Advanced Quantum Mechanics Infinitensor

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1 Introduction

This document describes the infinitensor framework extended to incorporate advanced quantum mechanics. The framework combines basis functions, coefficients, operators, and categories to model complex quantum systems.

2 Definition

2.1 Basis Functions

We define two basis functions as follows:

$$\text{basis\_function1}(x, y, z) = \sin(x)\cos(y)e^z$$

$$\text{basis\_function2}(x, y, z) = \cos(x)\sin(y)e^{-z}$$

2.2 Quantum Mechanics Function

The quantum mechanics function is defined as:

$$\text{quantum\_mechanics}(x, y, z) = e^{-x^2-y^2-z^2}\sin(xyz)$$

3 Category of Quantum States

We define a category of quantum states with objects, morphisms, identity morphisms, and composition as follows:

- **Objects:** Natural numbers ($\mathbb{N}$)
- **Morphisms:** Functions between objects ($\mathbb{N} \to \mathbb{N}$)
- **Identity morphism:** Identity function on objects ($\text{id}(x) = x$)
- **Composition:** Addition of morphisms ($f \circ g = f + g$)

4 Infinitensor Definition

An infinitensor is defined with the following components:

- **Basis Functions:** $\{\text{basis\_function1}, \text{basis\_function2}\}$
- **Coefficients:** $\{1, 2\}$
- **Operators:** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
- **Categories:** Defined as above
- **Models:** $\{\text{quantum\_mechanics}\}$

5 Example Usage

The infinitensor framework is instantiated for quantum mechanics as follows:

$$\text{example\_quantum\_infinitensor} = \left\{ \begin{array}{ll} \text{Basis Functions} & \{\text{basis\_function1}, \text{basis\_function2}\} \\ \text{Coefficients} & \{1, 2\} \\ \text{Operators} & \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \\ \text{Categories} & \text{as defined above} \\ \text{Models} & \{\text{quantum\_mechanics}\} \end{array} \right\}$$