Extending $\mathbb{Y}_n(\mathbb{Y}_m(F))$ Systems

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1 Introduction

In this work, we explore the implications and structure of $\mathbb{Y}_n(\mathbb{Y}_m(F))$, where both the indexing system and the field are defined by the Yang number systems $\mathbb{Y}_m(F)$. This creates a nested hierarchy of algebraic structures that generalizes classical fields and vector spaces.

2 Definitions and Basic Properties

Let F be a field. The system $\mathbb{Y}_n(\mathbb{Y}_m(F))$ is defined as a higher-order Yang system where the field is replaced by the structure $\mathbb{Y}_m(F)$. We consider the following properties:

2.1 Algebraic Operations

Define the addition and multiplication operations on $\mathbb{Y}_n(\mathbb{Y}_m(F))$ by extending those on $\mathbb{Y}_m(F)$. These operations adhere to the following axioms:

$$x + y \in \mathbb{Y}_n(\mathbb{Y}_m(F)) \tag{1}$$

$$x \cdot y \in \mathbb{Y}_n(\mathbb{Y}_m(F)) \tag{2}$$

3 Further Extensions

In future work, we aim to develop:

1. Tensor product structures in $\mathbb{Y}_n(\mathbb{Y}_m(F))$.
2. Interaction of $\mathbb{Y}_n$ with $\mathbb{Y}_m$-indexed systems.
3. Extensions to infinite-dimensional systems.