

**LECTURE NOTES FOR  
LMS-CMI RESEARCH SCHOOL ON  
“BOUNDED GAPS BETWEEN PRIMES”  
SEPTEMBER 22–26, 2014**

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1. INTRODUCTION TO PRIME NUMBER THEORY.  
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BY ANDREW GRANVILLE

2. INTRODUCTION TO PRIME NUMBER THEORY.  
 $\zeta$ - AND  $L$ -FUNCTIONS, THE PRIME NUMBER THEOREM (2/4)  
BY ANDREW GRANVILLE

3. THE BOMBIERI-VINOGRADOV THEOREM ABOUT DISTRIBUTION OF PRIMES IN  
PROGRESSIONS.

INTRODUCTION TO SIEVE THEORY (1/4)

BY KANNAN SOUNDARARAJAN

4. THE BOMBIERI-VINOGRADOV THEOREM ABOUT DISTRIBUTION OF PRIMES IN  
PROGRESSIONS.

INTRODUCTION TO SIEVE THEORY (2/4)

BY KANNAN SOUNDARARAJAN

5. INTRODUCTION TO PRIME NUMBER THEORY.  
 $\zeta$ - AND  $L$ -FUNCTIONS, THE PRIME NUMBER THEOREM (3/4)  
BY ANDREW GRANVILLE

6. INTRODUCTION TO PRIME NUMBER THEORY.  
 $\zeta$ - AND  $L$ -FUNCTIONS, THE PRIME NUMBER THEOREM (4/4)  
BY ANDREW GRANVILLE



7. THE BOMBIERI-VINOGRADOV THEOREM ABOUT DISTRIBUTION OF PRIMES IN  
PROGRESSIONS.

INTRODUCTION TO SIEVE THEORY (3/4)

BY KANNAN SOUNDARARAJAN

8. THE BOMBIERI-VINOGRADOV THEOREM ABOUT DISTRIBUTION OF PRIMES IN  
PROGRESSIONS.

INTRODUCTION TO SIEVE THEORY (4/4)

BY KANNAN SOUNDARARAJAN

9. PUBLIC LECTURE  
BY TERRY TAO

10. INPUTS FROM ALGEBRAIC GEOMETRY (1/4)  
BY EMMANUEL KOWALSKI

11. TUTORIAL FOR GRANVILLE'S LECTURE SERIES  
BY ADAM HARPER

12. TUTORIAL FOR SOUNDARARAJAN'S LECTURE SERIES  
BY JAMES MAYNARD

13. INPUTS FROM ALGEBRAIC GEOMETRY (2/4)  
BY EMMANUEL KOWALSKI

14. INPUTS FROM ALGEBRAIC GEOMETRY (3/4)  
BY EMMANUEL KOWALSKI



15. THE METHODS OF GOLDSTON, PINTZ AND YILDIRIM AND MAYNARD-TAO (1/3)  
BY JAMES MAYNARD

16. POLYMATH DISCUSSION  
LED BY BEN GREEN

17. PUBLIC LECTURE  
BY YITANG ZHANG

18. INPUTS FROM ALGEBRAIC GEOMETRY (4/4)  
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19. THE METHODS OF GOLDSTON, PINTZ AND YILDIRIM AND MAYNARD-TAO (2/3)  
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20. POSTER  
BY SÁVIO RIBAS

21. THE METHODS OF GOLDSTON, PINTZ AND YILDIRIM AND MAYNARD-TAO (3/3)  
BY JAMES MAYNARD

22. TUTORIAL FOR MAYNARD'S LECTURE SERIES  
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