FOUNDATIONS OF $\mathbb{Y}_3(\mathbb{Q}_p)$ ANALYSIS

PU JUSTIN SCARFY YANG

1. Introduction

In this work, we aim to rigorously develop a theory of analysis on the structure $\mathbb{Y}_3(\mathbb{Q}_p)$, avoiding conventional phenomena such as the Cauchy Integral Formula and the Cauchy-Riemann Equations. Instead, we explore whether new phenomena arise naturally in this context. This serves as the foundation for $\mathbb{Y}_3(\mathbb{Q}_p)$ analysis and opens pathways for novel mathematical discoveries.

2. Structure of $\mathbb{Y}_3(\mathbb{Q}_p)$

Let $\mathbb{Y}_3(\mathbb{Q}_p)$ denote a mathematical structure defined on the p-adic field $\mathbb{Q}_p$. We begin by setting up its algebraic properties, topological space, and any unique constructs that may arise from the interplay of $\mathbb{Y}_3$ with p-adic numbers.

2.1. Basic Properties.

Define $\mathbb{Y}_3(\mathbb{Q}_p)$ as a vector space over $\mathbb{Q}_p$ with the following properties:

- **(a)** $\mathbb{Y}_3(\mathbb{Q}_p)$ contains elements that respect the additive and scalar multiplication properties of vector spaces over $\mathbb{Q}_p$.
- **(b)** A distinguished basis $\{e_i\}_{i=1}^n$ for $\mathbb{Y}_3(\mathbb{Q}_p)$ such that every element can be uniquely expressed as a linear combination of the e_i's.

3. Metric and Topology

We define a metric $d : \mathbb{Y}_3(\mathbb{Q}_p) \times \mathbb{Y}_3(\mathbb{Q}_p) \to$ [parts that could be well-ordered in terms of] $\mathbb{Q}_p$ that respects p-adic properties and ensures $\mathbb{Y}_3(\mathbb{Q}_p)$ forms a complete metric space.

Definition 3.0.1 (Metric on $\mathbb{Y}_3(\mathbb{Q}_p)$)**.** *Let $x, y \in \mathbb{Y}_3(\mathbb{Q}_p)$. Define the distance $d(x, y)$ by*

$$d(x, y) = |x - y|_{\mathbb{Y}_3(\mathbb{Q}_p)}$$

where $|\cdot|_{\mathbb{Y}_3(\mathbb{Q}_p)}$ is an absolute value adapted to $\mathbb{Y}_3(\mathbb{Q}_p)$ that extends the p-adic norm on $\mathbb{Q}_p$ and maps to values in [parts that could be well-ordered in terms of] $\mathbb{Q}_p$.

Definition 3.0.2 (Topology of $\mathbb{Y}_3(\mathbb{Q}_p)$)**.** *The topology on $\mathbb{Y}_3(\mathbb{Q}_p)$ is induced by the metric d, with a basis of open sets given by open balls*

$$B(x, r) = \{y \in \mathbb{Y}_3(\mathbb{Q}_p) \mid d(x, y) < r\}.$$

4. Function Theory on $\mathbb{Y}_3(\mathbb{Q}_p)$

We now consider functions defined on $\mathbb{Y}_3(\mathbb{Q}_p)$ and examine conditions for differentiability and continuity in this context.

4.1. **Definitions of Continuity and Differentiability.**

Definition 4.1.1 (Continuity)**.** *A function $f : \mathbb{Y}_3(\mathbb{Q}_p) \to \mathbb{Y}_3(\mathbb{Q}_p)$ is said to be continuous at a point $x \in \mathbb{Y}_3(\mathbb{Q}_p)$ if, for every $\epsilon > 0$, there exists a $\delta > 0$ such that*

$$d(f(x), f(y)) < \epsilon \quad \textit{whenever} \quad d(x, y) < \delta.$$

Definition 4.1.2 (Differentiability)**.** *A function $f : \mathbb{Y}_3(\mathbb{Q}_p) \to \mathbb{Y}_3(\mathbb{Q}_p)$ is differentiable at x if there exists a linear map $Df_x : \mathbb{Y}_3(\mathbb{Q}_p) \to \mathbb{Y}_3(\mathbb{Q}_p)$ such that*

$$\lim_{y \to x} \frac{d(f(y) - f(x) - Df_x(y - x))}{d(y, x)} = 0.$$

5. Exploration of New Phenomena

As we proceed with developing this theory, we remain open to discovering new phenomena and properties that could emerge in $\mathbb{Y}_3(\mathbb{Q}_p)$ analysis. These may include new types of convergence, integral transformations, or differential properties unique to the p-adic setting of $\mathbb{Y}_3$.