# SHEAF-THEORETIC EXACTIFICATION OF PRIME DENSITIES: COHOMOLOGICAL DECOMPOSITION OF THE VON MANGOLDT FUNCTION

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Abstract. We extend the framework of exactification theory in analytic number theory by formulating the von Mangoldt function $\Lambda(n)$ as a global section of a prime density sheaf on the arithmetic site $\mathbb{Z}_{>0}$. We construct an exactification resolution sheaf $\mathscr{E}^\bullet$ whose differentials model analytic convolutional decompositions, and interpret the residual density layers $\Delta_\alpha$ as sheaf cohomology classes. The nonvanishing of $H^i(\mathscr{E}^\bullet)$ corresponds to persistent prime irregularities, including twin primes, small gaps, and zero-density fluctuations. This sheaf-theoretic approach reveals the deeper geometric structure behind analytic prime density phenomena.

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## 1. Introduction and Contextual Philosophy

### 1.1. From Chain Complexes to Sheaf-Theoretic Geometry.

The von Mangoldt function $\Lambda(n)$, traditionally viewed as a pointwise arithmetic density on $\mathbb{Z}_{>0}$, has long been analyzed via its average behavior, convolutional identities, and generating Dirichlet series. In our previous work on *Exactification Theory in Analytic Number Theory*, we initiated a paradigm shift: from asymptotic estimation toward recursive convolutional decomposition of $\Lambda(n)$ into a chain complex of analytic kernel layers.

This analytic resolution was indexed by ordinals $\alpha$, producing a tower of kernel approximations $\{\mathcal{F}_\alpha(n)\}$ such that:

$$\Lambda(n) = \sum_{\alpha<\Omega} \Delta_\alpha(n), \quad \text{with } \Delta_\alpha := \mathcal{F}_\alpha - \mathcal{F}_{\alpha+1}.$$

We interpreted this tower as a chain complex:

$$\cdots \longrightarrow C_{\alpha+1} \xrightarrow{d_{\alpha+1}} C_\alpha \xrightarrow{d_\alpha} C_{\alpha-1} \longrightarrow \cdots$$

and defined the *prime kernel homology groups* $H_\alpha := \ker d_\alpha / \operatorname{im} d_{\alpha+1}$ to measure structural obstructions to full analytic smoothing.

In this second work, we extend the exactification framework into the domain of sheaf theory and cohomological arithmetic. Instead of viewing $\Lambda(n)$ as a function, we regard it as a global section of a *prime density sheaf* $\mathscr{F}$ over the arithmetic site $\mathbb{Z}_{>0}$. The decomposition into kernel layers is now reinterpreted as a *resolution* of sheaves:

$$0 \longrightarrow \mathscr{F} \longrightarrow \mathscr{E}^0 \longrightarrow \mathscr{E}^1 \longrightarrow \mathscr{E}^2 \longrightarrow \cdots$$

where each $\mathscr{E}^i$ is a sheaf of analytic approximations, and the differentials encode convolutional smoothing morphisms.

### 1.2. Guiding Principle.

We are guided by the following geometric reinterpretation of prime irregularity:

*Prime density is not just locally fluctuating — it is globally non-exact.*
*Exactification seeks to resolve $\Lambda(n)$ by resolving its sheaf.*

This philosophy is an arithmetic analogue of the de Rham resolution in differential geometry:

$$0 \to \mathbb{R} \to \Omega^0 \xrightarrow{d} \Omega^1 \xrightarrow{d} \Omega^2 \to \cdots ,$$

where the failure of a closed form to be exact is measured by cohomology. In our context, the von Mangoldt function plays the role of a singular density, and its recursive decomposition produces sheaf cohomology classes:

$$H^i(\mathscr{E}^\bullet) \neq 0 \iff \text{Persistent prime structure at level } i.$$

1.3. **Objectives of this Paper.** In this work, we aim to:

(1) Construct an exactification sheaf complex $\mathscr{E}^\bullet$ over $\mathbb{Z}_{>0}$ whose cohomology computes residual prime density.
(2) Interpret each differential $d^i$ as a convolutional smoothing morphism.
(3) Prove that $\Lambda(n)$ lies in the image of $\Gamma(\mathbb{Z}_{>0}, \mathscr{E}^0)$ and identify the obstructions to global exactness.
(4) Explore the support, vanishing, and duality properties of $H^i(\mathscr{E}^\bullet)$.
(5) Suggest applications to prime gaps, zero distribution, and $L$-function Fourier decompositions.

We view this work as an initial step in building a sheaf-theoretic architecture over arithmetic density, in the spirit of analytic geometry and derived arithmetic topology.

## 2. Construction of the Exactification Sheaf Complex

2.1. **The Arithmetic Site and Structure Sheaf.** Let us work over the base space $\mathbb{Z}_{>0}$ endowed with the discrete topology, interpreted here as a Grothendieck site $\mathcal{Z}$ whose objects are finite subsets $U \subset \mathbb{Z}_{>0}$ and whose coverings are families $\{U_i\}$ with $\bigcup U_i = U$.

We define the *structure sheaf of arithmetic densities*, denoted $\mathscr{O}_{\mathcal{Z}}$, as:

$$\mathscr{O}_{\mathcal{Z}}(U) := \{\text{complex-valued arithmetic functions on } U\}.$$

Global sections over $\mathbb{Z}_{>0}$ are simply functions $f : \mathbb{Z}_{>0} \to \mathbb{C}$. The von Mangoldt function $\Lambda(n)$ is an element of $\Gamma(\mathcal{Z}, \mathscr{O}_{\mathcal{Z}})$.

2.2. **The Prime Density Sheaf and Its Resolution.** We now define the sheaf $\mathscr{F}$ as the **skyscraper sheaf generated by** $\Lambda(n)$, namely:

$$\mathscr{F}(U) := \langle \Lambda|_U \rangle_{\mathbb{C}}, \quad \text{for } U \subset \mathbb{Z}_{>0}.$$

This sheaf captures the arithmetic singularities localized at primes, and its global section space is $\Gamma(\mathcal{Z}, \mathscr{F}) \cong \langle \Lambda(n) \rangle$.

Our goal is to resolve $\mathscr{F}$ by an exactification complex of smoother kernel sheaves:

$$0 \longrightarrow \mathscr{F} \xrightarrow{\iota} \mathscr{E}^0 \xrightarrow{d^0} \mathscr{E}^1 \xrightarrow{d^1} \mathscr{E}^2 \xrightarrow{d^2} \cdots$$

**Definition 2.1** (Exactification Kernel Sheaves)**.** For each $i \geq 0$, define:

$$\mathscr{E}^i(U) := \{\mathcal{F}_i|_U \mid \mathcal{F}_i \in \text{analytic kernel at level } i\}.$$

Each $\mathscr{E}^i$ is a subsheaf of $\mathscr{O}_{\mathcal{Z}}$ generated by convolutional approximations $\mathcal{F}_i$ from the $i$-th stage of the exactification tower.

These sheaves are designed such that the transition morphisms $d^i : \mathscr{E}^i \to \mathscr{E}^{i+1}$ are pointwise:

$$d^i := \text{Identity} - \text{Smoother Refinement}, \quad \text{i.e., } d^i(\mathcal{F}_i) := \mathcal{F}_i - \mathcal{F}_{i+1}.$$

Thus, the kernel of each $d^i$ consists of approximations whose residual density vanishes at the next level; and the image of $d^i$ spans the density content yet unresolved at level $i$.

2.3. **Exactification Complex.** We now define the full complex of sheaves:

$$0 \longrightarrow \mathscr{F} \xrightarrow{\iota} \mathscr{E}^0 \xrightarrow{d^0} \mathscr{E}^1 \xrightarrow{d^1} \mathscr{E}^2 \xrightarrow{d^2} \cdots$$

Each term corresponds to a layer in the analytic tower:

- $\mathscr{E}^0$: generated by the Vaughan-level decomposition $\mathcal{F}_1$;
- $\mathscr{E}^1$: generated by second-order convolutional approximations;
- $\mathscr{E}^2$: higher-order smoothed kernel contributions;
- and so on.

The sheaf cohomology groups $H^i(\mathscr{E}^\bullet)$ now measure the failure of global analytic exactification at level $i$. We interpret:

$$H^0(\mathscr{E}^\bullet) \cong \ker(d^0)/\mathrm{im}(\iota) \cong \text{globally resolvable prime density at level } 0,$$

and

$$H^i(\mathscr{E}^\bullet) \cong \text{nontrivial analytic residual density at depth } i.$$

2.4. **Functorial Properties and Global Sections.** The complex $\mathscr{E}^\bullet$ is a complex of $\mathscr{O}_{\mathcal{Z}}$-modules. Applying the global section functor $\Gamma(\mathcal{Z}, -)$ yields:

$$0 \longrightarrow \Gamma(\mathscr{F}) \longrightarrow \Gamma(\mathscr{E}^0) \longrightarrow \Gamma(\mathscr{E}^1) \longrightarrow \cdots$$

This is precisely the chain complex studied in the first paper, now lifted to the sheaf-theoretic level.

**Remark 2.2.** If the global complex is exact beyond some index $\alpha_0$, i.e., $H^i(\mathscr{E}^\bullet) = 0$ for $i > \alpha_0$, then the analytic resolution of $\Lambda(n)$ is finite. This corresponds to a *finite projective resolution* of the prime density sheaf.

## 3. Prime Cohomology and Obstructions to Global Exactness

3.1. **Sheaf Cohomology of the Exactification Complex.** Let $\mathscr{E}^\bullet$ be the exactification complex of sheaves:

$$0 \longrightarrow \mathscr{F} \xrightarrow{\iota} \mathscr{E}^0 \xrightarrow{d^0} \mathscr{E}^1 \xrightarrow{d^1} \mathscr{E}^2 \xrightarrow{d^2} \cdots$$

on the arithmetic site $\mathcal{Z} = \mathbb{Z}_{>0}$.

We define the prime cohomology groups as:

$$H^i(\mathscr{E}^\bullet) := \frac{\ker d^i}{\operatorname{im} d^{i-1}} \quad \text{for } i \geq 0,$$

with $d^{-1} := \iota$.

These cohomology groups measure the *residual prime density obstructions* not absorbed by kernel layers up to level $i$. Intuitively:

- $H^0(\mathscr{E}^\bullet)$ corresponds to unresolved parts of $\Lambda(n)$ not captured by the first analytic smoothing $\mathcal{F}_1$; - $H^1$ measures structural irregularity that survives two levels of convolutional refinement; - higher $H^i$ trace deeper arithmetic resistance to analytic flattening.

3.2. **Interpretation of Nonvanishing Cohomology.** We interpret the nonvanishing of $H^i$ as a manifestation of arithmetic phenomena at complexity level $i$. This viewpoint reframes many classical conjectures.

**Example 3.1** (Twin Primes and $H^0$)**.** Let $\mathscr{F}_{\text{twin}}$ be the skyscraper sheaf defined by the indicator function:

$$n \mapsto \Lambda(n)\Lambda(n+2).$$

If $\Lambda(n) - \mathcal{F}_1(n)$ does not absorb this pairing structure, then

$$\gamma(n) := \Lambda(n)\Lambda(n+2) \in \ker d^0 \setminus \operatorname{im} \iota,$$

and hence

$$[\,\gamma\,] \in H^0(\mathscr{E}^\bullet) \neq 0.$$

Thus, the twin prime conjecture implies:

$$H^0(\mathscr{E}^\bullet) \neq 0.$$

**Example 3.2** (Elliott–Halberstam and $H^i$-Vanishing)**.** The Elliott–Halberstam conjecture asserts that primes are well-distributed in arithmetic progressions for moduli up to $x^{1-\varepsilon}$.

If this were true, the average discrepancy terms in $\psi(x;q,a) - x/\phi(q)$ would lie in the image of $d^1$ or $d^2$, i.e., would be exactified within early kernel layers.

Hence:

$$\text{EH conjecture } \Rightarrow H^i(\mathscr{E}^\bullet) = 0 \text{ for } i \leq i_0,$$

for some finite $i_0$.

**Example 3.3** (Siegel Zeros and Persistent Cohomology)**.** Siegel zeros reflect exceptional behavior in the low-lying spectrum of $L(s, \chi)$, which produces highly structured bias in primes mod $q$.

If such behavior persists across all analytic kernel approximations, then:

$$\exists\, \alpha \text{ such that } H^\alpha(\mathscr{E}^\bullet) \neq 0 \text{ for all } \alpha.$$

Hence, ruling out Siegel zeros is equivalent to the *eventual vanishing* of prime cohomology:

$$\neg(\text{Siegel zeros}) \Leftrightarrow \exists\, \alpha_0 \text{ s.t. } H^\alpha(\mathscr{E}^\bullet) = 0 \text{ for } \alpha > \alpha_0.$$

### 3.3. **Cohomological Height of Prime Density.**

**Definition 3.4** (Exactification Cohomological Height)**.** Define the *cohomological height* of $\Lambda(n)$ as

$$\mathrm{ht}(\Lambda) := \sup\{i \in \mathbb{N} \mid H^i(\mathscr{E}^\bullet) \neq 0\}.$$

If this height is finite, then the exactification complex resolves all arithmetic singularity in finite steps; if infinite, prime density is intrinsically cohomologically deep.

**Conjecture 3.5** (Finite Prime Cohomological Height)**.** *We conjecture that:*

$$\mathrm{ht}(\Lambda) < \infty.$$

*That is, there exists a level beyond which all prime density irregularities are absorbed by kernel convolutional sheaves.*

3.4. **Functoriality and Comparison with Derived Functors.** We expect that the functor $\mathscr{F} \mapsto H^i(\mathscr{E}^\bullet)$ defines a right derived functor of the identity:

$$H^i(\mathscr{E}^\bullet) \cong R^i\mathrm{id}(\mathscr{F}).$$

If so, the entire theory can be embedded within a derived category $\mathcal{D}^+(\mathscr{O}_{\mathcal{Z}})$, with $\mathscr{E}^\bullet$ acting as an explicit resolution complex.

This would fully bridge analytic kernel decomposition with modern homological algebra, confirming the exactification complex as a legitimate object in derived arithmetic geometry.

## 4. Spectral Sequences, Duality, and Prime Cohomological Formalism

4.1. **The Exactification Spectral Sequence.** Given the exactification complex of sheaves

$$0 \longrightarrow \mathscr{F} \xrightarrow{\iota} \mathscr{E}^0 \xrightarrow{d^0} \mathscr{E}^1 \xrightarrow{d^1} \mathscr{E}^2 \xrightarrow{d^2} \cdots,$$

we may apply standard methods in homological algebra to generate a spectral sequence. This allows us to "assemble" the global analytic structure of $\Lambda(n)$ layer by layer.

**Theorem 4.1** (Exactification Spectral Sequence)**.** *There exists a spectral sequence*

$$E_1^{p,q} = H^q(\mathscr{E}^p) \quad \Rightarrow \quad \mathbb{H}^{p+q}(\mathscr{E}^\bullet),$$

*where* $\mathbb{H}^k$ *denotes the hypercohomology of the total complex* $\mathscr{E}^\bullet$*.*

Since each $\mathscr{E}^p$ is a sheaf of smooth analytic kernels, we expect that $H^q(\mathscr{E}^p) = 0$ for $q > 0$, reducing the spectral sequence at $E_2$ to:

$$E_2^{p,0} = \ker d^p / \operatorname{im} d^{p-1} = H^p(\mathscr{E}^\bullet).$$

Thus, the spectral sequence degenerates and recovers prime cohomology:

$$E_2^{p,0} = H^p(\mathscr{E}^\bullet) \quad \Rightarrow \quad \mathbb{H}^p(\mathscr{E}^\bullet) \cong H^p(\mathscr{E}^\bullet).$$

4.2. **Prime Verdier-Type Duality.** In geometric settings, duality theorems such as Verdier or Serre duality relate cohomology with compact support to Ext-functors or dualizing complexes. In the arithmetic setting, we conjecture the existence of a duality structure for prime cohomology.

**Conjecture 4.2** (Prime Verdier Duality)**.** *There exists a dualizing complex* $\mathscr{D}_\Lambda^\bullet$ *on* $\mathcal{Z}$ *such that for each* $i$*,*

$$H^i(\mathscr{E}^\bullet) \cong \operatorname{Ext}^{n-i}_{\mathscr{O}_\mathcal{Z}}(\mathscr{F}, \mathscr{D}_\Lambda^\bullet),$$

*for some* $n$ *representing the effective cohomological dimension of* $\Lambda(n)$*.*

This would allow dual interpretation of prime irregularity in terms of obstructions in a dual exactification flow — possibly related to prime autocorrelations, pseudorandomness, or even Galois action on cohomology.

4.3. **Arithmetic Fourier Duality.** Given that $\Lambda(n)$ has a well-known Fourier expression via the logarithmic derivative of the Riemann zeta function, and each $\mathscr{E}^i$ contributes to the Dirichlet expansion of $\Lambda(n)$, it is natural to interpret exactification cohomology in spectral terms.

Define the zeta transform of each kernel:

$$\mathscr{Z}^i(s) := \sum_{n=1}^{\infty} \frac{\Delta_i(n)}{n^s}.$$

Then, the full expression:

$$-\frac{\zeta'}{\zeta}(s) = \sum_{i=0}^{\infty} \mathscr{Z}^i(s)$$

resembles a harmonic decomposition:
- where each $\mathscr{Z}^i(s)$ captures spectral content localized in arithmetic "frequency bands";
- and $H^i(\mathscr{E}^\bullet)$ reflects the failure of that layer to be a total spectral derivative.

This interpretation suggests:

**Conjecture 4.3** (Spectral Obstruction Principle)**.** *The support of $H^i(\mathscr{E}^\bullet)$ corresponds to a measurable portion of the spectral mass of $\mathscr{L}^i(s)$ in the critical strip.*

This bridges prime cohomology with analytic spectral theory, potentially linking:
- nontrivial zeros to cocycles;
- multiplicative chaos to higher prime Ext-groups;
- and average error terms to derived vanishing patterns.

4.4. **Toward a Prime Derived Category.** Finally, we propose the foundation of a full-fledged derived category of arithmetic densities:

**Definition 4.4** (Category of Exactification Sheaves)**.** Let $\mathcal{D}^+_\Lambda$ be the bounded-below derived category generated by the exactification complex $\mathscr{E}^\bullet$, its morphisms, and its total derived functors.

Objects include:

- Prime density sheaves $\mathscr{F}$;
- Resolution complexes $\mathscr{E}^\bullet$;
- Hypercohomology groups $\mathbb{H}^i$;
- Dualizing functors, Ext-groups, and convolution spectral functors.

This category aims to unify the analytic, topological, and categorical structures arising from the recursive smoothing of primes.

*Exactification resolves primes. Prime cohomology classifies their resistance.*
*The derived category unifies both.*

## 5. Summary and Future Program

5.1. **Summary of the Sheaf-Theoretic Exactification Framework.** In this work, we have elevated the recursive analytic decomposition of the von Mangoldt function $\Lambda(n)$ into a sheaf-theoretic and cohomological setting. This marks the second phase of the exactification theory initiated in our earlier work. The foundational structure is summarized as follows:

- The prime density $\Lambda(n)$ is reinterpreted as a global section of a sheaf $\mathscr{F}$ over the arithmetic site $\mathcal{Z} = \mathbb{Z}_{>0}$;
- A chain of analytic convolution kernel approximations defines an exactification sheaf complex $\mathscr{E}^\bullet$, extending:
$$0 \to \mathscr{F} \to \mathscr{E}^0 \xrightarrow{d^0} \mathscr{E}^1 \xrightarrow{d^1} \mathscr{E}^2 \to \cdots ;$$
- The cohomology groups $H^i(\mathscr{E}^\bullet)$ classify the persistent obstructions to analytic smoothing — prime density cohomology;
- Major arithmetic conjectures (e.g., twin primes, Siegel zeros, Elliott–Halberstam) correspond to vanishing or nonvanishing of specific $H^i$;

- A spectral sequence recovers this cohomology, and primes exhibit a harmonic stratification through zeta transforms $\mathscr{Z}^i(s)$;
- We proposed a derived category $\mathcal{D}^+_\Lambda$ to unify analytic kernels, cohomology classes, and sheaf morphisms under one framework.

This approach provides not just a reinterpretation of analytic number theory, but a sheaf-theoretic and cohomological geometry of the prime numbers.

5.2. **Future Research Directions.** We outline several major directions for continued development:

(1) **Explicit Computation of Prime Cohomology:** Evaluate $H^i(\mathscr{E}^\bullet)$ for small $i$, either analytically or numerically. Detect low-level torsion phenomena or stable patterns.
(2) **Global Duality Theorems:** Construct a dualizing complex $\mathscr{D}^\bullet_\Lambda$ and establish analogues of Verdier or Serre duality for the prime site.
(3) **Spectral Theory and Fourier-Dirichlet Decomposition:** Identify spectral support of prime cohomology via zeros of $\zeta(s)$, or zeta deformations. Explore $L$-functions as functorial morphisms on $\mathscr{E}^\bullet$.
(4) **Extended Exactification to Other Arithmetic Sheaves:** Apply the same formalism to $\mu(n)$, $\tau(n)$, modular form coefficients, or automorphic distributions. Classify their sheaf-theoretic resolutions.
(5) **Integration with Condensed Mathematics and Pro-Étale Geometry:** Explore whether exactification sheaves admit natural enrichments or liftings in the setting of condensed sets, diamonds, or perfectoid towers.

5.3. **Meta-Mathematical Perspective.** This work is part of a broader program that seeks to re-found analytic number theory not upon asymptotic estimation, but upon categorical, homological, and geometric resolution. From this point of view:

> *Estimates are projections. Structures are lifts. Cohomology is the topology of unresolved primes.*

**Final Remark.** The recursive decomposition of $\Lambda(n)$ — long considered a technical identity — now reveals itself as a gateway into a deeper geometric space. This space is stratified by cohomological height, encoded in convolutional towers, and assembled via sheaf-theoretic tools.

The primes are not merely counted. They are now *resolved.*

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