VOLUME VI: HYPERCATEGORICAL PERSISTENCE AND TRANSFINITE ARITHMETIC STRUCTURES
FROM COLLAPSE LOGIC TO ∞-STACKED EXISTENCE FIELDS

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Abstract. This volume develops a fully transfinite, hypercategorical framework for arithmetic structures derived from collapse-invariant geometry. Expanding on the ontoid–growth–torsor–gerbe structures from Volumes I–V, we define ∞-stacked persistence objects, recursive fixed-point sheaves, and trans-collapsing meta-logical spaces. Central to this theory is the notion of an *Existence Field*: an ontological base geometry defined as the fixed-point of recursive collapse under ∞-sheaf descent. We aim to unify arithmetic, logic, and geometry through formalized survival hierarchies of infinite depth.

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0. Symbol Dictionary for Hypercategorical Structures

This dictionary introduces the core notations and objects that govern ∞-stacked, recursively persistent, collapse-fixed, and meta-existential structures.

Persistence and Collapse.

- $\mathrm{Fil}_n^{\mathrm{ont}}\mathcal{F}$: ontological filtration at level n;
- $\mathrm{Fil}_{g(n)}^{\mathrm{ont}}\mathcal{F}$: growth-function stratified filtration;
- $\mathrm{Fil}_\infty^{\mathrm{ont}}\mathcal{F} := \bigcap_n \mathrm{Fil}_n^{\mathrm{ont}}\mathcal{F}$: stable core of $\mathcal{F}$;
- $\mathscr{C}_\varepsilon(\mathcal{F})$: collapse of non-persistent layers;
- $\mathscr{C}_\varepsilon^\infty(\mathcal{F})$: transfinite limit collapse;
- $\mathcal{E}_{\mathrm{exist}}(\mathcal{F})$: persistent core under ε-collapse;

Categorical Structures.

- $\mathbf{Ont}_{\varepsilon^\infty}$: category of ontoid spaces with ε-filtration;

- $\mathscr{S}_\infty$: ∞-stack over persistent filtered sites;
- $\mathsf{Sh}^{\infty\text{-meta}}(X)$: category of recursively defined ∞-sheaves over X;
- $\mathbb{E}^{[g]}_{\infty\text{-stack}}$: growth-indexed ∞-stack of stratified sheaf layers;
- **RecCollapse** : category of recursively collapsing towers;

Existence Fields and Onto-Meta Objects.

- $\mathbb{F}^{[\infty]}_{\text{exist}}$: the field object at fixed point of collapse—"field of pure persistence";
- $\mathbb{F}^{[\infty]}_{\text{core}}$: the intersection of all persistence fields across $g(n)$;
- $\mathcal{O}^{[\infty]}$: structure sheaf of the meta-existence spectrum;
- $\mathrm{Spec}^{[\infty]}(\mathbb{F}^{[\infty]}_{\text{exist}})$: spectrum of the Existence Field;

Fixed-Point Collapse and Meta-Stratification.

- $\mathcal{F}_{\text{fixed}}$: fixed point of a recursive collapse sequence on $\mathcal{F}$;
- $\mathcal{F}^{[\omega^\omega]}$: stability limit of iterated ε-collapse;
- $\mathrm{Fix}(\mathscr{C}^\infty_\varepsilon)$: functor returning collapse-stable ∞-objects;
- $\mathrm{gr}^{[\infty]}_n(\mathcal{F})$: meta-graded stratification at transfinite level n;

Recursive Universes and Hyperontologies.

- $\mathcal{U}^{[\infty]}$: universe of all ∞-persistent spaces;
- $\mathcal{T}_{\text{rec}}$: recursive tower topology over hyperontological base;
- $\mathscr{X}^{[\varepsilon^\infty]}_\infty$: object representing a trans-collapsing hyperontoid;
- **MetaTopos** : logical topos built from ∞-sheaf towers;

Conventions.

- All collapse operations are implicitly indexed over ε^∞ unless specified;
- Growth indexing extends into hypergrowth ($g(n) = n \uparrow^k n$, etc.);
- Recursive universes and logical fixed-points are interpreted internally via ∞-sheaf recursion;
- Fields are no longer algebraic: they are survival-class fields under layered meta-logic.

1. Transfinite Persistence Towers and Recursive Collapse

1.1. **From Finite Collapse to Transfinite Stratification.** Volumes I–V defined persistence by collapse filtrations indexed by $n \in \mathbb{N}$ or $g(n)$. We now extend this framework transfinely, replacing finite indexing with:

$$\alpha \in \omega,\ \omega^\omega,\ \varepsilon^\infty,\ \text{or } \mathrm{Ord}_{\text{meta}}.$$

Our goal is to define collapse towers of the form:

$$\cdots \subseteq \mathrm{Fil}^{\mathrm{ont}}_{\alpha+1}\mathcal{F} \subseteq \mathrm{Fil}^{\mathrm{ont}}_{\alpha}\mathcal{F} \subseteq \cdots \subseteq \mathrm{Fil}^{\mathrm{ont}}_{0}\mathcal{F},$$

where each $\mathrm{Fil}^{\mathrm{ont}}_{\alpha}$ records survival at ontological depth α.

1.2. **Recursive Collapse Tower.**

Definition 1.1 (Recursive Collapse Tower)**.** *Let $\mathcal{F}$ be an ∞-sheaf. Define:*

$$\mathrm{Fil}^{\mathrm{ont}}_{0}\mathcal{F} := \mathcal{F}, \quad \mathrm{Fil}^{\mathrm{ont}}_{\alpha+1}\mathcal{F} := \mathscr{C}_\varepsilon(\mathrm{Fil}^{\mathrm{ont}}_{\alpha}\mathcal{F}), \quad \mathrm{Fil}^{\mathrm{ont}}_{\lambda}\mathcal{F} := \bigcap_{\beta<\lambda} \mathrm{Fil}^{\mathrm{ont}}_{\beta}\mathcal{F} \text{ (for limit } \lambda).$$

This gives a transfinite tower of decreasing subobjects encoding meta-resilience.

1.3. **Collapse Fixed-Points and Onto-Existence.**

Definition 1.2 (Fixed-Point Object)**.** *We say $\mathcal{F}$ is* collapse-fixed *if:*

$$\mathrm{Fil}^{\mathrm{ont}}_{\alpha}\mathcal{F} = \mathrm{Fil}^{\mathrm{ont}}_{\alpha+1}\mathcal{F} \quad \text{for some } \alpha,$$

or equivalently:

$$\mathscr{C}_\varepsilon(\mathrm{Fil}^{\mathrm{ont}}_{\alpha}\mathcal{F}) = \mathrm{Fil}^{\mathrm{ont}}_{\alpha}\mathcal{F}.$$

Such $\mathcal{F}$ defines the emergence of an *Existence Field*, where no further collapse alters structure.

1.4. **Ontological Depth and Collapse Ordinals.**

Definition 1.3 (Collapse Ordinal)**.** *The* collapse ordinal $\delta(\mathcal{F})$ *is the smallest α such that:*

$$\mathrm{Fil}^{\mathrm{ont}}_{\alpha}\mathcal{F} = 0.$$

Alternatively, if this ordinal is not attained, $\mathcal{F}$ is said to be *infinitely persistent.*

1.5. **Collapse Sequences as Type-Theoretic Universes.**

We may view:

$$\mathrm{Fil}^{\mathrm{ont}}_{\alpha}\mathcal{F} \quad \text{as the } \alpha\text{-truncation of } \mathcal{F} \quad \text{(cf. HoTT } n\text{-types)}$$

Recursive collapse mimics universe stratification in homotopy type theory: each $\mathrm{Fil}^{\mathrm{ont}}_{\alpha}$ defines how deeply the object is proven to exist.

1.6. **Existence Spectrum and Survivability Index.**

Define the *existence spectrum* of $\mathcal{F}$:

$$\mathrm{Spec}^{[\infty]}(\mathcal{F}) := \left\{ \alpha \mid \mathrm{Fil}^{\mathrm{ont}}_{\alpha}\mathcal{F} \neq 0 \right\},$$

which encodes logical survival across all collapse depths.

1.7. **Limit Stabilization and Transfinite Cohomology.** Let:

$$\mathrm{Fil}_{\infty}^{\mathrm{ont}}\mathcal{F} := \bigcap_{\alpha} \mathrm{Fil}_{\alpha}^{\mathrm{ont}}\mathcal{F}$$

Then:

$$H_{\infty}^{i}(X, \mathcal{F}) := H^{i}(X, \mathrm{Fil}_{\infty}^{\mathrm{ont}}\mathcal{F}),$$

is the cohomological invariant of the recursively persistent core of $\mathcal{F}$.

1.8. **Collapse-Commutative Towers and Universality.** Let $\mathcal{F}$ and $\mathcal{G}$ be two towers. A morphism $f : \mathcal{F} \to \mathcal{G}$ is said to be:

$$\text{Collapse-commutative} \iff f(\mathrm{Fil}_{\alpha}^{\mathrm{ont}}\mathcal{F}) \subseteq \mathrm{Fil}_{\alpha}^{\mathrm{ont}}\mathcal{G}, \ \forall \alpha.$$

This induces functorial behavior across the category of transfinite persistence towers **RecCollapse**.

1.9. **Conclusion.** In this section, we have:

- Constructed transfinite collapse towers indexed by $\alpha \in \mathrm{Ord}$;
- Defined fixed-points as markers of meta-existence;
- Linked collapse stratification with type-theoretic universes;
- Introduced cohomology of infinitely persistent cores;
- Prepared for categorical constructions of ∞-stacked recursive objects.

In the next section, we define ∞-stacked ontoids and recursive universes, and formalize existence fields as fixed-points of $\mathscr{C}_{\varepsilon}^{\infty}$ over filtered sheaves.

2. ∞-Stacked Ontoids and Recursive Universes

2.1. **Motivation: Beyond Single Collapse Towers.** Transfinite persistence towers allow us to track collapse and survival through all levels of logical depth. But in higher categorical settings, one must also:
- Track families of towers;
- Stack persistence filtrations over filtrations;
- Organize fixed-points across ontological universes.

This motivates the concept of ∞-stacked ontoids.

2.2. Definition of ∞-Stacked Ontoid.

Definition 2.1 (∞-Stacked Ontoid)**.** *An* ∞-stacked ontoid *is a diagram:*

$$X^{[\infty]} := \left\{ (X_\alpha, \mathrm{Fil}^{\mathrm{ont}}_\beta \mathcal{F}_\alpha) \right\}_{\alpha, \beta < \varepsilon^\infty},$$

where:

- *Each* X_α *is an ontoid with its own filtration tower;*
- *There exist functorial collapse-compatible maps:*

$$f_{\alpha \to \alpha'} : X_\alpha \to X_{\alpha'}, \quad f^*_{\alpha \to \alpha'}(\mathrm{Fil}^{\mathrm{ont}}_\beta \mathcal{F}_{\alpha'}) \subseteq \mathrm{Fil}^{\mathrm{ont}}_\beta \mathcal{F}_\alpha;$$

- *The tower stabilizes at a transfinite collapse fixed-point.*

This creates a universe of nested persistence geometries.

2.3. Recursive Universes and Meta-Filtration.

Definition 2.2 (Recursive Universe)**.** *Let* $\mathcal{U}^{[\infty]}$ *be the class of all* ∞*-stacked ontoids. This universe is filtered by complexity of recursive collapse, i.e.,*

$$\mathcal{U}_n := \left\{ X^{[\infty]} \mid \delta(X_\alpha) \leq n, \ \forall \alpha \right\}.$$

As $n \to \infty$, these form an ascending tower of arithmetic ontologies.

2.4. Meta-Sheaves over Recursive Universes.

Let $\mathsf{Sh}^{\infty\text{-meta}}(\mathcal{U}^{[\infty]})$ be the category of sheaves $\mathscr{F}$ assigning to each X_α:

$$\mathscr{F}(X_\alpha) := \mathrm{Fil}^{\mathrm{ont}}_\bullet \mathcal{F}_\alpha,$$

along with transition maps compatible with both the collapse and ontological morphisms. Such $\mathscr{F}$ form the first layer of a recursive sheaf tower.

2.5. Recursive Collapse Stability.

Definition 2.3 (Collapse-Fixed ∞-Stack)**.** *We say* $X^{[\infty]}$ *is* universally persistent *if:*

$$\forall \alpha, \ \exists \lambda_\alpha < \varepsilon^\infty \ \textit{such that} \ \mathrm{Fil}^{\mathrm{ont}}_{\lambda_\alpha} \mathcal{F}_\alpha = \mathrm{Fil}^{\mathrm{ont}}_{\lambda_\alpha + 1} \mathcal{F}_\alpha.$$

This ensures all ∞-strata stabilize under recursive collapse.

2.6. Hyperontoid Fields and the Existence Field.

We define:

$$\mathbb{F}^{[\infty]}_{\mathrm{exist}} := \bigcap_\alpha \bigcap_\beta \mathrm{Fil}^{\mathrm{ont}}_\beta \mathcal{F}_\alpha \quad \in \quad \mathsf{Sh}^{\infty\text{-meta}}(\mathcal{U}^{[\infty]}),$$

This object represents the ultimate recursive survival core: a meta-fixed-point object stable across all layers of transfinite arithmetic.

2.7. Meta-Morphisms and ∞-Structural Functors. A *meta-morphism* $F : X^{[\infty]} \to Y^{[\infty]}$ consists of:

- Level-wise collapse-preserving functors F_α; - Natural transformations respecting collapse-fixed-point layers; - Commuting diagrams under limit towers.

The category $\mathscr{S}_\infty$ of ∞-stacked ontoids is thus enriched over $\mathsf{Sh}^{\infty\text{-meta}}$.

2.8. Conclusion. This section builds:

- The notion of ∞-stacked ontoids;
- Recursive universes $\mathcal{U}^{[\infty]}$ of stratified persistence spaces;
- The meta-sheaf category encoding towered survival;
- The Existence Field $\mathbb{F}_{\text{exist}}^{[\infty]}$ as the universal collapse-fixed object.

In the next section, we construct sheaves over ∞-sheaves and recursive fixed-point descent systems.

3. Sheaves over ∞-Sheaves: Recursive Fixed-Point Layers

3.1. Second-Order Ontology: Sheaves of Sheaf Towers. To capture meta-persistence and higher collapse behavior, we define sheaves not over spaces, but over towers of sheaves themselves. This gives rise to:

$$\textbf{Sheaves over } \boldsymbol{\infty}\textbf{-sheaves}, \quad \text{i.e.,} \quad \mathscr{F} \in \mathsf{Sh}^{\infty\text{-meta}}(\mathscr{G}),$$

where $\mathscr{G}$ is itself a filtered sheaf over a recursive ontoid universe.

3.2. Definition: Recursive Sheaf Tower.

Definition 3.1 (Recursive Sheaf Tower). *Let $\mathscr{G} = \left\{\mathrm{Fil}_\alpha^{\mathrm{ont}}\mathcal{F}\right\}_{\alpha<\lambda}$ be an ∞-filtered object. A* sheaf over $\mathscr{G}$ *is a diagram:*

$$\mathcal{S} : \mathrm{Fil}_\alpha^{\mathrm{ont}} \mapsto \mathcal{S}(\mathrm{Fil}_\alpha^{\mathrm{ont}}) \in \mathbf{Ab},$$

such that for all $\alpha \leq \beta$, there are morphisms:

$$\mathcal{S}(\mathrm{Fil}_\beta^{\mathrm{ont}}) \to \mathcal{S}(\mathrm{Fil}_\alpha^{\mathrm{ont}}), \quad \textit{satisfying } \mathcal{S}(\mathscr{C}_\varepsilon(\mathrm{Fil}_\beta^{\mathrm{ont}})) \simeq \mathscr{C}_\varepsilon(\mathcal{S}(\mathrm{Fil}_\beta^{\mathrm{ont}})).$$

These structures allow one to track collapse at both the base and sheaf level.

3.3. Meta-Persistence Functor. Define the functor:

$$\mathcal{P}^{[\infty]} : \mathscr{G} \mapsto \varprojlim_\alpha \mathcal{S}(\mathrm{Fil}_\alpha^{\mathrm{ont}}),$$

which outputs the stable core of the sheaf-over-sheaf. This functor is analogous to taking global sections in a classical topos, but indexed by recursive collapse layers.

3.4. Fixed-Point Sheaf Layers.

Definition 3.2 (Collapse Fixed-Point Layer)**.** *Let $\mathscr{S}$ be a sheaf over an ∞-stacked ontoid. The fixed-point layer is:*

$$\mathscr{S}_{\mathit{fixed}} := \left\{ s \in \mathscr{S} \mid \forall \alpha,\ \mathscr{C}_\varepsilon(s|_{\mathrm{Fil}^{\mathrm{ont}}_\alpha}) = s|_{\mathrm{Fil}^{\mathrm{ont}}_\alpha} \right\}.$$

These sections survive all levels of collapse inside a sheaf-of-sheaf structure.

3.5. **Recursive Descent Systems.** Given a diagram of ∞-sheaves $\{\mathscr{G}_i \to \mathscr{G}_j\}$, a recursive descent system is a collection of objects $\mathcal{S}_i$ with:
- Transition morphisms $\mathcal{S}_i \to \mathcal{S}_j$ compatible with both:
- ∞-sheaf base transitions;
- Internal collapse of $\mathcal{S}_j$.

Descent here is not classical glueing, but fixed-point coherence across ontological layers.

3.6. **Meta-Site and Higher Topology.** Define a *meta-site* $(\mathcal{U}^{[\infty]}, \tau^{[\infty]})$ where coverings are families:

$$\{f_i : \mathscr{G}_i \to \mathscr{G}\} \quad \text{such that} \quad \mathrm{Fil}^{\mathrm{ont}}_\alpha \mathscr{G} = \bigcap_i f_i^* \mathrm{Fil}^{\mathrm{ont}}_\alpha \mathscr{G}_i,\ \forall \alpha.$$

A stack on this site encodes descent data over stratified persistence universes.

3.7. **Collapse-Commutative Cohomology.** Let $\mathscr{S} \in \mathsf{Sh}^{\infty\text{-meta}}(\mathscr{G})$. Then meta-cohomology is defined by:

$$H^i_{\mathrm{meta}}(\mathscr{G}, \mathscr{S}) := \varprojlim_\alpha H^i(\mathrm{Fil}^{\mathrm{ont}}_\alpha \mathscr{G},\ \mathscr{S}(\mathrm{Fil}^{\mathrm{ont}}_\alpha)).$$

This measures not just section extension, but survival throughout recursive collapse.

3.8. **Conclusion.** This section defines:

- Sheaves over ∞-sheaves and recursive collapse tracking;
- Fixed-point layers as internal ontological survivors;
- Recursive descent systems over meta-sites;
- Meta-cohomology and recursive topological invariants.

4. Logical Fixed Points and ε-Inaccessible Collapses

4.1. **From Collapse Stability to Logical Self-Containment.** Collapse fixed-points describe structural invariants under survival stratification. We now consider fixed-points at the level of logical systems themselves.

Let $\mathcal{L}$ be a formal logic system (type theory, set theory, etc.) and let $\mathscr{C}_{\varepsilon\mathcal{L}}$ denote the collapse operation under provability and definability.

We aim to study:

$$\textbf{Logically fixed structures:} \quad \mathcal{F} \cong \mathscr{C}_{\varepsilon\mathcal{L}}(\mathcal{F}).$$

4.2. Definition: Logical Collapse Functor.

Definition 4.1 (Logical Collapse Functor). *Given a logic $\mathcal{L}$ and an object $\mathcal{F}$ in $\mathsf{Sh}^{\infty\text{-}meta}$, define:*

$$\mathscr{C}_{\varepsilon\mathcal{L}}(\mathcal{F}) := \textit{image of } \mathcal{F} \textit{ under all provable collapse-reductions in } \mathcal{L}.$$

This generalizes collapse from categorical structure to logical content and proof visibility.

4.3. Fixed-Point Logic and Meta-Stability.

Definition 4.2 (Logical Fixed Point). *We say $\mathcal{F}$ is* logically fixed *under $\mathcal{L}$ if:*

$$\mathcal{F} = \mathscr{C}_{\varepsilon\mathcal{L}}(\mathcal{F}),$$

and recursively:

$$\forall \mathcal{L}' \prec \mathcal{L}, \quad \mathscr{C}_{\varepsilon\mathcal{L}'}(\mathcal{F}) = \mathcal{F}.$$

This implies $\mathcal{F}$ is maximally stable under internal and external provability collapse.

4.4. ε-Inaccessible Stratification.

We define:

Definition 4.3 (ε-Inaccessible Collapse Level). *An ordinal δ is said to be ε-*inaccessible *for $\mathcal{F}$ if:*

$$\mathrm{Fil}^{\mathrm{ont}}_{\delta}\mathcal{F} \neq 0, \quad \textit{but} \quad \mathrm{Fil}^{\mathrm{ont}}_{\delta+1}\mathcal{F} = 0, \quad \textit{and } \delta \textit{ is inaccessible under collapse-definability in } \mathcal{L}.$$

These points represent collapse-resistant anomalies in logical universes.

4.5. Meta-Existence Conditions.

Define:

$$\mathcal{F}_{\text{meta-exist}} := \left\{ s \in \mathcal{F} \mid s \text{ is invariant under } \mathscr{C}^{n}_{\varepsilon\mathcal{L}} \text{ for all } n \right\}.$$

This is the ultimate fixed-point subobject under iterated logical collapse.

4.6. Collapse Hierarchies and Modal Logics.

Let $\mathcal{L}_\beta$ be a family of logics indexed by proof-theoretic strength (e.g., Peano, ZFC, HoTT, etc.). We define the *collapse hierarchy*:

$$\mathscr{C}_{\varepsilon\mathcal{L}_0}(\mathcal{F}) \supseteq \mathscr{C}_{\varepsilon\mathcal{L}_1}(\mathcal{F}) \supseteq \cdots \supseteq \mathscr{C}_{\varepsilon\mathcal{L}_\infty}(\mathcal{F}).$$

We say $\mathcal{F}$ is *collapse-modal stable* if it remains fixed across a family of logics.

4.7. **The Fixed-Point Topos.** Let **FixTopos** be the subcategory of $\mathsf{Sh}^{\infty\text{-meta}}$ consisting of collapse-logically fixed objects.

This category forms a closed structure under:
- pullbacks of logically fixed towers;
- collapse-invariant realization functors;
- internally definable cohomology.

4.8. **Conclusion.** In this section we:

- Extended collapse to logical content and proof-theoretic visibility;
- Defined fixed-points under logical systems $\mathcal{L}$;
- Introduced ε-inaccessible collapse levels;
- Built a collapse-modal topos **FixTopos** of self-surviving logical objects.

In the next section, we define Existence Fields as logical-collapse fixed-points and formulate their meta-cohomological classification.

5. Existence Fields and Meta-Stratified Cohomology

5.1. **Ultimate Collapse Stability and Existence Fields.** Throughout this volume, we have observed that certain objects—those stable under all forms of collapse, recursive filtration, and logical descent—emerge as foundational. We now isolate such objects and classify them as:

$$\textbf{Existence Fields}: \quad \text{Collapse-logically persistent core objects of meta-arithmetic geometry.}$$

5.2. **Definition: Existence Field.**

Definition 5.1 (Existence Field)**.** *An* Existence Field $\mathbb{F}_{\text{exist}}^{[\infty]}$ *is an object in* $\mathsf{Sh}^{\infty\text{-}meta}$ *satisfying:*

(1) $\mathbb{F}_{\text{exist}}^{[\infty]} = \mathscr{C}_{\varepsilon\mathcal{L}}(\mathbb{F}_{\text{exist}}^{[\infty]})$ *for all* $\mathcal{L}$ *in the modal logic hierarchy;*
(2) $\mathbb{F}_{\text{exist}}^{[\infty]} = \mathrm{Fil}_{\alpha}^{\mathrm{ont}}(\mathbb{F}_{\text{exist}}^{[\infty]})$ *for all* α *such that* $\mathrm{Fil}_{\alpha}^{\mathrm{ont}}(\mathbb{F}_{\text{exist}}^{[\infty]}) \neq 0$*;*
(3) $\mathbb{F}_{\text{exist}}^{[\infty]}$ *represents a terminal object in* **FixTopos** *under recursive sheaf morphisms.*

5.3. **Sheaf-Theoretic Field Structure.** The object $\mathbb{F}_{\text{exist}}^{[\infty]}$ supports the following meta-arithmetic structures:

- A collapse-invariant structure sheaf $\mathcal{O}^{[\infty]}$;
- Stable meta-ring operations: addition, multiplication over ∞-filtrations;
- Period morphisms: $\mathbb{F}_{\text{exist}}^{[\infty]} \to B_{\mathrm{dR}}^{[\infty]}$ via meta-realization functors.

5.4. **Meta-Stratified Cohomology.** Define cohomology groups:

$$H^i_{\infty,\mathrm{meta}}(\mathcal{U}^{[\infty]}, \mathbb{F}^{[\infty]}_{\mathrm{exist}}) := \varprojlim_{\alpha} H^i(\mathrm{Fil}^{\mathrm{ont}}_{\alpha}, \mathbb{F}^{[\infty]}_{\mathrm{exist}}),$$

where the transition maps are logical-collapse preserving.

These groups classify stable arithmetic invariants of recursive survival fields.

5.5. **Meta-Torsors and Moduli of Survival.** We define the category of $\mathbb{F}^{[\infty]}_{\mathrm{exist}}$-torsors $\mathscr{T}^{[\infty]}$:

$$\mathscr{T}^{[\infty]}(X) := \left\{ \mathcal{T} \text{ over } X \mid \mathcal{T} \times^{\mathbb{F}^{[\infty]}_{\mathrm{exist}}} \mathbb{F}^{[\infty]}_{\mathrm{exist}} \cong \mathbb{F}^{[\infty]}_{\mathrm{exist}} \right\},$$

These classify deformation classes under logic- and collapse-invariant auto-equivalences.

5.6. **Spectral Realization and Meta-Period Maps.** Let:

$$\mathrm{Spec}^{[\infty]}(\mathbb{F}^{[\infty]}_{\mathrm{exist}}) := \text{Space of meta-survivability types},$$

and define the period morphism:

$$\pi^{[\infty]} : \mathbb{F}^{[\infty]}_{\mathrm{exist}} \longrightarrow B^{[\infty]}_{\mathrm{dR}},$$

as the universal realization of fixed-point arithmetic structure.

5.7. **Collapse-Invariant Universal Coefficients.** For any sheaf $\mathscr{S}$ over $\mathbb{F}^{[\infty]}_{\mathrm{exist}}$, we have:

$$H^i_{\mathrm{meta}}(X, \mathscr{S}) \cong \mathrm{Ext}^i_{\mathbb{F}^{[\infty]}_{\mathrm{exist}}}(\mathcal{O}^{[\infty]}, \mathscr{S}),$$

where $\mathcal{O}^{[\infty]}$ is the canonical meta-structure sheaf.

This yields a universal cohomology theory entirely rooted in persistence.

5.8. **Conclusion.** This section identifies and formalizes:

- Existence Fields as collapse- and logic-fixed objects;
- Their meta-arithmetic and sheaf-theoretic structures;
- Meta-cohomology as a persistent invariant classifier;
- Moduli, torsors, and realization spectra over $\mathbb{F}^{[\infty]}_{\mathrm{exist}}$.

6. Conclusion: Toward a Formal Theory of Ultimate Arithmetic Ontology

6.1. **From Collapse to Being.** Arithmetic has been reinterpreted across six volumes—not as operations over numbers, but as layered survival of structured existence through:

- Recursive collapse;
- Logical visibility;
- Stratified cohomology;
- Ontological resistance.

This culminates in a unifying object: the **Existence Field**, defined not by algebraic closure or valuation, but by fixed-point survival under all structural, categorical, and logical descent.

6.2. **Volume-by-Volume Synthesis.**

- **Volume I**: From additive filtration to multiplicoid–knuthoid geometry;
- **Volume II**: From ε-stratification to transfinite motivic persistence;
- **Volume III**: From linear WMC to hypermonodromy conjectures;
- **Volume IV**: From topological covers to space-theoretic ontology;
- **Volume V**: From fields to growth-indexed categorical arithmetic;
- **Volume VI**: From collapse to existence, from logic to ultimate arithmetic.

Each layer embeds the previous into a broader survival framework.

6.3. **Collapse as Logic, Cohomology as Persistence.** Let us reinterpret classical invariants:

Concept	**Ontological Interpretation**
Field	Collapse-fixed survival object
Valuation	Rate of epistemic loss
Sheaf	Locally persistent structure
Cohomology	Measure of meta-survival
Period	Collapse-invariant projection
Torsor	Persistence deformation orbit
Galois group	Collapse automorphism tower
Logic	Collapse operator indexed by provability

6.4. **Meta-Existence as Formal Foundation.** We conclude with a formal axiom:

> **Axiom (Meta-Existence)**: *Only objects stable under all forms of collapse—categorical, logical, recursive, and epistemic— constitute the formal existence layer of arithmetic ontology.*

This defines a new mathematical universe built from internal proof-resilience.

6.5. Final Diagram: Collapse–Logic–Arithmetic Ontology.

$$
\begin{tikzcd}
\textbf{Collapse Theory} \arrow[rr, "\text{Indexed by } \varepsilon^\infty"] \arrow[dr] & & \textbf{Recursive Sheaf Geometr} \arrow[dl] \\
& \textbf{Ultimate Arithmetic Ontology} &
\end{tikzcd}
$$

Collapse is no longer loss—it is the formal engine of structure.

6.6. Future Frameworks.

- Extend Existence Fields into model theory and dependent type theory;
- Apply ∞-sheaf recursion to quantum geometry;
- Use meta-cohomology to classify generalized motives and logics;
- Formalize a universal topos of survival-indexed mathematical structures.

6.7. Closing Reflection.

> *We have transcended the finite, the algebraic, the spatial, and even the logical.*
> *In their place: persistence. Existence. Collapse-resilient being.*
> *This is no longer just arithmetic—it is a geometry of survival.*
>
> — *End of Volume VI and the Persistence Hexalogy*

References

[1] Peter Scholze, *Perfectoid spaces*, Publ. Math. Inst. Hautes Études Sci. **116** (2012), 245–313.

[2] J. Lurie, *Higher Topos Theory*, Annals of Mathematics Studies, Princeton University Press, 2009.

[3] J. Lurie, *Spectral Algebraic Geometry*, prepublication.

[4] V. Voevodsky, *Motivic cohomology with $\mathbb{Z}/l$-coefficients*, Annals of Mathematics, Vol. 174, No. 1 (2011), pp. 401–438.

[5] Steve Awodey, *Type theory and homotopy*, in Epistemology versus Ontology, Logic, Epistemology, and the Unity of Science, vol 27. Springer, 2012.

[6] M. Shulman, *Brouwer's fixed-point theorem in real-cohesive homotopy type theory*, Mathematical Structures in Computer Science, 28(6):856–941, 2018.

[7] A. Joyal and M. Tierney, *An extension of the Galois theory of Grothendieck*, Mem. Amer. Math. Soc., 51 (1984), no. 309.

[8] B. Toën and G. Vezzosi, *Homotopical Algebraic Geometry I–II*, Adv. Math. 193 (2005), 257–372; Mem. Amer. Math. Soc. 193 (2008), no. 902.

[9] S. Mac Lane and I. Moerdijk, *Sheaves in Geometry and Logic*, Springer-Verlag, 1992.

[10] Pu Justin Scarfy Yang, *Collapse Theory and Hyper-Stratified Geometries*, preprint series (2024–2025).

[11] Pu Justin Scarfy Yang, *Existence Fields and Ultimate Arithmetic Ontology*, forthcoming manuscript.