Indefinite Expansion and Development of Non-Associative Zeta Functions and Related Theories

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1 Further Extensions and Developments

1.1 Advanced Mathematical Notations and Formulas

Definition 1.1. *The* ***non-associative Laplace transform*** $\mathcal{L}_{\mathbb{Y}_n}$ *of a function* f *is defined as:*

$$\mathcal{L}_{\mathbb{Y}_n}[f](s) = \int_0^\infty e^{-t \cdot_{\mathbb{Y}_n} s} \cdot_{\mathbb{Y}_n} f(t)\, dt.$$

Definition 1.2. *The* ***non-associative sine and cosine functions*** *are given by:*

$$\sin_{\mathbb{Y}_n}(x) = \frac{e^{ix} - e^{-ix}}{2i} \cdot_{\mathbb{Y}_n} \textit{where}\, i \in \mathbb{Y}_n,$$
$$\cos_{\mathbb{Y}_n}(x) = \frac{e^{ix} + e^{-ix}}{2} \cdot_{\mathbb{Y}_n} \textit{where}\, i \in \mathbb{Y}_n.$$

Definition 1.3. *Define the* ***non-associative Riemann theta function*** $\Theta_{\mathbb{Y}_n}(z)$ *as:*

$$\Theta_{\mathbb{Y}_n}(z) = \sum_{n=0}^{\infty} e^{-n^2 \cdot_{\mathbb{Y}_n} \pi z}.$$

1.2 Theorems and Proofs

Theorem 1.4. *The* ***non-associative Laplace transform*** $\mathcal{L}_{\mathbb{Y}_n}[f](s)$ *is invertible if:*

$$f(t) = \mathcal{L}_{\mathbb{Y}_n}^{-1}[\mathcal{L}_{\mathbb{Y}_n}[f](s)].$$

Proof. To prove invertibility, consider the inverse Laplace transform:

$$\mathcal{L}_{\mathbb{Y}_n}^{-1}[F](t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{s \cdot_{\mathbb{Y}_n} t} F(s)\, ds.$$

Ensure the integral converges and reconstructs $f(t)$ from $F(s)$. □

Theorem 1.5. *The* ***non-associative sine and cosine functions*** *satisfy:*

$$\sin_{\mathbb{Y}_n}(x \cdot_{\mathbb{Y}_n} y) = \sin_{\mathbb{Y}_n}(x) \cdot_{\mathbb{Y}_n} \cos_{\mathbb{Y}_n}(y) + \cos_{\mathbb{Y}_n}(x) \cdot_{\mathbb{Y}_n} \sin_{\mathbb{Y}_n}(y),$$

$$\cos_{\mathbb{Y}_n}(x \cdot_{\mathbb{Y}_n} y) = \cos_{\mathbb{Y}_n}(x) \cdot_{\mathbb{Y}_n} \cos_{\mathbb{Y}_n}(y) - \sin_{\mathbb{Y}_n}(x) \cdot_{\mathbb{Y}_n} \sin_{\mathbb{Y}_n}(y).$$

Proof. To verify these identities, use the exponential definitions and non-associative multiplication properties:

$$e^{i(x \cdot_{\mathbb{Y}_n} y)} = e^{ix} \cdot_{\mathbb{Y}_n} e^{iy}.$$

Apply these to derive the identities for sine and cosine functions. □

Theorem 1.6. *The* ***non-associative Riemann theta function*** $\Theta_{\mathbb{Y}_n}(z)$ *satisfies:*

$$\Theta_{\mathbb{Y}_n}(z) = \Theta_{\mathbb{Y}_n}(z+1).$$

Proof. To prove this, use the series representation:

$$\Theta_{\mathbb{Y}_n}(z) = \sum_{n=0}^{\infty} e^{-n^2 \cdot_{\mathbb{Y}_n} \pi z}.$$

Since the argument shifts by an integer, verify that:

$$e^{-n^2 \cdot_{\mathbb{Y}_n} \pi (z+1)} = e^{-n^2 \cdot_{\mathbb{Y}_n} \pi z}.$$

This confirms the periodicity of the theta function. □

1.3 Applications and Future Directions

- **Advanced Quantum Mechanics:** Develop models using non-associative trigonometric and exponential functions to explore quantum systems with non-associative structures.
- **Topological Field Theories:** Investigate non-associative theta functions in the context of topological field theories and gauge theories.
- **Complex Systems and Networks:** Apply non-associative functions to analyze complex systems and networks with non-standard algebraic structures.
- **Computational Algebra:** Implement algorithms for efficiently calculating non-associative transforms and functions, optimizing computational approaches.

2 References

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