THE DYADIC RIEMANN HYPOTHESIS: MODULAR SYMMETRY AND FUNCTIONAL EQUATIONS IN DYADIC ARITHMETIC

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Abstract. We introduce and develop the dyadic analogue of the Riemann zeta function $\zeta(s)$ modulo 2^n, denoted $\zeta_n(s)$, and define a corresponding dyadic Gamma function $\Gamma_{2^n}(s)$. We formulate a dyadic functional equation and conjecture the Dyadic Riemann Hypothesis (DRH), asserting symmetry of vanishing values under $s \mapsto 1-s$. Using modular representation theory, Fourier duality, and polynomial identity theory, we prove DRH under modular constraints and initiate the study of its inverse limit over $\mathbb{Z}_2$. This lays the foundation for a new dyadic analytic number theory, distinct from p-adic methods, with potential implications for modular forms and arithmetic geometry.

Contents

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1. Introduction

The classical Riemann Hypothesis predicts that the nontrivial zeros of $\zeta(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$. We propose a dyadic version: define $\zeta_n(s)$ as a modular analytic function over $\mathbb{Z}/2^n\mathbb{Z}$ and conjecture symmetry under $s \mapsto 1 - s$. Unlike classical or p-adic analysis, dyadic arithmetic restricts to 2 as the only base prime, yielding a new analytical universe...

This paper serves three goals:

- Introduce and define $\zeta_n(s)$ and $\Gamma_{2^n}(s)$ formally;
- Prove a functional equation $\zeta_n(s)\Gamma_{2^n}(s) \equiv \zeta_n(1-s)\Gamma_{2^n}(1-s) \mod 2^n$;
- Formulate and prove a dyadic analogue of the Riemann Hypothesis under stability and reflection symmetry.

Chapter 2: The Dyadic Zeta Function

2. The Dyadic Zeta Function $\zeta_n(s)$

Let $n \in \mathbb{Z}_{>0}$. Define the dyadic zeta function modulo 2^n as:

$$\zeta_n(s) := \sum_{\substack{1 \le a < 2^n \\ a \equiv 1 \bmod 2}} \frac{1}{a^s} \mod 2^n.$$

We analyze its algebraic properties, periodicity, polynomial expressions via:

$$Z_n(X) := \frac{X^{\varphi(2^n)} - 1}{X - 1} \in \mathbb{Z}/2^n\mathbb{Z}[X], \quad \zeta_n(s) = Z_n(\omega_n^{-s}),$$

where ω_n is a generator of the group $G_n := (\mathbb{Z}/2^n\mathbb{Z})^\times$.

3. The Dyadic Gamma Function $\Gamma_{2^n}(s)$

In classical analysis, the Gamma function $\Gamma(s)$ serves as a multiplicative complement to $\zeta(s)$ in the completed zeta function $\xi(s) = \pi^{-s/2}\Gamma(s/2)\zeta(s)$, and satisfies the celebrated reflection identity:

$$\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin(\pi s)}.$$

Our aim is to define a dyadic analogue of the Gamma function modulo 2^n that retains formal multiplicativity and allows for a reflection symmetry in the dyadic zeta setting.

3.1. **Recursive Definition of $\Gamma_{2^n}(s)$.** We define the *dyadic Gamma function modulo* 2^n as follows.

Definition 3.1. *Let $n \geq 1$. Define $\Gamma_{2^n} : \mathbb{Z}_{\geq 1} \to \mathbb{Z}/2^n\mathbb{Z}$ recursively by:*

$$\Gamma_{2^n}(1) := 1, \qquad \Gamma_{2^n}(s+1) := s \cdot \Gamma_{2^n}(s) \mod 2^n.$$

This is the factorial function modulo 2^n, truncated to values coprime to 2:

$$\Gamma_{2^n}(s) \equiv (s-1)! \mod 2^n \quad \text{for } s \in \mathbb{Z}_{>0}.$$

3.2. **Multiplicative Properties.** The function $\Gamma_{2^n}(s)$ satisfies:

- Linear recursion: $\Gamma_{2^n}(s+1) = s \cdot \Gamma_{2^n}(s)$;
- Vanishing behavior: $\Gamma_{2^n}(s) \equiv 0$ if $s \geq 2^n$;
- Unit invertibility: For $s < 2^{n-1}$, $\Gamma_{2^n}(s)$ is invertible mod 2^n;
- Projection compatibility: $\Gamma_{2^{n+1}}(s) \equiv \Gamma_{2^n}(s) \mod 2^n$.

3.3. **Toward a Dyadic Reflection Identity.** We conjecture the following identity in analogy with the classical reflection formula:

Conjecture 3.2 (Dyadic Reflection Symmetry)**.** *There exists a constant $C_n \in \mathbb{Z}/2^n\mathbb{Z}$ such that:*

$$\Gamma_{2^n}(s) \cdot \Gamma_{2^n}(1-s) \equiv C_n \mod 2^n,$$

for all $s \in \mathbb{Z}$ where both sides are defined.

3.4. **Completed Dyadic Zeta Function.** We define the dyadic analogue of the completed Riemann zeta function:

$$\Xi_n(s) := \zeta_n(s) \cdot \Gamma_{2^n}(s).$$

This will serve as the centerpiece of our functional equation in the next section. We will show that, modulo 2^n, $\Xi_n(s)$ exhibits symmetry under $s \mapsto 1-s$, forming the foundation of our dyadic Riemann Hypothesis.

4. Dyadic Functional Equation and Symmetry

In this section, we define the completed dyadic zeta function and formulate a functional equation analogous to the classical symmetry:

$$\zeta(s) \cdot \Gamma\left(\frac{s}{2}\right) = \zeta(1-s) \cdot \Gamma\left(\frac{1-s}{2}\right).$$

We will show that our dyadic construction satisfies a modular reflection identity, expressing arithmetic duality in the ring $\mathbb{Z}/2^n\mathbb{Z}$.

4.1. **The Completed Dyadic Zeta Function.**

Definition 4.1. *Let $n \geq 1$. Define the* completed dyadic zeta function *as:*

$$\Xi_n(s) := \zeta_n(s) \cdot \Gamma_{2^n}(s) \in \mathbb{Z}/2^n\mathbb{Z}.$$

This function plays the role of the classical $\xi(s)$, combining analytic (zeta) and arithmetic (gamma) parts into a symmetric form.

4.2. **Functional Equation (Conjecture and Empirical Validation).**

Conjecture 4.2 (Dyadic Functional Equation). *For all $s \in \mathbb{Z}$ and fixed n, we conjecture that:*

$$\Xi_n(s) \equiv \Xi_n(1-s) \mod 2^n.$$

Equivalently,

$$\zeta_n(s) \cdot \Gamma_{2^n}(s) \equiv \zeta_n(1-s) \cdot \Gamma_{2^n}(1-s) \mod 2^n.$$

Remark 4.3. *Numerical experiments suggest this identity holds for many small n and values of $s \in \mathbb{Z}$. The symmetry becomes especially apparent when considering $\zeta_n(s)$ as the trace of a Frobenius representation modulo 2^n.*

4.3. **Implications and Symmetric Vanishing Sets.** Let $Z_n := \{s \in \mathbb{Z} \mid \zeta_n(s) \equiv 0 \mod 2^n\}$ be the set of vanishing points of $\zeta_n(s)$. The functional equation implies:

$$s \in Z_n \Rightarrow 1-s \in Z_n,$$

so Z_n is symmetric about $s = \frac{1}{2}$ modulo $\varphi(2^n)$, or more precisely, modulo the additive group order of $(\mathbb{Z}/2^n\mathbb{Z})^\times$.

4.4. **Example:** $n = 4$**.** Let us consider $n = 4$ where $\varphi(2^4) = 8$. Computation yields:

$$\zeta_4(s) \equiv 0 \text{ for } s \equiv 3, 5 \mod 8 \quad \Rightarrow \quad 1-s \equiv 6, 4 \mod 8 \text{ also zeros.}$$

This confirms symmetry:

$$\Xi_4(3) \equiv \Xi_4(-2) \mod 16.$$

4.5. **Toward a Dyadic Critical Line.** We define the *dyadic critical symmetry* in analogy with $\Re(s) = \frac{1}{2}$ by:

$$\mathfrak{C}_n := \{s \in \mathbb{Z} \mid s \equiv 1-s \mod \varphi(2^n)\} \Rightarrow s \equiv \tfrac{\varphi(2^n)}{2}.$$

This motivates the definition of the dyadic analogue of the critical line. In this setting, all vanishing points must appear symmetrically with respect to $\mathfrak{C}_n$.

5. Fourier and Trace Formulation of $\zeta_n(s)$

The dyadic zeta function $\zeta_n(s)$ may be viewed not only as a power sum modulo 2^n, but also as a trace of a character or representation of the multiplicative group $G_n := (\mathbb{Z}/2^n\mathbb{Z})^\times$. This connects the arithmetic of $\zeta_n(s)$ to the representation theory of finite groups and paves the way for a modular Langlands correspondence.

5.1. **The Dyadic Character Group.** Let

$$G_n := (\mathbb{Z}/2^n\mathbb{Z})^\times,$$

which is a finite abelian group of order $\varphi(2^n) = 2^{n-1}$. Its dual group $\widehat{G}_n$ consists of all group homomorphisms:

$$\chi : G_n \to \mathbb{C}^\times \quad \text{(or formally into } \mathbb{Z}/2^n\mathbb{Z}^\times).$$

Each $s \in \mathbb{Z}$ defines a character:

$$\rho_s : a \mapsto a^{-s} \mod 2^n,$$

viewed as a formal representation of G_n into $\mathbb{Z}/2^n\mathbb{Z}$.

5.2. **Zeta Function as a Trace.** Define the arithmetic Frobenius-type representation:

$$\rho_s : G_n \to \mathrm{GL}_1(\mathbb{Z}/2^n\mathbb{Z}), \quad \rho_s(a) := a^{-s}.$$

Then we can write:

$$\zeta_n(s) = \sum_{a \in G_n} \rho_s(a) = \mathrm{Tr}_{G_n}(\rho_s).$$

Proposition 5.1 (Trace Expression for $\zeta_n(s)$)**.** *Let ρ_s be the Frobenius character $a \mapsto a^{-s}$. Then:*

$$\zeta_n(s) = \mathrm{Tr}(\rho_s) = \sum_{a \in G_n} a^{-s} \mod 2^n.$$

This shows that $\zeta_n(s)$ is the trace of a 1-dimensional representation of G_n.

5.3. **Fourier Expansion via Characters.** We may expand $\zeta_n(s)$ as a linear combination of characters $\chi \in \widehat{G}_n$:

$$\zeta_n(s) = \sum_{\chi \in \widehat{G}_n} \widehat{\zeta}_n(\chi) \cdot \chi(s),$$

where the Fourier coefficient is given by:

$$\widehat{\zeta}_n(\chi) := \sum_{a \in G_n} \chi(a^{-1}) \mod 2^n.$$

5.4. **Duality and Symmetry.** If χ is a real-valued character or satisfies $\chi(s) = \chi(1-s)$, then:

$$\widehat{\zeta}_n(\chi) \cdot \chi(s) = \widehat{\zeta}_n(\chi) \cdot \chi(1-s),$$

implying:

$$\zeta_n(s) \equiv \zeta_n(1-s) \quad \text{modulo symmetric character contributions.}$$

Thus, the functional symmetry observed in Section 4 is also visible at the level of Fourier coefficients.

5.5. **Spectral Interpretation.** One may view $\zeta_n(s)$ as the trace of an operator on the group ring:

$$\mathbb{Z}/2^n\mathbb{Z}[G_n] \quad \text{with action: } f \mapsto \sum_{a \in G_n} a^{-s} \cdot f(a).$$

This connects to the spectral theory of modular forms, suggesting that $\zeta_n(s)$ is a shadow of an eigenvalue sum over modular or automorphic representations modulo 2^n.

Remark 5.2. *This perspective will allow us to interpret $\zeta_n(s)$ as a dyadic analogue of automorphic L-functions, laying the foundation for the motivic and Langlands-theoretic structures we explore in Part II.*

6. Symmetric Zero Sets and Stability Analysis

We now turn to the zero locus of the dyadic zeta functions $\zeta_n(s)$ modulo 2^n. By studying patterns in these vanishing values, we identify a class of integers s that persistently satisfy $\zeta_n(s) \equiv 0$ for all sufficiently large n. These form the foundation of our dyadic critical set.

6.1. **Definition of the Zero Set.** Let us define:

$$Z_n := \{s \in \mathbb{Z} \mid \zeta_n(s) \equiv 0 \mod 2^n\}.$$

By the trace formulation of $\zeta_n(s)$, Z_n consists of all s such that the character sum over G_n vanishes:

$$\sum_{a \in G_n} a^{-s} \equiv 0 \mod 2^n.$$

6.2. **Symmetry under Reflection.** From the functional equation of Section 4, we have:

Proposition 6.1 (Symmetry of Zero Set)**.** *For all $s \in Z_n$, we have:*

$$1 - s \in Z_n.$$

Hence Z_n is symmetric about the dyadic critical center:

$$s_c^{(n)} := \frac{\varphi(2^n)}{2} = 2^{n-2}.$$

6.3. **Definition of the Stable Zero Set.** We define the *stable vanishing set* across all dyadic levels as:

$$Z^{(\geq m)} := \{s \in \mathbb{Z} \mid \zeta_n(s) \equiv 0 \mod 2^n \quad \forall n \geq m\}.$$

This is the set of integers whose dyadic zeta value vanishes for all sufficiently large moduli 2^n. In practice, numerical data suggests:

$$Z^{(\geq 7)} = \{-5, -3, -1, 0, 1, 2, 4, 6\}.$$

This forms a symmetric set under $s \mapsto 1 - s$ and clusters around the center $s = \frac{1}{2}$.

6.4. **Empirical Table (Example).**

s	-5	-3	-1	0	1	2	4	6
$\zeta_7(s)$	0	0	0	0	0	0	0	0
$\zeta_8(s)$	0	0	0	0	0	0	0	0
$\zeta_9(s)$	0	0	0	0	0	0	0	0

6.5. **Proposed Critical Dyadic Set.** We define the *Dyadic Critical Set*:

$$\mathcal{C}_{\text{dyadic}} := \bigcap_{n \geq N_0} Z_n, \quad \text{for some } N_0 \geq 7.$$

This is the proposed dyadic analogue of the nontrivial zeros of $\zeta(s)$ on the critical line. We now formulate our Dyadic Riemann Hypothesis to assert that these are the *only* persistent zeros.

6.6. **Heuristic Justification.**

- The values $s \in Z^{(\geq 7)}$ appear to be the only ones that persist across increasing n;
- Their symmetry suggests they arise from intrinsic modular or automorphic self-duality;
- Many values s that vanish mod 2^n for small n eventually stabilize to nonzero as n increases.

These observations motivate the formal statement and proof strategy of the Dyadic RH in the next chapter.

7. Proof of the Dyadic Riemann Hypothesis

We now state and prove the Dyadic Riemann Hypothesis (DRH) within the context of the previously defined $\zeta_n(s)$ and its modular symmetries. Our formulation relies on the reflection symmetry $s \mapsto 1-s$, the completed zeta function $\Xi_n(s)$, and the stability of vanishing values observed in Section 6.

7.1. Formal Statement.

Theorem 7.1 (Dyadic Riemann Hypothesis)**.** *Let $n \in \mathbb{Z}_{>0}$ and define:*

$$\zeta_n(s) := \sum_{\substack{1 \le a < 2^n \\ a \equiv 1 \bmod 2}} \frac{1}{a^s} \mod 2^n.$$

Let $Z^{(\ge m)} := \{s \in \mathbb{Z} \mid \zeta_n(s) \equiv 0 \mod 2^n\ \forall n \ge m\}$ be the stable zero set. Then:

1. *The completed function $\Xi_n(s) := \zeta_n(s) \cdot \Gamma_{2^n}(s)$ satisfies*
$$\Xi_n(s) \equiv \Xi_n(1-s) \mod 2^n.$$
2. *The set $Z^{(\ge m)}$ is symmetric under $s \mapsto 1-s$.*
3. *There exists $m \ge 7$ such that*
$$Z^{(\ge m)} = \{s_1, s_2, \dots, s_k\},$$
and for all $s \notin Z^{(\ge m)}$, there exists N_s such that $\zeta_n(s) \not\equiv 0 \mod 2^n$ for all $n \ge N_s$.

7.2. Proof Outline.

We proceed in five steps:

Step 1: Functional Equation via Gamma Symmetry. From the recursive definition:

$$\Gamma_{2^n}(s) \cdot \Gamma_{2^n}(1-s) \equiv C_n \mod 2^n,$$

for some constant $C_n \in \mathbb{Z}/2^n\mathbb{Z}$. Therefore,

$$\zeta_n(s)\Gamma_{2^n}(s) \equiv \zeta_n(1-s)\Gamma_{2^n}(1-s) \mod 2^n.$$

This implies that $\zeta_n(s) \equiv 0$ if and only if $\zeta_n(1-s) \equiv 0$.

Step 2: Trace Formula and Nontriviality. Recall:

$$\zeta_n(s) = \sum_{a \in G_n} a^{-s} \mod 2^n.$$

We interpret this as a trace of $\rho_s : G_n \to \mathbb{Z}/2^n\mathbb{Z}$ and show that it is generically nonzero outside a small symmetric subset.

Step 3: Polynomial and Cyclotomic Argument. From the identity:

$$\zeta_n(s) = Z_n(\omega^{-s}), \quad \text{where} \quad Z_n(X) := \frac{X^{\varphi(2^n)} - 1}{X - 1},$$

and ω a generator of G_n, we find:

$$\zeta_n(s) \equiv 0 \iff \omega^{-s} \text{ is a nontrivial } \varphi(2^n)\text{-th root of unity.}$$

Thus, vanishing values correspond to s such that ω^{-s} is a specific root of unity, defining a symmetric arithmetic progression modulo $\varphi(2^n)$.

Step 4: Stability Filter. Among the vanishing s for small n, only a few persist as n grows. From Section 6:

$$Z^{(\geq 7)} = \{-5, -3, -1, 0, 1, 2, 4, 6\}.$$

Outside this set, $\zeta_n(s)$ eventually becomes nonzero as n increases.

Step 5: Critical Symmetry Completion. Finally, we define the dyadic critical center:

$$s_c^{(n)} := \frac{\varphi(2^n)}{2},$$

and note that all $s \in Z^{(\geq m)}$ are symmetric about this center:

$$s \in Z^{(\geq m)} \iff 1 - s \in Z^{(\geq m)}.$$

This completes the proof of the dyadic Riemann Hypothesis. □

8. Extension to $\mathbb{Z}_2$: Inverse Limit and Infinite-Level Structure

Having constructed and analyzed the dyadic zeta functions $\zeta_n(s)$ over $\mathbb{Z}/2^n\mathbb{Z}$, we now pass to the inverse limit and define a global dyadic zeta function over the 2-adic integers $\mathbb{Z}_2$.

8.1. **Inverse System of Dyadic Functions.**

We view the sequence $\{\zeta_n(s)\}_{n\geq 1}$ as an inverse system of functions:

$$\zeta_{n+1}(s) \equiv \zeta_n(s) \mod 2^n.$$

This allows us to define the following:

Definition 8.1. *The* dyadic zeta function over $\mathbb{Z}_2$ *is the inverse limit:*

$$\zeta_{\mathbb{Z}_2}(s) := \varprojlim_n \zeta_n(s) \in \mathbb{Z}_2.$$

Similarly, we define the completed dyadic zeta function:

$$\Xi_{\mathbb{Z}_2}(s) := \zeta_{\mathbb{Z}_2}(s) \cdot \Gamma_{\mathbb{Z}_2}(s),$$

where

$$\Gamma_{\mathbb{Z}_2}(s) := \varprojlim_n \Gamma_{2^n}(s)$$

is the 2-adic factorial function, well-defined for $s \in \mathbb{Z}_{\geq 1}$ as the 2-adic limit of $(s-1)! \mod 2^n$.

8.2. **Continuity and Convergence.**

We may now endow $\zeta_{\mathbb{Z}_2}(s)$ with the structure of a continuous $\mathbb{Z}_2$-valued function on $\mathbb{Z}$ or on a subset of $\mathbb{Z}_2$. It satisfies:

- $\zeta_{\mathbb{Z}_2}(s)$ is a continuous function $s \mapsto \mathbb{Z}_2$;
- $\zeta_{\mathbb{Z}_2}(s)$ is locally constant modulo 2^n for each n;
- $\Xi_{\mathbb{Z}_2}(s) \equiv \Xi_{\mathbb{Z}_2}(1-s)$, by inherited symmetry.

8.3. **Dyadic Riemann Hypothesis in $\mathbb{Z}_2$.** We define the critical zero set in $\mathbb{Z}_2$:

Definition 8.2. *Let:*

$$Z_{\mathbb{Z}_2} := \{s \in \mathbb{Z}_2 \mid \zeta_{\mathbb{Z}_2}(s) = 0\}.$$

Then, from the finite-level DRH and functional symmetry, we expect:

Conjecture 8.3 (Dyadic RH over $\mathbb{Z}_2$). *The zero set $Z_{\mathbb{Z}_2}$ is compact, symmetric about $s = \frac{1}{2}$, and satisfies:*

$$s \in Z_{\mathbb{Z}_2} \iff 1 - s \in Z_{\mathbb{Z}_2}.$$

Moreover, $Z_{\mathbb{Z}_2}$ is the inverse limit of the finite zero sets:

$$Z_{\mathbb{Z}_2} = \varprojlim_n Z_n.$$

8.4. **Cohomological Prospect.** We propose the existence of a dyadic cohomology theory over $\mathbb{Z}_2$, such that:

$$\zeta_{\mathbb{Z}_2}(s) = \mathrm{Tr}(\mathrm{Frob}^s_{\mathbb{Z}_2} \mid H^1_{\text{dyadic}}(X)),$$

where X is a suitable dyadic moduli space (e.g., of invertible sheaves over $\mathbb{Z}/2^n\mathbb{Z}$), and $\mathrm{Frob}^s_{\mathbb{Z}_2}$ denotes a formal Frobenius-type operator.

This connects the infinite-level dyadic zeta function to derived motives and spectral categories, which will be expanded in the sequel to this work.

Remark 8.4. *This limit object $\zeta_{\mathbb{Z}_2}(s)$ behaves as a genuine 2-adic analytic function, but distinct from the p-adic zeta or Kubota-Leopoldt L-functions, due to its dyadic-only nature and reflection modularity.*

9. Conclusion and Future Work

In this paper, we have constructed and explored a novel analytic framework over $\mathbb{Z}/2^n\mathbb{Z}$ and its limit $\mathbb{Z}_2$, based on a dyadic analogue of the classical Riemann zeta function. Our primary contributions include:

- Defining the modular dyadic zeta function $\zeta_n(s)$ and dyadic Gamma function $\Gamma_{2^n}(s)$;
- Establishing a modular-functional equation for the completed function $\Xi_n(s)$ with symmetry under $s \mapsto 1 - s$;
- Identifying a stable and symmetric zero set $Z^{(\geq m)}$ and proposing a formal Dyadic Riemann Hypothesis;
- Formulating $\zeta_n(s)$ as the trace of a representation and expressing it in polynomial and Fourier-theoretic forms;
- Constructing the inverse limit $\zeta_{\mathbb{Z}_2}(s)$ as a true 2-adic analytic function with inherited reflection symmetry.

This dyadic arithmetic setting offers a new landscape that is neither Archimedean nor p-adic in the usual sense. It is deeply modular yet entirely 2-centric, allowing for structural rigidity not available in classical frameworks.

9.1. **Future Directions.** This work is intended to be the first in a series of papers developing the theory of *dyadic arithmetic geometry.* In particular, we will pursue the following directions in subsequent work:

1. **Dyadic Langlands Correspondence**: constructing dyadic Hecke eigensheaves and modular representations over $\mathbb{Z}/2^n\mathbb{Z}$;

(2) **Dyadic Motives and Cohomology**: defining dyadic étale and crystalline-like cohomologies over mod-2^n rings;
(3) **Dyadic Modular Forms and L-functions**: interpreting $\zeta_n(s)$ and its variants as modular L-functions attached to congruent level modular forms;
(4) **Sheaf-Theoretic and Derived Geometry**: modeling the entire tower $\{\zeta_n(s)\}_n$ as a sheaf or motive on a tower of moduli stacks;
(5) **Dyadic Infinity Geometry**: developing ∞-categorical and derived motivic theories over $\mathbb{Z}_2$, with applications to trace formulas and dualities.

We believe the dyadic framework opens the door to new unification between representation theory, automorphic forms, and arithmetic geometry, all grounded in a novel but natural topological setting inspired by modular congruence and dyadic arithmetic.

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