Constructing Fields Larger than $\mathbb{C}$ Using Automorphic Forms, Motives, and L-functions

Pu Justin Scarfy Yang

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Course Description

This course provides an in-depth exploration of the construction of mathematical fields larger than $\mathbb{C}$, leveraging the rich interplay between automorphic forms, motives, and L-functions. The course is designed to be an ongoing development series, offering an indefinite number of lectures that allow for deep exploration, extensions, and generalizations of key concepts.

Course Objectives

- Understand the fundamentals of automorphic forms, motives, and L-functions and their role in constructing fields.
- Explore new mathematical fields larger than $\mathbb{C}$ through detailed analysis and innovative constructions.
- Develop rigorous proofs and new theoretical frameworks based on automorphic forms, motives, and L-functions.
- Extend the course indefinitely, incorporating new research findings, generalizations, and applications.

Course Format

- **Lecture Duration:** Each lecture will be approximately 3-6 hours long.
- **Lecture Frequency:** Lectures will be available daily, providing continuous learning opportunities.
- **Delivery Method:** Lectures will be delivered via YouTube using Beamer slides, with supplementary PDFs provided as handouts.

Course Structure: Part I - Foundational Concepts

- **Lecture 1-4:** Introduction to Automorphic Forms
- **Lecture 5-8:** Introduction to Motives
- **Lecture 9-12:** Introduction to L-functions

Part I: Foundational Concepts - Details

- **Automorphic Forms:**
  - History and development
  - Basic properties and examples
  - Automorphic representations
- **Motives:**
  - Motives in algebraic geometry
  - Connection to number theory
  - Motives and L-functions
- **L-functions:**
  - Definitions and examples
  - Dirichlet L-functions
  - Role in number theory

Course Structure: Part II - Constructing Fields Larger than $\mathbb{C}$

- **Lecture 13-20:** Field Constructions Using Automorphic Forms
- **Lecture 21-28:** Field Constructions Using Motives
- **Lecture 29-36:** Field Constructions Using L-functions

Part II: Constructing Fields - Details

- **Using Automorphic Forms:**
  - Automorphic forms and field extensions
  - Fields over $\mathbb{Q}$
  - Examples beyond $\mathbb{C}$
- **Using Motives:**
  - Motive-based field constructions
  - Extensions over $\mathbb{Q}$
  - Advanced techniques
- **Using L-functions:**
  - L-functions and field extensions
  - Larger fields via L-functions
  - Theoretical exploration

Course Structure: Part III - Extensions and Generalizations

- **Lecture 37-44:** Higher-Dimensional Automorphic Forms
- **Lecture 45-52:** Generalized Motives
- **Lecture 53-60:** Advanced L-functions

Part III: Extensions and Generalizations - Details

- **Higher-Dimensional Automorphic Forms:**
  - Higher-dimensional forms and field constructions
  - Generalization to other base fields
- **Generalized Motives:**
  - Extending motives to new objects
  - New fields from generalized motives
  - Theoretical implications
- **Advanced L-functions:**
  - New developments in L-functions
  - Constructing fields through advanced techniques
  - Open problems

Course Structure: Part IV - Infinite Extensions and Beyond

- **Lecture 61-70:** Exploring Infinite Extensions
- **Lecture 71-80:** Fields of Higher Cardinality
- **Lecture 81-90:** Ongoing Developments and New Directions

Part IV: Infinite Extensions and Beyond - Details

- **Exploring Infinite Extensions:**
  - Techniques for constructing infinite field extensions
  - Theoretical challenges and implications
  - Examples and applications
- **Fields of Higher Cardinality:**
  - Fields with cardinality greater than the continuum
  - Construction and applications
- **Ongoing Developments:**
  - Incorporating new research
  - Extending theory to new areas
  - Collaboration and discussion on open problems

Course Structure: Part V - Applications and Further Explorations

- **Lecture 91-100+:** Applications in Analysis, Geometry, and Physics

Part V: Applications - Details

- **Applications in Analysis, Geometry, and Physics:**
  - Utilizing new fields in various branches
  - Applications in geometric analysis and physical theories
  - Expanding scope through research

Course Resources

- **Primary Texts:**
  - Automorphic Forms on Adele Groups by Stephen Gelbart
  - Motives by Uwe Jannsen, Steven Kleiman, and Jean-Pierre Serre
  - Introduction to the Theory of L-functions and Eisenstein Series by James Arthur
- **Supplementary Materials:**
  - Detailed Beamer slides for each lecture
  - PDF handouts with proofs, examples, and exercises
  - Additional readings from research papers

Assessment and Evaluation

- **Continuous Assessment:**
  - Assignments reviewed periodically
  - Problem sets to test understanding
- **Final Project:**
  - Develop research projects based on course material

Course Development

This course is designed for continuous development. New lectures and materials will be added as research progresses, allowing for an indefinite expansion of content and exploration into new areas of field construction and application.

Course Conclusion

Students are encouraged to engage actively with the materials, contribute to ongoing research discussions, and apply the newly constructed fields to various mathematical and physical problems.