Automated Exploration of Large Cardinal Hierarchies through Axiom Generation

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1 Introduction

The aim of this project is to develop an automated system that explores the hierarchy of large cardinals by recursively generating and evaluating new axiom sets. The system draws inspiration from Conway's *Game of Life*, but operates in the domain of set theory, where it systematically constructs increasingly larger cardinal structures based on evolving axiom systems.

This proposal outlines the conceptual framework, computational requirements, and the long-term goals of the project. The system will be capable of running indefinitely, generating novel mathematical structures and large cardinals, potentially leading to new insights into the foundational aspects of set theory.

2 Mathematical Framework

2.1 Large Cardinals

Large cardinals are central objects in set theory and play a crucial role in understanding the structure of the mathematical universe. From inaccessible cardinals to supercompact cardinals and beyond, large cardinals exhibit increasingly powerful properties, often requiring the introduction of new axioms to establish their existence and characteristics.

2.2 Axiom Generation

The system will generate new sets of axioms using the following principles:

- Starting with the standard axioms of ZFC (Zermelo–Fraenkel set theory with the Axiom of Choice), the system will explore modifications and extensions based on known large cardinal axioms.
- New axioms will be generated recursively by introducing new reflection principles, embedding conditions, and compactness properties.

- Each new set of axioms will be tested for consistency using automated theorem provers such as *Coq*, *Lean*, or *Isabelle*.

3 Automated Cardinal Generation

For each valid set of axioms, the system will recursively generate large cardinals, beginning with inaccessible cardinals and progressing through the known large cardinal hierarchy. The system will:

- Automatically explore new properties and relationships between cardinals, based on the axiom sets.
- Store the relationships between cardinals and axioms in a graph-like structure, allowing for visualization of the cardinal hierarchy.
- Integrate machine learning techniques to detect patterns in the generated cardinals, potentially uncovering new mathematical properties.

4 Computational Requirements

The computational demands of this project are substantial, requiring scalable cloud resources or high-performance computing environments. The system will run continuously, generating new axiom sets and exploring the corresponding cardinal hierarchies.

4.1 Cloud Infrastructure

To handle the computational complexity, the system will leverage cloud platforms such as *Amazon Web Services (AWS)*, *Google Cloud Platform (GCP)*, or *Microsoft Azure*. These platforms provide scalable virtual machines and data storage solutions suitable for long-term exploration.

5 Conclusion

This project aims to create an indefinite exploration of the hierarchy of large cardinals through automated axiom generation and formal verification. By leveraging modern computational resources, we aim to push the boundaries of set theory and uncover new mathematical insights.

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