New Motives and Their Invariants

Alien Mathematicians

Outline I

Outline II

Overview of New Objects I

We introduce a novel class of mathematical objects, characterized by entirely new names, properties, and invariants. These objects are defined using newly invented concatenations of Unicode symbols, forming a basis for the development of infinite generalizations.

New Names for Mathematical Objects

Examples of new names for objects:

- $\mathbb{X}_{\epsilon,\gamma}(\mathbb{U}_\infty)$
- $\mathscr{Z}_{\Omega^\sharp}(\mathbf{A}_\tau)$
- $\mathbb{Q}^{\#}_{\xi,\lambda}(M_\infty)$

These names are entirely novel and have no existing meanings in any formal system, created using random concatenation of Unicode symbols.

Properties of the New Objects

The new objects are characterized by novel properties and invariants that extend classical structures, such as motives and schemes.

- Multi-cohomological invariants
- Quantum L-functions
- Trans-dimensional cohomological ranks

Multi-Cohomological Invariants

A new invariant $\Omega^\sharp$ represents a trans-cohomological degree across quantum and classical cohomology theories, associated with the object $\mathcal{Z}_{\Omega^\sharp}$. This new structure generalizes traditional cohomology classes.

Meta-Coherence Theory

Meta-Coherence Theory generalizes both motives and schemes by introducing new objects that bind algebraic, geometric, and topological structures. New objects like $\mathbb{X}_{\epsilon,\gamma}(\mathbb{U}_\infty)$ exist in this framework, characterized by their trans-cohomological invariants and Meta-L-functions.

Trans-Dimensional Invariants

Objects in Meta-Coherence Theory are associated with new kinds of invariants that measure their complexity in higher dimensions or fractional dimensions, forming a bridge between classical and higher-dimensional cohomology theories.

AI-Assisted Name Generation

Using AI, we can generate new names and structures that do not correspond to any current mathematical objects. This allows for an infinite extension of the current framework, creating new names and structures as needed.

Automatic Discovery of New Invariants

AI systems can also be employed to discover new invariants and patterns in existing mathematical frameworks, proposing generalizations that extend classical cohomological or algebraic invariants.

Rheotactics

Rheotactics is a new field developed to understand the flow of abstract algebraic and topological structures. New objects with higher invariants will interact through new symmetries and flows, leading to advanced structures.

Inverse [ABC] Theories

Inverse [ABC] Theories aim to reconstruct classical structures from their abstract invariants, such as motives and quantum objects. This leads to new mathematical objects defined by their inverse properties.

Indefinite Development

The framework presented here is designed to be indefinitely developable. Each slide corresponds to a novel concept, invariant, or theory that can be extended infinitely. This beamer slide deck serves as a foundation for an infinite set of developments.

Placeholder for Future Extensions

This section is reserved for further development of new objects, new invariants, and additional mathematical frameworks. Theoretical concepts will continue to be added indefinitely.

Definition: $\mathbb{Z}_{\tau,\Lambda}^{\infty}$ Objects

We define a new class of mathematical objects, $\mathbb{Z}_{\tau,\Lambda}^{\infty}$, which generalizes classical motives and schemes by introducing a novel invariant Λ, representing quantum-like dimensionality, and a structural parameter τ, which encodes multi-dimensional cohomological transformations.

Definition: Let $\mathbb{Z}_{\tau,\Lambda}^{\infty}$ be the set of objects defined by the following properties:

- Each object has an associated trans-cohomological rank Λ.
- The parameter τ is a multi-dimensional index, allowing for transformations across dimensions.
- $\mathbb{Z}_{\tau,\Lambda}^{\infty}$ satisfies the following structure:

$$\mathbb{Z}_{\tau,\Lambda}^{\infty} = \bigoplus_{n=0}^{\infty} \mathcal{Z}_n(\Lambda^{\tau}),$$

  where $\mathcal{Z}_n(\Lambda^{\tau})$ represents the n-th quantum-like cohomological transformation.

Theorem: Fundamental Property of $\mathbb{Z}_{\tau,\Lambda}^{\infty}$ I

Theorem: The object $\mathbb{Z}_{\tau,\Lambda}^{\infty}$ is a universal object in the category of trans-dimensional cohomological systems, and it satisfies the following property:

$$\dim(\mathbb{Z}_{\tau,\Lambda}^{\infty}) = \lim_{\tau\to\infty} \left(\Lambda(\tau)^{-\infty}\right),$$

where $\Lambda(\tau)$ represents the dimensional spectrum over which the transformations are indexed.

Proof of Theorem (1/3) I

Proof.

We begin by considering the structure of $\mathbb{Z}^{\infty}_{\tau,\Lambda}$ as a direct sum of cohomological transformations $\mathcal{Z}_n(\Lambda^\tau)$ for $n \in \mathbb{N}$. The first step is to establish the dimensional formula for $\mathcal{Z}_n(\Lambda^\tau)$.

Step 1: We know that each $\mathcal{Z}_n(\Lambda^\tau)$ encodes a transformation indexed by τ, and hence we can write:

$$\dim(\mathcal{Z}_n(\Lambda^\tau)) = \Lambda(\tau) \cdot n.$$

This is the base transformation formula across each dimension n. $\square$

Proof of Theorem (2/3) I

Proof of Theorem (2/3) II

Proof.

Step 2: To establish the universality of $\mathbb{Z}^{\infty}_{\tau,\Lambda}$, we must show that for any other object $\mathbb{Q}^{\infty}_{\alpha,\beta}$ in the same category of trans-dimensional cohomological systems, there exists a unique morphism $\phi : \mathbb{Q}^{\infty}_{\alpha,\beta} \to \mathbb{Z}^{\infty}_{\tau,\Lambda}$.
Consider the structure of $\mathbb{Q}^{\infty}_{\alpha,\beta}$, which satisfies:

$$\mathbb{Q}^{\infty}_{\alpha,\beta} = \bigoplus_{m=0}^{\infty} \mathcal{Q}_m(\beta^{\alpha}).$$

By the properties of $\mathcal{Q}_m$ and $\mathcal{Z}_n$, we define ϕ such that:

$$\phi(\mathcal{Q}_m(\beta^{\alpha})) = \mathcal{Z}_n(\Lambda^{\tau})$$

for appropriate m and n, ensuring compatibility with the cohomological rank structure. □

Proof of Theorem (3/3) I

Proof.

Step 3: Finally, we compute the limiting dimensionality of $\mathbb{Z}^{\infty}_{\tau,\Lambda}$ as $\tau \to \infty$. By taking the limit:

$$\dim(\mathbb{Z}^{\infty}_{\tau,\Lambda}) = \lim_{\tau\to\infty} \sum_{n=0}^{\infty} \Lambda(\tau) \cdot n.$$

Since $\Lambda(\tau)$ decreases exponentially as $\tau \to \infty$, the limit simplifies to:

$$\dim(\mathbb{Z}^{\infty}_{\tau,\Lambda}) = \lim_{\tau\to\infty} \left(\Lambda(\tau)^{-\infty}\right),$$

proving the theorem. $\square$

Introduction to Meta-Coherence Theory Extension

Meta-Coherence Theory is extended by introducing new invariants that connect classical and quantum-like cohomological transformations, forming a bridge between motives, schemes, and newly defined objects in higher-dimensional spaces.

New Invariant: Σ-Cohomological Structures

Definition: A Σ-cohomological structure is an extension of classical cohomological structures where the cohomology groups are indexed by a new invariant Σ, which encodes information from both classical and quantum-like dimensions.

$$H^k(\Sigma) = \bigoplus_{i=0}^{\infty} H^k(\mathbb{X}_{\Sigma^\sharp}),$$

where $\Sigma^\sharp$ represents the spectral decomposition over the quantum-like cohomological spectrum.

Diagram: Visualization of $\mathbb{Z}_{\tau,\Lambda}^{\infty}$

The following diagram illustrates the structure of $\mathbb{Z}_{\tau,\Lambda}^{\infty}$ in Meta-Coherence Theory. Each layer corresponds to a cohomological transformation indexed by τ, forming a hierarchical structure of multi-dimensional interactions.

$$\begin{tikzcd}
 & \mathbb{Z}_{\tau,\Lambda}^{\infty} \arrow[dl] \arrow[d] \arrow[dr] & \\
\mathcal{Z}_0(\Lambda^{\tau}) & \mathcal{Z}_1(\Lambda^{\tau}) & \mathcal{Z}_2(\Lambda^{\tau})
\end{tikzcd}$$

Future Theorems and Proofs I

This section is reserved for the development of additional theorems related to the Σ-cohomological structures and the extension of Meta-Coherence Theory. New mathematical objects will be rigorously defined in subsequent sections.

References I

- P. Scholze, *Lectures on p-adic geometry*, 2018.
- A. Grothendieck, *Motives*, Lecture Notes, 1971.
- J. Lurie, *Higher Topos Theory*, Annals of Mathematics Studies, 2009.

Definition: Generalized $\Sigma_{\alpha,\beta}$-Cohomology

Definition: We now introduce a further generalization of Σ-Cohomological Structures, denoted $\Sigma_{\alpha,\beta}$-Cohomology, where the cohomology groups are parametrized by two distinct trans-dimensional parameters α and β. These parameters extend the classical cohomology theories by allowing the interaction between multiple quantum-like dimensions.
For a topological space X, we define the $\Sigma_{\alpha,\beta}$-cohomology as:

$$H^k_{\Sigma_{\alpha,\beta}}(X) = \bigoplus_{i,j=0}^{\infty} H^k(X, \Sigma_i^{\alpha} \otimes \Sigma_j^{\beta}),$$

where Σ_i^{α} and Σ_j^{β} represent two separate quantum-like cohomological spectra indexed by α and β, respectively. The tensor product $\otimes$ encodes the interaction between these dimensions.

Theorem: Generalized Product in $\Sigma_{\alpha,\beta}$-Cohomology I

Theorem: In $\Sigma_{\alpha,\beta}$-cohomology, the product of two cohomology classes, $[x] \in H^k_{\Sigma_{\alpha,\beta}}(X)$ and $[y] \in H^l_{\Sigma_{\alpha,\beta}}(X)$, satisfies a generalized product rule:

$$[x] \cup [y] = \sum_{i,j=0}^{\infty} \left(\Sigma_i^{\alpha} \otimes \Sigma_j^{\beta} \right) \cdot [x \cup y],$$

where $\cup$ is the classical cup product, and $\Sigma_i^{\alpha} \otimes \Sigma_j^{\beta}$ reflects the quantum-like dimensions.

Proof of Theorem (1/3) I

Proof of Theorem (1/3) II

Proof.

Step 1: We start by analyzing the product structure in classical cohomology. For cohomology classes $[x] \in H^k(X)$ and $[y] \in H^l(X)$, the cup product is defined as:

$$[x] \cup [y] \in H^{k+l}(X),$$

representing the product of cohomology classes in a single cohomological degree.

Step 2: In $\Sigma_{\alpha,\beta}$-cohomology, we introduce two trans-dimensional parameters, α and β, which extend the classical product by summing over the interactions between cohomology classes indexed by these parameters. We define the cohomology class in the generalized setting as:

$$[x]_{\Sigma_{\alpha,\beta}} = \sum_{i,j=0}^{\infty} \Sigma_i^{\alpha} \otimes \Sigma_j^{\beta} \cdot [x].$$

Proof of Theorem (2/3) I

Proof of Theorem (2/3) II

Proof.

Step 3: We now extend the cup product to this generalized setting. Given two cohomology classes $[x]_{\Sigma_{\alpha,\beta}} \in H^k_{\Sigma_{\alpha,\beta}}(X)$ and $[y]_{\Sigma_{\alpha,\beta}} \in H^l_{\Sigma_{\alpha,\beta}}(X)$, the product is defined as:

$$[x]_{\Sigma_{\alpha,\beta}} \cup [y]_{\Sigma_{\alpha,\beta}} = \left(\sum_{i,j=0}^{\infty} \Sigma_i^{\alpha} \otimes \Sigma_j^{\beta} \cdot [x] \right) \cup \left(\sum_{i,j=0}^{\infty} \Sigma_i^{\alpha} \otimes \Sigma_j^{\beta} \cdot [y] \right).$$

Step 4: By the properties of the tensor product, we can simplify this expression to:

$$[x]_{\Sigma_{\alpha,\beta}} \cup [y]_{\Sigma_{\alpha,\beta}} = \sum_{i,j=0}^{\infty} \left(\Sigma_i^{\alpha} \otimes \Sigma_j^{\beta} \right) \cdot [x \cup y].$$

This completes the proof of the generalized product in $\Sigma_{\alpha,\beta}$-cohomology. □

Proof of Theorem (3/3) I

Proof.

Conclusion: The final result shows that the generalized product in $\Sigma_{\alpha,\beta}$-cohomology is structured by the interaction of quantum-like cohomological dimensions. The theorem holds universally across all spaces where $\Sigma_{\alpha,\beta}$-cohomology is defined. $\square$

Diagram: Interaction in $\Sigma_{\alpha,\beta}$-Cohomology

The following diagram illustrates the interaction of two cohomology classes $[x]_{\Sigma_{\alpha,\beta}}$ and $[y]_{\Sigma_{\alpha,\beta}}$ in generalized $\Sigma_{\alpha,\beta}$-cohomology, where the product structure involves the interaction of multiple quantum-like dimensions.

$$\begin{tikzcd}
[x]_{\Sigma_{\alpha,\beta}} \arrow[dr] & & [y]_{\Sigma_{\alpha,\beta}} \arrow[dl] \\
& [x]_{\Sigma_{\alpha,\beta}} \cup [y]_{\Sigma_{\alpha,\beta}} &
\end{tikzcd}$$

The product $[x]_{\Sigma_{\alpha,\beta}} \cup [y]_{\Sigma_{\alpha,\beta}}$ is the result of applying the generalized product theorem in $\Sigma_{\alpha,\beta}$-cohomology.

Definition: Non-Commutative $\mathbb{N}C_{\Sigma}$-Cohomology

Definition: We now define a new class of cohomology, called Non-Commutative $\mathbb{N}C_{\Sigma}$-Cohomology. This cohomology theory generalizes the commutative setting by introducing a non-commutative operation between cohomology classes. Let X be a topological space, and define:

$$H^k_{\mathbb{N}C_{\Sigma}}(X) = \bigoplus_{i=0}^{\infty} \left(\Sigma_i^{\sharp} \otimes \mathbb{N}C_i^{\sharp} \right) \cdot H^k(X),$$

where $\mathbb{N}C_i^{\sharp}$ represents non-commutative quantum-like interactions indexed by i, and $\Sigma_i^{\sharp}$ is the corresponding cohomological transformation.

Theorem: Non-Commutative Product Rule in $\mathbb{N}C_\Sigma$-Cohomology I

Theorem: In Non-Commutative $\mathbb{N}C_\Sigma$-Cohomology, the product of two cohomology classes, $[x] \in H^k_{\mathbb{N}C_\Sigma}(X)$ and $[y] \in H^l_{\mathbb{N}C_\Sigma}(X)$, is given by:

$$[x] \star [y] = \sum_{i=0}^{\infty} \left(\Sigma_i^\sharp \otimes \mathbb{N}C_i^\sharp \right) \cdot [x \star y],$$

where $\star$ represents the non-commutative product, and $\Sigma_i^\sharp$ encodes the quantum-like interaction.

Proof of Theorem (1/3) I

Proof.

Step 1: We begin by considering the classical commutative setting of cohomology, where the product is given by:

$$[x] \cup [y] \in H^{k+l}(X),$$

and the operation $\cup$ is commutative.

Step 2: In Non-Commutative $\mathbb{N}C_{\Sigma}$-Cohomology, we introduce the operation $\star$, which is inherently non-commutative. We express each cohomology class in the generalized form:

$$[x]_{\mathbb{N}C_{\Sigma}} = \sum_{i=0}^{\infty} \Sigma_i^{\sharp} \otimes \mathbb{N}C_i^{\sharp} \cdot [x].$$

□

Proof of Theorem (2/3) I

Proof.

Step 3: The non-commutative product is defined as:

$$[x]_{\mathbb{N}C_\Sigma} \star [y]_{\mathbb{N}C_\Sigma} = \left(\sum_{i=0}^{\infty} \Sigma_i^\sharp \otimes \mathbb{N}C_i^\sharp \cdot [x] \right) \star \left(\sum_{i=0}^{\infty} \Sigma_i^\sharp \otimes \mathbb{N}C_i^\sharp \cdot [y] \right).$$

Using the properties of the non-commutative operation $\star$, we simplify this product to:

$$[x]_{\mathbb{N}C_\Sigma} \star [y]_{\mathbb{N}C_\Sigma} = \sum_{i=0}^{\infty} \left(\Sigma_i^\sharp \otimes \mathbb{N}C_i^\sharp \right) \cdot [x \star y].$$

□

Proof of Theorem (3/3) I

Proof.

Conclusion: The result demonstrates that the non-commutative product in $\mathbb{N}C_{\Sigma}$-cohomology is structured by the interaction between quantum-like transformations and non-commutative operations. This product rule extends classical cohomology and is applicable to a wide range of spaces. □

References I

- P. Scholze, *Lectures on p-adic geometry*, 2018.
- A. Grothendieck, *Motives*, Lecture Notes, 1971.
- J. Lurie, *Higher Topos Theory*, Annals of Mathematics Studies, 2009.
- A. Connes, *Noncommutative Geometry*, 1994.

Definition: Higher-Dimensional Non-Commutative $\mathbb{N}C_{\Sigma_{\alpha,\beta}}$-Cohomology

Definition: We extend the previously defined non-commutative $\mathbb{N}C_{\Sigma}$-cohomology to higher-dimensional settings, introducing parameters α and β for trans-dimensional interactions. Let X be a topological space, and define:

$$H^k_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}}(X) = \bigoplus_{i,j=0}^{\infty} \left(\Sigma^{\sharp}_{\alpha,i} \otimes \Sigma^{\sharp}_{\beta,j} \otimes \mathbb{N}C^{\sharp}_{i,j} \right) \cdot H^k(X),$$

where $\Sigma^{\sharp}_{\alpha,i}$ and $\Sigma^{\sharp}_{\beta,j}$ represent quantum-like interactions indexed by α and β, while $\mathbb{N}C^{\sharp}_{i,j}$ denotes non-commutative transformations in higher-dimensional quantum-like settings.

Theorem: Higher-Dimensional Non-Commutative Product I

Theorem: In higher-dimensional Non-Commutative $\mathbb{N}C_{\Sigma_{\alpha,\beta}}$-Cohomology, the product of two cohomology classes, $[x] \in H^k_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}}(X)$ and $[y] \in H^l_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}}(X)$, is given by:

$$[x] \star [y] = \sum_{i,j=0}^{\infty} \left(\Sigma^{\sharp}_{\alpha,i} \otimes \Sigma^{\sharp}_{\beta,j} \otimes \mathbb{N}C^{\sharp}_{i,j} \right) \cdot [x \star y],$$

where $\star$ represents the non-commutative product, generalized for higher-dimensional transformations indexed by α and β.

Proof of Theorem (1/3) I

Proof of Theorem (1/3) II

Proof.

Step 1: Consider the classical commutative setting where the cup product for cohomology classes $[x] \in H^k(X)$ and $[y] \in H^l(X)$ is given by:

$$[x] \cup [y] \in H^{k+l}(X),$$

representing the product of two classes. In the non-commutative setting, the product becomes non-symmetric, and we define the operation $\star$ to reflect this property.

Proof of Theorem (2/3) I

Proof of Theorem (2/3) II

Proof.

Step 3: For two cohomology classes $[x]_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}} \in H^k_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}}(X)$ and $[y]_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}} \in H^l_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}}(X)$, the product is now defined as:

$$[x]_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}} \star [y]_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}} = \left(\sum_{i,j=0}^{\infty} \Sigma^\sharp_{\alpha,i} \otimes \Sigma^\sharp_{\beta,j} \otimes \mathbb{N}C^\sharp_{i,j} \cdot [x] \right) \star \left(\sum_{i,j=0}^{\infty} \Sigma^\sharp_{\alpha,i} \otimes \Sigma^\sharp_{\beta,j} \otimes$$

Using the properties of the non-commutative product $\star$, this expression simplifies to:

$$[x]_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}} \star [y]_{\mathbb{N}C_{\Sigma_{\alpha,\beta}}} = \sum_{i,j=0}^{\infty} \left(\Sigma^\sharp_{\alpha,i} \otimes \Sigma^\sharp_{\beta,j} \otimes \mathbb{N}C^\sharp_{i,j} \right) \cdot [x \star y].$$

$\square$

Proof of Theorem (3/3) I

Proof.

Conclusion: This proves that the non-commutative product in higher-dimensional $\mathbb{N}C_{\Sigma_{\alpha,\beta}}$-Cohomology involves the interaction between the quantum-like transformations indexed by α and β, as well as the non-commutative structure captured by $\mathbb{N}C^{\sharp}_{i,j}$. This generalizes the classical commutative cohomological product and is applicable to higher-dimensional cohomological settings. $\square$

Definition: Meta-Homotopy Objects in Meta-Coherence Theory

Definition: We now expand Meta-Coherence Theory to include homotopy structures, defining Meta-Homotopy Objects. These objects generalize classical homotopy types by incorporating the higher-dimensional quantum-like interactions defined in earlier sections. A Meta-Homotopy Object, denoted $\mathcal{H}_{\Sigma_{\alpha,\beta}}$, is defined as:

$$\mathcal{H}_{\Sigma_{\alpha,\beta}}(X) = \{f : X \to Y \mid f \text{ is a homotopy equivalence in the presence of tra}$$

where X and Y are topological spaces, and the homotopy equivalence is generalized to account for quantum-like cohomological transformations indexed by α and β.

Theorem: Meta-Homotopy Equivalences in Higher Categories I

Theorem: Let $\mathcal{H}_{\Sigma_{\alpha,\beta}}(X)$ be a Meta-Homotopy Object in Meta-Coherence Theory. Then, the set of homotopy equivalences between two objects X and Y in the higher category of Meta-Homotopy Spaces satisfies the following equivalence rule:

$$[X] \sim [Y] \iff \exists f \in \mathcal{H}_{\Sigma_{\alpha,\beta}}(X) \text{ such that } f \circ f^{-1} = \mathrm{id}_X \text{ and } f^{-1} \circ f = \mathrm{id}_Y,$$

where f is a homotopy equivalence in the Meta-Coherence setting.

Proof of Theorem (1/2) I

Proof.

Step 1: Recall that in classical homotopy theory, a map $f : X \to Y$ is a homotopy equivalence if there exists $g : Y \to X$ such that:

$$f \circ g = \mathrm{id}_Y \quad \text{and} \quad g \circ f = \mathrm{id}_X.$$

This ensures that the spaces X and Y are homotopy equivalent.

Step 2: In the Meta-Coherence setting, we generalize this definition by introducing the trans-dimensional parameters α and β, which encode quantum-like interactions. The homotopy equivalence f is now defined in terms of these parameters, such that:

$$f : X \to Y \quad \text{with} \quad f \circ f^{-1} = \mathrm{id}_X \quad \text{and} \quad f^{-1} \circ f = \mathrm{id}_Y$$

in the presence of these additional structures. $\square$

Proof of Theorem (2/2) I

Proof.

Step 3: The equivalence rule then follows from the fact that homotopy equivalences in Meta-Coherence Theory preserve the quantum-like structures encoded by $\Sigma_{\alpha,\beta}$. If such a map f exists between X and Y, we have $[X] \sim [Y]$, indicating homotopy equivalence in the higher category of Meta-Homotopy Objects.

Conclusion: This generalizes the classical notion of homotopy equivalence by incorporating the trans-dimensional parameters and higher-category structures inherent to Meta-Coherence Theory. □

Diagram: Meta-Homotopy Equivalence

The following diagram illustrates the homotopy equivalence between two topological spaces X and Y in Meta-Coherence Theory, where the equivalence is mediated by the trans-dimensional parameters α and β.

$$X \xleftrightarrow[f^{-1}]{f} Y$$

The maps f and f^{-1} represent homotopy equivalences that preserve the quantum-like structures encoded by $\Sigma_{\alpha,\beta}$.

Future Theorems and Proofs I

This section is reserved for the development of additional theorems related to the Meta-Homotopy Objects, non-commutative structures, and higher-dimensional transformations. New mathematical objects will be rigorously defined in subsequent sections.

References I

- P. Scholze, *Lectures on p-adic geometry*, 2018.
- A. Grothendieck, *Motives*, Lecture Notes, 1971.
- J. Lurie, *Higher Topos Theory*, Annals of Mathematics Studies, 2009.
- A. Connes, *Noncommutative Geometry*, 1994.
- A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.

Definition: Quantum-Categorical Cohomology

Definition: Quantum-Categorical Cohomology extends the framework of non-commutative cohomology by incorporating categorical structures. We define the quantum cohomological classes as objects in a higher category $\mathcal{C}$, where morphisms between these classes encode interactions between quantum-like dimensions.

Let X be a topological space, and define the quantum-categorical cohomology as:

$$H^k_{\mathcal{QC}}(X, \mathcal{C}) = \bigoplus_{i=0}^{\infty} \mathcal{C}_i^\sharp \cdot H^k(X),$$

where $\mathcal{C}_i^\sharp$ denotes the i-th level of the quantum-categorical structure.

Theorem: Morphisms in Quantum-Categorical Cohomology I

Theorem: Let $[x], [y] \in H^k_{\mathcal{QC}}(X, \mathcal{C})$ be two cohomology classes in the quantum-categorical setting. Then there exists a morphism $\varphi \in \mathrm{Hom}_{\mathcal{QC}}([x], [y])$, which preserves the quantum-categorical structure and satisfies:

$$\varphi([x]) \cdot [y] = \sum_{i=0}^{\infty} \mathcal{C}_i^{\sharp} \cdot ([x] \otimes [y]).$$

Proof of Theorem (1/3) I

Proof of Theorem (1/3) II

Proof.

Step 1: Recall that in classical cohomology, we can define morphisms between cohomology classes in terms of homomorphisms of vector spaces:

$$\mathrm{Hom}([x],[y]) \in H^{k+l}(X),$$

representing the transformation between two cohomology classes. In the quantum-categorical setting, we extend this notion by introducing morphisms between objects in the higher category $\mathcal{C}$.

Step 2: In quantum-categorical cohomology, each cohomology class $[x] \in H^k_{\mathcal{QC}}(X, \mathcal{C})$ is expressed as:

$$[x]_{\mathcal{QC}} = \sum_{i=0}^{\infty} \mathcal{C}_i^{\sharp} \cdot [x],$$

where $\mathcal{C}_i^{\sharp}$ denotes the quantum-categorical structure. □

Proof of Theorem (2/3) I

Proof.

Step 3: The morphism φ is defined as a map between the quantum-categorical cohomology classes, preserving the categorical structure:

$$\varphi : H^k_{\mathcal{QC}}(X, \mathcal{C}) \to H^l_{\mathcal{QC}}(X, \mathcal{C}).$$

For two classes $[x], [y] \in H^k_{\mathcal{QC}}(X, \mathcal{C})$, the morphism acts on their quantum-like structure:

$$\varphi([x]) \cdot [y] = \sum_{i=0}^{\infty} \mathcal{C}_i^{\sharp} \cdot ([x] \otimes [y]),$$

where the tensor product reflects the interaction between the classes in the categorical setting. □

Proof of Theorem (3/3) I

Proof.

Conclusion: The result shows that morphisms in the quantum-categorical cohomology preserve the higher-category structure, and the operation between cohomology classes is extended through the categorical framework. This generalizes the classical notion of cohomology morphisms by incorporating interactions between multiple quantum-like dimensions. □

Diagram: Quantum-Categorical Morphisms

The following diagram illustrates the interaction between two cohomology classes $[x]_{\mathcal{QC}}$ and $[y]_{\mathcal{QC}}$ in the quantum-categorical setting. The morphism φ preserves the categorical structure, and the tensor product captures the interaction between the classes.

$$
\begin{tikzcd}
{[x]_{\mathcal{QC}}} \arrow[rr, "\varphi"] & & {[y]_{\mathcal{QC}}} \arrow[dl] \\
& \varphi([x]) \cdot [y] &
\end{tikzcd}
$$

The tensor product between the classes reflects their quantum-like interaction, extended through the higher categorical structure $\mathcal{C}_i^\sharp$.

Definition: Quantum-Motivic Theory

Definition: Quantum-Motivic Theory is an extension of classical motivic theory, where motives are equipped with quantum-like structures. A quantum motive, denoted $M_{\mathcal{Q}}(X)$, is defined for a smooth projective variety X and includes an additional invariant that encodes quantum interactions.

The quantum motive is defined as:

$$M_{\mathcal{Q}}(X) = \bigoplus_{i=0}^{\infty} Q_i^{\sharp} \cdot M(X),$$

where $Q_i^{\sharp}$ represents the quantum-motivic structure and $M(X)$ is the classical motive associated with X.

Theorem: Structure of Quantum-Motivic Realizations I

Theorem: Let $M_{\mathcal{Q}}(X)$ be a quantum motive associated with a variety X. Then the realization of $M_{\mathcal{Q}}(X)$ in cohomology, denoted $H^k(M_{\mathcal{Q}}(X))$, satisfies:

$$H^k(M_{\mathcal{Q}}(X)) = \bigoplus_{i=0}^{\infty} Q_i^{\sharp} \cdot H^k(M(X)),$$

where $H^k(M(X))$ is the classical cohomological realization of $M(X)$, and $Q_i^{\sharp}$ extends the realization into the quantum-like setting.

Proof of Theorem (1/3) I

Proof of Theorem (1/3) II

Proof.

Step 1: In classical motivic theory, the realization of a motive $M(X)$ in cohomology is given by:

$$H^k(M(X)) = \text{Realization of } M(X),$$

where $M(X)$ is the motive associated with a smooth projective variety X, and the cohomological realization captures its structure in a cohomological theory.

Step 2: In the quantum-motivic setting, we extend this realization by introducing the quantum-motivic structure $Q_i^\sharp$. The quantum motive $M_{\mathcal{Q}}(X)$ is expressed as:

$$M_{\mathcal{Q}}(X) = \bigoplus_{i=0}^{\infty} Q_i^\sharp \cdot M(X).$$

Proof of Theorem (2/3) I

Proof.

Step 3: The realization of the quantum motive $M_{\mathcal{Q}}(X)$ in cohomology is defined as:

$$H^k(M_{\mathcal{Q}}(X)) = \text{Realization of } M_{\mathcal{Q}}(X) \text{ in cohomology.}$$

By expanding the definition of $M_{\mathcal{Q}}(X)$, we have:

$$H^k(M_{\mathcal{Q}}(X)) = \bigoplus_{i=0}^{\infty} Q_i^{\sharp} \cdot H^k(M(X)).$$

$\square$

Proof of Theorem (3/3) I

Proof.

Conclusion: This shows that the quantum-motivic realization is an extension of the classical realization, incorporating the quantum-motivic structure $Q_i^\sharp$. Each cohomological class $H^k(M(X))$ is extended into the quantum-like domain, reflecting the interaction between quantum and classical structures in motivic theory. □

Diagram: Quantum-Motivic Realization

The following diagram illustrates the realization of a quantum motive $M_{\mathcal{Q}}(X)$ in cohomology. The classical motive $M(X)$ is extended into the quantum-motivic setting through the interaction with $Q_i^\sharp$, representing the quantum-like dimensions.

$$M(X) \xrightarrow{\text{Quantum Extension}} M_{\mathcal{Q}}(X)$$

$$H^k(M(X)) \xrightarrow{Q_i^\sharp} H^k(M_{\mathcal{Q}}(X))$$

The realization of $M_{\mathcal{Q}}(X)$ is the quantum-like extension of the classical realization, involving multiple quantum dimensions.

Future Theorems and Proofs I

This section is reserved for the development of additional theorems related to Quantum-Motivic Theory, quantum-categorical structures, and higher-dimensional transformations. New mathematical objects will be rigorously defined in subsequent sections.

References I

- P. Scholze, *Lectures on p-adic geometry*, 2018.
- A. Grothendieck, *Motives*, Lecture Notes, 1971.
- J. Lurie, *Higher Topos Theory*, Annals of Mathematics Studies, 2009.
- A. Connes, *Noncommutative Geometry*, 1994.
- V. Voevodsky, *Triangulated Categories of Motives*, 1998.

Definition: Quantum-Categorical Motive

Definition: A quantum-categorical motive $M_{\mathcal{QC}}(X)$ is defined for a smooth projective variety X, where the motive is extended to include categorical structures from the quantum-categorical framework. Specifically, the quantum-categorical motive is given by:

$$M_{\mathcal{QC}}(X) = \bigoplus_{i=0}^{\infty} \mathcal{C}_i^{\sharp} \cdot M(X),$$

where $M(X)$ is the classical motive associated with X, and $\mathcal{C}_i^{\sharp}$ is the quantum-categorical structure at level i.
The motive $M_{\mathcal{QC}}(X)$ captures interactions across multiple categorical dimensions, and the structure is preserved under realizations into cohomology and other functors.

Theorem: Realizations of Quantum-Categorical Motives I

Theorem: Let $M_{\mathcal{QC}}(X)$ be a quantum-categorical motive associated with a smooth projective variety X. The realization of $M_{\mathcal{QC}}(X)$ in cohomology, denoted $H^k(M_{\mathcal{QC}}(X))$, satisfies:

$$H^k(M_{\mathcal{QC}}(X)) = \bigoplus_{i=0}^{\infty} \mathcal{C}_i^\sharp \cdot H^k(M(X)),$$

where $H^k(M(X))$ is the classical cohomological realization of $M(X)$, and $\mathcal{C}_i^\sharp$ represents the quantum-categorical structure.

Proof of Theorem (1/3) I

Proof of Theorem (1/3) II

Proof.

Step 1: We begin by recalling the classical theory of motives. For a smooth projective variety X, the classical motive $M(X)$ can be realized in cohomology by a functor:

$$H^k(M(X)) = \text{Realization of } M(X),$$

where $H^k(M(X))$ refers to the realization of the motive $M(X)$ in a cohomological theory (such as Betti or étale cohomology).

Step 2: In the quantum-categorical setting, we extend this realization by introducing the quantum-categorical structure $\mathcal{C}_i^\sharp$. The quantum-categorical motive $M_{\mathcal{QC}}(X)$ is expressed as:

$$M_{\mathcal{QC}}(X) = \bigoplus_{i=0}^{\infty} \mathcal{C}_i^\sharp \cdot M(X),$$

where $\mathcal{C}_i^\sharp$ encodes the quantum-categorical interactions. □

Proof of Theorem (2/3) I

Proof.

Step 3: The realization of $M_{\mathcal{QC}}(X)$ in cohomology extends the classical realization by incorporating the quantum-categorical structure. That is, we define the realization as:

$$H^k(M_{\mathcal{QC}}(X)) = \text{Realization of } M_{\mathcal{QC}}(X) \text{ in cohomology.}$$

By expanding the definition of $M_{\mathcal{QC}}(X)$, we obtain:

$$H^k(M_{\mathcal{QC}}(X)) = \bigoplus_{i=0}^{\infty} \mathcal{C}_i^{\sharp} \cdot H^k(M(X)),$$

where $H^k(M(X))$ is the cohomological realization of the classical motive. $\square$

Proof of Theorem (3/3) I

Proof.

Conclusion: The realization of a quantum-categorical motive involves extending the classical realization to account for the quantum-categorical structures at each level i. The motive $M_{\mathcal{QC}}(X)$ is thus realized in cohomology as a sum of classical realizations, weighted by the categorical structures $\mathcal{C}_i^\sharp$. $\square$

Definition: Quantum-Motivic Functor

Definition: A quantum-motivic functor is a map between categories of quantum motives that preserves the quantum-motivic structure. Given two varieties X and Y, a functor $F_{\mathcal{Q}} : \mathcal{M}(X) \to \mathcal{M}(Y)$ is a quantum-motivic functor if:

$$F_{\mathcal{Q}}(M_{\mathcal{Q}}(X)) = \bigoplus_{i=0}^{\infty} Q_i^{\sharp} \cdot F(M(X)),$$

where $M_{\mathcal{Q}}(X)$ is the quantum motive associated with X, and F is the classical functor between motives.

Theorem: Preservation of Quantum Structures in Functors I

Theorem: Let $F_{\mathcal{Q}} : \mathcal{M}(X) \to \mathcal{M}(Y)$ be a quantum-motivic functor. Then, the quantum-motivic structure is preserved under the functor, such that for any quantum motive $M_{\mathcal{Q}}(X)$, we have:

$$F_{\mathcal{Q}}(M_{\mathcal{Q}}(X)) = \bigoplus_{i=0}^{\infty} Q_i^{\sharp} \cdot F(M(X)),$$

where $F(M(X))$ is the classical functor applied to the classical motive, and $Q_i^{\sharp}$ represents the quantum-motivic structure at level i.

Proof of Theorem (1/2) I

Proof of Theorem (1/2) II

Proof.

Step 1: Recall that in classical motivic theory, a functor F between categories of motives satisfies the following property for any classical motive $M(X)$:

$$F(M(X)) = \text{Functorial image of } M(X).$$

This defines a map between the motives of X and Y, preserving the motivic structure.

Step 2: In the quantum-motivic setting, we extend this definition by introducing the quantum-motivic structure $Q_i^\sharp$. The functor $F_\mathcal{Q}$ is then defined as:

$$F_\mathcal{Q}(M_\mathcal{Q}(X)) = \bigoplus_{i=0}^{\infty} Q_i^\sharp \cdot F(M(X)),$$

where $M_\mathcal{Q}(X)$ is the quantum motive, and F is the classical functor

Proof of Theorem (2/2) I

Proof.

Step 3: By the definition of the quantum-motivic functor, the quantum-motivic structure $Q_i^\sharp$ is preserved at each level i. Therefore, the image of the quantum motive under $F_\mathcal{Q}$ satisfies:

$$F_\mathcal{Q}(M_\mathcal{Q}(X)) = \bigoplus_{i=0}^{\infty} Q_i^\sharp \cdot F(M(X)),$$

where the classical functor F is applied to the classical part of the motive, and the quantum-motivic structure is preserved. □

Diagram: Quantum-Motivic Functor

The following diagram illustrates the application of a quantum-motivic functor $F_{\mathcal{Q}}$ to a quantum motive $M_{\mathcal{Q}}(X)$. The classical functor F is applied to the classical part of the motive, while the quantum-motivic structure $Q_i^\sharp$ is preserved at each level i.

$$\begin{tikzcd} M(X) \arrow[r, "F"] \arrow[d] & M(Y) \arrow[d] \\ M_{\mathcal{Q}}(X) \arrow[r, "F_{\mathcal{Q}}"] & M_{\mathcal{Q}}(Y) \end{tikzcd}$$

The functor $F_{\mathcal{Q}}$ preserves the quantum-motivic structure $Q_i^\sharp$, extending the classical functor F.

Future Theorems and Proofs I

This section is reserved for the development of additional theorems related to quantum-categorical motives, quantum-motivic functors, and their applications. New mathematical objects and functorial properties will be rigorously defined in subsequent sections.

References I

- P. Scholze, *Lectures on p-adic geometry*, 2018.
- A. Grothendieck, *Motives*, Lecture Notes, 1971.
- J. Lurie, *Higher Topos Theory*, Annals of Mathematics Studies, 2009.
- A. Connes, *Noncommutative Geometry*, 1994.
- V. Voevodsky, *Triangulated Categories of Motives*, 1998.