THE ENTROPY–PERFECT MOLLIFIER FAMILY: CONVOLUTION IDENTITY, DUALITY WITH AMPLIFIERS, AND YANG–LANGLANDS ENTROPY INTEGRATION

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Abstract. We define the Entropy–Perfect Mollifier Family as a class of convolution kernels that suppress all non-spectral contributions outside a designated entropy-support band, and act as identity operators when composed with their amplifier duals. These mollifiers are classified by entropy-symmetry, convolutional minimality, and exact trace localization. We prove a convolution identity theorem, construct the amplifier–mollifier inverse* diagram, and integrate the mollifier family into the entropy–Langlands kernel stack hierarchy.

Contents

1. Introduction

In analytic number theory, mollifiers have long been employed to smooth L-functions, regularize moments, and isolate zero distributions. The entropy-theoretic perspective now offers a categorified refinement: a mollifier kernel not only damps noise, but also respects spectral entropy stratification.

In this paper, we define the **Entropy–Perfect Mollifier Family**—a class of mollifiers that:

- Perfectly suppress spectral components outside a designated entropy band;

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- Act as identity operators when composed with their amplifier duals;
- Are integrable into Yang-style convolution modules over Langlands period stacks.

We develop convolutional properties, prove identity theorems, and construct the duality diagram showing mollifier–amplifier interactions in the entropy-trace hierarchy.

2. Definition of Entropy–Perfect Mollifiers

Definition 2.1 (Entropy–Perfect Mollifier)**.** Let $\mathcal{H}$ be a spectral Hilbert space with orthonormal basis $\{\phi_\lambda\}$ and entropy weight function $H_Y(\lambda)$. An *entropy–perfect mollifier* is a kernel:

$$M^{(Y)}(x,y) := \sum_{\lambda\in\Lambda} e^{-H_Y(\lambda)}\phi_\lambda(x)\overline{\phi_\lambda(y)},$$

such that:

(M1) $e^{-H_Y(\lambda)} = 0$ for all $\lambda \notin \Lambda_Y$;
(M2) $e^{-H_Y(\lambda)} = 1$ for all $\lambda \in \Lambda_Y$;
(M3) Λ_Y is an entropy-filtered spectral band such that $M^{(Y)} * f = f$ if and only if $\mathrm{Spec}(f) \subset \Lambda_Y$.

Remark 2.2. This is a projection mollifier: it erases all spectral content outside Λ_Y and leaves target-band functions invariant.

3. Convolution Identity with Amplifiers

Theorem 3.1 (Mollifier–Amplifier Identity)**.** *Let $M^{(Y)}$ be an entropy–perfect mollifier supported on Λ_Y, and $A^{(Y)}$ an ultra amplifier supported on the same Λ_Y. Then:*

$$M^{(Y)} * A^{(Y)} = \mathrm{Id}_{\Lambda_Y} = A^{(Y)} * M^{(Y)}.$$

Proof. Both $M^{(Y)}$ and $A^{(Y)}$ are identity operators on $\mathrm{Span}\{\phi_\lambda \mid \lambda \in \Lambda_Y\}$, and zero outside. Their convolution thus equals the identity on that subspace, satisfying mutual inversion. □

Corollary 3.2. *Let f be any spectral function with $\mathrm{Spec}(f) \subset \Lambda_Y$. Then:*

$$M^{(Y)} * A^{(Y)} * f = f.$$

Hence, mollifier–amplifier composition forms a spectral identity chain.

4. Entropy Convolution Diagram and Yang–Langlands Functor

$$\begin{tikzcd}
A^{(Y)}\text{: Ultra Amplifier} \arrow[dr, "*M^{(Y)}"] & & M^{(Y)}\text{: Entropy Mollifier} \arrow[dl, "*A^{(Y)}"'] \\
& \mathrm{Id}_{\Lambda_Y}\text{: Spectral Identity} &
\end{tikzcd}$$

Figure 1. Amplifier–Mollifier Convolution Identity Diagram

5. Entropy Convolution in Yang–Langlands Stack

Theorem 5.1 (Stack Functorial Mollifier). *Let $\mathcal{M}_{BunG}$ be the moduli stack of G-bundles, and $\mathcal{F} \in \mathrm{Sh}(\mathcal{M}Bun_G)$ an automorphic sheaf with spectral support in Λ_Y. Then the entropy–perfect mollifier kernel*

$$\mathcal{M}^{(Y)} := \sum_{\lambda \in \Lambda_Y} \phi_\lambda \boxplus \phi_\lambda^\vee$$

acts via:

$$\mathcal{M}^{(Y)} \star \mathcal{F} = \mathcal{F}, \quad \textit{and} \quad \mathcal{M}^{(Y)} \star \mathcal{F} = 0 \quad \textit{for} \qquad \mathrm{Spec}(\mathcal{F}) \cap \Lambda_Y = \emptyset$$

6. Philosophical and Interdisciplinary Implications

The entropy–perfect mollifier constructs a precise filtration device: it projects functions or sheaves onto a designated spectral–entropic band while erasing all non-compatible entropy components. This mirrors information-theoretic compression and signal fidelity operations in computational neuroscience, AI, and quantum information.

In particular:

In physics, it corresponds to exact projection of quantum states onto energy eigenbands;

In machine learning, it mirrors selective attention or learned representation layers;

In philosophy, it instantiates the idea of pure intentionality—a mechanism that filters "meaning" from "noise."

7. Future Research and Expansion

Define entropy convolution algebras formed by compositions of mollifiers and amplifiers;

Introduce dynamic entropy stacks that evolve under convolution flows;

Formally connect zeta spectral dynamics with entropy flow categories;

Extend the entropy convolution framework to quantum differential stacks and categorified period integrals.

This foundational mollifier–amplifier module opens a full entropy-geometric perspective on the Langlands program and trace spectral field theories.

References

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