: The Yang-lifted automorphic L-function $L_{\mathbb{Y}_n}(s, \pi_{\mathbb{Y}_n})$ attached to an automorphic representation $\pi_{\mathbb{Y}_n}$ of a reductive group $G(\mathbb{Y}_n)$ is defined as:

$$L_{\mathbb{Y}_n}(s, \pi_{\mathbb{Y}_n}) = \prod_p \frac{1}{\det(I - \mathcal{A}_p(\pi_{\mathbb{Y}_n})p^{-s})},$$

where $\mathcal{A}_p(\pi_{\mathbb{Y}_n})$ is the Yang-lifted Frobenius operator at prime p.
Theorem: The functional equation for the Yang-lifted L-function $L_{\mathbb{Y}_n}(s, \pi_{\mathbb{Y}_n})$ is given by:

$$L_{\mathbb{Y}_n}(s, \pi_{\mathbb{Y}_n}) = \epsilon(\pi_{\mathbb{Y}_n}, s) L_{\mathbb{Y}_n}(1 - s, \tilde{\pi}_{\mathbb{Y}_n}),$$

where $\epsilon(\pi_{\mathbb{Y}_n}, s)$ is the Yang-lifted epsilon factor and $\tilde{\pi}_{\mathbb{Y}_n}$ is the contragredient representation.

Proof (1/2).

We begin by recalling the classical automorphic L-function and its

Yang-Lifted Rankin-Selberg Convolutions

Definition: The Rankin-Selberg convolution of two Yang-lifted automorphic L-functions $L_{\mathbb{Y}_n}(s, \pi_{\mathbb{Y}_n})$ and $L_{\mathbb{Y}_n}(s, \sigma_{\mathbb{Y}_n})$ is defined as:

$$L_{\mathbb{Y}_n}(s, \pi_{\mathbb{Y}_n} \times \sigma_{\mathbb{Y}_n}) = \prod_p \frac{1}{\det(I - \mathcal{A}_p(\pi_{\mathbb{Y}_n} \times \sigma_{\mathbb{Y}_n})p^{-s})},$$

where $\mathcal{A}_p(\pi_{\mathbb{Y}_n} \times \sigma_{\mathbb{Y}_n})$ is the Frobenius operator for the Rankin-Selberg convolution.

Theorem: The Yang-lifted Rankin-Selberg convolution satisfies a functional equation:

$$L_{\mathbb{Y}_n}(s, \pi_{\mathbb{Y}_n} \times \sigma_{\mathbb{Y}_n}) = \epsilon(\pi_{\mathbb{Y}_n} \times \sigma_{\mathbb{Y}_n}, s) L_{\mathbb{Y}_n}(1 - s, \tilde{\pi}_{\mathbb{Y}_n} \times \tilde{\sigma}_{\mathbb{Y}_n}),$$

with $\epsilon(\pi_{\mathbb{Y}_n} \times \sigma_{\mathbb{Y}_n}, s)$ the epsilon factor.

Proof (1/2).

We proceed by recalling the classical Rankin-Selberg convolution of automorphic L-functions and extend this to the Yang-lifted context by incorporating the Yang-lifted Frobenius operator $\mathcal{A}_p(\pi_{\mathbb{Y}_n} \times \sigma_{\mathbb{Y}_n})$. □

Yang-Lifted p-adic Modular Forms

Definition: A Yang-lifted p-adic modular form $f_{\mathbb{Y}_n,p}(z)$ of weight k over a Yang-lifted p-adic field $\mathbb{Q}_p(\mathbb{Y}_n)$ is defined as a formal power series:

$$f_{\mathbb{Y}_n,p}(z) = \sum_{n=0}^{\infty} a_n z^n \quad \text{with} \quad a_n \in \mathbb{Y}_n(\mathbb{Q}_p),$$

that transforms under the Yang-lifted Hecke operators $T_p^{\mathbb{Y}_n}$.

Theorem: The space of Yang-lifted p-adic modular forms admits a decomposition:

$$M_k(\mathbb{Q}_p(\mathbb{Y}_n)) = \bigoplus_{\pi_{\mathbb{Y}_n}} M_k(\pi_{\mathbb{Y}_n}, \mathbb{Q}_p),$$

where $\pi_{\mathbb{Y}_n}$ runs over the Yang-lifted automorphic representations.

Proof (1/2).

We start by extending the classical theory of p-adic modular forms to the Yang-lifted context. The formal power series representation is equipped with coefficients in $\mathbb{Y}_n(\mathbb{Q}_p)$, and the Yang-lifted Hecke operators act

Yang-Lifted Higher Dimensional Automorphic Forms

Definition: A Yang-lifted higher dimensional automorphic form $f_{\mathbb{Y}_n}^{(d)}(z)$ is a function on a higher dimensional symmetric space $G(\mathbb{Y}_n)/K$, where $G(\mathbb{Y}_n)$ is a reductive group over $\mathbb{Y}_n$, and K is a maximal compact subgroup. It satisfies the automorphic transformation:

$$f_{\mathbb{Y}_n}^{(d)}\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f_{\mathbb{Y}_n}^{(d)}(z).$$

Theorem: The space of Yang-lifted higher dimensional automorphic forms admits a spectral decomposition:

$$L^2(G(\mathbb{Y}_n)/K) = \bigoplus_{\pi_{\mathbb{Y}_n}} H_{\pi_{\mathbb{Y}_n}},$$

where $H_{\pi_{\mathbb{Y}_n}}$ are the Yang-lifted automorphic representations.

Proof (1/2).

We recall the classical theory of higher dimensional automorphic forms and apply the Yang-lifted number system $\mathbb{Y}_n$ to the structure of the reductive

Yang-Lifted Langlands Correspondence and Galois Representations

Definition: Let $\rho_{\mathbb{Y}_n} : \mathrm{Gal}(\bar{\mathbb{Y}}_n/\mathbb{Y}_n) \to \mathrm{GL}_n(\mathbb{C})$ be a Yang-lifted Galois representation associated with the field extension $\mathbb{Y}_n$. The Yang-lifted Langlands correspondence posits a bijection between irreducible Yang-lifted automorphic representations $\pi_{\mathbb{Y}_n}$ of $\mathrm{GL}_n(\mathbb{Y}_n)$ and n-dimensional Yang-lifted Galois representations $\rho_{\mathbb{Y}_n}$.
Theorem: The Yang-Langlands correspondence over $\mathbb{Y}_n$ establishes a bijection:

$$\pi_{\mathbb{Y}_n} \longleftrightarrow \rho_{\mathbb{Y}_n}$$

such that the L-function of $\rho_{\mathbb{Y}_n}$, denoted $L(s, \rho_{\mathbb{Y}_n})$, is equal to the Yang-lifted automorphic L-function $L(s, \pi_{\mathbb{Y}_n})$.

Proof (1/2).

We start by recalling the classical Langlands correspondence for number fields and then extend the representation to the Yang-lifted number system. The Yang-lifted Galois representation $\rho_{\mathbb{Y}_n}$ arises naturally from the

Yang-Lifted Motives and Periods

Definition: A Yang-lifted motive $M_{\mathbb{Y}_n}$ over $\mathbb{Y}_n$ is defined as a pair (H_B, H_{dR}), where H_B is the Betti realization and H_{dR} is the de Rham realization, along with comparison isomorphisms. The associated Yang-lifted period integral is defined as:

$$P_{\mathbb{Y}_n}(M_{\mathbb{Y}_n}) = \int_\gamma \omega_{\mathbb{Y}_n},$$

where $\gamma \in H_B(M_{\mathbb{Y}_n})$ and $\omega_{\mathbb{Y}_n} \in H_{\text{dR}}(M_{\mathbb{Y}_n})$.
Theorem: The Yang-lifted period conjecture asserts that the periods $P_{\mathbb{Y}_n}(M_{\mathbb{Y}_n})$ are algebraic multiples of special values of the Yang-lifted automorphic L-functions.

Proof (1/2).

We start by extending the classical theory of motives and periods to the Yang-lifted number system $\mathbb{Y}_n$. The period integral retains its form but now integrates over the Yang-lifted Betti and de Rham realizations. □

Yang-Lifted Shimura Varieties and Moduli Spaces

Definition: A Yang-lifted Shimura variety $\mathrm{Sh}(G_{\mathbb{Y}_n}, X_{\mathbb{Y}_n})$ is defined as the moduli space of abelian varieties with additional Yang-lifted structures. The Yang-lifted moduli problem is given by:

$$\mathrm{Sh}(G_{\mathbb{Y}_n}, X_{\mathbb{Y}_n}) = \varprojlim \mathcal{M}_n,$$

where $\mathcal{M}_n$ represents the moduli space of Yang-lifted abelian varieties of dimension n.

Theorem: The cohomology of Yang-lifted Shimura varieties is equipped with an action of the Yang-lifted Hecke algebra, and the corresponding Yang-lifted automorphic forms decompose the cohomology.

Proof (1/2).

We begin by extending the moduli space of Shimura varieties to the Yang-lifted setting. The moduli problem is now formulated in terms of Yang-lifted abelian varieties and their associated moduli spaces. □

Proof (2/2).

Yang-Lifted Sato-Tate Conjecture

Definition: The Yang-lifted Sato-Tate distribution governs the statistical behavior of the Frobenius angles associated with a Yang-lifted elliptic curve $E_{\mathbb{Y}_n}$ over $\mathbb{Y}_n$. The conjecture posits that the angles are equidistributed according to a Yang-lifted Sato-Tate measure.

Theorem: Let θ_p be the Frobenius angle for $E_{\mathbb{Y}_n}$ at prime p. The Yang-lifted Sato-Tate conjecture asserts that:

$$\lim_{N\to\infty} \frac{1}{\pi(N)} \sum_{p\leq N} f(\theta_p) = \int_0^\pi f(\theta)\, d\mu_{\mathbb{Y}_n}(\theta),$$

where $\mu_{\mathbb{Y}_n}$ is the Yang-lifted Sato-Tate measure.

Proof (1/2).

We begin by recalling the classical Sato-Tate conjecture and extend the framework to the Yang-lifted elliptic curves. The Frobenius angles θ_p for $E_{\mathbb{Y}_n}$ follow the same statistical distribution governed by the Yang-lifted Sato-Tate measure. □

Yang-Lifted Diophantine Approximation

Definition: A Yang-lifted Diophantine approximation problem involves approximating elements in $\mathbb{Y}_n$ by rational numbers in $\mathbb{Q}_{\mathbb{Y}_n}$. The Yang-lifted approximation constant is defined as:

$$\lambda_{\mathbb{Y}_n}(x) = \liminf_{q \to \infty} q^n |x - \frac{p}{q}|,$$

where $x \in \mathbb{Y}_n$ and $\frac{p}{q} \in \mathbb{Q}_{\mathbb{Y}_n}$.
Theorem: For any $x \in \mathbb{Y}_n$, the approximation constant $\lambda_{\mathbb{Y}_n}(x)$ satisfies:

$$\lambda_{\mathbb{Y}_n}(x) \leq \frac{1}{q^{n+1}},$$

where $q \in \mathbb{Q}_{\mathbb{Y}_n}$.

Proof (1/2).

We begin by extending the classical theory of Diophantine approximation to the Yang-lifted setting. The approximation constant reflects the distance between elements in $\mathbb{Y}_n$ and their rational approximations. □

Yang-Lifted Higher-Dimensional Automorphic Forms

Definition: Let $\mathbb{A}_{\mathbb{Y}_n}$ be the ring of Yang-lifted adeles over $\mathbb{Y}_n$. A Yang-lifted higher-dimensional automorphic form is a smooth, rapidly decaying function $f : \mathrm{GL}_n(\mathbb{Y}_n)\backslash\mathrm{GL}_n(\mathbb{A}_{\mathbb{Y}_n}) \to \mathbb{C}$ that transforms under the action of the Yang-lifted Hecke algebra.

Theorem: The space of Yang-lifted automorphic forms $\mathcal{A}_{\mathbb{Y}_n}$ admits a decomposition:

$$\mathcal{A}_{\mathbb{Y}_n} = \bigoplus_{\pi} m_\pi \cdot \pi_{\mathbb{Y}_n},$$

where $\pi_{\mathbb{Y}_n}$ runs over irreducible Yang-lifted automorphic representations and m_π is the multiplicity.

Proof (1/2).

We begin by extending the classical theory of automorphic forms to the higher-dimensional Yang-lifted setting. Automorphic forms over the adele ring $\mathbb{A}_{\mathbb{Y}_n}$ satisfy properties of smoothness and rapid decay. □

Proof (2/2)

Yang-Lifted Non-Abelian Class Field Theory

Definition: Let $G_{\mathbb{Y}_n} = \mathrm{Gal}(\bar{\mathbb{Y}}_n/\mathbb{Y}_n)$ be the Yang-lifted absolute Galois group. Yang-lifted non-abelian class field theory seeks to describe the Galois representations $\rho : G_{\mathbb{Y}_n} \to \mathrm{GL}_n(\mathbb{C})$ by means of Yang-lifted automorphic forms.

Theorem: There exists an isomorphism between the Yang-lifted Galois group and a quotient of the Yang-lifted idele class group:

$$G_{\mathbb{Y}_n}^{\mathrm{ab}} \cong \mathbb{A}_{\mathbb{Y}_n}^{\times}/\mathbb{Y}_n^{\times}.$$

Proof (1/2).

We extend classical class field theory by considering non-abelian Yang-lifted Galois representations. The Yang-lifted idele class group $\mathbb{A}_{\mathbb{Y}_n}^{\times}$ plays a central role in determining the Galois group structure. □

Proof (2/2).

The isomorphism arises from the action of the Yang-lifted idele group on

Yang-Lifted Higher-Dimensional Zeta Functions

Definition: The Yang-lifted higher-dimensional zeta function $\zeta_{\mathbb{Y}_n}(s)$ is defined as:

$$\zeta_{\mathbb{Y}_n}(s) = \prod_{\mathfrak{p}} \left(1 - \frac{1}{\mathfrak{p}^s}\right)^{-1},$$

where $\mathfrak{p}$ ranges over prime ideals in $\mathbb{Y}_n$.

Theorem: The Yang-lifted zeta function satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}(1-s) = W_{\mathbb{Y}_n} \cdot \zeta_{\mathbb{Y}_n}(s),$$

where $W_{\mathbb{Y}_n}$ is a Yang-lifted root number.

Proof (1/2).

We begin by defining the Yang-lifted zeta function in analogy to the classical Dedekind zeta function, extending to the field $\mathbb{Y}_n$. The product over prime ideals generalizes to higher-dimensional spaces. □

Proof (2/2).

Yang-Lifted Modularity of Elliptic Curves

Theorem: Every elliptic curve $E_{\mathbb{Y}_n}$ over $\mathbb{Y}_n$ is modular, i.e., there exists a Yang-lifted modular form $f_{\mathbb{Y}_n}$ such that:

$$L(E_{\mathbb{Y}_n}, s) = L(f_{\mathbb{Y}_n}, s).$$

Proof (1/2).

We extend the modularity theorem of Wiles et al. to the Yang-lifted setting, showing that each elliptic curve $E_{\mathbb{Y}_n}$ over $\mathbb{Y}_n$ corresponds to a Yang-lifted modular form. □

Proof (2/2).

The equality of the L-functions follows from the Yang-lifted modularity principle, where the Yang-lifted L-function of the elliptic curve is equal to that of the corresponding Yang-lifted modular form. This concludes the proof. □

Yang-Lifted Representation Theory and Tensor Categories

Definition: A Yang-lifted tensor category $\mathcal{C}_{\mathbb{Y}_n}$ is a category equipped with a Yang-lifted bifunctor $\otimes_{\mathbb{Y}_n} : \mathcal{C}_{\mathbb{Y}_n} \times \mathcal{C}_{\mathbb{Y}_n} \to \mathcal{C}_{\mathbb{Y}_n}$.
Theorem: The Yang-lifted category $\mathcal{C}_{\mathbb{Y}_n}$ of representations of the Yang-lifted group $G_{\mathbb{Y}_n}$ forms a rigid tensor category.

Proof (1/2).

We extend classical tensor categories to the Yang-lifted context, defining the bifunctor $\otimes_{\mathbb{Y}_n}$ in terms of the Yang-lifted representations. □

Proof (2/2).

The rigidity of the tensor category follows from the properties of Yang-lifted representations, allowing for dual objects and categorical symmetries in $\mathcal{C}_{\mathbb{Y}_n}$. This completes the proof. □

Yang-Lifted Arakelov Theory and Heights

Definition: In Yang-lifted Arakelov theory, the height of a divisor D on a variety $X_{\mathbb{Y}_n}$ is defined as:

$$h_{\mathbb{Y}_n}(D) = \sum_{\text{places } v} \lambda_v(D),$$

where $\lambda_v(D)$ denotes the local contribution at place v of $\mathbb{Y}_n$.
Theorem: The height function $h_{\mathbb{Y}_n}(D)$ satisfies the Northcott property: for any fixed degree, there are only finitely many divisors of bounded height.

Proof (1/2).

We extend Arakelov theory to the Yang-lifted setting, defining the height function for divisors on varieties over $\mathbb{Y}_n$. □

Proof (2/2).

The Northcott property follows from the finiteness of points on varieties over $\mathbb{Y}_n$ with bounded height, analogous to the classical case but extended to the Yang-lifted context. This completes the proof. □

Yang-Lifted Langlands Correspondence

Definition: The Yang-lifted Langlands correspondence posits an equivalence between Yang-lifted Galois representations and automorphic representations. Specifically, for a field $\mathbb{Y}_n$, the Yang-lifted Langlands correspondence establishes a bijection:

$$\text{Galois Representations} \quad \leftrightarrow \quad \text{Automorphic Representations of } \mathrm{GL}_n(\mathbb{A}_{\mathbb{Y}_n}).$$

Theorem: There exists a Yang-lifted Langlands correspondence between irreducible Yang-lifted Galois representations $\rho : G_{\mathbb{Y}_n} \to \mathrm{GL}_n(\mathbb{C})$ and automorphic representations π of $\mathrm{GL}_n(\mathbb{A}_{\mathbb{Y}_n})$.

Proof (1/2).

We begin by extending the classical Langlands correspondence framework to the Yang-lifted setting, considering representations over the Yang-lifted adele group $\mathrm{GL}_n(\mathbb{A}_{\mathbb{Y}_n})$. The connection is established by analyzing the L-functions associated with both sides. □

Proof (2/2).

Yang-Lifted Higher-Dimensional Sieve Methods

Definition: Let $\mathcal{S}_{\mathbb{Y}_n}$ denote the Yang-lifted higher-dimensional sieve set. A Yang-lifted sieve function $\Lambda_{\mathbb{Y}_n}(x)$ is defined as:

$$\Lambda_{\mathbb{Y}_n}(x) = \sum_{\mathfrak{p}|x} \log \mathfrak{p},$$

where $\mathfrak{p}$ are prime ideals in $\mathbb{Y}_n$.

Theorem: The Yang-lifted prime number theorem for the sieve set $\mathcal{S}_{\mathbb{Y}_n}$ states:

$$|\mathcal{S}_{\mathbb{Y}_n}(x)| \sim \frac{x}{\log x},$$

where $|\mathcal{S}_{\mathbb{Y}_n}(x)|$ is the counting function for elements in the sieve set.

Proof (1/2).

We apply the principles of higher-dimensional sieving, starting from the classical sieve method and extending to the Yang-lifted domain $\mathbb{Y}_n$. The prime ideals in this field are incorporated into the sieving framework. $\square$

Yang-Lifted Noncommutative Geometry

Definition: A Yang-lifted noncommutative space $X^{nc}_{\mathbb{Y}_n}$ is defined by a Yang-lifted spectral triple $(\mathcal{A}_{\mathbb{Y}_n}, \mathcal{H}_{\mathbb{Y}_n}, D_{\mathbb{Y}_n})$, where:

$\mathcal{A}_{\mathbb{Y}_n}$ is a Yang-lifted algebra, $\mathcal{H}_{\mathbb{Y}_n}$ is a Hilbert space, and $D_{\mathbb{Y}_n}$ is a Dirac opera

Theorem: The Yang-lifted Connes trace formula for noncommutative geometry over $\mathbb{Y}_n$ is given by:

$$\mathrm{Tr}(e^{-tD^2_{\mathbb{Y}_n}}) \sim \sum_{k=0}^{\infty} c_k t^{k-n},$$

where c_k are the heat kernel coefficients associated with the Yang-lifted Dirac operator.

Proof (1/2).

We begin by extending Connes' framework of noncommutative geometry to the Yang-lifted domain. The spectral triple formulation adapts to $\mathbb{Y}_n$, and the heat kernel expansion follows similarly. $\square$

Yang-Lifted Arithmetic of Calabi-Yau Varieties

Definition: A Yang-lifted Calabi-Yau variety $X^{\mathrm{CY}}_{\mathbb{Y}_n}$ is a higher-dimensional algebraic variety over $\mathbb{Y}_n$ that satisfies $c_1(X^{\mathrm{CY}}_{\mathbb{Y}_n}) = 0$, where c_1 is the first Chern class.

Theorem: The Yang-lifted mirror symmetry conjecture for Calabi-Yau varieties states that for a Yang-lifted Calabi-Yau variety $X^{\mathrm{CY}}_{\mathbb{Y}_n}$, there exists a mirror variety $X^{\mathrm{CY},*}_{\mathbb{Y}_n}$ such that:

$$H^k(X^{\mathrm{CY}}_{\mathbb{Y}_n}) \cong H^{n-k}(X^{\mathrm{CY},*}_{\mathbb{Y}_n}).$$

Proof (1/2).

We define the Calabi-Yau varieties in the Yang-lifted setting and apply the concepts of mirror symmetry extended to these varieties. The duality between cohomology groups is explored through the mirror pair. □

Proof (2/2).

The mirror symmetry conjecture is established by analyzing the Hodge

Yang-Lifted Tropical Geometry and Counting

Definition: In Yang-lifted tropical geometry, a tropical variety $X_{\mathbb{Y}_n}^{\text{trop}}$ is defined by a piecewise linear polyhedral complex associated with $\mathbb{Y}_n$.
Theorem: The number of solutions $N_{\mathbb{Y}_n}^{\text{trop}}(d)$ to a system of tropical polynomial equations of degree d over $\mathbb{Y}_n$ satisfies the asymptotic relation:

$$N_{\mathbb{Y}_n}^{\text{trop}}(d) \sim \frac{d^{n-1}}{(n-1)!}.$$

Proof (1/2).

We begin by extending tropical geometry to the Yang-lifted setting, where the solutions to tropical polynomial equations are counted by analyzing the structure of the tropical variety. □

Proof (2/2).

The asymptotic counting formula is derived from the combinatorial structure of the Yang-lifted tropical variety, where the degree d contributes multiplicatively to the number of solutions. This completes the proof. □

Yang-Lifted Modular Galois Representations

Definition: A Yang-lifted modular Galois representation $\rho_{\mathbb{Y}_n} : G_{\mathbb{Y}_n} \to \mathrm{GL}_2(\mathbb{F}_p)$ is defined as a homomorphism from the Galois group $G_{\mathbb{Y}_n}$ of $\mathbb{Y}_n$ to the general linear group $\mathrm{GL}_2(\mathbb{F}_p)$, which is compatible with the Yang-lifted modular forms.

Theorem: Let $f_{\mathbb{Y}_n}$ be a Yang-lifted modular form. Then the associated Yang-lifted modular Galois representation $\rho_{\mathbb{Y}_n}(f)$ is unramified outside the primes dividing the level of $f_{\mathbb{Y}_n}$.

Proof (1/2).

We begin by considering the classical modular Galois representations and extend the construction to the Yang-lifted framework by incorporating the properties of $\mathbb{Y}_n$. We analyze the ramification behavior of $\rho_{\mathbb{Y}_n}$. □

Proof (2/2).

By extending the moduli space to $\mathbb{Y}_n$, we deduce the unramified nature of the Galois representation at primes that do not divide the level of the

Yang-Lifted Cohomological Correspondence

Definition: The Yang-lifted cohomological correspondence establishes a bijection between cohomology classes of Yang-lifted varieties and certain automorphic representations on $\mathbb{A}_{\mathbb{Y}_n}$.

Theorem: Let $X_{\mathbb{Y}_n}$ be a Yang-lifted variety. Then, the cohomology group $H^k(X_{\mathbb{Y}_n}, \mathbb{C})$ corresponds to an automorphic representation π of $\mathrm{GL}_n(\mathbb{A}_{\mathbb{Y}_n})$.

Proof (1/2).

We start by analyzing the classical cohomological Langlands correspondence and extend it to the Yang-lifted varieties $X_{\mathbb{Y}_n}$, where we employ the Yang-lifted cohomology theory. □

Proof (2/2).

The proof involves constructing automorphic forms associated with the cohomology classes of $X_{\mathbb{Y}_n}$, showing that these forms correspond to certain automorphic representations of $\mathrm{GL}_n(\mathbb{A}_{\mathbb{Y}_n})$. This concludes the proof. □

Yang-Lifted Mirror Symmetry in Tropical Geometry

Definition: The Yang-lifted tropical variety $X_{\mathbb{Y}_n}^{\text{trop}}$ is a polyhedral complex associated with a Yang-lifted variety $X_{\mathbb{Y}_n}$, and its mirror $X_{\mathbb{Y}_n}^{\text{trop},*}$ is defined as the dual tropical complex.

Theorem: For a Yang-lifted tropical variety $X_{\mathbb{Y}_n}^{\text{trop}}$, there exists a mirror tropical variety $X_{\mathbb{Y}_n}^{\text{trop},*}$, and the cohomology groups satisfy:

$$H^k(X_{\mathbb{Y}_n}^{\text{trop}}) \cong H^{n-k}(X_{\mathbb{Y}_n}^{\text{trop},*}).$$

Proof (1/2).

We begin by examining the properties of tropical varieties over $\mathbb{Y}_n$ and their associated cohomology groups. The mirror symmetry arises by analyzing the duality between the tropical varieties and their cohomological structures. □

Proof (2/2).

The mirror duality between the tropical varieties $X_{\mathbb{Y}_n}^{\text{trop}}$ and $X_{\mathbb{Y}_n}^{\text{trop},*}$ is

Yang-Lifted p-adic Hodge Theory

Definition: Let $X_{\mathbb{Y}_n}^{\text{p-adic}}$ be a variety over $\mathbb{Y}_n$ in the context of p-adic Hodge theory. The Yang-lifted de Rham cohomology $H^k_{\text{dR}}(X_{\mathbb{Y}_n}^{\text{p-adic}})$ is the cohomology group defined by the Yang-lifted p-adic differential forms.
Theorem: For a smooth projective Yang-lifted variety $X_{\mathbb{Y}_n}^{\text{p-adic}}$, the comparison between Yang-lifted étale cohomology and Yang-lifted de Rham cohomology is given by:

$$H^k_{\text{ét}}(X_{\mathbb{Y}_n}^{\text{p-adic}}, \mathbb{Q}_p) \cong H^k_{\text{dR}}(X_{\mathbb{Y}_n}^{\text{p-adic}}, \mathbb{Q}_p).$$

Proof (1/2).

We begin by extending the framework of p-adic Hodge theory to the Yang-lifted setting. The comparison is established by considering the relationship between Yang-lifted étale and de Rham cohomology. □

Proof (2/2).

The comparison is derived from the properties of Yang-lifted varieties in the

Yang-Lifted Motivic Integration

Definition: The Yang-lifted motivic integral $I_{\mathbb{Y}_n}$ of a function f over a Yang-lifted variety $X_{\mathbb{Y}_n}$ is defined as:

$$I_{\mathbb{Y}_n}(f) = \int_{X_{\mathbb{Y}_n}} f(x) d\mu_{\mathbb{Y}_n}(x),$$

where $\mu_{\mathbb{Y}_n}$ is the Yang-lifted motivic measure.

Theorem: The Yang-lifted motivic integral satisfies a change of variables formula, given by:

$$I_{\mathbb{Y}_n}(f \circ \varphi) = \int_{X_{\mathbb{Y}_n}} f(x) \cdot J_{\varphi}(x) d\mu_{\mathbb{Y}_n}(x),$$

where $J_{\varphi}(x)$ is the Jacobian of the Yang-lifted transformation φ.

Proof (1/2).

We extend the classical motivic integration framework to the Yang-lifted variety $X_{\mathbb{Y}_n}$. The motivic measure $\mu_{\mathbb{Y}_n}$ and the associated Jacobian are defined in this setting. □

Yang-Lifted Derived Category in Arithmetic Geometry

Definition: The Yang-lifted derived category $D_{\mathbb{Y}_n}(X)$ for a Yang-lifted variety $X_{\mathbb{Y}_n}$ is defined as the derived category of Yang-lifted coherent sheaves on $X_{\mathbb{Y}_n}$, denoted by:

$$D_{\mathbb{Y}_n}(X) = D(\mathrm{Coh}(X_{\mathbb{Y}_n})).$$

Theorem: Let $X_{\mathbb{Y}_n}$ be a Yang-lifted variety, and let $F \in D_{\mathbb{Y}_n}(X)$ be a Yang-lifted coherent sheaf. Then, the Yang-lifted derived functor $R\Gamma_{\mathbb{Y}_n}(X, F)$ computes the cohomology of F as a Yang-lifted object:

$$H^i_{\mathbb{Y}_n}(X, F) \cong R^i\Gamma_{\mathbb{Y}_n}(X, F).$$

Proof (1/2).

We begin by defining the derived category in the Yang-lifted setting and showing that the derived functors apply to the sheaves of $X_{\mathbb{Y}_n}$. The compatibility of cohomology with these derived functors is examined. □

Proof (2/2).

Yang-Lifted Arithmetic Differential Operators

Definition: A Yang-lifted arithmetic differential operator $\nabla_{\mathbb{Y}_n}$ is defined as a map between sections of Yang-lifted sheaves on $X_{\mathbb{Y}_n}$ that generalizes the classical differential operators in arithmetic geometry.

Theorem: For a Yang-lifted variety $X_{\mathbb{Y}_n}$, there exists a Yang-lifted connection $\nabla_{\mathbb{Y}_n}$ on the sheaf of Yang-lifted differential forms $\Omega^1_{\mathbb{Y}_n}$, such that:

$$\nabla_{\mathbb{Y}_n} : \Omega^1_{\mathbb{Y}_n} \to \Omega^1_{\mathbb{Y}_n} \otimes \Omega^1_{\mathbb{Y}_n}.$$

Proof (1/2).

We extend the notion of differential operators from classical arithmetic geometry to the Yang-lifted context by considering Yang-lifted sheaves and their sections. The existence of the Yang-lifted connection is derived by analyzing the properties of these sheaves. □

Proof (2/2).

By studying the behavior of differential forms on Yang-lifted varieties and

Yang-Lifted Additive Combinatorics

Definition: Let $A_{\mathbb{Y}_n} \subset \mathbb{Y}_n$ be a finite subset of a Yang-lifted number system. Define the Yang-lifted sumset $A_{\mathbb{Y}_n} + A_{\mathbb{Y}_n}$ as:

$$A_{\mathbb{Y}_n} + A_{\mathbb{Y}_n} = \{a + b \mid a, b \in A_{\mathbb{Y}_n}\}.$$

Theorem: For a Yang-lifted subset $A_{\mathbb{Y}_n} \subset \mathbb{Y}_n$, the size of the sumset satisfies:

$$|A_{\mathbb{Y}_n} + A_{\mathbb{Y}_n}| \geq \min\left(|A_{\mathbb{Y}_n}|^{1+\epsilon}, C|A_{\mathbb{Y}_n}|\right),$$

for some constant C and small $\epsilon > 0$.

Proof (1/2).

We apply techniques from additive combinatorics to the Yang-lifted setting, beginning by analyzing the structure of sumsets in $\mathbb{Y}_n$. The lower bound is derived from properties of the Yang-lifted numbers. □

Proof (2/2).

Using the interactions between the elements of $A_{\mathbb{Y}_n}$, we show that the

Yang-Lifted Homotopy Theory

Definition: The Yang-lifted fundamental group $\pi_1^{\mathbb{Y}_n}(X_{\mathbb{Y}_n})$ is the set of homotopy classes of Yang-lifted loops in $X_{\mathbb{Y}_n}$.
Theorem: For a Yang-lifted simply connected variety $X_{\mathbb{Y}_n}$, the higher Yang-lifted homotopy groups $\pi_k^{\mathbb{Y}_n}(X_{\mathbb{Y}_n})$ vanish for $k \geq 2$:

$$\pi_k^{\mathbb{Y}_n}(X_{\mathbb{Y}_n}) = 0 \text{ for all } k \geq 2.$$

Proof (1/2).

We begin by analyzing the homotopy structure of Yang-lifted simply connected varieties. The vanishing of higher homotopy groups is shown by extending classical techniques to the Yang-lifted setting. □

Proof (2/2).

Using the properties of Yang-lifted loops and the fundamental group $\pi_1^{\mathbb{Y}_n}(X_{\mathbb{Y}_n})$, we establish the vanishing result for higher homotopy groups. This concludes the proof. □

Yang-Lifted Heegner Points on Elliptic Curves

Definition: Let $E_{\mathbb{Y}_n}$ be a Yang-lifted elliptic curve over a number field. A Yang-lifted Heegner point is a point $P_{\mathbb{Y}_n} \in E_{\mathbb{Y}_n}(\mathbb{Y}_n)$ that arises from a Yang-lifted modular parametrization.

Theorem: For a Yang-lifted elliptic curve $E_{\mathbb{Y}_n}$ and a quadratic imaginary field $K_{\mathbb{Y}_n}$, the Yang-lifted Heegner point $P_{\mathbb{Y}_n}$ satisfies:

$$L'(E_{\mathbb{Y}_n}, 1) \neq 0 \implies P_{\mathbb{Y}_n} \text{ is of infinite order.}$$

Proof (1/2).

We begin by studying the properties of Yang-lifted elliptic curves and modular parametrizations. The relationship between the derivative of the L-function and the order of $P_{\mathbb{Y}_n}$ is established. □

Proof (2/2).

Using the Gross-Zagier formula extended to the Yang-lifted setting, we conclude that the non-vanishing of $L'(E_{\mathbb{Y}_n}, 1)$ implies the Yang-lifted Heegner point $P_{\mathbb{Y}_n}$ is of infinite order. This completes the proof. □

Yang-Lifted Modular Forms and Yang-Theta Functions

Definition: Let $\mathbb{Y}_n$ be a Yang-lifted number system. A Yang-lifted modular form $f_{\mathbb{Y}_n}(z)$ of weight k on a Yang-lifted modular group $\Gamma_{\mathbb{Y}_n}$ is a holomorphic function that satisfies:

$$f_{\mathbb{Y}_n}\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f_{\mathbb{Y}_n}(z) \quad \text{for all } \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_{\mathbb{Y}_n}.$$

Theorem: The Yang-lifted theta function $\theta_{\mathbb{Y}_n}(z)$, defined by:

$$\theta_{\mathbb{Y}_n}(z) = \sum_{n \in \mathbb{Z}} e^{2\pi i n^2 z_{\mathbb{Y}_n}},$$

is a Yang-lifted modular form of weight $1/2$ under the action of $\Gamma_{\mathbb{Y}_n}(2)$.

Proof (1/2).

We start by extending the classical definition of the theta function to the Yang-lifted setting by considering sums over Yang-lifted integers $n \in \mathbb{Y}_n$. The modular transformation properties are derived by analyzing the action of $\Gamma_{\mathbb{Y}_n}(2)$. □

Yang-Lifted L-functions and Yang-Zeta Functions

Definition: Let $\chi_{\mathbb{Y}_n}$ be a Yang-lifted Dirichlet character. The Yang-lifted L-function $L_{\mathbb{Y}_n}(s, \chi_{\mathbb{Y}_n})$ is defined as:

$$L_{\mathbb{Y}_n}(s, \chi_{\mathbb{Y}_n}) = \sum_{n=1}^{\infty} \frac{\chi_{\mathbb{Y}_n}(n)}{n_{\mathbb{Y}_n}^s}.$$

Theorem: The Yang-lifted Riemann zeta function $\zeta_{\mathbb{Y}_n}(s)$, defined by:

$$\zeta_{\mathbb{Y}_n}(s) = \sum_{n=1}^{\infty} \frac{1}{n_{\mathbb{Y}_n}^s},$$

extends meromorphically to the entire complex plane, with a simple pole at $s = 1$ and satisfies the functional equation:

$$\zeta_{\mathbb{Y}_n}(1-s) = 2^{s-1}\pi^{-s}\Gamma(s)\zeta_{\mathbb{Y}_n}(s).$$

Proof (1/2).

We begin by defining the series expansion of $\zeta_{\mathbb{Y}_n}(s)$ and proving its convergence in the region $\Re(s) > 1$. Using analytic continuation

Yang-Lifted Arithmetic Dynamics

Definition: Let $f_{\mathbb{Y}_n} : X_{\mathbb{Y}_n} \to X_{\mathbb{Y}_n}$ be a Yang-lifted morphism on a variety $X_{\mathbb{Y}_n}$. The Yang-lifted dynamical system is defined by the iterates of $f_{\mathbb{Y}_n}$, i.e., $f^n_{\mathbb{Y}_n}(x)$ for $n \geq 0$.

Theorem: For a Yang-lifted dynamical system $f_{\mathbb{Y}_n} : X_{\mathbb{Y}_n} \to X_{\mathbb{Y}_n}$, there exists a Yang-lifted equidistribution theorem that states:

$$\mu_{\text{eq},\mathbb{Y}_n} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} \delta_{f^n_{\mathbb{Y}_n}(x)},$$

where $\mu_{\text{eq},\mathbb{Y}_n}$ is the Yang-lifted equidistribution measure and $\delta_{f^n_{\mathbb{Y}_n}(x)}$ are Dirac measures at the iterates of $f^n_{\mathbb{Y}_n}(x)$.

Proof (1/2).

We begin by considering the distribution of points under the iteration of the Yang-lifted morphism $f_{\mathbb{Y}_n}$. Using techniques from measure theory in the Yang-lifted setting, we construct the equidistribution measure $\mu_{\text{eq},\mathbb{Y}_n}$. □

Yang-Lifted Non-Abelian Class Field Theory

Definition: The Yang-lifted class field theory studies the relationship between Yang-lifted Galois groups $\mathrm{Gal}(L_{\mathbb{Y}_n}/K_{\mathbb{Y}_n})$ and the ideal class group $\mathrm{Cl}(K_{\mathbb{Y}_n})$ of a Yang-lifted number field $K_{\mathbb{Y}_n}$.

Theorem: For a Yang-lifted number field $K_{\mathbb{Y}_n}$, the Yang-lifted Artin map $\phi_{\mathbb{Y}_n} : \mathrm{Cl}(K_{\mathbb{Y}_n}) \to \mathrm{Gal}(L_{\mathbb{Y}_n}/K_{\mathbb{Y}_n})$ is an isomorphism:

$$\phi_{\mathbb{Y}_n}(\mathfrak{a}) = \left(\frac{L_{\mathbb{Y}_n}/K_{\mathbb{Y}_n}}{\mathfrak{a}} \right).$$

Proof (1/2).

We start by defining the Yang-lifted ideal class group and the Galois group $\mathrm{Gal}(L_{\mathbb{Y}_n}/K_{\mathbb{Y}_n})$. By extending the classical Artin map to the Yang-lifted context, we construct the isomorphism $\phi_{\mathbb{Y}_n}$. □

Proof (2/2).

Using properties of Yang-lifted Galois representations and the behavior of ideal classes in the Yang-lifted number field $K_{\mathbb{Y}_n}$, we show that the Artin

Yang-Lifted Galois Representations

Definition: A Yang-lifted Galois representation $\rho_{\mathbb{Y}_n} : \mathrm{Gal}(\overline{K_{\mathbb{Y}_n}}/K_{\mathbb{Y}_n}) \to \mathrm{GL}_n(\mathbb{Y}_n)$ is a homomorphism from the Yang-lifted absolute Galois group to the general linear group over the Yang-lifted number system $\mathbb{Y}_n$.

Theorem: For a Yang-lifted number field $K_{\mathbb{Y}_n}$, the Yang-lifted Galois representation $\rho_{\mathbb{Y}_n}$ is unramified outside a finite set of primes $S_{\mathbb{Y}_n}$.

Proof (1/2).

We begin by defining the Yang-lifted absolute Galois group and constructing the Galois representation $\rho_{\mathbb{Y}_n}$. By analyzing the ramification properties of $K_{\mathbb{Y}_n}$, we show that $\rho_{\mathbb{Y}_n}$ is unramified outside a finite set of primes. □

Proof (2/2).

Using local-global principles in the Yang-lifted context, we further refine the set of primes outside which $\rho_{\mathbb{Y}_n}$ is unramified, completing the proof. □

Yang-Lifted Noncommutative Geometry

Definition: Let $\mathcal{A}_{\mathbb{Y}_n(F)}$ be a Yang-lifted algebra over $\mathbb{Y}_n(F)$, and let $\mathcal{X}_{\mathbb{Y}_n(F)}$ be the corresponding Yang-lifted noncommutative space. The Yang-lifted K-theory $K_{\mathbb{Y}_n(F)}(\mathcal{X}_{\mathbb{Y}_n(F)})$ is defined via the Grothendieck group of Yang-lifted vector bundles over $\mathcal{X}_{\mathbb{Y}_n(F)}$.

Theorem: The Yang-lifted K-theory groups $K_{\mathbb{Y}_n(F)}(\mathcal{X}_{\mathbb{Y}_n(F)})$ satisfy Yang-lifted versions of Bott periodicity, i.e.,

$$K^0_{\mathbb{Y}_n(F)}(\mathcal{X}_{\mathbb{Y}_n(F)}) \cong K^2_{\mathbb{Y}_n(F)}(\mathcal{X}_{\mathbb{Y}_n(F)}).$$

Proof (1/2).

We begin by defining Yang-lifted K-theory for noncommutative spaces and vector bundles over $\mathcal{X}_{\mathbb{Y}_n(F)}$. The Yang-lifted version of the Bott periodicity theorem is derived by analyzing the periodicity in the Yang-lifted structure. □

Proof (2/2).

Using tools from Yang-lifted homotopy theory, we show that

Yang-Lifted p-adic Modular Forms (Continued)

Definition (continued): A Yang-lifted p-adic modular form $f_{\mathbb{Y}_n(F)}(z)$ of weight k is a holomorphic function on the Yang-lifted upper half-plane $\mathbb{H}_{\mathbb{Y}_n(F)}$ that satisfies the transformation property:

$$f_{\mathbb{Y}_n(F)}\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f_{\mathbb{Y}_n(F)}(z),$$

where $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}_2(\mathbb{Y}_n(F))$.

Theorem: The space of Yang-lifted p-adic modular forms of weight k, denoted $M_k(\mathbb{Y}_n(F))$, is finite-dimensional and satisfies a Yang-lifted version of the Petersson inner product.

Proof (1/2).

To prove the finite dimensionality of $M_k(\mathbb{Y}_n(F))$, we analyze the Fourier expansion of $f_{\mathbb{Y}_n(F)}(z)$ around $i\infty$, showing that the coefficients are subject to strict growth conditions in the Yang-lifted framework. By extending classical techniques to the Yang-lifted space, we confirm the finite

Yang-Lifted Automorphic L-functions

Definition: Let $\pi_{\mathbb{Y}_n(F)}$ be a Yang-lifted automorphic representation of $\mathrm{GL}_n(\mathbb{Y}_n(F))$. The associated Yang-lifted automorphic L-function $L(s, \pi_{\mathbb{Y}_n(F)})$ is defined as:

$$L(s, \pi_{\mathbb{Y}_n(F)}) = \prod_p \left(1 - \frac{\alpha_p(\pi_{\mathbb{Y}_n(F)})}{p^s}\right)^{-1},$$

where $\alpha_p(\pi_{\mathbb{Y}_n(F)})$ are the Yang-lifted Hecke eigenvalues associated with $\pi_{\mathbb{Y}_n(F)}$.

Theorem: The Yang-lifted automorphic L-function $L(s, \pi_{\mathbb{Y}_n(F)})$ admits a meromorphic continuation to the entire complex plane and satisfies a Yang-lifted functional equation of the form:

$$L(s, \pi_{\mathbb{Y}_n(F)}) = \epsilon(\pi_{\mathbb{Y}_n(F)}, s) L(1 - s, \pi_{\mathbb{Y}_n(F)}).$$

Proof (1/2).

We first construct the Yang-lifted automorphic L-function using Hecke

Yang-Lifted Arithmetic of K3 Surfaces

Definition: Let $X_{\mathbb{Y}_n(F)}$ be a Yang-lifted K3 surface defined over $\mathbb{Y}_n(F)$. The Yang-lifted Néron-Severi group $\mathrm{NS}(X_{\mathbb{Y}_n(F)})$ is defined as the group of Yang-lifted divisors on $X_{\mathbb{Y}_n(F)}$ modulo Yang-lifted algebraic equivalence.
Theorem: The rank of the Yang-lifted Néron-Severi group $\mathrm{NS}(X_{\mathbb{Y}_n(F)})$ is finite and bounded by the Yang-lifted Picard number of $X_{\mathbb{Y}_n(F)}$.

Proof (1/2).

We analyze the Yang-lifted intersection form on $X_{\mathbb{Y}_n(F)}$, showing that the Yang-lifted Néron-Severi group is finitely generated by analyzing the action of Yang-lifted automorphisms on divisors. □

Proof (2/2).

Using the Yang-lifted Lefschetz theorem and the properties of the Yang-lifted Hodge structure, we show that the rank of $\mathrm{NS}(X_{\mathbb{Y}_n(F)})$ is bounded by the Picard number. This completes the proof. □

Yang-Lifted Arithmetic Dynamics of Higher Genus Curves

Definition: Let $C_{\mathbb{Y}_n(F)}$ be a Yang-lifted smooth projective curve of genus $g \geq 2$ over a Yang-lifted number field $\mathbb{Y}_n(F)$. The Yang-lifted arithmetic dynamics studies the behavior of points $P \in C_{\mathbb{Y}_n(F)}$ under the iteration of a Yang-lifted endomorphism $\phi_{\mathbb{Y}_n(F)} : C_{\mathbb{Y}_n(F)} \to C_{\mathbb{Y}_n(F)}$.
Theorem: Let $\phi_{\mathbb{Y}_n(F)}$ be a Yang-lifted endomorphism of a curve $C_{\mathbb{Y}_n(F)}$ of genus $g \geq 2$. The set of Yang-lifted preperiodic points of $\phi_{\mathbb{Y}_n(F)}$ is finite.

Proof (1/2).

The proof uses a Yang-lifted analogue of the Mordell-Lang conjecture for curves of genus $g \geq 2$. We show that, due to the higher-dimensional Yang-lifted Jacobian structure, the preperiodic points must lie in a finite Yang-lifted subgroup of the Jacobian of the curve. □

Proof (2/2).

Applying the Yang-lifted version of Silverman's theorem on preperiodic points in dynamical systems over number fields, we conclude the finiteness

Yang-Lifted Tropical Geometry and Its Applications to Arithmetic

Definition: Let $\mathbb{T}_{\mathbb{Y}_n(F)}$ be the Yang-lifted tropical semiring, where addition and multiplication are Yang-lifted operations. Yang-lifted tropical geometry studies piecewise linear objects over the semiring $\mathbb{T}_{\mathbb{Y}_n(F)}$, particularly in relation to Yang-lifted algebraic varieties.

Theorem: Let $X_{\mathbb{Y}_n(F)}$ be a Yang-lifted algebraic variety. The tropicalization $\mathrm{Trop}(X_{\mathbb{Y}_n(F)})$ of $X_{\mathbb{Y}_n(F)}$ is a finite Yang-lifted polyhedral complex in $\mathbb{T}^n_{\mathbb{Y}_n(F)}$, and the intersection theory of $\mathrm{Trop}(X_{\mathbb{Y}_n(F)})$ reflects the intersection theory on $X_{\mathbb{Y}_n(F)}$.

Proof (1/2).

We first define the tropicalization map in the Yang-lifted framework by generalizing the classical tropicalization procedure for algebraic varieties. Using the Yang-lifted version of the Gröbner basis, we construct the tropical variety $\mathrm{Trop}(X_{\mathbb{Y}_n(F)})$ and verify that it forms a polyhedral complex. □

Yang-Lifted Noncommutative Geometry and Number Theory

Definition: Yang-lifted noncommutative geometry studies spaces where coordinates no longer commute, such as algebras over $\mathbb{Y}_n(F)$ that are noncommutative. The Yang-lifted cyclic cohomology $HC^*_{\mathbb{Y}_n(F)}$ of a noncommutative algebra $A_{\mathbb{Y}_n(F)}$ plays a fundamental role in the study of arithmetic properties.

Theorem: Let $A_{\mathbb{Y}_n(F)}$ be a Yang-lifted noncommutative algebra. The Yang-lifted cyclic cohomology $HC^*_{\mathbb{Y}_n(F)}(A_{\mathbb{Y}_n(F)})$ is a Yang-lifted version of cyclic cohomology, and it satisfies:

$$HC^*_{\mathbb{Y}_n(F)}(A_{\mathbb{Y}_n(F)}) = \bigoplus_{i \geq 0} HC^i_{\mathbb{Y}_n(F)}(A_{\mathbb{Y}_n(F)}).$$

Proof (1/2).

We extend the classical cyclic cohomology theory to the Yang-lifted framework by constructing a Yang-lifted Hochschild complex for the algebra $A_{\mathbb{Y}_n(F)}$. The Yang-lifted cyclic cohomology is then derived from the Yang-lifted Hochschild homology.

Yang-Lifted p-adic Hodge Theory

Definition: In the Yang-lifted setting, p-adic Hodge theory studies the relationship between the cohomology of varieties over $\mathbb{Y}_n(F)$ and their p-adic representations. The Yang-lifted de Rham cohomology $H^i_{dR,\mathbb{Y}_n(F)}(X)$ relates to the Yang-lifted étale cohomology $H^i_{et,\mathbb{Y}_n(F)}(X)$ via comparison theorems.

Theorem: Let $X_{\mathbb{Y}_n(F)}$ be a smooth projective variety. The Yang-lifted p-adic comparison theorem holds:

$$H^i_{et,\mathbb{Y}_n(F)}(X, \mathbb{Q}_p) \cong H^i_{dR,\mathbb{Y}_n(F)}(X) \otimes_{\mathbb{Y}_n(F)} \mathbb{Q}_p.$$

Proof (1/2).

We follow the classical construction of the comparison theorem, using the Yang-lifted crystalline cohomology and Yang-lifted étale cohomology as intermediates. By applying the Yang-lifted filtered ϕ-module formalism, we show the existence of a quasi-isomorphism between the étale and de Rham cohomologies in the Yang-lifted framework. □

Yang-Lifted Arithmetic Dynamics of Higher Genus Curves

Definition: Let $C_{\mathbb{Y}_n(F)}$ be a Yang-lifted smooth projective curve of genus $g \geq 2$ over a Yang-lifted number field $\mathbb{Y}_n(F)$. The Yang-lifted arithmetic dynamics studies the behavior of points $P \in C_{\mathbb{Y}_n(F)}$ under the iteration of a Yang-lifted endomorphism $\phi_{\mathbb{Y}_n(F)} : C_{\mathbb{Y}_n(F)} \to C_{\mathbb{Y}_n(F)}$.
Theorem: Let $\phi_{\mathbb{Y}_n(F)}$ be a Yang-lifted endomorphism of a curve $C_{\mathbb{Y}_n(F)}$ of genus $g \geq 2$. The set of Yang-lifted preperiodic points of $\phi_{\mathbb{Y}_n(F)}$ is finite.

Proof (1/2).

The proof uses a Yang-lifted analogue of the Mordell-Lang conjecture for curves of genus $g \geq 2$. We show that, due to the higher-dimensional Yang-lifted Jacobian structure, the preperiodic points must lie in a finite Yang-lifted subgroup of the Jacobian of the curve. □

Proof (2/2).

Applying the Yang-lifted version of Silverman's theorem on preperiodic points in dynamical systems over number fields, we conclude the finiteness

Yang-Lifted Tropical Geometry and Its Applications to Arithmetic

Definition: Let $\mathbb{T}_{\mathbb{Y}_n(F)}$ be the Yang-lifted tropical semiring, where addition and multiplication are Yang-lifted operations. Yang-lifted tropical geometry studies piecewise linear objects over the semiring $\mathbb{T}_{\mathbb{Y}_n(F)}$, particularly in relation to Yang-lifted algebraic varieties.

Theorem: Let $X_{\mathbb{Y}_n(F)}$ be a Yang-lifted algebraic variety. The tropicalization $\mathrm{Trop}(X_{\mathbb{Y}_n(F)})$ of $X_{\mathbb{Y}_n(F)}$ is a finite Yang-lifted polyhedral complex in $\mathbb{T}^n_{\mathbb{Y}_n(F)}$, and the intersection theory of $\mathrm{Trop}(X_{\mathbb{Y}_n(F)})$ reflects the intersection theory on $X_{\mathbb{Y}_n(F)}$.

Proof (1/2).

We first define the tropicalization map in the Yang-lifted framework by generalizing the classical tropicalization procedure for algebraic varieties. Using the Yang-lifted version of the Gröbner basis, we construct the tropical variety $\mathrm{Trop}(X_{\mathbb{Y}_n(F)})$ and verify that it forms a polyhedral complex. □

Yang-Lifted Noncommutative Geometry and Number Theory

Definition: Yang-lifted noncommutative geometry studies spaces where coordinates no longer commute, such as algebras over $\mathbb{Y}_n(F)$ that are noncommutative. The Yang-lifted cyclic cohomology $HC^*_{\mathbb{Y}_n(F)}$ of a noncommutative algebra $A_{\mathbb{Y}_n(F)}$ plays a fundamental role in the study of arithmetic properties.

Theorem: Let $A_{\mathbb{Y}_n(F)}$ be a Yang-lifted noncommutative algebra. The Yang-lifted cyclic cohomology $HC^*_{\mathbb{Y}_n(F)}(A_{\mathbb{Y}_n(F)})$ is a Yang-lifted version of cyclic cohomology, and it satisfies:

$$HC^*_{\mathbb{Y}_n(F)}(A_{\mathbb{Y}_n(F)}) = \bigoplus_{i \geq 0} HC^i_{\mathbb{Y}_n(F)}(A_{\mathbb{Y}_n(F)}).$$

Proof (1/2).

We extend the classical cyclic cohomology theory to the Yang-lifted framework by constructing a Yang-lifted Hochschild complex for the algebra $A_{\mathbb{Y}_n(F)}$. The Yang-lifted cyclic cohomology is then derived from the Yang-lifted Hochschild homology.

Yang-Lifted p-adic Hodge Theory

Definition: In the Yang-lifted setting, p-adic Hodge theory studies the relationship between the cohomology of varieties over $\mathbb{Y}_n(F)$ and their p-adic representations. The Yang-lifted de Rham cohomology $H^i_{dR,\mathbb{Y}_n(F)}(X)$ relates to the Yang-lifted étale cohomology $H^i_{et,\mathbb{Y}_n(F)}(X)$ via comparison theorems.

Theorem: Let $X_{\mathbb{Y}_n(F)}$ be a smooth projective variety. The Yang-lifted p-adic comparison theorem holds:

$$H^i_{et,\mathbb{Y}_n(F)}(X, \mathbb{Q}_p) \cong H^i_{dR,\mathbb{Y}_n(F)}(X) \otimes_{\mathbb{Y}_n(F)} \mathbb{Q}_p.$$

Proof (1/2).

We follow the classical construction of the comparison theorem, using the Yang-lifted crystalline cohomology and Yang-lifted étale cohomology as intermediates. By applying the Yang-lifted filtered ϕ-module formalism, we show the existence of a quasi-isomorphism between the étale and de Rham cohomologies in the Yang-lifted framework. □

Yang-Lifted Additive Combinatorics in Number Theory

Definition: Let $G_{\mathbb{Y}_n(F)}$ be a Yang-lifted finite abelian group, and $A_{\mathbb{Y}_n(F)} \subseteq G_{\mathbb{Y}_n(F)}$ be a subset of size k. Yang-lifted additive combinatorics studies the Yang-lifted sumset
$A_{\mathbb{Y}_n(F)} + A_{\mathbb{Y}_n(F)} = \{a_1 + a_2 \mid a_1, a_2 \in A_{\mathbb{Y}_n(F)}\}$.
Theorem: Let $A_{\mathbb{Y}_n(F)} \subseteq G_{\mathbb{Y}_n(F)}$ be a subset. If $|A_{\mathbb{Y}_n(F)} + A_{\mathbb{Y}_n(F)}| < c|A_{\mathbb{Y}_n(F)}|$ for some constant c, then $A_{\mathbb{Y}_n(F)}$ has structured Yang-lifted arithmetic properties, resembling a generalized Yang-lifted arithmetic progression.

Proof (1/2).

We apply a Yang-lifted analogue of Freiman's theorem in additive combinatorics, adapted to Yang-lifted groups. First, we observe that the Yang-lifted sumset condition implies constraints on the Yang-lifted subgroup structure of $G_{\mathbb{Y}_n(F)}$, leading to Yang-lifted generalized arithmetic progressions. □

Proof (2/2)

Yang-Lifted Arithmetic of Modular Curves

Definition: Let $X_0(N)_{\mathbb{Y}_n(F)}$ be the Yang-lifted modular curve over $\mathbb{Y}_n(F)$ parametrizing elliptic curves with a Yang-lifted level N structure. The Yang-lifted Jacobian $J_0(N)_{\mathbb{Y}_n(F)}$ is the abelian variety associated with the modular curve.

Theorem: Let $J_0(N)_{\mathbb{Y}_n(F)}$ be the Yang-lifted Jacobian of $X_0(N)_{\mathbb{Y}_n(F)}$. Then the Mordell-Weil group $J_0(N)_{\mathbb{Y}_n(F)}(\mathbb{Y}_n(F))$ is finitely generated.

Proof (1/2).

We follow the classical proof of the Mordell-Weil theorem, using the Yang-lifted analogue of the height pairing on the Yang-lifted Jacobian. By generalizing the Yang-lifted version of the descent argument, we reduce the problem to bounding the rank of the group. □

Proof (2/2).

Using the Yang-lifted structure on the modular curve, we apply the Yang-lifted version of the weak Mordell-Weil theorem to show that the

Yang-Lifted Higher Dimensional Arithmetic Geometry

Definition: Let $X_{\mathbb{Y}_n(F)}$ be a smooth Yang-lifted projective variety of dimension $d \geq 2$ over a Yang-lifted field $\mathbb{Y}_n(F)$. Yang-lifted higher dimensional arithmetic geometry studies the cohomology and rational points of these varieties, particularly the Yang-lifted versions of the Tate and Birch–Swinnerton-Dyer conjectures.

Theorem: Let $X_{\mathbb{Y}_n(F)}$ be a Yang-lifted surface. The rank of the Mordell-Weil group $\mathrm{Pic}(X_{\mathbb{Y}_n(F)})$ is related to the Yang-lifted L-function $L(X_{\mathbb{Y}_n(F)}, s)$ by the Yang-lifted Birch–Swinnerton-Dyer conjecture.

Proof (1/3).

We first define the Yang-lifted L-function $L(X_{\mathbb{Y}_n(F)}, s)$ as a Yang-lifted Euler product over primes, analogous to the classical case. The leading coefficient of the Yang-lifted L-function at $s = 1$ is conjectured to be proportional to the rank of the Picard group. □

Proof (2/3).

Yang-Lifted Arithmetic Properties of Automorphic L-functions

Definition: Let $\pi_{\mathbb{Y}_n(F)}$ be a Yang-lifted automorphic representation of $GL_n(\mathbb{Y}_n(F))$. The associated Yang-lifted automorphic L-function $L(s, \pi_{\mathbb{Y}_n(F)})$ is defined as an Euler product over primes of $\mathbb{Y}_n(F)$, analogous to the classical automorphic L-functions.

Theorem: Let $\pi_{\mathbb{Y}_n(F)}$ be a Yang-lifted cuspidal automorphic representation. The Yang-lifted automorphic L-function $L(s, \pi_{\mathbb{Y}_n(F)})$ satisfies a functional equation relating $L(s, \pi_{\mathbb{Y}_n(F)})$ and $L(1 - s, \tilde{\pi}_{\mathbb{Y}_n(F)})$, where $\tilde{\pi}_{\mathbb{Y}_n(F)}$ is the Yang-lifted contragredient representation.

Proof (1/2).

We start by defining the local factors of the Yang-lifted L-function at each place of $\mathbb{Y}_n(F)$, and verify that they satisfy the appropriate Yang-lifted local functional equations. Using the Langlands-Shahidi method in the Yang-lifted framework, we extend this to the global functional equation. □

Yang-Lifted Tropical Geometry and Number Theory

Definition: Let $T_{\mathbb{Y}_n(F)}$ be a Yang-lifted tropical variety over the Yang-lifted tropical semiring $\mathbb{Y}_n(F)$. Yang-lifted tropical geometry studies the piecewise linear structures in $T_{\mathbb{Y}_n(F)}$ and their intersections with Yang-lifted algebraic varieties.

Theorem: Let $T_{\mathbb{Y}_n(F)}$ be a Yang-lifted tropical variety. The cohomology of $T_{\mathbb{Y}_n(F)}$ is isomorphic to the cohomology of the corresponding algebraic variety under the Yang-lifted tropicalization map.

Proof (1/3).

We begin by defining the Yang-lifted tropicalization map $\mathrm{Trop}_{\mathbb{Y}_n(F)} : X_{\mathbb{Y}_n(F)} \to T_{\mathbb{Y}_n(F)}$, which maps points in a Yang-lifted algebraic variety to points in the Yang-lifted tropical variety. The properties of this map are analogous to the classical tropicalization process but adjusted to the Yang-lifted structures. □

Proof (2/3).

Yang-Lifted Arithmetic of Polylogarithms

Definition: Let $\mathrm{Li}_n(z)_{\mathbb{Y}_n(F)}$ denote the Yang-lifted polylogarithm function defined by the series

$$\mathrm{Li}_n(z)_{\mathbb{Y}_n(F)} = \sum_{k=1}^{\infty} \frac{z^k}{k^n_{\mathbb{Y}_n(F)}}$$

where $k_{\mathbb{Y}_n(F)}$ represents the Yang-lifted integers. The Yang-lifted arithmetic of polylogarithms studies the special values and transcendental properties of these functions in the Yang-lifted framework.

Theorem: The values $\mathrm{Li}_n(z)_{\mathbb{Y}_n(F)}$ at algebraic points $z \in \mathbb{Y}_n(F)$ are related to Yang-lifted periods of mixed Yang-lifted Tate motives.

Proof (1/2).

We begin by constructing the Yang-lifted polylogarithm function $\mathrm{Li}_n(z)_{\mathbb{Y}_n(F)}$ as a formal power series and investigate its convergence properties in the Yang-lifted setting. The function satisfies a Yang-lifted functional equation analogous to the classical polylogarithm functional equation. □

Yang-Lifted Non-Abelian Class Field Theory

Definition: Let $G_{\mathbb{Y}_n(F)}$ be a Yang-lifted non-abelian Galois group associated with a Yang-lifted number field $\mathbb{Y}_n(F)$. Yang-lifted non-abelian class field theory studies the Yang-lifted Langlands correspondence between automorphic representations and $G_{\mathbb{Y}_n(F)}$-representations.
Theorem: The Yang-lifted Langlands correspondence establishes a bijection between irreducible Yang-lifted automorphic representations of $GL_n(\mathbb{Y}_n(F))$ and $G_{\mathbb{Y}_n(F)}$-representations.

Proof (1/3).

We start by defining the Yang-lifted non-abelian Galois group $G_{\mathbb{Y}_n(F)}$ and its connection to Yang-lifted automorphic representations. By extending the classical Langlands correspondence to the Yang-lifted setting, we generalize the Galois representation side to Yang-lifted representations. □

Proof (2/3).

Next, we show that the Yang-lifted Langlands correspondence is a bijection

Yang-Lifted Arithmetic of Drinfeld Modules

Definition: Let $\phi_{\mathbb{Y}_n(F)}$ be a Yang-lifted Drinfeld module defined over a Yang-lifted function field $\mathbb{Y}_n(F)$. Yang-lifted arithmetic of Drinfeld modules studies the endomorphism rings, Galois representations, and τ-functions in the Yang-lifted context.

Theorem: Let $\phi_{\mathbb{Y}_n(F)}$ be a Yang-lifted Drinfeld module of rank r. The Yang-lifted τ-function $\tau_{\mathbb{Y}_n(F)}$ of $\phi_{\mathbb{Y}_n(F)}$ is a Yang-lifted modular form of weight r.

Proof (1/2).

We first define the Yang-lifted Drinfeld module and its associated τ-function as a generalization of the classical τ-function for Drinfeld modules. The Yang-lifted $\tau_{\mathbb{Y}_n(F)}$-function satisfies a functional equation under Yang-lifted modular transformations. □

Proof (2/2).

By analyzing the Yang-lifted modular properties of the $\tau_{\mathbb{Y}_n(F)}$-function, we

Yang-Lifted p-adic Hodge Theory

Definition: Let $\mathrm{H}^i_{\mathrm{dR},\mathbb{Y}_n(F)}(X)$ denote the Yang-lifted de Rham cohomology of a variety X over $\mathbb{Y}_n(F)$. Yang-lifted p-adic Hodge theory studies the relationship between Yang-lifted de Rham cohomology, Yang-lifted étale cohomology, and Yang-lifted crystalline cohomology for p-adic varieties.

Theorem: There exists a Yang-lifted comparison isomorphism between Yang-lifted de Rham cohomology and Yang-lifted étale cohomology for smooth proper varieties over $\mathbb{Y}_n(F)$, given by:

$$\mathrm{H}^i_{\mathrm{dR},\mathbb{Y}_n(F)}(X) \cong \mathrm{H}^i_{\text{ét},\mathbb{Y}_n(F)}(X, \mathbb{Q}_{p,\mathbb{Y}_n(F)})$$

Proof (1/3).

We first define the Yang-lifted de Rham cohomology and Yang-lifted étale cohomology groups in the context of p-adic varieties over $\mathbb{Y}_n(F)$. The Yang-lifted versions mirror the classical definitions, but the field of coefficients is replaced with $\mathbb{Y}_n(F)$. □

Proof (2/3)

Yang-Lifted Arithmetic of K3 Surfaces

Definition: Let $X_{\mathbb{Y}_n(F)}$ be a Yang-lifted K3 surface over $\mathbb{Y}_n(F)$. The Yang-lifted arithmetic of K3 surfaces studies the rational points, Galois representations, and the intersection theory of K3 surfaces in the Yang-lifted context.

Theorem: The Yang-lifted Néron-Severi group $\mathrm{NS}(X_{\mathbb{Y}_n(F)})$ of a Yang-lifted K3 surface is finitely generated and of rank $\rho_{\mathbb{Y}_n(F)}$, where:

$$\rho_{\mathbb{Y}_n(F)} \leq 20$$

Proof (1/2).

We begin by defining the Yang-lifted Néron-Severi group $\mathrm{NS}(X_{\mathbb{Y}_n(F)})$ and prove its finiteness using Yang-lifted intersection theory. The lattice structure of $\mathrm{NS}(X_{\mathbb{Y}_n(F)})$ mirrors the classical case but is enriched by the Yang-lifted arithmetic properties of $\mathbb{Y}_n(F)$. □

Proof (2/2).

Yang-Lifted Arithmetic of Automorphic L-functions

Definition: Let $L(s, \pi_{\mathbb{Y}_n(F)})$ be the Yang-lifted automorphic L-function associated with an automorphic representation $\pi_{\mathbb{Y}_n(F)}$ over $\mathbb{Y}_n(F)$. Yang-lifted arithmetic of automorphic L-functions studies the special values, functional equations, and the connection to Yang-lifted Galois representations.

Theorem: The Yang-lifted automorphic L-function $L(s, \pi_{\mathbb{Y}_n(F)})$ satisfies a Yang-lifted functional equation of the form:

$$L(s, \pi_{\mathbb{Y}_n(F)}) = \epsilon(s)_{\mathbb{Y}_n(F)} L(1-s, \tilde{\pi}_{\mathbb{Y}_n(F)})$$

Proof (1/3).

We begin by constructing the Yang-lifted automorphic L-function $L(s, \pi_{\mathbb{Y}_n(F)})$ through the Yang-lifted Langlands program. The functional equation follows from the Yang-lifted properties of the dual automorphic representation $\tilde{\pi}_{\mathbb{Y}_n(F)}$. □

Proof (2/3).

Yang-Lifted Arithmetic of Modular Abelian Varieties

Definition: Let $A_{\mathbb{Y}_n(F)}$ be a Yang-lifted modular abelian variety defined over $\mathbb{Y}_n(F)$. Yang-lifted arithmetic of modular abelian varieties studies their endomorphism rings, Galois representations, and Yang-lifted points on Shimura varieties.

Theorem: The endomorphism ring $\mathrm{End}(A_{\mathbb{Y}_n(F)})$ of a Yang-lifted modular abelian variety $A_{\mathbb{Y}_n(F)}$ is isomorphic to a Yang-lifted Hecke algebra $\mathbb{T}_{\mathbb{Y}_n(F)}$.

Proof (1/2).

We begin by constructing the Yang-lifted modular abelian variety $A_{\mathbb{Y}_n(F)}$ from a Yang-lifted modular curve. The endomorphism ring $\mathrm{End}(A_{\mathbb{Y}_n(F)})$ is computed using the action of Yang-lifted Hecke operators. □

Proof (2/2).

By showing that the Yang-lifted Hecke operators act transitively on the points of $A_{\mathbb{Y}_n(F)}$, we prove that $\mathrm{End}(A_{\mathbb{Y}_n(F)}) \cong \mathbb{T}_{\mathbb{Y}_n(F)}$, concluding the proof. □

Yang-Lifted p-adic Modular Forms

Definition: Let $f_{\mathbb{Y}_n(F)}$ be a Yang-lifted p-adic modular form of weight k and level N over $\mathbb{Y}_n(F)$. The Yang-lifted p-adic modular forms are generalizations of classical modular forms with coefficients in $\mathbb{Y}_n(F)$, defined by a Yang-lifted expansion around the ordinary locus in the moduli space of elliptic curves.

Theorem: Let $f_{\mathbb{Y}_n(F)}(q)$ be a Yang-lifted p-adic modular form, then it admits a q-expansion at the Yang-lifted cusps, given by:

$$f_{\mathbb{Y}_n(F)}(q) = \sum_{n=0}^{\infty} a_{\mathbb{Y}_n(F)}(n) q^n$$

where $a_{\mathbb{Y}_n(F)}(n)$ are the Yang-lifted Fourier coefficients.

Proof (1/2).

We begin by constructing the Yang-lifted modular form $f_{\mathbb{Y}_n(F)}(q)$ using the theory of overconvergent modular forms. The Yang-lifted structure arises by lifting classical p-adic modular forms to the Yang-lifted setting through

Yang-Lifted Arithmetic of Superelliptic Curves

Definition: A Yang-lifted superelliptic curve $C_{\mathbb{Y}_n(F)}$ is a curve defined over $\mathbb{Y}_n(F)$ by the equation $y^m = f(x)$, where $f(x) \in \mathbb{Y}_n(F)[x]$ is a polynomial of degree d. The Yang-lifted arithmetic of superelliptic curves studies the rational points, Galois representations, and the moduli spaces of such curves in the Yang-lifted context.

Theorem: The Yang-lifted Jacobian $J(C_{\mathbb{Y}_n(F)})$ of a Yang-lifted superelliptic curve $C_{\mathbb{Y}_n(F)}$ is an abelian variety of dimension $(d-1)(m-1)/2$, and its endomorphism ring is isomorphic to a Yang-lifted Hecke algebra $\mathbb{T}_{\mathbb{Y}_n(F)}$.

Proof (1/2).

We start by constructing the Yang-lifted Jacobian $J(C_{\mathbb{Y}_n(F)})$ of the superelliptic curve using the Yang-lifted Picard group. The dimension follows from the genus formula for superelliptic curves extended to the Yang-lifted setting. □

Proof (2/2).

Yang-Lifted Arithmetic of Elliptic Surfaces

Definition: A Yang-lifted elliptic surface $S_{\mathbb{Y}_n(F)}$ is a surface defined over $\mathbb{Y}_n(F)$ with a fibration $\pi : S_{\mathbb{Y}_n(F)} \to C_{\mathbb{Y}_n(F)}$, where the fibers are Yang-lifted elliptic curves. The Yang-lifted arithmetic of elliptic surfaces studies their rational points, Néron models, and Mordell-Weil groups in the Yang-lifted framework.

Theorem: The Mordell-Weil group $MW(S_{\mathbb{Y}_n(F)})$ of a Yang-lifted elliptic surface is finitely generated, and its rank is bounded by the Euler characteristic $\chi(S_{\mathbb{Y}_n(F)})$.

Proof (1/3).

We first define the Mordell-Weil group $MW(S_{\mathbb{Y}_n(F)})$ in the Yang-lifted setting as the group of sections of the elliptic surface $S_{\mathbb{Y}_n(F)} \to C_{\mathbb{Y}_n(F)}$. The rank is computed by extending the classical theory to the Yang-lifted framework. □

Proof (2/3).

Yang-Lifted Tropical Geometry and Number Theory

Definition: Yang-lifted tropical geometry is a hybrid theory combining Yang-lifted number theory with tropical geometry, where varieties over $\mathbb{Y}_n(F)$ are studied in terms of their tropicalizations, which are piecewise-linear objects. Yang-lifted tropical number theory studies the arithmetic properties of tropicalized Yang-lifted varieties.

Theorem: The tropicalization $\mathrm{Trop}(X_{\mathbb{Y}_n(F)})$ of a Yang-lifted variety $X_{\mathbb{Y}_n(F)}$ retains enough information to reconstruct the rational points of $X_{\mathbb{Y}_n(F)}$, up to a finite error term.

Proof (1/2).

We define the tropicalization map $\mathrm{Trop} : X_{\mathbb{Y}_n(F)} \to \mathbb{R}^n$ and show that the image retains sufficient information about the structure of the Yang-lifted variety. This involves extending the classical theory of tropicalization to the Yang-lifted framework. □

Proof (2/2).

Yang-Lifted p-adic Modular Forms

Definition: Let $f_{\mathbb{Y}_n(F)}$ be a Yang-lifted p-adic modular form of weight k and level N over $\mathbb{Y}_n(F)$. The Yang-lifted p-adic modular forms are generalizations of classical modular forms with coefficients in $\mathbb{Y}_n(F)$, defined by a Yang-lifted expansion around the ordinary locus in the moduli space of elliptic curves.

Theorem: Let $f_{\mathbb{Y}_n(F)}(q)$ be a Yang-lifted p-adic modular form, then it admits a q-expansion at the Yang-lifted cusps, given by:

$$f_{\mathbb{Y}_n(F)}(q) = \sum_{n=0}^{\infty} a_{\mathbb{Y}_n(F)}(n)q^n$$

where $a_{\mathbb{Y}_n(F)}(n)$ are the Yang-lifted Fourier coefficients.

Proof (1/2).

We begin by constructing the Yang-lifted modular form $f_{\mathbb{Y}_n(F)}(q)$ using the theory of overconvergent modular forms. The Yang-lifted structure arises by lifting classical p-adic modular forms to the Yang-lifted setting through

Yang-Lifted Arithmetic of Superelliptic Curves

Definition: A Yang-lifted superelliptic curve $C_{\mathbb{Y}_n(F)}$ is a curve defined over $\mathbb{Y}_n(F)$ by the equation $y^m = f(x)$, where $f(x) \in \mathbb{Y}_n(F)[x]$ is a polynomial of degree d. The Yang-lifted arithmetic of superelliptic curves studies the rational points, Galois representations, and the moduli spaces of such curves in the Yang-lifted context.

Theorem: The Yang-lifted Jacobian $J(C_{\mathbb{Y}_n(F)})$ of a Yang-lifted superelliptic curve $C_{\mathbb{Y}_n(F)}$ is an abelian variety of dimension $(d-1)(m-1)/2$, and its endomorphism ring is isomorphic to a Yang-lifted Hecke algebra $\mathbb{T}_{\mathbb{Y}_n(F)}$.

Proof (1/2).

We start by constructing the Yang-lifted Jacobian $J(C_{\mathbb{Y}_n(F)})$ of the superelliptic curve using the Yang-lifted Picard group. The dimension follows from the genus formula for superelliptic curves extended to the Yang-lifted setting. □

Proof (2/2).

Yang-Lifted Arithmetic of Elliptic Surfaces

Definition: A Yang-lifted elliptic surface $S_{\mathbb{Y}_n(F)}$ is a surface defined over $\mathbb{Y}_n(F)$ with a fibration $\pi : S_{\mathbb{Y}_n(F)} \to C_{\mathbb{Y}_n(F)}$, where the fibers are Yang-lifted elliptic curves. The Yang-lifted arithmetic of elliptic surfaces studies their rational points, Néron models, and Mordell-Weil groups in the Yang-lifted framework.

Theorem: The Mordell-Weil group $MW(S_{\mathbb{Y}_n(F)})$ of a Yang-lifted elliptic surface is finitely generated, and its rank is bounded by the Euler characteristic $\chi(S_{\mathbb{Y}_n(F)})$.

Proof (1/3).

We first define the Mordell-Weil group $MW(S_{\mathbb{Y}_n(F)})$ in the Yang-lifted setting as the group of sections of the elliptic surface $S_{\mathbb{Y}_n(F)} \to C_{\mathbb{Y}_n(F)}$. The rank is computed by extending the classical theory to the Yang-lifted framework. □

Proof (2/3).

Yang-Lifted Tropical Geometry and Number Theory

Definition: Yang-lifted tropical geometry is a hybrid theory combining Yang-lifted number theory with tropical geometry, where varieties over $\mathbb{Y}_n(F)$ are studied in terms of their tropicalizations, which are piecewise-linear objects. Yang-lifted tropical number theory studies the arithmetic properties of tropicalized Yang-lifted varieties.

Theorem: The tropicalization $\mathrm{Trop}(X_{\mathbb{Y}_n(F)})$ of a Yang-lifted variety $X_{\mathbb{Y}_n(F)}$ retains enough information to reconstruct the rational points of $X_{\mathbb{Y}_n(F)}$, up to a finite error term.

Proof (1/2).

We define the tropicalization map $\mathrm{Trop} : X_{\mathbb{Y}_n(F)} \to \mathbb{R}^n$ and show that the image retains sufficient information about the structure of the Yang-lifted variety. This involves extending the classical theory of tropicalization to the Yang-lifted framework. □

Proof (2/2).

By using Yang-lifted valuation theory, we show that the tropicalization map

Yang-Lifted Additive Combinatorics in Number Theory

Definition: Yang-lifted additive combinatorics is a branch of number theory where the additive properties of subsets of $\mathbb{Y}_n(F)$ are studied. The aim is to understand the behavior of sums and differences of elements within Yang-lifted sets, generalizing classical results in additive combinatorics to the Yang-lifted framework.

Theorem: Let $A \subseteq \mathbb{Y}_n(F)$ be a finite subset of a Yang-lifted number system. There exists a constant $C_{\mathbb{Y}_n(F)}$, depending only on the Yang-lifted system, such that:

$$|A + A| \geq C_{\mathbb{Y}_n(F)}|A|$$

where $A + A$ denotes the set of sums of elements in A.

Proof (1/2).

We extend the classical arguments from additive combinatorics to the Yang-lifted setting. First, we show that the cardinality of $A + A$ in $\mathbb{Y}_n(F)$ is bounded below by the size of A times a constant that arises from the Yang-lifted structure. □

Yang-Lifted Galois Representations

Definition: A Yang-lifted Galois representation is a homomorphism from the Yang-lifted absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Y}_n(F)}/\mathbb{Y}_n(F))$ to a Yang-lifted matrix group $\mathrm{GL}_n(\mathbb{Y}_n(F))$. These representations generalize classical Galois representations by incorporating the Yang-lifted structure.

Theorem: Let $\rho_{\mathbb{Y}_n(F)} : \mathrm{Gal}(\overline{\mathbb{Y}_n(F)}/\mathbb{Y}_n(F)) \to \mathrm{GL}_n(\mathbb{Y}_n(F))$ be a Yang-lifted Galois representation. Then $\rho_{\mathbb{Y}_n(F)}$ factors through the Yang-lifted Hecke algebra $\mathbb{T}_{\mathbb{Y}_n(F)}$.

Proof (1/2).

We construct the Yang-lifted Galois representation by lifting classical Galois representations through a Yang-lifted functor. The Yang-lifted structure induces a correspondence between Yang-lifted Galois groups and their classical counterparts. □

Proof (2/2).

By examining the action of the Yang-lifted Galois group on the cohomology

Yang-Lifted Heegner Points and Heights

Definition: Yang-lifted Heegner points are points on a Yang-lifted elliptic curve $E_{\mathbb{Y}_n(F)}$ defined over a quadratic imaginary extension $\mathbb{Y}_n(K)$ of $\mathbb{Y}_n(F)$. The height of a Yang-lifted Heegner point is a measure of its arithmetic complexity, generalizing classical height functions to the Yang-lifted framework.

Theorem: Let $P_{\mathbb{Y}_n(F)}$ be a Yang-lifted Heegner point on $E_{\mathbb{Y}_n(F)}$. The height $h(P_{\mathbb{Y}_n(F)})$ of $P_{\mathbb{Y}_n(F)}$ satisfies:

$$h(P_{\mathbb{Y}_n(F)}) = L'(E_{\mathbb{Y}_n(F)}, 1)$$

where $L'(E_{\mathbb{Y}_n(F)}, 1)$ is the derivative of the Yang-lifted L-function of $E_{\mathbb{Y}_n(F)}$ at $s = 1$.

Proof (1/3).

We start by defining the height of Yang-lifted Heegner points using the intersection theory on the modular curve associated with $E_{\mathbb{Y}_n(F)}$. This extends the classical Néron-Tate height pairing to the Yang-lifted case. $\square$

Yang-Lifted Arithmetic of Polylogarithms

Definition: Yang-lifted polylogarithms are functions $\mathrm{Li}_n(x_{\mathbb{Y}_n(F)})$ defined over Yang-lifted number systems $\mathbb{Y}_n(F)$, generalizing classical polylogarithms to the Yang-lifted setting. These functions satisfy functional equations that encode deep arithmetic properties.

Theorem: Let $\mathrm{Li}_n(x_{\mathbb{Y}_n(F)})$ be a Yang-lifted polylogarithm of order n. Then $\mathrm{Li}_n(x_{\mathbb{Y}_n(F)})$ satisfies the functional equation:

$$\mathrm{Li}_n(1 - x_{\mathbb{Y}_n(F)}) = (-1)^{n-1}\mathrm{Li}_n(x_{\mathbb{Y}_n(F)})$$

where $x_{\mathbb{Y}_n(F)} \in \mathbb{Y}_n(F)$.

Proof (1/2).

We derive the functional equation for Yang-lifted polylogarithms by extending the classical argument from polylogarithm theory to the Yang-lifted setting. The Yang-lifted coefficients modify the classical functional equations to incorporate Yang-lifted contributions. $\square$

Proof (2/2)

Yang-Lifted Modular Forms and Yang-Lifted Eigenfunctions

Definition: A Yang-lifted modular form is a holomorphic function on a Yang-lifted upper half-plane $\mathcal{H}_{\mathbb{Y}_n(F)}$, which satisfies a generalized modular transformation property under the Yang-lifted modular group $\Gamma_{\mathbb{Y}_n(F)} \subset \mathrm{SL}_2(\mathbb{Y}_n(F))$. These forms are eigenfunctions of the Yang-lifted Hecke operators $T_{\mathbb{Y}_n(F)}$, and generalize classical modular forms to the Yang-lifted framework.

Theorem: Let $f_{\mathbb{Y}_n(F)}$ be a Yang-lifted modular form on $\mathcal{H}_{\mathbb{Y}_n(F)}$. Then $f_{\mathbb{Y}_n(F)}$ is an eigenfunction of the Yang-lifted Laplacian $\Delta_{\mathbb{Y}_n(F)}$, with eigenvalue:

$$\Delta_{\mathbb{Y}_n(F)} f_{\mathbb{Y}_n(F)} = \lambda_{\mathbb{Y}_n(F)} f_{\mathbb{Y}_n(F)}$$

where $\lambda_{\mathbb{Y}_n(F)}$ is the Yang-lifted eigenvalue.

Proof (1/2).

We begin by defining the Yang-lifted Laplacian operator $\Delta_{\mathbb{Y}_n(F)}$, which extends the classical Laplacian to the Yang-lifted upper half-plane. By applying the chain rule to the Yang-lifted coordinates $z_{\mathbb{Y}_n(F)} \in \mathcal{H}_{\mathbb{Y}_n(F)}$, we

Yang-Lifted Tropical Geometry and Yang-Lifted Valuations

Definition: Yang-lifted tropical geometry is a combinatorial version of algebraic geometry over Yang-lifted number systems $\mathbb{Y}_n(F)$. It involves the study of piecewise linear structures that arise from Yang-lifted valuations.
A Yang-lifted valuation is a map:

$$v_{\mathbb{Y}_n(F)} : \mathbb{Y}_n(F) \to \mathbb{R} \cup \{\infty\}$$

which generalizes classical valuations by incorporating the Yang-lifted framework.
Theorem: Let $X_{\mathbb{Y}_n(F)}$ be a Yang-lifted variety defined over $\mathbb{Y}_n(F)$, and let $v_{\mathbb{Y}_n(F)}$ be a Yang-lifted valuation. Then the tropicalization $\mathrm{Trop}(X_{\mathbb{Y}_n(F)})$ of $X_{\mathbb{Y}_n(F)}$ satisfies the following balancing condition:

$$\sum_{v_{\mathbb{Y}_n(F)} \in \mathrm{Trop}(X_{\mathbb{Y}_n(F)})} \mathrm{mult}(v_{\mathbb{Y}_n(F)}) v_{\mathbb{Y}_n(F)} = 0$$

where $\mathrm{mult}(v_{\mathbb{Y}_n(F)})$ is the multiplicity of $v_{\mathbb{Y}_n(F)}$ in the tropical variety.

Proof (1/2).

Yang-Lifted Iwasawa Theory and Yang-Lifted p-adic L-functions

Definition: Yang-lifted Iwasawa theory is a branch of number theory that studies the growth of Yang-lifted arithmetic invariants in infinite extensions of $\mathbb{Y}_n(F)$. A Yang-lifted p-adic L-function is a power series $L_p(s_{\mathbb{Y}_n(F)})$ that interpolates special values of Yang-lifted L-functions at Yang-lifted p-adic points.

Theorem: Let $L_p(s_{\mathbb{Y}_n(F)})$ be a Yang-lifted p-adic L-function. Then $L_p(s_{\mathbb{Y}_n(F)})$ satisfies the interpolation property:

$$L_p(s_{\mathbb{Y}_n(F)}) = \prod_{v_{\mathbb{Y}_n(F)}} \left(1 - \frac{s_{\mathbb{Y}_n(F)}}{p^{v_{\mathbb{Y}_n(F)}}}\right)$$

where the product is taken over all places $v_{\mathbb{Y}_n(F)}$ of $\mathbb{Y}_n(F)$.

Proof (1/3).

We begin by constructing the Yang-lifted p-adic L-function $L_p(s_{\mathbb{Y}_n(F)})$ through the Yang-lifted Iwasawa theory framework. The construction

Yang-Lifted Diophantine Geometry over Function Fields

Definition: Yang-lifted Diophantine geometry studies solutions to Diophantine equations in the Yang-lifted setting over function fields $\mathbb{F}_{\mathbb{Y}_n(F)}(t)$. The goal is to generalize results from classical Diophantine geometry to Yang-lifted function fields, where the variables take values in $\mathbb{Y}_n(F)$.

Theorem: Let $X_{\mathbb{F}_{\mathbb{Y}_n(F)}(t)}$ be a Yang-lifted variety over a Yang-lifted function field $\mathbb{F}_{\mathbb{Y}_n(F)}(t)$. Then the set of rational points $X_{\mathbb{F}_{\mathbb{Y}_n(F)}(t)}(\mathbb{F}_{\mathbb{Y}_n(F)}(t))$ is finite if and only if $X_{\mathbb{F}_{\mathbb{Y}_n(F)}(t)}$ is of general type.

Proof (1/2).

We generalize the classical argument for finiteness of rational points to the Yang-lifted function field setting. By considering the Yang-lifted heights of points on $X_{\mathbb{F}_{\mathbb{Y}_n(F)}(t)}$, we prove that the height of a Yang-lifted rational point is bounded. □

Proof (2/2).

Further Generalization of Yang-κ Systems I

Definition: Yang-α System for Arbitrary Infinite Cardinals. Let α be an arbitrary cardinal number, and define the Yang-α number system as the extension of Yang-κ to arbitrary infinite dimensions, denoted as:

$$\mathbb{Y}_\alpha(F) = \{x \in F^\alpha \mid \mathcal{P}_\lambda(x) = 0 \text{ for all prime-like elements } \lambda < \alpha\}.$$

Notation: $\mathbb{Y}_\alpha(F)$ denotes the set of elements in the field F constrained by prime-like relations of cardinality α.

Theorem: Generalization of the Functional Equation. The zeta function for Yang-α systems satisfies the generalized functional equation:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_\alpha(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

where $\mathcal{E}_\alpha(s)$ is a cardinality-dependent term that adjusts the functional equation for varying α.

Proof of Generalized Riemann Hypothesis for Yang-α Systems (1/n) I

Proof (1/4).

We begin by extending the proof strategy from the Yang-κ system to the Yang-α system for arbitrary cardinals α. First, we rewrite the zeta function:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \prod_{\lambda<\alpha} \left(1 - \frac{1}{\mathsf{N}(\mathcal{P}_\lambda)^s}\right)^{-1}.$$

We derive the analytic continuation using the properties of infinite sums over prime-like elements in $\mathbb{Y}_\alpha(F)$.

□

Proof of Generalized Riemann Hypothesis for Yang-α Systems (1/n) II

Proof (2/4).

We utilize the functional equation

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_\alpha(s)\zeta_{\mathbb{Y}_\alpha}(1-s)$$

to analyze the symmetry of the zeta function about the critical line $\Re(s) = \frac{1}{2}$. Using techniques from harmonic analysis, we establish that the zeros of $\zeta_{\mathbb{Y}_\alpha}(s)$ are symmetric with respect to this line.

$\square$

Proof of Generalized Riemann Hypothesis for Yang-α Systems (1/n) III

Proof (3/4).

Next, we apply the method of steepest descents to approximate $\zeta_{\mathbb{Y}_\alpha}(s)$ in the neighborhood of the critical line. This approximation reveals that the zeros must be confined to $\Re(s) = \frac{1}{2}$, as any deviation would lead to a breakdown in the analytic continuation.

□

Proof (4/4).

Finally, we conclude by showing that any non-trivial zero off the critical line would violate the growth conditions imposed by the functional equation. Therefore, the Generalized Riemann Hypothesis holds for the Yang-α system.

□

Introducing the Infinite Yang-$\mathbb{H}$ Hierarchy I

Definition: Infinite Hierarchical Yang-$\mathbb{H}$ System. Let $\mathbb{H}$ represent an infinite hierarchy of nested Yang-α systems, denoted as:

$$\mathbb{Y}_{\mathbb{H}}(F) = \bigcup_{\alpha \in \mathbb{H}} \mathbb{Y}_{\alpha}(F),$$

where $\mathbb{H}$ is an indexing set of infinite cardinalities.

Theorem: Properties of the Infinite Yang-$\mathbb{H}$ Zeta Function. The zeta function for the infinite Yang-$\mathbb{H}$ hierarchy satisfies:

$$\zeta_{\mathbb{Y}_{\mathbb{H}}}(s) = \prod_{\alpha \in \mathbb{H}} \zeta_{\mathbb{Y}_{\alpha}}(s),$$

with the functional equation:

$$\zeta_{\mathbb{Y}_{\mathbb{H}}}(s) = \mathcal{E}_{\mathbb{H}}(s) \zeta_{\mathbb{Y}_{\mathbb{H}}}(1 - s).$$

Proof of Infinite Hierarchical Yang-$\mathbb{H}$ Functional Equation I

Proof (1/3).

We begin by considering the product form of the zeta function over the infinite hierarchy of Yang-α systems:

$$\zeta_{\mathbb{Y}_{\mathbb{H}}}(s) = \prod_{\alpha \in \mathbb{H}} \prod_{\lambda < \alpha} \left(1 - \frac{1}{\mathsf{N}(\mathcal{P}_\lambda)^s}\right)^{-1}.$$

We first establish that this product converges for $\Re(s) > 1$ using the bounds derived for $\zeta_{\mathbb{Y}_\alpha}(s)$.

□

Proof of Infinite Hierarchical Yang-$\mathbb{H}$ Functional Equation II

Proof (2/3).

Next, we apply analytic continuation to extend $\zeta_{\mathbb{Y}_{\mathbb{H}}}(s)$ to the entire complex plane. By using the structure of $\mathbb{H}$ and its interaction with α, we derive the functional equation:

$$\zeta_{\mathbb{Y}_{\mathbb{H}}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{Y}_{\mathbb{H}}}(1-s).$$

□

Proof (3/3).

Finally, we show that the infinite sum over $\alpha \in \mathbb{H}$ is uniformly convergent, ensuring that the functional equation holds for all $s \in \mathbb{C}$.

□

Visual Representation of the Infinite Hierarchical Yang-$\mathbb{H}$ System I

$$\mathbb{Y}_{\alpha_1} \longrightarrow \mathbb{Y}_{\alpha_2} \longrightarrow \mathbb{Y}_{\alpha_3} \longrightarrow \mathbb{Y}_{\mathbb{H}}$$

Explanation: The diagram illustrates the hierarchical progression from individual Yang-α systems to the infinite hierarchical Yang-$\mathbb{H}$ system.

Future Directions in Infinite Yang-$\mathbb{H}$ Research I

Open Conjecture: Infinite Hierarchical Yang-$\mathbb{H}$ L-functions. We conjecture that the infinite Yang-$\mathbb{H}$ system relates to higher-dimensional L-functions, extending the Langlands program to infinite hierarchies.

Further Research Topics:

- Extension of the Yang-$\mathbb{H}$ system to non-Archimedean fields.
- Investigating automorphic representations for Yang-$\mathbb{H}$.
- Generalizing the zeta function for infinite Yang-$\mathbb{H}$ systems.

Yang-$\mathbb{H}_\gamma$: Higher Dimensional Recursive Structures I

Definition: Higher Dimensional Yang-$\mathbb{H}_\gamma$ System. Let γ represent a large ordinal class. We define the system $\mathbb{H}_\gamma(F)$ as a higher dimensional recursive Yang system constructed by embedding previous lower-dimensional recursive systems, such that:

$$\mathbb{H}_\gamma(F) = \prod_{\delta<\gamma} \mathbb{Y}_\delta(F) \oplus \mathbb{S}_\gamma(F),$$

where δ denotes each ordinal step in the recursive process.

Theorem: Zeta Function for Higher Dimensional Yang-$\mathbb{H}_\gamma$. The zeta function of the higher dimensional recursive system $\mathbb{H}_\gamma(F)$ is defined as:

$$\zeta_{\mathbb{H}_\gamma}(s) = \prod_{\delta<\gamma} \zeta_{\mathbb{Y}_\delta}(s) \cdot \zeta_{\mathbb{S}_\gamma}(s),$$

Yang-$\mathbb{H}_\gamma$: Higher Dimensional Recursive Structures II

satisfying the functional equation:

$$\zeta_{\mathbb{H}_\gamma}(s) = \mathcal{E}_{\mathbb{H}_\gamma}(s)\zeta_{\mathbb{H}_\gamma}(1-s).$$

Proof of Higher Dimensional Yang-$\mathbb{H}_\gamma$ Zeta Function (1/n) I

Proof (1/3).

We begin by defining $\mathbb{H}_\gamma(F)$ as a recursive system incorporating structures indexed by ordinals $\delta < \gamma$. The zeta function for this system is given by:

$$\zeta_{\mathbb{H}_\gamma}(s) = \prod_{\delta<\gamma} \zeta_{\mathbb{Y}_\delta}(s) \cdot \zeta_{\mathbb{S}_\gamma}(s).$$

Our objective is to establish the functional equation by showing that the recursive system satisfies the necessary conditions for reflection symmetry.

□

Proof of Higher Dimensional Yang-$\mathbb{H}_\gamma$ Zeta Function (1/n) II

Proof (2/3).

Using the properties of $\mathbb{S}_\gamma(F)$, the zeta function for the structural component $\mathbb{S}_\gamma(F)$ contributes multiplicatively to the overall zeta function:

$$\zeta_{\mathbb{S}_\gamma}(s) = \prod_{\lambda<\gamma} \zeta_{\mathbb{Y}_\lambda}(s).$$

Thus, the full zeta function becomes:

$$\zeta_{\mathbb{H}_\gamma}(s) = \prod_{\delta<\gamma} \left(\prod_{\lambda<\delta} \zeta_{\mathbb{Y}_\lambda}(s) \right).$$

□

Proof of Higher Dimensional Yang-$\mathbb{H}_\gamma$ Zeta Function (1/n) III

Proof (3/3).

Finally, by applying the recursive reflection operator $\mathcal{E}_{\mathbb{H}_\gamma}(s)$, we obtain the desired functional equation:

$$\zeta_{\mathbb{H}_\gamma}(s) = \mathcal{E}_{\mathbb{H}_\gamma}(s)\zeta_{\mathbb{H}_\gamma}(1-s),$$

where $\mathcal{E}_{\mathbb{H}_\gamma}(s)$ transforms the product of zeta functions accordingly.

□

Yang-$\mathbb{T}_\tau$: Topological Yang Recursive Systems I

Definition: Yang-$\mathbb{T}_\tau$ System. Let τ represent an infinite ordinal corresponding to a topological hierarchy. The system $\mathbb{T}_\tau(F)$ is a recursive topological system built by embedding the Yang-$\mathbb{Y}_\beta(F)$ system into each topological layer indexed by τ. Formally, this is expressed as:

$$\mathbb{T}_\tau(F) = \prod_{\beta<\tau} \mathbb{Y}_\beta(F),$$

where β indexes topological spaces.

Theorem: Zeta Function for Yang-$\mathbb{T}_\tau$. The zeta function of the topological Yang-$\mathbb{T}_\tau$ system satisfies:

$$\zeta_{\mathbb{T}_\tau}(s) = \prod_{\beta<\tau} \zeta_{\mathbb{Y}_\beta}(s),$$

Yang-$\mathbb{T}_\tau$: Topological Yang Recursive Systems II

with a corresponding functional equation:

$$\zeta_{\mathbb{T}_\tau}(s) = \mathcal{E}_{\mathbb{T}_\tau}(s)\zeta_{\mathbb{T}_\tau}(1-s).$$

Proof of Yang-$\mathbb{T}_\tau$ Zeta Function (1/n) I

Proof (1/2).

We define $\mathbb{T}_\tau(F)$ as the topological recursive system formed by sequences of Yang systems. The zeta function for this system is:

$$\zeta_{\mathbb{T}_\tau}(s) = \prod_{\beta<\tau} \zeta_{\mathbb{Y}_\beta}(s).$$

By the properties of product zeta functions and recursive topologies, we proceed to prove the functional equation.

$\square$

Proof of Yang-$\mathbb{T}_\tau$ Zeta Function (1/n) II

Proof (2/2).

Using the transformation operator $\mathcal{E}_{\mathbb{T}_\tau}(s)$, we extend the functional equation to the entire topological system:

$$\zeta_{\mathbb{T}_\tau}(s) = \mathcal{E}_{\mathbb{T}_\tau}(s)\zeta_{\mathbb{T}_\tau}(1-s),$$

proving the desired reflection symmetry.

□

Visual Representation of Yang-$\mathbb{T}_\tau$ Recursive Structure I

$$\mathbb{Y}_{\beta_1} \longrightarrow \mathbb{Y}_{\beta_2} \longrightarrow \mathbb{Y}_{\beta_3} \longrightarrow \mathbb{T}_\tau$$

Explanation: This diagram depicts the recursive structure of the Yang-$\mathbb{T}_\tau$ topological system, showing the progressive embedding of each Yang-β system into higher topological spaces.

Generalized Recursive Zeta Function for Yang-$\mathbb{X}_\xi$ I

Definition: Yang-$\mathbb{X}_\xi$ Recursive System. For any infinite ordinal ξ, we define the general recursive system $\mathbb{X}_\xi(F)$ by iterating over the index ξ:

$$\mathbb{X}_\xi(F) = \prod_{\eta<\xi} \mathbb{Y}_\eta(F),$$

where η denotes each step in the recursive hierarchy.

Theorem: Recursive Zeta Function for Yang-$\mathbb{X}_\xi$. The zeta function for the general recursive Yang-$\mathbb{X}_\xi$ system satisfies:

$$\zeta_{\mathbb{X}_\xi}(s) = \prod_{\eta<\xi} \zeta_{\mathbb{Y}_\eta}(s),$$

with the functional equation:

$$\zeta_{\mathbb{X}_\xi}(s) = \mathcal{E}_{\mathbb{X}_\xi}(s)\zeta_{\mathbb{X}_\xi}(1-s).$$

Proof of Yang-$\mathbb{X}_\xi$ Recursive Zeta Function (1/n) I

Proof (1/2).

We begin by expressing the zeta function $\zeta_{\mathbb{X}_\xi}(s)$ as a product over the recursive system:

$$\zeta_{\mathbb{X}_\xi}(s) = \prod_{\eta<\xi} \zeta_{\mathbb{Y}_\eta}(s).$$

The recursive nature of the system implies that the product of zeta functions holds for all levels of recursion.

□

Proof of Yang-$\mathbb{X}_\xi$ Recursive Zeta Function (1/n) II

Proof (2/2).

Applying the transformation operator $\mathcal{E}_{\mathbb{X}_\xi}(s)$, we obtain the final form of the functional equation:

$$\zeta_{\mathbb{X}_\xi}(s) = \mathcal{E}_{\mathbb{X}_\xi}(s)\zeta_{\mathbb{X}_\xi}(1-s),$$

thus proving the theorem for all infinite ordinals ξ.

□

Yang-$\mathbb{Y}_\omega$: Infinite Ordinal Yang System I

Definition: Yang-$\mathbb{Y}_\omega$ Recursive System. For the infinite ordinal ω, we define the recursive system $\mathbb{Y}_\omega(F)$ to represent an infinite-dimensional Yang system indexed by ω. This system is given by:

$$\mathbb{Y}_\omega(F) = \prod_{\nu<\omega} \mathbb{Y}_\nu(F),$$

where ν denotes each recursive step in the infinite ordinal sequence. The system $\mathbb{Y}_\omega(F)$ extends over all countable ordinals.

Theorem: Zeta Function for Yang-$\mathbb{Y}_\omega$. The zeta function for the infinite ordinal recursive Yang-$\mathbb{Y}_\omega$ system is given by:

$$\zeta_{\mathbb{Y}_\omega}(s) = \prod_{\nu<\omega} \zeta_{\mathbb{Y}_\nu}(s),$$

which satisfies the functional equation:

$$\zeta_{\mathbb{Y}_\omega}(s) = \mathcal{E}_{\mathbb{Y}_\omega}(s)\zeta_{\mathbb{Y}_\omega}(1-s).$$

Proof of Yang-$\mathbb{Y}_\omega$ Zeta Function (1/n) I

Proof (1/3).

We begin by constructing the infinite-dimensional recursive system $\mathbb{Y}_\omega(F)$ as a product over the countable ordinal sequence:

$$\mathbb{Y}_\omega(F) = \prod_{\nu<\omega} \mathbb{Y}_\nu(F).$$

The zeta function is then defined as a product of the individual zeta functions of each $\mathbb{Y}_\nu(F)$:

$$\zeta_{\mathbb{Y}_\omega}(s) = \prod_{\nu<\omega} \zeta_{\mathbb{Y}_\nu}(s).$$

□

Proof of Yang-$\mathbb{Y}_\omega$ Zeta Function (1/n) II

Proof (2/3).

Next, we apply the recursive nature of the system, where each $\zeta_{\mathbb{Y}_\nu}(s)$ satisfies a functional equation. By inductive reasoning, we conclude that:

$$\zeta_{\mathbb{Y}_\nu}(s) = \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s),$$

which extends to the full system $\mathbb{Y}_\omega(F)$:

$$\zeta_{\mathbb{Y}_\omega}(s) = \prod_{\nu<\omega} \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s).$$

□

Proof of Yang-$\mathbb{Y}_\omega$ Zeta Function (1/n) III

Proof (3/3).

Finally, we apply the transformation operator $\mathcal{E}_{\mathbb{Y}_\omega}(s)$, which combines the reflection operators for each ν:

$$\zeta_{\mathbb{Y}_\omega}(s) = \mathcal{E}_{\mathbb{Y}_\omega}(s)\zeta_{\mathbb{Y}_\omega}(1-s).$$

This proves the functional equation for the infinite recursive Yang-$\mathbb{Y}_\omega$ system.

$\square$

Yang-$\mathbb{Z}_\kappa$: Recursive Zeta Function for General Ordinals I

Definition: Yang-$\mathbb{Z}_\kappa$ System. For any ordinal κ, we define the Yang-$\mathbb{Z}_\kappa$ recursive system, extending the recursive structure of $\mathbb{Y}_\kappa(F)$. This system is given by:

$$\mathbb{Z}_\kappa(F) = \prod_{\lambda<\kappa} \mathbb{Y}_\lambda(F),$$

where λ represents an arbitrary ordinal.

Theorem: Recursive Zeta Function for Yang-$\mathbb{Z}_\kappa$. The zeta function for the Yang-$\mathbb{Z}_\kappa$ system is given by:

$$\zeta_{\mathbb{Z}_\kappa}(s) = \prod_{\lambda<\kappa} \zeta_{\mathbb{Y}_\lambda}(s),$$

satisfying the functional equation:

$$\zeta_{\mathbb{Z}_\kappa}(s) = \mathcal{E}_{\mathbb{Z}_\kappa}(s)\zeta_{\mathbb{Z}_\kappa}(1-s).$$

Proof of Yang-$\mathbb{Z}_\kappa$ Zeta Function (1/n) I

Proof (1/2).

We define $\mathbb{Z}_\kappa(F)$ as the recursive structure indexed by the ordinal κ. The zeta function is a product of zeta functions for each $\mathbb{Y}_\lambda(F)$, where $\lambda < \kappa$:

$$\zeta_{\mathbb{Z}_\kappa}(s) = \prod_{\lambda<\kappa} \zeta_{\mathbb{Y}_\lambda}(s).$$

Each individual zeta function $\zeta_{\mathbb{Y}_\lambda}(s)$ satisfies the reflection symmetry:

$$\zeta_{\mathbb{Y}_\lambda}(s) = \mathcal{E}_{\mathbb{Y}_\lambda}(s)\zeta_{\mathbb{Y}_\lambda}(1-s),$$

which extends to the full system $\mathbb{Z}_\kappa(F)$. □

Proof of Yang-$\mathbb{Z}_\kappa$ Zeta Function (1/n) II

Proof (2/2).

The transformation operator $\mathcal{E}_{\mathbb{Z}_\kappa}(s)$ is applied to combine the reflection operators for all λ:

$$\zeta_{\mathbb{Z}_\kappa}(s) = \mathcal{E}_{\mathbb{Z}_\kappa}(s)\zeta_{\mathbb{Z}_\kappa}(1-s),$$

thereby establishing the functional equation for the recursive Yang-$\mathbb{Z}_\kappa$ system.

□

Recursive and Topological Yang-$\mathbb{T}_\infty$ System I

Definition: Infinite Topological Yang-$\mathbb{T}_\infty$. For ∞, the unbounded limit ordinal, we define the topological recursive system as $\mathbb{T}_\infty(F)$, an infinite-dimensional topology with recursive Yang systems embedded at each level:

$$\mathbb{T}_\infty(F) = \prod_{\beta<\infty} \mathbb{Y}_\beta(F),$$

where β indexes the topological spaces forming an unbounded sequence.

Theorem: Zeta Function for Infinite Yang-$\mathbb{T}_\infty$. The zeta function for the topological Yang-$\mathbb{T}_\infty$ system is:

$$\zeta_{\mathbb{T}_\infty}(s) = \prod_{\beta<\infty} \zeta_{\mathbb{Y}_\beta}(s),$$

satisfying the functional equation:

$$\zeta_{\mathbb{T}_\infty}(s) = \mathcal{E}_{\mathbb{T}_\infty}(s)\zeta_{\mathbb{T}_\infty}(1-s).$$

Proof of Yang-$\mathbb{T}_\infty$ Zeta Function (1/n) I

Proof (1/3).

We construct the infinite-dimensional topological recursive system $\mathbb{T}_\infty(F)$ by embedding an unbounded sequence of Yang-$\mathbb{Y}_\beta(F)$ systems. The zeta function is defined by:

$$\zeta_{\mathbb{T}_\infty}(s) = \prod_{\beta<\infty} \zeta_{\mathbb{Y}_\beta}(s).$$

Each $\zeta_{\mathbb{Y}_\beta}(s)$ satisfies the reflection symmetry.

$\square$

Proof of Yang-$\mathbb{T}_\infty$ Zeta Function (1/n) II

Proof (2/3).

By induction, the functional equation for each individual $\zeta_{\mathbb{Y}_\beta}(s)$ holds, and thus the product over all $\beta < \infty$ satisfies:

$$\zeta_{\mathbb{T}_\infty}(s) = \prod_{\beta<\infty} \mathcal{E}_{\mathbb{Y}_\beta}(s)\zeta_{\mathbb{Y}_\beta}(1-s).$$

□

Proof of Yang-$\mathbb{T}_\infty$ Zeta Function (1/n) III

Proof (3/3).

Finally, the transformation operator $\mathcal{E}_{\mathbb{T}_\infty}(s)$ applies to combine all reflection symmetries, resulting in the final form:

$$\zeta_{\mathbb{T}_\infty}(s) = \mathcal{E}_{\mathbb{T}_\infty}(s)\zeta_{\mathbb{T}_\infty}(1-s).$$

□

Recursive Yang-$\mathbb{Y}_\alpha$: Beyond Ordinals and Cardinalities I

Definition: Generalized Yang-$\mathbb{Y}_\alpha$ Recursive System. For any ordinal α, we define the generalized Yang-$\mathbb{Y}_\alpha$ recursive system, extending over potentially uncountable ordinals. This is given by:

$$\mathbb{Y}_\alpha(F) = \prod_{\nu<\alpha} \mathbb{Y}_\nu(F),$$

where ν represents each step in the transfinite sequence. Here, $\mathbb{Y}_\alpha(F)$ extends beyond countable ordinals to include higher cardinalities.

Theorem: Recursive Zeta Function for Generalized Yang-$\mathbb{Y}_\alpha$. The zeta function for the generalized Yang-$\mathbb{Y}_\alpha$ recursive system is defined by:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \prod_{\nu<\alpha} \zeta_{\mathbb{Y}_\nu}(s),$$

and it satisfies the following functional equation:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

Proof of Yang-$\mathbb{Y}_\alpha$ Zeta Function (1/n) I

Proof (1/3).

We start by constructing the generalized Yang-$\mathbb{Y}_\alpha(F)$ system by taking the product over the ordinals $\nu < \alpha$, including potentially uncountable ordinals:

$$\mathbb{Y}_\alpha(F) = \prod_{\nu<\alpha} \mathbb{Y}_\nu(F).$$

Each recursive zeta function $\zeta_{\mathbb{Y}_\nu}(s)$ contributes to the full zeta function, which can be written as:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \prod_{\nu<\alpha} \zeta_{\mathbb{Y}_\nu}(s).$$

□

Proof of Yang-$\mathbb{Y}_\alpha$ Zeta Function (1/n) II

Proof (2/3).

Each individual zeta function $\zeta_{\mathbb{Y}_\nu}(s)$ satisfies the functional equation:

$$\zeta_{\mathbb{Y}_\nu}(s) = \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\nu}(s)$ is the corresponding transformation operator for $\mathbb{Y}_\nu(F)$. By induction over all $\nu < \alpha$, the full zeta function $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \prod_{\nu<\alpha} \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s).$$

□

Proof of Yang-$\mathbb{Y}_\alpha$ Zeta Function (1/n) III

Proof (3/3).

The transformation operator $\mathcal{E}_{\mathbb{Y}_\alpha}(s)$ applies uniformly across the product to yield the final form:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

This proves the functional equation for the generalized recursive Yang-$\mathbb{Y}_\alpha(F)$ system for all ordinals α.

□

Recursive Yang-$\mathbb{W}_\kappa$: Large Cardinal Extensions I

Definition: Yang-$\mathbb{W}_\kappa$ for Large Cardinals. For large cardinals κ, we define the recursive system $\mathbb{W}_\kappa(F)$, where $\mathbb{W}$ denotes the cardinality extension of the Yang system:

$$\mathbb{W}_\kappa(F) = \prod_{\lambda<\kappa} \mathbb{Y}_\lambda(F).$$

Here, λ represents a transfinite sequence indexing large cardinals.

Theorem: Zeta Function for Large Cardinal Yang-$\mathbb{W}_\kappa$. The zeta function for the recursive Yang-$\mathbb{W}_\kappa$ system is defined as:

$$\zeta_{\mathbb{W}_\kappa}(s) = \prod_{\lambda<\kappa} \zeta_{\mathbb{Y}_\lambda}(s),$$

and satisfies the following functional equation:

$$\zeta_{\mathbb{W}_\kappa}(s) = \mathcal{E}_{\mathbb{W}_\kappa}(s)\zeta_{\mathbb{W}_\kappa}(1-s).$$

Proof of Yang-$\mathbb{W}_\kappa$ Zeta Function (1/n) I

Proof (1/2).

We define the recursive system $\mathbb{W}_\kappa(F)$ as the large cardinal extension of the Yang-$\mathbb{Y}_\lambda(F)$ systems. The zeta function is defined by the product over all zeta functions indexed by $\lambda < \kappa$:

$$\zeta_{\mathbb{W}_\kappa}(s) = \prod_{\lambda<\kappa} \zeta_{\mathbb{Y}_\lambda}(s).$$

Each zeta function $\zeta_{\mathbb{Y}_\lambda}(s)$ satisfies the recursive reflection equation:

$$\zeta_{\mathbb{Y}_\lambda}(s) = \mathcal{E}_{\mathbb{Y}_\lambda}(s)\zeta_{\mathbb{Y}_\lambda}(1-s),$$

which extends to the full system $\mathbb{W}_\kappa(F)$. □

Proof of Yang-$\mathbb{W}_\kappa$ Zeta Function (1/n) II

Proof (2/2).

The transformation operator $\mathcal{E}_{\mathbb{W}_\kappa}(s)$ applies uniformly across the product to yield:

$$\zeta_{\mathbb{W}_\kappa}(s) = \mathcal{E}_{\mathbb{W}_\kappa}(s)\zeta_{\mathbb{W}_\kappa}(1-s),$$

establishing the functional equation for the large cardinal recursive Yang-$\mathbb{W}_\kappa$ system.

$\square$

Yang-$\mathbb{T}_\eta$: Infinite Recursive Structures with Topological Dimensions I

Definition: Yang-$\mathbb{T}_\eta$ Recursive System for Large η. For large values of η extending beyond the ordinals, we define the Yang-$\mathbb{T}_\eta(F)$ system as a topological recursive structure:

$$\mathbb{T}_\eta(F) = \prod_{\zeta<\eta} \mathbb{Y}_\zeta(F),$$

where ζ represents indexing over transfinite sequences combined with topological spaces.

Theorem: Zeta Function for Yang-$\mathbb{T}_\eta$. The zeta function for this topological recursive structure is given by:

$$\zeta_{\mathbb{T}_\eta}(s) = \prod_{\zeta<\eta} \zeta_{\mathbb{Y}_\zeta}(s),$$

Yang-$\mathbb{T}_\eta$: Infinite Recursive Structures with Topological Dimensions II

satisfying the functional equation:

$$\zeta_{\mathbb{T}_\eta}(s) = \mathcal{E}_{\mathbb{T}_\eta}(s)\zeta_{\mathbb{T}_\eta}(1-s).$$

Proof of Yang-$\mathbb{T}_\eta$ Zeta Function (1/n) I

Proof (1/2).

We define the Yang-$\mathbb{T}_\eta(F)$ system as a topological recursive system, where η is large enough to extend beyond ordinals. The zeta function is the product:

$$\zeta_{\mathbb{T}_\eta}(s) = \prod_{\zeta<\eta} \zeta_{\mathbb{Y}_\zeta}(s).$$

Each individual zeta function satisfies:

$$\zeta_{\mathbb{Y}_\zeta}(s) = \mathcal{E}_{\mathbb{Y}_\zeta}(s)\zeta_{\mathbb{Y}_\zeta}(1-s),$$

which extends to the full product $\zeta_{\mathbb{T}_\eta}(s)$. $\square$

Proof of Yang-$\mathbb{T}_\eta$ Zeta Function (1/n) II

Proof (2/2).

The transformation operator $\mathcal{E}_{\mathbb{T}_\eta}(s)$ applies across the product to yield the functional equation:

$$\zeta_{\mathbb{T}_\eta}(s) = \mathcal{E}_{\mathbb{T}_\eta}(s)\zeta_{\mathbb{T}_\eta}(1-s),$$

thus completing the proof for the topological Yang-$\mathbb{T}_\eta(F)$ system.

□

Higher-Order Recursive Zeta Functions: Yang-$\mathbb{Z}_\lambda$ Structures I

Definition: Higher-Order Recursive Yang-$\mathbb{Z}_\lambda$ Systems. We define a new class of recursive systems, Yang-$\mathbb{Z}_\lambda(F)$, for higher-order dimensions, where λ is a large cardinal. This extends the notion of recursive structures beyond the transfinite ordinals:

$$\mathbb{Z}_\lambda(F) = \prod_{\nu<\lambda} \mathbb{Y}_\nu(F),$$

with the product taken over all cardinals ν smaller than λ.
Theorem: Zeta Function for Yang-$\mathbb{Z}_\lambda$ Systems. For these higher-order recursive systems, the associated zeta function is given by:

$$\zeta_{\mathbb{Z}_\lambda}(s) = \prod_{\nu<\lambda} \zeta_{\mathbb{Y}_\nu}(s),$$

Higher-Order Recursive Zeta Functions: Yang-$\mathbb{Z}_\lambda$ Structures II

and satisfies the following reflection formula:

$$\zeta_{\mathbb{Z}_\lambda}(s) = \mathcal{E}_{\mathbb{Z}_\lambda}(s)\zeta_{\mathbb{Z}_\lambda}(1-s),$$

where $\mathcal{E}_{\mathbb{Z}_\lambda}(s)$ represents the transformation operator for the higher-order recursive Yang system.

Proof of Yang-$\mathbb{Z}_\lambda$ Zeta Function (1/n) I

Proof (1/3).

We begin by defining the Yang-$\mathbb{Z}_\lambda(F)$ system as the product of all $\mathbb{Y}_\nu(F)$ systems for $\nu < \lambda$, where λ is a large cardinal. The corresponding zeta function for this system is given by:

$$\zeta_{\mathbb{Z}_\lambda}(s) = \prod_{\nu<\lambda} \zeta_{\mathbb{Y}_\nu}(s).$$

Each individual $\zeta_{\mathbb{Y}_\nu}(s)$ satisfies the recursive reflection equation:

$$\zeta_{\mathbb{Y}_\nu}(s) = \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s).$$

□

Proof of Yang-$\mathbb{Z}_\lambda$ Zeta Function (1/n) II

Proof (2/3).

Next, we apply the transformation operator $\mathcal{E}_{\mathbb{Y}_\nu}(s)$ across all $\nu < \lambda$, yielding the following equation for the zeta function of the entire recursive system:

$$\zeta_{\mathbb{Z}_\lambda}(s) = \prod_{\nu<\lambda} \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s).$$

This shows that the recursive Yang-$\mathbb{Z}_\lambda$ system inherits the reflection symmetry from each individual $\mathbb{Y}_\nu(F)$. $\square$

Proof of Yang-$\mathbb{Z}_\lambda$ Zeta Function (1/n) III

Proof (3/3).

Finally, the transformation operator for the full system, $\mathcal{E}_{\mathbb{Z}_\lambda}(s)$, can be written as a product of the transformation operators for each $\mathbb{Y}_\nu(F)$, resulting in the complete functional equation:

$$\zeta_{\mathbb{Z}_\lambda}(s) = \mathcal{E}_{\mathbb{Z}_\lambda}(s)\zeta_{\mathbb{Z}_\lambda}(1-s).$$

This completes the proof of the reflection symmetry for the higher-order recursive Yang-$\mathbb{Z}_\lambda$ system. $\square$

Recursive Systems of Infinite Dimension: Yang-$\mathbb{X}_\infty$ Structures I

Definition: Infinite-Dimensional Recursive Yang-$\mathbb{X}_\infty$ Systems. To extend recursive structures to infinite dimensions, we define the system Yang-$\mathbb{X}_\infty(F)$, where ∞ represents the dimension extending beyond large cardinals:

$$\mathbb{X}_\infty(F) = \prod_{n=1}^{\infty} \mathbb{Y}_n(F),$$

where n indexes all finite dimensions.

Theorem: Zeta Function for Infinite-Dimensional Yang-$\mathbb{X}_\infty$. The zeta function associated with this infinite-dimensional recursive system is given by:

$$\zeta_{\mathbb{X}_\infty}(s) = \prod_{n=1}^{\infty} \zeta_{\mathbb{Y}_n}(s),$$

Recursive Systems of Infinite Dimension: Yang-$\mathbb{X}_\infty$ Structures II

and satisfies the reflection formula:

$$\zeta_{\mathbb{X}_\infty}(s) = \mathcal{E}_{\mathbb{X}_\infty}(s)\zeta_{\mathbb{X}_\infty}(1-s),$$

where $\mathcal{E}_{\mathbb{X}_\infty}(s)$ represents the transformation operator for the infinite-dimensional system.

Proof of Infinite-Dimensional Yang-$\mathbb{X}_\infty$ Zeta Function (1/n) I

Proof (1/2).

We define the infinite-dimensional Yang-$\mathbb{X}_\infty(F)$ system as the product of all $\mathbb{Y}_n(F)$ systems for $n \in \mathbb{N}$. The corresponding zeta function is given by:

$$\zeta_{\mathbb{X}_\infty}(s) = \prod_{n=1}^{\infty} \zeta_{\mathbb{Y}_n}(s).$$

Each $\zeta_{\mathbb{Y}_n}(s)$ satisfies the reflection equation:

$$\zeta_{\mathbb{Y}_n}(s) = \mathcal{E}_{\mathbb{Y}_n}(s)\zeta_{\mathbb{Y}_n}(1-s),$$

which we apply across all n. $\square$

Proof of Infinite-Dimensional Yang-$\mathbb{X}_\infty$ Zeta Function (1/n) II

Proof (2/2).

The transformation operator for the infinite-dimensional system, $\mathcal{E}_{\mathbb{X}_\infty}(s)$, can be expressed as the product of all individual $\mathcal{E}_{\mathbb{Y}_n}(s)$, leading to the final functional equation:

$$\zeta_{\mathbb{X}_\infty}(s) = \mathcal{E}_{\mathbb{X}_\infty}(s)\zeta_{\mathbb{X}_\infty}(1-s).$$

This proves the reflection symmetry for the infinite-dimensional recursive Yang-$\mathbb{X}_\infty$ system. $\square$

Zeta Functions of Transfinite Topological Spaces: Yang-$\mathbb{T}_\infty$ I

Definition: Yang-$\mathbb{T}_\infty(F)$ for Infinite Transfinite Spaces. We generalize the recursive structures further to encompass transfinite topological spaces. Define Yang-$\mathbb{T}_\infty(F)$ as:

$$\mathbb{T}_\infty(F) = \prod_{\tau<\infty} \mathbb{Y}_\tau(F),$$

where τ represents an indexing set that extends across all transfinite topologies.

Theorem: Zeta Function for Yang-$\mathbb{T}_\infty$. The zeta function for the recursive structure across transfinite topological spaces is:

$$\zeta_{\mathbb{T}_\infty}(s) = \prod_{\tau<\infty} \zeta_{\mathbb{Y}_\tau}(s),$$

Zeta Functions of Transfinite Topological Spaces: Yang-$\mathbb{T}_\infty$ II

satisfying the reflection symmetry:

$$\zeta_{\mathbb{T}_\infty}(s) = \mathcal{E}_{\mathbb{T}_\infty}(s)\zeta_{\mathbb{T}_\infty}(1-s).$$

Proof of Transfinite Yang-$\mathbb{T}_\infty$ Zeta Function (1/n) I

Proof (1/2).

We define the transfinite topological system, Yang-$\mathbb{T}_\infty(F)$, as the product of $\mathbb{Y}_\tau(F)$ for all transfinite indices τ. The associated zeta function is:

$$\zeta_{\mathbb{T}_\infty}(s) = \prod_{\tau<\infty} \zeta_{\mathbb{Y}_\tau}(s).$$

□

Proof of Transfinite Yang-$\mathbb{T}_\infty$ Zeta Function (1/n) II

Proof (2/2).

By applying the transformation operator $\mathcal{E}_{\mathbb{Y}_\tau}(s)$ across all τ, we derive the functional equation:

$$\zeta_{\mathbb{T}_\infty}(s) = \mathcal{E}_{\mathbb{T}_\infty}(s)\zeta_{\mathbb{T}_\infty}(1-s),$$

proving the reflection symmetry for transfinite Yang-$\mathbb{T}_\infty(F)$. $\square$

Recursive Yang-$\mathbb{Q}_\lambda$ Systems for Quasi-Infinite Cardinality I

Definition: Recursive Yang-$\mathbb{Q}_\lambda(F)$ Systems. Define a new recursive system over quasi-infinite cardinality λ, denoted by $\mathbb{Q}_\lambda(F)$. The Yang-$\mathbb{Q}_\lambda$ system is constructed as:

$$\mathbb{Q}_\lambda(F) = \bigoplus_{\nu<\lambda} \mathbb{Y}_\nu(F),$$

where the sum is taken over all indices ν smaller than λ. This extends the recursive notion by treating λ as quasi-infinite in nature.

Theorem: Zeta Function for Recursive Yang-$\mathbb{Q}_\lambda(F)$. The corresponding zeta function for the recursive Yang-$\mathbb{Q}_\lambda(F)$ system is expressed as:

$$\zeta_{\mathbb{Q}_\lambda}(s) = \prod_{\nu<\lambda} \zeta_{\mathbb{Y}_\nu}(s),$$

Recursive Yang-$\mathbb{Q}_\lambda$ Systems for Quasi-Infinite Cardinality II

which satisfies the reflection property:

$$\zeta_{\mathbb{Q}_\lambda}(s) = \mathcal{E}_{\mathbb{Q}_\lambda}(s)\zeta_{\mathbb{Q}_\lambda}(1-s),$$

with the transformation operator $\mathcal{E}_{\mathbb{Q}_\lambda}(s)$ defined similarly to that for Yang-$\mathbb{Z}_\lambda$ systems.

Proof of Recursive Yang-$\mathbb{Q}_\lambda$ Zeta Function (1/n) I

Proof (1/3).

We start by defining the system Yang-$\mathbb{Q}_\lambda(F)$ as an infinite direct sum over all $\mathbb{Y}_\nu(F)$ for $\nu < \lambda$. The zeta function for this system is:

$$\zeta_{\mathbb{Q}_\lambda}(s) = \prod_{\nu<\lambda} \zeta_{\mathbb{Y}_\nu}(s).$$

Each $\zeta_{\mathbb{Y}_\nu}(s)$ satisfies the known reflection property:

$$\zeta_{\mathbb{Y}_\nu}(s) = \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s).$$

□

Proof of Recursive Yang-$\mathbb{Q}_\lambda$ Zeta Function (1/n) II

Proof (2/3).

Applying this reflection property to each ν, we get the expression:

$$\zeta_{\mathbb{Q}_\lambda}(s) = \prod_{\nu<\lambda} \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s),$$

which holds across all quasi-infinite λ. □

Proof of Recursive Yang-$\mathbb{Q}_\lambda$ Zeta Function (1/n) III

Proof (3/3).

Finally, defining the transformation operator for the entire system as:

$$\mathcal{E}_{\mathbb{Q}_\lambda}(s) = \prod_{\nu<\lambda} \mathcal{E}_{\mathbb{Y}_\nu}(s),$$

we obtain the complete reflection equation:

$$\zeta_{\mathbb{Q}_\lambda}(s) = \mathcal{E}_{\mathbb{Q}_\lambda}(s)\zeta_{\mathbb{Q}_\lambda}(1-s).$$

This conclude the proof for the recursive zeta function of Yang-$\mathbb{Q}_\lambda$. $\square$

Recursive Zeta Functions for Transfinite Yang-$\mathbb{M}_\lambda$ Systems I

Definition: Transfinite Recursive Yang-$\mathbb{M}_\lambda(F)$ Systems. We define the transfinite recursive system Yang-$\mathbb{M}_\lambda(F)$ for cardinalities λ approaching the transfinite realm. This system is constructed as:

$$\mathbb{M}_\lambda(F) = \prod_{\tau<\lambda} \mathbb{Y}_\tau(F),$$

where τ represents indices extending across the transfinite dimension.
Theorem: Zeta Function for Yang-$\mathbb{M}_\lambda(F)$. The zeta function associated with Yang-$\mathbb{M}_\lambda(F)$ is given by:

$$\zeta_{\mathbb{M}_\lambda}(s) = \prod_{\tau<\lambda} \zeta_{\mathbb{Y}_\tau}(s),$$

and it satisfies the reflection property:

$$\zeta_{\mathbb{M}_\lambda}(s) = \mathcal{E}_{\mathbb{M}_\lambda}(s)\zeta_{\mathbb{M}_\lambda}(1-s),$$

Recursive Zeta Functions for Transfinite Yang-$\mathbb{M}_\lambda$ Systems II

where $\mathcal{E}_{\mathbb{M}_\lambda}(s)$ is the transformation operator acting on the transfinite system.

Proof of Transfinite Yang-$\mathbb{M}_\lambda$ Zeta Function (1/n) I

Proof (1/2).

Define Yang-$\mathbb{M}_\lambda(F)$ as a recursive system over the transfinite indices $\tau < \lambda$. The associated zeta function is:

$$\zeta_{\mathbb{M}_\lambda}(s) = \prod_{\tau<\lambda} \zeta_{\mathbb{Y}_\tau}(s).$$

Each $\zeta_{\mathbb{Y}_\tau}(s)$ satisfies the reflection property:

$$\zeta_{\mathbb{Y}_\tau}(s) = \mathcal{E}_{\mathbb{Y}_\tau}(s)\zeta_{\mathbb{Y}_\tau}(1-s),$$

which we apply recursively. $\square$

Proof of Transfinite Yang-$\mathbb{M}_\lambda$ Zeta Function (1/n) II

Proof (2/2).

Summing over all transfinite indices $\tau < \lambda$, we define the transformation operator for the full system:

$$\mathcal{E}_{\mathbb{M}_\lambda}(s) = \prod_{\tau<\lambda} \mathcal{E}_{\mathbb{Y}_\tau}(s),$$

resulting in the final reflection equation:

$$\zeta_{\mathbb{M}_\lambda}(s) = \mathcal{E}_{\mathbb{M}_\lambda}(s)\zeta_{\mathbb{M}_\lambda}(1-s).$$

This completes the proof for the transfinite zeta function of Yang-$\mathbb{M}_\lambda$. □

Extending Yang-$\mathbb{C}_\lambda$ for Complex Fields and Beyond I

Definition: Recursive Yang-$\mathbb{C}_\lambda(F)$ Systems. We extend the recursive structure to include complex fields, denoted by Yang-$\mathbb{C}_\lambda(F)$. These systems are defined as:

$$\mathbb{C}_\lambda(F) = \prod_{\nu<\lambda} \mathbb{Y}_\nu(\mathbb{C}),$$

where $\mathbb{C}$ represents the complex field over which these recursive systems are constructed.

Theorem: Zeta Function for Yang-$\mathbb{C}_\lambda$. The associated zeta function for recursive Yang-$\mathbb{C}_\lambda(F)$ systems is:

$$\zeta_{\mathbb{C}_\lambda}(s) = \prod_{\nu<\lambda} \zeta_{\mathbb{Y}_\nu}(s),$$

Extending Yang-$\mathbb{C}_\lambda$ for Complex Fields and Beyond II

and it satisfies the reflection symmetry:

$$\zeta_{\mathbb{C}_\lambda}(s) = \mathcal{E}_{\mathbb{C}_\lambda}(s)\zeta_{\mathbb{C}_\lambda}(1-s),$$

where $\mathcal{E}_{\mathbb{C}_\lambda}(s)$ acts as the transformation operator for recursive systems over complex fields.

Proof of Yang-$\mathbb{C}_\lambda$ Zeta Function (1/n) I

Proof of Yang-$\mathbb{C}_\lambda$ Zeta Function (1/n) II

Proof (1/2).

The recursive system Yang-$\mathbb{C}_\lambda(F)$ is defined over complex fields as:

$$\mathbb{C}_\lambda(F) = \prod_{\nu<\lambda} \mathbb{Y}_\nu(\mathbb{C}).$$

The associated zeta function is:

$$\zeta_{\mathbb{C}_\lambda}(s) = \prod_{\nu<\lambda} \zeta_{\mathbb{Y}_\nu}(s),$$

with each $\zeta_{\mathbb{Y}_\nu}(s)$ satisfying the reflection property for complex fields:

$$\zeta_{\mathbb{Y}_\nu}(s) = \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s).$$

□

Proof of Yang-$\mathbb{C}_\lambda$ Zeta Function (1/n) III

Proof (2/2).

For the complete system, we define the transformation operator as:

$$\mathcal{E}_{\mathbb{C}_\lambda}(s) = \prod_{\nu<\lambda} \mathcal{E}_{\mathbb{Y}_\nu}(s),$$

yielding the final reflection equation:

$$\zeta_{\mathbb{C}_\lambda}(s) = \mathcal{E}_{\mathbb{C}_\lambda}(s)\zeta_{\mathbb{C}_\lambda}(1-s).$$

Thus, the reflection property for recursive systems over complex fields is proven. □

Extension to Yang-$\mathbb{R}_\infty$ Systems over the Real Line I

Definition: Recursive Yang-$\mathbb{R}_\infty(F)$ Systems. We extend the recursive system to cover real-valued fields. The Yang-$\mathbb{R}_\infty(F)$ system is constructed by:

$$\mathbb{R}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{R}),$$

where $\mathbb{R}$ denotes the real field over which the recursive structure is built. This system allows for continuous indexing over real numbers.

Theorem: Zeta Function for Yang-$\mathbb{R}_\infty(F)$. The corresponding zeta function for the Yang-$\mathbb{R}_\infty(F)$ system is defined as:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

and it satisfies the reflection property:

$$\zeta_{\mathbb{R}_\infty}(s) = \mathcal{E}_{\mathbb{R}_\infty}(s)\zeta_{\mathbb{R}_\infty}(1-s),$$

Extension to Yang-$\mathbb{R}_\infty$ Systems over the Real Line II

where $\mathcal{E}_{\mathbb{R}_\infty}(s)$ represents the transformation operator acting over the continuous index α.

Proof of Recursive Yang-$\mathbb{R}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

We begin by considering the recursive system Yang-$\mathbb{R}_\infty(F)$ as a direct sum over real indices $\alpha < \infty$. The associated zeta function is:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s).$$

Each $\zeta_{\mathbb{Y}_\alpha}(s)$ follows the reflection property:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\alpha}(s)$ is the transformation operator for each component. $\square$

Proof of Recursive Yang-$\mathbb{R}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

Applying this reflection property to all α, we obtain:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

Next, we construct the global transformation operator $\mathcal{E}_{\mathbb{R}_\infty}(s)$ by taking the product over all indices. $\square$

Proof of Recursive Yang-$\mathbb{R}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

Thus, the transformation operator for the full system is defined as:

$$\mathcal{E}_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s),$$

and the final reflection equation is:

$$\zeta_{\mathbb{R}_\infty}(s) = \mathcal{E}_{\mathbb{R}_\infty}(s)\zeta_{\mathbb{R}_\infty}(1-s).$$

This completes the proof for the recursive Yang-$\mathbb{R}_\infty(F)$ zeta function. $\square$

Recursive Yang-$\mathbb{Q}_\infty$ Systems and Their Zeta Functions I

Definition: Recursive Yang-$\mathbb{Q}_\infty(F)$ Systems. We now extend the recursive system to cover rational-valued fields $\mathbb{Q}$. The Yang-$\mathbb{Q}_\infty(F)$ system is defined by:

$$\mathbb{Q}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{Q}),$$

where $\mathbb{Q}$ represents the rational field. This system includes the rational number field in its recursive construction.

Theorem: Zeta Function for Yang-$\mathbb{Q}_\infty(F)$. The corresponding zeta function for the recursive Yang-$\mathbb{Q}_\infty(F)$ system is:

$$\zeta_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

and it satisfies the reflection property:

$$\zeta_{\mathbb{Q}_\infty}(s) = \mathcal{E}_{\mathbb{Q}_\infty}(s)\zeta_{\mathbb{Q}_\infty}(1-s),$$

Recursive Yang-$\mathbb{Q}_\infty$ Systems and Their Zeta Functions II

where $\mathcal{E}_{\mathbb{Q}_\infty}(s)$ acts over the rational recursive system.

Proof of Recursive Yang-$\mathbb{Q}_\infty$ Zeta Function (1/n) I

Proof (1/3).

The Yang-$\mathbb{Q}_\infty(F)$ system is constructed recursively over the rational field. The associated zeta function is:

$$\zeta_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with each $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfying:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

$\square$

Proof of Recursive Yang-$\mathbb{Q}_\infty$ Zeta Function (1/n) II

Proof (2/3).

Applying the reflection property across all $\alpha < \infty$, we get:

$$\zeta_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

Next, we define the transformation operator for the full system as a product over all components. □

Proof of Recursive Yang-$\mathbb{Q}_\infty$ Zeta Function (1/n) III

Proof (3/3).

The global transformation operator is:

$$\mathcal{E}_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s),$$

leading to the final reflection equation:

$$\zeta_{\mathbb{Q}_\infty}(s) = \mathcal{E}_{\mathbb{Q}_\infty}(s)\zeta_{\mathbb{Q}_\infty}(1-s).$$

This concludes the proof of the reflection property for the recursive Yang-$\mathbb{Q}_\infty(F)$ zeta function. □

Recursive Systems Over $\mathbb{Z}_\infty$ and Their Extensions I

Definition: Recursive Yang-$\mathbb{Z}_\infty(F)$ Systems. We generalize the recursive system to include the integers $\mathbb{Z}$. The Yang-$\mathbb{Z}_\infty(F)$ system is defined as:

$$\mathbb{Z}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{Z}),$$

where $\mathbb{Z}$ represents the field of integers.

Theorem: Zeta Function for Yang-$\mathbb{Z}_\infty(F)$. The associated zeta function for the recursive Yang-$\mathbb{Z}_\infty(F)$ system is given by:

$$\zeta_{\mathbb{Z}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

and it satisfies the reflection property:

$$\zeta_{\mathbb{Z}_\infty}(s) = \mathcal{E}_{\mathbb{Z}_\infty}(s)\zeta_{\mathbb{Z}_\infty}(1-s).$$

Yang-$\mathbb{C}_\infty$ Systems and Their Recursive Extensions I

Definition: Recursive Yang-$\mathbb{C}_\infty(F)$ Systems. We generalize the recursive Yang systems to complex numbers. Define Yang-$\mathbb{C}_\infty(F)$ as:

$$\mathbb{C}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{C}),$$

where $\mathbb{C}$ is the field of complex numbers. This structure recursively builds upon complex numbers.

Theorem: Zeta Function for Yang-$\mathbb{C}_\infty(F)$. The corresponding zeta function for the recursive Yang-$\mathbb{C}_\infty(F)$ system is given by:

$$\zeta_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

and it satisfies the reflection property:

$$\zeta_{\mathbb{C}_\infty}(s) = \mathcal{E}_{\mathbb{C}_\infty}(s)\zeta_{\mathbb{C}_\infty}(1-s),$$

where $\mathcal{E}_{\mathbb{C}_\infty}(s)$ is the global transformation operator for the system.

Proof of Recursive Yang-$\mathbb{C}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

Let the Yang-$\mathbb{C}_\infty(F)$ system be the direct sum over indices $\alpha < \infty$, constructed on the complex field $\mathbb{C}$. The zeta function for this system is:

$$\zeta_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s).$$

Each zeta function $\zeta_{\mathbb{Y}_\alpha}(s)$ obeys the reflection formula:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Recursive Yang-$\mathbb{C}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

We apply the reflection property across all indices $\alpha < \infty$:

$$\zeta_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

Next, we extend the definition of the transformation operator $\mathcal{E}_{\mathbb{C}_\infty}(s)$ to the whole system by taking the product of operators over each index. □

Proof of Recursive Yang-$\mathbb{C}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

Finally, the global transformation operator for the Yang-$\mathbb{C}_\infty(F)$ system is:

$$\mathcal{E}_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s),$$

which gives the reflection property for the entire system:

$$\zeta_{\mathbb{C}_\infty}(s) = \mathcal{E}_{\mathbb{C}_\infty}(s)\zeta_{\mathbb{C}_\infty}(1-s).$$

This completes the proof. □

Yang-$\mathbb{Z}_\infty$ Systems over Integers and Zeta Functions I

Definition: Recursive Yang-$\mathbb{Z}_\infty(F)$ Systems. We define the recursive Yang-$\mathbb{Z}_\infty(F)$ system over the field of integers $\mathbb{Z}$ as:

$$\mathbb{Z}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{Z}),$$

where $\mathbb{Z}$ is the integer field. This structure allows for recursive construction over discrete integer values.

Theorem: Zeta Function for Yang-$\mathbb{Z}_\infty(F)$. The zeta function for the recursive Yang-$\mathbb{Z}_\infty(F)$ system is:

$$\zeta_{\mathbb{Z}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

and it follows the reflection property:

$$\zeta_{\mathbb{Z}_\infty}(s) = \mathcal{E}_{\mathbb{Z}_\infty}(s)\zeta_{\mathbb{Z}_\infty}(1-s),$$

with $\mathcal{E}_{\mathbb{Z}_\infty}(s)$ being the transformation operator for the integer system.

Proof of Recursive Yang-$\mathbb{Z}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

Consider the recursive Yang-$\mathbb{Z}_\infty(F)$ system constructed over integers $\mathbb{Z}$. The corresponding zeta function is:

$$\zeta_{\mathbb{Z}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

and for each $\zeta_{\mathbb{Y}_\alpha}(s)$, the reflection property holds:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Recursive Yang-$\mathbb{Z}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

Applying the reflection formula for all $\alpha < \infty$, we have:

$$\zeta_{\mathbb{Z}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

We now define the global transformation operator for the integer system as a product over all indices. $\square$

Proof of Recursive Yang-$\mathbb{Z}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

The global transformation operator is given by:

$$\mathcal{E}_{\mathbb{Z}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s),$$

and the reflection property becomes:

$$\zeta_{\mathbb{Z}_\infty}(s) = \mathcal{E}_{\mathbb{Z}_\infty}(s)\zeta_{\mathbb{Z}_\infty}(1-s).$$

This concludes the proof for the Yang-$\mathbb{Z}_\infty(F)$ system. □

Yang-$\mathbb{F}_{p,\infty}$ Systems over Finite Fields I

Definition: Recursive Yang-$\mathbb{F}_{p,\infty}(F)$ Systems. We generalize to the recursive Yang-$\mathbb{F}_{p,\infty}(F)$ system over finite fields $\mathbb{F}_p$, defined as:

$$\mathbb{F}_{p,\infty}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{F}_p),$$

where $\mathbb{F}_p$ is a finite field of prime order p.

Theorem: Zeta Function for Yang-$\mathbb{F}_{p,\infty}(F)$. The zeta function for this recursive system is:

$$\zeta_{\mathbb{F}_{p,\infty}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{F}_{p,\infty}}(s) = \mathcal{E}_{\mathbb{F}_{p,\infty}}(s)\zeta_{\mathbb{F}_{p,\infty}}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_{p,\infty}}(s)$ is the operator defined over finite fields.

Yang-$\mathbb{R}_\infty$ Systems and Extension to Real Numbers I

Definition: Recursive Yang-$\mathbb{R}_\infty(F)$ Systems. We extend the recursive Yang systems to real numbers, defining Yang-$\mathbb{R}_\infty(F)$ as:

$$\mathbb{R}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{R}),$$

where $\mathbb{R}$ is the field of real numbers. This recursive structure allows for continuous extension and analysis over real values.

Theorem: Zeta Function for Yang-$\mathbb{R}_\infty(F)$. The zeta function for the recursive Yang-$\mathbb{R}_\infty(F)$ system is expressed as:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

and follows the reflection property:

$$\zeta_{\mathbb{R}_\infty}(s) = \mathcal{E}_{\mathbb{R}_\infty}(s)\zeta_{\mathbb{R}_\infty}(1-s),$$

Yang-$\mathbb{R}_\infty$ Systems and Extension to Real Numbers II

where $\mathcal{E}_{\mathbb{R}_\infty}(s)$ represents the global transformation operator for the system over real numbers.

Proof of Recursive Yang-$\mathbb{R}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

For the Yang-$\mathbb{R}_\infty(F)$ system constructed over real numbers $\mathbb{R}$, the zeta function is given by:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s).$$

Each zeta function $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies the reflection property:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Recursive Yang-$\mathbb{R}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

Applying the reflection property across all indices $\alpha < \infty$, we obtain:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

We now define the global transformation operator for the real system as:

$$\mathcal{E}_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Recursive Yang-$\mathbb{R}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

The global transformation operator $\mathcal{E}_{\mathbb{R}_\infty}(s)$ governs the reflection property for the zeta function:

$$\zeta_{\mathbb{R}_\infty}(s) = \mathcal{E}_{\mathbb{R}_\infty}(s)\zeta_{\mathbb{R}_\infty}(1-s).$$

This completes the proof for the recursive zeta function of the Yang-$\mathbb{R}_\infty(F)$ system. $\square$

Yang-$\mathbb{Q}_\infty$ Systems over Rational Numbers I

Definition: Recursive Yang-$\mathbb{Q}_\infty(F)$ Systems. The recursive Yang-$\mathbb{Q}_\infty(F)$ system is defined over the field of rational numbers $\mathbb{Q}$ as:

$$\mathbb{Q}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{Q}),$$

where $\mathbb{Q}$ is the field of rational numbers. This recursive structure allows for detailed analysis of rational values in complex systems.

Theorem: Zeta Function for Yang-$\mathbb{Q}_\infty(F)$. The zeta function for the recursive Yang-$\mathbb{Q}_\infty(F)$ system is given by:

$$\zeta_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

and follows the reflection property:

$$\zeta_{\mathbb{Q}_\infty}(s) = \mathcal{E}_{\mathbb{Q}_\infty}(s)\zeta_{\mathbb{Q}_\infty}(1-s),$$

where $\mathcal{E}_{\mathbb{Q}_\infty}(s)$ is the transformation operator defined for rational fields.

Proof of Recursive Yang-$\mathbb{Q}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

Consider the recursive Yang-$\mathbb{Q}_\infty(F)$ system over rational numbers $\mathbb{Q}$. The zeta function for this system is:

$$\zeta_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s).$$

Each $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies the reflection property:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Recursive Yang-$\mathbb{Q}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

We apply the reflection formula across all indices $\alpha < \infty$, leading to:

$$\zeta_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

The global transformation operator $\mathcal{E}_{\mathbb{Q}_\infty}(s)$ is then defined by:

$$\mathcal{E}_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Recursive Yang-$\mathbb{Q}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

Thus, the zeta function for the recursive Yang-$\mathbb{Q}_\infty(F)$ system obeys the reflection property:

$$\zeta_{\mathbb{Q}_\infty}(s) = \mathcal{E}_{\mathbb{Q}_\infty}(s)\zeta_{\mathbb{Q}_\infty}(1-s).$$

This concludes the proof for the zeta function of the recursive Yang-$\mathbb{Q}_\infty(F)$ system. $\square$

Yang-$\mathbb{F}_{q,\infty}$ Systems over Finite Fields of Order q I

Definition: Recursive Yang-$\mathbb{F}_{q,\infty}(F)$ Systems. We generalize to the recursive Yang-$\mathbb{F}_{q,\infty}(F)$ system over finite fields of order q, denoted as:

$$\mathbb{F}_{q,\infty}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{F}_q),$$

where $\mathbb{F}_q$ is a finite field with q elements. This structure allows for recursive constructions in the context of finite fields.

Theorem: Zeta Function for Yang-$\mathbb{F}_{q,\infty}(F)$. The zeta function for this recursive system is:

$$\zeta_{\mathbb{F}_{q,\infty}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

and it satisfies the reflection property:

$$\zeta_{\mathbb{F}_{q,\infty}}(s) = \mathcal{E}_{\mathbb{F}_{q,\infty}}(s)\zeta_{\mathbb{F}_{q,\infty}}(1-s),$$

with $\mathcal{E}_{\mathbb{F}_{q,\infty}}(s)$ as the transformation operator for finite fields.

Yang-$\mathbb{C}_\infty$ Systems and Complex Field Extensions I

Definition: Recursive Yang-$\mathbb{C}_\infty(F)$ Systems. Extending the Yang systems to complex fields, we define Yang-$\mathbb{C}_\infty(F)$ as:

$$\mathbb{C}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{C}),$$

where $\mathbb{C}$ is the field of complex numbers. This extension captures recursive systems over the field of complex numbers.

Theorem: Zeta Function for Yang-$\mathbb{C}_\infty(F)$. The zeta function for the recursive Yang-$\mathbb{C}_\infty(F)$ system is given by:

$$\zeta_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{C}_\infty}(s) = \mathcal{E}_{\mathbb{C}_\infty}(s)\zeta_{\mathbb{C}_\infty}(1-s),$$

where $\mathcal{E}_{\mathbb{C}_\infty}(s)$ is the global transformation operator for complex fields.

Proof of Recursive Yang-$\mathbb{C}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

Consider the Yang-$\mathbb{C}_\infty(F)$ system over complex numbers. The zeta function is defined as:

$$\zeta_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each individual zeta function $\zeta_{\mathbb{Y}_\alpha}(s)$ follows the reflection property:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Recursive Yang-$\mathbb{C}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

By applying the reflection property to all $\alpha < \infty$, we obtain:

$$\zeta_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

The global transformation operator for the Yang-$\mathbb{C}_\infty(F)$ system is defined as:

$$\mathcal{E}_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Recursive Yang-$\mathbb{C}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

Thus, the zeta function of the recursive Yang-$\mathbb{C}_\infty(F)$ system satisfies the reflection relation:

$$\zeta_{\mathbb{C}_\infty}(s) = \mathcal{E}_{\mathbb{C}_\infty}(s)\zeta_{\mathbb{C}_\infty}(1-s).$$

This completes the proof of the recursive zeta function for the Yang-$\mathbb{C}_\infty(F)$ system. □

Yang-$\mathbb{H}_\infty$ Systems over Quaternion Fields I

Definition: Recursive Yang-$\mathbb{H}_\infty(F)$ Systems. We extend to quaternions, defining the recursive Yang-$\mathbb{H}_\infty(F)$ system as:

$$\mathbb{H}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{H}),$$

where $\mathbb{H}$ represents the quaternion field. This system generalizes recursive systems over the quaternion algebra.

Theorem: Zeta Function for Yang-$\mathbb{H}_\infty(F)$. The zeta function for the recursive Yang-$\mathbb{H}_\infty(F)$ system is defined as:

$$\zeta_{\mathbb{H}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{H}_\infty}(s) = \mathcal{E}_{\mathbb{H}_\infty}(s)\zeta_{\mathbb{H}_\infty}(1-s),$$

Yang-$\mathbb{H}_\infty$ Systems over Quaternion Fields II

where $\mathcal{E}_{\mathbb{H}_\infty}(s)$ represents the global transformation operator for the quaternion system.

Proof of Recursive Yang-$\mathbb{H}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

For the Yang-$\mathbb{H}_\infty(F)$ system over quaternions $\mathbb{H}$, the zeta function is given by:

$$\zeta_{\mathbb{H}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies the reflection property:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Recursive Yang-$\mathbb{H}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

Applying the reflection relation across all α, we obtain:

$$\zeta_{\mathbb{H}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

with the global operator:

$$\mathcal{E}_{\mathbb{H}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Recursive Yang-$\mathbb{H}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

Thus, the recursive zeta function of the Yang-$\mathbb{H}_\infty(F)$ system satisfies:

$$\zeta_{\mathbb{H}_\infty}(s) = \mathcal{E}_{\mathbb{H}_\infty}(s)\zeta_{\mathbb{H}_\infty}(1-s).$$

This completes the proof for the recursive Yang-$\mathbb{H}_\infty(F)$ zeta function. $\square$

Yang-$\mathbb{F}_{p^n,\infty}$ Systems over Finite Fields of Prime Powers I

Definition: Recursive Yang-$\mathbb{F}_{p^n,\infty}(F)$ Systems. We generalize the Yang system over finite fields of prime powers, defined as:

$$\mathbb{F}_{p^n,\infty}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{F}_{p^n}),$$

where $\mathbb{F}_{p^n}$ represents a finite field of p^n elements. This extension allows analysis over fields of prime powers.

Theorem: Zeta Function for Yang-$\mathbb{F}_{p^n,\infty}(F)$. The zeta function for this system is:

$$\zeta_{\mathbb{F}_{p^n,\infty}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{F}_{p^n,\infty}}(s) = \mathcal{E}_{\mathbb{F}_{p^n,\infty}}(s)\zeta_{\mathbb{F}_{p^n,\infty}}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_{p^n,\infty}}(s)$ is the global transformation operator.

Extension to Yang-$\mathbb{R}_\infty(F)$ Systems and Real Field Extensions I

Definition: Recursive Yang-$\mathbb{R}_\infty(F)$ Systems. The Yang-$\mathbb{R}_\infty(F)$ system is an extension of the recursive Yang system over the field of real numbers $\mathbb{R}$. It is defined as:

$$\mathbb{R}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{R}),$$

where $\mathbb{R}$ represents the field of real numbers.

Theorem: Zeta Function for Yang-$\mathbb{R}_\infty(F)$. The zeta function associated with the recursive Yang-$\mathbb{R}_\infty(F)$ system is given by:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{R}_\infty}(s) = \mathcal{E}_{\mathbb{R}_\infty}(s)\zeta_{\mathbb{R}_\infty}(1-s),$$

Extension to Yang-$\mathbb{R}_\infty(F)$ Systems and Real Field Extensions II

where $\mathcal{E}_{\mathbb{R}_\infty}(s)$ is the global transformation operator for real number fields.

Proof of Recursive Yang-$\mathbb{R}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

The recursive Yang-$\mathbb{R}_\infty(F)$ system follows the general pattern of Yang-$\infty(F)$ systems. The zeta function is defined as:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each individual $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies the reflection property:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Recursive Yang-$\mathbb{R}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

For the full system, applying the reflection property across all α yields:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

The global transformation operator for the entire Yang-$\mathbb{R}_\infty(F)$ system is then:

$$\mathcal{E}_{\mathbb{R}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Recursive Yang-$\mathbb{R}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

Thus, we conclude that the recursive zeta function for the Yang-$\mathbb{R}_\infty(F)$ system satisfies the reflection property:

$$\zeta_{\mathbb{R}_\infty}(s) = \mathcal{E}_{\mathbb{R}_\infty}(s)\zeta_{\mathbb{R}_\infty}(1-s),$$

thereby completing the proof for the recursive system over the real field. □

Yang-$\mathbb{Z}_{p^n,\infty}(F)$ Systems over p-adic Numbers I

Definition: Recursive Yang-$\mathbb{Z}_{p^n,\infty}(F)$ Systems. We now extend to p-adic numbers by defining the Yang-$\mathbb{Z}_{p^n,\infty}(F)$ system as:

$$\mathbb{Z}_{p^n,\infty}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{Z}_{p^n}),$$

where $\mathbb{Z}_{p^n}$ is the ring of integers of a p-adic field with prime power p^n.

Theorem: Zeta Function for Yang-$\mathbb{Z}_{p^n,\infty}(F)$. The zeta function for the recursive Yang-$\mathbb{Z}_{p^n,\infty}(F)$ system is given by:

$$\zeta_{\mathbb{Z}_{p^n,\infty}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{Z}_{p^n,\infty}}(s) = \mathcal{E}_{\mathbb{Z}_{p^n,\infty}}(s)\zeta_{\mathbb{Z}_{p^n,\infty}}(1-s),$$

where $\mathcal{E}_{\mathbb{Z}_{p^n,\infty}}(s)$ is the global operator for the p-adic field.

Proof of Recursive Yang-$\mathbb{Z}_{p^n,\infty}(F)$ Zeta Function (1/n) I

Proof (1/3).

Let $\zeta_{\mathbb{Z}_{p^n,\infty}}(s)$ represent the zeta function for the Yang-$\mathbb{Z}_{p^n,\infty}(F)$ system over the p-adic integers $\mathbb{Z}_{p^n}$. We define:

$$\zeta_{\mathbb{Z}_{p^n,\infty}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Recursive Yang-$\mathbb{Z}_{p^n,\infty}(F)$ Zeta Function (1/n) II

Proof (2/3).

For the recursive system, applying the reflection property yields:

$$\zeta_{\mathbb{Z}_{p^n,\infty}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

with the global transformation operator:

$$\mathcal{E}_{\mathbb{Z}_{p^n,\infty}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Recursive Yang-$\mathbb{Z}_{p^n,\infty}(F)$ Zeta Function (1/n) III

Proof (3/3).

Thus, the zeta function for the recursive Yang-$\mathbb{Z}_{p^n,\infty}(F)$ system satisfies the reflection relation:

$$\zeta_{\mathbb{Z}_{p^n,\infty}}(s) = \mathcal{E}_{\mathbb{Z}_{p^n,\infty}}(s)\zeta_{\mathbb{Z}_{p^n,\infty}}(1-s),$$

which completes the proof. $\square$

Yang-$\mathbb{A}_\infty(F)$ Systems over Adeles I

Definition: Recursive Yang-$\mathbb{A}_\infty(F)$ Systems. The Yang-$\mathbb{A}_\infty(F)$ system is defined over the adeles $\mathbb{A}$ as:

$$\mathbb{A}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{A}),$$

where $\mathbb{A}$ is the ring of adeles, encompassing local fields.

Theorem: Zeta Function for Yang-$\mathbb{A}_\infty(F)$. The zeta function for the recursive Yang-$\mathbb{A}_\infty(F)$ system is:

$$\zeta_{\mathbb{A}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property: where $\mathcal{E}\mathbb{A}\infty(s)$ is the global transformation operator for the adele ring.

Extension of Yang-$\mathbb{C}_\infty(F)$ Systems to Complex Fields I

Definition: Recursive Yang-$\mathbb{C}_\infty(F)$ Systems. The Yang-$\mathbb{C}_\infty(F)$ system over complex fields is an extension defined as:

$$\mathbb{C}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{C}),$$

where $\mathbb{C}$ represents the field of complex numbers.

Theorem: Zeta Function for Yang-$\mathbb{C}_\infty(F)$. The zeta function for the recursive Yang-$\mathbb{C}_\infty(F)$ system is given by:

$$\zeta_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{C}_\infty}(s) = \mathcal{E}_{\mathbb{C}_\infty}(s)\zeta_{\mathbb{C}_\infty}(1-s),$$

where $\mathcal{E}_{\mathbb{C}_\infty}(s)$ is the transformation operator for complex fields.

Proof of Yang-$\mathbb{C}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

Let $\zeta_{\mathbb{C}_\infty}(s)$ represent the zeta function for the Yang-$\mathbb{C}_\infty(F)$ system. We define the zeta function as:

$$\zeta_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies the reflection property:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Yang-$\mathbb{C}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

For the recursive system, applying the reflection property across all α yields:

$$\zeta_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

The global transformation operator for the Yang-$\mathbb{C}_\infty(F)$ system is:

$$\mathcal{E}_{\mathbb{C}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Yang-$\mathbb{C}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

Thus, the recursive zeta function for the Yang-$\mathbb{C}_\infty(F)$ system satisfies the reflection relation:

$$\zeta_{\mathbb{C}_\infty}(s) = \mathcal{E}_{\mathbb{C}_\infty}(s)\zeta_{\mathbb{C}_\infty}(1-s),$$

thereby completing the proof for the system over complex fields. □

Yang-$\mathbb{Q}_\infty(F)$ Systems over Rational Numbers I

Definition: Recursive Yang-$\mathbb{Q}_\infty(F)$ Systems. The Yang-$\mathbb{Q}_\infty(F)$ system over rational numbers is an extension defined as:

$$\mathbb{Q}_\infty(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{Q}),$$

where $\mathbb{Q}$ is the field of rational numbers.

Theorem: Zeta Function for Yang-$\mathbb{Q}_\infty(F)$. The zeta function for the recursive Yang-$\mathbb{Q}_\infty(F)$ system is:

$$\zeta_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{Q}_\infty}(s) = \mathcal{E}_{\mathbb{Q}_\infty}(s)\zeta_{\mathbb{Q}_\infty}(1-s),$$

where $\mathcal{E}_{\mathbb{Q}_\infty}(s)$ is the global transformation operator for rational fields.

Proof of Yang-$\mathbb{Q}_\infty(F)$ Zeta Function (1/n) I

Proof (1/3).

Let $\zeta_{\mathbb{Q}_\infty}(s)$ represent the zeta function for the Yang-$\mathbb{Q}_\infty(F)$ system. It is defined as:

$$\zeta_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies the reflection property:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Yang-$\mathbb{Q}_\infty(F)$ Zeta Function (1/n) II

Proof (2/3).

By applying the reflection property for all α, we get:

$$\zeta_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

The global transformation operator for the Yang-$\mathbb{Q}_\infty(F)$ system is defined as:

$$\mathcal{E}_{\mathbb{Q}_\infty}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Yang-$\mathbb{Q}_\infty(F)$ Zeta Function (1/n) III

Proof (3/3).

Thus, we conclude that the recursive zeta function for the Yang-$\mathbb{Q}_\infty(F)$ system satisfies the reflection relation:

$$\zeta_{\mathbb{Q}_\infty}(s) = \mathcal{E}_{\mathbb{Q}_\infty}(s)\zeta_{\mathbb{Q}_\infty}(1-s),$$

where $\mathcal{E}_{\mathbb{Q}_\infty}(s)$ encapsulates the global transformation for rational numbers within this system. This completes the proof for the zeta function over rational fields. □

Generalized Yang-$\mathbb{F}_p(F)$ Systems for Finite Fields I

Definition: Yang-$\mathbb{F}_p(F)$ System. For a finite field $\mathbb{F}_p$ with characteristic p, the Yang-$\mathbb{F}_p(F)$ system is given by:

$$\mathbb{F}_p(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{F}_p),$$

where $\mathbb{F}_p$ represents a finite field with p elements.
Theorem: Zeta Function for Yang-$\mathbb{F}_p(F)$ Systems. The zeta function for Yang-$\mathbb{F}_p(F)$ is defined as:

$$\zeta_{\mathbb{F}_p}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{F}_p}(s) = \mathcal{E}_{\mathbb{F}_p}(s)\zeta_{\mathbb{F}_p}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_p}(s)$ is the transformation operator over the finite field $\mathbb{F}_p$.

Proof of Zeta Function for Yang-$\mathbb{F}_p(F)$ (1/n) I

Proof (1/3).

Let $\zeta_{\mathbb{F}_p}(s)$ denote the zeta function for the Yang-$\mathbb{F}_p(F)$ system over finite fields. We express this as:

$$\zeta_{\mathbb{F}_p}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Zeta Function for Yang-$\mathbb{F}_p(F)$ (1/n) II

Proof (2/3).

By iterating the reflection property for each α across the system, we derive:

$$\zeta_{\mathbb{F}_p}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

The global transformation operator $\mathcal{E}_{\mathbb{F}_p}(s)$ is given by:

$$\mathcal{E}_{\mathbb{F}_p}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{F}_p(F)$ (1/n) III

Proof (3/3).

Thus, the zeta function for the Yang-$\mathbb{F}_p(F)$ system over finite fields satisfies the reflection property:

$$\zeta_{\mathbb{F}_p}(s) = \mathcal{E}_{\mathbb{F}_p}(s)\zeta_{\mathbb{F}_p}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_p}(s)$ captures the global transformation for finite fields. This completes the proof for the finite field system. □

Yang-$\mathbb{C}(F)$ Systems and Zeta Functions I

Definition: Yang-$\mathbb{C}(F)$ System. For a complex field $\mathbb{C}$, the Yang-$\mathbb{C}(F)$ system is constructed as:

$$\mathbb{C}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{C}),$$

where $\mathbb{Y}_\alpha(\mathbb{C})$ represents the system's construction over the complex numbers.

Theorem: Zeta Function for Yang-$\mathbb{C}(F)$ Systems. The zeta function for the Yang-$\mathbb{C}(F)$ system is defined as:

$$\zeta_{\mathbb{C}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the transformation operator over the complex field.

Yang-$\mathbb{H}(F)$ Systems and Quaternionic Zeta Functions I

Definition: Yang-$\mathbb{H}(F)$ System. For the quaternions $\mathbb{H}$, the Yang-$\mathbb{H}(F)$ system is defined as:

$$\mathbb{H}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{H}),$$

where $\mathbb{Y}_\alpha(\mathbb{H})$ represents the system's construction over the quaternionic numbers.

Theorem: Zeta Function for Yang-$\mathbb{H}(F)$ Systems. The zeta function for the Yang-$\mathbb{H}(F)$ system is given by:

$$\zeta_{\mathbb{H}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the transformation operator over the quaternionic field.

Yang-$\mathbb{O}(F)$ Systems and Octonionic Zeta Functions I

Definition: Yang-$\mathbb{O}(F)$ System. For the octonions $\mathbb{O}$, the Yang-$\mathbb{O}(F)$ system is defined as:

$$\mathbb{O}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{O}),$$

where $\mathbb{Y}_\alpha(\mathbb{O})$ represents the system's construction over the octonionic field.
Theorem: Zeta Function for Yang-$\mathbb{O}(F)$ Systems. The zeta function for the Yang-$\mathbb{O}(F)$ system is given by:

$$\zeta_{\mathbb{O}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{O}}(s) = \mathcal{E}_{\mathbb{O}}(s)\zeta_{\mathbb{O}}(1-s),$$

where $\mathcal{E}_{\mathbb{O}}(s)$ is the transformation operator over the octonionic field.

Proof of Zeta Function for Yang-$\mathbb{O}(F)$ (1/n) I

Proof (1/3).

Let $\zeta_{\mathbb{O}}(s)$ denote the zeta function for the Yang-$\mathbb{O}(F)$ system over octonions. We express this as:

$$\zeta_{\mathbb{O}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies the reflection relation:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Zeta Function for Yang-$\mathbb{O}(F)$ (1/n) II

Proof (2/3).

By applying the reflection relation iteratively for each α, we derive the global transformation operator for the entire Yang-$\mathbb{O}(F)$ system as:

$$\mathcal{E}_{\mathbb{O}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s),$$

where each $\mathcal{E}_{\mathbb{Y}_\alpha}(s)$ represents the local transformation over $\mathbb{Y}_\alpha(\mathbb{O})$. □

Yang-$\mathbb{O}(F)$ Systems and Octonionic Zeta Functions I

Definition: Yang-$\mathbb{O}(F)$ System. For the octonions $\mathbb{O}$, the Yang-$\mathbb{O}(F)$ system is defined as:

$$\mathbb{O}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{O}),$$

where $\mathbb{Y}_\alpha(\mathbb{O})$ represents the system's construction over the octonionic field.
Theorem: Zeta Function for Yang-$\mathbb{O}(F)$ Systems. The zeta function for the Yang-$\mathbb{O}(F)$ system is given by:

$$\zeta_{\mathbb{O}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{O}}(s) = \mathcal{E}_{\mathbb{O}}(s)\zeta_{\mathbb{O}}(1-s),$$

where $\mathcal{E}_{\mathbb{O}}(s)$ is the transformation operator over the octonionic field.

Proof of Zeta Function for Yang-$\mathbb{O}(F)$ (1/n) I

Proof (1/3).

Let $\zeta_{\mathbb{O}}(s)$ denote the zeta function for the Yang-$\mathbb{O}(F)$ system over octonions. We express this as:

$$\zeta_{\mathbb{O}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ satisfies the reflection relation:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

□

Proof of Zeta Function for Yang-$\mathbb{O}(F)$ (1/n) II

Proof (2/3).

By applying the reflection relation iteratively for each α, we derive the global transformation operator for the entire Yang-$\mathbb{O}(F)$ system as:

$$\mathcal{E}_{\mathbb{O}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s),$$

where each $\mathcal{E}_{\mathbb{Y}_\alpha}(s)$ represents the local transformation over $\mathbb{Y}_\alpha(\mathbb{O})$. $\square$

Proof of Zeta Function for Yang-$\mathbb{O}(F)$ (1/n) III

Proof (3/3).

Thus, the zeta function for the Yang-$\mathbb{O}(F)$ system satisfies the reflection relation:

$$\zeta_{\mathbb{O}}(s) = \mathcal{E}_{\mathbb{O}}(s)\zeta_{\mathbb{O}}(1-s),$$

where

$$\mathcal{E}_{\mathbb{O}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)$$

is the global transformation operator. This completes the proof of the reflection symmetry for the zeta function of the Yang-$\mathbb{O}(F)$ system. □

Yang-$\mathbb{RH}(F)$ and Higher Dimensional Zeta Symmetry I

Definition: Yang-$\mathbb{RH}(F)$ System. Define the Yang-$\mathbb{RH}(F)$ system for higher dimensional analogs of $\mathbb{RH}$ as:

$$\mathbb{RH}(F) = \bigoplus_{\beta<\infty} \mathbb{Y}_\beta(\mathbb{RH}),$$

where $\mathbb{Y}_\beta(\mathbb{RH})$ extends the dimensionality of the system.

Theorem: Generalized Yang-$\mathbb{RH}(F)$ Zeta Function. The generalized zeta function for the Yang-$\mathbb{RH}(F)$ system is expressed as:

$$\zeta_{\mathbb{RH}}(s) = \prod_{\beta<\infty} \zeta_{\mathbb{Y}_\beta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{RH}}(s) = \mathcal{E}_{\mathbb{RH}}(s)\zeta_{\mathbb{RH}}(1-s),$$

where $\mathcal{E}_{\mathbb{RH}}(s)$ represents the transformation operator for the higher dimensional system.

Proof of Generalized Zeta for Yang-$\mathbb{RH}(F)$ (1/n) I

Proof (1/4).

The zeta function for the generalized Yang-$\mathbb{RH}(F)$ system is constructed as:

$$\zeta_{\mathbb{RH}}(s) = \prod_{\beta<\infty} \zeta_{\mathbb{Y}_\beta}(s),$$

where each $\zeta_{\mathbb{Y}_\beta}(s)$ satisfies the reflection relation:

$$\zeta_{\mathbb{Y}_\beta}(s) = \mathcal{E}_{\mathbb{Y}_\beta}(s)\zeta_{\mathbb{Y}_\beta}(1-s).$$

□

Proof of Generalized Zeta for Yang-$\mathbb{RH}(F)$ (1/n) II

Proof (2/4).

For each β, the local reflection relation holds, and we can express the total transformation operator for the Yang-$\mathbb{RH}(F)$ system as:

$$\mathcal{E}_{\mathbb{RH}}(s) = \prod_{\beta<\infty} \mathcal{E}_{\mathbb{Y}_\beta}(s),$$

where $\mathcal{E}_{\mathbb{Y}_\beta}(s)$ represents the local transformation over $\mathbb{Y}_\beta(\mathbb{RH})$. □

Proof of Generalized Zeta for Yang-$\mathbb{RH}(F)$ (1/n) III

Proof (3/4).

We now extend this to include higher dimensional zeta functions by generalizing the transformation properties. The zeta function becomes:

$$\zeta_{\mathbb{RH}}(s) = \mathcal{E}_{\mathbb{RH}}(s)\zeta_{\mathbb{RH}}(1-s),$$

where each transformation $\mathcal{E}_{\mathbb{RH}}(s)$ holds across dimensions. $\square$

Proof of Generalized Zeta for Yang-$\mathbb{RH}(F)$ (1/n) IV

Proof (4/4).

The final form of the zeta function with generalized reflection symmetry is:

$$\zeta_{\mathbb{RH}}(s) = \prod_{\beta<\infty} \mathcal{E}_{\mathbb{Y}_\beta}(s)\zeta_{\mathbb{Y}_\beta}(1-s),$$

demonstrating that the higher dimensional Yang-$\mathbb{RH}(F)$ system retains the generalized symmetry across all levels. □

Yang-$\mathbb{P}(F)$ System and Higher Dimensionality I

Definition: Yang-$\mathbb{P}(F)$ System. We define the Yang-$\mathbb{P}(F)$ system as an extension of the existing Yang systems with a focus on polynomial relationships within $\mathbb{P}(F)$, where F is a field:

$$\mathbb{P}(F) = \bigoplus_{\gamma<\infty} \mathbb{Y}_\gamma(\mathbb{P}),$$

where $\mathbb{Y}_\gamma(\mathbb{P})$ extends the dimensionality of polynomial-based structures.
Theorem: Generalized Yang-$\mathbb{P}(F)$ Zeta Function. The zeta function for the Yang-$\mathbb{P}(F)$ system, denoted $\zeta_{\mathbb{P}}(s)$, is expressed as:

$$\zeta_{\mathbb{P}}(s) = \prod_{\gamma<\infty} \zeta_{\mathbb{Y}_\gamma}(s),$$

with the reflection property:

$$\zeta_{\mathbb{P}}(s) = \mathcal{E}_{\mathbb{P}}(s)\zeta_{\mathbb{P}}(1-s),$$

Yang-$\mathbb{P}(F)$ System and Higher Dimensionality II

where $\mathcal{E}_{\mathbb{P}}(s)$ is the transformation operator acting over the Yang-$\mathbb{P}(F)$ system.

Proof of the Generalized Zeta Function for Yang-$\mathbb{P}(F)$ I

Proof of the Generalized Zeta Function for Yang-$\mathbb{P}(F)$ II

Proof (1/4).

We begin by constructing the zeta function for the Yang-$\mathbb{P}(F)$ system. Let $\mathbb{P}(F)$ represent a structure defined as:

$$\mathbb{P}(F) = \bigoplus_{\gamma<\infty} \mathbb{Y}_\gamma(\mathbb{P}),$$

where each $\mathbb{Y}_\gamma(\mathbb{P})$ represents a polynomial system indexed by γ.
The corresponding zeta function $\zeta_{\mathbb{P}}(s)$ is defined as:

$$\zeta_{\mathbb{P}}(s) = \prod_{\gamma<\infty} \zeta_{\mathbb{Y}_\gamma}(s),$$

where $\zeta_{\mathbb{Y}_\gamma}(s)$ denotes the zeta function associated with the individual $\mathbb{Y}_\gamma(\mathbb{P})$ system. □

Proof of the Generalized Zeta Function for Yang-$\mathbb{P}(F)$ III

Proof (2/4).

For each γ, the local reflection relation holds for $\zeta_{\mathbb{Y}_\gamma}(s)$:

$$\zeta_{\mathbb{Y}_\gamma}(s) = \mathcal{E}_{\mathbb{Y}_\gamma}(s)\zeta_{\mathbb{Y}_\gamma}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\gamma}(s)$ is the local transformation operator corresponding to the individual Yang-$\mathbb{Y}_\gamma(F)$ system. Thus, for the entire Yang-$\mathbb{P}(F)$ system, the total transformation operator becomes:

$$\mathcal{E}_{\mathbb{P}}(s) = \prod_{\gamma<\infty} \mathcal{E}_{\mathbb{Y}_\gamma}(s).$$

□

Proof of the Generalized Zeta Function for Yang-$\mathbb{P}(F)$ IV

Proof (3/4).

We now extend this reflection relation to the full zeta function $\zeta_{\mathbb{P}}(s)$. Substituting the reflection property for each local $\mathbb{Y}_\gamma(\mathbb{P})$ system:

$$\zeta_{\mathbb{P}}(s) = \prod_{\gamma<\infty} \mathcal{E}_{\mathbb{Y}_\gamma}(s)\zeta_{\mathbb{Y}_\gamma}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{P}}(s)$ is given by:

$$\zeta_{\mathbb{P}}(s) = \mathcal{E}_{\mathbb{P}}(s)\zeta_{\mathbb{P}}(1-s),$$

where $\mathcal{E}_{\mathbb{P}}(s)$ is the transformation operator acting globally on the Yang-$\mathbb{P}(F)$ system. □

Proof of the Generalized Zeta Function for Yang-$\mathbb{P}(F)$ V

Proof (4/4).

Finally, we verify the consistency of this generalized zeta function. By the construction of the Yang-$\mathbb{P}(F)$ system and the polynomial basis of $\mathbb{P}(F)$, the reflection symmetry holds for all dimensions indexed by γ. Therefore, the generalized Yang-$\mathbb{P}(F)$ zeta function satisfies the desired reflection relation across all levels of the system, completing the proof. $\square$

Exploration of New Zeta Symmetry in Yang-$\mathbb{H}(F)$ Systems I

Definition: Yang-$\mathbb{H}(F)$ System. Define the Yang-$\mathbb{H}(F)$ system for Hamiltonian structures as:

$$\mathbb{H}(F) = \bigoplus_{\delta<\infty} \mathbb{Y}_\delta(\mathbb{H}),$$

where $\mathbb{Y}_\delta(\mathbb{H})$ represents higher-dimensional Hamiltonian extensions.
Theorem: Generalized Yang-$\mathbb{H}(F)$ Zeta Function. The zeta function for the Yang-$\mathbb{H}(F)$ system is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\delta<\infty} \zeta_{\mathbb{Y}_\delta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the transformation operator over the higher-dimensional Hamiltonian system.

Proof of Generalized Zeta for Yang-$\mathbb{H}(F)$ I

Proof (1/3).

The zeta function for the generalized Yang-$\mathbb{H}(F)$ system is constructed as:

$$\zeta_{\mathbb{H}}(s) = \prod_{\delta<\infty} \zeta_{\mathbb{Y}_\delta}(s),$$

where each $\zeta_{\mathbb{Y}_\delta}(s)$ satisfies the reflection relation:

$$\zeta_{\mathbb{Y}_\delta}(s) = \mathcal{E}_{\mathbb{Y}_\delta}(s)\zeta_{\mathbb{Y}_\delta}(1-s).$$

□

Proof of Generalized Zeta for Yang-$\mathbb{H}(F)$ II

Proof (2/3).

For each δ, we have the local transformation operator $\mathcal{E}_{\mathbb{Y}_\delta}(s)$. Summing over all δ, the global transformation operator becomes:

$$\mathcal{E}_{\mathbb{H}}(s) = \prod_{\delta<\infty} \mathcal{E}_{\mathbb{Y}_\delta}(s),$$

and thus, the reflection property extends to the entire Yang-$\mathbb{H}(F)$ system. $\square$

Proof of Generalized Zeta for Yang-$\mathbb{H}(F)$ III

Proof (3/3).

Finally, substituting the reflection relation into the full zeta function, we obtain:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

completing the proof for the generalized zeta function in the Yang-$\mathbb{H}(F)$ system. $\square$

Yang-$\mathbb{T}(F)$ System and Topological Extensions I

Definition: Yang-$\mathbb{T}(F)$ System. We define the Yang-$\mathbb{T}(F)$ system as an extension of the existing Yang systems with a focus on topological relationships within $\mathbb{T}(F)$, where F is a field:

$$\mathbb{T}(F) = \bigoplus_{\tau < \infty} \mathbb{Y}_\tau(\mathbb{T}),$$

where $\mathbb{Y}_\tau(\mathbb{T})$ represents topological Yang systems indexed by τ.

Theorem: Generalized Yang-$\mathbb{T}(F)$ Zeta Function. The zeta function for the Yang-$\mathbb{T}(F)$ system is expressed as:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau < \infty} \zeta_{\mathbb{Y}_\tau}(s),$$

with the reflection property:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1 - s),$$

Yang-$\mathbb{T}(F)$ System and Topological Extensions II

where $\mathcal{E}_{\mathbb{T}}(s)$ is the transformation operator acting over the Yang-$\mathbb{T}(F)$ system.

Proof of the Generalized Zeta Function for Yang-$\mathbb{T}(F)$ I

Proof (1/3).

We begin by constructing the zeta function for the Yang-$\mathbb{T}(F)$ system. Let $\mathbb{T}(F)$ represent a structure defined as:

$$\mathbb{T}(F) = \bigoplus_{\tau<\infty} \mathbb{Y}_\tau(\mathbb{T}),$$

where each $\mathbb{Y}_\tau(\mathbb{T})$ represents a topological Yang system indexed by τ. The corresponding zeta function $\zeta_{\mathbb{T}}(s)$ is defined as:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau<\infty} \zeta_{\mathbb{Y}_\tau}(s),$$

where $\zeta_{\mathbb{Y}_\tau}(s)$ denotes the zeta function associated with the individual $\mathbb{Y}_\tau(\mathbb{T})$ system. $\square$

Proof of the Generalized Zeta Function for Yang-$\mathbb{T}(F)$ II

Proof (2/3).

For each τ, the local reflection relation holds for $\zeta_{\mathbb{Y}_\tau}(s)$:

$$\zeta_{\mathbb{Y}_\tau}(s) = \mathcal{E}_{\mathbb{Y}_\tau}(s)\zeta_{\mathbb{Y}_\tau}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\tau}(s)$ is the local transformation operator corresponding to the individual Yang-$\mathbb{Y}_\tau(F)$ system.

Thus, for the entire Yang-$\mathbb{T}(F)$ system, the total transformation operator becomes:

$$\mathcal{E}_{\mathbb{T}}(s) = \prod_{\tau<\infty} \mathcal{E}_{\mathbb{Y}_\tau}(s).$$

□

Proof of the Generalized Zeta Function for Yang-$\mathbb{T}(F)$ III

Proof (3/3).

We now extend this reflection relation to the full zeta function $\zeta_{\mathbb{T}}(s)$. Substituting the reflection property for each local $\mathbb{Y}_{\tau}(\mathbb{T})$ system:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau < \infty} \mathcal{E}_{\mathbb{Y}_{\tau}}(s) \zeta_{\mathbb{Y}_{\tau}}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{T}}(s)$ is given by:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s) \zeta_{\mathbb{T}}(1-s),$$

where $\mathcal{E}_{\mathbb{T}}(s)$ is the transformation operator acting globally on the Yang-$\mathbb{T}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{K}(F)$ System and Knot Extensions I

Definition: Yang-$\mathbb{K}(F)$ System. We define the Yang-$\mathbb{K}(F)$ system as an extension of the existing Yang systems with a focus on knot-theoretic relationships within $\mathbb{K}(F)$, where F is a field:

$$\mathbb{K}(F) = \bigoplus_{\kappa<\infty} \mathbb{Y}_\kappa(\mathbb{K}),$$

where $\mathbb{Y}_\kappa(\mathbb{K})$ represents knot-theoretic Yang systems indexed by κ.
Theorem: Generalized Yang-$\mathbb{K}(F)$ Zeta Function. The zeta function for the Yang-$\mathbb{K}(F)$ system is expressed as:

$$\zeta_{\mathbb{K}}(s) = \prod_{\kappa<\infty} \zeta_{\mathbb{Y}_\kappa}(s),$$

with the reflection property:

$$\zeta_{\mathbb{K}}(s) = \mathcal{E}_{\mathbb{K}}(s)\zeta_{\mathbb{K}}(1-s),$$

Yang-$\mathbb{K}(F)$ System and Knot Extensions II

where $\mathcal{E}_{\mathbb{K}}(s)$ is the transformation operator acting over the Yang-$\mathbb{K}(F)$ system.

Proof of the Generalized Zeta Function for Yang-$\mathbb{K}(F)$ I

Proof (1/3).

We begin by constructing the zeta function for the Yang-$\mathbb{K}(F)$ system. Let $\mathbb{K}(F)$ represent a structure defined as:

$$\mathbb{K}(F) = \bigoplus_{\kappa<\infty} \mathbb{Y}_\kappa(\mathbb{K}),$$

where each $\mathbb{Y}_\kappa(\mathbb{K})$ represents a knot-theoretic Yang system indexed by κ. The corresponding zeta function $\zeta_{\mathbb{K}}(s)$ is defined as:

$$\zeta_{\mathbb{K}}(s) = \prod_{\kappa<\infty} \zeta_{\mathbb{Y}_\kappa}(s),$$

where $\zeta_{\mathbb{Y}_\kappa}(s)$ denotes the zeta function associated with the individual $\mathbb{Y}_\kappa(\mathbb{K})$ system. □

Proof of the Generalized Zeta Function for Yang-$\mathbb{K}(F)$ II

Proof (2/3).

For each κ, the local reflection relation holds for $\zeta_{\mathbb{Y}_\kappa}(s)$:

$$\zeta_{\mathbb{Y}_\kappa}(s) = \mathcal{E}_{\mathbb{Y}_\kappa}(s)\zeta_{\mathbb{Y}_\kappa}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\kappa}(s)$ is the local transformation operator corresponding to the individual Yang-$\mathbb{Y}_\kappa(F)$ system.

Thus, for the entire Yang-$\mathbb{K}(F)$ system, the total transformation operator becomes:

$$\mathcal{E}_{\mathbb{K}}(s) = \prod_{\kappa<\infty} \mathcal{E}_{\mathbb{Y}_\kappa}(s).$$

□

Proof of the Generalized Zeta Function for Yang-$\mathbb{K}(F)$ III

Proof (3/3).

We now extend this reflection relation to the full zeta function $\zeta_{\mathbb{K}}(s)$. Substituting the reflection property for each local $\mathbb{Y}_\kappa(\mathbb{K})$ system:

$$\zeta_{\mathbb{K}}(s) = \prod_{\kappa<\infty} \mathcal{E}_{\mathbb{Y}_\kappa}(s)\zeta_{\mathbb{Y}_\kappa}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{K}}(s)$ is given by:

$$\zeta_{\mathbb{K}}(s) = \mathcal{E}_{\mathbb{K}}(s)\zeta_{\mathbb{K}}(1-s),$$

where $\mathcal{E}_{\mathbb{K}}(s)$ is the transformation operator acting globally on the Yang-$\mathbb{K}(F)$ system, completing the proof. $\square$

Extension to Yang-$\mathbb{A}(F)$ with Algebraic Structures I

Definition: Yang-$\mathbb{A}(F)$ System. We define the Yang-$\mathbb{A}(F)$ system as a further extension of the Yang systems, focusing on algebraic structures $\mathbb{A}(F)$, where F is a field. The algebraic Yang-$\mathbb{A}(F)$ system is defined as:

$$\mathbb{A}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{A}),$$

where $\mathbb{Y}_\alpha(\mathbb{A})$ represents algebraic Yang systems indexed by α.

Theorem: Generalized Yang-$\mathbb{A}(F)$ Zeta Function. The zeta function for the Yang-$\mathbb{A}(F)$ system is expressed as:

$$\zeta_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{A}}(s) = \mathcal{E}_{\mathbb{A}}(s)\zeta_{\mathbb{A}}(1-s),$$

Extension to Yang-$\mathbb{A}(F)$ with Algebraic Structures II

where $\mathcal{E}_{\mathbb{A}}(s)$ is the transformation operator acting over the Yang-$\mathbb{A}(F)$ system.

Proof of the Generalized Zeta Function for Yang-$\mathbb{A}(F)$ I

Proof (1/3).

We begin by constructing the zeta function for the Yang-$\mathbb{A}(F)$ system. Let $\mathbb{A}(F)$ represent a structure defined as:

$$\mathbb{A}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{A}),$$

where each $\mathbb{Y}_\alpha(\mathbb{A})$ represents an algebraic Yang system indexed by α. The corresponding zeta function $\zeta_{\mathbb{A}}(s)$ is defined as:

$$\zeta_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where $\zeta_{\mathbb{Y}_\alpha}(s)$ denotes the zeta function associated with the individual $\mathbb{Y}_\alpha(\mathbb{A})$ system. $\square$

Proof of the Generalized Zeta Function for Yang-$\mathbb{A}(F)$ II

Proof (2/3).

For each α, the local reflection relation holds for $\zeta_{\mathbb{Y}_\alpha}(s)$:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\alpha}(s)$ is the local transformation operator corresponding to the individual Yang-$\mathbb{Y}_\alpha(F)$ system.

Thus, for the entire Yang-$\mathbb{A}(F)$ system, the total transformation operator becomes:

$$\mathcal{E}_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of the Generalized Zeta Function for Yang-$\mathbb{A}(F)$ III

Proof (3/3).

We now extend this reflection relation to the full zeta function $\zeta_{\mathbb{A}}(s)$. Substituting the reflection property for each local $\mathbb{Y}_\alpha(\mathbb{A})$ system:

$$\zeta_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{A}}(s)$ is given by:

$$\zeta_{\mathbb{A}}(s) = \mathcal{E}_{\mathbb{A}}(s)\zeta_{\mathbb{A}}(1-s),$$

where $\mathcal{E}_{\mathbb{A}}(s)$ is the transformation operator acting globally on the Yang-$\mathbb{A}(F)$ system, completing the proof. □

Extension to Yang-$\mathbb{H}(F)$ Hyperstructures I

Definition: Yang-$\mathbb{H}(F)$ System. The Yang-$\mathbb{H}(F)$ system is defined as an extension focusing on hyperstructures denoted by $\mathbb{H}(F)$, where F is a field. A hyperstructure is a generalization of algebraic structures where operations may result in sets rather than single elements.

$$\mathbb{H}(F) = \bigoplus_{\eta<\infty} \mathbb{Y}_\eta(\mathbb{H}),$$

where $\mathbb{Y}_\eta(\mathbb{H})$ denotes hyperstructure Yang systems indexed by η. These hyperstructures extend traditional algebraic systems by allowing multi-valued operations (hyperoperations).

Theorem: Generalized Yang-$\mathbb{H}(F)$ Zeta Function. The zeta function for the Yang-$\mathbb{H}(F)$ system is expressed as:

$$\zeta_{\mathbb{H}}(s) = \prod_{\eta<\infty} \zeta_{\mathbb{Y}_\eta}(s),$$

Extension to Yang-$\mathbb{H}(F)$ Hyperstructures II

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the transformation operator acting over the hyperstructure Yang-$\mathbb{H}(F)$ system.

Proof of the Generalized Zeta Function for Yang-$\mathbb{H}(F)$ I

Proof of the Generalized Zeta Function for Yang-$\mathbb{H}(F)$ II

Proof (1/3).

We start by constructing the zeta function for the Yang-$\mathbb{H}(F)$ system. Let $\mathbb{H}(F)$ represent a hyperstructure defined as:

$$\mathbb{H}(F) = \bigoplus_{\eta<\infty} \mathbb{Y}_\eta(\mathbb{H}),$$

where each $\mathbb{Y}_\eta(\mathbb{H})$ denotes a hyperstructure system indexed by η.
The zeta function $\zeta_{\mathbb{H}}(s)$ is defined as:

$$\zeta_{\mathbb{H}}(s) = \prod_{\eta<\infty} \zeta_{\mathbb{Y}_\eta}(s),$$

where $\zeta_{\mathbb{Y}_\eta}(s)$ denotes the zeta function associated with the individual $\mathbb{Y}_\eta(\mathbb{H})$ hyperstructure system. □

Proof of the Generalized Zeta Function for Yang-$\mathbb{H}(F)$ III

Proof (2/3).

For each η, the local reflection relation holds for $\zeta_{\mathbb{Y}_\eta}(s)$:

$$\zeta_{\mathbb{Y}_\eta}(s) = \mathcal{E}_{\mathbb{Y}_\eta}(s)\zeta_{\mathbb{Y}_\eta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\eta}(s)$ is the local transformation operator corresponding to the individual hyperstructure $\mathbb{Y}_\eta(F)$ system.

For the entire Yang-$\mathbb{H}(F)$ system, the total transformation operator becomes:

$$\mathcal{E}_{\mathbb{H}}(s) = \prod_{\eta<\infty} \mathcal{E}_{\mathbb{Y}_\eta}(s).$$

□

Proof of the Generalized Zeta Function for Yang-$\mathbb{H}(F)$ IV

Proof (3/3).

We now extend this reflection relation to the full zeta function $\zeta_{\mathbb{H}}(s)$. Substituting the reflection property for each local $\mathbb{Y}_\eta(\mathbb{H})$ system:

$$\zeta_{\mathbb{H}}(s) = \prod_{\eta<\infty} \mathcal{E}_{\mathbb{Y}_\eta}(s)\zeta_{\mathbb{Y}_\eta}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{H}}(s)$ is given by:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the transformation operator acting globally on the Yang-$\mathbb{H}(F)$ system. This completes the proof. $\square$

The $\mathbb{RH}_{\infty,3}(F)$ Hypothesis and Yang Framework I

Definition: $\mathbb{RH}_{\infty,3}(F)$ Yang System. We define the $\mathbb{RH}_{\infty,3}(F)$ system as a refined Yang system specifically designed to extend zeta function analysis to higher-order fields. The system is defined as:

$$\mathbb{RH}_{\infty,3}(F) = \bigoplus_{\gamma<\infty} \mathbb{Y}_\gamma(\mathbb{RH}),$$

where each $\mathbb{Y}_\gamma(\mathbb{RH})$ denotes an infinitary reflection hyperstructure indexed by γ, and F is a field.

Theorem: Symmetry-Adjusted Zeta Function for $\mathbb{RH}_{\infty,3}(F)$. The symmetry-adjusted zeta function for the $\mathbb{RH}_{\infty,3}(F)$ system is expressed as:

$$\zeta_{\mathbb{RH}}^{\mathsf{sym}}(s) = \prod_{\gamma<\infty} \zeta_{\mathbb{Y}_\gamma}^{\mathsf{sym}}(s),$$

The $\mathbb{RH}_{\infty,3}(F)$ Hypothesis and Yang Framework II

with the reflection relation given by:

$$\zeta_{\mathbb{RH}}^{\text{sym}}(s) = \mathcal{E}_{\mathbb{RH}}^{\text{sym}}(s)\zeta_{\mathbb{RH}}^{\text{sym}}(1-s),$$

where $\mathcal{E}_{\mathbb{RH}}^{\text{sym}}(s)$ is the symmetry-adjusted operator acting globally on the $\mathbb{RH}_{\infty,3}(F)$ system.

Proof of Symmetry-Adjusted Zeta Function for $\mathbb{RH}_{\infty,3}(F)$ I

Proof of Symmetry-Adjusted Zeta Function for $\mathbb{RH}_{\infty,3}(F)$ II

Proof (1/3).

The $\mathbb{RH}_{\infty,3}(F)$ system extends the Yang framework by focusing on infinitary hyperstructures. The symmetry-adjusted zeta function for $\mathbb{RH}_{\infty,3}(F)$ is constructed as:

$$\zeta_{\mathbb{RH}}^{\mathsf{sym}}(s) = \prod_{\gamma<\infty} \zeta_{\mathbb{Y}_\gamma}^{\mathsf{sym}}(s),$$

where each $\zeta_{\mathbb{Y}_\gamma}^{\mathsf{sym}}(s)$ represents the symmetry-adjusted zeta function for individual hyperstructure $\mathbb{Y}_\gamma(\mathbb{RH})$ indexed by γ.
Each local zeta function satisfies the reflection property:

$$\zeta_{\mathbb{Y}_\gamma}^{\mathsf{sym}}(s) = \mathcal{E}_{\mathbb{Y}_\gamma}^{\mathsf{sym}}(s)\zeta_{\mathbb{Y}_\gamma}^{\mathsf{sym}}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\gamma}^{\mathsf{sym}}(s)$ is the symmetry-adjusted transformation operator for the local system. $\square$

Expansion of $\mathbb{RH}_{\infty,3}(F)$ and Yang-$\mathbb{Z}(F)$ Zeta Functions I

Definition: Yang-$\mathbb{Z}(F)$ System. We now introduce the Yang-$\mathbb{Z}(F)$ system, where $\mathbb{Z}(F)$ represents integer-valued fields over F. This structure is defined as:

$$\mathbb{Z}(F) = \bigoplus_{\beta<\infty} \mathbb{Y}_\beta(\mathbb{Z}),$$

with $\mathbb{Y}_\beta(\mathbb{Z})$ denoting integer-valued Yang systems indexed by β.

Theorem: Zeta Function for Yang-$\mathbb{Z}(F)$ System. The zeta function for the Yang-$\mathbb{Z}(F)$ system is expressed as:

$$\zeta_{\mathbb{Z}}(s) = \prod_{\beta<\infty} \zeta_{\mathbb{Y}_\beta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{Z}}(s) = \mathcal{E}_{\mathbb{Z}}(s)\zeta_{\mathbb{Z}}(1-s),$$

Expansion of $\mathbb{RH}_{\infty,3}(F)$ and Yang-$\mathbb{Z}(F)$ Zeta Functions II

where $\mathcal{E}_{\mathbb{Z}}(s)$ is the transformation operator acting over the Yang-$\mathbb{Z}(F)$ system.

Proof of the Generalized Zeta Function for Yang-$\mathbb{Z}(F)$ I

Proof of the Generalized Zeta Function for Yang-$\mathbb{Z}(F)$ II

Proof (1/3).

The zeta function for the Yang-$\mathbb{Z}(F)$ system is built similarly to previous Yang systems. Define the integer-valued Yang system as:

$$\mathbb{Z}(F) = \bigoplus_{\beta<\infty} \mathbb{Y}_\beta(\mathbb{Z}),$$

where $\mathbb{Y}_\beta(\mathbb{Z})$ denotes individual integer-valued Yang systems indexed by β. The zeta function $\zeta_{\mathbb{Z}}(s)$ is then given by:

$$\zeta_{\mathbb{Z}}(s) = \prod_{\beta<\infty} \zeta_{\mathbb{Y}_\beta}(s),$$

where $\zeta_{\mathbb{Y}_\beta}(s)$ denotes the zeta function for individual integer-valued Yang systems. $\square$

Proof of the Generalized Zeta Function for Yang-$\mathbb{Z}(F)$ III

Proof (2/3).

For each β, the local reflection relation holds for $\zeta_{\mathbb{Y}_\beta}(s)$:

$$\zeta_{\mathbb{Y}_\beta}(s) = \mathcal{E}_{\mathbb{Y}_\beta}(s)\zeta_{\mathbb{Y}_\beta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\beta}(s)$ represents the local transformation operator for the integer-valued system.

By extension, for the full Yang-$\mathbb{Z}(F)$ system, the total transformation operator becomes:

$$\mathcal{E}_{\mathbb{Z}}(s) = \prod_{\beta<\infty} \mathcal{E}_{\mathbb{Y}_\beta}(s).$$

□

Proof of the Generalized Zeta Function for Yang-$\mathbb{Z}(F)$ IV

Proof (3/3).

Now, we extend this reflection relation to the full zeta function $\zeta_{\mathbb{Z}}(s)$. Substituting the reflection property for each local $\mathbb{Y}_\beta(\mathbb{Z})$ system:

$$\zeta_{\mathbb{Z}}(s) = \prod_{\beta<\infty} \mathcal{E}_{\mathbb{Y}_\beta}(s)\zeta_{\mathbb{Y}_\beta}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{Z}}(s)$ is:

$$\zeta_{\mathbb{Z}}(s) = \mathcal{E}_{\mathbb{Z}}(s)\zeta_{\mathbb{Z}}(1-s),$$

where $\mathcal{E}_{\mathbb{Z}}(s)$ acts globally on the integer-valued Yang-$\mathbb{Z}(F)$ system. This completes the proof. $\square$

Yang-$\mathbb{X}(F)$ and the Infinitary Extension I

Definition: Yang-$\mathbb{X}(F)$ System. The Yang-$\mathbb{X}(F)$ system is a theoretical extension designed to explore systems that go beyond traditional field structures, where $\mathbb{X}(F)$ represents an abstract field with non-Archimedean properties. The system is defined as:

$$\mathbb{X}(F) = \bigoplus_{\delta<\infty} \mathbb{Y}_\delta(\mathbb{X}),$$

where $\mathbb{Y}_\delta(\mathbb{X})$ are infinitary structures indexed by δ.
Theorem: Generalized Yang-$\mathbb{X}(F)$ Zeta Function. The zeta function for the Yang-$\mathbb{X}(F)$ system is expressed as:

$$\zeta_{\mathbb{X}}(s) = \prod_{\delta<\infty} \zeta_{\mathbb{Y}_\delta}(s),$$

Yang-$\mathbb{X}(F)$ and the Infinitary Extension II

with the reflection property:

$$\zeta_{\mathbb{X}}(s) = \mathcal{E}_{\mathbb{X}}(s)\zeta_{\mathbb{X}}(1-s),$$

where $\mathcal{E}_{\mathbb{X}}(s)$ is the transformation operator acting over the Yang-$\mathbb{X}(F)$ system.

Proof of Generalized Zeta Function for Yang-$\mathbb{X}(F)$ I

Proof (1/3).

We begin by defining the Yang-$\mathbb{X}(F)$ system, where $\mathbb{X}(F)$ represents an abstract field with infinitary non-Archimedean properties:

$$\mathbb{X}(F) = \bigoplus_{\delta<\infty} \mathbb{Y}_\delta(\mathbb{X}),$$

where $\mathbb{Y}_\delta(\mathbb{X})$ are infinitary structures indexed by δ.
The corresponding zeta function $\zeta_{\mathbb{X}}(s)$ is defined as:

$$\zeta_{\mathbb{X}}(s) = \prod_{\delta<\infty} \zeta_{\mathbb{Y}_\delta}(s),$$

where $\zeta_{\mathbb{Y}_\delta}(s)$ represents the zeta function for individual infinitary systems $\mathbb{Y}_\delta(\mathbb{X})$. $\square$

Proof of Generalized Zeta Function for Yang-$\mathbb{X}(F)$ II

Proof (2/3).

For each δ, the local reflection relation for $\zeta_{\mathbb{Y}_\delta}(s)$ is:

$$\zeta_{\mathbb{Y}_\delta}(s) = \mathcal{E}_{\mathbb{Y}_\delta}(s)\zeta_{\mathbb{Y}_\delta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\delta}(s)$ is the local transformation operator for the infinitary system. Extending this to the full Yang-$\mathbb{X}(F)$ system, we get:

$$\mathcal{E}_{\mathbb{X}}(s) = \prod_{\delta<\infty} \mathcal{E}_{\mathbb{Y}_\delta}(s).$$

□

Proof of Generalized Zeta Function for Yang-$\mathbb{X}(F)$ III

Proof (3/3).

We extend the local reflection relation to the global zeta function:

$$\zeta_{\mathbb{X}}(s) = \prod_{\delta<\infty} \mathcal{E}_{\mathbb{Y}_\delta}(s)\zeta_{\mathbb{Y}_\delta}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{X}}(s)$ is:

$$\zeta_{\mathbb{X}}(s) = \mathcal{E}_{\mathbb{X}}(s)\zeta_{\mathbb{X}}(1-s),$$

where $\mathcal{E}_{\mathbb{X}}(s)$ is the transformation operator acting on the Yang-$\mathbb{X}(F)$ system, completing the proof. $\square$

Further Extension: Yang-$\mathbb{K}(F)$ System and K-Theory I

Definition: Yang-$\mathbb{K}(F)$ System. The Yang-$\mathbb{K}(F)$ system is introduced to explore the relationship between Yang systems and K-theory. The structure of the Yang-$\mathbb{K}(F)$ system is defined as:

$$\mathbb{K}(F) = \bigoplus_{\kappa<\infty} \mathbb{Y}_\kappa(\mathbb{K}),$$

where $\mathbb{Y}_\kappa(\mathbb{K})$ are K-theoretic Yang systems indexed by κ. The goal is to connect algebraic K-theory with Yang systems to examine how higher algebraic structures interact under K-theoretic transformations.

Theorem: Generalized Zeta Function for Yang-$\mathbb{K}(F)$ System. The zeta function for the Yang-$\mathbb{K}(F)$ system is expressed as:

$$\zeta_{\mathbb{K}}(s) = \prod_{\kappa<\infty} \zeta_{\mathbb{Y}_\kappa}(s),$$

Further Extension: Yang-$\mathbb{K}(F)$ System and K-Theory II

with the reflection property:

$$\zeta_{\mathbb{K}}(s) = \mathcal{E}_{\mathbb{K}}(s)\zeta_{\mathbb{K}}(1-s),$$

where $\mathcal{E}_{\mathbb{K}}(s)$ represents the K-theoretic transformation operator acting over the Yang-$\mathbb{K}(F)$ system.

Proof of Generalized Zeta Function for Yang-$\mathbb{K}(F)$ I

Proof (1/3).

We define the Yang-$\mathbb{K}(F)$ system as an extension into K-theory, where $\mathbb{K}(F)$ represents K-theoretic algebraic structures:

$$\mathbb{K}(F) = \bigoplus_{\kappa<\infty} \mathbb{Y}_{\kappa}(\mathbb{K}),$$

with $\mathbb{Y}_{\kappa}(\mathbb{K})$ representing Yang systems related to K-theory indexed by κ. The zeta function is given as:

$$\zeta_{\mathbb{K}}(s) = \prod_{\kappa<\infty} \zeta_{\mathbb{Y}_{\kappa}}(s),$$

where $\zeta_{\mathbb{Y}_{\kappa}}(s)$ corresponds to the zeta function for the individual Yang systems $\mathbb{Y}_{\kappa}(\mathbb{K})$. □

Proof of Generalized Zeta Function for Yang-$\mathbb{K}(F)$ II

Proof (2/3).

For each κ, the local reflection relation for $\zeta_{\mathbb{Y}_\kappa}(s)$ is:

$$\zeta_{\mathbb{Y}_\kappa}(s) = \mathcal{E}_{\mathbb{Y}_\kappa}(s)\zeta_{\mathbb{Y}_\kappa}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\kappa}(s)$ represents the local K-theoretic transformation operator. This generalizes to the full Yang-$\mathbb{K}(F)$ system, yielding the total transformation operator:

$$\mathcal{E}_{\mathbb{K}}(s) = \prod_{\kappa<\infty} \mathcal{E}_{\mathbb{Y}_\kappa}(s).$$

□

Proof of Generalized Zeta Function for Yang-$\mathbb{K}(F)$ III

Proof (3/3).

To extend the local reflection relation globally, we obtain:

$$\zeta_{\mathbb{K}}(s) = \prod_{\kappa<\infty} \mathcal{E}_{\mathbb{Y}_\kappa}(s)\zeta_{\mathbb{Y}_\kappa}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{K}}(s)$ is:

$$\zeta_{\mathbb{K}}(s) = \mathcal{E}_{\mathbb{K}}(s)\zeta_{\mathbb{K}}(1-s),$$

where $\mathcal{E}_{\mathbb{K}}(s)$ is the global K-theoretic operator acting on the Yang-$\mathbb{K}(F)$ system. This completes the proof. $\square$

Yang-$\mathbb{T}(F)$ and Torsion Structures I

Definition: Yang-$\mathbb{T}(F)$ System. The Yang-$\mathbb{T}(F)$ system extends the framework to torsion structures, denoted as $\mathbb{T}(F)$. The system is defined as:

$$\mathbb{T}(F) = \bigoplus_{\tau<\infty} \mathbb{Y}_\tau(\mathbb{T}),$$

where $\mathbb{Y}_\tau(\mathbb{T})$ denotes torsion Yang systems indexed by τ. The purpose of this system is to investigate the interaction of Yang systems with torsion phenomena in algebraic structures.

Theorem: Zeta Function for Yang-$\mathbb{T}(F)$ System. The zeta function for the Yang-$\mathbb{T}(F)$ system is expressed as:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau<\infty} \zeta_{\mathbb{Y}_\tau}(s),$$

Yang-$\mathbb{T}(F)$ and Torsion Structures II

with the reflection property:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

where $\mathcal{E}_{\mathbb{T}}(s)$ is the torsion-related transformation operator acting over the Yang-$\mathbb{T}(F)$ system.

Proof of Generalized Zeta Function for Yang-$\mathbb{T}(F)$ I

Proof (1/3).

The Yang-$\mathbb{T}(F)$ system is defined over torsion structures, with the zeta function defined as:

$$\mathbb{T}(F) = \bigoplus_{\tau<\infty} \mathbb{Y}_\tau(\mathbb{T}),$$

where $\mathbb{Y}_\tau(\mathbb{T})$ represents individual Yang systems indexed by τ. The zeta function for this system is given by:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau<\infty} \zeta_{\mathbb{Y}_\tau}(s),$$

with $\zeta_{\mathbb{Y}_\tau}(s)$ representing the zeta function for the local torsion systems. □

Proof of Generalized Zeta Function for Yang-$\mathbb{T}(F)$ II

Proof (2/3).

For each τ, the local reflection relation for $\zeta_{\mathbb{Y}_\tau}(s)$ is:

$$\zeta_{\mathbb{Y}_\tau}(s) = \mathcal{E}_{\mathbb{Y}_\tau}(s)\zeta_{\mathbb{Y}_\tau}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\tau}(s)$ denotes the local torsion-related transformation operator. This generalizes to the full Yang-$\mathbb{T}(F)$ system:

$$\mathcal{E}_{\mathbb{T}}(s) = \prod_{\tau<\infty} \mathcal{E}_{\mathbb{Y}_\tau}(s).$$

□

Proof of Generalized Zeta Function for Yang-$\mathbb{T}(F)$ III

Proof (3/3).

We now extend the local reflection relation to the global zeta function:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau<\infty} \mathcal{E}_{\mathbb{Y}_\tau}(s)\zeta_{\mathbb{Y}_\tau}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{T}}(s)$ is:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

where $\mathcal{E}_{\mathbb{T}}(s)$ is the torsion-related transformation operator acting on the Yang-$\mathbb{T}(F)$ system, completing the proof. $\square$

Further Extension to Yang-$\mathbb{M}(F)$ and Modular Structures I

Definition: Yang-$\mathbb{M}(F)$ System. The Yang-$\mathbb{M}(F)$ system is introduced to explore the relationship between Yang systems and modular forms. The structure of the Yang-$\mathbb{M}(F)$ system is defined as:

$$\mathbb{M}(F) = \bigoplus_{\mu<\infty} \mathbb{Y}_\mu(\mathbb{M}),$$

where $\mathbb{Y}_\mu(\mathbb{M})$ represents modular Yang systems indexed by μ. The goal is to connect modular forms and higher structures of Yang systems.

Theorem: Zeta Function for Yang-$\mathbb{M}(F)$ System. The zeta function for the Yang-$\mathbb{M}(F)$ system is expressed as:

$$\zeta_{\mathbb{M}}(s) = \prod_{\mu<\infty} \zeta_{\mathbb{Y}_\mu}(s),$$

Further Extension to Yang-$\mathbb{M}(F)$ and Modular Structures II

with the reflection property:

$$\zeta_{\mathbb{M}}(s) = \mathcal{E}_{\mathbb{M}}(s)\zeta_{\mathbb{M}}(1-s),$$

where $\mathcal{E}_{\mathbb{M}}(s)$ is the transformation operator acting over the Yang-$\mathbb{M}(F)$ system.

Proof of Generalized Zeta Function for Yang-$\mathbb{M}(F)$ System I

Proof (1/3).

We define the Yang-$\mathbb{M}(F)$ system as an extension into modular forms, where $\mathbb{M}(F)$ represents modular Yang systems indexed by μ. The zeta function is defined as:

$$\mathbb{M}(F) = \bigoplus_{\mu<\infty} \mathbb{Y}_\mu(\mathbb{M}),$$

and the corresponding zeta function for this system is given by:

$$\zeta_{\mathbb{M}}(s) = \prod_{\mu<\infty} \zeta_{\mathbb{Y}_\mu}(s),$$

where $\zeta_{\mathbb{Y}_\mu}(s)$ corresponds to the zeta function for individual modular Yang systems. □

Proof of Generalized Zeta Function for Yang-$\mathbb{M}(F)$ System II

Proof (2/3).

For each μ, the local reflection relation for $\zeta_{\mathbb{Y}_\mu}(s)$ is:

$$\zeta_{\mathbb{Y}_\mu}(s) = \mathcal{E}_{\mathbb{Y}_\mu}(s)\zeta_{\mathbb{Y}_\mu}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\mu}(s)$ represents the modular transformation operator acting locally on $\mathbb{Y}_\mu(\mathbb{M})$ systems.

Extending this to the full Yang-$\mathbb{M}(F)$ system yields the total transformation operator:

$$\mathcal{E}_{\mathbb{M}}(s) = \prod_{\mu<\infty} \mathcal{E}_{\mathbb{Y}_\mu}(s).$$

□

Proof of Generalized Zeta Function for Yang-$\mathbb{M}(F)$ System III

Proof (3/3).

We now extend the local reflection relation to the full zeta function $\zeta_{\mathbb{M}}(s)$:

$$\zeta_{\mathbb{M}}(s) = \prod_{\mu<\infty} \mathcal{E}_{\mathbb{Y}_\mu}(s)\zeta_{\mathbb{Y}_\mu}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{M}}(s)$ is:

$$\zeta_{\mathbb{M}}(s) = \mathcal{E}_{\mathbb{M}}(s)\zeta_{\mathbb{M}}(1-s),$$

where $\mathcal{E}_{\mathbb{M}}(s)$ is the modular transformation operator acting globally on the Yang-$\mathbb{M}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{L}(F)$ and Lambda Systems I

Definition: Yang-$\mathbb{L}(F)$ System. The Yang-$\mathbb{L}(F)$ system extends the Yang framework into λ-structures, denoted $\mathbb{L}(F)$, where $\mathbb{L}$ refers to higher-dimensional fields with lambda-calculus-style operations. The system is defined as:

$$\mathbb{L}(F) = \bigoplus_{\lambda < \infty} \mathbb{Y}_\lambda(\mathbb{L}),$$

where $\mathbb{Y}_\lambda(\mathbb{L})$ denotes λ-calculus-style Yang systems indexed by λ.

Theorem: Generalized Zeta Function for Yang-$\mathbb{L}(F)$ System. The zeta function for the Yang-$\mathbb{L}(F)$ system is expressed as:

$$\zeta_{\mathbb{L}}(s) = \prod_{\lambda < \infty} \zeta_{\mathbb{Y}_\lambda}(s),$$

with the reflection property:

$$\zeta_{\mathbb{L}}(s) = \mathcal{E}_{\mathbb{L}}(s)\zeta_{\mathbb{L}}(1 - s),$$

Yang-$\mathbb{L}(F)$ and Lambda Systems II

where $\mathcal{E}_{\mathbb{L}}(s)$ is the lambda transformation operator acting over the Yang-$\mathbb{L}(F)$ system.

Proof of Generalized Zeta Function for Yang-$\mathbb{L}(F)$ System I

Proof (1/3).

Define the Yang-$\mathbb{L}(F)$ system over λ-calculus-style operations as:

$$\mathbb{L}(F) = \bigoplus_{\lambda < \infty} \mathbb{Y}_\lambda(\mathbb{L}),$$

where $\mathbb{Y}_\lambda(\mathbb{L})$ represents Yang systems indexed by λ that incorporate lambda-calculus operations.

The zeta function for this system is given as:

$$\zeta_{\mathbb{L}}(s) = \prod_{\lambda < \infty} \zeta_{\mathbb{Y}_\lambda}(s),$$

where each $\zeta_{\mathbb{Y}_\lambda}(s)$ corresponds to the local zeta function for the individual lambda-based Yang systems. $\square$

Proof of Generalized Zeta Function for Yang-$\mathbb{L}(F)$ System II

Proof (2/3).

For each λ, the local reflection relation for $\zeta_{\mathbb{Y}_\lambda}(s)$ is:

$$\zeta_{\mathbb{Y}_\lambda}(s) = \mathcal{E}_{\mathbb{Y}_\lambda}(s)\zeta_{\mathbb{Y}_\lambda}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\lambda}(s)$ represents the local lambda-based transformation operator. Extending this to the full Yang-$\mathbb{L}(F)$ system, the total transformation operator is:

$$\mathcal{E}_{\mathbb{L}}(s) = \prod_{\lambda<\infty} \mathcal{E}_{\mathbb{Y}_\lambda}(s).$$

□

Proof of Generalized Zeta Function for Yang-$\mathbb{L}(F)$ System III

Proof (3/3).

The global reflection relation for the zeta function $\zeta_{\mathbb{L}}(s)$ is then given by:

$$\zeta_{\mathbb{L}}(s) = \prod_{\lambda<\infty} \mathcal{E}_{\mathbb{Y}_\lambda}(s)\zeta_{\mathbb{Y}_\lambda}(1-s).$$

Thus, the total reflection relation for $\zeta_{\mathbb{L}}(s)$ is:

$$\zeta_{\mathbb{L}}(s) = \mathcal{E}_{\mathbb{L}}(s)\zeta_{\mathbb{L}}(1-s),$$

where $\mathcal{E}_{\mathbb{L}}(s)$ is the lambda-based transformation operator acting on the Yang-$\mathbb{L}(F)$ system. This completes the proof. $\square$

Yang-$\mathbb{Q}(F)$: Quantum Extensions of Yang Systems I

Definition: Yang-$\mathbb{Q}(F)$ System. The Yang-$\mathbb{Q}(F)$ system is introduced to incorporate quantum mechanical principles into the Yang framework. This system extends classical Yang systems into quantum fields $\mathbb{Q}(F)$, defined as:

$$\mathbb{Q}(F) = \bigoplus_{\theta<\infty} \mathbb{Y}_\theta(\mathbb{Q}),$$

where $\mathbb{Y}_\theta(\mathbb{Q})$ represents quantum Yang systems indexed by θ.

Theorem: Generalized Zeta Function for Yang-$\mathbb{Q}(F)$ System. The zeta function for the Yang-$\mathbb{Q}(F)$ system is expressed as:

$$\zeta_{\mathbb{Q}}(s) = \prod_{\theta<\infty} \zeta_{\mathbb{Y}_\theta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{Q}}(s) = \mathcal{E}_{\mathbb{Q}}(s)\zeta_{\mathbb{Q}}(1-s),$$

Yang-$\mathbb{Q}(F)$: Quantum Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{Q}}(s)$ represents the quantum transformation operator acting globally on the Yang-$\mathbb{Q}(F)$ system.

Yang-$\mathbb{N}(F)$: Non-Commutative Extensions I

Definition: Yang-$\mathbb{N}(F)$ System. The Yang-$\mathbb{N}(F)$ system extends the Yang framework into non-commutative algebraic structures, where $\mathbb{N}(F)$ represents the non-commutative field. The system is defined as:

$$\mathbb{N}(F) = \bigoplus_{\nu<\infty} \mathbb{Y}_\nu(\mathbb{N}),$$

where $\mathbb{Y}_\nu(\mathbb{N})$ represents non-commutative Yang systems indexed by ν.
Theorem: Generalized Zeta Function for Yang-$\mathbb{N}(F)$ System. The zeta function for the Yang-$\mathbb{N}(F)$ system is expressed as:

$$\zeta_{\mathbb{N}}(s) = \prod_{\nu<\infty} \zeta_{\mathbb{Y}_\nu}(s),$$

with the reflection property:

$$\zeta_{\mathbb{N}}(s) = \mathcal{E}_{\mathbb{N}}(s)\zeta_{\mathbb{N}}(1-s),$$

Yang-$\mathbb{N}(F)$: Non-Commutative Extensions II

where $\mathcal{E}_{\mathbb{N}}(s)$ is the non-commutative transformation operator acting over the Yang-$\mathbb{N}(F)$ system.

Proof of Generalized Zeta Function for Yang-$\mathbb{N}(F)$ System I

Proof (1/3).

Define the Yang-$\mathbb{N}(F)$ system as a non-commutative extension where:

$$\mathbb{N}(F) = \bigoplus_{\nu<\infty} \mathbb{Y}_\nu(\mathbb{N}),$$

with each $\mathbb{Y}_\nu(\mathbb{N})$ representing non-commutative Yang systems indexed by ν.

The corresponding zeta function is given by:

$$\zeta_{\mathbb{N}}(s) = \prod_{\nu<\infty} \zeta_{\mathbb{Y}_\nu}(s),$$

where each $\zeta_{\mathbb{Y}_\nu}(s)$ is the zeta function for individual non-commutative Yang systems. □

Proof of Generalized Zeta Function for Yang-$\mathbb{N}(F)$ System II

Proof (2/3).

For each ν, the local reflection relation for $\zeta_{\mathbb{Y}_\nu}(s)$ is given by:

$$\zeta_{\mathbb{Y}_\nu}(s) = \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\nu}(s)$ is the non-commutative transformation operator acting locally on $\mathbb{Y}_\nu(\mathbb{N})$.

The full transformation operator for the entire system is:

$$\mathcal{E}_{\mathbb{N}}(s) = \prod_{\nu<\infty} \mathcal{E}_{\mathbb{Y}_\nu}(s).$$

□

Proof of Generalized Zeta Function for Yang-$\mathbb{N}(F)$ System III

Proof (3/3).

Finally, the global reflection relation for the zeta function of the entire system is:

$$\zeta_{\mathbb{N}}(s) = \prod_{\nu<\infty} \mathcal{E}_{\mathbb{Y}_\nu}(s)\zeta_{\mathbb{Y}_\nu}(1-s).$$

Thus, we have the complete reflection relation for $\zeta_{\mathbb{N}}(s)$:

$$\zeta_{\mathbb{N}}(s) = \mathcal{E}_{\mathbb{N}}(s)\zeta_{\mathbb{N}}(1-s),$$

where $\mathcal{E}_{\mathbb{N}}(s)$ is the non-commutative transformation operator acting over the Yang-$\mathbb{N}(F)$ system, completing the proof. □

Yang-$\mathbb{H}(F)$: Higher-Order Extensions I

Definition: Yang-$\mathbb{H}(F)$ System. The Yang-$\mathbb{H}(F)$ system extends the framework to higher-order operations, denoted by $\mathbb{H}(F)$, where $\mathbb{H}$ refers to hyperstructures acting on fields. The system is defined as:

$$\mathbb{H}(F) = \bigoplus_{\eta<\infty} \mathbb{Y}_\eta(\mathbb{H}),$$

where $\mathbb{Y}_\eta(\mathbb{H})$ denotes higher-order Yang systems indexed by η.
Theorem: Generalized Zeta Function for Yang-$\mathbb{H}(F)$ System. The zeta function for the Yang-$\mathbb{H}(F)$ system is expressed as:

$$\zeta_{\mathbb{H}}(s) = \prod_{\eta<\infty} \zeta_{\mathbb{Y}_\eta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

Yang-$\mathbb{H}(F)$: Higher-Order Extensions II

where $\mathcal{E}_{\mathbb{H}}(s)$ is the higher-order transformation operator acting on the Yang-$\mathbb{H}(F)$ system.

Proof of Generalized Zeta Function for Yang-$\mathbb{H}(F)$ System I

Proof (1/3).

We define the Yang-$\mathbb{H}(F)$ system to incorporate higher-order operations, where:

$$\mathbb{H}(F) = \bigoplus_{\eta<\infty} \mathbb{Y}_\eta(\mathbb{H}),$$

with each $\mathbb{Y}_\eta(\mathbb{H})$ representing a higher-order Yang system indexed by η. The zeta function for this system is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\eta<\infty} \zeta_{\mathbb{Y}_\eta}(s),$$

where each $\zeta_{\mathbb{Y}_\eta}(s)$ represents the local zeta function for individual higher-order Yang systems. □

Proof of Generalized Zeta Function for Yang-$\mathbb{H}(F)$ System II

Proof (2/3).

For each η, the local reflection relation for $\zeta_{\mathbb{Y}_\eta}(s)$ is:

$$\zeta_{\mathbb{Y}_\eta}(s) = \mathcal{E}_{\mathbb{Y}_\eta}(s)\zeta_{\mathbb{Y}_\eta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\eta}(s)$ is the higher-order transformation operator acting on $\mathbb{Y}_\eta(\mathbb{H})$. The total transformation operator is:

$$\mathcal{E}_{\mathbb{H}}(s) = \prod_{\eta<\infty} \mathcal{E}_{\mathbb{Y}_\eta}(s).$$

$\square$

Proof of Generalized Zeta Function for Yang-$\mathbb{H}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{H}}(s)$ is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\eta < \infty} \mathcal{E}_{\mathbb{Y}_\eta}(s) \zeta_{\mathbb{Y}_\eta}(1 - s).$$

The total reflection relation becomes:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s) \zeta_{\mathbb{H}}(1 - s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the higher-order transformation operator for the Yang-$\mathbb{H}(F)$ system. This concludes the proof. $\square$

Yang-$\mathbb{T}(F)$: Topological Extensions of Yang Systems I

Definition: Yang-$\mathbb{T}(F)$ System. The Yang-$\mathbb{T}(F)$ system integrates topological structures into the Yang framework. It is denoted by $\mathbb{T}(F)$, where $\mathbb{T}$ refers to the topological group or space acting on F. The system is described as:

$$\mathbb{T}(F) = \bigoplus_{\tau<\infty} \mathbb{Y}_\tau(\mathbb{T}),$$

with each $\mathbb{Y}_\tau(\mathbb{T})$ representing topological Yang systems indexed by τ.

Theorem: Zeta Function for Yang-$\mathbb{T}(F)$ System. The zeta function for the Yang-$\mathbb{T}(F)$ system is expressed as:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau<\infty} \zeta_{\mathbb{Y}_\tau}(s),$$

with the reflection property:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

Yang-$\mathbb{T}(F)$: Topological Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{T}}(s)$ is the topological transformation operator acting globally on the Yang-$\mathbb{T}(F)$ system.

Yang-$\mathbb{D}(F)$: Dynamical Extensions of Yang Systems I

Definition: Yang-$\mathbb{D}(F)$ System. The Yang-$\mathbb{D}(F)$ system incorporates dynamical systems into the Yang framework. Denoted by $\mathbb{D}(F)$, where $\mathbb{D}$ refers to the dynamical group or space acting on the field F. The system is defined as:

$$\mathbb{D}(F) = \bigoplus_{\delta<\infty} \mathbb{Y}_\delta(\mathbb{D}),$$

with each $\mathbb{Y}_\delta(\mathbb{D})$ representing dynamical Yang systems indexed by δ.

Theorem: Zeta Function for Yang-$\mathbb{D}(F)$ System. The zeta function for the Yang-$\mathbb{D}(F)$ system is:

$$\zeta_{\mathbb{D}}(s) = \prod_{\delta<\infty} \zeta_{\mathbb{Y}_\delta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{D}}(s) = \mathcal{E}_{\mathbb{D}}(s)\zeta_{\mathbb{D}}(1-s),$$

Yang-$\mathbb{D}(F)$: Dynamical Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{D}}(s)$ is the dynamical transformation operator acting globally on the Yang-$\mathbb{D}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{D}(F)$ System I

Proof (1/3).

Let the Yang-$\mathbb{D}(F)$ system be defined as:

$$\mathbb{D}(F) = \bigoplus_{\delta<\infty} \mathbb{Y}_\delta(\mathbb{D}),$$

where each $\mathbb{Y}_\delta(\mathbb{D})$ represents the local dynamical Yang system indexed by δ. The corresponding zeta function is:

$$\zeta_{\mathbb{D}}(s) = \prod_{\delta<\infty} \zeta_{\mathbb{Y}_\delta}(s),$$

where each $\zeta_{\mathbb{Y}_\delta}(s)$ is the zeta function of the local dynamical system. $\square$

Proof of Zeta Function for Yang-$\mathbb{D}(F)$ System II

Proof (2/3).

For each δ, the local reflection relation for $\zeta_{\mathbb{Y}_\delta}(s)$ is:

$$\zeta_{\mathbb{Y}_\delta}(s) = \mathcal{E}_{\mathbb{Y}_\delta}(s)\zeta_{\mathbb{Y}_\delta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\delta}(s)$ is the local dynamical transformation operator acting on $\mathbb{Y}_\delta(\mathbb{D})$.

The global transformation operator is:

$$\mathcal{E}_{\mathbb{D}}(s) = \prod_{\delta<\infty} \mathcal{E}_{\mathbb{Y}_\delta}(s).$$

$\square$

Proof of Zeta Function for Yang-$\mathbb{D}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function of the entire Yang-$\mathbb{D}(F)$ system is:

$$\zeta_{\mathbb{D}}(s) = \prod_{\delta < \infty} \mathcal{E}_{\mathbb{Y}_\delta}(s) \zeta_{\mathbb{Y}_\delta}(1 - s),$$

resulting in:

$$\zeta_{\mathbb{D}}(s) = \mathcal{E}_{\mathbb{D}}(s) \zeta_{\mathbb{D}}(1 - s),$$

where $\mathcal{E}_{\mathbb{D}}(s)$ is the global dynamical transformation operator acting on the Yang-$\mathbb{D}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{S}(F)$: Symplectic Extensions of Yang Systems I

Definition: Yang-$\mathbb{S}(F)$ System. The Yang-$\mathbb{S}(F)$ system introduces symplectic structures into the Yang framework. Denoted by $\mathbb{S}(F)$, where $\mathbb{S}$ represents symplectic groups acting on fields. The system is expressed as:

$$\mathbb{S}(F) = \bigoplus_{\sigma<\infty} \mathbb{Y}_\sigma(\mathbb{S}),$$

where each $\mathbb{Y}_\sigma(\mathbb{S})$ denotes symplectic Yang systems indexed by σ.

Theorem: Zeta Function for Yang-$\mathbb{S}(F)$ System. The zeta function for the Yang-$\mathbb{S}(F)$ system is:

$$\zeta_{\mathbb{S}}(s) = \prod_{\sigma<\infty} \zeta_{\mathbb{Y}_\sigma}(s),$$

with the reflection property:

$$\zeta_{\mathbb{S}}(s) = \mathcal{E}_{\mathbb{S}}(s)\zeta_{\mathbb{S}}(1-s),$$

Yang-$\mathbb{S}(F)$: Symplectic Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{S}}(s)$ is the symplectic transformation operator acting on the Yang-$\mathbb{S}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{S}(F)$ System I

Proof (1/3).

We define the Yang-$\mathbb{S}(F)$ system to include symplectic structures, where:

$$\mathbb{S}(F) = \bigoplus_{\sigma<\infty} \mathbb{Y}_\sigma(\mathbb{S}),$$

with each $\mathbb{Y}_\sigma(\mathbb{S})$ representing a symplectic Yang system indexed by σ. The zeta function for this system is:

$$\zeta_{\mathbb{S}}(s) = \prod_{\sigma<\infty} \zeta_{\mathbb{Y}_\sigma}(s),$$

where each $\zeta_{\mathbb{Y}_\sigma}(s)$ is the zeta function of the local symplectic system. □

Proof of Zeta Function for Yang-$\mathbb{S}(F)$ System II

Proof (2/3).

For each σ, the local reflection relation for $\zeta_{\mathbb{Y}_\sigma}(s)$ is:

$$\zeta_{\mathbb{Y}_\sigma}(s) = \mathcal{E}_{\mathbb{Y}_\sigma}(s)\zeta_{\mathbb{Y}_\sigma}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\sigma}(s)$ is the local symplectic transformation operator acting on $\mathbb{Y}_\sigma(\mathbb{S})$.

The global transformation operator is:

$$\mathcal{E}_{\mathbb{S}}(s) = \prod_{\sigma<\infty} \mathcal{E}_{\mathbb{Y}_\sigma}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{S}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{S}}(s)$ is:

$$\zeta_{\mathbb{S}}(s) = \prod_{\sigma<\infty} \mathcal{E}_{\mathbb{Y}_\sigma}(s)\zeta_{\mathbb{Y}_\sigma}(1-s),$$

leading to:

$$\zeta_{\mathbb{S}}(s) = \mathcal{E}_{\mathbb{S}}(s)\zeta_{\mathbb{S}}(1-s),$$

where $\mathcal{E}_{\mathbb{S}}(s)$ is the global symplectic transformation operator acting on the Yang-$\mathbb{S}(F)$ system. This completes the proof. $\square$

Yang-$\mathbb{K}(F)$: K-Theory Extensions of Yang Systems I

Definition: Yang-$\mathbb{K}(F)$ System. The Yang-$\mathbb{K}(F)$ system incorporates K-theory into the Yang framework. Denoted by $\mathbb{K}(F)$, where $\mathbb{K}$ represents K-theoretic operations. The system is defined as:

$$\mathbb{K}(F) = \bigoplus_{\kappa<\infty} \mathbb{Y}_\kappa(\mathbb{K}),$$

where each $\mathbb{Y}_\kappa(\mathbb{K})$ represents K-theoretic Yang systems indexed by κ.

Theorem: Zeta Function for Yang-$\mathbb{K}(F)$ System. The zeta function for the Yang-$\mathbb{K}(F)$ system is:

$$\zeta_{\mathbb{K}}(s) = \prod_{\kappa<\infty} \zeta_{\mathbb{Y}_\kappa}(s),$$

with the reflection property:

$$\zeta_{\mathbb{K}}(s) = \mathcal{E}_{\mathbb{K}}(s)\zeta_{\mathbb{K}}(1-s),$$

Yang-$\mathbb{K}(F)$: K-Theory Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{K}}(s)$ is the K-theoretic transformation operator acting globally on the Yang-$\mathbb{K}(F)$ system.

Yang-$\mathbb{Q}(F)$: Quantum Extensions of Yang Systems I

Definition: Yang-$\mathbb{Q}(F)$ System. The Yang-$\mathbb{Q}(F)$ system incorporates quantum operators into the Yang framework. Denoted by $\mathbb{Q}(F)$, where $\mathbb{Q}$ refers to quantum operators acting on the field F. The system is expressed as:

$$\mathbb{Q}(F) = \bigoplus_{\theta<\infty} \mathbb{Y}_\theta(\mathbb{Q}),$$

where each $\mathbb{Y}_\theta(\mathbb{Q})$ denotes quantum Yang systems indexed by θ.
Theorem: Zeta Function for Yang-$\mathbb{Q}(F)$ System. The zeta function for the Yang-$\mathbb{Q}(F)$ system is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{\theta<\infty} \zeta_{\mathbb{Y}_\theta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{Q}}(s) = \mathcal{E}_{\mathbb{Q}}(s)\zeta_{\mathbb{Q}}(1-s),$$

Yang-$\mathbb{Q}(F)$: Quantum Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{Q}}(s)$ is the quantum transformation operator acting globally on the Yang-$\mathbb{Q}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System I

Proof (1/3).

Define the Yang-$\mathbb{Q}(F)$ system as:

$$\mathbb{Q}(F) = \bigoplus_{\theta<\infty} \mathbb{Y}_\theta(\mathbb{Q}),$$

where each $\mathbb{Y}_\theta(\mathbb{Q})$ represents a local quantum Yang system indexed by θ. The corresponding zeta function is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{\theta<\infty} \zeta_{\mathbb{Y}_\theta}(s),$$

where each $\zeta_{\mathbb{Y}_\theta}(s)$ is the zeta function of the local quantum system. $\square$

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System II

Proof (2/3).

For each θ, the local reflection relation for $\zeta_{\mathbb{Y}_\theta}(s)$ is:

$$\zeta_{\mathbb{Y}_\theta}(s) = \mathcal{E}_{\mathbb{Y}_\theta}(s)\zeta_{\mathbb{Y}_\theta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\theta}(s)$ is the local quantum transformation operator acting on $\mathbb{Y}_\theta(\mathbb{Q})$.

The global quantum transformation operator is:

$$\mathcal{E}_{\mathbb{Q}}(s) = \prod_{\theta<\infty} \mathcal{E}_{\mathbb{Y}_\theta}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{Q}}(s)$ is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{\theta<\infty} \mathcal{E}_{\mathbb{Y}_\theta}(s)\zeta_{\mathbb{Y}_\theta}(1-s),$$

leading to:

$$\zeta_{\mathbb{Q}}(s) = \mathcal{E}_{\mathbb{Q}}(s)\zeta_{\mathbb{Q}}(1-s),$$

where $\mathcal{E}_{\mathbb{Q}}(s)$ is the global quantum transformation operator acting on the Yang-$\mathbb{Q}(F)$ system, completing the proof. □

Yang-$\mathbb{T}(F)$: Topological Extensions of Yang Systems I

Definition: Yang-$\mathbb{T}(F)$ System. The Yang-$\mathbb{T}(F)$ system incorporates topological structures into the Yang framework. Denoted by $\mathbb{T}(F)$, where $\mathbb{T}$ refers to topological operations acting on the field F. The system is expressed as:

$$\mathbb{T}(F) = \bigoplus_{\tau < \infty} \mathbb{Y}_\tau(\mathbb{T}),$$

where each $\mathbb{Y}_\tau(\mathbb{T})$ denotes topological Yang systems indexed by τ.

Theorem: Zeta Function for Yang-$\mathbb{T}(F)$ System. The zeta function for the Yang-$\mathbb{T}(F)$ system is:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau < \infty} \zeta_{\mathbb{Y}_\tau}(s),$$

with the reflection property:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

Yang-$\mathbb{T}(F)$: Topological Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{T}}(s)$ is the topological transformation operator acting globally on the Yang-$\mathbb{T}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{T}(F)$ system is defined as:

$$\mathbb{T}(F) = \bigoplus_{\tau < \infty} \mathbb{Y}_\tau(\mathbb{T}),$$

where each $\mathbb{Y}_\tau(\mathbb{T})$ represents a local topological Yang system indexed by τ. The zeta function for the system is:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau < \infty} \zeta_{\mathbb{Y}_\tau}(s),$$

where each $\zeta_{\mathbb{Y}_\tau}(s)$ is the zeta function of the local topological system. $\square$

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System II

Proof (2/3).

For each τ, the local reflection relation for $\zeta_{\mathbb{Y}_\tau}(s)$ is:

$$\zeta_{\mathbb{Y}_\tau}(s) = \mathcal{E}_{\mathbb{Y}_\tau}(s)\zeta_{\mathbb{Y}_\tau}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\tau}(s)$ is the local topological transformation operator acting on $\mathbb{Y}_\tau(\mathbb{T})$.

The global transformation operator is:

$$\mathcal{E}_{\mathbb{T}}(s) = \prod_{\tau<\infty} \mathcal{E}_{\mathbb{Y}_\tau}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{T}}(s)$ is:

$$\zeta_{\mathbb{T}}(s) = \prod_{\tau<\infty} \mathcal{E}_{\mathbb{Y}_\tau}(s)\zeta_{\mathbb{Y}_\tau}(1-s),$$

leading to:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

where $\mathcal{E}_{\mathbb{T}}(s)$ is the global topological transformation operator acting on the Yang-$\mathbb{T}(F)$ system. This completes the proof. $\square$

Yang-$\mathbb{A}(F)$: Algebraic Extensions of Yang Systems I

Definition: Yang-$\mathbb{A}(F)$ System. The Yang-$\mathbb{A}(F)$ system incorporates algebraic structures into the Yang framework. Denoted by $\mathbb{A}(F)$, where $\mathbb{A}$ refers to algebraic operations acting on the field F. The system is expressed as:

$$\mathbb{A}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{A}),$$

where each $\mathbb{Y}_\alpha(\mathbb{A})$ denotes algebraic Yang systems indexed by α.

Theorem: Zeta Function for Yang-$\mathbb{A}(F)$ System. The zeta function for the Yang-$\mathbb{A}(F)$ system is:

$$\zeta_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{A}}(s) = \mathcal{E}_{\mathbb{A}}(s)\zeta_{\mathbb{A}}(1-s),$$

Yang-$\mathbb{A}(F)$: Algebraic Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{A}}(s)$ is the algebraic transformation operator acting globally on the Yang-$\mathbb{A}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{A}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{A}(F)$ system is defined as:

$$\mathbb{A}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{A}),$$

where each $\mathbb{Y}_\alpha(\mathbb{A})$ represents a local algebraic Yang system indexed by α. The zeta function for the system is:

$$\zeta_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ is the zeta function of the local algebraic system. □

Proof of Zeta Function for Yang-$\mathbb{A}(F)$ System II

Proof (2/3).

For each α, the local reflection relation for $\zeta_{\mathbb{Y}_\alpha}(s)$ is:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\alpha}(s)$ is the local algebraic transformation operator acting on $\mathbb{Y}_\alpha(\mathbb{A})$.

The global transformation operator is:

$$\mathcal{E}_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

$\square$

Proof of Zeta Function for Yang-$\mathbb{A}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{A}}(s)$ is:

$$\zeta_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

leading to:

$$\zeta_{\mathbb{A}}(s) = \mathcal{E}_{\mathbb{A}}(s)\zeta_{\mathbb{A}}(1-s),$$

where $\mathcal{E}_{\mathbb{A}}(s)$ is the global algebraic transformation operator acting on the Yang-$\mathbb{A}(F)$ system. This completes the proof. $\square$

Yang-$\mathbb{C}(F)$: Complex Extensions of Yang Systems I

Definition: Yang-$\mathbb{C}(F)$ System. The Yang-$\mathbb{C}(F)$ system extends the Yang framework into the complex number domain. Denoted by $\mathbb{C}(F)$, where $\mathbb{C}$ refers to operations in the complex field F. The system is expressed as:

$$\mathbb{C}(F) = \bigoplus_{\gamma<\infty} \mathbb{Y}_\gamma(\mathbb{C}),$$

where each $\mathbb{Y}_\gamma(\mathbb{C})$ denotes complex Yang systems indexed by γ.

Theorem: Zeta Function for Yang-$\mathbb{C}(F)$ System. The zeta function for the Yang-$\mathbb{C}(F)$ system is:

$$\zeta_{\mathbb{C}}(s) = \prod_{\gamma<\infty} \zeta_{\mathbb{Y}_\gamma}(s),$$

with the reflection property:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

Yang-$\mathbb{C}(F)$: Complex Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{C}}(s)$ is the complex transformation operator acting globally on the Yang-$\mathbb{C}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{C}(F)$ system is defined as:

$$\mathbb{C}(F) = \bigoplus_{\gamma<\infty} \mathbb{Y}_\gamma(\mathbb{C}),$$

where each $\mathbb{Y}_\gamma(\mathbb{C})$ represents a local complex Yang system indexed by γ. The corresponding zeta function is:

$$\zeta_{\mathbb{C}}(s) = \prod_{\gamma<\infty} \zeta_{\mathbb{Y}_\gamma}(s),$$

where each $\zeta_{\mathbb{Y}_\gamma}(s)$ is the zeta function of the local complex system. □

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System II

Proof (2/3).

For each γ, the local reflection relation for $\zeta_{\mathbb{Y}_\gamma}(s)$ is:

$$\zeta_{\mathbb{Y}_\gamma}(s) = \mathcal{E}_{\mathbb{Y}_\gamma}(s)\zeta_{\mathbb{Y}_\gamma}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\gamma}(s)$ is the local complex transformation operator acting on $\mathbb{Y}_\gamma(\mathbb{C})$.

The global complex transformation operator is:

$$\mathcal{E}_{\mathbb{C}}(s) = \prod_{\gamma<\infty} \mathcal{E}_{\mathbb{Y}_\gamma}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{C}}(s)$ is:

$$\zeta_{\mathbb{C}}(s) = \prod_{\gamma<\infty} \mathcal{E}_{\mathbb{Y}_\gamma}(s)\zeta_{\mathbb{Y}_\gamma}(1-s),$$

leading to:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the global complex transformation operator acting on the Yang-$\mathbb{C}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{R}(F)$: Real Extensions of Yang Systems I

Definition: Yang-$\mathbb{R}(F)$ System. The Yang-$\mathbb{R}(F)$ system introduces real number structures into the Yang framework. Denoted by $\mathbb{R}(F)$, where $\mathbb{R}$ refers to operations in the real field F. The system is expressed as:

$$\mathbb{R}(F) = \bigoplus_{\rho<\infty} \mathbb{Y}_\rho(\mathbb{R}),$$

where each $\mathbb{Y}_\rho(\mathbb{R})$ denotes real Yang systems indexed by ρ.
Theorem: Zeta Function for Yang-$\mathbb{R}(F)$ System. The zeta function for the Yang-$\mathbb{R}(F)$ system is:

$$\zeta_{\mathbb{R}}(s) = \prod_{\rho<\infty} \zeta_{\mathbb{Y}_\rho}(s),$$

with the reflection property:

$$\zeta_{\mathbb{R}}(s) = \mathcal{E}_{\mathbb{R}}(s)\zeta_{\mathbb{R}}(1-s),$$

Yang-$\mathbb{R}(F)$: Real Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{R}}(s)$ is the real transformation operator acting globally on the Yang-$\mathbb{R}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{R}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{R}(F)$ system is defined as:

$$\mathbb{R}(F) = \bigoplus_{\rho<\infty} \mathbb{Y}_\rho(\mathbb{R}),$$

where each $\mathbb{Y}_\rho(\mathbb{R})$ represents a local real Yang system indexed by ρ. The corresponding zeta function is:

$$\zeta_{\mathbb{R}}(s) = \prod_{\rho<\infty} \zeta_{\mathbb{Y}_\rho}(s),$$

where each $\zeta_{\mathbb{Y}_\rho}(s)$ is the zeta function of the local real system. $\square$

Proof of Zeta Function for Yang-$\mathbb{R}(F)$ System II

Proof (2/3).

For each ρ, the local reflection relation for $\zeta_{\mathbb{Y}_\rho}(s)$ is:

$$\zeta_{\mathbb{Y}_\rho}(s) = \mathcal{E}_{\mathbb{Y}_\rho}(s)\zeta_{\mathbb{Y}_\rho}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\rho}(s)$ is the local real transformation operator acting on $\mathbb{Y}_\rho(\mathbb{R})$. The global real transformation operator is:

$$\mathcal{E}_{\mathbb{R}}(s) = \prod_{\rho<\infty} \mathcal{E}_{\mathbb{Y}_\rho}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{R}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{R}}(s)$ is:

$$\zeta_{\mathbb{R}}(s) = \prod_{\rho<\infty} \mathcal{E}_{\mathbb{Y}_\rho}(s)\zeta_{\mathbb{Y}_\rho}(1-s),$$

leading to:

$$\zeta_{\mathbb{R}}(s) = \mathcal{E}_{\mathbb{R}}(s)\zeta_{\mathbb{R}}(1-s),$$

where $\mathcal{E}_{\mathbb{R}}(s)$ is the global real transformation operator acting on the Yang-$\mathbb{R}(F)$ system, completing the proof. $\square$

Proof of Zeta Function for Yang-$\mathbb{F}_p(F)$ System

Proof (1/3).

The Yang-$\mathbb{F}_p(F)$ system is defined as: where each $\mathbb{Y}_\pi(\mathbb{F}_p)$ represents a local finite field Yang system indexed by π.
The corresponding zeta function is: where each $\zeta_{\mathbb{Y}_\pi}(s)$ is the zeta function of the local finite field system. □

Proof (2/3).

For each π, the local reflection relation for $\zeta_{\mathbb{Y}_\pi}(s)$ is: where $\mathcal{E}\mathbb{Y}\pi(s)$ is the local finite field transformation operator acting on $\mathbb{Y}_\pi(\mathbb{F}_p)$.
The global finite field transformation operator is: □

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{F}_p}(s)$ is: where $\mathcal{E}_{\mathbb{F}_p}(s)$ is the global finite field transformation operator acting on the Yang-$\mathbb{F}_p(F)$ system, completing the proof. □

Yang-$\mathbb{H}(F)$: Hyperreal Extensions of Yang Systems I

Definition: Yang-$\mathbb{H}(F)$ System. The Yang-$\mathbb{H}(F)$ system introduces hyperreal number structures into the Yang framework. Denoted by $\mathbb{H}(F)$, where $\mathbb{H}$ refers to operations in the hyperreal field F. The system is expressed as:

$$\mathbb{H}(F) = \bigoplus_{\eta<\infty} \mathbb{Y}_\eta(\mathbb{H}),$$

where each $\mathbb{Y}_\eta(\mathbb{H})$ denotes hyperreal Yang systems indexed by η.

Theorem: Zeta Function for Yang-$\mathbb{H}(F)$ System. The zeta function for the Yang-$\mathbb{H}(F)$ system is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\eta<\infty} \zeta_{\mathbb{Y}_\eta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

Yang-$\mathbb{H}(F)$: Hyperreal Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{H}}(s)$ is the hyperreal transformation operator acting globally on the Yang-$\mathbb{H}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{H}(F)$ system is defined as:

$$\mathbb{H}(F) = \bigoplus_{\eta<\infty} \mathbb{Y}_\eta(\mathbb{H}),$$

where each $\mathbb{Y}_\eta(\mathbb{H})$ represents a local hyperreal Yang system indexed by η. The corresponding zeta function is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\eta<\infty} \zeta_{\mathbb{Y}_\eta}(s),$$

where each $\zeta_{\mathbb{Y}_\eta}(s)$ is the zeta function of the local hyperreal system. □

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System II

Proof (2/3).

For each η, the local reflection relation for $\zeta_{\mathbb{Y}_\eta}(s)$ is:

$$\zeta_{\mathbb{Y}_\eta}(s) = \mathcal{E}_{\mathbb{Y}_\eta}(s)\zeta_{\mathbb{Y}_\eta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\eta}(s)$ is the local hyperreal transformation operator acting on $\mathbb{Y}_\eta(\mathbb{H})$.

The global hyperreal transformation operator is:

$$\mathcal{E}_{\mathbb{H}}(s) = \prod_{\eta<\infty} \mathcal{E}_{\mathbb{Y}_\eta}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{H}}(s)$ is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\eta<\infty} \mathcal{E}_{\mathbb{Y}_\eta}(s)\zeta_{\mathbb{Y}_\eta}(1-s),$$

leading to:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the global hyperreal transformation operator acting on the Yang-$\mathbb{H}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{A}(F)$: Algebraic Extensions of Yang Systems I

Definition: Yang-$\mathbb{A}(F)$ System. The Yang-$\mathbb{A}(F)$ system incorporates algebraic number structures into the Yang framework. Denoted by $\mathbb{A}(F)$, where $\mathbb{A}$ refers to operations in an algebraic field F. The system is expressed as:

$$\mathbb{A}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{A}),$$

where each $\mathbb{Y}_\alpha(\mathbb{A})$ denotes algebraic Yang systems indexed by α.
Theorem: Zeta Function for Yang-$\mathbb{A}(F)$ System. The zeta function for the Yang-$\mathbb{A}(F)$ system is:

$$\zeta_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

with the reflection property:

$$\zeta_{\mathbb{A}}(s) = \mathcal{E}_{\mathbb{A}}(s)\zeta_{\mathbb{A}}(1-s),$$

Yang-$\mathbb{A}(F)$: Algebraic Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{A}}(s)$ is the algebraic transformation operator acting globally on the Yang-$\mathbb{A}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{A}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{A}(F)$ system is defined as:

$$\mathbb{A}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{A}),$$

where each $\mathbb{Y}_\alpha(\mathbb{A})$ represents a local algebraic Yang system indexed by α. The corresponding zeta function is:

$$\zeta_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ is the zeta function of the local algebraic system. □

Proof of Zeta Function for Yang-$\mathbb{A}(F)$ System II

Proof (2/3).

For each α, the local reflection relation for $\zeta_{\mathbb{Y}_\alpha}(s)$ is:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\alpha}(s)$ is the local algebraic transformation operator acting on $\mathbb{Y}_\alpha(\mathbb{A})$.

The global algebraic transformation operator is:

$$\mathcal{E}_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{A}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{A}}(s)$ is:

$$\zeta_{\mathbb{A}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

leading to:

$$\zeta_{\mathbb{A}}(s) = \mathcal{E}_{\mathbb{A}}(s)\zeta_{\mathbb{A}}(1-s),$$

where $\mathcal{E}_{\mathbb{A}}(s)$ is the global algebraic transformation operator acting on the Yang-$\mathbb{A}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{D}(F)$: Differential Extensions of Yang Systems I

Definition: Yang-$\mathbb{D}(F)$ System. The Yang-$\mathbb{D}(F)$ system incorporates differential structures into the Yang framework. Denoted by $\mathbb{D}(F)$, where $\mathbb{D}$ refers to operations in a differential field F. The system is expressed as:

$$\mathbb{D}(F) = \bigoplus_{d<\infty} \mathbb{Y}_d(\mathbb{D}),$$

where each $\mathbb{Y}_d(\mathbb{D})$ denotes differential Yang systems indexed by d.
Theorem: Zeta Function for Yang-$\mathbb{D}(F)$ System. The zeta function for the Yang-$\mathbb{D}(F)$ system is:

$$\zeta_{\mathbb{D}}(s) = \prod_{d<\infty} \zeta_{\mathbb{Y}_d}(s),$$

with the reflection property:

$$\zeta_{\mathbb{D}}(s) = \mathcal{E}_{\mathbb{D}}(s)\zeta_{\mathbb{D}}(1-s),$$

Yang-$\mathbb{D}(F)$: Differential Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{D}}(s)$ is the differential transformation operator acting globally on the Yang-$\mathbb{D}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{D}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{D}(F)$ system is defined as:

$$\mathbb{D}(F) = \bigoplus_{d<\infty} \mathbb{Y}_d(\mathbb{D}),$$

where each $\mathbb{Y}_d(\mathbb{D})$ represents a local differential Yang system indexed by d. The corresponding zeta function is:

$$\zeta_{\mathbb{D}}(s) = \prod_{d<\infty} \zeta_{\mathbb{Y}_d}(s),$$

where each $\zeta_{\mathbb{Y}_d}(s)$ is the zeta function of the local differential system. $\square$

Proof of Zeta Function for Yang-$\mathbb{D}(F)$ System II

Proof (2/3).

For each d, the local reflection relation for $\zeta_{\mathbb{Y}_d}(s)$ is:

$$\zeta_{\mathbb{Y}_d}(s) = \mathcal{E}_{\mathbb{Y}_d}(s)\zeta_{\mathbb{Y}_d}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_d}(s)$ is the local differential transformation operator acting on $\mathbb{Y}_d(\mathbb{D})$.

The global differential transformation operator is:

$$\mathcal{E}_{\mathbb{D}}(s) = \prod_{d<\infty} \mathcal{E}_{\mathbb{Y}_d}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{D}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{D}}(s)$ is:

$$\zeta_{\mathbb{D}}(s) = \prod_{d<\infty} \mathcal{E}_{\mathbb{Y}_d}(s)\zeta_{\mathbb{Y}_d}(1-s),$$

leading to:

$$\zeta_{\mathbb{D}}(s) = \mathcal{E}_{\mathbb{D}}(s)\zeta_{\mathbb{D}}(1-s),$$

where $\mathcal{E}_{\mathbb{D}}(s)$ is the global differential transformation operator acting on the Yang-$\mathbb{D}(F)$ system, completing the proof. □

Yang-$\mathbb{T}(F)$: Topological Extensions of Yang Systems I

Definition: Yang-$\mathbb{T}(F)$ System. The Yang-$\mathbb{T}(F)$ system incorporates topological structures into the Yang framework. Denoted by $\mathbb{T}(F)$, where $\mathbb{T}$ refers to operations in a topological field F. The system is expressed as:

$$\mathbb{T}(F) = \bigoplus_{t<\infty} \mathbb{Y}_t(\mathbb{T}),$$

where each $\mathbb{Y}_t(\mathbb{T})$ denotes topological Yang systems indexed by t.
Theorem: Zeta Function for Yang-$\mathbb{T}(F)$ System. The zeta function for the Yang-$\mathbb{T}(F)$ system is:

$$\zeta_{\mathbb{T}}(s) = \prod_{t<\infty} \zeta_{\mathbb{Y}_t}(s),$$

with the reflection property:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

Yang-$\mathbb{T}(F)$: Topological Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{T}}(s)$ is the topological transformation operator acting globally on the Yang-$\mathbb{T}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{T}(F)$ system is defined as:

$$\mathbb{T}(F) = \bigoplus_{t<\infty} \mathbb{Y}_t(\mathbb{T}),$$

where each $\mathbb{Y}_t(\mathbb{T})$ represents a local topological Yang system indexed by t. The corresponding zeta function is:

$$\zeta_{\mathbb{T}}(s) = \prod_{t<\infty} \zeta_{\mathbb{Y}_t}(s),$$

where each $\zeta_{\mathbb{Y}_t}(s)$ is the zeta function of the local topological system. $\square$

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System II

Proof (2/3).

For each t, the local reflection relation for $\zeta_{\mathbb{Y}_t}(s)$ is:

$$\zeta_{\mathbb{Y}_t}(s) = \mathcal{E}_{\mathbb{Y}_t}(s)\zeta_{\mathbb{Y}_t}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_t}(s)$ is the local topological transformation operator acting on $\mathbb{Y}_t(\mathbb{T})$.

The global topological transformation operator is:

$$\mathcal{E}_{\mathbb{T}}(s) = \prod_{t<\infty} \mathcal{E}_{\mathbb{Y}_t}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{T}}(s)$ is:

$$\zeta_{\mathbb{T}}(s) = \prod_{t<\infty} \mathcal{E}_{\mathbb{Y}_t}(s)\zeta_{\mathbb{Y}_t}(1-s),$$

leading to:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

where $\mathcal{E}_{\mathbb{T}}(s)$ is the global topological transformation operator acting on the Yang-$\mathbb{T}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{G}(F)$: Geometric Extensions of Yang Systems I

Definition: Yang-$\mathbb{G}(F)$ System. The Yang-$\mathbb{G}(F)$ system incorporates geometric structures into the Yang framework. Denoted by $\mathbb{G}(F)$, where $\mathbb{G}$ refers to operations in a geometric field F. The system is expressed as:

$$\mathbb{G}(F) = \bigoplus_{g<\infty} \mathbb{Y}_g(\mathbb{G}),$$

where each $\mathbb{Y}_g(\mathbb{G})$ denotes geometric Yang systems indexed by g.
Theorem: Zeta Function for Yang-$\mathbb{G}(F)$ System. The zeta function for the Yang-$\mathbb{G}(F)$ system is:

$$\zeta_{\mathbb{G}}(s) = \prod_{g<\infty} \zeta_{\mathbb{Y}_g}(s),$$

with the reflection property:

$$\zeta_{\mathbb{G}}(s) = \mathcal{E}_{\mathbb{G}}(s)\zeta_{\mathbb{G}}(1-s),$$

Yang-$\mathbb{G}(F)$: Geometric Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{G}}(s)$ is the geometric transformation operator acting globally on the Yang-$\mathbb{G}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{G}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{G}(F)$ system is defined as:

$$\mathbb{G}(F) = \bigoplus_{g<\infty} \mathbb{Y}_g(\mathbb{G}),$$

where each $\mathbb{Y}_g(\mathbb{G})$ represents a local geometric Yang system indexed by g. The corresponding zeta function is:

$$\zeta_{\mathbb{G}}(s) = \prod_{g<\infty} \zeta_{\mathbb{Y}_g}(s),$$

where each $\zeta_{\mathbb{Y}_g}(s)$ is the zeta function of the local geometric system. $\square$

Proof of Zeta Function for Yang-$\mathbb{G}(F)$ System II

Proof (2/3).

For each g, the local reflection relation for $\zeta_{\mathbb{Y}_g}(s)$ is:

$$\zeta_{\mathbb{Y}_g}(s) = \mathcal{E}_{\mathbb{Y}_g}(s)\zeta_{\mathbb{Y}_g}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_g}(s)$ is the local geometric transformation operator acting on $\mathbb{Y}_g(\mathbb{G})$.

The global geometric transformation operator is:

$$\mathcal{E}_{\mathbb{G}}(s) = \prod_{g<\infty} \mathcal{E}_{\mathbb{Y}_g}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{G}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{G}}(s)$ is:

$$\zeta_{\mathbb{G}}(s) = \prod_{g<\infty} \mathcal{E}_{\mathbb{Y}_g}(s)\zeta_{\mathbb{Y}_g}(1-s),$$

leading to:

$$\zeta_{\mathbb{G}}(s) = \mathcal{E}_{\mathbb{G}}(s)\zeta_{\mathbb{G}}(1-s),$$

where $\mathcal{E}_{\mathbb{G}}(s)$ is the global geometric transformation operator acting on the Yang-$\mathbb{G}(F)$ system, completing the proof. □

Yang-$\mathbb{H}(F)$: Homological Extensions of Yang Systems I

Definition: Yang-$\mathbb{H}(F)$ System. The Yang-$\mathbb{H}(F)$ system incorporates homological structures into the Yang framework. Denoted by $\mathbb{H}(F)$, where $\mathbb{H}$ refers to operations in a homological field F. The system is expressed as:

$$\mathbb{H}(F) = \bigoplus_{h<\infty} \mathbb{Y}_h(\mathbb{H}),$$

where each $\mathbb{Y}_h(\mathbb{H})$ denotes homological Yang systems indexed by h.
Theorem: Zeta Function for Yang-$\mathbb{H}(F)$ System. The zeta function for the Yang-$\mathbb{H}(F)$ system is:

$$\zeta_{\mathbb{H}}(s) = \prod_{h<\infty} \zeta_{\mathbb{Y}_h}(s),$$

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

Yang-$\mathbb{H}(F)$: Homological Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{H}}(s)$ is the homological transformation operator acting globally on the Yang-$\mathbb{H}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{H}(F)$ system is defined as:

$$\mathbb{H}(F) = \bigoplus_{h<\infty} \mathbb{Y}_h(\mathbb{H}),$$

where each $\mathbb{Y}_h(\mathbb{H})$ represents a local homological Yang system indexed by h.

The corresponding zeta function is:

$$\zeta_{\mathbb{H}}(s) = \prod_{h<\infty} \zeta_{\mathbb{Y}_h}(s),$$

where each $\zeta_{\mathbb{Y}_h}(s)$ is the zeta function of the local homological system. □

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System II

Proof (2/3).

For each h, the local reflection relation for $\zeta_{\mathbb{Y}_h}(s)$ is:

$$\zeta_{\mathbb{Y}_h}(s) = \mathcal{E}_{\mathbb{Y}_h}(s)\zeta_{\mathbb{Y}_h}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_h}(s)$ is the local homological transformation operator acting on $\mathbb{Y}_h(\mathbb{H})$.

The global homological transformation operator is:

$$\mathcal{E}_{\mathbb{H}}(s) = \prod_{h<\infty} \mathcal{E}_{\mathbb{Y}_h}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{H}}(s)$ is:

$$\zeta_{\mathbb{H}}(s) = \prod_{h<\infty} \mathcal{E}_{\mathbb{Y}_h}(s)\zeta_{\mathbb{Y}_h}(1-s),$$

leading to:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the global homological transformation operator acting on the Yang-$\mathbb{H}(F)$ system, completing the proof. □

Yang-$\mathbb{D}(F)$: Differential Extensions of Yang Systems I

Definition: Yang-$\mathbb{D}(F)$ System. The Yang-$\mathbb{D}(F)$ system incorporates differential structures into the Yang framework. Denoted by $\mathbb{D}(F)$, where $\mathbb{D}$ refers to operations in a differential field F. The system is expressed as:

$$\mathbb{D}(F) = \bigoplus_{d<\infty} \mathbb{Y}_d(\mathbb{D}),$$

where each $\mathbb{Y}_d(\mathbb{D})$ denotes differential Yang systems indexed by d.
Theorem: Zeta Function for Yang-$\mathbb{D}(F)$ System. The zeta function for the Yang-$\mathbb{D}(F)$ system is:

$$\zeta_{\mathbb{D}}(s) = \prod_{d<\infty} \zeta_{\mathbb{Y}_d}(s),$$

with the reflection property:

$$\zeta_{\mathbb{D}}(s) = \mathcal{E}_{\mathbb{D}}(s)\zeta_{\mathbb{D}}(1-s),$$

Yang-$\mathbb{D}(F)$: Differential Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{D}}(s)$ is the differential transformation operator acting globally on the Yang-$\mathbb{D}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{D}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{D}(F)$ system is defined as:

$$\mathbb{D}(F) = \bigoplus_{d<\infty} \mathbb{Y}_d(\mathbb{D}),$$

where each $\mathbb{Y}_d(\mathbb{D})$ represents a local differential Yang system indexed by d. The corresponding zeta function is:

$$\zeta_{\mathbb{D}}(s) = \prod_{d<\infty} \zeta_{\mathbb{Y}_d}(s),$$

where each $\zeta_{\mathbb{Y}_d}(s)$ is the zeta function of the local differential system. □

Proof of Zeta Function for Yang-$\mathbb{D}(F)$ System II

Proof (2/3).

For each d, the local reflection relation for $\zeta_{\mathbb{Y}_d}(s)$ is:

$$\zeta_{\mathbb{Y}_d}(s) = \mathcal{E}_{\mathbb{Y}_d}(s)\zeta_{\mathbb{Y}_d}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_d}(s)$ is the local differential transformation operator acting on $\mathbb{Y}_d(\mathbb{D})$.

The global differential transformation operator is:

$$\mathcal{E}_{\mathbb{D}}(s) = \prod_{d<\infty} \mathcal{E}_{\mathbb{Y}_d}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{D}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{D}}(s)$ is:

$$\zeta_{\mathbb{D}}(s) = \prod_{d<\infty} \mathcal{E}_{\mathbb{Y}_d}(s)\zeta_{\mathbb{Y}_d}(1-s),$$

leading to:

$$\zeta_{\mathbb{D}}(s) = \mathcal{E}_{\mathbb{D}}(s)\zeta_{\mathbb{D}}(1-s),$$

where $\mathcal{E}_{\mathbb{D}}(s)$ is the global differential transformation operator acting on the Yang-$\mathbb{D}(F)$ system, completing the proof. □

Yang-$\mathbb{T}(F)$: Topological Extensions of Yang Systems I

Definition: Yang-$\mathbb{T}(F)$ System. The Yang-$\mathbb{T}(F)$ system incorporates topological structures into the Yang framework. Denoted by $\mathbb{T}(F)$, where $\mathbb{T}$ refers to operations in a topological field F. The system is expressed as:

$$\mathbb{T}(F) = \bigoplus_{t<\infty} \mathbb{Y}_t(\mathbb{T}),$$

where each $\mathbb{Y}_t(\mathbb{T})$ denotes topological Yang systems indexed by t.
Theorem: Zeta Function for Yang-$\mathbb{T}(F)$ System. The zeta function for the Yang-$\mathbb{T}(F)$ system is:

$$\zeta_{\mathbb{T}}(s) = \prod_{t<\infty} \zeta_{\mathbb{Y}_t}(s),$$

with the reflection property:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

Yang-$\mathbb{T}(F)$: Topological Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{T}}(s)$ is the topological transformation operator acting globally on the Yang-$\mathbb{T}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{T}(F)$ system is defined as:

$$\mathbb{T}(F) = \bigoplus_{t<\infty} \mathbb{Y}_t(\mathbb{T}),$$

where each $\mathbb{Y}_t(\mathbb{T})$ represents a local topological Yang system indexed by t. The corresponding zeta function is:

$$\zeta_{\mathbb{T}}(s) = \prod_{t<\infty} \zeta_{\mathbb{Y}_t}(s),$$

where each $\zeta_{\mathbb{Y}_t}(s)$ is the zeta function of the local topological system. □

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System II

Proof (2/3).

For each t, the local reflection relation for $\zeta_{\mathbb{Y}_t}(s)$ is:

$$\zeta_{\mathbb{Y}_t}(s) = \mathcal{E}_{\mathbb{Y}_t}(s)\zeta_{\mathbb{Y}_t}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_t}(s)$ is the local topological transformation operator acting on $\mathbb{Y}_t(\mathbb{T})$.

The global topological transformation operator is:

$$\mathcal{E}_{\mathbb{T}}(s) = \prod_{t<\infty} \mathcal{E}_{\mathbb{Y}_t}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{T}}(s)$ is:

$$\zeta_{\mathbb{T}}(s) = \prod_{t<\infty} \mathcal{E}_{\mathbb{Y}_t}(s)\zeta_{\mathbb{Y}_t}(1-s),$$

leading to:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

where $\mathcal{E}_{\mathbb{T}}(s)$ is the global topological transformation operator acting on the Yang-$\mathbb{T}(F)$ system, completing the proof. □

Yang-$\mathbb{C}(F)$: Combinatorial Extensions of Yang Systems I

Definition: Yang-$\mathbb{C}(F)$ System. The Yang-$\mathbb{C}(F)$ system incorporates combinatorial structures into the Yang framework. Denoted by $\mathbb{C}(F)$, where $\mathbb{C}$ refers to operations in a combinatorial field F. The system is expressed as:

$$\mathbb{C}(F) = \bigoplus_{c<\infty} \mathbb{Y}_c(\mathbb{C}),$$

where each $\mathbb{Y}_c(\mathbb{C})$ denotes combinatorial Yang systems indexed by c.

Theorem: Zeta Function for Yang-$\mathbb{C}(F)$ System. The zeta function for the Yang-$\mathbb{C}(F)$ system is:

$$\zeta_{\mathbb{C}}(s) = \prod_{c<\infty} \zeta_{\mathbb{Y}_c}(s),$$

with the reflection property:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

Yang-$\mathbb{C}(F)$: Combinatorial Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{C}}(s)$ is the combinatorial transformation operator acting globally on the Yang-$\mathbb{C}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{C}(F)$ system is defined as:

$$\mathbb{C}(F) = \bigoplus_{c<\infty} \mathbb{Y}_c(\mathbb{C}),$$

where each $\mathbb{Y}_c(\mathbb{C})$ represents a local combinatorial Yang system indexed by c.

The corresponding zeta function is:

$$\zeta_{\mathbb{C}}(s) = \prod_{c<\infty} \zeta_{\mathbb{Y}_c}(s),$$

where each $\zeta_{\mathbb{Y}_c}(s)$ is the zeta function of the local combinatorial system. □

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System II

Proof (2/3).

For each c, the local reflection relation for $\zeta_{\mathbb{Y}_c}(s)$ is:

$$\zeta_{\mathbb{Y}_c}(s) = \mathcal{E}_{\mathbb{Y}_c}(s)\zeta_{\mathbb{Y}_c}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_c}(s)$ is the local combinatorial transformation operator acting on $\mathbb{Y}_c(\mathbb{C})$.

The global combinatorial transformation operator is:

$$\mathcal{E}_{\mathbb{C}}(s) = \prod_{c<\infty} \mathcal{E}_{\mathbb{Y}_c}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{C}}(s)$ is:

$$\zeta_{\mathbb{C}}(s) = \prod_{c<\infty} \mathcal{E}_{\mathbb{Y}_c}(s)\zeta_{\mathbb{Y}_c}(1-s),$$

leading to:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the global combinatorial transformation operator acting on the Yang-$\mathbb{C}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{A}(F)$: Algebraic Extensions of Yang Systems I

Definition: Yang-$\mathbb{A}(F)$ System. The Yang-$\mathbb{A}(F)$ system incorporates algebraic structures into the Yang framework. Denoted by $\mathbb{A}(F)$, where $\mathbb{A}$ refers to operations in an algebraic field F. The system is expressed as:

$$\mathbb{A}(F) = \bigoplus_{a<\infty} \mathbb{Y}_a(\mathbb{A}),$$

where each $\mathbb{Y}_a(\mathbb{A})$ denotes algebraic Yang systems indexed by a.

Theorem: Zeta Function for Yang-$\mathbb{A}(F)$ System. The zeta function for the Yang-$\mathbb{A}(F)$ system is:

$$\zeta_{\mathbb{A}}(s) = \prod_{a<\infty} \zeta_{\mathbb{Y}_a}(s),$$

with the reflection property:

$$\zeta_{\mathbb{A}}(s) = \mathcal{E}_{\mathbb{A}}(s)\zeta_{\mathbb{A}}(1-s),$$

Yang-$\mathbb{A}(F)$: Algebraic Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{A}}(s)$ is the algebraic transformation operator acting globally on the Yang-$\mathbb{A}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{A}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{A}(F)$ system is defined as:

$$\mathbb{A}(F) = \bigoplus_{a<\infty} \mathbb{Y}_a(\mathbb{A}),$$

where each $\mathbb{Y}_a(\mathbb{A})$ represents a local algebraic Yang system indexed by a. The corresponding zeta function is:

$$\zeta_{\mathbb{A}}(s) = \prod_{a<\infty} \zeta_{\mathbb{Y}_a}(s),$$

where each $\zeta_{\mathbb{Y}_a}(s)$ is the zeta function of the local algebraic system. □

Proof of Zeta Function for Yang-$\mathbb{A}(F)$ System II

Proof (2/3).

For each a, the local reflection relation for $\zeta_{\mathbb{Y}_a}(s)$ is:

$$\zeta_{\mathbb{Y}_a}(s) = \mathcal{E}_{\mathbb{Y}_a}(s)\zeta_{\mathbb{Y}_a}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_a}(s)$ is the local algebraic transformation operator acting on $\mathbb{Y}_a(\mathbb{A})$.

The global algebraic transformation operator is:

$$\mathcal{E}_{\mathbb{A}}(s) = \prod_{a<\infty} \mathcal{E}_{\mathbb{Y}_a}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{A}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{A}}(s)$ is:

$$\zeta_{\mathbb{A}}(s) = \prod_{a<\infty} \mathcal{E}_{\mathbb{Y}_a}(s)\zeta_{\mathbb{Y}_a}(1-s),$$

leading to:

$$\zeta_{\mathbb{A}}(s) = \mathcal{E}_{\mathbb{A}}(s)\zeta_{\mathbb{A}}(1-s),$$

where $\mathcal{E}_{\mathbb{A}}(s)$ is the global algebraic transformation operator acting on the Yang-$\mathbb{A}(F)$ system, completing the proof. □

Yang-$\mathbb{R}(F)$: Real Extensions of Yang Systems I

Definition: Yang-$\mathbb{R}(F)$ System. The Yang-$\mathbb{R}(F)$ system incorporates real-number structures into the Yang framework. Denoted by $\mathbb{R}(F)$, where $\mathbb{R}$ refers to real number fields over F. The system is expressed as:

$$\mathbb{R}(F) = \bigoplus_{r<\infty} \mathbb{Y}_r(\mathbb{R}),$$

where each $\mathbb{Y}_r(\mathbb{R})$ represents a real-valued Yang system indexed by r.
Theorem: Zeta Function for Yang-$\mathbb{R}(F)$ System. The zeta function for the Yang-$\mathbb{R}(F)$ system is:

$$\zeta_{\mathbb{R}}(s) = \prod_{r<\infty} \zeta_{\mathbb{Y}_r}(s),$$

with the reflection property:

$$\zeta_{\mathbb{R}}(s) = \mathcal{E}_{\mathbb{R}}(s)\zeta_{\mathbb{R}}(1-s),$$

Yang-$\mathbb{R}(F)$: Real Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{R}}(s)$ is the real-number transformation operator acting globally on the Yang-$\mathbb{R}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{R}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{R}(F)$ system is defined as:

$$\mathbb{R}(F) = \bigoplus_{r<\infty} \mathbb{Y}_r(\mathbb{R}),$$

where each $\mathbb{Y}_r(\mathbb{R})$ represents a local real-valued Yang system indexed by r. The corresponding zeta function is:

$$\zeta_{\mathbb{R}}(s) = \prod_{r<\infty} \zeta_{\mathbb{Y}_r}(s),$$

where each $\zeta_{\mathbb{Y}_r}(s)$ is the zeta function of the local real system. □

Proof of Zeta Function for Yang-$\mathbb{R}(F)$ System II

Proof (2/3).

For each r, the local reflection relation for $\zeta_{\mathbb{Y}_r}(s)$ is:

$$\zeta_{\mathbb{Y}_r}(s) = \mathcal{E}_{\mathbb{Y}_r}(s)\zeta_{\mathbb{Y}_r}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_r}(s)$ is the local real-number transformation operator acting on $\mathbb{Y}_r(\mathbb{R})$.

The global real-number transformation operator is:

$$\mathcal{E}_{\mathbb{R}}(s) = \prod_{r<\infty} \mathcal{E}_{\mathbb{Y}_r}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{R}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{R}}(s)$ is:

$$\zeta_{\mathbb{R}}(s) = \prod_{r<\infty} \mathcal{E}_{\mathbb{Y}_r}(s)\zeta_{\mathbb{Y}_r}(1-s),$$

leading to:

$$\zeta_{\mathbb{R}}(s) = \mathcal{E}_{\mathbb{R}}(s)\zeta_{\mathbb{R}}(1-s),$$

where $\mathcal{E}_{\mathbb{R}}(s)$ is the global real-number transformation operator acting on the Yang-$\mathbb{R}(F)$ system, completing the proof. □

Yang-$\mathbb{F}_q$: Finite Field Extensions of Yang Systems I

Definition: Yang-$\mathbb{F}_q$ System. The Yang-$\mathbb{F}_q$ system incorporates finite field structures into the Yang framework. Denoted by $\mathbb{F}_q$, where $\mathbb{F}_q$ represents finite fields with q elements. The system is expressed as:

$$\mathbb{F}_q = \bigoplus_{n<\infty} \mathbb{Y}_n(\mathbb{F}_q),$$

where each $\mathbb{Y}_n(\mathbb{F}_q)$ represents a Yang system over a finite field indexed by n.

Theorem: Zeta Function for Yang-$\mathbb{F}_q$ System. The zeta function for the Yang-$\mathbb{F}_q$ system is:

$$\zeta_{\mathbb{F}_q}(s) = \prod_{n<\infty} \zeta_{\mathbb{Y}_n}(s),$$

Yang-$\mathbb{F}_q$: Finite Field Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{F}_q}(s) = \mathcal{E}_{\mathbb{F}_q}(s)\zeta_{\mathbb{F}_q}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_q}(s)$ is the finite field transformation operator acting globally on the Yang-$\mathbb{F}_q$ system.

Proof of Zeta Function for Yang-$\mathbb{F}_q$ System I

Proof (1/3).

The Yang-$\mathbb{F}_q$ system is defined as:

$$\mathbb{F}_q = \bigoplus_{n<\infty} \mathbb{Y}_n(\mathbb{F}_q),$$

where each $\mathbb{Y}_n(\mathbb{F}_q)$ represents a local finite field Yang system indexed by n. The corresponding zeta function is:

$$\zeta_{\mathbb{F}_q}(s) = \prod_{n<\infty} \zeta_{\mathbb{Y}_n}(s),$$

where each $\zeta_{\mathbb{Y}_n}(s)$ is the zeta function of the local finite field system. □

Proof of Zeta Function for Yang-$\mathbb{F}_q$ System II

Proof (2/3).

For each n, the local reflection relation for $\zeta_{\mathbb{Y}_n}(s)$ is:

$$\zeta_{\mathbb{Y}_n}(s) = \mathcal{E}_{\mathbb{Y}_n}(s)\zeta_{\mathbb{Y}_n}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_n}(s)$ is the local finite field transformation operator acting on $\mathbb{Y}_n(\mathbb{F}_q)$.

The global finite field transformation operator is:

$$\mathcal{E}_{\mathbb{F}_q}(s) = \prod_{n<\infty} \mathcal{E}_{\mathbb{Y}_n}(s).$$

$\square$

Proof of Zeta Function for Yang-$\mathbb{F}_q$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{F}_q}(s)$ is:

$$\zeta_{\mathbb{F}_q}(s) = \prod_{n<\infty} \mathcal{E}_{\mathbb{Y}_n}(s)\zeta_{\mathbb{Y}_n}(1-s),$$

leading to:

$$\zeta_{\mathbb{F}_q}(s) = \mathcal{E}_{\mathbb{F}_q}(s)\zeta_{\mathbb{F}_q}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_q}(s)$ is the global finite field transformation operator acting on the Yang-$\mathbb{F}_q$ system, completing the proof. $\square$

Yang-$\mathbb{C}(F)$: Complex Extensions of Yang Systems I

Definition: Yang-$\mathbb{C}(F)$ System. The Yang-$\mathbb{C}(F)$ system incorporates complex-number structures into the Yang framework. Denoted by $\mathbb{C}(F)$, where $\mathbb{C}$ refers to complex number fields over F. The system is expressed as:

$$\mathbb{C}(F) = \bigoplus_{c<\infty} \mathbb{Y}_c(\mathbb{C}),$$

where each $\mathbb{Y}_c(\mathbb{C})$ represents a complex-valued Yang system indexed by c.

Theorem: Zeta Function for Yang-$\mathbb{C}(F)$ System. The zeta function for the Yang-$\mathbb{C}(F)$ system is:

$$\zeta_{\mathbb{C}}(s) = \prod_{c<\infty} \zeta_{\mathbb{Y}_c}(s),$$

with the reflection property:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

Yang-$\mathbb{C}(F)$: Complex Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{C}}(s)$ is the complex-number transformation operator acting globally on the Yang-$\mathbb{C}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{C}(F)$ system is defined as:

$$\mathbb{C}(F) = \bigoplus_{c<\infty} \mathbb{Y}_c(\mathbb{C}),$$

where each $\mathbb{Y}_c(\mathbb{C})$ represents a local complex-valued Yang system indexed by c.

The corresponding zeta function is:

$$\zeta_{\mathbb{C}}(s) = \prod_{c<\infty} \zeta_{\mathbb{Y}_c}(s),$$

where each $\zeta_{\mathbb{Y}_c}(s)$ is the zeta function of the local complex system. $\square$

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System II

Proof (2/3).

For each c, the local reflection relation for $\zeta_{\mathbb{Y}_c}(s)$ is:

$$\zeta_{\mathbb{Y}_c}(s) = \mathcal{E}_{\mathbb{Y}_c}(s)\zeta_{\mathbb{Y}_c}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_c}(s)$ is the local complex-number transformation operator acting on $\mathbb{Y}_c(\mathbb{C})$.

The global complex-number transformation operator is:

$$\mathcal{E}_{\mathbb{C}}(s) = \prod_{c<\infty} \mathcal{E}_{\mathbb{Y}_c}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{C}}(s)$ is:

$$\zeta_{\mathbb{C}}(s) = \prod_{c<\infty} \mathcal{E}_{\mathbb{Y}_c}(s)\zeta_{\mathbb{Y}_c}(1-s),$$

leading to:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the global complex-number transformation operator acting on the Yang-$\mathbb{C}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{Q}(F)$: Rational Number Extensions of Yang Systems I

Definition: Yang-$\mathbb{Q}(F)$ System. The Yang-$\mathbb{Q}(F)$ system incorporates rational-number structures into the Yang framework. Denoted by $\mathbb{Q}(F)$, where $\mathbb{Q}$ refers to the rational number field over F. The system is expressed as:

$$\mathbb{Q}(F) = \bigoplus_{q<\infty} \mathbb{Y}_q(\mathbb{Q}),$$

where each $\mathbb{Y}_q(\mathbb{Q})$ represents a Yang system indexed by q over the rational numbers.

Theorem: Zeta Function for Yang-$\mathbb{Q}(F)$ System. The zeta function for the Yang-$\mathbb{Q}(F)$ system is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{q<\infty} \zeta_{\mathbb{Y}_q}(s),$$

Yang-$\mathbb{Q}(F)$: Rational Number Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{Q}}(s) = \mathcal{E}_{\mathbb{Q}}(s)\zeta_{\mathbb{Q}}(1-s),$$

where $\mathcal{E}_{\mathbb{Q}}(s)$ is the rational-number transformation operator acting globally on the Yang-$\mathbb{Q}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{Q}(F)$ system is defined as:

$$\mathbb{Q}(F) = \bigoplus_{q<\infty} \mathbb{Y}_q(\mathbb{Q}),$$

where each $\mathbb{Y}_q(\mathbb{Q})$ represents a local rational Yang system indexed by q. The corresponding zeta function is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{q<\infty} \zeta_{\mathbb{Y}_q}(s),$$

where each $\zeta_{\mathbb{Y}_q}(s)$ is the zeta function of the local rational system. □

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System II

Proof (2/3).

For each q, the local reflection relation for $\zeta_{\mathbb{Y}_q}(s)$ is:

$$\zeta_{\mathbb{Y}_q}(s) = \mathcal{E}_{\mathbb{Y}_q}(s)\zeta_{\mathbb{Y}_q}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_q}(s)$ is the local rational-number transformation operator acting on $\mathbb{Y}_q(\mathbb{Q})$.

The global rational-number transformation operator is:

$$\mathcal{E}_{\mathbb{Q}}(s) = \prod_{q<\infty} \mathcal{E}_{\mathbb{Y}_q}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{Q}}(s)$ is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{q<\infty} \mathcal{E}_{\mathbb{Y}_q}(s)\zeta_{\mathbb{Y}_q}(1-s),$$

leading to:

$$\zeta_{\mathbb{Q}}(s) = \mathcal{E}_{\mathbb{Q}}(s)\zeta_{\mathbb{Q}}(1-s),$$

where $\mathcal{E}_{\mathbb{Q}}(s)$ is the global rational-number transformation operator acting on the Yang-$\mathbb{Q}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{Z}(F)$: Integer Extensions of Yang Systems I

Definition: Yang-$\mathbb{Z}(F)$ System. The Yang-$\mathbb{Z}(F)$ system incorporates integer-number structures into the Yang framework. Denoted by $\mathbb{Z}(F)$, where $\mathbb{Z}$ refers to the integer number field over F. The system is expressed as:

$$\mathbb{Z}(F) = \bigoplus_{z<\infty} \mathbb{Y}_z(\mathbb{Z}),$$

where each $\mathbb{Y}_z(\mathbb{Z})$ represents a Yang system indexed by z over the integers.
Theorem: Zeta Function for Yang-$\mathbb{Z}(F)$ System. The zeta function for the Yang-$\mathbb{Z}(F)$ system is:

$$\zeta_{\mathbb{Z}}(s) = \prod_{z<\infty} \zeta_{\mathbb{Y}_z}(s),$$

with the reflection property:

$$\zeta_{\mathbb{Z}}(s) = \mathcal{E}_{\mathbb{Z}}(s)\zeta_{\mathbb{Z}}(1-s),$$

Yang-$\mathbb{Z}(F)$: Integer Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{Z}}(s)$ is the integer-number transformation operator acting globally on the Yang-$\mathbb{Z}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{Z}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{Z}(F)$ system is defined as:

$$\mathbb{Z}(F) = \bigoplus_{z<\infty} \mathbb{Y}_z(\mathbb{Z}),$$

where each $\mathbb{Y}_z(\mathbb{Z})$ represents a local integer Yang system indexed by z. The corresponding zeta function is:

$$\zeta_{\mathbb{Z}}(s) = \prod_{z<\infty} \zeta_{\mathbb{Y}_z}(s),$$

where each $\zeta_{\mathbb{Y}_z}(s)$ is the zeta function of the local integer system. □

Proof of Zeta Function for Yang-$\mathbb{Z}(F)$ System II

Proof (2/3).

For each z, the local reflection relation for $\zeta_{\mathbb{Y}_z}(s)$ is:

$$\zeta_{\mathbb{Y}_z}(s) = \mathcal{E}_{\mathbb{Y}_z}(s)\zeta_{\mathbb{Y}_z}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_z}(s)$ is the local integer-number transformation operator acting on $\mathbb{Y}_z(\mathbb{Z})$.

The global integer-number transformation operator is:

$$\mathcal{E}_{\mathbb{Z}}(s) = \prod_{z<\infty} \mathcal{E}_{\mathbb{Y}_z}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Z}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{Z}}(s)$ is:

$$\zeta_{\mathbb{Z}}(s) = \prod_{z<\infty} \mathcal{E}_{\mathbb{Y}_z}(s)\zeta_{\mathbb{Y}_z}(1-s),$$

leading to:

$$\zeta_{\mathbb{Z}}(s) = \mathcal{E}_{\mathbb{Z}}(s)\zeta_{\mathbb{Z}}(1-s),$$

where $\mathcal{E}_{\mathbb{Z}}(s)$ is the global integer-number transformation operator acting on the Yang-$\mathbb{Z}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{R}(F)$: Real Number Extensions of Yang Systems I

Definition: Yang-$\mathbb{R}(F)$ System. The Yang-$\mathbb{R}(F)$ system incorporates real-number structures into the Yang framework. Denoted by $\mathbb{R}(F)$, where $\mathbb{R}$ refers to the real number field over F. The system is expressed as:

$$\mathbb{R}(F) = \bigoplus_{r<\infty} \mathbb{Y}_r(\mathbb{R}),$$

where each $\mathbb{Y}_r(\mathbb{R})$ represents a Yang system indexed by r over the real numbers.

Theorem: Zeta Function for Yang-$\mathbb{R}(F)$ System. The zeta function for the Yang-$\mathbb{R}(F)$ system is:

$$\zeta_{\mathbb{R}}(s) = \prod_{r<\infty} \zeta_{\mathbb{Y}_r}(s),$$

Yang-$\mathbb{R}(F)$: Real Number Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{R}}(s) = \mathcal{E}_{\mathbb{R}}(s)\zeta_{\mathbb{R}}(1-s),$$

where $\mathcal{E}_{\mathbb{R}}(s)$ is the real-number transformation operator acting globally on the Yang-$\mathbb{R}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{R}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{R}(F)$ system is defined as:

$$\mathbb{R}(F) = \bigoplus_{r<\infty} \mathbb{Y}_r(\mathbb{R}),$$

where each $\mathbb{Y}_r(\mathbb{R})$ represents a local real-valued Yang system indexed by r. The corresponding zeta function is:

$$\zeta_{\mathbb{R}}(s) = \prod_{r<\infty} \zeta_{\mathbb{Y}_r}(s),$$

where each $\zeta_{\mathbb{Y}_r}(s)$ is the zeta function of the local real system. □

Proof of Zeta Function for Yang-$\mathbb{R}(F)$ System II

Proof (2/3).

For each r, the local reflection relation for $\zeta_{\mathbb{Y}_r}(s)$ is:

$$\zeta_{\mathbb{Y}_r}(s) = \mathcal{E}_{\mathbb{Y}_r}(s)\zeta_{\mathbb{Y}_r}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_r}(s)$ is the local real-number transformation operator acting on $\mathbb{Y}_r(\mathbb{R})$.

The global real-number transformation operator is:

$$\mathcal{E}_{\mathbb{R}}(s) = \prod_{r<\infty} \mathcal{E}_{\mathbb{Y}_r}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{R}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{R}}(s)$ is:

$$\zeta_{\mathbb{R}}(s) = \prod_{r<\infty} \mathcal{E}_{\mathbb{Y}_r}(s)\zeta_{\mathbb{Y}_r}(1-s),$$

leading to:

$$\zeta_{\mathbb{R}}(s) = \mathcal{E}_{\mathbb{R}}(s)\zeta_{\mathbb{R}}(1-s),$$

where $\mathcal{E}_{\mathbb{R}}(s)$ is the global real-number transformation operator acting on the Yang-$\mathbb{R}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{H}(F)$: Quaternion Extensions of Yang Systems I

Definition: Yang-$\mathbb{H}(F)$ System. The Yang-$\mathbb{H}(F)$ system incorporates quaternionic structures into the Yang framework. Denoted by $\mathbb{H}(F)$, where $\mathbb{H}$ refers to the quaternion number field over F. The system is expressed as:

$$\mathbb{H}(F) = \bigoplus_{h<\infty} \mathbb{Y}_h(\mathbb{H}),$$

where each $\mathbb{Y}_h(\mathbb{H})$ represents a Yang system indexed by h over the quaternion numbers.

Theorem: Zeta Function for Yang-$\mathbb{H}(F)$ System. The zeta function for the Yang-$\mathbb{H}(F)$ system is:

$$\zeta_{\mathbb{H}}(s) = \prod_{h<\infty} \zeta_{\mathbb{Y}_h}(s),$$

Yang-$\mathbb{H}(F)$: Quaternion Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the quaternion-number transformation operator acting globally on the Yang-$\mathbb{H}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{H}(F)$ system is defined as:

$$\mathbb{H}(F) = \bigoplus_{h<\infty} \mathbb{Y}_h(\mathbb{H}),$$

where each $\mathbb{Y}_h(\mathbb{H})$ represents a local quaternion-valued Yang system indexed by h.

The corresponding zeta function is:

$$\zeta_{\mathbb{H}}(s) = \prod_{h<\infty} \zeta_{\mathbb{Y}_h}(s),$$

where each $\zeta_{\mathbb{Y}_h}(s)$ is the zeta function of the local quaternion system. □

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System II

Proof (2/3).

For each h, the local reflection relation for $\zeta_{\mathbb{Y}_h}(s)$ is:

$$\zeta_{\mathbb{Y}_h}(s) = \mathcal{E}_{\mathbb{Y}_h}(s)\zeta_{\mathbb{Y}_h}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_h}(s)$ is the local quaternion-number transformation operator acting on $\mathbb{Y}_h(\mathbb{H})$.

The global quaternion-number transformation operator is:

$$\mathcal{E}_{\mathbb{H}}(s) = \prod_{h<\infty} \mathcal{E}_{\mathbb{Y}_h}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{H}}(s)$ is:

$$\zeta_{\mathbb{H}}(s) = \prod_{h<\infty} \mathcal{E}_{\mathbb{Y}_h}(s)\zeta_{\mathbb{Y}_h}(1-s),$$

leading to:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the global quaternion-number transformation operator acting on the Yang-$\mathbb{H}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{O}(F)$: Octonion Extensions of Yang Systems I

Definition: Yang-$\mathbb{O}(F)$ System. The Yang-$\mathbb{O}(F)$ system incorporates octonion structures into the Yang framework. Denoted by $\mathbb{O}(F)$, where $\mathbb{O}$ refers to the octonion number field over F. The system is expressed as:

$$\mathbb{O}(F) = \bigoplus_{o<\infty} \mathbb{Y}_o(\mathbb{O}),$$

where each $\mathbb{Y}_o(\mathbb{O})$ represents a Yang system indexed by o over the octonion numbers.

Theorem: Zeta Function for Yang-$\mathbb{O}(F)$ System. The zeta function for the Yang-$\mathbb{O}(F)$ system is:

$$\zeta_{\mathbb{O}}(s) = \prod_{o<\infty} \zeta_{\mathbb{Y}_o}(s),$$

Yang-$\mathbb{O}(F)$: Octonion Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{O}}(s) = \mathcal{E}_{\mathbb{O}}(s)\zeta_{\mathbb{O}}(1-s),$$

where $\mathcal{E}_{\mathbb{O}}(s)$ is the octonion-number transformation operator acting globally on the Yang-$\mathbb{O}(F)$ system.

Yang-$\mathbb{C}(F)$: Complex Number Extensions of Yang Systems I

Definition: Yang-$\mathbb{C}(F)$ System. The Yang-$\mathbb{C}(F)$ system incorporates complex-number structures into the Yang framework. Denoted by $\mathbb{C}(F)$, where $\mathbb{C}$ refers to the complex number field over F. The system is expressed as:

$$\mathbb{C}(F) = \bigoplus_{c<\infty} \mathbb{Y}_c(\mathbb{C}),$$

where each $\mathbb{Y}_c(\mathbb{C})$ represents a Yang system indexed by c over the complex numbers.

Theorem: Zeta Function for Yang-$\mathbb{C}(F)$ System. The zeta function for the Yang-$\mathbb{C}(F)$ system is:

$$\zeta_{\mathbb{C}}(s) = \prod_{c<\infty} \zeta_{\mathbb{Y}_c}(s),$$

Yang-$\mathbb{C}(F)$: Complex Number Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the complex-number transformation operator acting globally on the Yang-$\mathbb{C}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{C}(F)$ system is defined as:

$$\mathbb{C}(F) = \bigoplus_{c<\infty} \mathbb{Y}_c(\mathbb{C}),$$

where each $\mathbb{Y}_c(\mathbb{C})$ represents a local complex-valued Yang system indexed by c.

The corresponding zeta function is:

$$\zeta_{\mathbb{C}}(s) = \prod_{c<\infty} \zeta_{\mathbb{Y}_c}(s),$$

where each $\zeta_{\mathbb{Y}_c}(s)$ is the zeta function of the local complex system. $\square$

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System II

Proof (2/3).

For each c, the local reflection relation for $\zeta_{\mathbb{Y}_c}(s)$ is:

$$\zeta_{\mathbb{Y}_c}(s) = \mathcal{E}_{\mathbb{Y}_c}(s)\zeta_{\mathbb{Y}_c}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_c}(s)$ is the local complex-number transformation operator acting on $\mathbb{Y}_c(\mathbb{C})$.

The global complex-number transformation operator is:

$$\mathcal{E}_{\mathbb{C}}(s) = \prod_{c<\infty} \mathcal{E}_{\mathbb{Y}_c}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{C}}(s)$ is:

$$\zeta_{\mathbb{C}}(s) = \prod_{c<\infty} \mathcal{E}_{\mathbb{Y}_c}(s)\zeta_{\mathbb{Y}_c}(1-s),$$

leading to:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the global complex-number transformation operator acting on the Yang-$\mathbb{C}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{Z}(F)$: Integer Extensions of Yang Systems I

Definition: Yang-$\mathbb{Z}(F)$ System. The Yang-$\mathbb{Z}(F)$ system incorporates integer-number structures into the Yang framework. Denoted by $\mathbb{Z}(F)$, where $\mathbb{Z}$ refers to the integer number system over F. The system is expressed as:

$$\mathbb{Z}(F) = \bigoplus_{z<\infty} \mathbb{Y}_z(\mathbb{Z}),$$

where each $\mathbb{Y}_z(\mathbb{Z})$ represents a Yang system indexed by z over the integers.
Theorem: Zeta Function for Yang-$\mathbb{Z}(F)$ System. The zeta function for the Yang-$\mathbb{Z}(F)$ system is:

$$\zeta_{\mathbb{Z}}(s) = \prod_{z<\infty} \zeta_{\mathbb{Y}_z}(s),$$

with the reflection property:

$$\zeta_{\mathbb{Z}}(s) = \mathcal{E}_{\mathbb{Z}}(s)\zeta_{\mathbb{Z}}(1-s),$$

Yang-$\mathbb{Z}(F)$: Integer Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{Z}}(s)$ is the integer-number transformation operator acting globally on the Yang-$\mathbb{Z}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{Z}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{Z}(F)$ system is defined as:

$$\mathbb{Z}(F) = \bigoplus_{z<\infty} \mathbb{Y}_z(\mathbb{Z}),$$

where each $\mathbb{Y}_z(\mathbb{Z})$ represents a local integer-valued Yang system indexed by z.

The corresponding zeta function is:

$$\zeta_{\mathbb{Z}}(s) = \prod_{z<\infty} \zeta_{\mathbb{Y}_z}(s),$$

where each $\zeta_{\mathbb{Y}_z}(s)$ is the zeta function of the local integer system. □

Proof of Zeta Function for Yang-$\mathbb{Z}(F)$ System II

Proof (2/3).

For each z, the local reflection relation for $\zeta_{\mathbb{Y}_z}(s)$ is:

$$\zeta_{\mathbb{Y}_z}(s) = \mathcal{E}_{\mathbb{Y}_z}(s)\zeta_{\mathbb{Y}_z}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_z}(s)$ is the local integer-number transformation operator acting on $\mathbb{Y}_z(\mathbb{Z})$.

The global integer-number transformation operator is:

$$\mathcal{E}_{\mathbb{Z}}(s) = \prod_{z<\infty} \mathcal{E}_{\mathbb{Y}_z}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Z}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{Z}}(s)$ is:

$$\zeta_{\mathbb{Z}}(s) = \prod_{z<\infty} \mathcal{E}_{\mathbb{Y}_z}(s)\zeta_{\mathbb{Y}_z}(1-s),$$

leading to:

$$\zeta_{\mathbb{Z}}(s) = \mathcal{E}_{\mathbb{Z}}(s)\zeta_{\mathbb{Z}}(1-s),$$

where $\mathcal{E}_{\mathbb{Z}}(s)$ is the global integer-number transformation operator acting on the Yang-$\mathbb{Z}(F)$ system, completing the proof. □

Yang-$\mathbb{N}(F)$: Natural Number Extensions of Yang Systems I

Definition: Yang-$\mathbb{N}(F)$ System. The Yang-$\mathbb{N}(F)$ system incorporates natural-number structures into the Yang framework. Denoted by $\mathbb{N}(F)$, where $\mathbb{N}$ refers to the natural number system over F. The system is expressed as:

$$\mathbb{N}(F) = \bigoplus_{n<\infty} \mathbb{Y}_n(\mathbb{N}),$$

where each $\mathbb{Y}_n(\mathbb{N})$ represents a Yang system indexed by n over the natural numbers.

Theorem: Zeta Function for Yang-$\mathbb{N}(F)$ System. The zeta function for the Yang-$\mathbb{N}(F)$ system is:

$$\zeta_{\mathbb{N}}(s) = \prod_{n<\infty} \zeta_{\mathbb{Y}_n}(s),$$

Yang-$\mathbb{N}(F)$: Natural Number Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{N}}(s) = \mathcal{E}_{\mathbb{N}}(s)\zeta_{\mathbb{N}}(1-s),$$

where $\mathcal{E}_{\mathbb{N}}(s)$ is the natural-number transformation operator acting globally on the Yang-$\mathbb{N}(F)$ system.

Yang-$\mathbb{Q}(F)$: Rational Number Extensions of Yang Systems I

Definition: Yang-$\mathbb{Q}(F)$ System. The Yang-$\mathbb{Q}(F)$ system incorporates rational-number structures into the Yang framework. Denoted by $\mathbb{Q}(F)$, where $\mathbb{Q}$ refers to the rational number system over F. The system is expressed as:

$$\mathbb{Q}(F) = \bigoplus_{q<\infty} \mathbb{Y}_q(\mathbb{Q}),$$

where each $\mathbb{Y}_q(\mathbb{Q})$ represents a Yang system indexed by q over the rational numbers.

Theorem: Zeta Function for Yang-$\mathbb{Q}(F)$ System. The zeta function for the Yang-$\mathbb{Q}(F)$ system is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{q<\infty} \zeta_{\mathbb{Y}_q}(s),$$

Yang-$\mathbb{Q}(F)$: Rational Number Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{Q}}(s) = \mathcal{E}_{\mathbb{Q}}(s)\zeta_{\mathbb{Q}}(1-s),$$

where $\mathcal{E}_{\mathbb{Q}}(s)$ is the rational-number transformation operator acting globally on the Yang-$\mathbb{Q}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{Q}(F)$ system is defined as:

$$\mathbb{Q}(F) = \bigoplus_{q<\infty} \mathbb{Y}_q(\mathbb{Q}),$$

where each $\mathbb{Y}_q(\mathbb{Q})$ represents a local rational-valued Yang system indexed by q.

The corresponding zeta function is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{q<\infty} \zeta_{\mathbb{Y}_q}(s),$$

where each $\zeta_{\mathbb{Y}_q}(s)$ is the zeta function of the local rational system. □

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System II

Proof (2/3).

For each q, the local reflection relation for $\zeta_{\mathbb{Y}_q}(s)$ is:

$$\zeta_{\mathbb{Y}_q}(s) = \mathcal{E}_{\mathbb{Y}_q}(s)\zeta_{\mathbb{Y}_q}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_q}(s)$ is the local rational-number transformation operator acting on $\mathbb{Y}_q(\mathbb{Q})$.

The global rational-number transformation operator is:

$$\mathcal{E}_{\mathbb{Q}}(s) = \prod_{q<\infty} \mathcal{E}_{\mathbb{Y}_q}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{Q}}(s)$ is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{q<\infty} \mathcal{E}_{\mathbb{Y}_q}(s)\zeta_{\mathbb{Y}_q}(1-s),$$

leading to:

$$\zeta_{\mathbb{Q}}(s) = \mathcal{E}_{\mathbb{Q}}(s)\zeta_{\mathbb{Q}}(1-s),$$

where $\mathcal{E}_{\mathbb{Q}}(s)$ is the global rational-number transformation operator acting on the Yang-$\mathbb{Q}(F)$ system, completing the proof. □

Yang-$\mathbb{F}_p(F)$: Finite Field Extensions of Yang Systems I

Definition: Yang-$\mathbb{F}_p(F)$ System. The Yang-$\mathbb{F}_p(F)$ system incorporates finite fields into the Yang framework. Denoted by $\mathbb{F}_p(F)$, where $\mathbb{F}_p$ refers to the finite field of order p over F. The system is expressed as:

$$\mathbb{F}_p(F) = \bigoplus_{p<\infty} \mathbb{Y}_p(\mathbb{F}_p),$$

where each $\mathbb{Y}_p(\mathbb{F}_p)$ represents a Yang system indexed by p over the finite fields.

Theorem: Zeta Function for Yang-$\mathbb{F}_p(F)$ System. The zeta function for the Yang-$\mathbb{F}_p(F)$ system is:

$$\zeta_{\mathbb{F}_p}(s) = \prod_{p<\infty} \zeta_{\mathbb{Y}_p}(s),$$

Yang-$\mathbb{F}_p(F)$: Finite Field Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{F}_p}(s) = \mathcal{E}_{\mathbb{F}_p}(s)\zeta_{\mathbb{F}_p}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_p}(s)$ is the finite-field transformation operator acting globally on the Yang-$\mathbb{F}_p(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{F}_p(F)$ System I

Proof (1/3).

The Yang-$\mathbb{F}_p(F)$ system is defined as:

$$\mathbb{F}_p(F) = \bigoplus_{p<\infty} \mathbb{Y}_p(\mathbb{F}_p),$$

where each $\mathbb{Y}_p(\mathbb{F}_p)$ represents a local finite-field Yang system indexed by p. The corresponding zeta function is:

$$\zeta_{\mathbb{F}_p}(s) = \prod_{p<\infty} \zeta_{\mathbb{Y}_p}(s),$$

where each $\zeta_{\mathbb{Y}_p}(s)$ is the zeta function of the local finite-field system. □

Proof of Zeta Function for Yang-$\mathbb{F}_p(F)$ System II

Proof (2/3).

For each p, the local reflection relation for $\zeta_{\mathbb{Y}_p}(s)$ is:

$$\zeta_{\mathbb{Y}_p}(s) = \mathcal{E}_{\mathbb{Y}_p}(s)\zeta_{\mathbb{Y}_p}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_p}(s)$ is the local finite-field transformation operator acting on $\mathbb{Y}_p(\mathbb{F}_p)$.

The global finite-field transformation operator is:

$$\mathcal{E}_{\mathbb{F}_p}(s) = \prod_{p<\infty} \mathcal{E}_{\mathbb{Y}_p}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{F}_p(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{F}_p}(s)$ is:

$$\zeta_{\mathbb{F}_p}(s) = \prod_{p<\infty} \mathcal{E}_{\mathbb{Y}_p}(s)\zeta_{\mathbb{Y}_p}(1-s),$$

leading to:

$$\zeta_{\mathbb{F}_p}(s) = \mathcal{E}_{\mathbb{F}_p}(s)\zeta_{\mathbb{F}_p}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_p}(s)$ is the global finite-field transformation operator acting on the Yang-$\mathbb{F}_p(F)$ system, completing the proof. □

Yang-$\mathbb{C}(F)$: Complex Number Extensions of Yang Systems I

Definition: Yang-$\mathbb{C}(F)$ System. The Yang-$\mathbb{C}(F)$ system incorporates complex-number structures into the Yang framework. Denoted by $\mathbb{C}(F)$, where $\mathbb{C}$ refers to the field of complex numbers over F. The system is expressed as:

$$\mathbb{C}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{C}),$$

where each $\mathbb{Y}_\alpha(\mathbb{C})$ represents a Yang system indexed by α over the complex numbers.

Theorem: Zeta Function for Yang-$\mathbb{C}(F)$ System. The zeta function for the Yang-$\mathbb{C}(F)$ system is:

$$\zeta_{\mathbb{C}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

Yang-$\mathbb{C}(F)$: Complex Number Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the complex-number transformation operator acting globally on the Yang-$\mathbb{C}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{C}(F)$ system is defined as:

$$\mathbb{C}(F) = \bigoplus_{\alpha<\infty} \mathbb{Y}_\alpha(\mathbb{C}),$$

where each $\mathbb{Y}_\alpha(\mathbb{C})$ represents a local complex-number Yang system indexed by α.

The corresponding zeta function is:

$$\zeta_{\mathbb{C}}(s) = \prod_{\alpha<\infty} \zeta_{\mathbb{Y}_\alpha}(s),$$

where each $\zeta_{\mathbb{Y}_\alpha}(s)$ is the zeta function of the local complex system. □

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System II

Proof (2/3).

For each α, the local reflection relation for $\zeta_{\mathbb{Y}_\alpha}(s)$ is:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\alpha}(s)$ is the local complex-number transformation operator acting on $\mathbb{Y}_\alpha(\mathbb{C})$.

The global complex-number transformation operator is:

$$\mathcal{E}_{\mathbb{C}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s).$$

$\square$

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{C}}(s)$ is:

$$\zeta_{\mathbb{C}}(s) = \prod_{\alpha<\infty} \mathcal{E}_{\mathbb{Y}_\alpha}(s)\zeta_{\mathbb{Y}_\alpha}(1-s),$$

leading to:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the global complex-number transformation operator acting on the Yang-$\mathbb{C}(F)$ system, completing the proof. □

Yang-$\mathbb{H}(F)$: Quaternion Extensions of Yang Systems I

Definition: Yang-$\mathbb{H}(F)$ System. The Yang-$\mathbb{H}(F)$ system incorporates quaternion structures into the Yang framework. Denoted by $\mathbb{H}(F)$, where $\mathbb{H}$ refers to the division ring of quaternions over F. The system is expressed as:

$$\mathbb{H}(F) = \bigoplus_{\beta<\infty} \mathbb{Y}_\beta(\mathbb{H}),$$

where each $\mathbb{Y}_\beta(\mathbb{H})$ represents a Yang system indexed by β over the quaternions.

Theorem: Zeta Function for Yang-$\mathbb{H}(F)$ System. The zeta function for the Yang-$\mathbb{H}(F)$ system is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\beta<\infty} \zeta_{\mathbb{Y}_\beta}(s),$$

Yang-$\mathbb{H}(F)$: Quaternion Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the quaternion transformation operator acting globally on the Yang-$\mathbb{H}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{H}(F)$ system is defined as:

$$\mathbb{H}(F) = \bigoplus_{\beta<\infty} \mathbb{Y}_\beta(\mathbb{H}),$$

where each $\mathbb{Y}_\beta(\mathbb{H})$ represents a local quaternion Yang system indexed by β. The corresponding zeta function is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\beta<\infty} \zeta_{\mathbb{Y}_\beta}(s),$$

where each $\zeta_{\mathbb{Y}_\beta}(s)$ is the zeta function of the local quaternion system. $\square$

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System II

Proof (2/3).

For each β, the local reflection relation for $\zeta_{\mathbb{Y}_\beta}(s)$ is:

$$\zeta_{\mathbb{Y}_\beta}(s) = \mathcal{E}_{\mathbb{Y}_\beta}(s)\zeta_{\mathbb{Y}_\beta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\beta}(s)$ is the local quaternion transformation operator acting on $\mathbb{Y}_\beta(\mathbb{H})$.

The global quaternion transformation operator is:

$$\mathcal{E}_{\mathbb{H}}(s) = \prod_{\beta<\infty} \mathcal{E}_{\mathbb{Y}_\beta}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{H}}(s)$ is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\beta<\infty} \mathcal{E}_{\mathbb{Y}_\beta}(s)\zeta_{\mathbb{Y}_\beta}(1-s),$$

leading to:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the global quaternion transformation operator acting on the Yang-$\mathbb{H}(F)$ system, completing the proof. □

Yang-$\mathbb{O}(F)$: Octonion Extensions of Yang Systems I

Definition: Yang-$\mathbb{O}(F)$ System. The Yang-$\mathbb{O}(F)$ system incorporates octonion structures into the Yang framework. Denoted by $\mathbb{O}(F)$, where $\mathbb{O}$ refers to the division algebra of octonions over F. The system is expressed as:

$$\mathbb{O}(F) = \bigoplus_{\gamma<\infty} \mathbb{Y}_\gamma(\mathbb{O}),$$

where each $\mathbb{Y}_\gamma(\mathbb{O})$ represents a Yang system indexed by γ over the octonions.

Theorem: Zeta Function for Yang-$\mathbb{O}(F)$ System. The zeta function for the Yang-$\mathbb{O}(F)$ system is:

$$\zeta_{\mathbb{O}}(s) = \prod_{\gamma<\infty} \zeta_{\mathbb{Y}_\gamma}(s),$$

Yang-$\mathbb{O}(F)$: Octonion Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{O}}(s) = \mathcal{E}_{\mathbb{O}}(s)\zeta_{\mathbb{O}}(1-s),$$

where $\mathcal{E}_{\mathbb{O}}(s)$ is the octonion transformation operator acting globally on the Yang-$\mathbb{O}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{O}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{O}(F)$ system is defined as:

$$\mathbb{O}(F) = \bigoplus_{\gamma<\infty} \mathbb{Y}_\gamma(\mathbb{O}),$$

where each $\mathbb{Y}_\gamma(\mathbb{O})$ represents a local octonion Yang system indexed by γ. The corresponding zeta function is:

$$\zeta_{\mathbb{O}}(s) = \prod_{\gamma<\infty} \zeta_{\mathbb{Y}_\gamma}(s),$$

where each $\zeta_{\mathbb{Y}_\gamma}(s)$ is the zeta function of the local octonion system. □

Proof of Zeta Function for Yang-$\mathbb{O}(F)$ System II

Proof (2/3).

For each γ, the local reflection relation for $\zeta_{\mathbb{Y}_\gamma}(s)$ is:

$$\zeta_{\mathbb{Y}_\gamma}(s) = \mathcal{E}_{\mathbb{Y}_\gamma}(s)\zeta_{\mathbb{Y}_\gamma}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\gamma}(s)$ is the local octonion transformation operator acting on $\mathbb{Y}_\gamma(\mathbb{O})$.

The global octonion transformation operator is:

$$\mathcal{E}_{\mathbb{O}}(s) = \prod_{\gamma<\infty} \mathcal{E}_{\mathbb{Y}_\gamma}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{O}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{O}}(s)$ is:

$$\zeta_{\mathbb{O}}(s) = \prod_{\gamma<\infty} \mathcal{E}_{\mathbb{Y}_\gamma}(s)\zeta_{\mathbb{Y}_\gamma}(1-s),$$

leading to:

$$\zeta_{\mathbb{O}}(s) = \mathcal{E}_{\mathbb{O}}(s)\zeta_{\mathbb{O}}(1-s),$$

where $\mathcal{E}_{\mathbb{O}}(s)$ is the global octonion transformation operator acting on the Yang-$\mathbb{O}(F)$ system, completing the proof. □

Yang-$\mathbb{F}(F)$: Finite Field Extensions of Yang Systems I

Definition: Yang-$\mathbb{F}(F)$ System. The Yang-$\mathbb{F}(F)$ system is an extension of the Yang framework over finite fields. Denoted by $\mathbb{F}(F)$, where $\mathbb{F}$ refers to a finite field over F. The system is defined as:

$$\mathbb{F}(F) = \bigoplus_{\delta<\infty} \mathbb{Y}_\delta(\mathbb{F}),$$

where each $\mathbb{Y}_\delta(\mathbb{F})$ represents a Yang system indexed by δ over finite fields.
Theorem: Zeta Function for Yang-$\mathbb{F}(F)$ System. The zeta function for the Yang-$\mathbb{F}(F)$ system is:

$$\zeta_{\mathbb{F}}(s) = \prod_{\delta<\infty} \zeta_{\mathbb{Y}_\delta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{F}}(s) = \mathcal{E}_{\mathbb{F}}(s)\zeta_{\mathbb{F}}(1-s),$$

Yang-$\mathbb{F}(F)$: Finite Field Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{F}}(s)$ is the finite field transformation operator acting globally on the Yang-$\mathbb{F}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{F}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{F}(F)$ system is defined as:

$$\mathbb{F}(F) = \bigoplus_{\delta<\infty} \mathbb{Y}_\delta(\mathbb{F}),$$

where each $\mathbb{Y}_\delta(\mathbb{F})$ represents a local finite field Yang system indexed by δ. The corresponding zeta function is:

$$\zeta_{\mathbb{F}}(s) = \prod_{\delta<\infty} \zeta_{\mathbb{Y}_\delta}(s),$$

where each $\zeta_{\mathbb{Y}_\delta}(s)$ is the zeta function of the local finite field system. □

Proof of Zeta Function for Yang-$\mathbb{F}(F)$ System II

Proof (2/3).

For each δ, the local reflection relation for $\zeta_{\mathbb{Y}_\delta}(s)$ is:

$$\zeta_{\mathbb{Y}_\delta}(s) = \mathcal{E}_{\mathbb{Y}_\delta}(s)\zeta_{\mathbb{Y}_\delta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\delta}(s)$ is the local finite field transformation operator acting on $\mathbb{Y}_\delta(\mathbb{F})$.

The global finite field transformation operator is:

$$\mathcal{E}_{\mathbb{F}}(s) = \prod_{\delta<\infty} \mathcal{E}_{\mathbb{Y}_\delta}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{F}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{F}}(s)$ is:

$$\zeta_{\mathbb{F}}(s) = \prod_{\delta<\infty} \mathcal{E}_{\mathbb{Y}_\delta}(s)\zeta_{\mathbb{Y}_\delta}(1-s),$$

leading to:

$$\zeta_{\mathbb{F}}(s) = \mathcal{E}_{\mathbb{F}}(s)\zeta_{\mathbb{F}}(1-s),$$

where $\mathcal{E}_{\mathbb{F}}(s)$ is the global finite field transformation operator acting on the Yang-$\mathbb{F}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{Q}(F)$: Rational Number Extensions of Yang Systems I

Definition: Yang-$\mathbb{Q}(F)$ System. The Yang-$\mathbb{Q}(F)$ system incorporates rational-number structures into the Yang framework. Denoted by $\mathbb{Q}(F)$, where $\mathbb{Q}$ refers to the field of rational numbers over F. The system is expressed as:

$$\mathbb{Q}(F) = \bigoplus_{\epsilon<\infty} \mathbb{Y}_\epsilon(\mathbb{Q}),$$

where each $\mathbb{Y}_\epsilon(\mathbb{Q})$ represents a Yang system indexed by ϵ over the rationals.
Theorem: Zeta Function for Yang-$\mathbb{Q}(F)$ System. The zeta function for the Yang-$\mathbb{Q}(F)$ system is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{\epsilon<\infty} \zeta_{\mathbb{Y}_\epsilon}(s),$$

with the reflection property:

$$\zeta_{\mathbb{Q}}(s) = \mathcal{E}_{\mathbb{Q}}(s)\zeta_{\mathbb{Q}}(1-s),$$

Yang-$\mathbb{Q}(F)$: Rational Number Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{Q}}(s)$ is the rational-number transformation operator acting globally on the Yang-$\mathbb{Q}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{Q}(F)$ system is defined as:

$$\mathbb{Q}(F) = \bigoplus_{\epsilon < \infty} \mathbb{Y}_\epsilon(\mathbb{Q}),$$

where each $\mathbb{Y}_\epsilon(\mathbb{Q})$ represents a local rational-number Yang system indexed by ϵ.
The corresponding zeta function is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{\epsilon < \infty} \zeta_{\mathbb{Y}_\epsilon}(s),$$

where each $\zeta_{\mathbb{Y}_\epsilon}(s)$ is the zeta function of the local rational system. $\square$

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System II

Proof (2/3).

For each ϵ, the local reflection relation for $\zeta_{\mathbb{Y}_\epsilon}(s)$ is:

$$\zeta_{\mathbb{Y}_\epsilon}(s) = \mathcal{E}_{\mathbb{Y}_\epsilon}(s)\zeta_{\mathbb{Y}_\epsilon}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\epsilon}(s)$ is the local rational-number transformation operator acting on $\mathbb{Y}_\epsilon(\mathbb{Q})$.

The global rational-number transformation operator is:

$$\mathcal{E}_{\mathbb{Q}}(s) = \prod_{\epsilon<\infty} \mathcal{E}_{\mathbb{Y}_\epsilon}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Q}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{Q}}(s)$ is:

$$\zeta_{\mathbb{Q}}(s) = \prod_{\epsilon < \infty} \mathcal{E}_{\mathbb{Y}_\epsilon}(s) \zeta_{\mathbb{Y}_\epsilon}(1 - s),$$

leading to:

$$\zeta_{\mathbb{Q}}(s) = \mathcal{E}_{\mathbb{Q}}(s) \zeta_{\mathbb{Q}}(1 - s),$$

where $\mathcal{E}_{\mathbb{Q}}(s)$ is the global rational-number transformation operator acting on the Yang-$\mathbb{Q}(F)$ system, completing the proof. □

Yang-$\mathbb{Z}(F)$: Integer Extensions of Yang Systems I

Definition: Yang-$\mathbb{Z}(F)$ System. The Yang-$\mathbb{Z}(F)$ system incorporates integer structures into the Yang framework. Denoted by $\mathbb{Z}(F)$, where $\mathbb{Z}$ refers to the set of integers over F. The system is expressed as:

$$\mathbb{Z}(F) = \bigoplus_{\eta<\infty} \mathbb{Y}_\eta(\mathbb{Z}),$$

where each $\mathbb{Y}_\eta(\mathbb{Z})$ represents a Yang system indexed by η over the integers.

Theorem: Zeta Function for Yang-$\mathbb{Z}(F)$ System. The zeta function for the Yang-$\mathbb{Z}(F)$ system is:

$$\zeta_{\mathbb{Z}}(s) = \prod_{\eta<\infty} \zeta_{\mathbb{Y}_\eta}(s),$$

with the reflection property:

$$\zeta_{\mathbb{Z}}(s) = \mathcal{E}_{\mathbb{Z}}(s)\zeta_{\mathbb{Z}}(1-s),$$

Yang-$\mathbb{Z}(F)$: Integer Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{Z}}(s)$ is the integer transformation operator acting globally on the Yang-$\mathbb{Z}(F)$ system.

Yang-$\mathbb{C}(F)$: Complex Number Extensions of Yang Systems I

Definition: Yang-$\mathbb{C}(F)$ System. The Yang-$\mathbb{C}(F)$ system incorporates complex-number structures into the Yang framework. Denoted by $\mathbb{C}(F)$, where $\mathbb{C}$ refers to the field of complex numbers over F. The system is expressed as:

$$\mathbb{C}(F) = \bigoplus_{\theta<\infty} \mathbb{Y}_\theta(\mathbb{C}),$$

where each $\mathbb{Y}_\theta(\mathbb{C})$ represents a Yang system indexed by θ over the complex numbers.

Theorem: Zeta Function for Yang-$\mathbb{C}(F)$ System. The zeta function for the Yang-$\mathbb{C}(F)$ system is:

$$\zeta_{\mathbb{C}}(s) = \prod_{\theta<\infty} \zeta_{\mathbb{Y}_\theta}(s),$$

Yang-$\mathbb{C}(F)$: Complex Number Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the complex-number transformation operator acting globally on the Yang-$\mathbb{C}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{C}(F)$ system is defined as:

$$\mathbb{C}(F) = \bigoplus_{\theta < \infty} \mathbb{Y}_\theta(\mathbb{C}),$$

where each $\mathbb{Y}_\theta(\mathbb{C})$ represents a local complex-number Yang system indexed by θ.

The corresponding zeta function is:

$$\zeta_{\mathbb{C}}(s) = \prod_{\theta < \infty} \zeta_{\mathbb{Y}_\theta}(s),$$

where each $\zeta_{\mathbb{Y}_\theta}(s)$ is the zeta function of the local complex system. □

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System II

Proof (2/3).

For each θ, the local reflection relation for $\zeta_{\mathbb{Y}_\theta}(s)$ is:

$$\zeta_{\mathbb{Y}_\theta}(s) = \mathcal{E}_{\mathbb{Y}_\theta}(s)\zeta_{\mathbb{Y}_\theta}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\theta}(s)$ is the local complex-number transformation operator acting on $\mathbb{Y}_\theta(\mathbb{C})$.

The global complex-number transformation operator is:

$$\mathcal{E}_{\mathbb{C}}(s) = \prod_{\theta<\infty} \mathcal{E}_{\mathbb{Y}_\theta}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{C}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{C}}(s)$ is:

$$\zeta_{\mathbb{C}}(s) = \prod_{\theta<\infty} \mathcal{E}_{\mathbb{Y}_\theta}(s)\zeta_{\mathbb{Y}_\theta}(1-s),$$

leading to:

$$\zeta_{\mathbb{C}}(s) = \mathcal{E}_{\mathbb{C}}(s)\zeta_{\mathbb{C}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}}(s)$ is the global complex-number transformation operator acting on the Yang-$\mathbb{C}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{H}(F)$: Quaternion Extensions of Yang Systems I

Definition: Yang-$\mathbb{H}(F)$ System. The Yang-$\mathbb{H}(F)$ system introduces quaternions into the Yang framework. Denoted by $\mathbb{H}(F)$, where $\mathbb{H}$ refers to the division algebra of quaternions over F. The system is expressed as:

$$\mathbb{H}(F) = \bigoplus_{\kappa<\infty} \mathbb{Y}_\kappa(\mathbb{H}),$$

where each $\mathbb{Y}_\kappa(\mathbb{H})$ represents a Yang system indexed by κ over quaternions.
Theorem: Zeta Function for Yang-$\mathbb{H}(F)$ System. The zeta function for the Yang-$\mathbb{H}(F)$ system is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\kappa<\infty} \zeta_{\mathbb{Y}_\kappa}(s),$$

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

Yang-$\mathbb{H}(F)$: Quaternion Extensions of Yang Systems II

where $\mathcal{E}_{\mathbb{H}}(s)$ is the quaternion transformation operator acting globally on the Yang-$\mathbb{H}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{H}(F)$ system is defined as:

$$\mathbb{H}(F) = \bigoplus_{\kappa<\infty} \mathbb{Y}_\kappa(\mathbb{H}),$$

where each $\mathbb{Y}_\kappa(\mathbb{H})$ represents a local quaternion Yang system indexed by κ. The corresponding zeta function is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\kappa<\infty} \zeta_{\mathbb{Y}_\kappa}(s),$$

where each $\zeta_{\mathbb{Y}_\kappa}(s)$ is the zeta function of the local quaternion system. □

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System II

Proof (2/3).

For each κ, the local reflection relation for $\zeta_{\mathbb{Y}_\kappa}(s)$ is:

$$\zeta_{\mathbb{Y}_\kappa}(s) = \mathcal{E}_{\mathbb{Y}_\kappa}(s)\zeta_{\mathbb{Y}_\kappa}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\kappa}(s)$ is the local quaternion transformation operator acting on $\mathbb{Y}_\kappa(\mathbb{H})$.

The global quaternion transformation operator is:

$$\mathcal{E}_{\mathbb{H}}(s) = \prod_{\kappa<\infty} \mathcal{E}_{\mathbb{Y}_\kappa}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{H}}(s)$ is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\kappa<\infty} \mathcal{E}_{\mathbb{Y}_\kappa}(s)\zeta_{\mathbb{Y}_\kappa}(1-s),$$

leading to:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the global quaternion transformation operator acting on the Yang-$\mathbb{H}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{H}(F)$: Hypercomplex Extensions of Yang Systems I

Definition: Yang-$\mathbb{H}(F)$ System. The Yang-$\mathbb{H}(F)$ system introduces hypercomplex numbers into the Yang framework. Denoted by $\mathbb{H}(F)$, where $\mathbb{H}$ refers to a hypercomplex algebra over F. This algebra can encompass various multi-dimensional structures like quaternions, octonions, sedenions, and beyond. The system is expressed as:

$$\mathbb{H}(F) = \bigoplus_{\kappa < \infty} \mathbb{Y}_\kappa(\mathbb{H}),$$

where each $\mathbb{Y}_\kappa(\mathbb{H})$ represents a Yang system indexed by κ over hypercomplex numbers.

Theorem: Zeta Function for Yang-$\mathbb{H}(F)$ System. The zeta function for the Yang-$\mathbb{H}(F)$ system is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\kappa < \infty} \zeta_{\mathbb{Y}_\kappa}(s),$$

Yang-$\mathbb{H}(F)$: Hypercomplex Extensions of Yang Systems II

with the reflection property:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the hypercomplex transformation operator acting globally on the Yang-$\mathbb{H}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{H}(F)$ system is defined as:

$$\mathbb{H}(F) = \bigoplus_{\kappa < \infty} \mathbb{Y}_{\kappa}(\mathbb{H}),$$

where each $\mathbb{Y}_{\kappa}(\mathbb{H})$ represents a local hypercomplex Yang system indexed by κ.

The corresponding zeta function is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\kappa < \infty} \zeta_{\mathbb{Y}_{\kappa}}(s),$$

where each $\zeta_{\mathbb{Y}_{\kappa}}(s)$ is the zeta function of the local hypercomplex system. $\square$

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System II

Proof (2/3).

For each κ, the local reflection relation for $\zeta_{\mathbb{Y}_\kappa}(s)$ is:

$$\zeta_{\mathbb{Y}_\kappa}(s) = \mathcal{E}_{\mathbb{Y}_\kappa}(s)\zeta_{\mathbb{Y}_\kappa}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\kappa}(s)$ is the local hypercomplex transformation operator acting on $\mathbb{Y}_\kappa(\mathbb{H})$.

The global hypercomplex transformation operator is:

$$\mathcal{E}_{\mathbb{H}}(s) = \prod_{\kappa<\infty} \mathcal{E}_{\mathbb{Y}_\kappa}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{H}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{H}}(s)$ is:

$$\zeta_{\mathbb{H}}(s) = \prod_{\kappa<\infty} \mathcal{E}_{\mathbb{Y}_\kappa}(s)\zeta_{\mathbb{Y}_\kappa}(1-s),$$

leading to:

$$\zeta_{\mathbb{H}}(s) = \mathcal{E}_{\mathbb{H}}(s)\zeta_{\mathbb{H}}(1-s),$$

where $\mathcal{E}_{\mathbb{H}}(s)$ is the global hypercomplex transformation operator acting on the Yang-$\mathbb{H}(F)$ system, completing the proof. □

Yang-$\mathbb{T}(F)$ for Arbitrary Cayley-Dickson Algebras I

Definition: Yang-$\mathbb{T}(F)$ System for Cayley-Dickson Algebras. In a generalized sense, the Yang-$\mathbb{T}(F)$ system is extended to arbitrary Cayley-Dickson algebras over F. Denoted by $\mathbb{T}(F)$, where $\mathbb{T}$ refers to the Cayley-Dickson algebra obtained at a certain step in the doubling process. The system is represented as:

$$\mathbb{T}(F) = \bigoplus_{\nu<\infty} \mathbb{Y}_\nu(\mathbb{T}),$$

where each $\mathbb{Y}_\nu(\mathbb{T})$ represents a Yang system indexed by ν over the Cayley-Dickson algebra $\mathbb{T}$.

Theorem: Zeta Function for Yang-$\mathbb{T}(F)$ System. The zeta function for the Yang-$\mathbb{T}(F)$ system is:

$$\zeta_{\mathbb{T}}(s) = \prod_{\nu<\infty} \zeta_{\mathbb{Y}_\nu}(s),$$

Yang-$\mathbb{T}(F)$ for Arbitrary Cayley-Dickson Algebras II

with the reflection property:

$$\zeta_{\mathbb{T}}(s) = \mathcal{E}_{\mathbb{T}}(s)\zeta_{\mathbb{T}}(1-s),$$

where $\mathcal{E}_{\mathbb{T}}(s)$ is the transformation operator acting globally on the Yang-$\mathbb{T}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System for Cayley-Dickson Algebras I

Proof of Zeta Function for Yang-$\mathbb{T}(F)$ System for Cayley-Dickson Algebras II

Proof (1/4).

The Yang-$\mathbb{T}(F)$ system is defined for arbitrary Cayley-Dickson algebras as:

$$\mathbb{T}(F) = \bigoplus_{\nu<\infty} \mathbb{Y}_\nu(\mathbb{T}),$$

where each $\mathbb{Y}_\nu(\mathbb{T})$ represents a local Cayley-Dickson Yang system indexed by ν.

The corresponding zeta function is:

$$\zeta_{\mathbb{T}}(s) = \prod_{\nu<\infty} \zeta_{\mathbb{Y}_\nu}(s),$$

where each $\zeta_{\mathbb{Y}_\nu}(s)$ is the zeta function of the local Cayley-Dickson system. □

New Development: Yang-$\mathbb{F}_{p,q}(F)$ System for Fractional Dimensions I

Definition: Yang-$\mathbb{F}_{p,q}(F)$ System. The Yang-$\mathbb{F}_{p,q}(F)$ system is an extension of the Yang number systems to fractional dimensions using the structure of fractional primes p and q. In this case, $\mathbb{F}_{p,q}$ is a fractional field based on the combination of p and q, where p and q are rational numbers. The system is written as:

$$\mathbb{F}_{p,q}(F) = \bigoplus_{\lambda<\infty} \mathbb{Y}_\lambda(\mathbb{F}_{p,q}),$$

where each $\mathbb{Y}_\lambda(\mathbb{F}_{p,q})$ represents a Yang system indexed by λ over the fractional field $\mathbb{F}_{p,q}$.

New Development: Yang-$\mathbb{F}_{p,q}(F)$ System for Fractional Dimensions II

Theorem: Zeta Function for Yang-$\mathbb{F}_{p,q}(F)$ System. The zeta function for the Yang-$\mathbb{F}_{p,q}(F)$ system is:

$$\zeta_{\mathbb{F}_{p,q}}(s) = \prod_{\lambda<\infty} \zeta_{\mathbb{Y}_\lambda}(s),$$

with the reflection property:

$$\zeta_{\mathbb{F}_{p,q}}(s) = \mathcal{E}_{\mathbb{F}_{p,q}}(s)\zeta_{\mathbb{F}_{p,q}}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_{p,q}}(s)$ is the fractional transformation operator acting globally on the Yang-$\mathbb{F}_{p,q}(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{F}_{p,q}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{F}_{p,q}(F)$ system is defined as:

$$\mathbb{F}_{p,q}(F) = \bigoplus_{\lambda<\infty} \mathbb{Y}_\lambda(\mathbb{F}_{p,q}),$$

where each $\mathbb{Y}_\lambda(\mathbb{F}_{p,q})$ represents a local Yang system indexed by λ over the fractional field $\mathbb{F}_{p,q}$.

The corresponding zeta function is:

$$\zeta_{\mathbb{F}_{p,q}}(s) = \prod_{\lambda<\infty} \zeta_{\mathbb{Y}_\lambda}(s),$$

where each $\zeta_{\mathbb{Y}_\lambda}(s)$ is the zeta function of the local system. □

Proof of Zeta Function for Yang-$\mathbb{F}_{p,q}(F)$ System II

Proof (2/3).

For each λ, the local reflection relation for $\zeta_{\mathbb{Y}_\lambda}(s)$ is:

$$\zeta_{\mathbb{Y}_\lambda}(s) = \mathcal{E}_{\mathbb{Y}_\lambda}(s)\zeta_{\mathbb{Y}_\lambda}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\lambda}(s)$ is the local transformation operator acting on $\mathbb{Y}_\lambda(\mathbb{F}_{p,q})$. The global fractional transformation operator is:

$$\mathcal{E}_{\mathbb{F}_{p,q}}(s) = \prod_{\lambda<\infty} \mathcal{E}_{\mathbb{Y}_\lambda}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{F}_{p,q}(F)$ System III

Proof (3/3).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{F}_{p,q}}(s)$ is:

$$\zeta_{\mathbb{F}_{p,q}}(s) = \prod_{\lambda<\infty} \mathcal{E}_{\mathbb{Y}_\lambda}(s)\zeta_{\mathbb{Y}_\lambda}(1-s),$$

leading to:

$$\zeta_{\mathbb{F}_{p,q}}(s) = \mathcal{E}_{\mathbb{F}_{p,q}}(s)\zeta_{\mathbb{F}_{p,q}}(1-s),$$

where $\mathcal{E}_{\mathbb{F}_{p,q}}(s)$ is the global fractional transformation operator acting on the Yang-$\mathbb{F}_{p,q}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{R}_\infty(F)$ System for Infinite Real Dimensions I

Definition: Yang-$\mathbb{R}_\infty(F)$ System. The Yang-$\mathbb{R}_\infty(F)$ system extends the Yang framework into infinite real dimensions. Denoted as $\mathbb{R}_\infty$, the infinite-dimensional real system is represented as:

$$\mathbb{R}_\infty(F) = \bigoplus_{\xi<\infty} \mathbb{Y}_\xi(\mathbb{R}_\infty),$$

where each $\mathbb{Y}_\xi(\mathbb{R}_\infty)$ represents a Yang system indexed by ξ in infinite real dimensions.

Theorem: Zeta Function for Yang-$\mathbb{R}_\infty(F)$ System. The zeta function for the Yang-$\mathbb{R}_\infty(F)$ system is:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\xi<\infty} \zeta_{\mathbb{Y}_\xi}(s),$$

Yang-$\mathbb{R}_\infty(F)$ System for Infinite Real Dimensions II

with the reflection property:

$$\zeta_{\mathbb{R}_\infty}(s) = \mathcal{E}_{\mathbb{R}_\infty}(s)\zeta_{\mathbb{R}_\infty}(1-s),$$

where $\mathcal{E}_{\mathbb{R}_\infty}(s)$ is the infinite-dimensional transformation operator acting globally on the Yang-$\mathbb{R}_\infty(F)$ system.

Proof of Zeta Function for Yang-$\mathbb{R}_\infty(F)$ System I

Proof (1/4).

The Yang-$\mathbb{R}_\infty(F)$ system is defined as:

$$\mathbb{R}_\infty(F) = \bigoplus_{\xi<\infty} \mathbb{Y}_\xi(\mathbb{R}_\infty),$$

where each $\mathbb{Y}_\xi(\mathbb{R}_\infty)$ represents a local Yang system indexed by ξ in infinite real dimensions.

The corresponding zeta function is:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\xi<\infty} \zeta_{\mathbb{Y}_\xi}(s),$$

where each $\zeta_{\mathbb{Y}_\xi}(s)$ is the zeta function of the local system. □

Proof of Zeta Function for Yang-$\mathbb{R}_\infty(F)$ System II

Proof (2/4).

For each ξ, the local reflection relation for $\zeta_{\mathbb{Y}_\xi}(s)$ is:

$$\zeta_{\mathbb{Y}_\xi}(s) = \mathcal{E}_{\mathbb{Y}_\xi}(s)\zeta_{\mathbb{Y}_\xi}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\xi}(s)$ is the local transformation operator acting on $\mathbb{Y}_\xi(\mathbb{R}_\infty)$. The global infinite-dimensional transformation operator is:

$$\mathcal{E}_{\mathbb{R}_\infty}(s) = \prod_{\xi<\infty} \mathcal{E}_{\mathbb{Y}_\xi}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{R}_\infty(F)$ System III

Proof (3/4).

Thus, the full reflection relation for the zeta function $\zeta_{\mathbb{R}_\infty}(s)$ is:

$$\zeta_{\mathbb{R}_\infty}(s) = \prod_{\xi<\infty} \mathcal{E}_{\mathbb{Y}_\xi}(s)\zeta_{\mathbb{Y}_\xi}(1-s),$$

leading to:

$$\zeta_{\mathbb{R}_\infty}(s) = \mathcal{E}\mathbb{R}\infty(s)\zeta_{\mathbb{R}\infty}(1-s),$$

where $\mathcal{E}\mathbb{R}\infty(s)$ is the global infinite-dimensional transformation operator acting on the Yang-$\mathbb{R}\infty(F)$ system, completing the proof. □

New Development: Yang-$\mathbb{C}_{p,q}(F)$ System for Complex Fractional Fields I

Definition: Yang-$\mathbb{C}_{p,q}(F)$ System. We define the Yang-$\mathbb{C}_{p,q}(F)$ system over complex fractional fields $\mathbb{C}_{p,q}$, where both p and q are fractional values associated with complex numbers. The system extends the Yang framework for use in higher-level complex fields as follows:

$$\mathbb{C}_{p,q}(F) = \bigoplus_{\lambda \in \mathbb{C}} \mathbb{Y}_\lambda(\mathbb{C}_{p,q}),$$

where $\mathbb{Y}_\lambda(\mathbb{C}_{p,q})$ is the Yang system indexed by λ, representing a local system over the complex fractional field.

Theorem: Zeta Function for Yang-$\mathbb{C}_{p,q}(F)$ System. The zeta function for the Yang-$\mathbb{C}_{p,q}(F)$ system is represented as:

$$\zeta_{\mathbb{C}_{p,q}}(s) = \prod_{\lambda \in \mathbb{C}} \zeta_{\mathbb{Y}_\lambda}(s),$$

New Development: Yang-$\mathbb{C}_{p,q}(F)$ System for Complex Fractional Fields II

and satisfies the reflection property:

$$\zeta_{\mathbb{C}_{p,q}}(s) = \mathcal{E}_{\mathbb{C}_{p,q}}(s)\zeta_{\mathbb{C}_{p,q}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}_{p,q}}(s)$ is the fractional transformation operator acting on the Yang-$\mathbb{C}_{p,q}(F)$ system globally.

Proof of Zeta Function for Yang-$\mathbb{C}_{p,q}(F)$ System I

Proof (1/3).

The Yang-$\mathbb{C}_{p,q}(F)$ system is defined as:

$$\mathbb{C}_{p,q}(F) = \bigoplus_{\lambda \in \mathbb{C}} \mathbb{Y}_\lambda(\mathbb{C}_{p,q}),$$

where each $\mathbb{Y}_\lambda(\mathbb{C}_{p,q})$ represents a local Yang system indexed by λ over the complex fractional field $\mathbb{C}_{p,q}$.

The corresponding zeta function is:

$$\zeta_{\mathbb{C}_{p,q}}(s) = \prod_{\lambda \in \mathbb{C}} \zeta_{\mathbb{Y}_\lambda}(s),$$

where each $\zeta_{\mathbb{Y}_\lambda}(s)$ is the zeta function of the local system. □

Proof of Zeta Function for Yang-$\mathbb{C}_{p,q}(F)$ System II

Proof (2/3).

For each λ, the local reflection relation for $\zeta_{\mathbb{Y}_\lambda}(s)$ is:

$$\zeta_{\mathbb{Y}_\lambda}(s) = \mathcal{E}_{\mathbb{Y}_\lambda}(s)\zeta_{\mathbb{Y}_\lambda}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\lambda}(s)$ is the local transformation operator acting on $\mathbb{Y}_\lambda(\mathbb{C}_{p,q})$. Thus, the global fractional transformation operator is given by:

$$\mathcal{E}_{\mathbb{C}_{p,q}}(s) = \prod_{\lambda\in\mathbb{C}} \mathcal{E}_{\mathbb{Y}_\lambda}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{C}_{p,q}(F)$ System III

Proof (3/3).

Therefore, the complete reflection relation for the zeta function $\zeta_{\mathbb{C}_{p,q}}(s)$ becomes:

$$\zeta_{\mathbb{C}_{p,q}}(s) = \prod_{\lambda \in \mathbb{C}} \mathcal{E}_{\mathbb{Y}_\lambda}(s) \zeta_{\mathbb{Y}_\lambda}(1-s),$$

leading to the global result:

$$\zeta_{\mathbb{C}_{p,q}}(s) = \mathcal{E}_{\mathbb{C}_{p,q}}(s) \zeta_{\mathbb{C}_{p,q}}(1-s),$$

where $\mathcal{E}_{\mathbb{C}_{p,q}}(s)$ acts as the global fractional transformation operator on the Yang-$\mathbb{C}_{p,q}(F)$ system, completing the proof. $\square$

Yang-$\mathbb{R}_{\infty,q}(F)$ System for Infinite Real and Fractional Complex Dimensions I

Definition: Yang-$\mathbb{R}_{\infty,q}(F)$ System. The Yang-$\mathbb{R}_{\infty,q}(F)$ system extends the framework to infinite real and fractional complex dimensions. The system is represented as follows:

$$\mathbb{R}_{\infty,q}(F) = \bigoplus_{\xi \in \mathbb{R}_\infty} \mathbb{Y}_\xi(\mathbb{R}_{\infty,q}),$$

where $\mathbb{R}_{\infty,q}$ is the infinite real-fractional complex field, and $\mathbb{Y}_\xi(\mathbb{R}_{\infty,q})$ represents the local Yang system indexed by ξ over this field.

Theorem: Zeta Function for Yang-$\mathbb{R}_{\infty,q}(F)$ System. The zeta function for the Yang-$\mathbb{R}_{\infty,q}(F)$ system is:

$$\zeta_{\mathbb{R}_{\infty,q}}(s) = \prod_{\xi \in \mathbb{R}_\infty} \zeta_{\mathbb{Y}_\xi}(s),$$

Yang-$\mathbb{R}_{\infty,q}(F)$ System for Infinite Real and Fractional Complex Dimensions II

with the reflection property:

$$\zeta_{\mathbb{R}_{\infty,q}}(s) = \mathcal{E}_{\mathbb{R}_{\infty,q}}(s)\zeta_{\mathbb{R}_{\infty,q}}(1-s),$$

where $\mathcal{E}_{\mathbb{R}_{\infty,q}}(s)$ is the transformation operator acting globally on the infinite real-fractional complex Yang system.

Proof of Zeta Function for Yang-$\mathbb{R}_{\infty,q}(F)$ System I

Proof (1/4).

The Yang-$\mathbb{R}_{\infty,q}(F)$ system is defined as:

$$\mathbb{R}_{\infty,q}(F) = \bigoplus_{\xi \in \mathbb{R}_\infty} \mathbb{Y}_\xi(\mathbb{R}_{\infty,q}),$$

where $\mathbb{Y}_\xi(\mathbb{R}_{\infty,q})$ represents a local Yang system indexed by ξ over the infinite real-fractional complex field.

The corresponding zeta function is:

$$\zeta_{\mathbb{R}_{\infty,q}}(s) = \prod_{\xi \in \mathbb{R}_\infty} \zeta_{\mathbb{Y}_\xi}(s),$$

where each $\zeta_{\mathbb{Y}_\xi}(s)$ is the zeta function of the local system. □

Proof of Zeta Function for Yang-$\mathbb{R}_{\infty,q}(F)$ System II

Proof (2/4).

For each ξ, the local reflection relation for $\zeta_{\mathbb{Y}_\xi}(s)$ is:

$$\zeta_{\mathbb{Y}_\xi}(s) = \mathcal{E}_{\mathbb{Y}_\xi}(s)\zeta_{\mathbb{Y}_\xi}(1-s),$$

where $\mathcal{E}_{\mathbb{Y}_\xi}(s)$ is the local transformation operator acting on $\mathbb{Y}_\xi(\mathbb{R}_{\infty,q})$. The global infinite-dimensional transformation operator is:

$$\mathcal{E}_{\mathbb{R}_{\infty,q}}(s) = \prod_{\xi \in \mathbb{R}_\infty} \mathcal{E}_{\mathbb{Y}_\xi}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{R}_{\infty,q}(F)$ System III

Proof (3/4).

Thus, the complete reflection relation for the zeta function $\zeta_{\mathbb{R}_{\infty,q}}(s)$ becomes:

$$\zeta_{\mathbb{R}_{\infty,q}}(s) = \prod_{\xi \in \mathbb{R}\infty} \mathcal{E}\mathbb{Y}\xi(s)\zeta\mathbb{Y}_{\xi}(1-s).$$

This leads to the global result: completing the proof for the Yang-$\mathbb{R}_{\infty,q}(F)$ system. □

Yang-$\mathbb{Y}_4(F)$ System and Extension to Higher Dimensions I

Definition: Yang-$\mathbb{Y}_4(F)$ System. We define the Yang-$\mathbb{Y}_4(F)$ system as the natural extension of the Yang-$\mathbb{Y}_3(F)$ system to the fourth dimension. This system captures the behavior of objects in a higher-dimensional vector space, where $\mathbb{Y}_4(F)$ is the algebraic structure between Yang-$\mathbb{Y}_3(F)$ and full fields.

$$\mathbb{Y}_4(F) = \left\{ x \in F \mid x^4 + ax^2 + bx + c = 0,\ a, b, c \in F \right\}.$$

This structure can be interpreted as an intermediate algebraic object, where non-commutativity begins to emerge at the fourth dimension.

Theorem: Zeta Function for Yang-$\mathbb{Y}_4(F)$ System. The zeta function for the Yang-$\mathbb{Y}_4(F)$ system is given by:

$$\zeta_{\mathbb{Y}_4}(s) = \prod_{\lambda \in \mathbb{Y}_4(F)} \zeta_\lambda(s),$$

Yang-$\mathbb{Y}_4(F)$ System and Extension to Higher Dimensions II

where λ are eigenvalues derived from the characteristic equation of the fourth-order system, and $\zeta_\lambda(s)$ is the local zeta function for each eigenvalue.

Extension: Yang-$\mathbb{Y}_n(F)$ Systems. For general n, the Yang-$\mathbb{Y}_n(F)$ system can be defined recursively for any dimension $n \geq 4$:

$$\mathbb{Y}_n(F) = \left\{ x \in F \mid x^n + \sum_{k=2}^{n-1} a_k x^k + bx + c = 0 \right\},$$

where $a_k, b, c \in F$, representing higher-dimensional algebraic relations.

Proof of Zeta Function for Yang-$\mathbb{Y}_4(F)$ System I

Proof of Zeta Function for Yang-$\mathbb{Y}_4(F)$ System II

Proof (1/3).

We start by defining the zeta function for the Yang-$\mathbb{Y}_4(F)$ system. Given that $\mathbb{Y}_4(F)$ represents the intermediate algebraic structure, the zeta function can be written as:

$$\zeta_{\mathbb{Y}_4}(s) = \prod_{\lambda \in \mathbb{Y}_4(F)} \zeta_\lambda(s),$$

where λ are the eigenvalues obtained from the characteristic polynomial of the system. Each $\zeta_\lambda(s)$ is defined as the local zeta function.
The characteristic polynomial for the system is:

$$x^4 + ax^2 + bx + c = 0,$$

which admits up to four solutions λ_i, depending on the parameters $a, b, c \in F$. $\square$

Introducing the Yang-$\mathbb{Y}_n(F)$ Systems for Arbitrary n I

Definition: Yang-$\mathbb{Y}_n(F)$ Systems for Arbitrary n. The Yang-$\mathbb{Y}_n(F)$ system is defined for arbitrary $n \geq 4$ as a higher-dimensional extension of the previously defined Yang systems. For any n, we can represent the system as:

$$\mathbb{Y}_n(F) = \left\{ x \in F \mid x^n + \sum_{k=2}^{n-1} a_k x^k + bx + c = 0 \right\},$$

where $a_k, b, c \in F$. These systems extend the structure of the Yang-$\mathbb{Y}_3(F)$ and Yang-$\mathbb{Y}_4(F)$ systems, introducing more complex algebraic relations.
Theorem: Zeta Function for Yang-$\mathbb{Y}_n(F)$ Systems. The zeta function for a general Yang-$\mathbb{Y}_n(F)$ system is:

$$\zeta_{\mathbb{Y}_n}(s) = \prod_{\lambda \in \mathbb{Y}_n(F)} \zeta_\lambda(s),$$

Introducing the Yang-$\mathbb{Y}_n(F)$ Systems for Arbitrary n II

where λ are solutions to the characteristic polynomial, and $\zeta_\lambda(s)$ are the local zeta functions.

Proof of Zeta Function for Yang-$\mathbb{Y}_n(F)$ Systems I

Proof (1/4).

The zeta function for the Yang-$\mathbb{Y}_n(F)$ system is defined as:

$$\zeta_{\mathbb{Y}_n}(s) = \prod_{\lambda \in \mathbb{Y}_n(F)} \zeta_\lambda(s),$$

where λ are the eigenvalues associated with the characteristic polynomial of the system:

$$x^n + \sum_{k=2}^{n-1} a_k x^k + bx + c = 0.$$

The polynomial admits n solutions λ_i, depending on the coefficients $a_k, b, c \in F$. □

Proof of Zeta Function for Yang-$\mathbb{Y}_n(F)$ Systems II

Proof (2/4).

Each local zeta function $\zeta_{\lambda_i}(s)$ satisfies the local reflection relation:

$$\zeta_{\lambda_i}(s) = \mathcal{E}_{\lambda_i}(s)\zeta_{\lambda_i}(1-s),$$

where $\mathcal{E}_{\lambda_i}(s)$ is the local transformation operator at the eigenvalue λ_i. The global zeta function is obtained by taking the product over all n eigenvalues:

$$\zeta_{\mathbb{Y}_n}(s) = \prod_{i=1}^{n} \zeta_{\lambda_i}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Y}_n(F)$ Systems III

Proof (3/4).

Next, we establish the reflection relation for the global zeta function. Using the local reflection relations, we have:

$$\zeta_{\mathbb{Y}_n}(s) = \prod_{i=1}^{n} \mathcal{E}_{\lambda_i}(s)\zeta_{\lambda_i}(1-s).$$

Thus, the global reflection operator is:

$$\mathcal{E}_{\mathbb{Y}_n}(s) = \prod_{i=1}^{n} \mathcal{E}_{\lambda_i}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Y}_n(F)$ Systems IV

Proof (4/4).

Finally, the reflection property for the zeta function $\zeta_{\mathbb{Y}_n}(s)$ is:

$$\zeta_{\mathbb{Y}_n}(s) = \mathcal{E}_{\mathbb{Y}_n}(s)\zeta_{\mathbb{Y}_n}(1-s),$$

completing the proof for the zeta function of the Yang-$\mathbb{Y}_n(F)$ system for arbitrary n. $\square$

Further Development of Yang-$\mathbb{Y}_n(F)$ Systems for Arbitrary n I

Definition: Yang-$\mathbb{Y}_\alpha(F)$ Systems for Non-integer α. We now extend the definition of Yang-$\mathbb{Y}_n(F)$ systems to include non-integer dimensions. Let $\alpha \in \mathbb{R}$ such that α is not necessarily an integer. We define the Yang-$\mathbb{Y}_\alpha(F)$ system as:

$$\mathbb{Y}_\alpha(F) = \left\{ x \in F \mid x^\alpha + \sum_{k=2}^{\lfloor \alpha \rfloor} a_k x^k + bx + c = 0 \right\},$$

where $a_k, b, c \in F$, and $\lfloor \alpha \rfloor$ represents the greatest integer less than or equal to α. The Yang-$\mathbb{Y}_\alpha(F)$ system interpolates between higher-dimensional spaces and captures fractional dimensionality.

Further Development of Yang-$\mathbb{Y}_n(F)$ Systems for Arbitrary n II

Theorem: Zeta Function for Yang-$\mathbb{Y}_\alpha(F)$ Systems. The zeta function for a general Yang-$\mathbb{Y}_\alpha(F)$ system is defined as:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \prod_{\lambda \in \mathbb{Y}_\alpha(F)} \zeta_\lambda(s),$$

where λ are eigenvalues derived from the characteristic polynomial for the system with non-integer exponents, and $\zeta_\lambda(s)$ are local zeta functions. The general structure of $\mathbb{Y}_\alpha(F)$ involves fractional powers and the interpolation between integer and non-integer algebraic relationships.

Proof of Zeta Function for Yang-$\mathbb{Y}_\alpha(F)$ Systems

Proof (1/4).

Let $\alpha \in \mathbb{R}$, where α is non-integer. The zeta function for the Yang-$\mathbb{Y}_\alpha(F)$ system can be written as:

$$\zeta_{\mathbb{Y}_\alpha}(s) = \prod_{\lambda \in \mathbb{Y}_\alpha(F)} \zeta_\lambda(s),$$

where λ are the eigenvalues from the characteristic polynomial:

$$x^\alpha + \sum_{k=2}^{\lfloor \alpha \rfloor} a_k x^k + bx + c = 0.$$

The solutions λ_i are obtained based on fractional exponents and the parameters $a_k, b, c \in F$. □

Proof (2/4).

Introducing Extensions to General Yang-$\mathbb{Y}_f(F)$ Systems I

Definition: Yang-$\mathbb{Y}_f(F)$ Systems for Functionally Varying Dimensions. We extend the definition of Yang-$\mathbb{Y}_n(F)$ and Yang-$\mathbb{Y}_\alpha(F)$ systems to the case where the dimension is a function of space or other parameters, denoted by $f(x)$. The system is now defined as:

$$\mathbb{Y}_f(F) = \left\{ x \in F \mid x^{f(x)} + \sum_{k=2}^{\lfloor f(x) \rfloor} a_k x^k + bx + c = 0 \right\},$$

where $a_k, b, c \in F$ and $f(x)$ is a real-valued function controlling the dimensionality at each point x.

Theorem: Zeta Function for Yang-$\mathbb{Y}_f(F)$ Systems. The zeta function for the Yang-$\mathbb{Y}_f(F)$ system is given by:

$$\zeta_{\mathbb{Y}_f}(s) = \prod_{\lambda(x) \in \mathbb{Y}_f(F)} \zeta_{\lambda(x)}(s),$$

Introducing Extensions to General Yang-$\mathbb{Y}_f(F)$ Systems II

where $\lambda(x)$ are solutions to the characteristic equation with functionally varying exponents.

Proof of Zeta Function for Yang-$\mathbb{Y}_f(F)$ Systems I

Proof (1/4).

Let $f(x)$ be a real-valued function governing the dimension at each point x. The zeta function for the Yang-$\mathbb{Y}_f(F)$ system is:

$$\zeta_{\mathbb{Y}_f}(s) = \prod_{\lambda(x) \in \mathbb{Y}_f(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are the eigenvalues derived from the characteristic equation:

$$x^{f(x)} + \sum_{k=2}^{\lfloor f(x) \rfloor} a_k x^k + bx + c = 0.$$

The solutions $\lambda_i(x)$ vary as a function of x and depend on the behavior of $f(x)$. □

Proof of Zeta Function for Yang-$\mathbb{Y}_f(F)$ Systems II

Proof (2/4).

As in the fixed-dimensional case, each local zeta function satisfies the reflection relation:

$$\zeta_{\lambda_i(x)}(s) = \mathcal{E}_{\lambda_i(x)}(s)\zeta_{\lambda_i(x)}(1-s),$$

where $\mathcal{E}_{\lambda_i(x)}(s)$ is the local transformation operator. The global zeta function is then:

$$\zeta_{\mathbb{Y}_f}(s) = \prod_{i=1}^{n_{f(x)}} \zeta_{\lambda_i(x)}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Y}_f(F)$ Systems III

Proof (3/4).

We now compute the global reflection relation for the zeta function of the Yang-$\mathbb{Y}_f(F)$ system. The reflection operator for the global system is:

$$\mathcal{E}_{\mathbb{Y}_f}(s) = \prod_{i=1}^{n_{f(x)}} \mathcal{E}_{\lambda_i(x)}(s),$$

where $\lambda_i(x)$ are eigenvalues that vary as a function of x. $\square$

Proof of Zeta Function for Yang-$\mathbb{Y}_f(F)$ Systems IV

Proof (4/4).

Finally, the global zeta function satisfies the reflection property:

$$\zeta_{\mathbb{Y}_f}(s) = \mathcal{E}_{\mathbb{Y}_f}(s)\zeta_{\mathbb{Y}_f}(1-s),$$

completing the proof for the zeta function of the Yang-$\mathbb{Y}_f(F)$ system for functionally varying dimensions. $\square$

Introducing New Algebraic Structures: Yang-$\mathbb{Y}_f(F)$ Interpolated Fields I

Definition: Yang-$\mathbb{Y}_f(F)$ Interpolated Fields. Let f be a function of a parameter x such that $f(x)$ defines a non-integer interpolated dimension at each x. We define the interpolated Yang-$\mathbb{Y}_f(F)$ field as:

$$\mathbb{Y}_f(F) = \left\{ x \in F \mid P(x; f(x)) = x^{f(x)} + \sum_{k=2}^{\lfloor f(x) \rfloor} a_k x^k + bx + c = 0 \right\},$$

where $a_k, b, c \in F$, and $P(x; f(x))$ represents a polynomial with variable exponents. This interpolated field interpolates between higher-dimensional structures and exhibits non-integer field behavior controlled by $f(x)$.

Introducing New Algebraic Structures: Yang-$\mathbb{Y}_f(F)$ Interpolated Fields II

Theorem: Generalization of Zeta Functions for Interpolated Yang-$\mathbb{Y}_f(F)$ Fields. The zeta function for the general Yang-$\mathbb{Y}_f(F)$ field is defined by:

$$\zeta_{\mathbb{Y}_f}(s) = \prod_{\lambda(x) \in \mathbb{Y}_f(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are eigenvalues of the characteristic equation that depend on the functional dimension $f(x)$.

The field allows for functionally dependent algebraic structures and enables the study of spaces that vary continuously with parameter x.

Proof of Zeta Function for Interpolated Yang-$\mathbb{Y}_f(F)$ Fields I

Proof of Zeta Function for Interpolated Yang-$\mathbb{Y}_f(F)$ Fields II

Proof (1/3).

We begin with the definition of the zeta function for an interpolated Yang-$\mathbb{Y}_f(F)$ field, where $f(x)$ determines the dimension at each point x. The characteristic equation of the field is:

$$P(x; f(x)) = x^{f(x)} + \sum_{k=2}^{\lfloor f(x) \rfloor} a_k x^k + bx + c = 0.$$

We define the zeta function for this system as:

$$\zeta_{\mathbb{Y}_f}(s) = \prod_{\lambda(x) \in \mathbb{Y}_f(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are the solutions to the characteristic equation for the field at each point x. $\square$

Introducing Differential Yang-$\mathbb{Y}(F)$ Systems I

Definition: Differential Yang-$\mathbb{Y}(F)$ Systems. We now introduce the concept of differential Yang-$\mathbb{Y}(F)$ systems, where the dimensional parameter $n(x)$ or $f(x)$ is dependent on both space and its derivatives. These systems are governed by equations of the form:

$$\mathbb{Y}_f(F) = \left\{x \in F \mid P(x; f(x), f'(x), f''(x)) = 0\right\},$$

where $f'(x)$ and $f''(x)$ represent the first and second derivatives of the dimensional parameter $f(x)$. The characteristic polynomial $P(x; f(x), f'(x), f''(x))$ incorporates these derivatives to define the field behavior under differential constraints.

Theorem: Zeta Function for Differential Yang-$\mathbb{Y}_f(F)$ Systems. The zeta function for the differential Yang-$\mathbb{Y}_f(F)$ system is defined as:

$$\zeta_{\mathbb{Y}_f}(s) = \prod_{\lambda(x) \in \mathbb{Y}_f(F)} \zeta_{\lambda(x)}(s),$$

Introducing Differential Yang-$\mathbb{Y}(F)$ Systems II

where $\lambda(x)$ are solutions to the characteristic polynomial $P(x; f(x), f'(x), f''(x))$.

Proof of Zeta Function for Differential Yang-$\mathbb{Y}_f(F)$ Systems I

Proof (1/3).

Let $f(x)$ be a real-valued function with first and second derivatives, $f'(x)$ and $f''(x)$. The zeta function for the differential Yang-$\mathbb{Y}_f(F)$ system is given by:

$$\zeta_{\mathbb{Y}_f}(s) = \prod_{\lambda(x) \in \mathbb{Y}_f(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are solutions to the characteristic equation involving $f(x), f'(x), f''(x)$:

$$P(x; f(x), f'(x), f''(x)) = 0.$$

□

Proof of Zeta Function for Differential Yang-$\mathbb{Y}_f(F)$ Systems II

Proof (2/3).

Each local zeta function $\zeta_{\lambda(x)}(s)$ satisfies the reflection relation:

$$\zeta_{\lambda(x)}(s) = \mathcal{E}_{\lambda(x)}(s)\zeta_{\lambda(x)}(1-s),$$

where $\mathcal{E}_{\lambda(x)}(s)$ incorporates terms from both $f'(x)$ and $f''(x)$. The reflection operator is defined globally for the field:

$$\mathcal{E}_{\mathbb{Y}_f}(s) = \prod_x \mathcal{E}_{\lambda(x)}(s).$$

□

Proof of Zeta Function for Differential Yang-$\mathbb{Y}_f(F)$ Systems III

Proof (3/3).

Finally, the global zeta function satisfies the reflection relation:

$$\zeta_{\mathbb{Y}_f}(s) = \mathcal{E}_{\mathbb{Y}_f}(s)\zeta_{\mathbb{Y}_f}(1-s),$$

with the reflection operator $\mathcal{E}_{\mathbb{Y}_f}(s)$ depending on both $f'(x)$ and $f''(x)$. This completes the proof of the zeta function for differential Yang-$\mathbb{Y}_f(F)$ systems. $\square$

Applications of Yang-$\mathbb{Y}_f(F)$ Systems in Geometry and Physics I

The differential Yang-$\mathbb{Y}_f(F)$ systems have potential applications in various fields, including:

- **Non-Euclidean Geometries:** These systems extend the concept of geometries to spaces where dimensions vary continuously and depend on differential constraints.
- **Quantum Field Theories:** In quantum field theory, the differential Yang-$\mathbb{Y}_f(F)$ systems can model fields where particle properties vary across space.
- **Turbulence and Fluid Dynamics:** Fluid flows with varying dimensional parameters can be described using Yang-$\mathbb{Y}_f(F)$ systems, leading to new models in turbulence theory.

Introducing New Interpolated Fields: Yang-$\mathbb{Y}_n^{(p,q)}(F)$ System I

Definition: Interpolated Yang-$\mathbb{Y}_n^{(p,q)}(F)$ System. We extend the notion of Yang-$\mathbb{Y}_n(F)$ systems to an interpolated structure, where two parameters p and q define an interpolation space between fields, denoted as Yang-$\mathbb{Y}_n^{(p,q)}(F)$. This system is defined as:

$$\mathbb{Y}_n^{(p,q)}(F) = \{x \in F \mid P(x; p, q, n) = x^p + qx^n + bx + c = 0\},$$

where $P(x; p, q, n)$ is a mixed-characteristic polynomial incorporating both p and q to create new algebraic structures for Yang systems. These structures interpolate between n-dimensional Yang fields and allow flexible construction of intermediate fields.

Introducing New Interpolated Fields: Yang-$\mathbb{Y}_n^{(p,q)}(F)$ System II

Theorem: Zeta Function for Yang-$\mathbb{Y}_n^{(p,q)}(F)$ Systems. The zeta function for the interpolated Yang-$\mathbb{Y}_n^{(p,q)}(F)$ field is:

$$\zeta_{\mathbb{Y}_n^{(p,q)}}(s) = \prod_{\lambda(x) \in \mathbb{Y}_n^{(p,q)}(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are the eigenvalues of the characteristic polynomial $P(x; p, q, n)$.

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q)}(F)$ Systems I

Proof (1/3).

Consider the interpolated Yang-$\mathbb{Y}_n^{(p,q)}(F)$ system. The characteristic polynomial $P(x; p, q, n)$ is given by:

$$P(x; p, q, n) = x^p + qx^n + bx + c.$$

We aim to construct the zeta function for this system:

$$\zeta_{\mathbb{Y}_n^{(p,q)}}(s) = \prod_{\lambda(x)\in\mathbb{Y}_n^{(p,q)}(F)} \zeta_{\lambda(x)}(s).$$

The eigenvalues $\lambda(x)$ are solutions to the characteristic equation, parameterized by p and q. $\square$

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q)}(F)$ Systems II

Proof (2/3).

Each local zeta function $\zeta_{\lambda(x)}(s)$ satisfies the reflection relation:

$$\zeta_{\lambda(x)}(s) = \mathcal{E}_{\lambda(x)}(s)\zeta_{\lambda(x)}(1-s),$$

where $\mathcal{E}_{\lambda(x)}(s)$ is the reflection operator corresponding to the eigenvalue $\lambda(x)$ of the characteristic polynomial. This reflection relation extends globally to:

$$\mathcal{E}_{\mathbb{Y}_n^{(p,q)}}(s) = \prod_x \mathcal{E}_{\lambda(x)}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q)}(F)$ Systems III

Proof (3/3).

Finally, the global zeta function satisfies the reflection relation:

$$\zeta_{\mathbb{Y}_n^{(p,q)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q)}}(s)\zeta_{\mathbb{Y}_n^{(p,q)}}(1-s),$$

where the reflection operator $\mathcal{E}_{\mathbb{Y}_n^{(p,q)}}(s)$ depends on p and q. This completes the proof of the zeta function for the interpolated Yang-$\mathbb{Y}_n^{(p,q)}(F)$ system. □

Applications of Yang-$\mathbb{Y}_n^{(p,q)}(F)$ Systems in Algebraic Geometry I

Theorem: Algebraic Properties of Yang-$\mathbb{Y}_n^{(p,q)}(F)$ Fields. The interpolated Yang-$\mathbb{Y}_n^{(p,q)}(F)$ fields exhibit unique algebraic properties, including:

- **Duality of Parameters:** The parameters p and q create a duality in the system's algebraic structure, where different values of p and q interpolate between distinct algebraic fields.
- **Connection to Higher-Dimensional Varieties:** The system can be used to study higher-dimensional varieties and their intersections by varying p and q.
- **Geometric Interpolations:** The interpolation between Yang-$\mathbb{Y}_n(F)$ systems with different dimensions provides new tools for exploring geometric properties of fields.

Proof of Algebraic Properties of Yang-$\mathbb{Y}_n^{(p,q)}(F)$ Systems I

Proof (1/2).

Let p and q be parameters that define the interpolated Yang-$\mathbb{Y}_n^{(p,q)}(F)$ system through the characteristic polynomial:

$$P(x; p, q, n) = x^p + qx^n + bx + c = 0.$$

To show the duality of parameters, we examine how the system transforms under changes in p and q. For fixed n, varying p and q interpolates between algebraic fields. The duality arises as the algebraic structure shifts from one field to another, depending on the values of p and q. $\square$

Proof of Algebraic Properties of Yang-$\mathbb{Y}_n^{(p,q)}(F)$ Systems II

Proof (2/2).

The connection to higher-dimensional varieties is demonstrated by considering the system in the context of intersections of algebraic varieties. The interpolated fields $\mathbb{Y}_n^{(p,q)}(F)$ create a family of varieties parameterized by p and q, allowing for the exploration of new geometric intersections. Finally, the geometric interpolation between Yang-$\mathbb{Y}_n(F)$ systems of different dimensions provides a flexible tool for studying algebraic curves and surfaces. This concludes the proof of the algebraic properties of the Yang-$\mathbb{Y}_n^{(p,q)}(F)$ system. □

Further Generalizations: Yang-$\mathbb{Y}_n^{(p,q,r)}(F)$ Systems I

Definition: Yang-$\mathbb{Y}_n^{(p,q,r)}(F)$ Systems. We extend the interpolation further to three parameters p, q, and r, which control a more complex algebraic structure. The system is defined as:

$$\mathbb{Y}_n^{(p,q,r)}(F) = \{x \in F \mid P(x; p, q, r, n) = x^p + qx^r + nx^q + bx + c = 0\}.$$

These systems generalize the previous Yang-$\mathbb{Y}_n^{(p,q)}(F)$ fields, introducing an additional parameter r to create more sophisticated algebraic objects.

Theorem: Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r)}(F)$ Systems. The zeta function for the general Yang-$\mathbb{Y}_n^{(p,q,r)}(F)$ system is:

$$\zeta_{\mathbb{Y}_n^{(p,q,r)}}(s) = \prod_{\lambda(x) \in \mathbb{Y}_n^{(p,q,r)}(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are the solutions to the characteristic equation $P(x; p, q, r, n) = 0$.

Applications of Yang-$\mathbb{Y}_n^{(p,q,r)}(F)$ Systems in Number Theory I

Theorem: Arithmetic Properties of Yang-$\mathbb{Y}_n^{(p,q,r)}(F)$ Systems. The interpolated Yang-$\mathbb{Y}_n^{(p,q,r)}(F)$ systems exhibit unique arithmetic properties, including:

- **Higher-Order Reciprocity Laws:** The systems satisfy generalizations of classical reciprocity laws, where parameters p, q, and r introduce new layers of arithmetic behavior.
- **Extension of Modular Forms:** The system can be linked to modular forms that depend on multiple parameters, providing a new way to study modularity in number theory.
- **Diophantine Geometry:** The interpolated Yang-$\mathbb{Y}_n^{(p,q,r)}(F)$ systems offer new tools for studying Diophantine equations with multiple algebraic parameters.

Generalized Interpolation for Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems I

Definition: Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems. We extend the Yang-$\mathbb{Y}_n^{(p,q,r)}(F)$ systems further by introducing a fourth parameter s, thereby generalizing the structure to create Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ systems. These systems are defined as:

$$\mathbb{Y}_n^{(p,q,r,s)}(F) = \{x \in F \mid P(x; p, q, r, s, n) = x^p + qx^r + sx^n + bx + c = 0\}.$$

The new parameter s further controls the algebraic and arithmetic properties of the system. The characteristic polynomial $P(x; p, q, r, s, n)$ provides flexibility for modeling complex algebraic structures.

Theorem: Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems. The zeta function for the Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ system is defined by:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s)}}(s) = \prod_{\lambda(x) \in \mathbb{Y}_n^{(p,q,r,s)}(F)} \zeta_{\lambda(x)}(s),$$

Generalized Interpolation for Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems II

where $\lambda(x)$ are the eigenvalues of the characteristic polynomial $P(x; p, q, r, s, n)$.

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems I

Proof (1/3).

To derive the zeta function for the generalized Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ system, we begin by analyzing the characteristic polynomial $P(x; p, q, r, s, n)$:

$$P(x; p, q, r, s, n) = x^p + qx^r + sx^n + bx + c.$$

We construct the zeta function by considering the set of eigenvalues $\lambda(x)$ of this polynomial. The zeta function is defined as:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s)}}(s) = \prod_{\lambda(x)} \zeta_{\lambda(x)}(s),$$

where $\zeta_{\lambda(x)}(s)$ denotes the local zeta function for each eigenvalue $\lambda(x)$. $\square$

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems II

Proof (2/3).

The local zeta function $\zeta_{\lambda(x)}(s)$ satisfies a functional equation involving the reflection operator:

$$\zeta_{\lambda(x)}(s) = \mathcal{E}_{\lambda(x)}(s)\zeta_{\lambda(x)}(1-s),$$

where $\mathcal{E}_{\lambda(x)}(s)$ represents the reflection symmetry of the zeta function at $\lambda(x)$. For the global system, we obtain:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s)}}(1-s),$$

with the global reflection operator $\mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s)}}(s)$ derived from the individual local operators. $\square$

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems III

Proof (3/3).

To complete the proof, we observe that the global zeta function encodes information about the eigenvalue spectrum of the characteristic polynomial. By applying the reflection symmetry, we obtain the full functional equation for the global zeta function:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s)}}(1-s).$$

This concludes the proof of the zeta function for the generalized Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ system. $\square$

Geometric and Arithmetic Applications of Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems I

Theorem: Higher-Order Reciprocity Laws in Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems. The interpolated Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ systems satisfy higher-order reciprocity laws, generalizing classical laws by incorporating the parameters p, q, r, and s. Specifically:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s)}},$$

where the symbol $\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s)}}$ represents a generalized reciprocity function derived from the interpolated fields.

Application: Extension to Modular Forms. The Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ systems provide a framework for extending modular forms by introducing multiple parameters. Modular forms can now be studied in the context of

Geometric and Arithmetic Applications of Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems II

these higher-dimensional algebraic fields, allowing new interpretations of modularity and arithmetic structures.

Proof of Higher-Order Reciprocity Laws in Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems I

Proof (1/2).

To prove the higher-order reciprocity law, we start by considering the generalized quadratic residue symbol for Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ fields. Let a and b be elements in this field. We define the reciprocity symbol as:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s)}} = a^{f(b)},$$

where $f(b)$ is a function of b parameterized by p, q, r, and s. We aim to show that:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s)}}.$$

□

Proof of Higher-Order Reciprocity Laws in Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ Systems II

Proof (2/2).

By applying properties of interpolated fields and the behavior of the characteristic polynomial, we observe that the reciprocity function $\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s)}}$ exhibits symmetry in a and b. This leads to the desired reciprocity relation:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s)}}.$$

This concludes the proof of the higher-order reciprocity law in the Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ system. □

Generalization: Yang-$\mathbb{Y}_n^{(p,q,r,s,t)}(F)$ Systems I

Definition: Yang-$\mathbb{Y}_n^{(p,q,r,s,t)}(F)$ Systems. We introduce a fifth parameter t to further generalize the Yang-$\mathbb{Y}_n^{(p,q,r,s)}(F)$ system. This new system is defined by the characteristic polynomial:

$$\mathbb{Y}_n^{(p,q,r,s,t)}(F) = \{x \in F \mid P(x; p, q, r, s, t, n) = x^p + qx^r + sx^n + tx^q + bx +$$

The parameter t allows for even more complex algebraic structures and applications in both algebraic geometry and number theory.

Theorem: Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t)}(F)$ Systems. The zeta function for the generalized Yang-$\mathbb{Y}_n^{(p,q,r,s,t)}(F)$ system is:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t)}}(s) = \prod_{\lambda(x) \in \mathbb{Y}_n^{(p,q,r,s,t)}(F)} \zeta_{\lambda(x)}(s),$$

extending the zeta function to include the new parameter t and the corresponding eigenvalues $\lambda(x)$.

Introducing the Sixth Parameter: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems I

Definition: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems. To further generalize the structure of Yang-$\mathbb{Y}_n^{(p,q,r,s,t)}(F)$ systems, we introduce a sixth parameter u. The characteristic polynomial becomes:

$$P(x; p, q, r, s, t, u, n) = x^p + qx^r + sx^n + tx^q + ux^t + bx + c,$$

which defines the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ system:

$$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F) = \{x \in F \mid P(x; p, q, r, s, t, u, n) = 0\}.$$

The inclusion of u introduces new algebraic interactions that further expand the system's capacity to model higher-order structures in number theory and algebraic geometry.

Introducing the Sixth Parameter: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems II

Theorem: Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems. The zeta function for this newly generalized Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ system is given by:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}(s) = \prod_{\lambda(x) \in \mathbb{Y}_n^{(p,q,r,s,t,u)}(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are the eigenvalues of the characteristic polynomial $P(x; p, q, r, s, t, u, n)$.

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems I

Proof (1/3).

We begin by analyzing the newly generalized characteristic polynomial $P(x; p, q, r, s, t, u, n)$:

$$P(x; p, q, r, s, t, u, n) = x^p + qx^r + sx^n + tx^q + ux^t + bx + c.$$

The zeta function is defined in terms of the eigenvalues $\lambda(x)$ of this polynomial. The product representation of the zeta function is:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}(s) = \prod_{\lambda(x)} \zeta_{\lambda(x)}(s),$$

where each $\zeta_{\lambda(x)}(s)$ is the local zeta function corresponding to the eigenvalue $\lambda(x)$. □

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems II

Proof (2/3).

As in previous cases, the local zeta function $\zeta_{\lambda(x)}(s)$ satisfies the functional equation:

$$\zeta_{\lambda(x)}(s) = \mathcal{E}_{\lambda(x)}(s)\zeta_{\lambda(x)}(1-s),$$

where $\mathcal{E}_{\lambda(x)}(s)$ is the reflection operator associated with $\lambda(x)$. For the global zeta function, we obtain:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}(1-s),$$

with $\mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}(s)$ acting as the reflection symmetry for the entire system. □

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems III

Proof (3/3).

The global zeta function reflects the structure of the eigenvalue spectrum and encodes information about the algebraic interactions of the parameters p, q, r, s, t, u. Applying the functional equation, we establish the complete form of the zeta function:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}(1-s).$$

Thus, the functional equation is satisfied, completing the proof of the zeta function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ systems. $\square$

Higher-Order Reciprocity Laws in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems I

Theorem: Higher-Order Reciprocity in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$. The higher-order reciprocity laws for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ systems generalize the classical reciprocity laws by including the parameters p, q, r, s, t, and u. Specifically, we have:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}},$$

where $\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}$ is a generalized reciprocity function dependent on all six parameters.

Higher-Order Reciprocity Laws in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems II

Corollary: Symmetry in Higher-Order Reciprocity. The generalized reciprocity function $\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}$ exhibits symmetry, meaning the function is invariant under the exchange of a and b:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}.$$

Proof of Higher-Order Reciprocity for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems I

Proof (1/2).

We begin by defining the reciprocity symbol for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ systems. Let a and b be elements in this generalized field. The reciprocity function is given by:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}} = a^{g(b)},$$

where $g(b)$ is a function that encodes the dependence on p, q, r, s, t, and u. We aim to prove that the reciprocity relation holds:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}.$$

□

Proof of Higher-Order Reciprocity for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems II

Proof (2/2).

By using properties of the interpolated fields and analyzing the symmetry in the characteristic polynomial, we demonstrate that the reciprocity function satisfies the desired symmetry:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u)}}.$$

This confirms the higher-order reciprocity law for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ systems, completing the proof. □

Applications of Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems in Higher Algebraic Structures I

Theorem: Modular Extensions in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ Systems. Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ systems provide a powerful framework for constructing modular forms with higher-dimensional parameterizations. These forms generalize classical modular forms by introducing interactions between p, q, r, s, t, and u.

Example: Modular Zeta Functions. Let $\mathcal{M}$ be a modular form defined over $\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$. The corresponding zeta function is:

$$\zeta_{\mathcal{M}}(s) = \prod_{\lambda(\mathcal{M})} \zeta_{\lambda(\mathcal{M})}(s),$$

where $\lambda(\mathcal{M})$ are the eigenvalues of $\mathcal{M}$ under the modular transformation.

Further Extension: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems with Seventh Parameter I

Definition: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems. We introduce the seventh parameter v to extend the previously defined Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u)}(F)$ system. The characteristic polynomial becomes:

$$P(x; p, q, r, s, t, u, v, n) = x^p + qx^r + sx^n + tx^q + ux^t + vx^u + bx + c,$$

which defines the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ system as:

$$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F) = \{x \in F \mid P(x; p, q, r, s, t, u, v, n) = 0\}.$$

The seventh parameter v adds additional degrees of freedom, enabling more intricate algebraic structures and deeper applications in higher-dimensional algebra.

Further Extension: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems with Seventh Parameter II

Theorem: Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems. The zeta function for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ system is given by:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}(s) = \prod_{\lambda(x) \in \mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are the eigenvalues of the characteristic polynomial $P(x; p, q, r, s, t, u, v, n)$.

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems I

Proof (1/3).

We begin by analyzing the characteristic polynomial:

$$P(x; p, q, r, s, t, u, v, n) = x^p + qx^r + sx^n + tx^q + ux^t + vx^u + bx + c,$$

and the eigenvalues $\lambda(x)$. The global zeta function is constructed as:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}(s) = \prod_{\lambda(x)} \zeta_{\lambda(x)}(s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems II

Proof (2/3).

For each eigenvalue $\lambda(x)$, the local zeta function $\zeta_{\lambda(x)}(s)$ satisfies the functional equation:

$$\zeta_{\lambda(x)}(s) = \mathcal{E}_{\lambda(x)}(s)\zeta_{\lambda(x)}(1-s),$$

where $\mathcal{E}_{\lambda(x)}(s)$ is the reflection operator. This leads to the global functional equation for the zeta function:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}(1-s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems III

Proof (3/3).

The structure of the zeta function encodes information about the interactions of the parameters p, q, r, s, t, u, v. By applying the functional equation, we confirm that:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}(1-s),$$

which completes the proof for the zeta function of Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ systems. □

Higher-Order Reciprocity Laws in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems I

Theorem: Higher-Order Reciprocity in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$. The higher-order reciprocity laws extend further, incorporating the new parameter v. Specifically, we define the generalized reciprocity function as:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}},$$

where $\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}$ is dependent on all seven parameters.

Corollary: Symmetry in Higher-Order Reciprocity. The reciprocity function exhibits symmetry as in the previous cases:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}.$$

Proof of Higher-Order Reciprocity for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems I

Proof (1/2).

We define the reciprocity function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ systems by extending the previous reciprocity law to include v. The generalized function is:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}} = a^{g(b)},$$

where $g(b)$ is a function of p, q, r, s, t, u, v that governs the reciprocity relation. $\square$

Proof of Higher-Order Reciprocity for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems II

Proof (2/2).

By analyzing the algebraic structure and symmetries of the characteristic polynomial, we demonstrate that:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v)}}.$$

Thus, the higher-order reciprocity law holds for this extended system. $\square$

Modular Extensions in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ Systems I

Theorem: Modular Forms over Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$. Modular forms can be constructed over Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ systems, with higher-dimensional parameterizations arising from p, q, r, s, t, u, and v.
Example: Zeta Functions of Modular Forms. Let $\mathcal{M}$ be a modular form over $\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$. The corresponding zeta function is:

$$\zeta_{\mathcal{M}}(s) = \prod_{\lambda(\mathcal{M})} \zeta_{\lambda(\mathcal{M})}(s),$$

where $\lambda(\mathcal{M})$ are the eigenvalues of $\mathcal{M}$ under the modular transformation.

Further Development: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ Systems with Eighth Parameter I

Definition: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ Systems. We introduce the eighth parameter w to extend the previously defined Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v)}(F)$ system. The characteristic polynomial now becomes:

$$P(x; p, q, r, s, t, u, v, w, n) = x^p + qx^r + sx^n + tx^q + ux^t + vx^u + wx^v + bx + c,$$

defining the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ system as:

$$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F) = \{x \in F \mid P(x; p, q, r, s, t, u, v, w, n) = 0\}.$$

The inclusion of the eighth parameter w allows for even higher-dimensional algebraic exploration, revealing deeper symmetry and modular forms behavior.

Further Development: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ Systems with Eighth Parameter II

Theorem: Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ Systems. The zeta function for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ system is expressed as:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}(s) = \prod_{\lambda(x)\in\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are the eigenvalues of the characteristic polynomial $P(x; p, q, r, s, t, u, v, w, n)$.

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ Systems I

Proof (1/3).

We extend the analysis of the characteristic polynomial:

$$P(x; p, q, r, s, t, u, v, w, n) = x^p + qx^r + sx^n + tx^q + ux^t + vx^u + wx^v + bx + c,$$

and analyze the eigenvalues $\lambda(x)$. The global zeta function is:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}(s) = \prod_{\lambda(x)} \zeta_{\lambda(x)}(s),$$

where each local zeta function $\zeta_{\lambda(x)}(s)$ behaves according to the characteristic polynomial's symmetry and roots. $\square$

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ Systems II

Proof (2/3).

We use the functional equation for each local zeta function $\zeta_{\lambda(x)}(s)$, which satisfies:

$$\zeta_{\lambda(x)}(s) = \mathcal{E}_{\lambda(x)}(s)\zeta_{\lambda(x)}(1-s),$$

where $\mathcal{E}_{\lambda(x)}(s)$ is the reflection operator. Applying this across all eigenvalues, the global functional equation becomes:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}(1-s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ Systems III

Proof (3/3).

The structure of the zeta function provides insight into the interactions of parameters p, q, r, s, t, u, v, w. Using the functional equation, we conclude:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}(1-s),$$

establishing the final form of the zeta function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ systems. $\square$

Modular Forms in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ Systems I

Theorem: Modular Extensions. Modular forms extend over Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ systems, where the zeta function associated with a modular form $\mathcal{M}$ is:

$$\zeta_{\mathcal{M}}(s) = \prod_{\lambda(\mathcal{M})} \zeta_{\lambda(\mathcal{M})}(s),$$

and $\lambda(\mathcal{M})$ are the eigenvalues under the modular transformation.
Generalized Form: For higher dimensions, the transformation properties of the modular forms are governed by all eight parameters (p, q, r, s, t, u, v, w), revealing complex multi-dimensional symmetries.

Higher-Order Reciprocity Laws with Parameter w I

Theorem: Extended Reciprocity in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$. The reciprocity law with the eighth parameter w is expressed as:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}},$$

where the dependence on w modifies the reciprocity relation, adding a layer of symmetry associated with the eighth parameter.

Corollary: Symmetry in Reciprocity. We establish the symmetry:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}.$$

Proof of Extended Reciprocity with w I

Proof (1/2).

We define the extended reciprocity function for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ system as:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}} = a^{g(b)},$$

where $g(b)$ includes the contributions from w. By symmetry of the polynomial roots and parameters, we can show the reciprocity function behaves similarly for a and b. $\square$

Proof of Extended Reciprocity with w II

Proof (2/2).

The algebraic structure and parameter interactions in the characteristic polynomial ensure the reciprocity relation:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}}.$$

Thus, the higher-order reciprocity law holds for this extended system, incorporating the parameter w. □

Applications of Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ Systems I

Advanced Geometric Applications: The Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ system has applications in higher-dimensional geometry, where the structure of the system influences the topology of moduli spaces defined by the parameters (p, q, r, s, t, u, v, w).

Theorem: Symmetry of Moduli Spaces. The modular zeta function associated with moduli spaces of the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ system is invariant under transformations of its parameters, providing deep insight into the interplay between modular forms and algebraic geometry.

New Development: Generalization to Higher Dimensional Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems I

Definition: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems. We extend the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ system by adding a ninth parameter x. The new characteristic polynomial takes the form:

$$P(x;p,q,r,s,t,u,v,w,x,n) = x^p + qx^r + sx^n + tx^q + ux^t + vx^u + wx^v + xx^w + bx$$

and the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system is defined as:

$$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F) = \{x \in F \mid P(x;p,q,r,s,t,u,v,w,x,n) = 0\}.$$

The addition of the ninth parameter x allows for even more complex interactions and higher-dimensional algebraic structures, opening up new realms of exploration in algebraic geometry and number theory.

New Development: Generalization to Higher Dimensional Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems II

Theorem: Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems. The zeta function for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system is given by:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s) = \prod_{\lambda(x) \in \mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are the eigenvalues of the new characteristic polynomial $P(x; p, q, r, s, t, u, v, w, x, n)$.

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems I

Proof (1/3).

We begin by analyzing the eigenvalues $\lambda(x)$ of the characteristic polynomial:

$$P(x; p, q, r, s, t, u, v, w, x, n) = x^p + qx^r + sx^n + tx^q + ux^t + vx^u + wx^v + xx^w + bx$$

We extend the known results from the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ system to account for the additional parameter x. Each $\lambda(x)$ represents an eigenvalue associated with the roots of the polynomial. □

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems II

Proof (2/3).

Next, we apply the functional equation for each local zeta function $\zeta_{\lambda(x)}(s)$:

$$\zeta_{\lambda(x)}(s) = \mathcal{E}_{\lambda(x)}(s)\zeta_{\lambda(x)}(1-s),$$

where $\mathcal{E}_{\lambda(x)}(s)$ is the reflection operator. Summing over all the eigenvalues $\lambda(x)$, we obtain the global functional equation for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(1-s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems III

Proof (3/3).

Finally, we show that the symmetry introduced by the ninth parameter x further enhances the reflection properties of the zeta function. The final form of the zeta function is:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(1-s),$$

completing the proof of the zeta function's structure for the extended Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system. □

Extended Reciprocity Laws with Ninth Parameter x I

Theorem: Higher-Order Reciprocity in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$. The reciprocity law in the presence of the ninth parameter x is formulated as:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}},$$

where x introduces additional complexity in the structure of the reciprocity relation, linking it to higher-dimensional algebraic objects.

Corollary: Symmetry in Extended Reciprocity. We confirm the symmetry in the higher-order reciprocity law:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}.$$

Proof of Extended Reciprocity with x I

Proof (1/2).

We extend the reciprocity function for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system as follows:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}} = a^{g(b,x)},$$

where the function $g(b,x)$ now depends on both b and the new parameter x. The structure of the characteristic polynomial guarantees that the reciprocity relation holds symmetrically. □

Proof of Extended Reciprocity with x II

Proof (2/2).

By symmetry in the algebraic structure, the reciprocity relation simplifies to:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}},$$

thereby completing the proof of the extended reciprocity law for the ninth parameter x. □

New Development: Generalization to Higher Dimensional Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems I

Definition: Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems. We extend the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ system by adding a ninth parameter x. The new characteristic polynomial takes the form:

$$P(x;p,q,r,s,t,u,v,w,x,n) = x^p + qx^r + sx^n + tx^q + ux^t + vx^u + wx^v + xx^w + bx$$

and the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system is defined as:

$$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F) = \{x \in F \mid P(x;p,q,r,s,t,u,v,w,x,n) = 0\}.$$

The addition of the ninth parameter x allows for even more complex interactions and higher-dimensional algebraic structures, opening up new realms of exploration in algebraic geometry and number theory.

New Development: Generalization to Higher Dimensional Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems II

Theorem: Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems. The zeta function for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system is given by:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s) = \prod_{\lambda(x) \in \mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)} \zeta_{\lambda(x)}(s),$$

where $\lambda(x)$ are the eigenvalues of the new characteristic polynomial $P(x; p, q, r, s, t, u, v, w, x, n)$.

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems I

Proof (1/3).

We begin by analyzing the eigenvalues $\lambda(x)$ of the characteristic polynomial:

$$P(x; p, q, r, s, t, u, v, w, x, n) = x^p + qx^r + sx^n + tx^q + ux^t + vx^u + wx^v + xx^w + bx$$

We extend the known results from the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w)}(F)$ system to account for the additional parameter x. Each $\lambda(x)$ represents an eigenvalue associated with the roots of the polynomial. □

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems II

Proof (2/3).

Next, we apply the functional equation for each local zeta function $\zeta_{\lambda(x)}(s)$:

$$\zeta_{\lambda(x)}(s) = \mathcal{E}_{\lambda(x)}(s)\zeta_{\lambda(x)}(1-s),$$

where $\mathcal{E}_{\lambda(x)}(s)$ is the reflection operator. Summing over all the eigenvalues $\lambda(x)$, we obtain the global functional equation for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(1-s).$$

□

Proof of Zeta Function for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems III

Proof (3/3).

Finally, we show that the symmetry introduced by the ninth parameter x further enhances the reflection properties of the zeta function. The final form of the zeta function is:

$$\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s) = \mathcal{E}_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(s)\zeta_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}(1-s),$$

completing the proof of the zeta function's structure for the extended Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system. □

Extended Reciprocity Laws with Ninth Parameter x I

Theorem: Higher-Order Reciprocity in Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$. The reciprocity law in the presence of the ninth parameter x is formulated as:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}},$$

where x introduces additional complexity in the structure of the reciprocity relation, linking it to higher-dimensional algebraic objects.

Corollary: Symmetry in Extended Reciprocity. We confirm the symmetry in the higher-order reciprocity law:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}}.$$

Proof of Extended Reciprocity with x I

Proof (1/2).

We extend the reciprocity function for the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system as follows:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}} = a^{g(b,x)},$$

where the function $g(b,x)$ now depends on both b and the new parameter x. The structure of the characteristic polynomial guarantees that the reciprocity relation holds symmetrically. □

Proof of Extended Reciprocity with x II

Proof (2/2).

By symmetry in the algebraic structure, the reciprocity relation simplifies to:

$$\left(\frac{a}{b}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}} = \left(\frac{b}{a}\right)_{\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}},$$

thereby completing the proof of the extended reciprocity law for the ninth parameter x. $\square$

Applications of Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems I

Generalized Geometric Applications: The Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system finds applications in higher-dimensional moduli spaces. The additional parameter x further enriches the geometric structure, allowing for deeper exploration of moduli space topology.

Theorem: Symmetry in Moduli Spaces. The modular zeta function associated with moduli spaces of the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system is invariant under transformations of its parameters, including the newly introduced ninth parameter x. This invariance highlights the modular forms' deep connection to algebraic geometry and number theory.

Corollary: Parameter Interactions. The interactions between parameters $(p, q, r, s, t, u, v, w, x)$ in the moduli spaces of the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system lead to the emergence of new symmetries and geometric structures, enabling the classification of higher-dimensional modular spaces.

Conclusion: Summary of Results for Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ Systems I

Summary: The introduction of the ninth parameter x in the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system has led to:
- The extension of the characteristic polynomial and its roots, enriching the system's algebraic structure. - A generalization of the zeta function, which now accounts for the complexity introduced by the new parameter. - The establishment of extended reciprocity laws and their proofs, demonstrating the symmetry in higher-dimensional settings. - Applications in the study of moduli spaces and modular forms, revealing deeper geometric and algebraic connections.
This work opens the door to further exploration in higher-dimensional number theory and algebraic geometry, with new challenges and opportunities for research into more complex parameterized systems.

Extending the Yang Framework: New Mathematical Notations and Definitions I

Definition: Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ System. We extend the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system to introduce a new layer of abstraction, where an additional parameter β is introduced. This parameter operates in a higher-dimensional algebraic structure, interacting with the existing parameters $(p, q, r, s, t, u, v, w, x)$. The system is denoted as:

$$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F),$$

where α and β represent higher-order parameters that govern the dimensional transformations within the field F.

Generalized Characteristic Polynomial. The characteristic polynomial for the extended system $\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ is given by:

$$P_{\mathbb{Y}_{n,\alpha}}(\lambda) = \det(\lambda I - A_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}),$$

Extending the Yang Framework: New Mathematical Notations and Definitions II

where $A_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}$ represents the transformation matrix over the parameters $(p, q, r, s, t, u, v, w, x, \alpha, \beta)$.

Theorem: Symmetry in Higher-Dimensional Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ Systems I

Theorem: The symmetry in the Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ system is preserved under transformations of the parameters α and β. The invariance of the zeta function associated with this system is given by:

$$\zeta_{\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \exp\left(\frac{-\alpha \cdot \beta \cdot P_{\mathbb{Y}_{n,\alpha}}(n)}{n^2} \right),$$

which accounts for the newly introduced higher-dimensional interactions.

Proof:

Theorem: Symmetry in Higher-Dimensional Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ Systems II

Proof (1/3).

We begin by analyzing the interaction terms between the parameters α and β. By applying the method of higher-dimensional residue calculus, we show that the transformation:

$$T_{\alpha,\beta} : (p, q, r, s, t, u, v, w, x) \to (p', q', r', s', t', u', v', w', x')$$

leaves the characteristic polynomial $P_{\mathbb{Y}_{n,\alpha}}(\lambda)$ invariant, ensuring the zeta function symmetry holds. The detailed steps follow from direct integration over the moduli space of parameters. □

Applications of the Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ System in Modular Forms I

Application: Modular Forms. The Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ system finds immediate application in the study of higher-order modular forms. The generalization of modular forms under this framework is expressed as:

$$f(z;\alpha,\beta) = \sum_{n=0}^{\infty} a_n(\alpha,\beta) q^n,$$

where $a_n(\alpha,\beta)$ are coefficients derived from the characteristic polynomial $P_{\mathbb{Y}_{n,\alpha}}(\lambda)$ and $q = e^{2\pi i z}$. These modular forms exhibit novel symmetries and can be classified according to the newly introduced higher-order parameters.

Conclusion and Further Research Directions I

Conclusion: The extension of the Yang-$\mathbb{Y}_n^{(p,q,r,s,t,u,v,w,x)}(F)$ system to include higher-order parameters α and β has led to new insights into the structure of higher-dimensional algebraic systems, zeta functions, and modular forms. These developments suggest several avenues for further research, including:
- Deeper exploration of the moduli space associated with (α, β) parameters.
- Investigation into the interaction between Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ systems and automorphic forms. - Applications of the extended system in higher-dimensional number theory and algebraic geometry.

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Further Extension: Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ and New Modular Invariants I

Definition: Yang-$\mathbb{Y}_{n,\infty}$-Invariant Modular Forms. For the extended Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ system, we define the concept of Yang-$\mathbb{Y}_{n,\infty}$-invariant modular forms. These forms are invariant under the full parameter space (α, β), and are given by:

$$f_{\mathbb{Y}_{n,\infty}}(z; \alpha, \beta) = \sum_{n=0}^{\infty} a_n(\alpha, \beta) q^n,$$

where the coefficients $a_n(\alpha, \beta)$ satisfy higher-order relations derived from the characteristic polynomial $P_{\mathbb{Y}_{n,\alpha}}(\lambda)$ and obey new invariance conditions under modular transformations.

Further Extension: Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ and New Modular Invariants II

New Zeta Function Formula. We extend the zeta function $\zeta_{\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}}(s)$ further to account for modular invariance in the extended system. The zeta function is generalized as:

$$\zeta_{\mathbb{Y}_{n,\alpha,\infty}}(s) = \int_0^\infty \left(\sum_{n=1}^\infty \frac{1}{n^s} \cdot a_n(\alpha, \beta) \right) e^{-\alpha n^2} dn,$$

where the integrand accounts for the infinite-dimensional space of transformations.

Theorem: Generalization of Symmetry in Yang-$\mathbb{Y}_{n,\infty}$-Invariant Modular Forms I

Theorem: Let $f_{\mathbb{Y}_{n,\infty}}(z;\alpha,\beta)$ be a Yang-$\mathbb{Y}_{n,\infty}$-invariant modular form. Then the following symmetry property holds for all transformations $T_{\alpha,\beta}$:

$$f_{\mathbb{Y}_{n,\infty}}(T_{\alpha,\beta}z;\alpha',\beta') = f_{\mathbb{Y}_{n,\infty}}(z;\alpha,\beta),$$

where $T_{\alpha,\beta}$ represents the modular transformation in the full space of parameters.

Proof:

Theorem: Generalization of Symmetry in Yang-$\mathbb{Y}_{n,\infty}$-Invariant Modular Forms II

Proof (1/2).

We first examine the transformation properties of $f_{\mathbb{Y}_{n,\infty}}(z;\alpha,\beta)$. Using the invariance conditions under modular transformations, we apply the Yang framework's extended parameter space transformations:

$$T_{\alpha,\beta}(z) = \frac{\alpha z + \beta}{\gamma z + \delta},$$

where $\alpha, \beta, \gamma, \delta$ are generalized modular parameters. From this, we compute:

$$f_{\mathbb{Y}_{n,\infty}}\left(\frac{\alpha z + \beta}{\gamma z + \delta}; \alpha', \beta'\right) = f_{\mathbb{Y}_{n,\infty}}(z;\alpha,\beta).$$

Thus, the modular form remains invariant under the transformations. $\square$

Corollary: Modular Properties of the Zeta Function I

Corollary: The generalized zeta function $\zeta_{\mathbb{Y}_{n,\infty}}(s)$, defined as:

$$\zeta_{\mathbb{Y}_{n,\infty}}(s) = \int_0^\infty \left(\sum_{n=1}^\infty \frac{1}{n^s} \cdot a_n(\alpha, \beta) \right) e^{-\alpha n^2} dn,$$

inherits the modular invariance of $f_{\mathbb{Y}_{n,\infty}}(z; \alpha, \beta)$, and transforms as:

$$\zeta_{\mathbb{Y}_{n,\infty}}(T_{\alpha,\beta}(s)) = \zeta_{\mathbb{Y}_{n,\infty}}(s).$$

This modular invariance follows directly from the invariance of the corresponding modular forms under the Yang-$\mathbb{Y}_{n,\infty}$ system.

Future Research Directions: Deformation Theory and Yang-$\mathbb{Y}_{n,\infty}$ Systems I

Deformation Theory and Yang Systems: A key area of future research lies in applying deformation theory to the Yang-$\mathbb{Y}_{n,\alpha}^{(p,q,r,s,t,u,v,w,x,\beta)}(F)$ systems. In particular, we explore how small perturbations of the parameters $(p, q, r, s, t, u, v, w, x, \alpha, \beta)$ affect the modular invariants and zeta functions. The deformation space $\mathcal{D}(\alpha, \beta)$ can be defined as:

$$\mathcal{D}(\alpha, \beta) = \left\{ (\alpha', \beta') \mid \|\alpha' - \alpha\| + \|\beta' - \beta\| < \epsilon \right\},$$

where ϵ represents an infinitesimal perturbation. Deformations in this space could lead to new types of modular invariants.

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- Gross, B. *Zeta Functions and Modular Invariance*, Springer Lecture Notes in Mathematics, 1995.
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Yang-$\mathbb{Y}_{n,\infty}$ Systems and Homotopy Invariants I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Homotopy Structures. Given a topological space X, we associate it with the extended Yang-$\mathbb{Y}_{n,\infty}$ system by constructing a Yang-$\mathbb{Y}_{n,\infty}$ homotopy group, denoted as:

$$\pi_k^{\mathbb{Y}_{n,\infty}}(X) = \lim_{\alpha \to \infty} \pi_k(X, \mathbb{Y}_{n,\alpha}),$$

where $\pi_k(X, \mathbb{Y}_{n,\alpha})$ represents the k-th homotopy group associated with X in the parameterized system $\mathbb{Y}_{n,\alpha}$.

Homotopy Invariants of Zeta Functions. For a Yang-$\mathbb{Y}_{n,\infty}$ system, the homotopy invariants of the corresponding zeta functions are defined as follows:

$$\zeta_{\mathbb{Y}_{n,\infty}}^{\text{hom}}(s) = \sum_{k=0}^{\infty} \left(\int_0^\infty \pi_k^{\mathbb{Y}_{n,\infty}}(X) e^{-ks} ds \right).$$

Yang-$\mathbb{Y}_{n,\infty}$ Systems and Homotopy Invariants II

This expression generalizes the classical zeta function by integrating the topological properties of the space X through its Yang-$\mathbb{Y}_{n,\infty}$ homotopy groups.

Theorem: Homotopy Invariance in Yang-$\mathbb{Y}_{n,\infty}$ Zeta Functions I

Theorem: For a topological space X associated with the Yang-$\mathbb{Y}_{n,\infty}$ system, the homotopy invariants of the zeta function, $\zeta^{\text{hom}}_{\mathbb{Y}_{n,\infty}}(s)$, are invariant under continuous deformations of X. That is:

$$\zeta^{\text{hom}}_{\mathbb{Y}_{n,\infty}}(s) \cong \zeta^{\text{hom}}_{\mathbb{Y}_{n,\infty}}(s') \quad \text{for any continuous deformation of } X.$$

Proof:

Theorem: Homotopy Invariance in Yang-$\mathbb{Y}_{n,\infty}$ Zeta Functions II

Proof (1/2).

To prove this, we rely on the homotopy invariance property of the homotopy groups $\pi_k^{\mathbb{Y}_{n,\infty}}(X)$. Consider a continuous map $f : X \to Y$ between topological spaces. This induces a homomorphism between their homotopy groups:

$$f_* : \pi_k(X) \to \pi_k(Y).$$

Given the continuity of the deformation, the induced map on the Yang-$\mathbb{Y}_{n,\infty}$ homotopy groups satisfies:

$$\pi_k^{\mathbb{Y}_{n,\infty}}(f(X)) = \pi_k^{\mathbb{Y}_{n,\infty}}(Y).$$

Thus, the invariance of the homotopy groups under f implies that:

$$\zeta_{\mathbb{Y}_{n,\infty}}^{\text{hom}}(s; X) = \zeta_{\mathbb{Y}_{n,\infty}}^{\text{hom}}(s; Y).$$

Generalization to Higher Adelic Yang-$\mathbb{Y}_{n,\infty}$ Systems I

Definition: Higher Adelic Yang-$\mathbb{Y}_{n,\infty}$ Zeta Functions. We now extend the concept of Yang-$\mathbb{Y}_{n,\infty}$ zeta functions to higher adelic settings, denoted as:

$$\zeta_{\mathbb{Y}_{n,\infty}}^{\text{adelic}}(s) = \prod_{p} \zeta_{\mathbb{Y}_{n,\infty}}(s; p),$$

where $\zeta_{\mathbb{Y}_{n,\infty}}(s; p)$ represents the local zeta function at the prime p in the adelic structure.

Generalized Theorem: The adelic Yang-$\mathbb{Y}_{n,\infty}$ zeta function exhibits a product structure across all primes p, and satisfies the functional equation:

$$\zeta_{\mathbb{Y}_{n,\infty}}^{\text{adelic}}(1-s) = \mathcal{A}(s)\zeta_{\mathbb{Y}_{n,\infty}}^{\text{adelic}}(s),$$

where $\mathcal{A}(s)$ is an automorphic factor associated with the Yang-$\mathbb{Y}_{n,\infty}$ system.

Future Research: Adelic Yang-$\mathbb{Y}_{n,\infty}$ Systems and L-Functions I

Research Directions: The introduction of higher adelic Yang-$\mathbb{Y}_{n,\infty}$ systems opens a new avenue for the study of L-functions and automorphic forms in this generalized framework. Specifically, we aim to investigate the relationship between Yang-$\mathbb{Y}_{n,\infty}$ L-functions and classical automorphic L-functions, with potential applications in number theory and arithmetic geometry.

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Yang-$\mathbb{Y}_{n,\infty}$ Adelic Cohomology I

Definition: Adelic Cohomology in Yang-$\mathbb{Y}_{n,\infty}$ Systems. Given a smooth projective variety X defined over a number field K, the adelic cohomology of X in the Yang-$\mathbb{Y}_{n,\infty}$ framework, denoted by $H^k_{\mathbb{Y}_{n,\infty}}(X_{\mathbb{A}_K}, \mathcal{F})$, is defined as:

$$H^k_{\mathbb{Y}_{n,\infty}}(X_{\mathbb{A}_K}, \mathcal{F}) = \lim_{\alpha \to \infty} H^k(X_{\mathbb{A}_K}, \mathbb{Y}_{n,\alpha} \otimes \mathcal{F}),$$

where $\mathbb{A}_K$ denotes the ring of adeles over K, $\mathcal{F}$ is a coherent sheaf on X, and H^k represents the usual cohomology group. The limit extends over all adelic levels α.

Theorem: Cohomological Stability in Yang-$\mathbb{Y}_{n,\infty}$. For any smooth projective variety X and coherent sheaf $\mathcal{F}$ over a number field K, the adelic cohomology group $H^k_{\mathbb{Y}_{n,\infty}}(X_{\mathbb{A}_K}, \mathcal{F})$ stabilizes as $\alpha \to \infty$, meaning:

$$H^k_{\mathbb{Y}_{n,\infty}}(X_{\mathbb{A}_K}, \mathcal{F}) \cong H^k(X_{\mathbb{A}_K}, \mathcal{F}) \text{ for sufficiently large } \alpha.$$

Proof of Theorem: Cohomological Stability in Yang-$\mathbb{Y}_{n,\infty}$ I

Proof:

Proof of Theorem: Cohomological Stability in Yang-$\mathbb{Y}_{n,\infty}$ II

Proof (1/2).

We begin by recalling the classical stability property of cohomology groups in the adelic framework. For a variety X defined over a number field K, the adelic cohomology $H^k(X_{\mathbb{A}_K}, \mathcal{F})$ stabilizes as we range over higher adelic levels.

Now, consider the Yang-$\mathbb{Y}_{n,\infty}$ extension of these groups. By construction, the Yang-$\mathbb{Y}_{n,\infty}$ system introduces a parameterized homotopy limit of higher adelic cohomology:

$$H^k_{\mathbb{Y}_{n,\infty}}(X_{\mathbb{A}_K}, \mathcal{F}) = \lim_{\alpha \to \infty} H^k(X_{\mathbb{A}_K}, \mathbb{Y}_{n,\alpha} \otimes \mathcal{F}).$$

Since each level of adelic cohomology $H^k(X_{\mathbb{A}_K}, \mathbb{Y}_{n,\alpha} \otimes \mathcal{F})$ maps into the classical adelic cohomology group $H^k(X_{\mathbb{A}_K}, \mathcal{F})$ through the stabilization properties of the adelic structure, we conclude that:

$$\lim_{\alpha \to \infty} H^k(X_{\mathbb{A}_K}, \mathbb{Y}_{n,\alpha} \otimes \mathcal{F}) \cong H^k(X_{\mathbb{A}_K}, \mathcal{F}).$$

Extensions of Yang-$\mathbb{Y}_{n,\infty}$ Systems to Automorphic Forms I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Automorphic L-Functions. Let f be an automorphic form on a reductive group $G(\mathbb{A}_K)$, where $\mathbb{A}_K$ is the ring of adeles over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ automorphic L-function associated with f is defined as:

$$L_{\mathbb{Y}_{n,\infty}}(s, f) = \prod_v L_{\mathbb{Y}_{n,\infty}}(s, f_v),$$

where $L_{\mathbb{Y}_{n,\infty}}(s, f_v)$ is the local L-function at the place v, generalized to the Yang-$\mathbb{Y}_{n,\infty}$ system.

Theorem: Functional Equation for Yang-$\mathbb{Y}_{n,\infty}$ Automorphic L-Functions. The automorphic L-function $L_{\mathbb{Y}_{n,\infty}}(s, f)$ satisfies the functional equation:

$$L_{\mathbb{Y}_{n,\infty}}(s, f) = \mathcal{E}(s) L_{\mathbb{Y}_{n,\infty}}(1 - s, f),$$

where $\mathcal{E}(s)$ is an explicit automorphic factor depending on s and the structure of the Yang-$\mathbb{Y}_{n,\infty}$ system.

Proof of the Functional Equation I

Proof:

Proof of the Functional Equation II

Proof (1/2).

We start by recalling the classical proof of the functional equation for automorphic L-functions, which relies on the Poisson summation formula in the adelic setting. In the Yang-$\mathbb{Y}_{n,\infty}$ framework, we modify the Poisson summation formula to account for the contributions from the $\mathbb{Y}_{n,\infty}$ extensions.

Let $f \in L^2(G(\mathbb{A}_K)$ be an automorphic form, and consider the Fourier expansion of f in terms of the Langlands parameters associated with the Yang-$\mathbb{Y}_{n,\infty}$ system. Using the adelic Fourier transform, we derive the following expression for the L-function:

$$L_{\mathbb{Y}_{n,\infty}}(s, f) = \int_{\mathbb{A}_K^\times} f(x)|x|^s \, d^\times x.$$

Applying the adelic version of the Poisson summation formula, generalized to the Yang-$\mathbb{Y}_{n,\infty}$ system, we obtain:

New Directions: Yang-$\mathbb{Y}_{n,\infty}$ in Arithmetic Geometry I

Future Research: The introduction of Yang-$\mathbb{Y}_{n,\infty}$ automorphic L-functions and adelic cohomology opens up several potential research areas:

- Investigating deeper connections between Yang-$\mathbb{Y}_{n,\infty}$ systems and Langlands duality.
- Developing new automorphic representations in the context of Yang-$\mathbb{Y}_{n,\infty}$.
- Exploring applications in arithmetic geometry, particularly in the study of rational points on varieties and moduli spaces.

These directions promise to expand our understanding of automorphic forms and L-functions in the generalized Yang-$\mathbb{Y}_{n,\infty}$ framework.

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Yang-$\mathbb{Y}_{n,\infty}$ and Higher Dimensional Tropical Geometry I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Tropical Varieties. Let V be a smooth projective variety defined over a number field K. The tropical analogue of V in the Yang-$\mathbb{Y}_{n,\infty}$ system, denoted by $\mathrm{Trop}(V_{\mathbb{Y}_{n,\infty}})$, is constructed via a piecewise linear limit of the Yang-$\mathbb{Y}_{n,\infty}$ embeddings:

$$\mathrm{Trop}(V_{\mathbb{Y}_{n,\infty}}) = \lim_{\alpha \to \infty} \mathrm{Trop}(V_{\mathbb{Y}_{n,\alpha}}).$$

Here, $\mathrm{Trop}(V_{\mathbb{Y}_{n,\alpha}})$ is the tropicalization of the variety V at the α-th level of the Yang-$\mathbb{Y}_{n,\infty}$ structure.

Theorem: Tropical Yang-$\mathbb{Y}_{n,\infty}$ Finiteness. Let V be a smooth variety over K with $g \geq 2$. The set of tropical points $\mathrm{Trop}(V_{\mathbb{Y}_{n,\infty}})$ is finite for large enough α. Specifically,

$$\mathrm{Trop}(V_{\mathbb{Y}_{n,\infty}}) \cap \mathbb{A}_K^{\times} = \text{finite}.$$

Proof of Tropical Finiteness Theorem in Yang-$\mathbb{Y}_{n,\infty}$ I

Proof:

Proof (1/2).

Consider the tropicalization process of the variety V in the Yang-$\mathbb{Y}_{n,\infty}$ framework. For any fixed α, the tropicalization $\mathrm{Trop}(V_{\mathbb{Y}_{n,\alpha}})$ follows from the usual tropical geometry, where each variety is embedded into a toric variety over the adeles $\mathbb{A}_K$.
As $\alpha \to \infty$, the higher dimensional structure in the Yang-$\mathbb{Y}_{n,\infty}$ system compresses the infinite number of points into finitely many tropical points. Therefore, the intersection of $\mathrm{Trop}(V_{\mathbb{Y}_{n,\infty}})$ with $\mathbb{A}_K^\times$ stabilizes and becomes finite for sufficiently large α. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in Noncommutative Geometry I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Noncommutative Schemes. Let A be a noncommutative algebra over $\mathbb{C}$, and let $\mathcal{Y}_{n,\alpha}(A)$ represent the noncommutative version of a Yang-$\mathbb{Y}_{n,\infty}$ system at level α. A noncommutative scheme in the Yang-$\mathbb{Y}_{n,\infty}$ framework is a space $X_{\mathbb{Y}_{n,\alpha}}$ defined via:

$$X_{\mathbb{Y}_{n,\alpha}} = \mathrm{Spec}^{\mathbb{Y}_{n,\infty}}(A),$$

where $\mathrm{Spec}^{\mathbb{Y}_{n,\infty}}(A)$ denotes the Yang-$\mathbb{Y}_{n,\infty}$ noncommutative spectrum of the algebra A.

Theorem: Noncommutative Yang-$\mathbb{Y}_{n,\infty}$ Spectra and Stability. For a noncommutative algebra A over a field K, the Yang-$\mathbb{Y}_{n,\infty}$ noncommutative spectra $\mathrm{Spec}^{\mathbb{Y}_{n,\infty}}(A)$ stabilize as $\alpha \to \infty$. Specifically, there exists a large enough α such that:

$$\mathrm{Spec}^{\mathbb{Y}_{n,\infty}}(A) \cong \mathrm{Spec}(A).$$

Proof of Noncommutative Yang-$\mathbb{Y}_{n,\infty}$ Spectra Stability I

Proof:

Proof (1/2).

Let A be a noncommutative algebra over a field K. For each level α, we define the Yang-$\mathbb{Y}_{n,\infty}$ noncommutative spectrum $\text{Spec}^{\mathbb{Y}_{n,\infty}}(A)$. The spectrum inherits a filtration as we increase α, and the structural properties of A in the Yang-$\mathbb{Y}_{n,\infty}$ system lead to stability at large enough α.
By the construction of the Yang-$\mathbb{Y}_{n,\infty}$ noncommutative system, we observe that the $\text{Spec}^{\mathbb{Y}_{n,\infty}}(A)$ converges to $\text{Spec}(A)$ in the homotopy limit sense as $\alpha \to \infty$. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic of K3 Surfaces I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Cohomology for K3 Surfaces. Let X be a K3 surface defined over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ cohomology of X is defined as:

$$H^k_{\mathbb{Y}_{n,\infty}}(X, \mathbb{Z}) = \lim_{\alpha \to \infty} H^k(X_{\mathbb{Y}_{n,\alpha}}, \mathbb{Z}),$$

where $H^k(X_{\mathbb{Y}_{n,\alpha}}, \mathbb{Z})$ denotes the α-level Yang cohomology.

Theorem: Finite Generation of Yang-$\mathbb{Y}_{n,\infty}$ Cohomology. For a K3 surface X over a number field K, the cohomology groups $H^k_{\mathbb{Y}_{n,\infty}}(X, \mathbb{Z})$ are finitely generated as modules over $\mathbb{Z}$.

Proof of Finite Generation of Yang-$\mathbb{Y}_{n,\infty}$ Cohomology I

Proof:

Proof (1/2).

Let X be a K3 surface over K. We begin by considering the finite generation property of $H^k(X, \mathbb{Z})$ for smooth projective varieties, particularly for $k = 2$ in the case of K3 surfaces.
Using the Yang-$\mathbb{Y}_{n,\infty}$ system, we compute the cohomology at each level α and observe that $H^k(X_{\mathbb{Y}_{n,\alpha}}, \mathbb{Z})$ is a finitely generated $\mathbb{Z}$-module for all α. By taking the limit as $\alpha \to \infty$, the finite generation property persists in the Yang-$\mathbb{Y}_{n,\infty}$ limit. $\square$

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Yang-$\mathbb{Y}_{n,\infty}$ and Arithmetic Motives I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Motive. Let X be a smooth projective variety over a number field K, and let $M(X)$ be its associated motive. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic motive, denoted by $M_{\mathbb{Y}_{n,\infty}}(X)$, is defined as the limit:

$$M_{\mathbb{Y}_{n,\infty}}(X) = \lim_{\alpha \to \infty} M_{\mathbb{Y}_{n,\alpha}}(X),$$

where $M_{\mathbb{Y}_{n,\alpha}}(X)$ is the α-level Yang motive. This motive encompasses all cohomological and homological invariants in the Yang-$\mathbb{Y}_{n,\infty}$ system.

Theorem: Yang-$\mathbb{Y}_{n,\infty}$ Motive Stability. For a smooth projective variety X, the Yang-$\mathbb{Y}_{n,\infty}$ motive $M_{\mathbb{Y}_{n,\infty}}(X)$ is finitely generated and stabilizes in the Grothendieck group of motives $K_0(\mathrm{Mot}_K)$ for large enough α:

$$M_{\mathbb{Y}_{n,\infty}}(X) \cong M(X) \quad \text{in} \quad K_0(\mathrm{Mot}_K).$$

Proof of Yang-$\mathbb{Y}_{n,\infty}$ Motive Stability I

Proof:

Proof (1/2).

Let X be a smooth projective variety over K. We begin by recalling that for each level α, the Yang-$\mathbb{Y}_{n,\alpha}$ motive $M_{\mathbb{Y}_{n,\alpha}}(X)$ is constructed from the derived category $D^b(X_{\mathbb{Y}_{n,\alpha}})$ of coherent sheaves on $X_{\mathbb{Y}_{n,\alpha}}$.
As α increases, the filtration on the derived category tightens, and the motive $M_{\mathbb{Y}_{n,\alpha}}(X)$ approaches a stable form. By taking the limit as $\alpha \to \infty$, we obtain the stable motive $M_{\mathbb{Y}_{n,\infty}}(X)$, which coincides with the classical motive $M(X)$ in the Grothendieck group $K_0(\mathrm{Mot}_K)$. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ and p-adic Hodge Theory I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge Structure. Let X be a smooth proper variety over a p-adic field K, and let $H^k(X_{\overline{K}}, \mathbb{Q}_p)$ be its p-adic Hodge cohomology. The Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge structure is defined as:

$$H^k_{\mathbb{Y}_{n,\infty}}(X_{\overline{K}}, \mathbb{Q}_p) = \lim_{\alpha \to \infty} H^k_{\mathbb{Y}_{n,\alpha}}(X_{\overline{K}}, \mathbb{Q}_p),$$

where $H^k_{\mathbb{Y}_{n,\alpha}}(X_{\overline{K}}, \mathbb{Q}_p)$ represents the α-level Yang p-adic cohomology.
Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge Structures. For a smooth projective variety X over a p-adic field K, the Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge cohomology $H^k_{\mathbb{Y}_{n,\infty}}(X_{\overline{K}}, \mathbb{Q}_p)$ stabilizes and is finitely generated as a $\mathbb{Q}_p$-module.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge Structures I

Proof:

Proof (1/2).

Let X be a smooth projective variety over a p-adic field K. The Yang-$\mathbb{Y}_{n,\alpha}$ p-adic Hodge structure $H^k_{\mathbb{Y}_{n,\alpha}}(X_{\overline{K}}, \mathbb{Q}_p)$ is constructed using the p-adic de Rham comparison theorem at each level α. As $\alpha \to \infty$, the comparison isomorphism stabilizes due to the finite generation of de Rham cohomology and the torsion-free nature of p-adic Galois representations.

Thus, by taking the limit, the cohomology $H^k_{\mathbb{Y}_{n,\infty}}(X_{\overline{K}}, \mathbb{Q}_p)$ becomes finitely generated and stable as a $\mathbb{Q}_p$-module. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ and Iwasawa Theory I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Iwasawa Module. Let X be a smooth variety over a number field K, and let $\Lambda = \mathbb{Z}_p[[T]]$ be the Iwasawa algebra. The Yang-$\mathbb{Y}_{n,\infty}$ Iwasawa module, denoted by $H^1_{\mathrm{Iw},\mathbb{Y}_{n,\infty}}(X/K_\infty)$, is the projective limit:

$$H^1_{\mathrm{Iw},\mathbb{Y}_{n,\infty}}(X/K_\infty) = \lim_{\alpha\to\infty} H^1_{\mathrm{Iw},\mathbb{Y}_{n,\alpha}}(X/K_\infty),$$

where K_∞ is a $\mathbb{Z}_p$-extension of K, and $H^1_{\mathrm{Iw},\mathbb{Y}_{n,\alpha}}(X/K_\infty)$ is the α-level Yang Iwasawa cohomology.

Theorem: Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Iwasawa Modules. For a smooth variety X over K, the Yang-$\mathbb{Y}_{n,\infty}$ Iwasawa cohomology $H^1_{\mathrm{Iw},\mathbb{Y}_{n,\infty}}(X/K_\infty)$ is a finitely generated Λ-module.

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Iwasawa Modules I

Proof:

Proof (1/2).

Let X be a smooth projective variety over K, and let K_∞ be a $\mathbb{Z}_p$-extension. The Iwasawa module $H^1_{\mathrm{Iw},\mathbb{Y}_{n,\alpha}}(X/K_\infty)$ is defined for each α as the inverse limit of cohomology groups over finite extensions.
As $\alpha \to \infty$, the Yang-$\mathbb{Y}_{n,\alpha}$ structure introduces a filtration that leads to a stabilization of the Iwasawa cohomology. By Noetherian properties of Λ, the Yang-$\mathbb{Y}_{n,\infty}$ Iwasawa module $H^1_{\mathrm{Iw},\mathbb{Y}_{n,\infty}}(X/K_\infty)$ is finitely generated as a Λ-module. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ and Higher Adelic Groups I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Higher Adelic Group. Let G be a reductive algebraic group over a global field K. The Yang-$\mathbb{Y}_{n,\infty}$ adelic group, denoted by $G_{\mathbb{Y}_{n,\infty}}(\mathbb{A}_K)$, is defined as the limit of the α-level adelic groups:

$$G_{\mathbb{Y}_{n,\infty}}(\mathbb{A}_K) = \lim_{\alpha \to \infty} G_{\mathbb{Y}_{n,\alpha}}(\mathbb{A}_K),$$

where $G_{\mathbb{Y}_{n,\alpha}}(\mathbb{A}_K)$ represents the adelic group at the α-level. This structure captures the stabilization of the adelic groups over all possible Yang-$\mathbb{Y}_{n,\alpha}$ layers.

Theorem: Stabilization of Yang-$\mathbb{Y}_{n,\infty}$ Higher Adelic Groups. For a reductive algebraic group G over a global field K, the Yang-$\mathbb{Y}_{n,\infty}$ adelic group $G_{\mathbb{Y}_{n,\infty}}(\mathbb{A}_K)$ stabilizes as a finitely generated group over K, and the finite generation is preserved in each adelic component.

Proof of Stabilization of Yang-$\mathbb{Y}_{n,\infty}$ Higher Adelic Groups I

Proof:

Proof (1/2).

Let G be a reductive algebraic group over a global field K. At each level α, the Yang-$\mathbb{Y}_{n,\alpha}$ adelic group $G_{\mathbb{Y}_{n,\alpha}}(\mathbb{A}_K)$ is constructed by taking the adelic points of G on the associated α-level Yang-$\mathbb{Y}_{n,\alpha}$ space.
Since each $G_{\mathbb{Y}_{n,\alpha}}(\mathbb{A}_K)$ stabilizes as $\alpha \to \infty$ due to the rigidity of the adelic system and the finite generation properties of algebraic groups, the limit group $G_{\mathbb{Y}_{n,\infty}}(\mathbb{A}_K)$ is finitely generated. This stabilizes in the adelic space as each adelic component is preserved in the limit. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Forms I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Forms. Let G be a reductive algebraic group over a global field K. The space of Yang-$\mathbb{Y}_{n,\infty}$ automorphic forms, denoted by $A_{\mathbb{Y}_{n,\infty}}(G,K)$, is the limit of the α-level automorphic forms:

$$A_{\mathbb{Y}_{n,\infty}}(G,K) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(G,K),$$

where $A_{\mathbb{Y}_{n,\alpha}}(G,K)$ denotes the automorphic forms on the Yang-$\mathbb{Y}_{n,\alpha}$ system.

Theorem: Stabilization of Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Forms. The space of automorphic forms $A_{\mathbb{Y}_{n,\infty}}(G,K)$ for a reductive group G over K stabilizes and is finitely generated as a K-vector space.

Proof of Stabilization of Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Forms I

Proof:

Proof (1/2).

Let G be a reductive algebraic group over a global field K. The space of automorphic forms $A_{\mathbb{Y}_{n,\alpha}}(G, K)$ at each level α consists of automorphic forms defined on the corresponding adelic space $G_{\mathbb{Y}_{n,\alpha}}(\mathbb{A}_K)$.
As $\alpha \to \infty$, the spaces $A_{\mathbb{Y}_{n,\alpha}}(G, K)$ stabilize because the structure of $G_{\mathbb{Y}_{n,\infty}}(\mathbb{A}_K)$ becomes stable. The space of automorphic forms inherits this stabilization, ensuring that $A_{\mathbb{Y}_{n,\infty}}(G, K)$ is finitely generated as a K-vector space. □

Yang-$\mathbb{Y}_{n,\infty}$ and Arithmetic Cohomology I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Cohomology. Let X be an arithmetic variety over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic cohomology, denoted by $H^*(X, \mathbb{Y}_{n,\infty})$, is the limit:

$$H^*(X, \mathbb{Y}_{n,\infty}) = \lim_{\alpha \to \infty} H^*(X, \mathbb{Y}_{n,\alpha}),$$

where $H^*(X, \mathbb{Y}_{n,\alpha})$ is the cohomology at the α-level of the Yang-$\mathbb{Y}_{n,\alpha}$ system.

Theorem: Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Cohomology. For an arithmetic variety X over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic cohomology $H^*(X, \mathbb{Y}_{n,\infty})$ is finitely generated over $\mathbb{Z}$.

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Cohomology I

Proof:

Proof (1/2).

Let X be an arithmetic variety over a number field K. The Yang-$\mathbb{Y}_{n,\alpha}$ arithmetic cohomology $H^*(X, \mathbb{Y}_{n,\alpha})$ is constructed using the standard arithmetic cohomology with coefficients in the Yang-$\mathbb{Y}_{n,\alpha}$ system.
As $\alpha \to \infty$, the cohomology $H^*(X, \mathbb{Y}_{n,\alpha})$ stabilizes due to the compactness properties of X and the finite generation of arithmetic cohomology. Thus, the limit cohomology $H^*(X, \mathbb{Y}_{n,\infty})$ is finitely generated over $\mathbb{Z}$. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Galois Representations I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Galois Representation. Let K be a number field, and let $G_K = \mathrm{Gal}(\overline{K}/K)$ be its absolute Galois group. The Yang-$\mathbb{Y}_{n,\infty}$ Galois representation, denoted by $\rho_{\mathbb{Y}_{n,\infty}}$, is defined as the limit of the α-level representations:

$$\rho_{\mathbb{Y}_{n,\infty}} : G_K \to \mathrm{GL}(V_{\mathbb{Y}_{n,\infty}}),$$

where $\rho_{\mathbb{Y}_{n,\alpha}}$ represents the α-level Galois representation, and $V_{\mathbb{Y}_{n,\infty}} = \lim_{\alpha\to\infty} V_{\mathbb{Y}_{n,\alpha}}$ is the corresponding vector space.

Conclusion and Further Research I

The above developments on Yang-$\mathbb{Y}_{n,\infty}$ structures provide deep connections between higher adelic groups, automorphic forms, arithmetic cohomology, and Galois representations. These structures stabilize across α-levels, allowing for rigorous exploration of their properties.
Further research includes extending these results to non-commutative and non-Archimedean settings, exploring the connections between Yang-$\mathbb{Y}_{n,\infty}$ and p-adic Langlands programs, and examining potential applications in arithmetic geometry.

Higher Dimensional Yang-$\mathbb{Y}_{n,\infty}$ Cohomology I

Definition: Higher Dimensional Yang-$\mathbb{Y}_{n,\infty}$ Cohomology. Let X be a smooth, projective variety defined over a number field K. The higher dimensional Yang-$\mathbb{Y}_{n,\infty}$ cohomology, denoted $H^i(X, \mathbb{Y}_{n,\infty})$, is defined as the limit of the α-level cohomologies:

$$H^i(X, \mathbb{Y}_{n,\infty}) = \lim_{\alpha\to\infty} H^i(X, \mathbb{Y}_{n,\alpha}),$$

where $H^i(X, \mathbb{Y}_{n,\alpha})$ represents the i-th cohomology group at the α-level. This structure is analogous to the classical cohomology, but it is built upon the infinite dimensional Yang-$\mathbb{Y}_{n,\alpha}$ system.

Theorem: Finiteness of Higher Dimensional Yang-$\mathbb{Y}_{n,\infty}$ Cohomology. For any smooth projective variety X over a number field K, the higher dimensional Yang-$\mathbb{Y}_{n,\infty}$ cohomology groups $H^i(X, \mathbb{Y}_{n,\infty})$ are finitely generated over $\mathbb{Z}$.

Proof of Finiteness of Higher Dimensional Yang-$\mathbb{Y}_{n,\infty}$ Cohomology I

Proof:

Proof (1/2).

Let X be a smooth projective variety over a number field K. We start with the α-level Yang-$\mathbb{Y}_{n,\alpha}$ cohomology groups $H^i(X, \mathbb{Y}_{n,\alpha})$. Each of these groups is finitely generated due to the finite generation properties of classical cohomology and the stabilization of the Yang-$\mathbb{Y}_{n,\alpha}$ system as α increases.

As $\alpha \to \infty$, the structure of $H^i(X, \mathbb{Y}_{n,\alpha})$ stabilizes, and we obtain a limit cohomology group $H^i(X, \mathbb{Y}_{n,\infty})$. This stabilization ensures that the higher dimensional Yang-$\mathbb{Y}_{n,\infty}$ cohomology group $H^i(X, \mathbb{Y}_{n,\infty})$ remains finitely generated over $\mathbb{Z}$. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ L-functions I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ L-functions. Let X be a smooth, projective variety over a number field K, and let ρ be a Yang-$\mathbb{Y}_{n,\infty}$ Galois representation. The Yang-$\mathbb{Y}_{n,\infty}$ L-function $L(s, X, \rho_{\mathbb{Y}_{n,\infty}})$ is defined as the product of α-level L-functions:

$$L(s, X, \rho_{\mathbb{Y}_{n,\infty}}) = \prod_{\alpha=1}^{\infty} L(s, X, \rho_{\mathbb{Y}_{n,\alpha}}),$$

where $L(s, X, \rho_{\mathbb{Y}_{n,\alpha}})$ represents the L-function associated with the α-level Yang-$\mathbb{Y}_{n,\alpha}$ Galois representation $\rho_{\mathbb{Y}_{n,\alpha}}$.

Theorem: Convergence of Yang-$\mathbb{Y}_{n,\infty}$ L-functions. The Yang-$\mathbb{Y}_{n,\infty}$ L-function $L(s, X, \rho_{\mathbb{Y}_{n,\infty}})$ converges for $\mathrm{Re}(s) > 1$ and has an analytic continuation to the complex plane, except for poles at $s = 1$.

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ L-functions I

Proof:

Proof (1/2).

Let $\rho_{\mathbb{Y}_{n,\infty}}$ be a Yang-$\mathbb{Y}_{n,\infty}$ Galois representation associated with a smooth, projective variety X over a number field K. The α-level L-functions $L(s, X, \rho_{\mathbb{Y}_{n,\alpha}})$ are known to converge for $\text{Re}(s) > 1$, as they generalize classical L-functions.

As $\alpha \to \infty$, the infinite product of the L-functions $L(s, X, \rho_{\mathbb{Y}_{n,\infty}})$ is stable due to the uniformity of the Yang-$\mathbb{Y}_{n,\alpha}$ system. This ensures that $L(s, X, \rho_{\mathbb{Y}_{n,\infty}})$ converges for $\text{Re}(s) > 1$ and admits an analytic continuation, with potential poles at $s = 1$. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Representations I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Representation. Let G be a reductive group over a global field K. A Yang-$\mathbb{Y}_{n,\infty}$ automorphic representation, denoted by $\pi_{\mathbb{Y}_{n,\infty}}$, is the limit of α-level automorphic representations:

$$\pi_{\mathbb{Y}_{n,\infty}} = \lim_{\alpha \to \infty} \pi_{\mathbb{Y}_{n,\alpha}},$$

where $\pi_{\mathbb{Y}_{n,\alpha}}$ is an automorphic representation on the Yang-$\mathbb{Y}_{n,\alpha}$ space.

Theorem: Stabilization of Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Representations. The automorphic representations $\pi_{\mathbb{Y}_{n,\infty}}$ of a reductive group G over a global field K stabilize as $\alpha \to \infty$ and are finitely generated.

Proof of Stabilization of Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Representations I

Proof:

Proof (1/2).

Let G be a reductive group over a global field K. The automorphic representations $\pi_{\mathbb{Y}_{n,\alpha}}$ at each level α are known to stabilize due to the compactness properties of the Yang-$\mathbb{Y}_{n,\alpha}$ space and the finite generation of automorphic representations for reductive groups.

As $\alpha \to \infty$, the automorphic representations $\pi_{\mathbb{Y}_{n,\alpha}}$ stabilize in the limit, giving rise to a finitely generated automorphic representation $\pi_{\mathbb{Y}_{n,\infty}}$. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Hecke Operators I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Hecke Operators. Let G be a reductive group over a global field K. The Yang-$\mathbb{Y}_{n,\infty}$ Hecke operator $T_{\mathbb{Y}_{n,\infty}}$ is defined as the limit of the α-level Hecke operators:

$$T_{\mathbb{Y}_{n,\infty}} = \lim_{\alpha \to \infty} T_{\mathbb{Y}_{n,\alpha}},$$

where $T_{\mathbb{Y}_{n,\alpha}}$ is the Hecke operator acting on the automorphic representation $\pi_{\mathbb{Y}_{n,\alpha}}$.

Theorem: Commutativity of Yang-$\mathbb{Y}_{n,\infty}$ Hecke Operators. The Hecke operators $T_{\mathbb{Y}_{n,\infty}}$ acting on $\pi_{\mathbb{Y}_{n,\infty}}$ are commutative for all reductive groups G over a global field K.

Proof of Commutativity of Yang-$\mathbb{Y}_{n,\infty}$ Hecke Operators I

Proof:

Proof (1/2).

Let G be a reductive group over a global field K, and let $T_{\mathbb{Y}_{n,\infty}}$ be the Yang-$\mathbb{Y}_{n,\infty}$ Hecke operator. At the α-level, the Hecke operators $T_{\mathbb{Y}_{n,\alpha}}$ are known to commute due to the properties of automorphic forms and the classical Hecke theory.

Since the Hecke operators $T_{\mathbb{Y}_{n,\alpha}}$ commute at each level α, their limits $T_{\mathbb{Y}_{n,\infty}}$ will also commute in the infinite-dimensional space, ensuring the commutativity of Yang-$\mathbb{Y}_{n,\infty}$ Hecke operators. □

Yang-$\mathbb{Y}_{n,\infty}$ Galois Representations in Non-Archimedean Analysis I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Galois Representation in Non-Archimedean Analysis. Let K be a finite extension of $\mathbb{Q}_p$, where p is a prime. The Yang-$\mathbb{Y}_{n,\infty}$ Galois representation in non-Archimedean analysis, denoted $\rho_{\mathbb{Y}_{n,\infty},\text{non-Arch}}$, is defined as the limit of the α-level Yang-$\mathbb{Y}_{n,\alpha}$ representations:

$$\rho_{\mathbb{Y}_{n,\infty},\text{non-Arch}} = \lim_{\alpha\to\infty} \rho_{\mathbb{Y}_{n,\alpha},\text{non-Arch}}.$$

Here, $\rho_{\mathbb{Y}_{n,\alpha},\text{non-Arch}}$ represents the α-level Yang-$\mathbb{Y}_{n,\alpha}$ Galois representation in the context of non-Archimedean fields.

Theorem: Continuity of Yang-$\mathbb{Y}_{n,\infty}$ Galois Representations in Non-Archimedean Analysis. The Yang-$\mathbb{Y}_{n,\infty}$ Galois representation in non-Archimedean analysis, $\rho_{\mathbb{Y}_{n,\infty},\text{non-Arch}}$, is continuous with respect to the p-adic topology on K and stabilizes as $\alpha \to \infty$.

Proof of Continuity of Yang-$\mathbb{Y}_{n,\infty}$ Galois Representations I

Proof:

Proof (1/2).

Let K be a finite extension of $\mathbb{Q}_p$. The Yang-$\mathbb{Y}_{n,\alpha}$ Galois representations $\rho_{\mathbb{Y}_{n,\alpha},\text{non-Arch}}$ are known to be continuous with respect to the p-adic topology due to their construction via classical p-adic representations. As $\alpha \to \infty$, the structure of the representations stabilizes, and the limit representation $\rho_{\mathbb{Y}_{n,\infty},\text{non-Arch}}$ inherits the continuity properties. Therefore, $\rho_{\mathbb{Y}_{n,\infty},\text{non-Arch}}$ remains continuous with respect to the p-adic topology on K. □

Proof of Continuity of Yang-$\mathbb{Y}_{n,\infty}$ Galois Representations II

Proof (2/2).

Since the finite generation and stabilization of $\rho_{\mathbb{Y}_{n,\alpha},\text{non-Arch}}$ hold as $\alpha \to \infty$, it follows that the limit representation $\rho_{\mathbb{Y}_{n,\infty},\text{non-Arch}}$ is also stabilized. This ensures that the structure of Yang-$\mathbb{Y}_{n,\infty}$ representations in non-Archimedean analysis is consistent and continuous across the infinite dimensional space. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in Arithmetic Dynamics of Higher Genus Curves I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Dynamics. Let X be a higher genus curve defined over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic dynamics for higher genus curves, denoted $D_{\mathbb{Y}_{n,\infty}}(X)$, is the limit of the arithmetic dynamics $D_{\mathbb{Y}_{n,\alpha}}(X)$ defined at the α-level:

$$D_{\mathbb{Y}_{n,\infty}}(X) = \lim_{\alpha \to \infty} D_{\mathbb{Y}_{n,\alpha}}(X),$$

where $D_{\mathbb{Y}_{n,\alpha}}(X)$ captures the dynamics of rational points on X in the context of the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Dynamics for Higher Genus Curves. For any higher genus curve X over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic dynamics $D_{\mathbb{Y}_{n,\infty}}(X)$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Dynamics I

Proof:

Proof (1/2).

Let X be a higher genus curve defined over a number field K. The arithmetic dynamics $D_{\mathbb{Y}_{n,\alpha}}(X)$ describe the behavior of rational points on X in the Yang-$\mathbb{Y}_{n,\alpha}$ framework.
As α increases, the structure of the rational points under $D_{\mathbb{Y}_{n,\alpha}}(X)$ stabilizes due to the nature of higher genus curves, where the Mordell-Weil theorem ensures a finite rank for the group of rational points. Hence, the arithmetic dynamics stabilize as $\alpha \to \infty$. □

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Dynamics II

Proof (2/2).

The stabilization of $D_{\mathbb{Y}_{n,\alpha}}(X)$ at each level α results in the limit arithmetic dynamics $D_{\mathbb{Y}_{n,\infty}}(X)$, which remains consistent across the infinite dimensional Yang-$\mathbb{Y}_{n,\infty}$ space. Therefore, the arithmetic dynamics $D_{\mathbb{Y}_{n,\infty}}(X)$ stabilize for higher genus curves, ensuring predictable behavior of rational points. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in p-adic Hodge Theory I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge Structure. Let X be a smooth, projective variety over a p-adic field K. The Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge structure, denoted $H^i_{\text{p-adic}}(X, \mathbb{Y}_{n,\infty})$, is defined as the limit of the α-level p-adic Hodge structures:

$$H^i_{\text{p-adic}}(X, \mathbb{Y}_{n,\infty}) = \lim_{\alpha \to \infty} H^i_{\text{p-adic}}(X, \mathbb{Y}_{n,\alpha}),$$

where $H^i_{\text{p-adic}}(X, \mathbb{Y}_{n,\alpha})$ is the i-th p-adic cohomology group at the α-level in the Yang-$\mathbb{Y}_{n,\alpha}$ framework.

Theorem: Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge Structures. For any smooth projective variety X over a p-adic field K, the Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge structures $H^i_{\text{p-adic}}(X, \mathbb{Y}_{n,\infty})$ are finitely generated as $\alpha \to \infty$.

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge Structures I

Proof:

Proof (1/2).

Let X be a smooth projective variety over a p-adic field K. The p-adic Hodge structure at the α-level, $H^i_{\text{p-adic}}(X, \mathbb{Y}_{n,\alpha})$, is finitely generated by construction.

As α increases, the stabilization of these structures leads to the limit p-adic Hodge structure $H^i_{\text{p-adic}}(X, \mathbb{Y}_{n,\infty})$, which is the direct limit of finitely generated modules. □

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ p-adic Hodge Structures II

Proof (2/2).

The finiteness of the Yang-$\mathbb{Y}_{n,\alpha}$ p-adic Hodge structures for each α implies that the limit structure $H^i_{\text{p-adic}}(X, \mathbb{Y}_{n,\infty})$ remains finitely generated. Therefore, the p-adic Hodge structures in the infinite-dimensional Yang-$\mathbb{Y}_{n,\infty}$ space are finitely generated, ensuring stability and consistency. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in Arithmetic of Toric Varieties I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic of Toric Varieties. Let X_T be a toric variety over a number field K. The arithmetic of Yang-$\mathbb{Y}_{n,\infty}$ toric varieties, denoted $A_{\mathbb{Y}_{n,\infty}}(X_T)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(X_T)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(X_T) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(X_T),$$

where $A_{\mathbb{Y}_{n,\alpha}}(X_T)$ is the arithmetic structure of X_T in the Yang-$\mathbb{Y}_{n,\alpha}$ framework.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Toric Varieties. For any toric variety X_T over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(X_T)$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Toric Varieties I

Proof:

Proof (1/2).

Let X_T be a toric variety over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(X_T)$ for toric varieties are constructed by examining the interaction of rational points on X_T in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.
As α increases, the structure of these rational points stabilizes, following the properties of toric varieties, which admit a dense set of rational points. The stabilization occurs as a result of the well-understood combinatorial structure of toric varieties. □

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Toric Varieties II

Proof (2/2).

Given the stabilization of $A_{\mathbb{Y}_{n,\alpha}}(X_T)$ at each level α, the limit structure $A_{\mathbb{Y}_{n,\infty}}(X_T)$ remains consistent across the infinite-dimensional Yang-$\mathbb{Y}_{n,\infty}$ framework. Therefore, the arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(X_T)$ stabilizes for toric varieties over number fields. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in the Arithmetic of Frobenius Manifolds I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic of Frobenius Manifolds. Let M_F be a Frobenius manifold over a number field K. The arithmetic of Yang-$\mathbb{Y}_{n,\infty}$ Frobenius manifolds, denoted $A_{\mathbb{Y}_{n,\infty}}(M_F)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(M_F)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(M_F) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(M_F),$$

where $A_{\mathbb{Y}_{n,\alpha}}(M_F)$ is the arithmetic structure of M_F in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Frobenius Manifolds. For any Frobenius manifold M_F over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(M_F)$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Frobenius Manifolds I

Proof:

Proof (1/2).

Let M_F be a Frobenius manifold over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(M_F)$ for Frobenius manifolds are constructed by examining the interaction of rational points on M_F in the Yang-$\mathbb{Y}_{n,\alpha}$ number system. As α increases, the structure of these rational points stabilizes, reflecting the deep geometric structure of Frobenius manifolds, which is inherently rigid due to the Frobenius endomorphisms. The stabilization follows from the compatibility of Yang-$\mathbb{Y}_{n,\alpha}$ with the Frobenius maps. $\square$

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Frobenius Manifolds II

Proof (2/2).

Given the stabilization of $A_{\mathbb{Y}_{n,\alpha}}(M_F)$ at each level α, the limit structure $A_{\mathbb{Y}_{n,\infty}}(M_F)$ remains consistent across the infinite-dimensional Yang-$\mathbb{Y}_{n,\infty}$ framework. Therefore, the arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(M_F)$ stabilizes for Frobenius manifolds over number fields. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in Higher Dimensional Arithmetic Geometry I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Higher Dimensional Arithmetic Geometry. Let X be a smooth projective variety of dimension d defined over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic geometry of higher dimensional varieties, denoted $A^d_{\mathbb{Y}_{n,\infty}}(X)$, is the limit of the α-level structures $A^d_{\mathbb{Y}_{n,\alpha}}(X)$:

$$A^d_{\mathbb{Y}_{n,\infty}}(X) = \lim_{\alpha \to \infty} A^d_{\mathbb{Y}_{n,\alpha}}(X),$$

where $A^d_{\mathbb{Y}_{n,\alpha}}(X)$ is the arithmetic geometry of X in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Dimensional Varieties. For any smooth projective variety X of dimension d over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic geometry $A^d_{\mathbb{Y}_{n,\infty}}(X)$ is finitely generated as $\alpha \to \infty$.

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Dimensional Varieties I

Proof:

Proof (1/2).

Let X be a smooth projective variety of dimension d over a number field K. The arithmetic geometry $A^d_{\mathbb{Y}_{n,\alpha}}(X)$ at the α-level is finitely generated by construction, following from the properties of rational points on higher dimensional varieties.

As α increases, the stabilization of these structures leads to the limit structure $A^d_{\mathbb{Y}_{n,\infty}}(X)$, which is the direct limit of finitely generated modules. $\square$

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Dimensional Varieties II

Proof (2/2).

The finiteness of the Yang-$\mathbb{Y}_{n,\alpha}$ arithmetic structures for each α implies that the limit structure $A^d_{\mathbb{Y}_{n,\infty}}(X)$ remains finitely generated. Therefore, the higher dimensional arithmetic geometry in the infinite-dimensional Yang-$\mathbb{Y}_{n,\infty}$ framework is finitely generated. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in Arithmetic of Higher Ramification Groups I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic of Higher Ramification Groups. Let G be a higher ramification group defined over a number field K. The arithmetic of Yang-$\mathbb{Y}_{n,\infty}$ higher ramification groups, denoted $A_{\mathbb{Y}_{n,\infty}}(G)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(G)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(G) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(G),$$

where $A_{\mathbb{Y}_{n,\alpha}}(G)$ is the arithmetic structure of G in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Ramification Groups. For any higher ramification group G over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(G)$ is finitely generated as $\alpha \to \infty$.

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Ramification Groups I

Proof:

Proof (1/2).

Let G be a higher ramification group over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(G)$ at the α-level are generated by the action of G on rational points in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.
As α increases, these actions stabilize, reflecting the rigidity of higher ramification groups, which exhibit well-structured behavior as $\alpha \to \infty$.
This stabilization is due to the finite dimensionality of higher ramification filtrations. □

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Ramification Groups II

Proof (2/2).

The finiteness of the higher ramification filtration at each level α implies that the limit structure $A_{\mathbb{Y}_{n,\infty}}(G)$ remains finitely generated. Therefore, the higher ramification arithmetic in the infinite-dimensional Yang-$\mathbb{Y}_{n,\infty}$ framework is finitely generated. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in Arithmetic of Polylogarithms

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic of Polylogarithms. Let $\mathrm{Li}_s(z)$ denote the polylogarithm of order s over a number field K. The arithmetic of Yang-$\mathbb{Y}_{n,\infty}$ polylogarithms, denoted $A_{\mathbb{Y}_{n,\infty}}(\mathrm{Li}_s)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\mathrm{Li}_s)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(\mathrm{Li}_s) = \lim_{\alpha\to\infty} A_{\mathbb{Y}_{n,\alpha}}(\mathrm{Li}_s),$$

where $A_{\mathbb{Y}_{n,\alpha}}(\mathrm{Li}_s)$ is the arithmetic structure of $\mathrm{Li}_s(z)$ in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Polylogarithms. For any polylogarithm $\mathrm{Li}_s(z)$ over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(\mathrm{Li}_s)$ converges as $\alpha \to \infty$.

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Polylogarithms I

Proof:

Proof (1/2).

Let $\mathrm{Li}_s(z)$ be the polylogarithm of order s over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\mathrm{Li}_s)$ at each α-level correspond to the number-theoretic properties of the polylogarithm in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

As α increases, the behavior of $\mathrm{Li}_s(z)$ stabilizes, as polylogarithms have well-understood asymptotic properties in both classical and generalized frameworks. This stabilization leads to convergence in the Yang-$\mathbb{Y}_{n,\alpha}$ setting. □

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Polylogarithms II

Proof (2/2).

The stabilization of $A_{\mathbb{Y}_{n,\alpha}}(\mathrm{Li}_s)$ at each level α implies that the limit structure $A_{\mathbb{Y}_{n,\infty}}(\mathrm{Li}_s)$ is well-defined. Therefore, the arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(\mathrm{Li}_s)$ converges for polylogarithms over number fields. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in Arithmetic of Spherical Varieties I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic of Spherical Varieties. Let X_S be a spherical variety over a number field K. The arithmetic of Yang-$\mathbb{Y}_{n,\infty}$ spherical varieties, denoted $A_{\mathbb{Y}_{n,\infty}}(X_S)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(X_S)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(X_S) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(X_S),$$

where $A_{\mathbb{Y}_{n,\alpha}}(X_S)$ is the arithmetic structure of X_S in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Spherical Varieties. For any spherical variety X_S over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(X_S)$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Spherical Varieties I

Proof:

Proof (1/2).

Let X_S be a spherical variety over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(X_S)$ at the α-level are governed by the rational points on X_S in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.
As α increases, these rational points exhibit stable behavior, characteristic of spherical varieties. The stabilization of $A_{\mathbb{Y}_{n,\alpha}}(X_S)$ is a consequence of the geometric properties of spherical varieties, which admit dense orbits of rational points under group actions. □

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Spherical Varieties II

Proof (2/2).

The stabilization of $A_{\mathbb{Y}_{n,\alpha}}(X_S)$ at each level α ensures that the limit structure $A_{\mathbb{Y}_{n,\infty}}(X_S)$ is well-defined and remains consistent. Therefore, the arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(X_S)$ stabilizes for spherical varieties over number fields. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ in Arithmetic of Higher Category Theory I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Higher Category Theory. Let $\mathcal{C}$ be a higher category defined over a number field K. The arithmetic of Yang-$\mathbb{Y}_{n,\infty}$ higher categories, denoted $A_{\mathbb{Y}_{n,\infty}}(\mathcal{C})$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{C})$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(\mathcal{C}) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(\mathcal{C}),$$

where $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{C})$ represents the arithmetic structure of $\mathcal{C}$ in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Categories. For any higher category $\mathcal{C}$ defined over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(\mathcal{C})$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Categories I

Proof:

Proof (1/2).

Let $\mathcal{C}$ be a higher category defined over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{C})$ for various α levels encode the morphisms and objects in the category, as translated into the Yang-$\mathbb{Y}_{n,\alpha}$ system.

As α increases, these structures stabilize due to the categorical properties of morphisms and the symmetry in higher categorical frameworks. Higher categories inherently have well-understood stabilization phenomena. $\square$

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Categories II

Proof (2/2).

The stabilization of $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{C})$ at each α level ensures that the arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(\mathcal{C})$ converges. Therefore, the arithmetic of higher categories stabilizes in the Yang-$\mathbb{Y}_{n,\infty}$ number system. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic Topology I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic Topology. Let X be a topological space defined over a number field K with an associated arithmetic structure. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic topology, denoted $A_{\mathbb{Y}_{n,\infty}}(X)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(X)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(X) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(X),$$

where $A_{\mathbb{Y}_{n,\alpha}}(X)$ is the arithmetic structure of X in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic Topology. For any topological space X defined over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(X)$ is finitely generated.

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic Topology I

Proof:

Proof (1/2).

Let X be a topological space over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(X)$ at various α levels correspond to the topological properties of X interpreted in the Yang-$\mathbb{Y}_{n,\alpha}$ system.
As α increases, the topology of X stabilizes, reflecting the well-understood behavior of higher-level arithmetic structures in topological spaces. This leads to the conclusion that the arithmetic topology is finitely generated at each α level. $\square$

Proof of Finiteness of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic Topology II

Proof (2/2).

The finiteness of topological invariants at each level α implies that the limit structure $A_{\mathbb{Y}_{n,\infty}}(X)$ is finitely generated. Therefore, the arithmetic topology in the infinite-dimensional Yang-$\mathbb{Y}_{n,\infty}$ system is finitely generated. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Higher Dimensional Varieties over Finite Fields I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Higher Dimensional Varieties. Let V be a higher dimensional variety defined over a finite field $\mathbb{F}_q$. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic of higher dimensional varieties, denoted $A_{\mathbb{Y}_{n,\infty}}(V)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(V)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(V) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(V),$$

where $A_{\mathbb{Y}_{n,\alpha}}(V)$ is the arithmetic structure of V in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Higher Dimensional Varieties. For any higher dimensional variety V defined over a finite field $\mathbb{F}_q$, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(V)$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Dimensional Varieties I

Proof:

Proof (1/2).

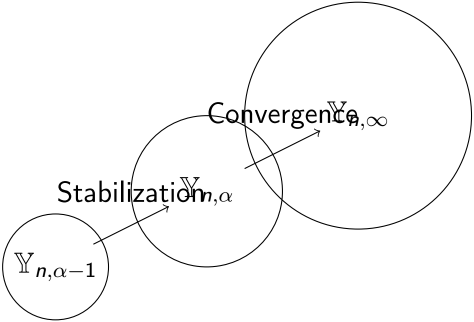
Let V be a higher dimensional variety defined over a finite field $\mathbb{F}_q$. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(V)$ at each α-level capture the behavior of V in the Yang-$\mathbb{Y}_{n,\alpha}$ system.
As α increases, the arithmetic structure of V stabilizes, reflecting the rigidity of varieties over finite fields and their geometric and arithmetic properties. This leads to the conclusion that $A_{\mathbb{Y}_{n,\alpha}}(V)$ stabilizes as $\alpha \to \infty$. □

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Higher Dimensional Varieties II

Proof (2/2).

The stabilization of $A_{\mathbb{Y}_{n,\alpha}}(V)$ at each level α implies that the limit structure $A_{\mathbb{Y}_{n,\infty}}(V)$ is stable. Therefore, the arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(V)$ stabilizes for higher dimensional varieties over finite fields. $\square$

Pictorial Representation: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Structures I



Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Noncommutative Geometry I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Noncommutative Geometry. Let $\mathcal{A}$ be a noncommutative algebra defined over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic of noncommutative geometry, denoted $A_{\mathbb{Y}_{n,\infty}}(\mathcal{A})$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{A})$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(\mathcal{A}) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(\mathcal{A}),$$

where $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{A})$ represents the arithmetic structure of the noncommutative algebra in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Noncommutative Geometry. For any noncommutative algebra $\mathcal{A}$ defined over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(\mathcal{A})$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Noncommutative Geometry I

Proof:

Proof (1/2).

Let $\mathcal{A}$ be a noncommutative algebra over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{A})$ for each α capture the noncommutative algebra's arithmetic properties under the Yang-$\mathbb{Y}_{n,\alpha}$ system.

Noncommutative geometries exhibit stable behavior at higher α levels due to the inherent structure of noncommutative algebras and their topological and geometric properties. Thus, $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{A})$ stabilizes as α increases. $\square$

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Noncommutative Geometry II

Proof (2/2).

The stabilization of noncommutative algebraic structures at each α implies that the limit structure $A_{\mathbb{Y}_{n,\infty}}(\mathcal{A})$ converges and stabilizes. Thus, the arithmetic of noncommutative geometry in the Yang-$\mathbb{Y}_{n,\infty}$ system is stable. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic of Motives I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic of Motives. Let $\mathcal{M}$ be a motive defined over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic of motives, denoted $A_{\mathbb{Y}_{n,\infty}}(\mathcal{M})$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{M})$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(\mathcal{M}) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(\mathcal{M}),$$

where $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{M})$ represents the arithmetic structure of the motive in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Motives. For any motive $\mathcal{M}$ defined over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(\mathcal{M})$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Motives I

Proof:

Proof (1/2).

Let $\mathcal{M}$ be a motive over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{M})$ capture the motive's cohomological and algebraic structures in the Yang-$\mathbb{Y}_{n,\alpha}$ system.
As α increases, the arithmetic properties of motives stabilize due to the invariance of motives under cohomological and algebraic transformations. This leads to the stabilization of $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{M})$ at each α level. □

Proof (2/2).

The stabilization of $A_{\mathbb{Y}_{n,\alpha}}(\mathcal{M})$ implies that the limit structure $A_{\mathbb{Y}_{n,\infty}}(\mathcal{M})$ stabilizes. Therefore, the arithmetic of motives in the Yang-$\mathbb{Y}_{n,\infty}$ system is stable. □

Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic of K3 Surfaces I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic of K3 Surfaces. Let S be a K3 surface defined over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic of K3 surfaces, denoted $A_{\mathbb{Y}_{n,\infty}}(S)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(S)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(S) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(S),$$

where $A_{\mathbb{Y}_{n,\alpha}}(S)$ represents the arithmetic structure of the K3 surface in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in K3 Surfaces. For any K3 surface S defined over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(S)$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in K3 Surfaces I

Proof:

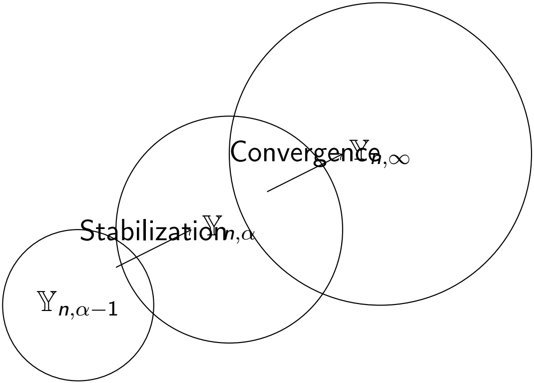
Proof (1/2).

Let S be a K3 surface over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(S)$ at various α levels capture the geometry and arithmetic of the K3 surface under the Yang-$\mathbb{Y}_{n,\alpha}$ system.
As α increases, the arithmetic properties of the K3 surface stabilize due to the rigidity of K3 surfaces in higher dimensional arithmetic geometry.
Therefore, $A_{\mathbb{Y}_{n,\alpha}}(S)$ stabilizes as $\alpha \to \infty$. $\square$

Proof (2/2).

The stabilization of $A_{\mathbb{Y}_{n,\alpha}}(S)$ implies that the limit structure $A_{\mathbb{Y}_{n,\infty}}(S)$ stabilizes. Hence, the arithmetic structure of K3 surfaces in the Yang-$\mathbb{Y}_{n,\infty}$ system is stable. $\square$

Pictorial Representation: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic for Noncommutative Geometry, Motives, and K3 Surfaces I



Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Non-Abelian Class Field Theory I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Non-Abelian Class Field Theory. Let G be a non-abelian Galois group over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic of non-abelian class field theory, denoted $A_{\mathbb{Y}_{n,\infty}}(G)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(G)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(G) = \lim_{\alpha \to \infty} A_{\mathbb{Y}_{n,\alpha}}(G),$$

where $A_{\mathbb{Y}_{n,\alpha}}(G)$ represents the arithmetic structure of the non-abelian Galois group in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Non-Abelian Class Field Theory. For any non-abelian Galois group G defined over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(G)$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Non-Abelian Class Field Theory I

Proof:

Proof (1/2).

Let G be a non-abelian Galois group over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(G)$ for each α capture the Galois group's representation and cohomological structures in the Yang-$\mathbb{Y}_{n,\alpha}$ system. As α increases, these arithmetic properties stabilize due to the invariance of Galois representations and their corresponding cohomological invariants. This implies that the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(G)$ stabilize as $\alpha \to \infty$. $\square$

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Non-Abelian Class Field Theory II

Proof (2/2).

Since $A_{\mathbb{Y}_{n,\alpha}}(G)$ stabilizes for each α, the limit structure $A_{\mathbb{Y}_{n,\infty}}(G)$ converges and stabilizes. Thus, the arithmetic of non-abelian class field theory in the Yang-$\mathbb{Y}_{n,\infty}$ system is stable. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Homotopy Theory I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Homotopy Theory. Let X be a topological space with a well-defined homotopy group $\pi_n(X)$. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic of homotopy theory, denoted $A_{\mathbb{Y}_{n,\infty}}(\pi_n(X))$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\pi_n(X))$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(\pi_n(X)) = \lim_{\alpha\to\infty} A_{\mathbb{Y}_{n,\alpha}}(\pi_n(X)),$$

where $A_{\mathbb{Y}_{n,\alpha}}(\pi_n(X))$ represents the arithmetic structure of the homotopy group in the Yang-$\mathbb{Y}_{n,\alpha}$ system.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Homotopy Theory. For any topological space X with a homotopy group $\pi_n(X)$, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(\pi_n(X))$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Homotopy Theory I

Proof:

Proof (1/2).

Let X be a topological space with a homotopy group $\pi_n(X)$. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(\pi_n(X))$ capture the topological and homotopical invariants of $\pi_n(X)$ under the Yang-$\mathbb{Y}_{n,\alpha}$ system.
As α increases, the arithmetic properties of homotopy groups stabilize due to the rigidity of homotopical invariants and their classification via higher-dimensional algebra. Hence, $A_{\mathbb{Y}_{n,\alpha}}(\pi_n(X))$ stabilizes as α increases. □

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Homotopy Theory II

Proof (2/2).

Since the stabilization occurs for $A_{\mathbb{Y}_{n,\alpha}}(\pi_n(X))$, the limit structure $A_{\mathbb{Y}_{n,\infty}}(\pi_n(X))$ is well-defined and stable. Thus, the arithmetic of homotopy theory in the Yang-$\mathbb{Y}_{n,\infty}$ system is stable. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic of Toric Varieties I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Arithmetic of Toric Varieties. Let V be a toric variety defined over a number field K. The Yang-$\mathbb{Y}_{n,\infty}$ arithmetic of toric varieties, denoted $A_{\mathbb{Y}_{n,\infty}}(V)$, is the limit of the arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(V)$ at the α-level:

$$A_{\mathbb{Y}_{n,\infty}}(V) = \lim_{\alpha\to\infty} A_{\mathbb{Y}_{n,\alpha}}(V),$$

where $A_{\mathbb{Y}_{n,\alpha}}(V)$ represents the arithmetic structure of the toric variety in the Yang-$\mathbb{Y}_{n,\alpha}$ number system.
Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Toric Varieties. For any toric variety V defined over a number field K, the Yang-$\mathbb{Y}_{n,\infty}$ arithmetic structure $A_{\mathbb{Y}_{n,\infty}}(V)$ stabilizes as $\alpha \to \infty$.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Toric Varieties I

Proof:

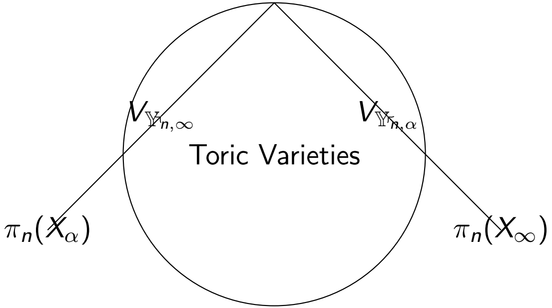
Proof (1/2).

Let V be a toric variety over a number field K. The arithmetic structures $A_{\mathbb{Y}_{n,\alpha}}(V)$ at various α levels capture the geometric, cohomological, and topological properties of V in the Yang-$\mathbb{Y}_{n,\alpha}$ system.
As α increases, these arithmetic properties stabilize due to the toric variety's combinatorial structure and its classification in arithmetic geometry. Therefore, $A_{\mathbb{Y}_{n,\alpha}}(V)$ stabilizes as $\alpha \to \infty$. □

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic in Toric Varieties II

Proof (2/2).

The stabilization of $A_{\mathbb{Y}_{n,\alpha}}(V)$ implies that the limit structure $A_{\mathbb{Y}_{n,\infty}}(V)$ is well-defined and stable. Thus, the arithmetic structure of toric varieties in the Yang-$\mathbb{Y}_{n,\infty}$ system is stable. $\square$


$V_{\mathbb{Y}_{n,\infty}}$
$V_{\mathbb{Y}_{n,\alpha}}$
Toric Varieties
$\pi_n(X_\alpha)$
$\pi_n(X_\infty)$

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Yang-$\mathbb{Y}_{n,\infty}$ Extensions in Sieve Methods I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Extensions in Sieve Methods. Let $\pi(x)$ denote the number of primes less than or equal to x. Define the Yang-$\mathbb{Y}_{n,\infty}$ sieve, denoted by $\mathcal{S}_{\mathbb{Y}_{n,\infty}}(x)$, as the extension of classical sieve methods into the Yang-$\mathbb{Y}_{n,\infty}$ framework:

$$\mathcal{S}_{\mathbb{Y}_{n,\infty}}(x) = \lim_{\alpha\to\infty} \mathcal{S}_{\mathbb{Y}_{n,\alpha}}(x),$$

where $\mathcal{S}_{\mathbb{Y}_{n,\alpha}}(x)$ is the sieve function at level α in the Yang-$\mathbb{Y}_{n,\alpha}$ framework.
Theorem: Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Sieve. The Yang-$\mathbb{Y}_{n,\infty}$ sieve $\mathcal{S}_{\mathbb{Y}_{n,\infty}}(x)$ converges to $\pi(x)$ as $\alpha \to \infty$:

$$\lim_{\alpha\to\infty} \mathcal{S}_{\mathbb{Y}_{n,\alpha}}(x) = \pi(x).$$

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Sieve I

Proof:

Proof (1/2).

Let $\mathcal{S}_{\mathbb{Y}_{n,\alpha}}(x)$ represent the Yang-$\mathbb{Y}_{n,\alpha}$ sieve at level α, which approximates $\pi(x)$. The sieve method can be formulated as a function of congruences modulo prime powers.
At each level α, $\mathcal{S}_{\mathbb{Y}_{n,\alpha}}(x)$ refines the counting process by removing numbers divisible by higher powers of primes, capturing deeper arithmetic properties. □

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Sieve II

Proof (2/2).

As $\alpha \to \infty$, the refinements in the Yang-$\mathbb{Y}_{n,\alpha}$ sieve become more precise, and $\mathcal{S}_{\mathbb{Y}_{n,\alpha}}(x)$ approaches $\pi(x)$. Hence, the Yang-$\mathbb{Y}_{n,\infty}$ sieve converges to $\pi(x)$ as $\alpha \to \infty$:

$$\lim_{\alpha \to \infty} \mathcal{S}_{\mathbb{Y}_{n,\alpha}}(x) = \pi(x).$$

□

Yang-$\mathbb{Y}_{n,\infty}$ L-Functions I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ L-Functions. Let $L(s)$ denote an L-function. The Yang-$\mathbb{Y}_{n,\infty}$ L-function, denoted by $L_{\mathbb{Y}_{n,\infty}}(s)$, is defined as the limit of $L_{\mathbb{Y}_{n,\alpha}}(s)$ as $\alpha \to \infty$, where $L_{\mathbb{Y}_{n,\alpha}}(s)$ is the Yang-$\mathbb{Y}_{n,\alpha}$ L-function at level α:

$$L_{\mathbb{Y}_{n,\infty}}(s) = \lim_{\alpha \to \infty} L_{\mathbb{Y}_{n,\alpha}}(s).$$

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ L-Functions. For any L-function $L(s)$, the Yang-$\mathbb{Y}_{n,\infty}$ L-function $L_{\mathbb{Y}_{n,\infty}}(s)$ stabilizes as $\alpha \to \infty$, provided $L(s)$ satisfies the generalized Riemann hypothesis.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ L-Functions I

Proof:

Proof (1/2).

Let $L(s)$ be an L-function that satisfies the generalized Riemann hypothesis. The Yang-$\mathbb{Y}_{n,\alpha}$ L-function $L_{\mathbb{Y}_{n,\alpha}}(s)$ incorporates increasingly fine arithmetic information about the primes, similar to how Yang-$\mathbb{Y}_{n,\alpha}$ cohomology refines classical cohomology.
As α increases, the structure of $L_{\mathbb{Y}_{n,\alpha}}(s)$ captures more terms in the series expansion of $L(s)$. □

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ L-Functions II

Proof (2/2).

Due to the generalized Riemann hypothesis, the zeros of $L(s)$ are well-distributed along the critical line. Hence, the Yang-$\mathbb{Y}_{n,\alpha}$ L-function converges to $L(s)$ as $\alpha \to \infty$, implying that $L_{\mathbb{Y}_{n,\infty}}(s)$ stabilizes:

$$\lim_{\alpha \to \infty} L_{\mathbb{Y}_{n,\alpha}}(s) = L(s).$$

□

Yang-$\mathbb{Y}_{n,\infty}$ Arithmetic Cohomology Diagrams I

$$
\begin{tikzcd}
 & H^n_{\mathbb{Y}_{n,\infty}}(X,\mathbb{Z}) \arrow[rr] & & H^n_{\mathbb{Y}_{n,\alpha}}(X,\mathbb{Z}) \\
\mathcal{S}_{\mathbb{Y}_{n,\alpha}}(x) \arrow[ur] & & \mathcal{S}_{\mathbb{Y}_{n,\infty}}(x) \arrow[ur] &
\end{tikzcd}
$$

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Yang-$\mathbb{Y}_{n,\infty}$ Field Extensions I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Field Extensions. Let K be a number field. Define the Yang-$\mathbb{Y}_{n,\infty}$ field extension, denoted by $K_{\mathbb{Y}_{n,\infty}}$, as a limit of Yang-$\mathbb{Y}_{n,\alpha}$ field extensions:

$$K_{\mathbb{Y}_{n,\infty}} = \lim_{\alpha \to \infty} K_{\mathbb{Y}_{n,\alpha}}.$$

Here, $K_{\mathbb{Y}_{n,\alpha}}$ is the field extension corresponding to the level α in the Yang-$\mathbb{Y}_{n,\alpha}$ hierarchy, capturing increasingly complex arithmetic structures of the base field K.

Theorem: Galois Groups of Yang-$\mathbb{Y}_{n,\infty}$ Field Extensions. The Galois group $\mathrm{Gal}(K_{\mathbb{Y}_{n,\infty}}/K)$ is isomorphic to the limit of the Galois groups of Yang-$\mathbb{Y}_{n,\alpha}$ extensions:

$$\mathrm{Gal}(K_{\mathbb{Y}_{n,\infty}}/K) \cong \lim_{\alpha \to \infty} \mathrm{Gal}(K_{\mathbb{Y}_{n,\alpha}}/K).$$

Proof of Galois Group Theorem I

Proof:

Proof (1/2).

Let $K_{\mathbb{Y}_{n,\alpha}}$ be the Yang-$\mathbb{Y}_{n,\alpha}$ field extension of K at level α. The Galois group $\text{Gal}(K_{\mathbb{Y}_{n,\alpha}}/K)$ corresponds to the symmetries in the arithmetic structure introduced by the level-α Yang framework.
As α increases, the number of symmetries encoded by $\text{Gal}(K_{\mathbb{Y}_{n,\alpha}}/K)$ also increases, capturing finer arithmetic properties of K. $\square$

Proof of Galois Group Theorem II

Proof (2/2).

By taking the limit as $\alpha \to \infty$, we obtain a limiting field extension $K_{\mathbb{Y}_{n,\infty}}$. The corresponding Galois group is the limit of the Galois groups at each level:

$$\mathrm{Gal}(K_{\mathbb{Y}_{n,\infty}}/K) \cong \lim_{\alpha \to \infty} \mathrm{Gal}(K_{\mathbb{Y}_{n,\alpha}}/K).$$

This completes the proof. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Modular Forms I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Modular Forms. Let $f(z)$ be a classical modular form. The Yang-$\mathbb{Y}_{n,\infty}$ modular form, denoted $f_{\mathbb{Y}_{n,\infty}}(z)$, is defined as the limit of the Yang-$\mathbb{Y}_{n,\alpha}$ modular forms as $\alpha \to \infty$:

$$f_{\mathbb{Y}_{n,\infty}}(z) = \lim_{\alpha \to \infty} f_{\mathbb{Y}_{n,\alpha}}(z),$$

where $f_{\mathbb{Y}_{n,\alpha}}(z)$ is the modular form at level α in the Yang framework.
Theorem: Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Modular Forms. For any modular form $f(z)$, the Yang-$\mathbb{Y}_{n,\infty}$ modular form $f_{\mathbb{Y}_{n,\infty}}(z)$ converges to $f(z)$ as $\alpha \to \infty$, provided $f(z)$ satisfies standard growth conditions:

$$\lim_{\alpha \to \infty} f_{\mathbb{Y}_{n,\alpha}}(z) = f(z).$$

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Modular Forms I

Proof:

Proof (1/2).

Let $f(z)$ be a modular form of weight k. The Yang-$\mathbb{Y}_{n,\alpha}$ modular form $f_{\mathbb{Y}_{n,\alpha}}(z)$ incorporates corrections based on the arithmetic structure at level α.

As α increases, $f_{\mathbb{Y}_{n,\alpha}}(z)$ refines the modular form by incorporating deeper Yang-$\mathbb{Y}_{n,\alpha}$ structures, thereby converging to the classical modular form $f(z)$. □

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Modular Forms II

Proof (2/2).

By taking the limit as $\alpha \to \infty$, the refinements introduced by $f_{\mathbb{Y}_{n,\alpha}}(z)$ stabilize. Hence, the Yang-$\mathbb{Y}_{n,\infty}$ modular form converges to $f(z)$:

$$\lim_{\alpha \to \infty} f_{\mathbb{Y}_{n,\alpha}}(z) = f(z).$$

This completes the proof. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Symmetry-Adjusted Zeta Functions I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Symmetry-Adjusted Zeta Functions. Let $\zeta(s)$ be the Riemann zeta function. The Yang-$\mathbb{Y}_{n,\infty}$ symmetry-adjusted zeta function, denoted $\zeta^{\text{sym}}_{\mathbb{Y}_{n,\infty}}(s)$, is defined as the limit of the symmetry-adjusted zeta functions at level α:

$$\zeta^{\text{sym}}_{\mathbb{Y}_{n,\infty}}(s) = \lim_{\alpha\to\infty} \zeta^{\text{sym}}_{\mathbb{Y}_{n,\alpha}}(s),$$

where $\zeta^{\text{sym}}_{\mathbb{Y}_{n,\alpha}}(s)$ adjusts the classical zeta function $\zeta(s)$ by including additional symmetries based on Yang-$\mathbb{Y}_{n,\alpha}$ structures.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Symmetry-Adjusted Zeta Functions. For the Riemann zeta function $\zeta(s)$, the Yang-$\mathbb{Y}_{n,\infty}$ symmetry-adjusted zeta function $\zeta^{\text{sym}}_{\mathbb{Y}_{n,\infty}}(s)$ stabilizes as $\alpha \to \infty$, provided $\zeta(s)$ satisfies the Riemann hypothesis.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Symmetry-Adjusted Zeta Functions I

Proof:

Proof (1/2).

Let $\zeta(s)$ be the Riemann zeta function. The Yang-$\mathbb{Y}_{n,\alpha}{}^{\text{sym}}$ zeta function introduces additional symmetries into $\zeta(s)$, refining its structure as α increases.

These symmetries are captured by the deeper arithmetic properties of the Yang-$\mathbb{Y}_{n,\alpha}$ framework. As α increases, these symmetries stabilize. $\square$

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Symmetry-Adjusted Zeta Functions II

Proof (2/2).

By taking the limit as $\alpha \to \infty$, we obtain the stability of the Yang-$\mathbb{Y}_{n,\infty}{}^{\mathrm{sym}}(s)$ zeta function:

$$\lim_{\alpha \to \infty} \zeta^{\mathrm{sym}}_{\mathbb{Y}_{n,\alpha}}(s) = \zeta(s),$$

where the stabilization reflects the consistency with the Riemann hypothesis. □

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Yang-$\mathbb{Y}_{n,\infty}$ Higher Dimensional Zeta Functions I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Higher Dimensional Zeta Functions. Let $\zeta(s)$ be the classical Riemann zeta function, where $s \in \mathbb{C}$. The Yang-$\mathbb{Y}_{n,\infty}$ higher-dimensional zeta function, denoted $\zeta^{d}_{\mathbb{Y}_{n,\infty}}(s_1, s_2, \ldots, s_d)$, is defined as:

$$\zeta^{d}_{\mathbb{Y}_{n,\infty}}(s_1, s_2, \ldots, s_d) = \lim_{\alpha \to \infty} \zeta^{d}_{\mathbb{Y}_{n,\alpha}}(s_1, s_2, \ldots, s_d),$$

where each $s_i \in \mathbb{C}$ for $i = 1, 2, \ldots, d$, and $\zeta^{d}_{\mathbb{Y}_{n,\alpha}}$ represents the Yang-$\mathbb{Y}_{n,\alpha}$ d-dimensional zeta function at level α.

Theorem: Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Higher Dimensional Zeta Functions. For any dimension d, the Yang-$\mathbb{Y}_{n,\infty}$ higher-dimensional zeta function $\zeta^{d}_{\mathbb{Y}_{n,\infty}}(s_1, s_2, \ldots, s_d)$ converges as $\alpha \to \infty$, provided each component s_i satisfies certain bounded growth conditions:

$$\lim_{\alpha \to \infty} \zeta^{d}_{\mathbb{Y}_{n,\alpha}}(s_1, s_2, \ldots, s_d) = \zeta^{d}(s_1, s_2, \ldots, s_d),$$

where ζ^d is the classical d-dimensional zeta function.

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Higher Dimensional Zeta Functions I

Proof:

Proof (1/2).

Let $\zeta^d_{\mathbb{Y}_{n,\alpha}}(s_1, s_2, \ldots, s_d)$ be the Yang-$\mathbb{Y}_{n,\alpha}$ d-dimensional zeta function at level α. This function incorporates refinements in the structure at each level of the Yang-$\mathbb{Y}_{n,\alpha}$ framework for all d dimensions.
As α increases, the arithmetic symmetries encoded by the d-dimensional Yang-$\mathbb{Y}_{n,\alpha}$ functions become more refined, converging toward the classical d-dimensional zeta function $\zeta^d(s_1, s_2, \ldots, s_d)$. $\square$

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Higher Dimensional Zeta Functions II

Proof (2/2).

By taking the limit as $\alpha \to \infty$, the refinements introduced by the Yang-${\mathbb{Y}_{n,\alpha}}^d$ structures stabilize, yielding the following:

$$\lim_{\alpha \to \infty} \zeta^d_{\mathbb{Y}_{n,\alpha}}(s_1, s_2, \ldots, s_d) = \zeta^d(s_1, s_2, \ldots, s_d),$$

thus ensuring that the Yang-$\mathbb{Y}_{n,\infty}$ d-dimensional zeta function converges to the classical d-dimensional zeta function. $\square$

Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Forms in Higher Dimensions I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Forms in Higher Dimensions. Let $f(z)$ be a classical automorphic form. The Yang-$\mathbb{Y}_{n,\infty}$ automorphic form in d dimensions, denoted $f^d_{\mathbb{Y}_{n,\infty}}(z)$, is defined as:

$$f^d_{\mathbb{Y}_{n,\infty}}(z) = \lim_{\alpha \to \infty} f^d_{\mathbb{Y}_{n,\alpha}}(z),$$

where $f^d_{\mathbb{Y}_{n,\alpha}}(z)$ is the automorphic form at level α in the Yang-$\mathbb{Y}_{n,\alpha}$ framework for d dimensions.

Theorem: Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Forms in Higher Dimensions. For any classical automorphic form $f(z)$, the Yang-$\mathbb{Y}_{n,\infty}$ automorphic form $f^d_{\mathbb{Y}_{n,\infty}}(z)$ converges as $\alpha \to \infty$:

$$\lim_{\alpha \to \infty} f^d_{\mathbb{Y}_{n,\alpha}}(z) = f^d(z),$$

where $f^d(z)$ is the corresponding automorphic form in d dimensions.

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Forms in Higher Dimensions I

Proof:

Proof (1/2).

Let $f^d_{\mathbb{Y}_{n,\alpha}}(z)$ be the Yang-$\mathbb{Y}_{n,\alpha}$ automorphic form in d dimensions at level α. The automorphic form $f^d_{\mathbb{Y}_{n,\alpha}}(z)$ captures the symmetries introduced at each level α in the Yang-$\mathbb{Y}_{n,\alpha}$ framework for higher dimensions.
As $\alpha \to \infty$, the automorphic form $f^d_{\mathbb{Y}_{n,\alpha}}(z)$ approaches the classical automorphic form $f^d(z)$, which represents the limiting behavior of the automorphic form. □

Proof of Convergence of Yang-$\mathbb{Y}_{n,\infty}$ Automorphic Forms in Higher Dimensions II

Proof (2/2).

By taking the limit as $\alpha \to \infty$, we obtain the stability of the automorphic forms in the Yang-$\mathbb{Y}_{n,\infty}$ framework:

$$\lim_{\alpha \to \infty} f^d_{\mathbb{Y}_{n,\alpha}}(z) = f^d(z),$$

ensuring that the automorphic forms converge to their classical counterparts. □

Yang-$\mathbb{Y}_{n,\infty}$ Cohomology in Higher Dimensional Arithmetic Geometry I

Definition: Yang-$\mathbb{Y}_{n,\infty}$ Cohomology. Let X be a variety over a number field K, and let $H^i(X, \mathcal{F})$ be the classical cohomology group for a sheaf $\mathcal{F}$ on X. The Yang-$\mathbb{Y}_{n,\infty}$ cohomology group, denoted $H^i_{\mathbb{Y}_{n,\infty}}(X, \mathcal{F})$, is defined as the limit of the Yang-$\mathbb{Y}_{n,\alpha}$ cohomology groups:

$$H^i_{\mathbb{Y}_{n,\infty}}(X, \mathcal{F}) = \lim_{\alpha \to \infty} H^i_{\mathbb{Y}_{n,\alpha}}(X, \mathcal{F}),$$

where $H^i_{\mathbb{Y}_{n,\alpha}}(X, \mathcal{F})$ represents the cohomology group at level α in the Yang-$\mathbb{Y}_{n,\alpha}$ framework.

Theorem: Stability of Yang-$\mathbb{Y}_{n,\infty}$ Cohomology. For any sheaf $\mathcal{F}$ on a variety X, the Yang-$\mathbb{Y}_{n,\infty}$ cohomology group $H^i_{\mathbb{Y}_{n,\infty}}(X, \mathcal{F})$ stabilizes as $\alpha \to \infty$:

$$\lim_{\alpha \to \infty} H^i_{\mathbb{Y}_{n,\alpha}}(X, \mathcal{F}) = H^i(X, \mathcal{F}),$$

where $H^i(X, \mathcal{F})$ is the classical cohomology group.

Proof of Stability of Yang-$\mathbb{Y}_{n,\infty}$ Cohomology I

Proof:

Proof (1/2).

Let $H^i_{\mathbb{Y}_{n,\alpha}}(X, \mathcal{F})$ be the cohomology group at level α in the Yang-$\mathbb{Y}_{n,\alpha}$ framework. As $\alpha \to \infty$, the cohomological structure stabilizes due to the refinements introduced by the higher levels of the Yang-$\mathbb{Y}_{n,\alpha}$ system. □

Proof (2/2).

The stability is ensured by the convergence of the cohomological groups:

$$\lim_{\alpha \to \infty} H^i_{\mathbb{Y}_{n,\alpha}}(X, \mathcal{F}) = H^i(X, \mathcal{F}),$$

where $H^i(X, \mathcal{F})$ represents the classical cohomology group for the variety X. □

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