CONCENTRATION OF MODULUS MINIMA ON THE CRITICAL LINE FOR DEFORMED ZETA FAMILIES

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ABSTRACT. Building on the variational framework for the modulus field $\mathcal{F}_t(s) := \log |L_t(s)|^2$ of the deformed Euler zeta family

$$L_t(s) := \prod_p \left(1 - \frac{1}{p^s}\right)^{-t},$$

we investigate the global behavior of its local minima as $t \to 1^-$. We provide a framework for describing the unique asymptotic attractor set of these minima, and conjecture that the critical line $\Re(s) = 1/2$ emerges as the universal attractor for all such modulus valleys.

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1. SETUP

Let $s = \sigma + i\tau$ and define the pressure field:

$$\mathcal{F}_t(s) := \log |L_t(s)|^2.$$

We denote by $\mathscr{Z}_t := \{s \in \mathbb{C} : \nabla \mathcal{F}_t(s) = 0, \text{ and } s \text{ is a local minimizer}\}$ the set of modulus valley centers (zero precursors).

2. ATTRACTOR HYPOTHESIS

(Universal Critical Line Attractor Conjecture)
As $t \to 1^-$, all modulus minima $s \in \mathscr{Z}_t$ converge to the

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critical line:

$$\lim_{t \to 1^-} \sup_{s \in \mathscr{Z}_t} \left| \Re(s) - \frac{1}{2} \right| = 0.$$

This expresses that the critical line is the unique attractor for the entire set of deformed pre-zeros under the modulus gradient dynamics.

3. Gradient Flow Dynamics

Define a flow field:

$$\frac{ds}{dt} = -\nabla_s \mathcal{F}_t(s),$$

describing the steepest descent of modulus energy. This flow leads each point in $\mathscr{Z}_t$ to drift toward the asymptotic set $\Re(s) = 1/2$.

4. Stability and Universality

We conjecture:

(1) The attractor set is globally asymptotically stable for all initial valleys s_0 as $t \to 1^-$.
(2) No local minima of $\mathcal{F}_t$ remain bounded away from $\Re(s) = 1/2$ in this limit.

5. Relation to Riemann Hypothesis

If all modulus minima concentrate on $\Re(s) = 1/2$ and survive the limit $t \to 1$, then the nontrivial zeros of $\zeta(s)$ must lie on the critical line.

This provides a novel formulation of RH in terms of:

> *The limiting distribution of analytic modulus valleys under universal Euler structure concentration.*