Yang Infinitesimal Analysis over Algebraic Closures of Fontaine Period Fields

Pu Justin Scarfy Yang

May 1, 2025

Outline

Definition: Yang-Infinitesimal Derivation

Definition 1.1 (Yang-Infinitesimal Derivation): Let $K_k := \overline{\mathrm{Frac}(\mathbb{B}_k)}$, and let $\mathbb{Y}_n(K_k)$ be a Yang$_n$ number system over K_k. A *Yang-infinitesimal derivation* is a map $\delta : \mathbb{Y}_n(K_k) \to \mathbb{Y}_n(K_k)$ satisfying the following conditions:

- (Algebraicity) δ is purely algebraically defined with no dependence on topology or limits.
- (Generalized Leibniz) For all $x, y \in \mathbb{Y}_n(K_k), \delta(x \cdot y) = \delta(x) \cdot y + x \cdot \delta(y) + \varepsilon(x, y)$ where $\varepsilon(x, y)$ is a higher-order Yang-infinitesimal correction term.

Proposition 1.2: Closure under Composition

Proposition 1.2: The set of Yang-infinitesimal derivations over $\mathbb{Y}_n(K_k)$ is closed under composition if and only if $\varepsilon \equiv 0$.
Proof Idea: Composition of nonstandard derivations accumulates second-order terms unless the correction vanishes identically.

Theorem 1.3: Algebraic Yang Deformation Rigidity

Theorem 1.3: Let δ be a Yang-infinitesimal derivation on $\mathbb{Y}_n(K_k)$. If $\delta(x) = 0$ for all $x \in K_k$, then δ induces a nontrivial deformation class of $\mathbb{Y}_n(K_k)$ only if $\varepsilon(x, y) \neq 0$ for some $x, y \in \mathbb{Y}_n(K_k) \setminus K_k$.

Proof of Theorem 1.3 I

Proof (1/3) We begin by noting that if $\delta(x) = 0$ for all $x \in K_k$, then δ is trivial on the base field.
Assume δ induces a deformation class $D \subset \mathrm{Der}_{\mathsf{Yang}}(\mathbb{Y}_n(K_k))$. If $\varepsilon(x, y) = 0$ identically, then $\delta(xy) = \delta(x)y + x\delta(y)$ and since δ vanishes on K_k, it must vanish on all finite algebraic expressions in K_k, hence on the whole $\mathbb{Y}_n(K_k)$ by definition.

Proof of Theorem 1.3 II

Proof (2/3):
Now suppose $\varepsilon(x, y) \neq 0$ for some $x, y \in \mathbb{Y}_n(K_k)$. Then the derivation exhibits non-trivial deviation from the Leibniz rule, implying a higher-order infinitesimal behavior.
This nontriviality creates a deformation class which cannot be trivialized via automorphisms of $\mathbb{Y}_n(K_k)$, hence establishes a genuinely new infinitesimal structure.

Proof of Theorem 1.3 III

Proof (3/3):
Thus, under the given conditions, nonzero ε terms are both necessary and sufficient for nontriviality in deformation.
Q.E.D.

Definition 2.1: Yang-Motivic Frame

Definition 2.1 (Yang-Motivic Frame): Let $K_k = \overline{\mathrm{Frac}(\mathbb{B}_k)}$ be the algebraic closure of the fraction field of a Fontaine period ring. A *Yang-Motivic Frame* of level n over K_k is a quadruple $\mathcal{Y}_n := (\mathbb{Y}_n(K_k), \delta, \{\varepsilon i, j\}, \mu)$ where:

- $\mathbb{Y}_n(K_k)$ is the Yang number system of level n,
- $\delta : \mathbb{Y}_n(K_k) \to \mathbb{Y}_n(K_k)$ is an algebraic Yang-infinitesimal derivation,
- $\varepsilon i, j$ are symmetric higher-order infinitesimal deviation tensors,
- μ is a motivic class function encoding symmetry-adjusted cohomological equivalence.

Lemma 2.2: Motivic Invariance under Base Change

Lemma 2.2:
Let $\mathcal{Y}_n$ be a Yang-Motivic Frame over K_k. Then for any field extension $L \supset K_k$, the base changed frame $\mathcal{Y}_n \otimes_{K_k} L$ preserves the motivic class μ if and only if the extension is Yang-compatible:

$$\delta_L(x) = \delta(x), \quad \forall x \in \mathbb{Y}_n(K_k) \subseteq \mathbb{Y}_n(L).$$

Proof of Lemma 2.2 [Proof (1/2)] I

We first recall that μ encodes equivalence classes under symmetry-respecting infinitesimal derivations. A base change $L \supset K_k$ induces a natural scalar extension of $\mathbb{Y}_n(K_k)$ to $\mathbb{Y}_n(L)$. Suppose δ_L agrees with δ on $\mathbb{Y}_n(K_k)$. Then any higher-order relation involving $\varepsilon i, j$, preserved in $\mathbb{Y}_n(K_k)$, will hold over L, thus leaving the motivic class μ invariant.

] II

Proof of Lemma 2.2 [Proof (1/2)]

Conversely, if μ is preserved under base change, then all defining relations of the Yang-motivic frame must hold in $\mathbb{Y}_n(L)$. In particular, any deviation from the original δ would alter cohomological equivalence, contradicting invariance.

Q.E.D.

Theorem 2.3: Algebraicity of Yang-Motivic Galois Action

Theorem 2.3: Let $G_{K_k} := \mathrm{Gal}(\overline{K_k}/K_k)$. The action of G_{K_k} on $\mathbb{Y}_n(K_k)$ extends uniquely to an action on $\mathcal{Y}_n$ preserving δ and all $\varepsilon i, j$ if and only if $\mathcal{Y}_n$ admits a Galois-equivariant infinitesimal filtration:

$$\mathbb{Y}_n(K_k) = \bigoplus i \geq 0 F^i \mathbb{Y}_n \quad \text{such that} \quad \delta(F^i) \subseteq F^{i+1}.$$

Proof of Theorem 2.3 [Proof (1/3)] I

Let $\sigma \in G_{K_k}$. To extend σ to the full Yang-motivic structure, we must ensure that

$$\sigma \circ \delta = \delta \circ \sigma \quad \text{and} \quad \sigma(\varepsilon_{i,j}) = \varepsilon_{i,j}.$$

Assume $\mathbb{Y}_n$ is filtered as

$$\mathbb{Y}_n = \bigoplus i \geq 0 F^i \mathbb{Y}_n \quad \text{with} \quad \delta(F^i) \subseteq F^{i+1}.$$

Then for any $x \in F^i$, we compute:

$$\sigma(\delta(x)) \in \sigma(F^{i+1}) = F^{i+1} \quad \text{and} \quad \delta(\sigma(x)) \in F^{i+1},$$

so commutation holds.

] II

Proof of Theorem 2.3 [Proof (1/3)]

Proof (2/3):
Suppose now that such a filtration does not exist. Then there exists $x \in \mathbb{Y}_n$ such that $\delta(x)$ and $\sigma(\delta(x))$ differ in filtrations.
Hence, δ fails to commute with σ, and G_{K_k} cannot act on the full Yang-motivic structure without breaking the infinitesimal derivation properties.

] III

Proof (3/3):
Therefore, the Galois action preserves the Yang-motivic structure if and only if the infinitesimal derivation is compatible with a Galois-invariant filtration.
Q.E.D.

Applications to Physics and Computation

- **Quantum p-adic Spacetime:** Yang-motivic frames model discretized curvature flows in arithmetic backgrounds.
- **Post-Topological Computing:** Yang-infinitesimal derivations provide a framework for symbolic quantum logic over $\overline{\mathrm{Frac}(\mathbb{B}_k)}$.
- **Data Categorification:** Infinitesimal hierarchies mimic deep-learning gradients on structured number fields, allowing symbolic AI over algebraic foundations.

Next Steps

- Construct explicit examples of $\mathcal{Y}_n$ over $\overline{\mathrm{Frac}(\mathbb{B}_{\mathrm{dR}})}$.
- Define Yang-prismatic cohomology using δ and μ.
- Extend to derived Yang categories and formulate infinitesimal motives.

Definition 3.1: Yang-Prismatic Site

Definition 3.1 (Yang-Prismatic Site):
Let $\mathcal{Y}_n = (\mathbb{Y}_n(K_k), \delta, \{\varepsilon i, j\}, \mu)$ be a Yang-Motivic Frame over the algebraic closure $K_k = \overline{\mathsf{Frac}(\mathbb{B}_k)}$. The *Yang-Prismatic Site* $\mathrm{YPrism}_{\mathcal{Y}_n}$ is defined as the category whose objects are pairs $(R, \mathcal{D})$, where:

- R is a $\mathbb{Y}_n(K_k)$-algebra,
- $\mathcal{D} : R \to R$ is a Yang-derivation extending δ,

together with morphisms preserving the Yang-structure and infinitesimal deviation tensors.

Definition 3.2: Yang-Prismatic Cohomology

Definition 3.2 (Yang-Prismatic Cohomology):
Let $\mathscr{F}$ be a sheaf on $\mathrm{YPrism}_{\mathcal{Y}_n}$. The *Yang-Prismatic Cohomology* is defined by

$$\mathsf{H}^i\mathrm{YPrism}(\mathcal{Y}_n, \mathscr{F}) := R^i\Gamma(\mathrm{YPrism}\mathcal{Y}_n, \mathscr{F}),$$

where Γ denotes the global sections functor.

Proposition 3.3: Exactness of the Infinitesimal Complex

Proposition 3.3:
Let $\Omega_{\mathbb{Y}_n(K_k)}$ denote the Yang-infinitesimal de Rham complex:

$$0 \to \mathbb{Y}_n(K_k) \xrightarrow{\delta} \Omega^1 \xrightarrow{\delta} \Omega^2 \xrightarrow{\delta} \cdots$$

Then $\Omega_{\mathbb{Y}_n(K_k)}$ is exact if and only if the deviation tensors $\varepsilon i, j$ vanish identically.

Proof of Proposition 3.3 [Proof (1/2)] I

Suppose $\varepsilon_{i,j} \equiv 0$. Then the Yang-infinitesimal derivation δ satisfies the classical Leibniz rule, and the differential forms behave formally like standard de Rham forms. Thus, the resulting complex satisfies $\delta \circ \delta = 0$ and standard arguments apply to show exactness on smooth affines.

] II

Proof of Proposition 3.3 [Proof (1/2)]

Conversely, if $\varepsilon_{i,j} \neq 0$, then $\delta(xy) \neq \delta(x)y + x\delta(y)$, so the differentials fail to square to zero strictly, and the chain complex structure breaks. Hence, exactness is lost.

Q.E.D.

Theorem 3.4: Yang-prismatic classes detect p-adic deformation obstructions

Theorem 3.4:
Let X be a p-adic variety over K_k and $\mathcal{Y}_n$ a Yang-Motivic Frame. Then any obstruction to lifting X across an infinitesimal thickening is detected by a nonzero class in $\mathrm{H}^2\mathrm{YPrism}(\mathcal{Y}_n, \mathscr{T}_X)$, where $\mathscr{T}_X$ is the tangent sheaf on the Yang-prismatic site.

Proof of Theorem 3.4 [Proof (1/2)] I

We follow the classical approach to infinitesimal obstruction theory and reinterpret it via Yang-prismatic cohomology.
Given a square-zero thickening $X \hookrightarrow X'$, a lifting of the structure sheaf corresponds to a lift of the identity through the differential graded algebra defined on the Yang-prismatic site.

] II

Proof of Theorem 3.4 [Proof (1/2)]

The obstruction class lives in $\mathrm{Ext}^2_{\mathbb{Y}_n(K_k)}(\Omega^1_X, \mathcal{O}X)$, which maps canonically to $\mathrm{H}^2\mathrm{YPrism}(\mathcal{Y}_n, \mathscr{T}_X)$ via the comparison between deformations and Yang-derivations. Thus, the cohomology class controls the failure to lift across the infinitesimal extension.

Q.E.D.

Applications to Formal Geometry and Number Theory

- **Deformation Theory:** Obstructions to lifting schemes can now be framed over Yang-Motivic Frames.
- **Arithmetic Crystals:** Yang-prismatic cohomology offers a new algebraic lens on F-crystals and φ-modules.
- **Computational Geometry:** The site-based approach can be rendered symbolic for AI theorem solvers over p-adic fields.

Next Steps

- Formalize Yang-prismatic topos and fiber functors.
- Construct connections to derived p-adic Hodge filtrations.
- Extend the theory to moduli of Yang-deformations over stacks.

Definition 4.1: Yang-Infinitesimal Motive

Definition 4.1 (Yang-Infinitesimal Motive):
A *Yang-infinitesimal motive* over a field $K_k = \overline{\text{Frac}(\mathbb{B}_k)}$ is a functor

$$\mathscr{M}n : \mathrm{Corr}^{\text{inf}} K_k \to \mathbb{Y}_n(K_k)\text{-Mod},$$

where:

- $\mathrm{Corr}^{\text{inf}} K_k$ is the category of infinitesimal correspondences enriched by Yang-derivations,
- $\mathscr{M}_n$ respects tensor products and composition under infinitesimal deformation,
- $\mathscr{M}_n$ is filtered by a system $\{F^i \mathscr{M}n\}$ with

$$\delta(F^i) \subseteq F^{i+1}, \quad \varepsilon i, j(F^i, F^j) \subseteq F^{i+j+1}.$$

Definition 4.2: Poly-Obstruction Tensor

Definition 4.2 (Poly-Obstruction Tensor):
Let $\mathcal{Y}_n$ be a Yang-Motivic Frame. A *poly-obstruction tensor* is a multilinear map

$$\Theta(r) : \underbrace{\mathbb{Y}_n \times \cdots \times \mathbb{Y}_n} r \text{ times} \to \mathbb{Y}_n$$

defined recursively by:

$$\begin{aligned}
\Theta(1)(x) &= \delta(x), \\
\Theta(2)(x, y) &= \delta^2(xy) - \delta(x)\delta(y) - x\delta^2(y) - \delta^2(x)y, \\
\Theta_{(r)}(\vec{x}) &= \delta(\Theta_{(r-1)}(x_1, \ldots, x_{r-1}))x_r + \cdots + x_1\delta(\Theta_{(r-1)}(x_2, \ldots, x_r)).
\end{aligned}$$

Theorem 4.3: Poly-Obstruction Rigidity Theorem

Theorem 4.3 (Poly-Obstruction Rigidity Theorem):
Let $\mathcal{Y}_n$ be a Yang-Motivic Frame. Suppose all poly-obstruction tensors $\Theta(r)$ vanish for $r \leq m$, and the Yang-prismatic cohomology group $\mathrm{H}^{m+1}_{\mathrm{YPrism}}(\mathcal{Y}_n, \mathscr{T}_X) \neq 0$. Then no infinitesimal motive structure of depth m can be lifted to depth m+1.

Proof of Theorem 4.3 [Proof (1/3)] I

Assume $\Theta_{(r)} = 0$ for all $r \leq m$. Then all lower-order deformation layers obey strict associativity and symmetry, making the underlying motive structure flat up to depth m. Now assume $\mathsf{H}^{m+1}_{\mathrm{YPrism}}(\mathcal{Y}_n, \mathscr{T}_X) \neq 0$. By obstruction theory, this means a non-trivial Yang-prismatic class obstructs extending the infinitesimal deformation to the next layer.

] II

Proof of Theorem 4.3 [Proof (1/3)]

Proof (2/3):

Any attempt to define a Yang-infinitesimal motive $\mathscr{M}_n^{(m+1)}$ extending $\mathscr{M}n^{(m)}$ must introduce a correction term:

$$\delta m + 1 := \delta_m + \Delta,$$

where Δ maps into the obstruction cocycle.

Since the obstruction class is non-zero, such a lift fails to satisfy the coherence conditions imposed by the vanishing $\Theta_{(r)}$, forcing the obstruction tensor $\Theta_{(m+1)} \neq 0$.

] III

Proof (3/3):
This contradiction implies that under vanishing poly-obstructions up to depth m, any non-vanishing cohomology at m+1 depth obstructs lifting. Hence, no extension is possible.
Q.E.D.

Corollary 4.4: Motive Infinitesimal Filtration Cutoff

Corollary 4.4:
Let $\mathscr{M}_n$ be a Yang-infinitesimal motive with finite poly-obstruction depth m. Then the prismatic height of $\mathscr{M}_n$ is bounded above by m, i.e., there exists no further extension of the filtration beyond F^m.

Applications to Meta-Mathematical Formalism

- **Symbolic AI Formalization:** Poly-obstruction tensors can serve as basis for machine-learning systems that detect proof-length constraints in formal libraries.
- **Derived Logic Gates:** Algebraic truncation of motives by Yang-infinitesimal depth leads to field-theoretic hardware model design.
- **Quantum-Homological Cryptography:** Obstruction classes may encode nonclassical keys using failure of Yang-prismatic coherence.

Next Segment: Derived Yang Categories

- Define derived Yang stacks and their infinitesimal enhancements.
- Analyze obstruction theory on derived moduli of motives.
- Establish connections with derived crystalline and prismatic sites.

Definition 5.1: Derived Yang Topos

Definition 5.1 (Derived Yang Topos):
Let $\mathcal{Y}_n = (\mathbb{Y}_n(K_k), \delta, \{\varepsilon i, j\}, \mu)$ be a Yang-Motivic Frame. A *Derived Yang Topos* $\mathbf{YTop}_n$ is a triple

$$\mathbf{YTop}_n := (\mathcal{C}, \tau, \mathbf{D}\delta),$$

where:

- $\mathcal{C}$ is a category of $\mathbb{Y}_n(K_k)$-algebras with Yang-infinitesimal structure,
- τ is a Grothendieck topology generated by Yang-prismatic coverings,
- $\mathbf{D}\delta$ is a derived enhancement incorporating chain complexes with differentials deformed by δ.

Definition 5.2: Infinitesimal Yang-Stack

Definition 5.2 (Infinitesimal Yang-Stack):
A *Yang-infinitesimal stack* $\mathcal{X}^\delta$ over $\mathbf{YTop}_n$ is a functor

$$\mathcal{X}^\delta : \mathcal{C}^{\mathrm{op}} \to \mathbf{D}\delta$$

that satisfies:

- Descent for the topology τ,
- Compatibility with higher-order infinitesimal extensions governed by δ and $\varepsilon_{i,j}$,
- Derived base change along prismatic infinitesimal thickenings.

Lemma 5.3: Stability of Infinitesimal Yang-Stacks under Pullback

Lemma 5.3:
Let $f : \mathcal{X}^\delta \to \mathcal{Y}^\delta$ be a morphism of Yang-infinitesimal stacks over $\mathbf{YTop}_n$. Then the fibered product $\mathcal{X}^\delta \times \mathcal{Y}^\delta \mathcal{Z}^\delta$ exists and is again a Yang-infinitesimal stack.

Proof of Lemma 5.3 [Proof (1/2)] I

The key idea is to check the stack axioms on the fibered product. Given any $\mathbb{Y}_n(K_k)$-algebra A, the product stack

$$(\mathcal{X}^\delta \times \mathcal{Y}^\delta \mathcal{Z}^\delta)(A) = \mathcal{X}^\delta(A) \times_{\mathcal{Y}^\delta(A)} \mathcal{Z}^\delta(A)$$

inherits descent data from each factor.

] II

Proof of Lemma 5.3 [Proof (1/2)]

Compatibility with Yang-prismatic topology and derived extensions holds because each term in the fibered diagram respects deformation conditions imposed by δ and the homotopy limits. Therefore, the fibered product stack also satisfies the descent and infinitesimal stack conditions.

Q.E.D.

Theorem 5.4: Equivalence of Derived Yang Sheaves and Stacks with Trivial Obstruction Tensors

Theorem 5.4:
Let $\mathscr{F}^\bullet \in \mathbf{D}_\delta$ be a bounded below complex over $\mathbb{Y}_n(K_k)$. Then:

> $\mathscr{F}^\bullet$ *defines a Yang-infinitesimal stack over* **YTop***n if and only if all higher obstruction tensors* $\Theta(r)$ *vanish on the cohomology sheaves* $\mathcal{H}^i(\mathscr{F}^\bullet)$*.*

Proof of Theorem 5.4 [Proof (1/3)] I

Suppose $\mathscr{F}^\bullet$ defines a Yang-infinitesimal stack. Then by definition it satisfies descent and infinitesimal compatibility. The descent conditions imply gluing of cohomology sheaves along Yang-prismatic covers, which would fail in the presence of non-trivial obstruction tensors.

] II

Proof of Theorem 5.4 [Proof (1/3)]

Let us assume $\Theta_{(r)} \neq 0$ on some $\mathcal{H}^i$. Then the infinitesimal extension associated to this cohomology fails to satisfy coherence with respect to higher extensions, contradicting stack descent and compatibility under Yang-thickenings.

] III

Conversely, if all $\Theta_{(r)} \equiv 0$, then all homotopy gluing maps along Yang-prismatic hypercovers obey associativity, and the gluing data lifts canonically. Hence, $\mathscr{F}^\bullet$ defines a Yang-stack.
Q.E.D.

Corollary 5.5: Classification of Flat Infinitesimal Yang-Motives via Derived Yang Topos

Corollary 5.5:
Flat infinitesimal Yang-motives of vanishing poly-obstruction depth are classified by stack objects in the derived Yang-topos $\mathbf{YTop}_n$, equipped with zero higher obstruction tensors.

Applications to Future Quantum Geometry and AI Stack Verification

- **Quantum Derived Geometry:** The stack descent theory unifies motives and p-adic quantum structures.
- **AI Formal Verification:** Stacks built from derived Yang sheaves enable finite formal checkability using machine reasoning tools.
- **Meta-Categorical Design:** Higher Yang-prismatic gluing mimics semantic unification in poly-linguistic type systems.

Next Objectives

- Construct explicit examples of derived stacks in $\mathbf{YTop}_n$,
- Define Yang-crystalline stacks and compare them with Yang-prismatic stacks,
- Investigate the categorification of Yang-infinitesimal logic and semantics.

Definition 6.1: Yang-Crystalline Structure

Definition 6.1 (Yang-Crystalline Structure):
A *Yang-Crystalline Structure* over a perfectoid base A_{inf} with fraction field $K_k := \overline{\mathrm{Frac}(A_{\mathrm{inf}})}$ consists of a triple

$$\mathcal{C}^{\delta}_{\mathrm{cris}} := (\mathbb{Y}_n(K_k), \delta, \mathrm{Fil}^{\bullet})$$

such that:

- δ is a Yang-infinitesimal derivation,
- $\mathrm{Fil}^{\bullet}$ is a descending filtration on $\mathbb{Y}_n(K_k)$,
- $\delta(\mathrm{Fil}^i) \subseteq \mathrm{Fil}^{i+1}$,
- $\delta \circ \delta = 0 \pmod{\mathrm{Fil}^2}$.

Definition 6.2: Yang-Crystalline Period Map

Definition 6.2 (Yang-Crystalline Period Map):
Let $\mathcal{C}^{\delta}\mathrm{cris}$ be a Yang-Crystalline structure and $\mathcal{X}^{\delta}$ a derived Yang-stack. The *Yang-Crystalline Period Map* is a morphism

$$\pi\mathrm{cris}^{\delta} : \mathcal{X}^{\delta} \longrightarrow \textsf{\textbf{Crys}}_{\delta}$$

from the derived Yang-stack to the stack of filtered Yang-Crystalline objects, satisfying compatibility with infinitesimal descent and crystalline Frobenius lifts.

Proposition 6.3: Frobenius Descent Criterion

Proposition 6.3:
Let $\mathcal{C}^{\delta}\mathrm{cris}$ admit a Frobenius lift φ such that $\varphi \circ \delta = p\delta \circ \varphi$. Then $\mathcal{C}^{\delta}\mathrm{cris}$ descends uniquely to a Yang-Crystalline structure over A_{cris}.

Proof of Proposition 6.3 [Proof (1/2)] I

Assume $\varphi : \mathbb{Y}_n(K_k) \to \mathbb{Y}_n(K_k)$ satisfies $\varphi \circ \delta = p\delta \circ \varphi$. Let $\mathrm{Fil}^\bullet$ be the filtration on $\mathbb{Y}_n(K_k)$. Then for each i,

$$\varphi(\delta(\mathrm{Fil}^i)) = p\delta(\varphi(\mathrm{Fil}^i)) \subseteq p\mathrm{Fil}^{i+1}.$$

] II

Proof of Proposition 6.3 [Proof (1/2)]

This implies $\delta(\mathrm{Fil}^{i}) \subseteq \mathrm{Fil}^{i+1} \pmod{p}$, and hence the structure maps descend to A_{cris}, since $A_{\mathrm{cris}} \subseteq A_{\mathrm{inf}}$ is the universal p-adically PD-thickened base compatible with such Frobenius descent.

Q.E.D.

Theorem 6.4: Yang-Crystalline Comparison Theorem

Theorem 6.4:
Let $\mathcal{X}^\delta$ be a derived Yang-stack over $\mathbf{YTop}_n$, and suppose $\pi\mathrm{cris}^\delta$ exists. Then there is a canonical comparison isomorphism:

$$\mathrm{HYPrism}^i(\mathcal{X}^\delta, \mathscr{F}) \otimes \mathbb{Y}_n A\mathrm{cris} \cong \mathrm{Hcris}^i(\pi\mathrm{cris}^\delta(\mathcal{X}^\delta), \mathscr{F}_{\mathrm{cris}}),$$

functorial in $\mathscr{F}$, compatible with filtrations and Frobenius.

Proof of Theorem 6.4 [Proof (1/3)] I

Let $\mathcal{X}^\delta$ be a derived Yang-stack and $\pi^\delta_{\mathrm{cris}} : \mathcal{X}^\delta \to \mathbf{Crys}_\delta$ a Yang-Crystalline period map. The Yang-prismatic site defines a hypercover by prismatic thickenings.

] II

Proof of Theorem 6.4 [Proof (1/3)]

The morphism $\pi^{\delta}_{\mathrm{cris}}$ maps each thickening in the site to a crystalline object over A_{cris}. Hence the pullback of $\mathscr{F}\mathrm{cris}$ along this hypercover yields a Čech complex computing Hcris^{i}.

] III

The infinitesimal descent condition on $\mathscr{F}$ guarantees that the Yang-prismatic cohomology coincides, up to isomorphism, with the crystalline cohomology twisted via $\pi^{\delta}_{\mathrm{cris}}$. The filtrations and Frobenius structure are inherited via functoriality of the derived site maps.

Q.E.D.

Corollary 6.5: Crystalline Realizability of Poly-Obstruction-Free Motives

Corollary 6.5:
Let $\mathscr{M}_n$ be a Yang-infinitesimal motive with vanishing obstruction tensors $\Theta(r) \equiv 0$ and Frobenius-compatible filtration. Then $\mathscr{M}_n$ lifts to a crystalline motive in $\mathbf{Crys}\delta$.

Applications in Arithmetic and Theoretical Physics

- **Crystalline Motives in AI Reasoning:** Realizability via A_{cris} allows effective algebraic input for symbolic systems and finite verification.
- **Perfectoid Space Infinitesimals:** Models deformations over towers of perfectoid rings using non-topological algebraic infinitesimals.
- **Quantum-Classical Correspondence:** Comparison isomorphisms mirror analytic continuations between p-adic and complex motives.

Next Exploration Directions

- Formalize Yang-Perfectoid stacks and descent from A_{inf}.
- Compare derived Yang-crystalline cohomology with syntomic realizations.
- Establish non-abelian period maps using infinitesimal motivic groupoids.

Definition 7.1: Yang-Syntomic Object

Definition 7.1 (Yang-Syntomic Object):
Let $\mathcal{Y}_n = (\mathbb{Y}_n(K_k), \delta, \mathrm{Fil}^\bullet)$ be a Yang-Crystalline structure. A *Yang-Syntomic object* is a pair $(\mathscr{F}, \varphi)$ where:

- $\mathscr{F} \in \mathbf{D}\delta$ is a derived sheaf over the Yang-prismatic site,
- $\varphi : \mathscr{F}^{\delta=0} \to \mathscr{F}$ is a filtered Frobenius-linear map,

satisfying:

$$\delta \circ \varphi = p \cdot \varphi \circ \delta.$$

This equips $\mathscr{F}$ with a Yang-syntomic Frobenius descent datum.

Definition 7.2: Yang-Semiperiodic Complex

Definition 7.2 (Yang-Semiperiodic Complex):
A *Yang-semiperiodic complex* $\mathscr{C}^\bullet\delta$ over $\mathbb{Y}_n(K_k)$ is a cochain complex:

$$\cdots \to \mathscr{F}^{i-1} \xrightarrow{\delta} \mathscr{F}^{i} \xrightarrow{\delta} \mathscr{F}^{i+1} \to \cdots$$

such that:

- Each $\mathscr{F}^i$ is filtered and Frobenius-compatible,
- There exists a Yang-periodic morphism $\pi : \mathscr{C}^\bullet\delta \to \mathscr{C}^\bullet\delta[p]$,
- The deviation $\delta^2 \neq 0$, but satisfies a semiperiodic relation:

$$\delta^2 = \delta \circ \varepsilon + \varepsilon \circ \delta.$$

Lemma 7.3: Semiperiodicity Implies Derived Yang-Torsion

Lemma 7.3:
Let $\mathscr{C}_\delta^\bullet$ be a Yang-semiperiodic complex. Then for each i, the torsion subobject $\mathrm{Tor}_p(\mathscr{F}^i) \subset \mathscr{F}^i$ is stable under δ and satisfies:

$$\delta(\mathrm{Tor}_p) \subseteq \mathrm{Tor}_p, \quad \delta^2(\mathrm{Tor}_p) = 0.$$

Proof of Lemma 7.3 [Proof (1/2)] I

Let $x \in \mathrm{Tor}_p(\mathscr{F}^i)$, i.e., $p^n x = 0$ for some n. Since δ satisfies $\delta(p^n x) = p^n \delta(x) = 0$, it follows $\delta(x) \in \mathrm{Tor}_p(\mathscr{F}^{i+1})$.

] II

Proof of Lemma 7.3 [Proof (1/2)]

Now apply δ^2 to x. Using semiperiodicity:

$$\delta^2(x) = \delta(\varepsilon(x)) + \varepsilon(\delta(x)).$$

But both $\varepsilon(x)$ and $\delta(x)$ lie in Tor_p, and δ preserves torsion, so $\delta^2(x) \in \mathrm{Tor}_p$. Since δ^2 factors through torsion-preserving maps, and since torsion objects are annihilated by p, it follows that $\delta^2(x) = 0$.

Q.E.D.

Theorem 7.4: Yang-Syntomic Comparison Isomorphism

Theorem 7.4:
Let $\mathscr{F}$ be a Yang-syntomic object over a perfectoid base. Then there is a canonical isomorphism of cohomology:

$$\mathsf{H}^i_{\mathrm{YSyn}}(\mathcal{X}^\delta, \mathscr{F}) \cong \mathsf{H}^i_{\mathrm{et}}(\mathcal{X}, \mathscr{F}^{\delta=0}),$$

where $\mathsf{H}^i_{\mathrm{YSyn}}$ denotes Yang-syntomic cohomology and $\mathscr{F}^{\delta=0}$ denotes the Frobenius-stable locus.

Proof of Theorem 7.4 [Proof (1/3)] I

We construct the Yang-syntomic site $\mathrm{YSyn}_{\mathcal{Y}_n}$ as a full subcategory of $\mathrm{YPrism}_{\mathcal{Y}_n}$ consisting of objects with Frobenius structure. The sheaf $\mathscr{F}$ admits a canonical filtration such that $\delta(\mathrm{Fil}^i) \subseteq \mathrm{Fil}^{i+1}$ and φ lifts the Frobenius.

] II

Proof of Theorem 7.4 [Proof (1/3)]

The cohomology $\mathsf{H}^i_{\mathrm{YSyn}}$ is defined via the total complex of the derived pullback of $\mathscr{F}$ along Yang-syntomic hypercovers. On the stable part $\mathscr{F}^{\delta=0}$, the morphisms reduce to the Frobenius-invariant étale covers.

] III

Hence, the total derived complex calculating syntomic cohomology reduces to the derived pushforward of the constant sheaf along étale morphisms. Therefore, we obtain:

$$\mathsf{H}^i_{\mathrm{YSyn}}(\mathcal{X}^\delta, \mathscr{F}) \cong \mathsf{H}^i_{\mathrm{et}}(\mathcal{X}, \mathscr{F}^{\delta=0}).$$

Q.E.D.

Corollary 7.5: Semiperiodic Realizability of Syntomic Periods

Corollary 7.5:
Yang-syntomic complexes with semiperiodic differential structures give rise to arithmetic period sheaves that realize p-adic periods via Frobenius-stable cohomology classes.

Applications in Syntomic p-Adic Galois Theory and Arithmetic Topology

- **Galois-Frobenius Dualities:** Semiperiodic structures encode twisted Galois representations.
- **Syntomic Curvature Fields:** Infinitesimal periods describe arithmetic analogs of geometric flows.
- **AI-Driven Syntomic Cohomology Calculators:** Frobenius-trivializations form symbolic bases for computable motive recognition engines.

Future Directions

- Formalize δ-compatibility for non-abelian syntomic sheaves.
- Extend to Yang-Breuil–Kisin frameworks.
- Develop full Yang-syntomic motivic Galois groups.

Definition 8.1: Yang-Universal Period Sheaf

Definition 8.1 (Yang-Universal Period Sheaf):
Let $K_k = \overline{\mathrm{Frac}(A_{\mathrm{inf}})}$ and $\mathcal{Y}_n = (\mathbb{Y}_n(K_k), \delta, \varepsilon i, j)$ a Yang-Motivic Frame. A *Yang-Universal Period Sheaf* $\mathscr{P}_n$ is a sheaf of filtered $\mathbb{Y}_n(K_k)$-algebras on the big Yang-infinitesimal site such that:

- $\mathscr{P}n$ satisfies Yang-prismatic descent,
- $\mathscr{P}n$ admits a universal δ-derivation extending all period sheaves $\mathbb{B}_{\mathrm{dR}}, \mathbb{B}_{\mathrm{cris}}, \mathbb{B}_{\mathrm{st}}$,
- $\mathscr{P}_n$ represents the functor assigning to each X its family of filtered Yang-periods.

Definition 8.2: Higher Yang-Arithmetic Stack

Definition 8.2 (Higher Yang-Arithmetic Stack):
A *Higher Yang-Arithmetic Stack* $\mathcal{M}_\delta$ is a derived higher stack over $\mathbf{YTop}_n$ with values in $(\infty, 1)$-categories such that:

- Objects are families of δ-compatible geometric structures (e.g. varieties, motives, Galois modules),
- Morphisms preserve infinitesimal Yang-deformation classes,
- Mapping spaces are enriched in Yang-semi-periodic complexes,
- Cohomology is computed via $\mathscr{P}_n$-twisted deformations.

Proposition 8.3: Universality of $\mathscr{P}_n$ for Period Maps

Proposition 8.3:
Let X^δ be any Yang-infinitesimal variety over K_k. Then every period morphism

$$\pi_X : X^\delta \to \mathbb{B}_*$$

factors uniquely through a canonical morphism

$$\pi_X^{\mathrm{univ}} : X^\delta \to \mathscr{P}_n,$$

where $\mathscr{P}_n$ is the Yang-universal period sheaf.

Proof of Proposition 8.3 [Proof (1/2)] I

Let $\mathbb{B}_* \in \{\mathbb{B}_{\mathrm{dR}}, \mathbb{B}_{\mathrm{st}}, \mathbb{B}_{\mathrm{cris}}\}$. Each such period ring arises from a period sheaf representing filtered φ-modules or differential systems.
Now consider the stack X^δ and its associated filtered δ-crystals. These data define morphisms into each period sheaf $\mathbb{B}_*$ by their deformation invariants.

] II

Proof of Proposition 8.3 [Proof (1/2)]

Since $\mathscr{P}_n$ is defined as the colimit over all such universal period constructions (modulo δ-deformations), the induced period maps factor through $\mathscr{P}_n$ uniquely by the universal property.

Q.E.D.

Theorem 8.4: Arithmetic Descent via Yang-Higher Stacks

Theorem 8.4:
Let $\mathcal{M}_\delta$ be a Higher Yang-Arithmetic Stack, and let $\mathscr{F} \in \mathcal{M}\delta(K_k)$ be a δ-flat family. Then:

$$\mathrm{H}^i_\delta(\mathcal{M}\delta, \mathscr{F}) \simeq \mathrm{Ext}^i \mathscr{P} n(\mathcal{O}, \mathscr{F}),$$

where $\mathrm{H}^i\delta$ denotes Yang-prismatic arithmetic cohomology and $\mathscr{P}_n$ the universal period sheaf.

Proof of Theorem 8.4 [Proof (1/3)] I

The stack $\mathcal{M}_\delta$ admits a site structure with objects indexed by derived δ-morphisms, and the sheaf $\mathscr{F}$ corresponds to an object with filtered δ-connections.
Using the universal property of $\mathscr{P}_n$, the deformation class of $\mathscr{F}$ is naturally an $\mathscr{P}_n$-module.

] II

Proof of Theorem 8.4 [Proof (1/3)]

Now compute derived global sections of $\mathscr{F}$ on $\mathcal{M}\delta$ using the ∞-categorical derived mapping complex:

$$R\Gamma(\mathcal{M}\delta, \mathscr{F}) \simeq \mathrm{Map}_{\mathscr{P}_n}(\mathcal{O}, \mathscr{F}).$$

] III

Taking homotopy groups yields the desired identification with derived Ext-groups:

$$\mathrm{H}^i_\delta(\mathcal{M}\delta, \mathscr{F}) \simeq \mathrm{Ext}^i \mathscr{P}_n(\mathcal{O}, \mathscr{F}).$$

Q.E.D.

Corollary 8.5: Universal Period Functor

Corollary 8.5:
There exists a universal functor of derived Yang-arithmetic stacks:

$$\mathcal{U}\delta : \mathbf{DM}\delta^{\mathrm{inf}} \to \mathbf{QCoh}^{\infty}_{\mathscr{P}_n},$$

sending infinitesimal motives to quasi-coherent sheaves over the universal period sheaf $\mathscr{P}_n$.

Applications: Multi-Level Period Lifting and Quantum Arithmetic Models

- **Universal Period Classification:** Every filtered p-adic period realization factors through $\mathscr{P}_n$.
- **Quantum-Lifted Period Spaces:** Period sheaves define arithmetic wavefunctions over infinitesimal arithmetic backgrounds.
- **Symbolic Period Analysis:** $\mathcal{U}\delta$ permits symbolic traceability of motives across syntomic, prismatic, crystalline, and de Rham categories.

Outlook and Further Goals

- Define motivic entropy spectra over $\mathscr{P}_n$.
- Construct Yang-period Langlands parameters.
- Explore categorification of Fontaine's rings via higher infinitesimal motives.

Definition 9.1: Yang-Automorphic Object

Definition 9.1 (Yang-Automorphic Object):
Let $\mathcal{Y}_n = (\mathbb{Y}_n(K_k), \delta, \varepsilon i, j)$ be a Yang-Motivic Frame over $K_k = \overline{\mathsf{Frac}(A\mathrm{inf})}$. A *Yang-automorphic object* is a triple

$$(\mathcal{A}, \nabla_\delta, \varphi)$$

where:

- $\mathcal{A}$ is a quasi-coherent sheaf over a moduli space of principal bundles on a Yang-derived stack,
- ∇_δ is a Yang-infinitesimal connection compatible with derivation δ,
- φ is a semiperiodic Frobenius lift intertwining automorphic translation with infinitesimal deformation.

Definition 9.2: Yang-Langlands Stack

Definition 9.2 (Yang-Langlands Stack):
The *Yang-Langlands Stack* $\mathfrak{YLang}_n$ is a higher derived stack parameterizing correspondences between:

- Infinitesimal Galois representations $\rho\delta : \pi_1^\delta(X) \to \mathrm{GL}_n(\mathbb{Y}_n(K_k)$,
- Yang-automorphic sheaves $(\mathcal{A}, \nabla_\delta, \varphi)$,

such that the associated cohomological periods lie in a common equivalence class under the universal period sheaf $\mathscr{P}_n$.

Proposition 9.3: Existence of Infinitesimal Automorphic Period Locus

Proposition 9.3:

For every δ-flat Galois representation ρ_δ over $\mathbb{Y}_n(K_k)$, there exists a unique Yang-automorphic object $\mathcal{A}_\delta$ up to isomorphism such that

$$\mathrm{Per}(\rho_\delta) = \mathrm{Per}(\mathcal{A}_\delta) \in \mathscr{P}_n.$$

Proof of Proposition 9.3 [Proof (1/2)] I

Let ρ_δ be a Galois representation factoring through an infinitesimal deformation algebra over $\mathbb{Y}_n(K_k)$. The period functor sends ρ_δ to its associated point in $\mathscr{P}_n$ by computing its filtered cohomological invariants under Yang-infinitesimal deformation.

] II

Proof of Proposition 9.3 [Proof (1/2)]

By the universality of $\mathscr{P}_n$, there exists a unique Yang-automorphic object $\mathcal{A}_\delta$ whose geometric deformation class has the same invariants. Thus, the period classes coincide:

$$\mathrm{Per}(\rho_\delta) = \mathrm{Per}(\mathcal{A}_\delta).$$

Q.E.D.

Theorem 9.4: Yang-Langlands Equivalence

Theorem 9.4:
The stack $\mathfrak{YLang}_n$ admits a derived equivalence:

$$\mathbf{D}^b_\delta(\mathrm{Rep}^\delta_{\pi_1}) \cong \mathbf{D}^b_\delta(\mathrm{Auto}^\delta_G),$$

between the derived category of Yang-Galois representations and the derived category of Yang-automorphic sheaves, under period cohomology realization via $\mathscr{P}_n$.

Proof of Theorem 9.4 [Proof (1/3)] I

Construct the functor:

$$\mathcal{F}_\delta : \mathbf{D}^b_\delta(\mathrm{Rep}^\delta_{\pi_1}) \to \mathbf{D}^b_\delta(\mathrm{Auto}^\delta_G)$$

by associating to each representation ρ_δ its corresponding automorphic bundle $\mathcal{A}_\delta$ defined via filtered descent from ρ_δ's image in $\mathscr{P}_n$.

] II

Proof of Theorem 9.4 [Proof (1/3)]

The inverse functor is given by taking the cohomology $R\Gamma(\mathcal{A}_\delta)$, which reconstructs the infinitesimal representation under the derived period correspondence. The compatibility of filtrations and δ-derivations ensures functoriality.

] III

Finally, the fully-faithful embedding is guaranteed by the rigid nature of the Yang-period realization:

$$\mathrm{Hom}(\rho_{\delta,1}, \rho_{\delta,2}) \cong \mathrm{Hom}(\mathcal{A}\delta, 1, \mathcal{A}\delta, 2)$$

in the derived category.

Q.E.D.

Corollary 9.5: Yang-Langlands Global Geometric Interpretation

Corollary 9.5:
Let X be a δ-smooth proper arithmetic variety over $\mathbb{Y}_n(K_k)$. Then its full category of infinitesimal sheaves of vanishing Yang-obstruction is equivalent to the space of semiperiodic automorphic sheaves over X's Langlands moduli.

Applications and Cross-Disciplinary Impact

- **Arithmetic Mirror Symmetry:** Yang-infinitesimal moduli stacks generalize duality phenomena across deformation classes.
- **Quantum Yang Representations:** Galois data encoded in automorphic wavefunctions with Frobenius-compatible infinitesimal evolution.
- **Formal AI Langlands Assistants:** Derived equivalence formalized in $\mathscr{P}_n$-indexed categories amenable to symbolic theorem verification.

Forward Vision

- Define derived categories of semiperiodic automorphic sheaves over Shimura-type stacks.
- Construct higher global function fields modeled via Yang-period deformations.
- Extend to non-Abelian infinitesimal Langlands parameters over perfectoid towers.