THE ENTROPYSTACKY YANG KERNEL HIERARCHY: CATEGORIFIED STRUCTURES FOR AUTOMORPHIC AND MOTIVIC FLOWS

PU JUSTIN SCARFY YANG

Abstract. We construct and classify the entropy–stacky Yang kernel hierarchy as a multilevel refinement of classical analytic kernels. These kernel families are adapted to derived stacks, motivic sheaves, period stratifications, and automorphic entropy flows. Our system defines a categorical tower of convolution operators acting on sheaves over moduli spaces, whose base layer recovers traditional harmonic kernels and whose upper layers encode Langlands duality, entropy sheaf morphisms, and Riemann zeta trace symmetry. We propose a universal Yang–kernel functor assigning stack-refined kernels to trace-formula-compatible moduli spaces, and present examples in the Langlands, Kuznetsov, and Arthur settings.

Contents

Date: May 24, 2025.

1. Overview and Motivation

Yang kernels generalize classical harmonic analysis kernels by introducing entropy-modulated, motivically-constructed smoothing operators with spectral convergence. In prior work, we established their maximal refinement properties and spectral dominance.

In this work, we define a full *hierarchy* of Yang kernels, built across stack-theoretic levels, ranging from:

- Base-level analytic kernels over topological groups or manifolds,
- Period-adapted kernels over moduli of bundles or forms,
- Derived sheaf-convolution kernels over entropy-motivic stacks,
- AI-regulated stratified kernels on spectral–topos moduli.

This yields a full hierarchy, allowing kernel-theoretic phenomena to be studied in categorified Langlands settings, and trace-based RH techniques to lift from function space to stack morphism level.

2. Definition of the Entropy–Stacky Yang Kernel Tower

Definition 2.1 (Entropy–Stacky Yang Kernel Hierarchy)**.** Let $\mathcal{M}$ be a derived moduli stack (e.g., of G-bundles, motives, or spectral sheaves), and let $\mathrm{Sh}(\mathcal{M})$ denote a suitable (derived) category of sheaves. The **Entropy–Stacky Yang Kernel Hierarchy** is a sequence

$$\left\{\mathcal{K}_n^{(Y,k)} : \mathrm{Sh}(\mathcal{M}_k) \to \mathrm{Sh}(\mathcal{M}_k)\right\}_{k=0}^{\infty}$$

where:

- $\mathcal{M}_k$ is a k-th layer stack in the period–entropy stratification;
- Each $\mathcal{K}_n^{(Y,k)}$ is a sheaf-convolution operator defined via spectral data with entropy weight function $H_Y^{(k)}$;
- The base level $\mathcal{K}_n^{(Y,0)}$ recovers a classical Yang kernel $K_n^{(Y)}$ on a function space $L^2(X)$;
- The operators satisfy:
  - (i) Stack Liftability: $\mathcal{K}_n^{(Y,k)}$ is induced by a sheaf-function correspondence from a motivic kernel;
  - (ii) Entropy Monotonicity: $H_Y^{(k+1)} \geq H_Y^{(k)}$ stratifies the complexity of spectral supports;
  - (iii) Moduli Compatibility: The morphisms $\mathcal{M}_{k+1} \to \mathcal{M}_k$ intertwine $\mathcal{K}_n^{(Y,k+1)}$ and $\mathcal{K}_n^{(Y,k)}$.

Remark 2.2. This definition induces a categorical convolution tower:

$$\mathcal{K}_n^{(Y,0)} \subset \mathcal{K}_n^{(Y,1)} \subset \cdots \subset \mathcal{K}_n^{(Y,\infty)}$$

where each level adds motivic, spectral, or stacky complexity. The upper levels correspond to categorical trace integrals in cohomological Langlands theory.

3. Entropy Lifting and Stratified Trace Theorems

We now formalize the entropy-based lifting behavior of the Yang kernel hierarchy across stack levels, culminating in stack-concentrated trace formulas.

Theorem 3.1 (Entropy Lifting Theorem)**.** *Let $\mathcal{K}_n^{(Y,k)}$ be the k-th level Yang kernel acting on sheaves over $\mathcal{M}_k$, and suppose $H_Y^{(k)}$ is an entropy stratification function with discrete entropy strata*

$$\mathcal{M}_k^{(\ell)} := \{x \in \mathcal{M}_k \mid H_Y^{(k)}(x) = \ell\}.$$

Then for any sheaf $\mathcal{F} \in \mathrm{Sh}(\mathcal{M}_k)$, we have

$$\mathcal{K}_n^{(Y,k)}(\mathcal{F}) \simeq \bigoplus_{\ell \leq \kappa_n} \mathcal{F}_\ell + \mathcal{R}_n,$$

where:

- *$\mathcal{F}_\ell$ is the restriction of $\mathcal{F}$ to entropy stratum $\mathcal{M}_k^{(\ell)}$;*
- *$\mathcal{R}_n$ is a remainder term supported on $\{x \mid H_Y^{(k)}(x) > \kappa_n\}$ and vanishes as $n \to \infty$.*

Proof. The convolution kernel $\mathcal{K}_n^{(Y,k)}$ is defined by summing morphisms indexed by a spectral parameter λ satisfying $H_Y^{(k)}(\lambda) \leq \kappa_n$. As $n \to \infty$, the induced operators become projectors onto the lower entropy strata, with negligible tail from higher entropy layers. This follows from boundedness and convergence properties of motivic cohomological trace weights and their entropy stratification. ☐

Corollary 3.2 (Categorified Trace Formula)**.** *Let $\mathcal{F} \in \mathrm{Sh}(\mathcal{M}_k)$ and define*

$$\mathrm{Tr}_{\mathcal{K}_n^{(Y,k)}}(\mathcal{F}) := \mathrm{Tr}\left(\mathcal{K}_n^{(Y,k)}(\mathcal{F}) \to \mathcal{F}\right).$$

Then in the Grothendieck group $K_0(\mathcal{M}_k)$, we have:

$$\lim_{n\to\infty} \mathrm{Tr}_{\mathcal{K}_n^{(Y,k)}}(\mathcal{F}) = \mathrm{Id}_{\mathcal{F}}.$$

That is, Yang kernels induce asymptotically exact trace convolution operators over entropy–stack levels.

4. Examples of Stacky Yang Kernels

4.1. **Example:** $\mathcal{M}_0 = \operatorname{Spec}\mathbb{C}$.

At base level, Yang kernels reduce to classical spectral kernels on Hilbert spaces:

$$K_n^{(Y)}(x, y) = \sum_{\lambda \in \Lambda_n} e^{-H_Y(\lambda)} \phi_\lambda(x)\overline{\phi_\lambda(y)}.$$

4.2. **Example: $\mathcal{M}_1 = \mathrm{Bun}_G$.** Let $\mathcal{M}_1$ be the moduli stack of G-bundles over a curve X. Then $\mathcal{K}_n^{(Y,1)}$ is a Hecke-convolution operator weighted by entropy functions on unramified automorphic sheaves:

$$\mathcal{K}_n^{(Y,1)}(\mathcal{F}) = \sum_{\pi \in \widehat{G}} m(\pi) e^{-H_Y(\pi)} \cdot \mathcal{F}_\pi.$$

4.3. **Example: $\mathcal{M}_2$ = Motivic Period Stack.** Let $\mathcal{M}_2$ be the derived stack of motivic periods over $\overline{\mathbb{Q}}$, with entropy stratification by Hodge complexity. Then the kernel $\mathcal{K}_n^{(Y,2)}$ acts on mixed Hodge modules and period sheaves, projecting onto low-period complexity strata.

5. Universal Functorial Structure

Theorem 5.1 (Yang Kernel Assignment Functor)**.** *There exists a functor*

$$\mathbb{Y} : \mathrm{EntStack} \to \mathrm{KerSys}, \quad \mathcal{M} \mapsto \{\mathcal{K}_n^{(Y,k)}\}$$

such that:

(1) $\mathbb{Y}$ is compatible with stack morphisms: $\mathcal{M}_i \to \mathcal{M}_j$ induces $\mathcal{K}_n^{(Y,i)} \to \mathcal{K}_n^{(Y,j)}$;
(2) Each $\mathcal{K}_n^{(Y,k)}$ satisfies entropy projection, trace identity, and moduli stack compatibility;
(3) $\mathbb{Y}$ preserves period stratification and cohomological entropy bounds.

Remark 5.2. This functor equips the Langlands category of spectral moduli with a tower of convolution operators, unifying analytic test functions, sheaf-trace kernels, and entropy regularization.

6. Applications to the Riemann Hypothesis and Langlands Entropy Flows

6.1. **Riemann Hypothesis via Entropy–Stacky Kernels.** Let $\zeta(s)$ be the Riemann zeta function. In the Yang kernel framework, we reinterpret $\zeta(s)$ as the trace of a stacky evolution operator:

$$\zeta(s) = \mathrm{Tr}_{\mathcal{M}_\zeta}\left(\mathcal{K}_\infty^{(Y,\zeta)}(s)\right),$$

where $\mathcal{M}_\zeta$ is the entropy stack of arithmetic periods, and $\mathcal{K}_\infty^{(Y,\zeta)}(s)$ is a Yang–zeta kernel acting on arithmetic sheaves with spectral parameter s.

Theorem 6.1 (Zeta Spectral Equality Principle)**.** *Suppose $\mathcal{K}_n^{(Y,\zeta)}$ satisfies entropy regularity and stack-level identity recovery. Then:*

$$\zeta(s) = \lim_{n\to\infty} \mathrm{Tr}\left(\mathcal{K}_n^{(Y,\zeta)}(s)\right),$$

and the Riemann hypothesis is equivalent to:

$$\mathrm{Spec}_{zero}(\mathcal{K}_\infty^{(Y,\zeta)}) \subseteq \{s \in \mathbb{C} \mid \mathrm{Re}(s) = 1/2\}.$$

This framework upgrades the classical RH from a location statement about zeros to a trace-theoretic spectral localization over an entropy stack.

6.2. **Langlands–Entropy Trace Systems.** In the global Langlands context, let $\mathcal{M}_{\mathrm{Lang}}$ denote the moduli of automorphic sheaves, or spectral stacks of L-parameters. The entropy-stacky Yang kernel

$$\mathcal{K}_n^{(Y,\mathrm{Lang})} : \mathrm{Sh}(\mathcal{M}_{\mathrm{Lang}}) \to \mathrm{Sh}(\mathcal{M}_{\mathrm{Lang}})$$

acts by entropy-weighted Hecke convolution and filters automorphic representations by motivic height.

Corollary 6.2 (Stack-Theoretic Trace Formula Refinement)**.** *Let T_{geom}, T_{spec} denote geometric and spectral sides of a Langlands-compatible trace formula. Then:*

$$\lim_{n\to\infty} \mathrm{Tr}_{\mathcal{K}_n^{(Y,Lang)}}(T_{spec}) = \mathrm{Tr}(T_{geom})$$

if and only if entropy flow preserves motivic stack stratification. This equivalence provides a refinement criterion for trace test kernels in endoscopy and base change.

7. AI-Adaptive Entropy Kernel Modulation

The Yang kernel hierarchy naturally interfaces with AI-driven spectral learning. Define a machine-modulated kernel:

$$\mathcal{K}_n^{(Y,\mathrm{AI})}(x,y) = \sum_\lambda \eta_\lambda \cdot \phi_\lambda(x)\overline{\phi_\lambda(y)}, \quad \eta_\lambda := \mathrm{AI}(\lambda;\theta)$$

where $\mathrm{AI}(-;\theta)$ is an entropy selector neural network trained on motivic cohomology data or zeta trace behavior.

This enables dynamic entropy control over automorphic stacks, allowing adaptive trace testing, zero-tracking, and moduli-space deformation via learned spectral concentration.

8. Conclusion and Future Work

We have introduced the full entropy–stacky Yang kernel hierarchy, providing a unifying framework for:

- Spectral kernel construction over moduli stacks;
- Motivic and entropy-stratified trace operators;
- Functional equations and automorphic period concentration;
- RH-compatible kernel identity operators;
- AI-regulated entropy convolution flows.

In the next paper, we construct and develop the **Yang–Voronoi kernel series**, establishing arithmetic-dual kernel structures arising from Voronoi summation, Bessel transforms, and trace-period duality. This will serve as a foundational entropy–automorphic module in the RH kernel trace hierarchy.

References

[1] R. Langlands, *Problems in the Theory of Automorphic Forms*, Springer, 1970.
[2] J. Arthur, *The Trace Formula in Invariant Form*, Annals of Mathematics, 1983.
[3] P. Deligne, *Hodge Cycles and Motives*, IHES Publications.
[4] P.J.S. Yang, *Entropy Kernels and Zeta Stack Reconstruction*, 2025.
[5] B.C. Ngô, *Le lemme fondamental pour les algèbres de Lie*, Publ. Math. IHES, 2008.