TOWARD A PROOF FRAMEWORK FOR THE RIEMANN HYPOTHESIS VIA DEFORMED EULER FIELDS

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Abstract. We initiate a formal structure for analyzing the deformation family

$$L_t(s) := \prod_p \left(1 - \frac{1}{p^s}\right)^{-t}, \quad t \in [0,1],$$

and its associated modulus field $\mathcal{F}_t(s) := \log |L_t(s)|^2$. Our aim is to develop a pathway toward proving that $\Re(s) = \frac{1}{2}$ is the universal attractor for modulus valleys as $t \to 1^-$, yielding a novel formulation and approach to the Riemann Hypothesis.

Contents

1. Key Definitions

Definition 1. *For fixed $t \in (0,1]$, define the* ***modulus field****:*

$$\mathcal{F}_t(s) := \log |L_t(s)|^2,$$

and the set of ***modulus minima****:*

$$\mathscr{Z}_t := \left\{ s \in \mathbb{C} : \nabla \mathcal{F}_t(s) = 0 \text{ and } \mathcal{F}_t(s) < \mathcal{F}_t(s') \text{ for all } s' \in \mathcal{N}_\epsilon(s) \right\}.$$

2. Main Theorem Goal

Theorem 1 (Critical Line Attractor Principle)**.** *Let $\mathscr{Z}_t$ be as above. Then:*

$$\lim_{t \to 1^-} \sup_{s \in \mathscr{Z}_t} \left| \Re(s) - \frac{1}{2} \right| = 0.$$

Proof Sketch (To be expanded). The strategy is to study:

(1) The gradient field $\nabla \mathcal{F}_t(s)$ near any fixed s.
(2) Show that flowlines of the deformation (as t increases) move local minima toward $\Re(s) = \frac{1}{2}$.

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(3) Prove that no stable minima can persist away from this line in the $t \to 1^-$ limit.

$\square$

3. Technical Lemmas

Lemma 1 (Gradient Estimate)**.** *There exists $C_t > 0$ such that:*

$$\left|\frac{\partial \mathcal{F}_t}{\partial \sigma}(s)\right| \geq C_t \cdot \left|\Re(s) - \frac{1}{2}\right| + o(1) \quad \text{as } t \to 1^-.$$

Lemma 2 (Curvature Positivity)**.** *For each $s \in \mathscr{Z}_t$, the Hessian matrix of $\mathcal{F}_t(s)$ satisfies:*

$$\mathit{Hess}(\mathcal{F}_t)(s) \succ 0.$$

Conjecture 1 (Zero Flow Stability)**.** *The zero precursor flow governed by $\frac{ds}{dt} = -\nabla \mathcal{F}_t(s)$ has a unique global attractor at $\Re(s) = 1/2$.*

4. Conclusion

Establishing this attractor framework provides an entirely new path toward a rigorous, dynamical-analytic proof of the Riemann Hypothesis.