Yang-Langlands Program: A Higher Dimensional Extension

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1 Introduction

The Yang-Langlands Program is an extended framework that integrates the Yang $\mathbb{Y}_*(F)$ number systems with Langlands' principles, providing a higher-dimensional generalization of arithmetic dualities.

2 Yang-Galois Groups and Correspondences

We define the **Yang-Galois group** as:

$$G_{\mathbb{Y}_n(F)} = \mathrm{Gal}(Q_{\mathbb{Y}_n,v\alpha}/\mathbb{Y}_n(F))$$

where $Q_{\mathbb{Y}_n,v\alpha}$ is the **generalized valuation field** associated with $\mathbb{Y}_n(F)$.

The Yang-Langlands correspondence establishes a non-trivial bijection:

$$G_{\mathbb{Y}_n(F)} \longleftrightarrow A_{\mathbb{Y}_n(F)}$$

where $A_{\mathbb{Y}_n(F)}$ represents the Yang-Langlands automorphic forms.

3 Generalized Valuations and L-Functions

The structure of $\mathbb{Y}_n(F)$ leads to the definition of an extended zeta function:

$$\zeta_{\mathbb{Y}_n}(s) = \prod_{v\alpha} \zeta_{Q_{\mathbb{Y}_n,v\alpha}}(s)$$

This function extends classical Langlands L-functions into the Yang-Langlands framework, capturing deeper arithmetic properties.

4 Higher Hecke Algebras

We introduce a Hecke algebra associated with $\mathbb{Y}_n(F)$:

$$H_n(A_{\mathbb{Y}_n(F)})$$

which acts on automorphic forms in a non-commutative setting, allowing for higher categorical structures in the Langlands program.

5 Geometric Yang-Langlands

The extension into geometric settings involves defining a derived category of sheaves over Nocturnis moduli spaces:

$$D^b(Y_{\mathbb{Y}_n}) \longleftrightarrow \mathrm{Rep}(G_{\mathbb{Y}_n})$$

which establishes a deeper connection between arithmetic geometry and representation theory.

6 Future Directions

Future work will focus on:

- Developing the representation theory of $G_{\mathbb{Y}_n(F)}$.
- Constructing explicit instances of $\mathbb{Y}_n(F)$ structures in arithmetic geometry.
- Extending the program to a quantum Yang-Langlands setting.