HARMONIC ANALYSIS AND ANALYTIC NUMBER THEORY ON $\mathbb{Y}_3$ NUMBER SYSTEMS

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1. Introduction

This document explores harmonic analysis and analytic number theory within the framework of $\mathbb{Y}_3$ number systems. We aim to generalize classical results, including the Riemann zeta function, to this non-associative setting.

2. Definition and Properties of $\mathbb{Y}_3$

2.1. **Definition.** Define the $\mathbb{Y}_3$ number system. This system is characterized by its non-associative nature, and we represent its elements and operations as follows:

Definition 2.1.1. *Let $\mathbb{Y}_3$ be a set with a binary operation $*$ that satisfies the following properties:*

- *Non-associativity: $(x * y) * z \neq x * (y * z)$ for some $x, y, z \in \mathbb{Y}_3$.*
- *Other specific axioms unique to $\mathbb{Y}_3$.*

2.2. **Harmonic Analysis on $\mathbb{Y}_3$.** Harmonic analysis traditionally involves studying functions over groups, but $\mathbb{Y}_3$ is not associative. Thus, we extend harmonic analysis as follows:

Definition 2.2.1. *Define the $\mathbb{Y}_3$-Fourier transform $\mathcal{F}_{\mathbb{Y}_3}$ for a function $f : \mathbb{Y}_3 \to \mathbb{C}$ as*

$$\mathcal{F}_{\mathbb{Y}_3}(u) = \sum_{x \in \mathbb{Y}_3} f(x)\phi_u(x),$$

where ϕ_u is a character of $\mathbb{Y}_3$, if such characters exist.

Theorem 2.2.2. *The $\mathbb{Y}_3$-Fourier transform satisfies Parseval's identity:*

$$\|f\|^2 = \|\mathcal{F}_{\mathbb{Y}_3}(f)\|^2,$$

where $\|\cdot\|$ denotes the norm in the appropriate space.

3. Analytic Number Theory with $\mathbb{Y}_3$

3.1. **Generalized Zeta Function.** Construct a zeta function $\zeta_{\mathbb{Y}_3}$ for the $\mathbb{Y}_3$ number system:

Definition 3.1.1. *Define the $\mathbb{Y}_3$-zeta function as*

$$\zeta_{\mathbb{Y}_3}(s) = \sum_{x \in \mathbb{Y}_3} \frac{1}{x^s},$$

where s is a complex parameter generalized to the $\mathbb{Y}_3$ framework.

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3.2. **Properties of $\zeta_{\mathbb{Y}_3}$.** Investigate properties such as functional equations and analytic continuation:

Theorem 3.2.1. *$\zeta_{\mathbb{Y}_3}$ satisfies a generalized functional equation of the form*

$$\zeta_{\mathbb{Y}_3}(s) = \Phi(s)\zeta_{\mathbb{Y}_3}(1-s),$$

where $\Phi(s)$ is an appropriate function in the $\mathbb{Y}_3$ context.

4. Implications for the Riemann Hypothesis

Examine how the new $\mathbb{Y}_3$-zeta function relates to the classical Riemann Hypothesis:

Definition 4.0.1. *Define the $\mathbb{Y}_3$-Riemann Hypothesis as*

$$\textit{All non-trivial zeros of } \zeta_{\mathbb{Y}_3}(s) \textit{ lie on the critical line } \Re(s) = \frac{1}{2}.$$

5. Conclusion

Summarize the findings and potential implications for number theory and the Riemann Hypothesis.