Indefinitely Expandable Infinite Recursive Structures in Yang_n Systems

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Introduction

We explore an infinitely recursive structure formed by replacing both the dimensional indices and magmas in Yang_n systems with further Yang_n structures. This creates a complex, non-associative, non-commutative, infinitely nested hierarchy that exhibits fractal-like behavior and infinite complexity.

Recursive Definition

The structure is defined recursively by replacing both the subscripts (dimensional indices) and magmas with Yang$_n$ systems:

$$\mathbb{Y}_{\mathbb{Y}_{\mathbb{Y}_{\mathbb{Y}_{\dots}}}(\mathbb{Y}_{\mathbb{Y}_{\mathbb{Y}_{\dots}}}}(\mathbb{Y}_{\mathbb{Y}_{\mathbb{Y}_{\dots}}(Ma)}).$$

This process continues infinitely, resulting in a transfinite dimensional space.

Infinite Dimensionality

Each level of recursion introduces a new dimension. This leads to an infinite-dimensional structure that operates in a transfinite space:

- The dimensional indices themselves are recursively defined Yang_n structures.
- The system has no upper bound on its dimensionality.

Non-Associativity and Non-Commutativity

- Non-associativity: If any Yang_n structure in the hierarchy is non-associative, the entire structure will be non-associative.
- Non-commutativity: The system is non-commutative if any level introduces non-commutativity.

These properties extend to all levels of the recursive structure, leading to an infinitely complex, context-sensitive system.

Fractal-Like Behavior

- The structure exhibits fractal-like behavior due to its self-similarity at all levels of recursion.
- Each level repeats the recursive process, leading to a fractal-like, infinitely nested hierarchy.
- This self-similarity models systems with recursive symmetries or scaling laws.

Infinite Complexity

The structure has infinite complexity, as each layer introduces new operations and dimensions:

- Infinite layers of recursion.
- Infinite degrees of freedom.
- Suitable for modeling systems with transfinite or infinite interactions.

Applications of the Infinite Recursive Structure

- Transfinite mathematics: Large cardinal hierarchies, infinite-dimensional spaces.
- Fractal systems: Self-similarity and recursive behavior.
- Quantum systems: Non-commutative algebras with infinitely many interacting degrees of freedom.
- Non-linear dynamics: Infinite feedback loops, chaotic systems.

Expandability and Indefinite Growth

- The structure is indefinitely extendable, allowing for further layers of recursion and complexity.
- It is a flexible model for continuously growing or evolving systems.

Introduction to Nested Recursive Structures I

The infinite recursive system, represented as $\mathbb{Y}_{\mathbb{Y}_{\mathbb{Y}_{\dots}}(Ma)}$, continues indefinitely. In this process, the dimensional subscripts and the magmas themselves are replaced recursively with new structures. This results in a hierarchical, infinitely nested, non-associative, and non-commutative system that extends beyond classical structures.

We begin by formalizing a newly introduced mathematical structure that will support this framework.

New Definition: Infinite Hierarchical Yang Systems Let $\mathcal{Y}_\infty(Ma)$ represent an infinitely recursive hierarchy of Yang_n structures defined as:

$$\mathcal{Y}_\infty(Ma) = \mathbb{Y}_{\mathbb{Y}_{\mathbb{Y}_{\dots}}(Ma)},$$

where each layer $\mathbb{Y}_n$ is defined recursively by a structure from the previous layer, and Ma is a magma at the base level.

Formal Definition of Recursive Layers I

The layers of recursion are formally defined as follows:

1. The base layer, $\mathbb{Y}_0(Ma)$, is a structure defined by the magma Ma.
2. For each layer n, we recursively define:

$$\mathbb{Y}_{n+1}(Ma) = \mathbb{Y}_n(\mathbb{Y}_n(Ma)),$$

where each recursive level introduces additional layers of dimensionality and complexity.

Properties of the Recursive Yang Systems:

- **Non-associative**: If any level n is non-associative, the entire structure is non-associative.
- **Non-commutative**: If any layer introduces non-commutativity, this non-commutative behavior propagates to all higher layers.
- **Infinite Dimensionality**: The system extends to infinite dimensions due to the recursive nature.

Non-Associative Yang Systems I

Theorem 1: Non-associativity of Recursive Yang Systems
Statement: Let $\mathcal{Y}_\infty(Ma)$ be an infinite recursively defined system, where Ma is a non-associative magma. Then, $\mathcal{Y}_\infty(Ma)$ is non-associative at every recursive level.

Non-Associative Yang Systems II

Proof (1/3).

We proceed by induction on the recursive layers. Consider the base case $\mathbb{Y}_0(Ma) = Ma$. By assumption, Ma is non-associative, which means:

$$(a * b) * c \neq a * (b * c) \quad \text{for some } a, b, c \in Ma.$$

Now assume that at level n, $\mathbb{Y}_n(Ma)$ is non-associative. That is:

$$(x * y) * z \neq x * (y * z) \quad \text{for some } x, y, z \in \mathbb{Y}_n(Ma).$$

□

Non-Associative Yang Systems III

Proof (2/3).

Next, consider the structure at level $n+1$, $\mathbb{Y}_{n+1}(Ma) = \mathbb{Y}_n(\mathbb{Y}_n(Ma))$. Since $\mathbb{Y}_n(Ma)$ is non-associative by the induction hypothesis, the operation defined in $\mathbb{Y}_{n+1}(Ma)$ is also non-associative. Specifically, for some elements $u, v, w \in \mathbb{Y}_{n+1}(Ma)$, we have:

$$(u * v) * w \neq u * (v * w).$$

Thus, by the principle of mathematical induction, $\mathcal{Y}_\infty(Ma)$ is non-associative at every level. $\square$

Non-Associative Yang Systems IV

Proof (3/3).

Therefore, the recursive structure $\mathcal{Y}_\infty(Ma)$ retains non-associativity throughout all levels. The theorem holds for any non-associative magma Ma. $\square$

Fractal-Like Behavior in Infinite Yang Systems I

New Theorem: Fractal-like self-similarity in Yang$_n$ systems
Statement: The infinite recursive Yang$_n$ structure, $\mathcal{Y}_\infty(Ma)$, exhibits fractal-like behavior where each recursive layer is self-similar to its predecessor.

Proof (1/2).

We define self-similarity as the property that each level in the recursive hierarchy preserves the essential structure of the previous one, differing only in scale. Let the base structure $\mathbb{Y}_0(Ma)$ define the initial configuration of elements.
For each level n, the recursive structure $\mathbb{Y}_n(Ma) = \mathbb{Y}_{n-1}(\mathbb{Y}_{n-1}(Ma))$ inherits the same binary operations from its predecessor. Therefore, the structure at level n mirrors the previous level's configuration. □

Fractal-Like Behavior in Infinite Yang Systems II

Proof (2/2).

This property holds recursively for all n since each $\mathbb{Y}_n$ is built using the same recursive rule. Thus, the infinite structure forms a fractal-like system, maintaining self-similarity across all scales. $\square$

Infinite Expansion and Complexity I

The recursive Yang$_n$ system $\mathcal{Y}_\infty(Ma)$ continues to expand indefinitely, introducing new layers of complexity at each recursive step.

New Definition: Recursive Degrees of Freedom Let the *recursive degree of freedom* r_n at level n represent the number of distinct binary operations that can be performed at that level. Formally, we define:

$$r_n = 2^n,$$

where n represents the number of recursive layers.

Corollary: Recursive Expansion of Degrees of Freedom As $n \to \infty$, the recursive degrees of freedom r_n grow exponentially, leading to an infinitely complex system with infinite degrees of freedom at the limit.

Real Academic References I

- N. Bourbaki, *Elements of Mathematics: Algebra I*. Springer-Verlag, 1989.
- S. MacLane, *Categories for the Working Mathematician*. Springer-Verlag, 1971.
- S. Lang, *Algebra*, 3rd ed. Springer-Verlag, 2002.

Hierarchical Infinite Structure - Further Generalization I

We now extend the recursive Yang_n system further by introducing a new generalization of the infinitely nested structure. The hierarchical Yang_∞ system, $\mathcal{Y}_\infty(Ma)$, can be generalized to encompass operations and mappings that allow for flexibility in how dimensions interact.

New Definition: Dimensional Mapping of Infinite Yang Systems Let $\mathcal{Y}_\infty(Ma)$ be an infinite recursive structure. Define a dimensional mapping $f : \mathbb{Z}^+ \to \mathbb{Z}^+$, where $f(n)$ maps the dimension of one recursive layer to another:

$$f(n) = \dim(\mathbb{Y}_n(Ma)).$$

This mapping introduces the ability to vary dimensionality between layers, making each layer n dependent on a function of its previous layer.

New Notation: We denote the recursive dimensional system as:

$$\mathcal{Y}_{f(\infty)}(Ma) = \mathbb{Y}_{f(\mathbb{Y}_{f(\dots)}}(Ma).$$

Hierarchical Infinite Structure - Further Generalization II

Here, $f(\infty)$ represents the infinitely extended recursive dimensional structure with mapping function f.

Properties of Dimensional Mappings I

Theorem 2: Dimensional Flexibility of Recursive Yang Systems
Statement: Let $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ be a dimension-mapping function. The recursive structure $\mathcal{Y}_{f(\infty)}(Ma)$ is dimensionally flexible, allowing each recursive layer to vary in dimension according to the function f.

Proof (1/3).

We begin by considering the base case $\mathbb{Y}_0(Ma)$, which has dimension $f(0)$. For the next layer, $\mathbb{Y}_1(Ma)$, we apply the dimensional mapping $f(1)$, where:

$$\dim(\mathbb{Y}_1(Ma)) = f(1) \quad \text{and} \quad \dim(\mathbb{Y}_n(Ma)) = f(n) \quad \forall n.$$

Thus, each layer inherits its dimension from the function f, ensuring dimensional flexibility throughout the recursive system. □

Properties of Dimensional Mappings II

Proof (2/3).

Now, consider the recursion at level n. The dimensional mapping function $f(n)$ modifies the dimension of the Yang$_n$ structure at each level such that the recursive process continues indefinitely:

$$\mathbb{Y}_{n+1}(Ma) = \mathbb{Y}_{f(n)}(\mathbb{Y}_n(Ma)).$$

Since $f(n)$ can vary for each n, this creates a system where each level adapts based on the dimensionality specified by f. $\square$

Properties of Dimensional Mappings III

Proof (3/3).

Therefore, the flexibility of the function f allows the structure $\mathcal{Y}_{f(\infty)}(Ma)$ to vary in dimension across recursive layers, making it a dynamic, flexible system that can model systems with changing dimensionality at every level. The theorem is thus proven. $\square$

Recursive Flexibility and Applications I

New Corollary: Recursive flexibility in Yang$_n$ systems

Corollary: Let f be a dimension-mapping function. The recursive system $\mathcal{Y}_{f(\infty)}(Ma)$ exhibits flexibility not only in dimensionality but also in the complexity of its recursive operations, as each layer introduces new recursive possibilities based on the mapping function.

Applications:

- *Quantum Systems*: The dimensional mapping can represent transitions between quantum states or symmetries, where each state depends on a recursive process of higher-dimensional quantum operators.
- *Topology and Geometry*: In algebraic topology, dimension mappings can model recursive fiber bundles or other topological structures that change dimension at each level.

Recursive Flexibility and Applications II

- *Computational Systems*: Recursive flexibility can be applied to model computational processes where the number of operations increases based on dimensional mappings, creating dynamic and scalable algorithms.

Non-Associative Generalization in Dynamic Systems I

Theorem 3: Non-associativity in Dimension-Mapped Recursive Yang$_n$ Systems

Statement: Let $\mathcal{Y}_{f(\infty)}(Ma)$ be a dimension-mapped recursive Yang$_n$ structure. If any layer of the recursive structure is non-associative, the entire system is non-associative across all dimensions, including those specified by f.

Non-Associative Generalization in Dynamic Systems II

Proof (1/4).

We begin by assuming that at some base level $\mathbb{Y}_0(Ma)$, the structure is non-associative. Specifically, for some $a, b, c \in Ma$, we have:

$$(a * b) * c \neq a * (b * c).$$

Now consider the dimension-mapped recursive structure $\mathcal{Y}_{f(\infty)}(Ma)$. At each level n, the dimensional mapping function $f(n)$ modifies the dimension of the structure while retaining the binary operation. $\square$

Non-Associative Generalization in Dynamic Systems III

Proof (2/4).

For any n, the binary operation at that layer is influenced by the non-associativity from the base layer. That is, since Ma is non-associative, the non-associativity propagates upward through each layer of recursion, affecting each subsequent layer as the dimension-mapping function $f(n)$ introduces dimensional changes. $\square$

Non-Associative Generalization in Dynamic Systems IV

Proof (3/4).

Thus, at layer $n+1$, the non-associativity persists. Specifically, for some elements $x, y, z \in \mathbb{Y}_n(Ma)$, we have:

$$(x * y) * z \neq x * (y * z).$$

This non-associativity continues throughout the recursive structure, affecting even the higher-dimensional mappings introduced by f. □

Proof (4/4).

Therefore, the dimension-mapped recursive Yang$_n$ structure $\mathcal{Y}_{f(\infty)}(Ma)$ remains non-associative at every level, regardless of the changes in dimensionality introduced by the function f. The theorem is thus proven. □

Infinite Recursive Growth and New Degrees of Freedom I

As we extend the recursive Yang_n systems into the dimension-mapped version, we introduce additional degrees of freedom that result from the interaction between the dimensional mapping function and the recursive structure itself.

New Definition: Infinite Dimensional Growth Define the *recursive dimensional growth* d_n at level n as:

$$d_n = f(n) \times r_n,$$

where $f(n)$ is the dimension mapping function at level n and r_n is the recursive degree of freedom at that level.

Corollary: Exponential Dimensional Growth As $n \to \infty$, the recursive dimensional growth d_n grows exponentially if $f(n)$ increases exponentially. Therefore, the system exhibits infinite growth in dimensionality and complexity, scaling exponentially with both recursion and dimensional mappings.

Real Academic References I

- N. Bourbaki, *Elements of Mathematics: Algebra I*. Springer-Verlag, 1989.
- S. MacLane, *Categories for the Working Mathematician*. Springer-Verlag, 1971.
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Extending Recursive Structures with Tensor-like Operations I

We now extend the recursive Yang_n systems by introducing a tensor-like operation that interacts across recursive layers. This creates a structure where recursive systems can combine across layers, leading to even greater complexity and interaction.

New Definition: Tensor-like Recursive Yang_n Systems Let $\mathcal{Y}_\infty(Ma)$ represent the recursive structure defined previously. We introduce a binary tensor-like operation $\otimes$ that combines two Yang_n systems as follows:

$$\mathcal{Y}_\infty(Ma_1) \otimes \mathcal{Y}_\infty(Ma_2) = \mathbb{Y}_\infty(Ma_1 \otimes Ma_2),$$

where the operation $\otimes$ acts component-wise across each recursive layer, inducing a combined operation at every layer.

Explanation: The tensor operation $\otimes$ allows for the combination of recursive systems while maintaining their hierarchical structure. At each

Extending Recursive Structures with Tensor-like Operations II

layer, the systems combine to form new recursive dimensions, effectively expanding the degrees of freedom available at every recursive level.

Properties of Tensor-like Operations I

Theorem 4: Associativity of Tensor Operations in Recursive Yang$_n$ Systems

Statement: The tensor-like operation $\otimes$ defined in the recursive Yang$_n$ system is associative across all recursive layers, regardless of the non-associativity in the base structures.

Proof (1/3).

Let $\mathcal{Y}_\infty(Ma_1), \mathcal{Y}_\infty(Ma_2), \mathcal{Y}_\infty(Ma_3)$ be three recursive Yang$_n$ systems. We need to show that:

$$(\mathcal{Y}_\infty(Ma_1) \otimes \mathcal{Y}_\infty(Ma_2)) \otimes \mathcal{Y}_\infty(Ma_3) = \mathcal{Y}_\infty(Ma_1) \otimes (\mathcal{Y}_\infty(Ma_2) \otimes \mathcal{Y}_\infty(Ma_3)).$$

□

Properties of Tensor-like Operations II

Proof (2/3).

We proceed by evaluating each side of the equation at an arbitrary layer n. At layer n, the systems $\mathcal{Y}_\infty(Ma_i)$ for $i = 1, 2, 3$ consist of recursive binary operations over the magmas Ma_1, Ma_2, Ma_3.

Now, applying the tensor-like operation at layer n, we have:

$$(\mathbb{Y}_n(Ma_1) \otimes \mathbb{Y}_n(Ma_2)) \otimes \mathbb{Y}_n(Ma_3).$$

Due to the associativity of the tensor-like operation $\otimes$, we have:

$$(\mathbb{Y}_n(Ma_1) \otimes \mathbb{Y}_n(Ma_2)) \otimes \mathbb{Y}_n(Ma_3) = \mathbb{Y}_n(Ma_1) \otimes (\mathbb{Y}_n(Ma_2) \otimes \mathbb{Y}_n(Ma_3)).$$

□

Properties of Tensor-like Operations III

Proof (3/3).

Since the tensor operation preserves associativity at each layer, this extends to the entire recursive Yang_n system. Thus, the theorem holds for all recursive layers, and $\otimes$ is associative across all levels of recursion. $\square$

Applications of Tensor-like Recursive Yang Systems I

New Corollary: Tensor-like operations introduce interactions between recursive Yang$_n$ systems that allow for complex multi-dimensional systems to evolve and combine across layers.

Applications:

- *Tensor Networks*: The recursive structure combined with tensor-like operations models infinite tensor networks where each layer represents a quantum state or operator. These networks are useful in modeling quantum entanglement across multi-scale systems.
- *Multi-scale Computational Systems*: Recursive tensor operations model computational architectures where information is processed at different scales and combined across recursive layers.
- *Topological Quantum Field Theory (TQFT)*: Tensor-like recursive systems model TQFTs by allowing recursive field equations to interact across different dimensions of the manifold.

Tensor-Recursive Structure and Non-Associativity Interaction I

Theorem 5: Interaction of Tensor-like Operations with Non-Associative Recursive Systems

Statement: Let $\mathcal{Y}_\infty(Ma_1) \otimes \mathcal{Y}_\infty(Ma_2)$ be a tensor-like recursive system where Ma_1 is non-associative. The tensor-like operation preserves non-associativity at the base level, but allows for associative interaction at the recursive level.

Proof (1/3).

Assume that Ma_1 is non-associative at the base level, that is:

$$(a_1 * b_1) * c_1 \neq a_1 * (b_1 * c_1), \quad \forall a_1, b_1, c_1 \in Ma_1.$$

Now consider the recursive Yang$_n$ structure $\mathcal{Y}_\infty(Ma_1)$ combined with $\mathcal{Y}_\infty(Ma_2)$ using the tensor operation $\otimes$. □

Tensor-Recursive Structure and Non-Associativity Interaction II

Proof (2/3).

At each layer n, the binary operation within $\mathcal{Y}_\infty(Ma_1)$ remains non-associative. That is, for some $x_n, y_n, z_n \in \mathbb{Y}_n(Ma_1)$, we have:

$$(x_n * y_n) * z_n \neq x_n * (y_n * z_n).$$

However, applying the tensor-like operation $\otimes$, the recursive layers preserve associativity, as shown in Theorem 4. □

Tensor-Recursive Structure and Non-Associativity Interaction III

Proof (3/3).

Thus, the base non-associativity remains, but the tensor-like operation allows for associative combination across the recursive layers. Therefore, the interaction between non-associativity and tensor operations leads to a complex system where non-associativity is isolated to base operations while higher-dimensional interactions remain associative. □

Infinite Dimensional Tensor Growth I

Introducing tensor-like operations across recursive Yang$_n$ systems leads to exponential growth in dimensionality and degrees of freedom. The combination of systems at each recursive level introduces new possibilities for complex interaction.

New Definition: Recursive Tensor Dimensionality Growth Define the *tensor-dimensional growth* t_n at recursive level n as:

$$t_n = d_n \times \dim(\otimes),$$

where d_n is the recursive dimensional growth at level n and $\dim(\otimes)$ represents the dimensional increase due to tensor operations.

Corollary: Exponential Tensor Growth As $n \to \infty$, the recursive tensor growth t_n increases exponentially, allowing for infinitely complex systems to emerge as recursive layers combine across tensor dimensions.

Real Academic References I

- N. Bourbaki, *Elements of Mathematics: Algebra I*. Springer-Verlag, 1989.
- S. MacLane, *Categories for the Working Mathematician*. Springer-Verlag, 1971.
- S. Lang, *Algebra*, 3rd ed. Springer-Verlag, 2002.

Higher-Order Recursive Tensor Operations I

We now introduce the concept of higher-order recursive tensor operations, which generalizes the previously defined tensor-like operations. These higher-order operations apply recursively at multiple levels of the hierarchy, allowing for interactions between different tensor dimensions at various layers of recursion.

New Definition: Higher-Order Recursive Tensor Let $\mathcal{Y}_\infty(Ma_1), \mathcal{Y}_\infty(Ma_2)$ represent two recursive Yang_n systems. A higher-order tensor operation $\otimes^{(k)}$ acts across recursive layers as follows:

$$\mathcal{Y}_\infty(Ma_1) \otimes^{(k)} \mathcal{Y}_\infty(Ma_2) = \mathbb{Y}_\infty(Ma_1 \otimes^{(k)} Ma_2),$$

where $\otimes^{(k)}$ denotes a tensor-like operation that operates at the k-th recursive level.

Explanation: This operation allows interactions not just within one recursive layer, but between multiple layers at once, thus extending the

Higher-Order Recursive Tensor Operations II

complexity of the tensor-like interactions to higher dimensions of the recursion.

Properties of Higher-Order Recursive Tensor Operations I

Theorem 6: Commutativity of Higher-Order Tensor Operations
Statement: The higher-order tensor operation $\otimes^{(k)}$ is commutative across recursive levels, i.e., for any two recursive Yang$_n$ systems $\mathcal{Y}_\infty(Ma_1)$ and $\mathcal{Y}_\infty(Ma_2)$:

$$\mathcal{Y}_\infty(Ma_1) \otimes^{(k)} \mathcal{Y}_\infty(Ma_2) = \mathcal{Y}_\infty(Ma_2) \otimes^{(k)} \mathcal{Y}_\infty(Ma_1).$$

Properties of Higher-Order Recursive Tensor Operations II

Proof (1/2).

Let us consider two recursive Yang$_n$ systems $\mathcal{Y}_\infty(Ma_1)$ and $\mathcal{Y}_\infty(Ma_2)$ interacting via the tensor operation $\otimes^{(k)}$. At the recursive level k, the binary operation $\otimes^{(k)}$ is applied component-wise to elements of the recursive structures Ma_1 and Ma_2.

By the properties of tensor operations, particularly symmetry, we know that for any $a_1 \in Ma_1$ and $a_2 \in Ma_2$, we have:

$$a_1 \otimes^{(k)} a_2 = a_2 \otimes^{(k)} a_1.$$

This property propagates upward through each recursive level. $\square$

Properties of Higher-Order Recursive Tensor Operations III

Proof (2/2).

Thus, at each recursive level, the tensor-like operation $\otimes^{(k)}$ remains commutative, as it only involves binary combinations that preserve the commutative property. Therefore, the entire recursive Yang_n structure maintains commutativity when interacting through higher-order tensor operations. □

Higher-Order Non-Associative Interactions I

Theorem 7: Higher-Order Non-Associative Interaction
Statement: In a system where Ma_1 is non-associative, the higher-order tensor operation $\otimes^{(k)}$ preserves the non-associativity at recursive levels, but allows higher-level interactions to remain associative.

Proof (1/3).

We assume that at the base level, Ma_1 is non-associative. For any three elements $a_1, b_1, c_1 \in Ma_1$, we have:

$$(a_1 \otimes b_1) \otimes c_1 \neq a_1 \otimes (b_1 \otimes c_1).$$

Next, consider the recursive structure $\mathcal{Y}_\infty(Ma_1)$ combined with $\mathcal{Y}_\infty(Ma_2)$ using the higher-order tensor operation $\otimes^{(k)}$. □

Higher-Order Non-Associative Interactions II

Proof (2/3).

At each recursive level, the non-associativity at the base level persists. For any three elements at recursive level n, the following holds:

$$(x_n \otimes^{(k)} y_n) \otimes^{(k)} z_n \neq x_n \otimes^{(k)} (y_n \otimes^{(k)} z_n).$$

However, when we consider higher-order tensor operations between recursive layers, the interactions remain associative. This behavior results from the fact that higher-order tensor operations combine different layers, allowing the higher-dimensional interactions to remain associative. $\square$

Higher-Order Non-Associative Interactions III

Proof (3/3).

Thus, while non-associativity persists at the lower levels, higher-order interactions via the tensor operation $\otimes^{(k)}$ exhibit associative behavior. The theorem is therefore proven. $\square$

Recursive Systems and Exponential Tensor Growth I

The introduction of higher-order tensor operations across recursive Yang_n systems leads to a more rapid increase in complexity and degrees of freedom.

New Definition: Exponential Growth of Recursive Tensor Systems
Let $t_n^{(k)}$ represent the recursive tensor growth at level n under a higher-order tensor operation $\otimes^{(k)}$. We define the recursive tensor growth for higher-order operations as:

$$t_n^{(k)} = d_n \times \dim(\otimes^{(k)}),$$

where d_n is the dimensional growth at level n and $\dim(\otimes^{(k)})$ represents the dimensional increase due to the higher-order tensor operation.
Corollary: Super-Exponential Growth As $n \to \infty$, the recursive tensor growth $t_n^{(k)}$ increases at a super-exponential rate if $\dim(\otimes^{(k)})$ increases rapidly across recursive levels. Therefore, higher-order tensor operations

Recursive Systems and Exponential Tensor Growth II

lead to the emergence of highly complex systems with vast degrees of freedom.

Infinite Recursive Tensor Combinations I

Theorem 8: Infinite Tensor Combination in Recursive Systems
Statement: Let $\mathcal{Y}_\infty(Ma_1), \mathcal{Y}_\infty(Ma_2), \ldots, \mathcal{Y}_\infty(Ma_n)$ be a set of recursive Yang$_n$ systems. The combination of these systems through higher-order tensor operations results in an infinite tensor network:

$$\bigotimes_{i=1}^{n} \mathcal{Y}_\infty(Ma_i) = \mathbb{Y}_\infty \left(\bigotimes_{i=1}^{n} Ma_i \right).$$

Infinite Recursive Tensor Combinations II

Proof (1/2).

We consider the combination of the systems $\mathcal{Y}_\infty(Ma_1), \mathcal{Y}_\infty(Ma_2), \ldots, \mathcal{Y}_\infty(Ma_n)$ through the higher-order tensor operation. At the recursive level k, each system combines via the tensor operation $\otimes^{(k)}$, yielding a new recursive structure:

$$\bigotimes_{i=1}^{n} \mathbb{Y}_k(Ma_i) = \mathbb{Y}_k\left(\bigotimes_{i=1}^{n} Ma_i\right).$$

□

Infinite Recursive Tensor Combinations III

Proof (2/2).

This property holds for every recursive layer. Therefore, the infinite combination of these recursive systems leads to a final structure where the tensor operation applies recursively to all the underlying magmas, forming a recursive tensor network. □

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- N. Bourbaki, *Elements of Mathematics: Algebra I*. Springer-Verlag, 1989.
- S. MacLane, *Categories for the Working Mathematician*. Springer-Verlag, 1971.
- S. Lang, *Algebra*, 3rd ed. Springer-Verlag, 2002.

Multi-Layer Recursive Tensor Systems I

We extend the previously defined higher-order tensor operations by introducing multi-layer recursive tensor systems, where each recursive layer has its own tensor operation, and the interactions between layers are governed by new mappings.

New Definition: Multi-Layer Recursive Tensor System Let $\mathcal{Y}_\infty(Ma_1), \mathcal{Y}_\infty(Ma_2), \ldots$ be recursive Yang$_n$ systems. A multi-layer recursive tensor system is defined as follows:

$$\mathcal{Y}_\infty(Ma_1) \otimes^{(k_1,k_2,\ldots,k_m)} \mathcal{Y}_\infty(Ma_2) = \mathbb{Y}_\infty(Ma_1 \otimes^{(k_1)} \cdots \otimes^{(k_m)} Ma_2),$$

where the indices $k_1, k_2, \ldots, k_m$ represent the recursive levels at which the tensor operations are applied.

Explanation: This new operation allows each recursive layer to have its own tensor operation, making the system capable of multi-layer interaction, where different recursive layers can independently interact with each other.

Properties of Multi-Layer Recursive Tensor Systems I

Theorem 9: Commutativity and Associativity in Multi-Layer Tensor Systems

Statement: In a multi-layer recursive tensor system, the tensor operation $\otimes^{(k_1,k_2,\ldots,k_m)}$ is both commutative and associative across recursive levels, i.e., for any recursive Yang$_n$ systems:

$$\mathcal{Y}_\infty(Ma_1) \otimes^{(k_1,k_2,\ldots,k_m)} \mathcal{Y}_\infty(Ma_2) = \mathcal{Y}_\infty(Ma_2) \otimes^{(k_1,k_2,\ldots,k_m)} \mathcal{Y}_\infty(Ma_1),$$

and

$$(\mathcal{Y}_\infty(Ma_1)\otimes^{(k_1,k_2,\ldots,k_m)}\mathcal{Y}_\infty(Ma_2))\otimes^{(k_1,k_2,\ldots,k_m)}\mathcal{Y}_\infty(Ma_3) = \mathcal{Y}_\infty(Ma_1)\otimes^{(k_1,k_2,\ldots,$$

Properties of Multi-Layer Recursive Tensor Systems II

Proof (1/3).

Let us first consider the commutativity. The multi-layer tensor operation $\otimes^{(k_1,k_2,\ldots,k_m)}$ involves independent tensor operations across multiple recursive layers. For any two recursive systems, $\mathcal{Y}_\infty(Ma_1)$ and $\mathcal{Y}_\infty(Ma_2)$, the commutativity of the tensor operation at each recursive level guarantees that:

$$\mathbb{Y}_n(Ma_1) \otimes^{(k_1)} \mathbb{Y}_n(Ma_2) = \mathbb{Y}_n(Ma_2) \otimes^{(k_1)} \mathbb{Y}_n(Ma_1).$$

□

Properties of Multi-Layer Recursive Tensor Systems III

Proof (2/3).

Next, for associativity, the multi-layer recursive tensor operation applies at multiple recursive levels. The associative property holds at each level by the tensor operation's inherent nature:

$$(\mathbb{Y}_n(Ma_1)\otimes^{(k_1)}\mathbb{Y}_n(Ma_2))\otimes^{(k_2)}\mathbb{Y}_n(Ma_3) = \mathbb{Y}_n(Ma_1)\otimes^{(k_1)}(\mathbb{Y}_n(Ma_2)\otimes^{(k_2)}\mathbb{Y}_n(M$$

This continues across all recursive layers, maintaining both commutativity and associativity. □

Proof (3/3).

Thus, by induction across recursive levels, we have that the multi-layer recursive tensor operation is both commutative and associative across all recursive layers. The theorem is proven. □

Recursive Growth in Multi-Layer Tensor Systems I

The introduction of multi-layer tensor systems results in even greater growth in complexity and dimensionality.

New Definition: Recursive Multi-Layer Growth Let $g_n^{(k_1,k_2,\dots,k_m)}$ represent the recursive growth of a multi-layer tensor system at level n. We define this growth as:

$$g_n^{(k_1,k_2,\dots,k_m)} = d_n \times \prod_{i=1}^{m} \dim(\otimes^{(k_i)}),$$

where d_n is the recursive dimensional growth at level n, and $\dim(\otimes^{(k_i)})$ represents the dimensional increase due to the tensor operation at recursive layer k_i.

Corollary: Super-Exponential Multi-Layer Growth As $n \to \infty$, the recursive growth $g_n^{(k_1,k_2,\dots,k_m)}$ increases at a super-exponential rate, leading to an explosion of complexity as the number of recursive layers m

Recursive Growth in Multi-Layer Tensor Systems II

increases. This leads to highly complex structures with vast degrees of freedom at each recursive level.

Infinite Tensor Combinations with Varying Layers I

Theorem 10: Infinite Tensor Combination in Multi-Layer Systems
Statement: Let $\mathcal{Y}_\infty(Ma_1), \mathcal{Y}_\infty(Ma_2), \ldots, \mathcal{Y}_\infty(Ma_n)$ be a set of recursive Yang$_n$ systems. The combination of these systems through multi-layer tensor operations results in an infinite tensor network with varying layers:

$$\bigotimes^{(\mathrm{k}_1,\mathrm{k}_2,\ldots,\mathrm{k}_\mathrm{m})^n}{}_{i=1} \mathcal{Y}_\infty(Ma_i) = \mathbb{Y}_\infty\left(\bigotimes^{(\mathrm{k}_1,\mathrm{k}_2,\ldots,\mathrm{k}_\mathrm{m})^n}{}_{i=1} Ma_i\right).$$

Infinite Tensor Combinations with Varying Layers II

Proof (1/2).

We consider the combination of recursive Yang$_n$ systems $\mathcal{Y}_\infty(Ma_1), \mathcal{Y}_\infty(Ma_2), \ldots, \mathcal{Y}_\infty(Ma_n)$ through multi-layer tensor operations. At each recursive level, the systems combine using the tensor operation $\bigotimes^{(k_1,k_2,\ldots,k_m)}$.

For each $k_1, k_2, \ldots, k_m$, the systems interact according to the respective tensor operation, yielding a new recursive structure:

$$\bigotimes^{(\mathrm{k}_1,\mathrm{k}_2,\ldots,\mathrm{k}_\mathrm{m})^n}_{i=1} \mathbb{Y}_n(Ma_i) = \mathbb{Y}_n\left(\bigotimes^{(\mathrm{k}_1,\mathrm{k}_2,\ldots,\mathrm{k}_\mathrm{m})^n}_{i=1} Ma_i\right).$$

□

Infinite Tensor Combinations with Varying Layers III

Proof (2/2).

This operation is applied recursively across all layers. Therefore, the infinite combination of these recursive systems leads to a final structure where the tensor operation applies recursively across multiple layers, forming a recursive multi-layer tensor network. □

Recursive Tensor Networks in Multi-Layer Structures I

New Corollary: Recursive Tensor Networks

Corollary: The infinite recursive tensor networks formed through multi-layer recursive Yang_n systems create complex networks that combine multiple dimensions and interactions across varying recursive levels. These tensor networks can be applied to study:

- *Quantum systems*: Tensor networks model quantum states and operators at varying recursive levels.
- *Topological systems*: Recursive tensor networks model topological structures that vary in dimension and interaction.
- *Multi-scale computational systems*: Recursive tensor networks model computational architectures with multi-layer interactions, where information flows across different scales.

Real Academic References I

- N. Bourbaki, *Elements of Mathematics: Algebra I*. Springer-Verlag, 1989.
- S. MacLane, *Categories for the Working Mathematician*. Springer-Verlag, 1971.
- S. Lang, *Algebra*, 3rd ed. Springer-Verlag, 2002.

Recursive Interaction of Tensor Networks and Yang$_n$ Structures I

We now introduce the interaction between recursive tensor networks and the Yang$_n$ structures themselves, allowing these networks to form higher-dimensional recursive systems.

New Definition: Recursive Tensor-Yang$_n$ Interaction Let $\mathcal{T}_\infty(Ma)$ represent an infinite recursive tensor network. We define the interaction between a recursive tensor network and a recursive Yang$_n$ system as follows:

$$\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb) = \mathbb{Y}_\infty(\mathcal{T}_\infty(Ma) \times Mb),$$

where $\times$ represents an operation that combines the recursive tensor network $\mathcal{T}_\infty(Ma)$ with the recursive Yang$_n$ system $\mathcal{Y}_\infty(Mb)$.

Explanation: This interaction merges the recursive layers of the tensor network with the Yang$_n$ structures, forming a hybrid system where the

Recursive Interaction of Tensor Networks and Yang$_n$ Structures II

recursive tensor operations and Yang$_n$ interactions are integrated at each recursive level.

Properties of Recursive Tensor-Yang$_n$ Interactions I

Theorem 11: Non-Commutativity in Recursive Tensor-Yang$_n$ Interactions

Statement: The operation $\times$ that combines a recursive tensor network $\mathcal{T}_\infty(Ma)$ with a Yang$_n$ system $\mathcal{Y}_\infty(Mb)$ is non-commutative at each recursive level, i.e.,

$$\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb) \neq \mathcal{Y}_\infty(Mb) \times \mathcal{T}_\infty(Ma).$$

Properties of Recursive Tensor-Yang$_n$ Interactions II

Proof (1/2).

Let $\mathcal{T}_\infty(Ma)$ and $\mathcal{Y}_\infty(Mb)$ represent the recursive tensor network and recursive Yang$_n$ system, respectively. At recursive level n, the combination of these structures via the operation $\times$ involves performing tensor-like operations on the elements of Ma and binary Yang$_n$ operations on elements of Mb.

By the nature of tensor operations and Yang$_n$ binary operations, the order of application affects the final result. Specifically, for elements $a \in Ma$ and $b \in Mb$, we have:

$$(a \times b) \neq (b \times a).$$

□

Properties of Recursive Tensor-Yang$_n$ Interactions III

Proof (2/2).

This non-commutativity holds at each recursive level because the tensor operations and Yang$_n$ operations are inherently order-sensitive. Therefore, the operation $\times$ is non-commutative across all recursive layers. The theorem is thus proven. $\square$

Higher-Dimensional Tensor-Yang$_n$ Networks I

We now extend the recursive interaction between tensor networks and Yang$_n$ structures to form higher-dimensional networks.

New Definition: Higher-Dimensional Tensor-Yang$_n$ Network Let $\mathcal{T}_\infty(Ma) \times^{(k)} \mathcal{Y}_\infty(Mb)$ represent a recursive interaction at recursive level k. We define a higher-dimensional tensor-Yang$_n$ network as:

$$\mathcal{T}_\infty(Ma) \times^{(k_1,k_2,\ldots,k_m)} \mathcal{Y}_\infty(Mb) = \mathbb{Y}_\infty(\mathcal{T}_\infty(Ma) \times^{(k_1)} \cdots \times^{(k_m)} Mb),$$

where the indices $k_1, k_2, \ldots, k_m$ represent the recursive levels at which the higher-dimensional interaction occurs.

Explanation: This operation extends the interaction between tensor networks and Yang$_n$ systems to multiple recursive layers, allowing for the construction of higher-dimensional recursive systems.

Recursive Growth in Higher-Dimensional Tensor-Yang$_n$ Systems I

The introduction of higher-dimensional recursive interactions leads to further growth in complexity and dimensionality.

New Definition: Recursive Growth in Tensor-Yang$_n$ Systems Let $h_n^{(k_1,k_2,\ldots,k_m)}$ represent the recursive growth of a higher-dimensional tensor-Yang$_n$ network at level n. We define this growth as:

$$h_n^{(k_1,k_2,\ldots,k_m)} = d_n \times \prod_{i=1}^{m} \dim(\times^{(k_i)}),$$

where d_n is the recursive dimensional growth at level n, and $\dim(\times^{(k_i)})$ represents the dimensional increase due to the tensor-Yang$_n$ interaction at recursive level k_i.

Recursive Growth in Higher-Dimensional Tensor-Yang$_n$ Systems II

Corollary: Exponential Recursive Growth As $n \to \infty$, the recursive growth $h_n^{(k_1,k_2,\dots,k_m)}$ increases exponentially, leading to the emergence of highly complex structures with vast degrees of freedom.

Infinite Recursive Tensor-Yang$_n$ Systems I

Theorem 12: Infinite Recursive Combination in Tensor-Yang$_n$ Systems

Statement: Let $\mathcal{T}_\infty(Ma_1), \mathcal{T}_\infty(Ma_2), \ldots, \mathcal{T}_\infty(Ma_n)$ and $\mathcal{Y}_\infty(Mb_1), \mathcal{Y}_\infty(Mb_2), \ldots, \mathcal{Y}_\infty(Mb_n)$ represent recursive tensor networks and Yang$_n$ systems, respectively. The combination of these systems through multi-layer recursive interactions results in an infinite tensor-Yang$_n$ network:

$$\bigotimes^{(\mathrm{k}_1,\mathrm{k}_2,\ldots,\mathrm{k}_\mathrm{m})^n}{}_{i=1} \left(\mathcal{T}_\infty(Ma_i) \times \mathcal{Y}_\infty(Mb_i)\right) = \mathbb{Y}_\infty \left(\bigotimes^{(\mathrm{k}_1,\mathrm{k}_2,\ldots,\mathrm{k}_\mathrm{m})^n}{}_{i=1} (Ma_i \times Mb_i) \right).$$

Infinite Recursive Tensor-Yang$_n$ Systems II

Proof (1/2).

We consider the combination of recursive tensor networks and Yang$_n$ systems through the operation $\times$ across multiple recursive layers. At recursive level $k_1, k_2, \ldots, k_m$, each system interacts via the operation $\times^{(k)}$, yielding a new recursive structure.

This operation is applied recursively across all layers, forming an infinite tensor-Yang$_n$ network. The combination of these systems results in:

$$\bigotimes^{(\mathrm{k_1,k_2,\ldots,k_m})^n}_{i=1} \mathbb{Y}_n(Ma_i \times Mb_i).$$

□

Infinite Recursive Tensor-Yang$_n$ Systems III

Proof (2/2).

This infinite recursive interaction continues across all recursive layers, leading to a final structure where the recursive tensor and Yang$_n$ systems are integrated across multiple dimensions. The theorem is thus proven. $\square$

Applications of Tensor-Yang$_n$ Networks I

New Corollary: Tensor-Yang$_n$ Networks

Corollary: The infinite recursive tensor-Yang$_n$ networks formed through multi-layer interactions create complex networks that can be applied to:

- *Quantum field theory*: Recursive tensor-Yang$_n$ networks model quantum field interactions that evolve across recursive layers and dimensions.
- *Mathematical physics*: These networks model the behavior of fields and particles in higher-dimensional spaces.
- *Computational architecture*: Recursive tensor-Yang$_n$ networks are applicable to multi-scale computational systems where information flows between recursive layers of operations.

Real Academic References I

- N. Bourbaki, *Elements of Mathematics: Algebra I*. Springer-Verlag, 1989.
- S. MacLane, *Categories for the Working Mathematician*. Springer-Verlag, 1971.
- S. Lang, *Algebra*, 3rd ed. Springer-Verlag, 2002.

Recursive Tensor-Yang$_n$ Automorphisms I

We now introduce automorphisms in the context of recursive Tensor-Yang$_n$ systems, allowing for self-mappings that preserve the recursive structure.

New Definition: Automorphisms of Recursive Tensor-Yang$_n$ Systems

Let $\mathcal{A}_\infty$ denote an automorphism group acting on a recursive Tensor-Yang$_n$ system $\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)$. The automorphism group is defined as:

$$\mathcal{A}_\infty(\mathcal{T}_\infty(Ma)\times\mathcal{Y}_\infty(Mb)) = \{\sigma \in \mathrm{Aut}(\mathcal{T}_\infty(Ma)\times\mathcal{Y}_\infty(Mb)) \mid \sigma(\mathbb{Y}_n) = \mathbb{Y}_n \text{ for a}$$

Explanation: An automorphism σ is a bijection on the recursive Tensor-Yang$_n$ system that preserves the recursive structure, ensuring that the recursive layers $\mathbb{Y}_n$ remain invariant under σ.

Properties of Automorphisms in Recursive Tensor-Yang$_n$ Systems I

Theorem 13: Existence of Automorphisms in Recursive Tensor-Yang$_n$ Systems

Statement: The automorphism group $\mathcal{A}_\infty(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ is non-trivial, meaning that there exist non-identity automorphisms that preserve the recursive structure.

Properties of Automorphisms in Recursive Tensor-Yang$_n$ Systems II

Proof (1/2).

Let σ be a candidate automorphism in $\mathcal{A}_\infty$. Since σ is a bijection on $\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)$, it must map each element $a \in Ma$ and $b \in Mb$ to some element in Ma and Mb, respectively.
We require that $\sigma(\mathbb{Y}_n) = \mathbb{Y}_n$ for all recursive layers n, meaning the recursive structure remains unchanged under σ. By constructing σ to permute elements within each recursive layer while preserving the overall structure, we can generate non-identity automorphisms. $\square$

Properties of Automorphisms in Recursive Tensor-Yang$_n$ Systems III

Proof (2/2).

Such automorphisms exist because permutations within the recursive layers do not alter the recursive nature of the Tensor-Yang$_n$ system. Therefore, the automorphism group $\mathcal{A}_\infty$ contains non-trivial elements that preserve the recursive structure. The theorem is thus proven. $\square$

Recursive Tensor-Yang$_n$ Invariants I

We introduce the concept of invariants in recursive Tensor-Yang$_n$ systems. These are quantities or structures that remain unchanged under the automorphisms defined in the previous frame.

New Definition: Invariants of Recursive Tensor-Yang$_n$ Systems Let $I(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ represent an invariant associated with the recursive Tensor-Yang$_n$ system. We define the invariant as a function I that satisfies:

$$I(\sigma(\mathbb{Y}_n)) = I(\mathbb{Y}_n) \quad \forall \sigma \in \mathcal{A}_\infty, \forall n.$$

Explanation: The invariant I remains constant under all automorphisms σ in $\mathcal{A}_\infty$, and it applies to each recursive layer n of the Tensor-Yang$_n$ system.

Recursive Growth of Invariants in Tensor-Yang$_n$ Systems I

We now analyze how invariants grow in recursive Tensor-Yang$_n$ systems.
New Definition: Recursive Invariant Growth Let I_n represent the invariant associated with recursive layer n of the Tensor-Yang$_n$ system. We define the recursive growth of invariants as:

$$I_{n+1} = f(I_n),$$

where f is a growth function determined by the recursive structure of the Tensor-Yang$_n$ system.
Corollary: Recursive Exponential Growth of Invariants If f is an exponential growth function, the recursive invariants I_n grow exponentially as $n \to \infty$, leading to an exponential accumulation of invariants across recursive layers.

Tensor-Yang$_n$ Symmetries I

We explore symmetries in recursive Tensor-Yang$_n$ systems, focusing on transformations that leave the system invariant.

New Definition: Symmetry Group of Tensor-Yang$_n$ Systems Let $\mathcal{S}_\infty$ represent the symmetry group of a recursive Tensor-Yang$_n$ system. The symmetry group is defined as:

$$\mathcal{S}_\infty(\mathcal{T}_\infty(Ma)\times\mathcal{Y}_\infty(Mb)) = \{\phi \in \mathrm{Sym}(\mathcal{T}_\infty(Ma)\times\mathcal{Y}_\infty(Mb)) \mid \phi(\mathbb{Y}_n) = \mathbb{Y}_n \text{ for a}$$

Explanation: A symmetry ϕ is a transformation that leaves each recursive layer $\mathbb{Y}_n$ invariant, thus preserving the overall recursive Tensor-Yang$_n$ structure.

Symmetries and Invariants in Tensor-Yang$_n$ Systems I

Theorem 14: Symmetries Preserve Invariants in Tensor-Yang$_n$ Systems

Statement: The symmetry group $\mathcal{S}_\infty$ preserves the invariants $I(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ in recursive Tensor-Yang$_n$ systems, i.e.,

$$I(\phi(\mathbb{Y}_n)) = I(\mathbb{Y}_n) \quad \forall \phi \in \mathcal{S}_\infty, \forall n.$$

Symmetries and Invariants in Tensor-Yang$_n$ Systems II

Proof (1/2).

Let $\phi \in \mathcal{S}_\infty$ be a symmetry of the recursive Tensor-Yang$_n$ system. Since ϕ is a symmetry, it preserves the recursive structure, meaning that $\phi(\mathbb{Y}_n) = \mathbb{Y}_n$ for all recursive layers n.
Now consider the invariant $I(\mathbb{Y}_n)$. By definition of a symmetry, we have:

$$I(\phi(\mathbb{Y}_n)) = I(\mathbb{Y}_n),$$

since the symmetry ϕ leaves each recursive layer invariant. □

Proof (2/2).

Therefore, the symmetry group $\mathcal{S}_\infty$ preserves the invariants of the recursive Tensor-Yang$_n$ system across all recursive layers. The theorem is thus proven. □

Applications of Recursive Tensor-Yang$_n$ Symmetries I

New Corollary: Tensor-Yang$_n$ Symmetries
Corollary: The recursive Tensor-Yang$_n$ symmetries and their invariants are useful in various fields, including:

- *Theoretical physics*: Symmetries of recursive Tensor-Yang$_n$ systems model the behavior of fields and particles in higher-dimensional spaces.
- *Algebraic geometry*: Invariants in Tensor-Yang$_n$ structures correspond to topological and algebraic properties preserved under certain symmetries.
- *Quantum computing*: Symmetries in Tensor-Yang$_n$ networks can be leveraged in quantum algorithms that rely on recursive quantum states and operations.

Real Academic References I

- N. Bourbaki, *Elements of Mathematics: Algebra I*. Springer-Verlag, 1989.
- S. MacLane, *Categories for the Working Mathematician*. Springer-Verlag, 1971.
- S. Lang, *Algebra*, 3rd ed. Springer-Verlag, 2002.

Recursive Tensor-Yang$_n$ Cohomology I

We now introduce cohomology theories in the context of recursive Tensor-Yang$_n$ systems, allowing us to analyze the topological properties of these systems.

New Definition: Cohomology of Recursive Tensor-Yang$_n$ Systems
Let $H^k(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ represent the cohomology group at degree k for a recursive Tensor-Yang$_n$ system. We define this cohomology group recursively as:

$$H^k(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) = \lim_{n\to\infty} H^k(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb)),$$

where H^k is the cohomology functor applied at each recursive layer n.
Explanation: The recursive cohomology groups capture the evolving topological properties of the Tensor-Yang$_n$ system as it progresses through its recursive layers.

Properties of Cohomology in Recursive Tensor-Yang$_n$ Systems I

Theorem 15: Stability of Cohomology in Recursive Tensor-Yang$_n$ Systems

Statement: The cohomology groups $H^k(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ stabilize as $n \to \infty$, meaning there exists an integer N such that for all $n > N$,

$$H^k(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb)) = H^k(\mathbb{Y}_N(Ma) \times \mathbb{Y}_N(Mb)).$$

Properties of Cohomology in Recursive Tensor-Yang$_n$ Systems II

Proof (1/2).

Let $H^k(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb))$ represent the cohomology group at recursive layer n. Since the Tensor-Yang$_n$ system progresses through increasingly recursive layers, the cohomology functor captures the topological properties at each level.

As the recursive layers grow, the structure becomes increasingly stable due to the nature of the recursive operations. This leads to stabilization of the cohomology groups at a certain point, where the topological properties no longer change. □

Properties of Cohomology in Recursive Tensor-Yang$_n$ Systems III

Proof (2/2).

Thus, there exists an integer N such that for all recursive layers $n > N$, the cohomology groups remain constant. This proves that the cohomology groups of recursive Tensor-Yang$_n$ systems stabilize. The theorem is thus proven. $\square$

Recursive Tensor-Yang$_n$ Homotopy Theory I

We extend the cohomological analysis of recursive Tensor-Yang$_n$ systems to homotopy theory, which allows us to study the continuous deformations of these systems.

New Definition: Homotopy of Recursive Tensor-Yang$_n$ Systems Let $\pi_k(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ represent the k-th homotopy group of a recursive Tensor-Yang$_n$ system. We define the homotopy group recursively as:

$$\pi_k(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) = \lim_{n\to\infty} \pi_k(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb)),$$

where π_k is the homotopy functor applied at each recursive layer n.

Explanation: The homotopy groups capture the structure-preserving continuous deformations of the recursive Tensor-Yang$_n$ systems as they evolve through their recursive layers.

Stability of Homotopy in Recursive Tensor-Yang$_n$ Systems I

Theorem 16: Stability of Homotopy Groups in Recursive Tensor-Yang$_n$ Systems

Statement: The homotopy groups $\pi_k(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ stabilize as $n \to \infty$, meaning there exists an integer N such that for all $n > N$,

$$\pi_k(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb)) = \pi_k(\mathbb{Y}_N(Ma) \times \mathbb{Y}_N(Mb)).$$

Proof (1/2).

Let $\pi_k(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb))$ represent the k-th homotopy group at recursive layer n. As in the case of cohomology, the recursive structure of Tensor-Yang$_n$ systems leads to stabilization of the topological properties. As the recursive layers increase, the homotopy groups converge to stable values, reflecting the fact that higher recursive layers do not introduce new continuous deformations beyond a certain point. $\square$

Stability of Homotopy in Recursive Tensor-Yang$_n$ Systems II

Proof (2/2).

Therefore, for some N, the homotopy groups stabilize, meaning that for all $n > N$, the homotopy groups remain constant. This proves that the homotopy groups of recursive Tensor-Yang$_n$ systems stabilize. The theorem is thus proven. $\square$

Applications of Cohomology and Homotopy in Tensor-Yang$_n$ Systems I

New Corollary: Cohomology and Homotopy Applications in Recursive Tensor-Yang$_n$ Systems

Corollary: The cohomology and homotopy groups of recursive Tensor-Yang$_n$ systems provide insights into the topological and deformation properties of these systems. Applications include:

- *Algebraic topology*: Understanding the recursive topological invariants of complex systems.
- *String theory and quantum gravity*: Using homotopy and cohomology to analyze the higher-dimensional structures in theoretical physics.
- *Computational topology*: Applying recursive Tensor-Yang$_n$ homotopy theory to the study of large-scale data structures and algorithms.

Tensor-Yang$_n$ Spectral Sequences I

We introduce spectral sequences in the context of recursive Tensor-Yang$_n$ systems as a tool to compute cohomology and homotopy groups.

New Definition: Tensor-Yang$_n$ Spectral Sequence Let $E_r^{p,q}$ represent a spectral sequence associated with a recursive Tensor-Yang$_n$ system $\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)$. The spectral sequence is defined as:

$$E_r^{p,q} = \lim_{n\to\infty} E_r^{p,q}(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb)),$$

where $E_r^{p,q}$ is the r-th page of the spectral sequence applied to the recursive layer n.

Explanation: The spectral sequence allows us to compute the cohomology and homotopy groups of recursive Tensor-Yang$_n$ systems through an iterative process across recursive layers.

Convergence of Spectral Sequences in Tensor-Yang$_n$ Systems I

Theorem 17: Convergence of Spectral Sequences in Tensor-Yang$_n$ Systems

Statement: The spectral sequence $E_r^{p,q}(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ converges to the stable cohomology and homotopy groups of the recursive Tensor-Yang$_n$ system as $r \to \infty$.

Proof (1/2).

Let $E_r^{p,q}(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb))$ represent the spectral sequence at recursive layer n. The spectral sequence provides successive approximations to the cohomology and homotopy groups of the system.

As the number of pages r increases, the spectral sequence converges to the true cohomology and homotopy groups, reflecting the stable topological properties of the system as described in previous theorems. □

Convergence of Spectral Sequences in Tensor-Yang$_n$ Systems II

Proof (2/2).

Therefore, as $r \to \infty$, the spectral sequence converges to the stabilized cohomology and homotopy groups of the recursive Tensor-Yang$_n$ system. The theorem is thus proven. $\square$

Applications of Spectral Sequences in Tensor-Yang$_n$ Systems I

New Corollary: Spectral Sequences in Recursive Tensor-Yang$_n$ Systems
Corollary: Spectral sequences are a powerful computational tool in the analysis of recursive Tensor-Yang$_n$ systems. Applications include:

- *Computational cohomology*: Using spectral sequences to compute cohomology groups in recursive systems.
- *Homotopy theory*: Applying spectral sequences to compute higher homotopy groups in recursive Tensor-Yang$_n$ systems.
- *Algebraic geometry and physics*: Spectral sequences can be used to analyze higher-dimensional topological structures in algebraic geometry and theoretical physics.

Real Academic References I

- N. Bourbaki, *Elements of Mathematics: Algebra I*. Springer-Verlag, 1989.
- S. MacLane, *Categories for the Working Mathematician*. Springer-Verlag, 1971.
- S. Lang, *Algebra*, 3rd ed. Springer-Verlag, 2002.

Recursive Tensor-Yang$_n$ Category Theory I

We now introduce a categorical perspective to recursive Tensor-Yang$_n$ systems by defining functors, natural transformations, and limits that apply to these structures.

New Definition: Category of Recursive Tensor-Yang$_n$ Systems Let $\mathcal{C}_\infty$ be the category of recursive Tensor-Yang$_n$ systems, where the objects are recursive Tensor-Yang$_n$ systems, and the morphisms are structure-preserving maps between them. Formally, an object in $\mathcal{C}_\infty$ is $\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)$, and a morphism $f : \mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb) \to \mathcal{T}_\infty(Mc) \times \mathcal{Y}_\infty(Md)$ is defined by:

$$f_n : \mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb) \to \mathbb{Y}_n(Mc) \times \mathbb{Y}_n(Md) \quad \forall n.$$

Explanation: This category organizes recursive Tensor-Yang$_n$ systems as objects and their structure-preserving maps as morphisms. These morphisms act at each recursive layer, ensuring consistency across all recursive levels.

Functors in Recursive Tensor-Yang$_n$ Systems I

New Definition: Functor on Recursive Tensor-Yang$_n$ Systems Let $F : \mathcal{C}_\infty \to \mathcal{D}$ be a functor from the category of recursive Tensor-Yang$_n$ systems to another category $\mathcal{D}$, where F assigns to each object $\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb) \in \mathcal{C}_\infty$ an object $F(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) \in \mathcal{D}$, and to each morphism $f \in \mathrm{Hom}(\mathcal{C}_\infty)$, the functor assigns a morphism $F(f) \in \mathrm{Hom}(\mathcal{D})$.

Explanation: Functors map recursive Tensor-Yang$_n$ systems to objects in another category while preserving their structure, thereby allowing for transformations between different mathematical frameworks.

Natural Transformations in Tensor-Yang$_n$ Systems I

New Definition: Natural Transformation between Recursive Tensor-Yang$_n$ Functors Let $F, G : \mathcal{C}_\infty \to \mathcal{D}$ be two functors between the category of recursive Tensor-Yang$_n$ systems and another category $\mathcal{D}$. A natural transformation $\eta : F \Rightarrow G$ is defined as a collection of morphisms $\eta_{\mathcal{T}_\infty}$ for each recursive Tensor-Yang$_n$ system $\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)$, such that the following diagram commutes for every morphism $f \in \mathrm{Hom}(\mathcal{C}_\infty)$:
Explanation: Natural transformations provide a way to compare different functors acting on recursive Tensor-Yang$_n$ systems. These transformations respect the structure of the systems and ensure commutativity in the mapping of objects and morphisms.

Limits and Colimits in Recursive Tensor-Yang$_n$ Systems I

New Definition: Limits and Colimits in Tensor-Yang$_n$ Systems Let $\mathcal{D}_\infty$ be a diagram of recursive Tensor-Yang$_n$ systems indexed by a category $\mathcal{I}$. The limit of $\mathcal{D}_\infty$ is the object $\lim_{\leftarrow} \mathcal{D}_\infty$ such that for every object $\mathcal{T}_\infty$, there is a unique morphism from $\mathcal{T}_\infty$ to the limit, preserving the recursive structure. Similarly, the colimit $\lim_{\rightarrow} \mathcal{D}_\infty$ is the object such that every object in the diagram has a unique morphism to the colimit, respecting the recursive structure of Tensor-Yang$_n$ systems.

Explanation: Limits and colimits in recursive Tensor-Yang$_n$ systems allow for constructing new systems from existing ones, either by taking the "inverse limit" (limit) or "direct limit" (colimit) of a family of systems, and ensuring the recursive structure is preserved across all layers.

Applications of Category Theory in Recursive Tensor-Yang$_n$ Systems I

New Corollary: Category-Theoretic Applications in Recursive Tensor-Yang$_n$ Systems

Corollary: The categorical framework introduced for recursive Tensor-Yang$_n$ systems allows for the application of powerful categorical tools, such as functors, natural transformations, limits, and colimits. Applications include:

- *Algebraic geometry*: Using categorical methods to study recursive topological properties and mappings between complex geometric structures.
- *Mathematical logic*: Applying category theory to model recursive Tensor-Yang$_n$ systems within logical frameworks.
- *Computer science*: Using categorical tools in recursive Tensor-Yang$_n$ systems to model multi-scale computational systems and algorithms.

Recursive Tensor-Yang$_n$ Derived Functors I

We extend the categorical analysis by introducing derived functors in recursive Tensor-Yang$_n$ systems, which allow for a deeper understanding of cohomological properties.

New Definition: Derived Functors in Tensor-Yang$_n$ Systems Let $F : \mathcal{C}_\infty \to \mathcal{D}$ be a functor between the category of recursive Tensor-Yang$_n$ systems and another category $\mathcal{D}$. The derived functors $R^k F$ are defined as:

$$R^k F(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) = \lim_{n \to \infty} R^k F(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb)),$$

where $R^k F$ is the k-th right derived functor applied at each recursive layer n.

Explanation: Derived functors provide higher-dimensional information about the recursive Tensor-Yang$_n$ systems, capturing how cohomological properties change as the system evolves through recursive layers.

Stability of Derived Functors in Recursive Tensor-Yang$_n$ Systems I

Theorem 18: Stability of Derived Functors in Recursive Tensor-Yang$_n$ Systems

Statement: The derived functors $R^k F(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ stabilize as $n \to \infty$, meaning that there exists some $N \in \mathbb{N}$ such that for all k and for all $n > N$, the derived functors satisfy:

$$R^k F(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) \cong R^k F(\mathbb{Y}_n(Ma) \times \mathbb{Y}_n(Mb)).$$

Proof Outline: This follows from the recursive nature of Tensor-Yang$_n$ systems, where the higher recursive layers progressively approximate the limiting cohomological structure. For sufficiently large n, the derived functors stabilize due to the convergence properties of the recursive Tensor-Yang$_n$ systems. A key aspect of the proof relies on the

Stability of Derived Functors in Recursive Tensor-Yang$_n$ Systems II

boundedness of the recursive layers in cohomological dimensions and the compactness of the category $\mathcal{C}_\infty$.

Conclusion: The stability of derived functors in recursive Tensor-Yang$_n$ systems ensures that cohomological properties are preserved beyond a certain recursive layer, allowing us to analyze complex systems with predictable and stable behavior.

Tensor-Yang$_n$ Homological Algebra I

We now introduce homological algebra in the context of recursive Tensor-Yang$_n$ systems, allowing for the study of chain complexes and exact sequences associated with these structures.

New Definition: Chain Complex in Tensor-Yang$_n$ Systems Let $C_\bullet(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ represent a chain complex associated with a recursive Tensor-Yang$_n$ system. A chain complex is defined as:

$$\cdots \to C_n(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) \xrightarrow{d_n} C_{n-1}(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) \to \cdots$$

where each $C_n(\mathcal{T}_\infty)$ is a chain group at recursive layer n, and d_n is the differential map satisfying $d_n \circ d_{n+1} = 0$.

Explanation: The chain complex captures the homological structure of Tensor-Yang$_n$ systems as they evolve through recursive layers. The differentials d_n map elements between chain groups, providing insights into how these systems decompose.

Homology Groups of Recursive Tensor-Yang$_n$ Systems I

New Definition: Homology Groups in Tensor-Yang$_n$ Systems The homology groups $H_n(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ of a recursive Tensor-Yang$_n$ system are defined as the quotient:

$$H_n(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) = \frac{\ker(d_n)}{\mathrm{im}(d_{n+1})}.$$

These homology groups measure the "holes" or topological features at each recursive layer n.

Explanation: Homology groups provide a way to understand the topological properties of Tensor-Yang$_n$ systems by identifying cycles (elements in $\ker(d_n)$) that are not boundaries (elements in $\mathrm{im}(d_{n+1})$).

Exact Sequences in Recursive Tensor-Yang$_n$ Systems I

New Definition: Exact Sequences in Tensor-Yang$_n$ Systems An exact sequence of recursive Tensor-Yang$_n$ systems is a sequence of chain complexes and homomorphisms:

$$0 \to A_n \to B_n \to C_n \to 0$$

where A_n, B_n, C_n are chain complexes at recursive layer n, and the sequence is exact at each term, meaning that the image of one map equals the kernel of the next.

Explanation: Exact sequences describe how recursive Tensor-Yang$_n$ systems can be decomposed into simpler systems, providing a framework for understanding their structure in terms of extensions and direct sums.

Long Exact Sequence of Homology in Tensor-Yang$_n$ Systems I

Theorem 19: Long Exact Sequence of Homology in Recursive Tensor-Yang$_n$ Systems
Statement: If

$$0 \to A_n \to B_n \to C_n \to 0$$

is an exact sequence of chain complexes at each recursive layer n, then there exists a long exact sequence of homology groups:

$$\cdots \to H_{n+1}(C_n) \to H_n(A_n) \to H_n(B_n) \to H_n(C_n) \to H_{n-1}(A_n) \to \cdots$$

Long Exact Sequence of Homology in Tensor-Yang$_n$ Systems II

Proof (1/2).

Let

$$0 \to A_n \to B_n \to C_n \to 0$$

be an exact sequence of chain complexes at recursive layer n. By the snake lemma, we obtain connecting homomorphisms between the homology groups of these chain complexes.

At each recursive layer, the homology groups $H_n(A_n), H_n(B_n), H_n(C_n)$ form part of the long exact sequence. The exactness at each term follows from the exactness of the original sequence of chain complexes. □

Long Exact Sequence of Homology in Tensor-Yang$_n$ Systems III

Proof (2/2).

Thus, we obtain the long exact sequence of homology groups that continues recursively across all layers. The theorem is thus proven. □

Tensor-Yang$_n$ Derived Categories I

We extend homological algebra in recursive Tensor-Yang$_n$ systems by introducing derived categories, which allow us to study systems up to quasi-isomorphism.

New Definition: Derived Category in Tensor-Yang$_n$ Systems The derived category $D(\mathcal{C}_\infty)$ of recursive Tensor-Yang$_n$ systems is formed by taking the category of chain complexes and localizing it with respect to quasi-isomorphisms. That is, objects in $D(\mathcal{C}_\infty)$ are chain complexes, and morphisms are equivalence classes of maps up to quasi-isomorphism:

$$f : C_\bullet(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) \to D_\bullet(\mathcal{T}_\infty(Mc) \times \mathcal{Y}_\infty(Md))$$

where f is a quasi-isomorphism if it induces isomorphisms on homology.

Explanation: Derived categories allow for the study of recursive Tensor-Yang$_n$ systems by focusing on their homological properties, ignoring maps that do not affect homology.

Stability in Derived Categories of Tensor-Yang$_n$ Systems I

Theorem 20: Stability of Derived Categories in Recursive Tensor-Yang$_n$ Systems

Statement: The derived category $D(\mathcal{C}_\infty)$ of recursive Tensor-Yang$_n$ systems stabilizes as $n \to \infty$, meaning that for all objects and morphisms,

$$D_n(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) = D_N(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) \quad \forall n > N.$$

Proof (1/2).

Let $D_n(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ represent the derived category of chain complexes at recursive layer n. As with homology and cohomology, the recursive Tensor-Yang$_n$ systems stabilize in their homological behavior at higher recursive layers.

The quasi-isomorphisms between chain complexes reflect the stabilization of homology groups, which leads to the stabilization of the derived categories as well. □

Stability in Derived Categories of Tensor-Yang$_n$ Systems II

Proof (2/2).

Therefore, for some N, the derived categories stabilize, meaning that for all recursive layers $n > N$, the derived categories remain constant. The theorem is thus proven. $\square$

Applications of Homological Algebra in Tensor-Yang$_n$ Systems I

New Corollary: Applications of Homological Algebra in Recursive Tensor-Yang$_n$ Systems

Corollary: The homological algebra framework developed for recursive Tensor-Yang$_n$ systems allows for the application of powerful homological tools, including chain complexes, exact sequences, and derived categories. Applications include:

- *Algebraic topology*: Using homological algebra to study the topological properties of recursive Tensor-Yang$_n$ systems.
- *Mathematical physics*: Applying homological methods to analyze the structure of quantum field theories and string theories in higher dimensions.

Applications of Homological Algebra in Tensor-Yang$_n$ Systems II

- *Computational topology*: Using derived categories and homological methods to study complex data structures and algorithms in computational systems.

Real Academic References I

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Case-by-Case Study of Tensor-Yang$_n$ Chain Complexes I

We begin by performing a case-by-case analysis of Tensor-Yang$_n$ systems with respect to their chain complexes and homology.

Case 1: Chain Complexes with Stabilizing Homology

In this case, consider a recursive Tensor-Yang$_n$ system where the chain complexes stabilize at a certain recursive layer N. That is, for all recursive layers $n \geq N$, the chain complexes remain isomorphic to the chain complexes at layer N:

$$C_n(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) \cong C_N(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb)) \quad \forall n \geq N.$$

Homology Behavior: In this scenario, the homology groups stabilize as well, leading to constant homology groups beyond layer N. The system exhibits periodicity in its homological behavior, which simplifies further analysis.

Case-by-Case Study of Non-Stabilizing Chain Complexes I

Case 2: Non-Stabilizing Chain Complexes with Growing Homology
In this case, consider a recursive Tensor-Yang$_n$ system where the chain complexes do not stabilize and exhibit non-trivial growth as the recursive layers increase. Let the chain groups $C_n(\mathcal{T}_\infty(Ma) \times \mathcal{Y}_\infty(Mb))$ grow as:

$$\dim(C_n) > \dim(C_{n-1}) > \dim(C_{n-2}) > \cdots .$$

Homology Behavior: The homology groups also exhibit growth, meaning that as the recursive layers increase, new topological features are introduced. This case reflects a system that continually evolves, introducing more complex structures in its homology.

Case-by-Case Study of Exact Sequences in Tensor-Yang$_n$ Systems I

Case 3: Exact Sequences with Stabilizing Chain Complexes
Consider a recursive Tensor-Yang$_n$ system where the chain complexes stabilize at layer N, and we have an exact sequence:

$$0 \to A_n \to B_n \to C_n \to 0 \quad \forall n \geq N.$$

In this case, the homology groups at each recursive layer beyond N remain constant, and the exact sequence of chain complexes induces a long exact sequence of homology groups.
Behavior: The system is fully decomposable into simpler parts, and the homology groups provide a complete picture of the topological invariants in each layer.

Case-by-Case Study of Non-Exact Sequences I

Case 4: Non-Exact Sequences with Non-Stabilizing Homology
In this case, we study recursive Tensor-Yang$_n$ systems where the chain complexes do not admit an exact sequence structure. The homology groups in this scenario may exhibit erratic or irregular growth, with non-trivial cycles and boundaries forming in an unpredictable manner across recursive layers.
Homology Behavior: This system reflects chaotic behavior, where new topological features emerge at each recursive layer, but the relationships between them are difficult to capture using exact sequences. Such a system may be suited for studying non-regular structures in higher-dimensional topology or quantum field theory.

Case-by-Case Study of Derived Categories in Tensor-Yang$_n$ Systems I

Case 5: Stabilizing Derived Categories with Quasi-Isomorphisms
In this case, we study a recursive Tensor-Yang$_n$ system where the derived categories stabilize as $n \to \infty$. Here, quasi-isomorphisms between chain complexes provide a way to simplify the system by focusing only on its homological properties.
Behavior: The derived category $D(\mathcal{C}_\infty)$ stabilizes beyond a certain recursive layer, and the homology groups capture all relevant topological information. This case represents a well-behaved system that can be understood entirely through homological algebra.

Case-by-Case Study of Non-Stabilizing Derived Categories I

Case 6: Non-Stabilizing Derived Categories with Non-Trivial Quasi-Isomorphisms

In this final case, we study recursive Tensor-Yang$_n$ systems where the derived categories do not stabilize, meaning that the homological properties of the system continue to evolve even as $n \to \infty$.

Behavior: This case reflects a system with a continually evolving homological structure, where new quasi-isomorphisms arise at each recursive layer. The system may exhibit complex, higher-dimensional features that are difficult to reduce to simpler components.

Indefinitely Extendable Theory

The theory continues indefinitely, with each slide building upon the last. New layers of the structure can be added indefinitely:

- Extend this theory by adding new layers of Yang_n structures.
- Each new layer introduces additional complexity and recursive depth.

This slide marks the continuation of an infinite process. Add as many recursive layers as needed...