ARITHOGEOMETIC MEAN

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Given any elliptic curve E with Weistrauß equation

$$y^2 = x(x+r)(x+s),$$

assume neither r nor r is real and nonnegative.

Let $L \subset \mathbb{C}$ be the lattice with period $\omega = \frac{dx}{2y}$, and let $\Phi : \mathbb{C}/L \xrightarrow{\cong} E(\mathbb{C})$ be isomorphism with $\Phi^{-1}(\omega) = dz$, where z is parameter on $\mathbb{C}$.

Now write $L = \mathbb{Z}\gamma + \mathbb{Z}\delta$ where $\gamma = \int_0^\infty \omega \in \mathbb{R}$, we know that $\Phi[0, \gamma] \subseteq E(\mathbb{R})$ with $\Phi(0) = \infty$, and so $\Phi(\gamma/2) = (0, 0)$.

Other points of order two are $(-r, 0)$ and $(-s, 0)$, so we assume $\Phi(\delta/2) = (-r, 0)$ and $\Phi(\delta/2 + \gamma/2) = (-s, 0)$, $\Big($OR $\Phi(\delta/2) = (-s, 0)$ and $\Phi(\delta/2 + \gamma/2) = (-r, 0)\Big)$.

Let Λ be the lattice $\mathbb{Z}\gamma + \mathbb{Z}\delta \subseteq \mathbb{C}$. Invariant under complex conjugation (why?), and hence corresponds to a real elliptic curve. Find Weistrauß equation

$$v^2 = u(u+R)(u+S)$$

and an isomorphism $\Psi : \mathbb{C}/\Lambda \xrightarrow{\cong} F(\mathbb{C})$ (why F here?). Constant c in $\Psi^{-1}(\frac{du}{2v}) = c\,dz$

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