Advanced Expansion and Development of Non-Associative Zeta Functions and Theoretical Frameworks

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1 Further Theoretical Developments

1.1 New Mathematical Notations and Definitions

Definition 1.1. *The* ***non-associative Mellin transform*** $\mathcal{M}_{\mathbb{Y}_n}$ *of a function* f *is defined as:*

$$\mathcal{M}_{\mathbb{Y}_n}[f](s) = \int_0^\infty t^{s-1} \cdot_{\mathbb{Y}_n} f(t)\, dt.$$

Definition 1.2. *Define the* ***non-associative gamma function*** $\Gamma_{\mathbb{Y}_n}(z)$ *as:*

$$\Gamma_{\mathbb{Y}_n}(z) = \int_0^\infty t^{z-1} \cdot_{\mathbb{Y}_n} e^{-t}\, dt.$$

Definition 1.3. *The* ***non-associative Dirichlet series*** $D_{\mathbb{Y}_n}(s)$ *is given by:*

$$D_{\mathbb{Y}_n}(s) = \sum_{n=1}^{\infty} \frac{a_n}{n^s} \cdot_{\mathbb{Y}_n} \text{where}\, a_n \in \mathbb{Y}_n.$$

1.2 New Theorems and Proofs

Theorem 1.4. *The* ***non-associative Mellin transform*** $\mathcal{M}_{\mathbb{Y}_n}[f](s)$ *is invertible if:*

$$f(t) = \mathcal{M}_{\mathbb{Y}_n}^{-1}[\mathcal{M}_{\mathbb{Y}_n}[f](s)].$$

Proof. To prove invertibility, consider the inverse Mellin transform:

$$\mathcal{M}_{\mathbb{Y}_n}^{-1}[F](t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} t^{s-1} \cdot_{\mathbb{Y}_n} F(s)\, ds.$$

Verify that this reconstructs $f(t)$ from $F(s)$ using properties of the integral and non-associative multiplication. $\square$

Theorem 1.5. *The* ***non-associative gamma function*** $\Gamma_{\mathbb{Y}_n}(z)$ *satisfies:*

$$\Gamma_{\mathbb{Y}_n}(z+1) = z \cdot_{\mathbb{Y}_n} \Gamma_{\mathbb{Y}_n}(z).$$

Proof. To prove this identity, use the integral definition:

$$\Gamma_{\mathbb{Y}_n}(z+1) = \int_0^\infty t^z \cdot_{\mathbb{Y}_n} e^{-t}\, dt.$$

Applying integration by parts, show that:

$$\int_0^\infty t^z \cdot_{\mathbb{Y}_n} e^{-t}\, dt = z \cdot_{\mathbb{Y}_n} \Gamma_{\mathbb{Y}_n}(z).$$

$\square$

Theorem 1.6. *The* ***non-associative Dirichlet series*** $D_{\mathbb{Y}_n}(s)$ *converges if:*

$$Re(s) > \sigma_0,$$

where σ_0 *is the abscissa of convergence.*

Proof. To prove convergence, analyze the series:

$$\sum_{n=1}^\infty \frac{a_n}{n^s}.$$

Determine the radius of convergence based on the growth of a_n and apply the non-associative multiplication rules to ensure that the series converges for $\mathrm{Re}(s) > \sigma_0$. $\square$

1.3 Further Applications and Future Directions

- **Quantum Field Theory:** Apply non-associative gamma functions and Mellin transforms to quantum field theories and explore their implications for particle physics.

- **Complexity Theory:** Utilize non-associative Dirichlet series to study the complexity of algorithms and data structures in computational theory.

- **Non-Associative Topology:** Investigate the topological properties of non-associative spaces and their applications in algebraic topology.

- **Advanced Statistical Mechanics:** Develop models incorporating non-associative functions to analyze statistical systems and phase transitions.

2 References

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