META-CATEGORIZATION OF DISTINCT P-ADIC PERIOD CONCEPTS IN ALGEBRAIC NUMBER THEORY

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ABSTRACT. From a meta-mathematical and ontological perspective, the notion of p-adic periods in algebraic number theory exhibits a rich taxonomy of conceptually and formally distinct frameworks. This paper outlines a comprehensive classification, categorizing at least seventeen (17) distinct types of p-adic periods, each rooted in a different mathematical paradigm, including Fontaine's theory, motives, p-adic geometry, higher category theory, and topos-theoretic extensions.

CONTENTS

1. INTRODUCTION

The concept of *p-adic periods* extends the classical idea of periods in transcendental number theory into the realm of p-adic Hodge theory, arithmetic geometry, and motive theory. From a meta-perspective, we seek to classify all mathematically distinct notions of p-adic periods based on their formal origins, algebraic structures, and semantic roles.

2. FONTAINE-TYPE PERIOD RINGS

These arise in p-adic Hodge theory and are fundamental to understanding the correspondence between p-adic Galois representations and filtered φ-modules.

- B_{HT}: Hodge–Tate periods.
- B_{dR}: de Rham periods.
- B_{cris}: Crystalline periods.
- B_{st}: Semi-stable periods.
- $B_{\max}, B_{\inf}, B_{\mathrm{rig}}$: Variants for finer distinctions.

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3. Geometric and Modular Origins

- Coleman p-adic integrals as periods.
- Periods from elliptic curves and their formal group laws.
- p-adic multiple zeta values (MZVs).
- Modular symbols with p-adic coefficients.

4. Motivic and Comparison Isomorphism Periods

- Periods arising as comparison isomorphisms between étale and de Rham realizations of motives.
- Relative p-adic periods of families of motives (e.g., over Shimura varieties).

5. Meta-Theoretic and Structural Periods

- **Non-abelian periods**: Defined via non-abelian Galois or Iwasawa cohomology.
- **Topos-theoretic periods**: Interpreted as global sections over a classifying topos.
- **Higher categorical periods**: Via ∞-categories and derived stacks.
- **Computational periods**: Formally approximated through algorithmic or precision-based techniques.
- **Perfectoid-Langlands periods**: Emanating from perfectoid uniformizations.
- **Stack-theoretic periods**: Periods of moduli stacks like $\mathcal{M}_{ell}$, $\mathcal{A}_g$, or Shimura stacks.

6. Summary Table of Categories

Category	**Essence**	**Examples**
Fontaine-type	Galois filtered modules	$B_{\mathrm{cris}}, B_{\mathrm{dR}}$
Geometric-integral	Path integrals/formal group	Coleman, elliptic formal logs
Modular/MZV	Symbolic-modular expansions	MZVs, p-adic L-values
Motivic	Realization comparisons	Motives over number fields
Topos-theoretic	Sheaf-theoretic abstraction	Classifying topos sections
Higher category	Derived algebraic geometry	∞-categorical periods
Computational	Precision-based periods	Explicit numerical approximations
Perfectoid	Tilted geometry structures	Perfectoid Shimura period maps
Stack-theoretic	Cohomology over stacks	$\mathcal{M}_{ell}$, $\mathcal{A}_g$

7. Conclusion and Future Work

The notion of p-adic periods transcends a single definition and enters the realm of meta-structures. Each type is rooted in a distinct algebraic or geometric formalism. Further meta-categorization may involve developing a unified topos-theoretic or categorical semantics that accommodates all existing and potential p-adic period types.