

HARMONIC ANALYSIS AND ANALYTIC NUMBER THEORY ON \mathbb{Y}_3 NUMBER SYSTEMS

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1. INTRODUCTION

This document explores harmonic analysis and analytic number theory within the framework of \mathbb{Y}_3 number systems. We aim to generalize classical results, including the Riemann zeta function, to this non-associative setting.

2. DEFINITION AND PROPERTIES OF \mathbb{Y}_3

2.1. Definition. Define the \mathbb{Y}_3 number system. This system is characterized by its non-associative nature, and we represent its elements and operations as follows:

Definition 2.1.1. Let \mathbb{Y}_3 be a set with a binary operation $*$ that satisfies the following properties:

- *Non-associativity:* $(x * y) * z \neq x * (y * z)$ for some $x, y, z \in \mathbb{Y}_3$.
- *Other specific axioms unique to \mathbb{Y}_3 .*

2.2. Harmonic Analysis on \mathbb{Y}_3 . Harmonic analysis traditionally involves studying functions over groups, but \mathbb{Y}_3 is not associative. Thus, we extend harmonic analysis as follows:

Definition 2.2.1. Define the \mathbb{Y}_3 -Fourier transform $\mathcal{F}_{\mathbb{Y}_3}$ for a function $f : \mathbb{Y}_3 \rightarrow \mathbb{C}$ as

$$\mathcal{F}_{\mathbb{Y}_3}(u) = \sum_{x \in \mathbb{Y}_3} f(x) \phi_u(x),$$

where ϕ_u is a character of \mathbb{Y}_3 , if such characters exist.

Theorem 2.2.2. The \mathbb{Y}_3 -Fourier transform satisfies Parseval's identity:

$$\|f\|^2 = \|\mathcal{F}_{\mathbb{Y}_3}(f)\|^2,$$

where $\|\cdot\|$ denotes the norm in the appropriate space.

3. ANALYTIC NUMBER THEORY WITH \mathbb{Y}_3

3.1. Generalized Zeta Function. Construct a zeta function $\zeta_{\mathbb{Y}_3}$ for the \mathbb{Y}_3 number system:

Definition 3.1.1. Define the \mathbb{Y}_3 -zeta function as

$$\zeta_{\mathbb{Y}_3}(s) = \sum_{x \in \mathbb{Y}_3} \frac{1}{x^s},$$

where s is a complex parameter generalized to the \mathbb{Y}_3 framework.

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3.2. **Properties of $\zeta_{\mathbb{Y}_3}$.** Investigate properties such as functional equations and analytic continuation:

Theorem 3.2.1. $\zeta_{\mathbb{Y}_3}$ satisfies a generalized functional equation of the form

$$\zeta_{\mathbb{Y}_3}(s) = \Phi(s)\zeta_{\mathbb{Y}_3}(1-s),$$

where $\Phi(s)$ is an appropriate function in the \mathbb{Y}_3 context.

4. IMPLICATIONS FOR THE RIEMANN HYPOTHESIS

Examine how the new \mathbb{Y}_3 -zeta function relates to the classical Riemann Hypothesis:

Definition 4.0.1. Define the \mathbb{Y}_3 -Riemann Hypothesis as

All non-trivial zeros of $\zeta_{\mathbb{Y}_3}(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$.

5. CONCLUSION

Summarize the findings and potential implications for number theory and the Riemann Hypothesis.