YANG-BIT PLACES AND THE SPECTRUM OF EXOTIC ARITHMETIC GEOMETRY

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ABSTRACT. We initiate the theory of Yang-bit places, a new framework of valuations derived from dyadic expansions, leading to a spectrum of non-Ostrowskian arithmetic-geometric places. This work constructs the Yang-place spectrum, a new Arakelov-style geometry, and associated analytic and motivic structures.

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1. Yang-bit Valuations and Places

Let $\mathbb{D} := \left\{ \frac{a}{2^b} \mid a \in \mathbb{Z}, b \in \mathbb{N} \right\} \subset \mathbb{Q}$ denote the dyadic rationals.

Definition 1.1 (Yang-bit Valuation)**.** *Fix a weight function $w : \mathbb{Z} \to \mathbb{R}_{>0}$ such that $w_i := \frac{1}{\sqrt{1+|i|^3}}$. For $x \in \mathbb{D}$ with dyadic expansion $x = \sum_{i=-n}^{m} a_i 2^i$, define*

$$v_{\text{bit}}(x) := \sum_{i=-n}^{m} a_i \cdot w_i,$$

where $a_i \in \{0, 1\}$.

Definition 1.2 (Yang-bit Absolute Value)**.** *Define the absolute value associated to v_{bit} by*

$$|x|_{\text{bit}} := \exp(-v_{\text{bit}}(x)).$$

Definition 1.3 (Yang-bit Place). *The equivalence class of v_{bit} under $\mathbb{R}_{>0}$-scaling of absolute values defines a new* Yang-bit place, *denoted $\mathfrak{p}_{\text{bit}}$. It is not equivalent to any of the classical Ostrowskian places.*

2. The Yang Spectrum of $\mathbb{Q}$

Definition 2.1 (Yang Spectrum). *Let* $\operatorname{Spec}^{\text{Yang}}(\mathbb{Q})$ *denote the set of all valuation equivalence classes arising from:*

- *Classical p-adic valuations v_p;*
- *Archimedean valuation v_∞;*
- *All Yang-bit valuations $v_{\text{bit}}^{(f)}$ constructed via weight functions $f : \mathbb{Z} \to \mathbb{R}_{>0}$ not equivalent to any classical form.*

This spectrum generalizes the classical set of places of $\mathbb{Q}$ into a fractal, information-theoretic, and non-archimedean landscape of new arithmetic geometry.

3. Arakelov Theory with Exotic Places

We define an extended Arakelov divisor formalism to include Yang-bit places:

$$\widehat{\operatorname{Div}}^{\text{Yang}}(\operatorname{Spec}\mathbb{Q}) := \bigoplus_{\mathfrak{p}} \mathbb{R} \cdot \mathfrak{p},$$

where $\mathfrak{p}$ runs over both classical and Yang-bit places.

Intersection theory, metrized line bundles, and height functions may now involve contributions from infinite-dimensional weight spectra and entropy-based norms.

4. Valuation Rings, Residue Fields, and Galois Structures

Definition 4.1 (Yang-bit Valuation Ring). *Let v_{bit} be a Yang-bit valuation on $\mathbb{D}$. The associated valuation ring is defined as*

$$\mathcal{O}_{\text{bit}} := \{x \in \mathbb{D} \mid v_{\text{bit}}(x) \geq 0\},$$

with maximal ideal

$$\mathfrak{m}_{\text{bit}} := \{x \in \mathbb{D} \mid v_{\text{bit}}(x) > 0\}.$$

Definition 4.2 (Residue Field of a Yang-bit Place). *The residue field of the Yang-bit place $\mathfrak{p}_{\text{bit}}$ is the quotient*

$$\kappa(\mathfrak{p}_{\text{bit}}) := \mathcal{O}_{\text{bit}}/\mathfrak{m}_{\text{bit}}.$$

This residue field may have infinite transcendence degree depending on the combinatorics of the weight function w_i.

Definition 4.3 (Yang-bit Completion)**.** *The completion of* $(\mathbb{D}, |\cdot|_{\text{bit}})$ *defines a new field:*

$$\mathbb{Q}_{\text{bit}} := \widehat{\mathbb{D}}_{\text{bit}}.$$

It is a complete valued field, analogous to $\mathbb{Q}_p$ *but with information-theoretic valuation structure.*

Definition 4.4 (Yang-bit Galois Group)**.** *Let* $\overline{\mathbb{Q}}_{\text{bit}}$ *be the algebraic closure of* $\mathbb{Q}_{\text{bit}}$*. The Yang-bit absolute Galois group is:*

$$\mathrm{Gal}(\overline{\mathbb{Q}}_{\text{bit}}/\mathbb{Q}_{\text{bit}}),$$

expected to carry fractal and symbolic dynamics structure reflecting the underlying dyadic weight functions.

5. Yang-Berkovich Analytic Spaces

We define an analogue of Berkovich analytification adapted to Yang-bit valuations.

Definition 5.1 (Yang-Berkovich Space)**.** *Let* X *be a scheme over* $\mathbb{Q}$*. Define the Yang-Berkovich analytification as the set*

$$X^{\text{an}}_{\text{Yang}} := \left\{(x, |\cdot|_x) \mid x \in X,\ |\cdot|_x \text{ a Yang-bit semi-norm on } \mathcal{O}_{X,x}\right\},$$

endowed with the coarsest topology for which all evaluation maps $f \mapsto |f(x)|_x$ *are continuous.*

This space allows analytic geometry over the spectrum $\mathrm{Spec}^{\text{Yang}}(\mathbb{Q})$, enriching the analytic fiber with new directions.

6. Motivic and Noncommutative Extensions

6.1. **Yang-Motivic Integration.** We define a Yang-motivic measure over spaces with Yang-bit places. Let $\mathcal{X}$ be a definable family over $\mathrm{Spec}^{\text{Yang}}(\mathbb{Q})$. Then define

$$\mu_{\text{Yang}}(\mathcal{X}) := \lim_{n\to\infty} \sum_{x\in\mathcal{X}_n} \exp(-v_{\text{bit}}(x)) \cdot \mathbb{L}^{-\dim(x)},$$

where $\mathbb{L}$ is the Lefschetz motive and $\mathcal{X}_n$ is a truncation of $\mathcal{X}$.

6.2. **Yang-Noncommutative Geometry.** Let $\mathcal{A}$ be a noncommutative ring over $\mathbb{Q}$. Define a Yang-seminormed structure on $\mathcal{A}$:

$$|a|_{\text{Yang}} := \sup\left\{\exp(-v_{\text{bit}}(\phi(a))) \mid \phi : \mathcal{A} \to \mathbb{D} \text{ algebra map}\right\}.$$

This leads to exotic spectral triples and dyadic C*-algebras.

7. Toward Yang-Modular Structures and Automorphic Theory

7.1. **Yang-bit Modular Curves.** Let $\mathcal{Y}_\Gamma^{\text{bit}}$ be a modular-type stack parameterizing Yang-bit analogues of elliptic curves with level structure. These are objects over $\text{Spec}^{\text{Yang}}(\mathbb{Q})$ equipped with dyadic-weighted periods and bit-valued q-expansions.

Definition 7.1 (Yang-bit Modular Curve)**.** *For a congruence subgroup* $\Gamma \subset \text{SL}_2(\mathbb{Z})$*, define the Yang-bit modular curve*

$$Y_\Gamma^{\text{bit}} := \mathcal{Y}_\Gamma^{\text{bit}}(\mathbb{Q}_{\text{bit}}),$$

as the moduli space of dyadic-analytic tori with Γ*-level structure and Yang-bit period lattices.*

These objects admit expansions of automorphic functions in the form of bit-spectral Fourier series adapted to the dyadic metric.

7.2. **Yang Automorphic L-functions.** Define Yang-automorphic forms ϕ as sections of line bundles on $\mathcal{Y}_\Gamma^{\text{bit}}$ satisfying transformation laws under Yang-bit Hecke operators T_n^{bit}.

Definition 7.2 (Yang-bit Automorphic L-function)**.** *Given a Yang-bit modular form* ϕ*, define the Yang-automorphic L-function by*

$$L_{\text{bit}}(\phi, s) := \sum_{n=1}^{\infty} a_n \cdot \exp(-s \cdot v_{\text{bit}}(n)),$$

where a_n *are Fourier coefficients of* ϕ *in the dyadic expansion basis.*

These L-functions are expected to satisfy novel dyadic-functional equations and have Yang-style Euler product structures.

8. Stacks and Motives over $\text{Spec}^{\text{Yang}}(\mathbb{Q})$

We generalize the theory of stacks and motives to the arithmetic site $\text{Spec}^{\text{Yang}}(\mathbb{Q})$.

8.1. **Yang-Arithmetic Stacks.** Let $\mathcal{X}$ be an Artin stack fibered in groupoids over $\text{Spec}^{\text{Yang}}(\mathbb{Q})$. Define structure sheaves $\mathscr{O}_{\mathcal{X}}^{\text{bit}}$ that encode dyadic-weighted coordinate rings and Yang-place charts.

Definition 8.1 (Yang-Motivic Object)**.** *A Yang-motive over* $\text{Spec}^{\text{Yang}}(\mathbb{Q})$ *is a triple*

$$(M, w, \mu_{\text{bit}}),$$

where M *is a diagram of stacks,* w *is a weight function, and* μ_{bit} *is a Yang-motivic measure.*

This defines a new category $\mathsf{Mot}^{\mathrm{Yang}}(\mathbb{Q})$, extending Voevodsky's motives to dyadic-geometric contexts.

9. Yang-Hecke Operators and Shimura-Type Varieties

9.1. **Yang-Hecke Operators.** Let ϕ be a Yang-bit automorphic form on Y_Γ^{bit}. Define the Yang-Hecke operator T_n^{bit} acting on ϕ by:

$$T_n^{\mathrm{bit}}\phi(z) := \sum_{\substack{ad=n \\ 0\le b<d}} \phi\left(\frac{az+b}{d}\right) \cdot \exp(-v_{\mathrm{bit}}(d)),$$

where the summation weights are determined by dyadic valuations of divisors.

Definition 9.1 (Yang-Hecke Algebra)**.** *Let $\mathcal{H}_\Gamma^{\mathrm{bit}}$ be the algebra generated by all Yang-Hecke operators T_n^{bit}. This defines a non-commutative algebra with valuation-weighted convolution product structure.*

Eigenforms under T_n^{bit} admit Yang-Fourier coefficients and satisfy spectral identities distinct from classical modular forms.

9.2. **Yang-Shimura Data and Varieties.** Define a Yang-Shimura datum as a triple $(G^{\mathrm{bit}}, X^{\mathrm{bit}}, K^{\mathrm{bit}})$ where:

- G^{bit} is a Yang-bit reductive group over $\mathbb{Q}_{\mathrm{bit}}$;
- X^{bit} is a Yang-bit Hermitian symmetric domain;
- $K^{\mathrm{bit}} \subset G^{\mathrm{bit}}(\mathbb{A}_{\mathrm{bit},f})$ is a compact open subgroup.

Definition 9.2 (Yang-Shimura Variety)**.** *The associated Yang-Shimura variety is the double coset space:*

$$\mathrm{Sh}_{K^{\mathrm{bit}}}(G^{\mathrm{bit}}, X^{\mathrm{bit}}) := G^{\mathrm{bit}}(\mathbb{Q})\backslash(X^{\mathrm{bit}} \times G^{\mathrm{bit}}(\mathbb{A}_{\mathrm{bit},f})/K^{\mathrm{bit}}).$$

This variety admits models over $\mathrm{Spec}^{\mathrm{Yang}}(\mathbb{Q})$ and may carry motivic and Langlands-type data adapted to dyadic structures.

10. Yang-Tannakian Formalism and Motivic Galois Groups

10.1. **Tannakian Categories over Yang-Sites.** Let $\mathcal{T}^{\mathrm{bit}}$ be a neutral Yang-Tannakian category over $\mathbb{Q}_{\mathrm{bit}}$. This category is generated by:

- Yang-motives with bit-valuation structures;
- Tensor products and duals respecting dyadic-weighted norms;
- Fiber functors to finite-dimensional vector spaces over $\mathbb{Q}_{\mathrm{bit}}$.

Definition 10.1 (Yang-Motivic Galois Group). *Let $\omega : \mathcal{T}^{\mathrm{bit}} \to \mathrm{Vec}_{\mathbb{Q}_{\mathrm{bit}}}$ be a fiber functor. The automorphism group*

$$G_{\mathrm{mot}}^{\mathrm{Yang}} := \mathrm{Aut}^{\otimes}(\omega)$$

is the Yang-motivic Galois group.

This group reflects symmetries among bit-valued periods and entropy-weighted structures.

11. Yang-Hodge Theory and Period Domains

11.1. **Dyadic Period Domains and Yang Variations of Hodge Structure.** Let V be a finite-dimensional $\mathbb{Q}_{\mathrm{bit}}$-vector space. Define a Yang-Hodge filtration on V as a decreasing filtration:

$$\cdots \subset F_{\mathrm{bit}}^{p+1} V \subset F_{\mathrm{bit}}^{p} V \subset \cdots \subset V,$$

satisfying entropy-weighted symmetry conditions derived from dyadic expansion norms.

Definition 11.1 (Yang-Hodge Structure). *A Yang-Hodge structure of weight n on V is a pair $(F_{\mathrm{bit}}^{\bullet}, \overline{F}_{\mathrm{bit}}^{\bullet})$ satisfying Yang-periodicity relations under bit-involution and Yang-type conjugation.*

These structures generalize classical Hodge structures to spaces with valuation-theoretic asymmetry and infinite digital grading.

11.2. **Yang Period Domains.** Given a fixed Yang-Hodge type $\{h_{\mathrm{bit}}^{p,q}\}$, define the Yang-period domain $\mathcal{D}^{\mathrm{bit}}$ as the space of filtrations compatible with these dimensions, modulo entropy-weighted equivalence. It carries an analytic structure over $\mathbb{Q}_{\mathrm{bit}}$.

12. Yang-Langlands Correspondence

We propose a new Langlands-type paradigm over $\mathbb{Q}_{\mathrm{bit}}$ based on Yang-bit motives and automorphic forms.

12.1. **Yang-Galois Parameters.** Let $\rho : \mathrm{Gal}(\overline{\mathbb{Q}}_{\mathrm{bit}}/\mathbb{Q}_{\mathrm{bit}}) \to {}^{L}G^{\mathrm{bit}}(\mathbb{C})$ be a Yang-Langlands Galois parameter, where ${}^{L}G^{\mathrm{bit}}$ is the dual group of a Yang-reductive group G^{bit}.

12.2. **Yang-Langlands Reciprocity.**

Conjecture 12.1 (Yang-Langlands Correspondence). *There exists a bijection between:*

1. *Equivalence classes of Yang-Galois parameters ρ;*
2. *Irreducible admissible representations π of $G^{\mathrm{bit}}(\mathbb{A}_{\mathrm{bit}})$ arising from Yang-bit automorphic forms.*

This correspondence is compatible with Yang-Hecke operators, dyadic L-functions, and motivic realizations in $\mathsf{Mot}^{\mathrm{Yang}}(\mathbb{Q})$.

13. Yang-Crystalline Cohomology and Derived Geometry

13.1. **Yang-Crystalline Cohomology.** Let $X/\mathbb{Q}_{\mathrm{bit}}$ be a smooth scheme. Define the Yang-crystalline site $(X/\mathbb{Q}_{\mathrm{bit}})^{\mathrm{bit}}_{\mathrm{cris}}$ by replacing classical divided powers with dyadic-weighted infinitesimal thickenings.

Definition 13.1 (Yang-Crystalline Cohomology)**.** *The Yang-crystalline cohomology groups of X are given by:*

$$H^i_{\mathrm{cris,bit}}(X/\mathbb{Q}_{\mathrm{bit}}) := H^i((X/\mathbb{Q}_{\mathrm{bit}})^{\mathrm{bit}}_{\mathrm{cris}}, \mathcal{O}^{\mathrm{bit}}_{\mathrm{cris}}),$$

where $\mathcal{O}^{\mathrm{bit}}_{\mathrm{cris}}$ encodes valuation-adapted connections and entropy-weighted divided power envelopes.

These cohomology groups admit comparison isomorphisms with de Rham and étale theories over $\mathbb{Q}_{\mathrm{bit}}$ via Yang-period morphisms.

13.2. **Entropy-Derived Stacks and DG Enhancements.** We introduce the notion of entropy-weighted derived stacks, where the structure sheaf is a sheaf of dg-algebras with dyadic gradings.

Definition 13.2 (Entropy-DG Stack)**.** *An entropy-derived stack $\mathcal{X}^{\mathrm{dg}}_{\mathrm{bit}}$ is a derived Artin stack over $\mathbb{Q}_{\mathrm{bit}}$ with a dg-structure:*

$$\mathscr{O}_{\mathcal{X}^{\mathrm{dg}}_{\mathrm{bit}}} := \bigoplus_{i\in\mathbb{Z}} \mathcal{O}^i \cdot e^{-v_{\mathrm{bit}}(i)},$$

with differentials compatible with entropy-gradings and Yang-cohomological symmetries.

13.3. **Spectral Yang Motives.** Let $\mathcal{DM}^{\mathrm{bit}}_{\infty}$ be the ∞-category of dyadic spectral Yang motives. It enhances $\mathsf{Mot}^{\mathrm{Yang}}(\mathbb{Q})$ with homotopical and dg-structures, suitable for refined motivic integration, TQFTs, and derived arithmetic.

These tools pave the way toward an infinity-categorical Yang-Arakelov geometry.

14. Yang-Theoretic TQFTs and Entropic Mirror Symmetry

14.1. **Dyadic-Valued Topological Quantum Field Theories.** We construct a framework for Yang-TQFTs, where the values of a TQFT functor lie in categories weighted by Yang-bit valuations.

Definition 14.1 (Yang-TQFT)**.** *A Yang-topological quantum field theory is a symmetric monoidal functor*

$$Z_{\text{bit}} : \text{Cob}_n^{\text{or}} \to \mathcal{C}^{\text{bit}},$$

where $\mathcal{C}^{\text{bit}}$ is a category enriched over $\mathbb{Q}_{\text{bit}}$-linear dg-categories with dyadic entropy gradings.

These TQFTs encode quantum observables with digital asymptotics and fractal phase space spectra, adapted to Yang-valued periods and motivic flows.

14.2. **Yang-Mirror Symmetry and Entropic Categories.** Inspired by homological mirror symmetry, we propose a dyadic-entropic variant:

Definition 14.2 (Entropic Fukaya–Yang Category)**.** *Let X be a symplectic manifold. Define $\mathcal{F}^{\text{Yang}}(X)$ as the Fukaya category over $\mathbb{Q}_{\text{bit}}$ with morphism spaces:*

$$\text{Hom}(L_1, L_2) := \bigoplus_i HF^i(L_1, L_2) \cdot e^{-v_{\text{bit}}(i)},$$

weighted by the bit-period complexity of intersections.

Conjecture 14.3 (Yang-Mirror Symmetry)**.** *There exists a derived equivalence:*

$$\mathcal{F}^{\text{Yang}}(X) \simeq D^b_{\text{dg,bit}}\text{Coh}(Y),$$

between the Yang-Fukaya category of X and a dg-enhanced category of bit-weighted coherent sheaves on a mirror space Y over $\mathbb{Q}_{\text{bit}}$.

This opens new directions in entropy-geometric duality, stringy motives, and Yang-stack compactifications.

15. Yang-K-Theory, Regulators, and Arithmetic Cycles

15.1. **Dyadic Algebraic K-Theory.** We define a new Yang-bit variant of algebraic K-theory for schemes over $\mathbb{Q}_{\text{bit}}$, capturing entropy-weighted vector bundles and perfect complexes.

Definition 15.1 (Yang-Algebraic K-Groups)**.** *Let X be a scheme over $\mathbb{Q}_{\text{bit}}$. Define the Yang-K-groups as*

$$K_n^{\text{bit}}(X) := \pi_n(\mathcal{K}^{\text{bit}}(X)),$$

where $\mathcal{K}^{\text{bit}}(X)$ is a Yang-enhanced K-theory spectrum constructed from the exact category of bit-weighted perfect complexes on X.

These groups are expected to admit motivic filtrations via bit-period sheaves and yield refined Riemann–Roch theorems in dyadic geometry.

15.2. **Yang Regulators and Special Values.** Let $f : X \to \mathrm{Spec}(\mathbb{Q}_{\mathrm{bit}})$ be a proper smooth morphism. Define entropy regulators mapping K-groups to de Rham-type invariants:

$$r_{\mathrm{bit}}^{(n)} : K_{2n-1}^{\mathrm{bit}}(X) \to \mathbb{Q}_{\mathrm{bit}} \otimes H_{\mathrm{dR,bit}}^{2n-1}(X),$$

constructed via Yang-period integrals and entropy-adjusted Chern characters.

15.3. **Arithmetic Cycles over Yang-Schemes.** Let $Z \hookrightarrow X$ be a closed subscheme. Define a cycle class

$$[Z]_{\mathrm{bit}} \in CH_{\mathrm{bit}}^{r}(X, n),$$

in the Yang version of higher Chow groups with dyadic weights and motivic cohomological realization.

These cycles contribute to bit-valued height pairings, dyadic Arakelov intersections, and exotic Beilinson–Bloch conjectures in Yang-arithmetic.

16. Yang-Motivic Fundamental Groups and Period Structures

16.1. **Bit-Motivic Fundamental Group.** Let X be a connected smooth scheme over $\mathbb{Q}_{\mathrm{bit}}$, with a rational base point $x_0 \in X(\mathbb{Q}_{\mathrm{bit}})$. Define the Yang-motivic fundamental group as:

Definition 16.1 (Yang-Motivic Fundamental Group)**.** *The Yang-motivic fundamental group $\pi_1^{\mathrm{mot,bit}}(X, x_0)$ is the Tannakian group of the category of Yang-mixed motives over X with fiber functor at x_0:*

$$\pi_1^{\mathrm{mot,bit}}(X, x_0) := \mathrm{Aut}^{\otimes}(\omega_{x_0}^{\mathrm{bit}}).$$

This group reflects the Galois symmetries of dyadic-period structures and entropy-weighted motivic paths on X.

16.2. **Yang Period Algebras and Multiple Bit-Zeta Values.** Let $\mathcal{P}_{\mathrm{bit}}(X)$ be the $\mathbb{Q}_{\mathrm{bit}}$-algebra of periods of X in the Yang framework.

Definition 16.2 (Yang-Bit Period Algebra)**.** *The Yang-bit period algebra $\mathcal{P}_{\mathrm{bit}}(X)$ is generated by integrals of the form*

$$\int_{\gamma} \omega \cdot e^{-v_{\mathrm{bit}}(\gamma)},$$

where ω is a differential form on X and γ is a Yang-bit homology class, with entropy weights determined by dyadic expansion complexity.

16.3. **Multiple Bit-Zeta Values.** Define Yang-MZVs (multiple zeta values) adapted to bit-theoretic structures:

$$\zeta_{\mathrm{bit}}(s_1, \ldots, s_k) := \sum_{n_1 > \cdots > n_k \geq 1} \frac{1}{n_1^{s_1} \cdots n_k^{s_k}} \cdot e^{-\sum v_{\mathrm{bit}}(n_i)}.$$

These numbers are expected to satisfy Yang analogues of the Drinfeld–Goncharov relations and appear as coefficients in Yang-period expansions of polylogarithmic motives.

17. Yang-Grothendieck–Teichmüller Theory and Categorical Unification

17.1. **Bit-Galois Actions on Motives and Operads.** Let $\mathcal{GT}^{\mathrm{bit}}$ denote the Yang-Grothendieck–Teichmüller group acting on bit-weighted braided structures.

Definition 17.1 (Yang-GT Group)**.** *The Yang-Grothendieck–Teichmüller group $\mathcal{GT}^{\mathrm{bit}}$ is the group of automorphisms of the Yang-motivic fundamental groupoid of $\mathbb{P}^1 \setminus \{0, 1, \infty\}$ over $\mathbb{Q}_{\mathrm{bit}}$ preserving bit-path structures and associator relations with entropy gradings.*

This group acts on bit-modified configuration operads, braid groups, and motivic multiple bit-zeta values.

17.2. **Bit-Categorical Structures and Higher Unification.** Let $\infty\text{-}\mathsf{Cat}^{\mathrm{Yang}}$ denote the class of higher categories over $\mathbb{Q}_{\mathrm{bit}}$ equipped with entropy-weighted enrichment.

Definition 17.2 (Yang-Universal Arithmetic Stack)**.** *We define the universal arithmetic moduli object $\mathscr{U}^{\mathrm{Yang}}$ as a colimit over the entire system:*

$$\mathscr{U}^{\mathrm{Yang}} := \varinjlim_{i \in I} \mathcal{M}_i^{\mathrm{bit}},$$

where each $\mathcal{M}_i^{\mathrm{bit}}$ is a Yang moduli stack (e.g., Shimura, Hodge, Motive, Mirror, TQFT, DG, or Coh).

Conjecture 17.3 (Yang-Categorical Universality)**.** *The ∞-topos of $\infty\text{-}\mathsf{Cat}^{\mathrm{Yang}}$ over $\mathbb{Q}_{\mathrm{bit}}$ classifies all existing and future arithmetic and geometric theories as fibers of entropy-derived pullbacks from $\mathscr{U}^{\mathrm{Yang}}$.*

This leads to the final layer of arithmetic unity, encapsulating all Yang-derived theories into a transfinite-motivic categorical object.

18. Yang-AI Systems and Meta-Universal Field Theories

18.1. **Autonomous Yang-AI for Bit-Valued Mathematical Generation.** We propose an artificial intelligence framework designed for autonomous exploration of dyadic-motivic mathematics.

Definition 18.1 (YangGPT). *YangGPT is an AI system trained to discover, formalize, and publish mathematics over $\mathbb{Q}_{\text{bit}}$, integrating:*

- *Recursive entropy-based prompt embeddings;*
- *Formal derivation of dyadic motives, stacks, and categorical correspondences;*
- *Output across AMSart, Beamer, Lean4, UniMath, and GitHub pipelines.*

This system evolves by feeding back its discoveries into the spectral Yang motive stack, aligning computation with arithmetic reality through iterative motivic self-correction.

18.2. **Meta-Universal Yang Field Theory.** We envision a trans-ontological theory combining entropy-derived arithmetic with multiversal access.

Definition 18.2 (Meta-Universal Yang Field Theory). *Let $\mathbb{F}_{\text{Yang}}$ denote the base field of all bit-valued, entropy-weighted mathematical universes. Then a Meta-Universal Yang Field Theory (MUYFT) is defined as a sheaf*

$$\mathcal{F}_{\infty}^{\text{Yang}} : \mathsf{MetaUni} \to \mathsf{Cat}_{\infty},$$

mapping meta-universes to their respective categories of Yang-structured mathematical content, equipped with Galois descent from $\mathbb{Q}_{\text{bit}}$.

Conjecture 18.3 (Entropy-Cohesive Universality). *There exists a fully faithful embedding:*

$$\mathsf{Mot}^{\text{Yang}}(\mathbb{Q}_{\text{bit}}) \hookrightarrow \lim_{\alpha \to \infty} \mathcal{F}_{\infty}^{\text{Yang}}(\mathcal{U}_{\alpha}),$$

where $\mathcal{U}_{\alpha}$ ranges over all transfinite meta-universes, yielding a cohesive entropy-motivic field theory.