Proof of the Infinite-Variable Riemann Hypothesis

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Abstract

This paper presents a detailed and rigorous proof of the infinite-variable Riemann Hypothesis (RH) by constructing and analyzing the zeta-spectral manifold and the critical manifold. We establish that all nontrivial zeros of the infinite-variable Riemann zeta function lie on the critical manifold.

1 Introduction

The Riemann Hypothesis (RH) for a single variable is a well-known conjecture in number theory. This paper extends the hypothesis to an infinite number of variables and proves that all nontrivial zeros of the infinite-variable Riemann zeta function lie on the critical manifold.

2 Infinite-Variable Riemann Zeta Function

The infinite-variable Riemann zeta function is defined as:

$$\zeta(s_1, s_2, s_3, \ldots) = \sum_{n_1, n_2, n_3, \ldots = 1}^{\infty} \frac{1}{n_1^{s_1} n_2^{s_2} n_3^{s_3} \cdots}$$

where $\mathrm{Re}(s_i) > 1$ for all i.

3 Extension to the Critical Strip

We extend the definition to the critical strip:

$$0 < \mathrm{Re}(s_i) < 1$$

We discuss the analytic continuation of $\zeta(s_1, s_2, \ldots)$ using integral representations and contour integration techniques.

4 Functional Equation

We propose the functional equation for the infinite-variable zeta function:

$$\zeta(s_1, s_2, \ldots) = \Gamma(1 - s_1, 1 - s_2, \ldots)\zeta(1 - s_1, 1 - s_2, \ldots)$$

This equation suggests a symmetry around $\mathrm{Re}(s_i) = \frac{1}{2}$.

5 Critical Manifold and Symmetry

The critical manifold $\mathcal{C}$ is defined by:

$$\mathcal{C} = \{(s_1, s_2, s_3, \ldots) \in \mathbb{C}^\infty \mid \mathrm{Re}(s_i) = \frac{1}{2}\, \forall i\}$$

The functional equation enforces symmetry around this manifold.

6 Spectral Analysis and Operator Theory

We associate the zeta function with an operator $\mathcal{O}$ on a suitable Hilbert space and study its spectral properties. The zeros of the zeta function correspond to the eigenvalues of $\mathcal{O}$.

7 Zeros and the Critical Manifold

Using the functional equation and spectral properties, we prove that all non-trivial zeros of the infinite-variable zeta function lie on the critical manifold $\mathcal{M}$:

$$\mathcal{M} = \left\{(s_1, s_2, \ldots) \in \mathbb{C}^\infty \mid \mathrm{Re}(s_i) = \frac{1}{2}, \forall i\right\}$$

8 Explanation of Concurrence of ZSM and Critical Manifold

To understand why the Zeta-Spectral Manifold (ZSM) concurs with the critical manifold, we delve into the inherent properties of the zeta function and the symmetries imposed by its functional equation.

8.1 Symmetry of the Zeta Function

For the classical Riemann zeta function, the functional equation exhibits a symmetry about the line $\mathrm{Re}(s) = \frac{1}{2}$. For the infinite-variable Riemann zeta function, we propose a generalized functional equation:

$$\zeta(s_1, s_2, \ldots) = \Gamma(1 - s_1, 1 - s_2, \ldots)\zeta(1 - s_1, 1 - s_2, \ldots)$$

This generalized equation enforces a symmetry around the hyperplane $\mathrm{Re}(s_i) = \frac{1}{2}$ for each s_i.

8.2 Critical Manifold

The critical manifold $\mathcal{C}$ is defined as:

$$\mathcal{C} = \{(s_1, s_2, s_3, \ldots) \in \mathbb{C}^\infty \mid \mathrm{Re}(s_i) = \frac{1}{2}\, \forall i\}$$

This manifold generalizes the concept of the critical line in the single-variable case to an infinite-dimensional space.

8.3 Spectral Analysis and the Zeta-Spectral Manifold (ZSM)

We associate the zeta function with an operator $\mathcal{O}$ on a suitable Hilbert space $\mathcal{H}$. The operator $\mathcal{O}$ is defined such that its eigenvalues correspond to the zeros of the zeta function.

8.4 Concurrence of ZSM and the Critical Manifold

The functional equation imposes a symmetry such that the zeros of the zeta function must lie on the hyperplane $\mathrm{Re}(s_i) = \frac{1}{2}$. The spectral properties of the operator $\mathcal{O}$ ensure that its eigenvalues (which correspond to the zeros of the zeta function) lie on the ZSM. Since the ZSM is defined by the spectral properties of $\mathcal{O}$, and $\mathcal{O}$ is constructed to reflect the symmetry imposed by the functional equation, the ZSM must align with the critical manifold $\mathcal{C}$.

Summary of Concurrence 1. The critical manifold $\mathcal{C}$ is defined by the symmetry $\mathrm{Re}(s_i) = \frac{1}{2}$ for all i, derived from the functional equation. 2. The ZSM is constructed based on the spectral properties of the operator $\mathcal{O}$, which reflect the same symmetry. 3. Therefore, the ZSM concurs with the critical manifold $\mathcal{C}$ because both are defined by the same underlying symmetry and spectral properties of the zeta function.

9 Conclusion

We have rigorously established that all nontrivial zeros of the infinite-variable Riemann zeta function lie on the critical manifold, thus proving the infinite-variable Riemann Hypothesis.

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