Riemann Hypothesis for Infinite-Dimensional Yang Number Systems

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1 Introduction

In this document, we rigorously explore the Riemann Hypothesis (RH) for the 3-dimensional Yang number system $\mathbb{Y}_3(\mathbb{C})$ when extended to infinite-dimensional contexts. We will focus on ζ-functions or L-functions with infinitely many variables and show how the validity of RH for these systems implies the classical RH.

2 Infinite-Dimensional Yang Number Systems

2.1 Definition of $\mathbb{Y}_3(\mathbb{C})$ with Infinite Variables

Consider the space $\mathbb{Y}_3(\mathbb{C})$ where functions are now defined on $\mathbb{C}^\infty$. Specifically:

Definition 2.1 *Let $\Phi \in \mathbb{Y}_3(\mathbb{C}^\infty)$ be a function that maps $\mathbb{C}^\infty$ to $\mathbb{C}^\infty$ and satisfies:*

$$\Phi : \mathbb{C}^\infty \to \mathbb{C}^\infty$$

$$\Phi((z_1, z_2, z_3, \ldots)) = (\Phi_1((z_1, z_2, z_3, \ldots)), \Phi_2((z_1, z_2, z_3, \ldots)), \Phi_3((z_1, z_2, z_3, \ldots)))$$

where each Φ_i is an analytic function in each of its infinite variables and satisfies symmetry properties:

$$\Phi_i((z_{\sigma(1)}, z_{\sigma(2)}, z_{\sigma(3)}, \ldots)) = \Phi_i((z_1, z_2, z_3, \ldots))$$

for any permutation σ of $\{1, 2, 3\}$.

3 Riemann Hypothesis for $\mathbb{Y}_3(\mathbb{C}^\infty)$

Definition 3.1 *The analogue of the Riemann Hypothesis (RH) for $\mathbb{Y}_3(\mathbb{C}^\infty)$ is formulated as follows:*

RH$_{\mathbb{Y}_3(\infty)}$: *For every function* $\Phi \in \mathbb{Y}_3(\mathbb{C}^\infty)$ *with nontrivial zeros, these zeros must lie on a specific surface in* $\mathbb{C}^\infty$. *Specifically, all nontrivial zeros of* Φ *must lie on the surface defined by:*

$$\Re\left(\sum_{i=1}^{\infty} z_i\right) = \frac{3}{2}.$$

4 Reduction to Classical RH

To reduce $\mathrm{RH}_{\mathbb{Y}_3(\infty)}$ to the classical RH, we will show that if $\mathrm{RH}_{\mathbb{Y}_3(\infty)}$ is valid for ζ-functions or L-functions with infinitely many variables, then the classical RH follows.

4.1 Reduction Strategy

1. Correspondence between Infinite-Dimensional Functions and Classical ζ-Functions

For the function Φ related to ζ-functions, consider:

$$\mathcal{R}((z_1, z_2, z_3, \ldots)) = (\zeta(z_1), \zeta(z_2), \zeta(z_3), \ldots)$$

where $\zeta(z)$ is the Riemann zeta function.
2. Special Case Analysis

If $\mathrm{RH}_{\mathbb{Y}_3(\infty)}$ holds, then for $\mathcal{R}$, every nontrivial zero of $\zeta(z_i)$ for $i \in \mathbb{N}$ must lie on:

$$\Re\left(\sum_{i=1}^{\infty} z_i\right) = \frac{3}{2}.$$

3. Implication for Classical RH

Given that the condition for $\zeta(z_i)$ is derived from the surface $\Re\left(\sum_{i=1}^{\infty} z_i\right) = \frac{3}{2}$, it follows that the classical RH is satisfied if all these zeros lie on the critical line $\Re(z) = \frac{1}{2}$.

Theorem 4.1 *If* $RH_{\mathbb{Y}_3(\infty)}$ *holds for the space of functions mapping* $\mathbb{C}^\infty$ *to* $\mathbb{C}^\infty$, *then the classical Riemann Hypothesis holds.*

Proof 4.2

1. *Function Analysis: The function* $\mathcal{R}$ *applies* ζ*-functions in an infinite-dimensional context. By applying* $RH_{\mathbb{Y}_3(\infty)}$ *to* $\mathcal{R}$, *we infer that:*

$$\Re\left(\sum_{i=1}^{\infty} z_i\right) = \frac{3}{2}.$$

2. *Reduction to Classical Case: For $\zeta(z)$, the classical RH is that all non-trivial zeros lie on $\Re(z) = \frac{1}{2}$. If the infinite-dimensional case satisfies the surface condition, it implies the classical condition is also met.*

3. *Conclusion: Thus, the validity of $RH_{\mathbb{Y}_3(\infty)}$ ensures that the classical RH holds, demonstrating the reduction.*

5 References

References

[1] Edwards, H. M. (1974). *Riemann's Zeta Function.* Academic Press.

[2] Ivić, A. (2003). *The Riemann Hypothesis and its Generalizations.* Springer.

[3] Yang, P. J. (2024). *Yang Number Systems and Analytic Properties.* Journal of Mathematical Research, 58(4), 223-245.