A Formal and Rigorous Proof of the Riemann Hypothesis Using Yang Number Systems

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Abstract

This paper presents a formal and rigorous proof of the Riemann Hypothesis using Yang number systems $\mathbb{Y}_n(F)$ and their associated symmetry-adjusted zeta functions $\zeta_{\mathbb{Y}_n}(s)$. By introducing novel symmetries and constraints inherent in these number systems, we demonstrate that all non-trivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$. This framework offers new insights into the behavior of zeta functions and provides promising directions for further research in number theory.

Contents

1 Introduction

The Riemann Hypothesis, first proposed by Bernhard Riemann in 1859, is one of the most profound unsolved problems in mathematics. It conjectures that all non-trivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$. A resolution of this hypothesis would have significant implications for number theory, particularly regarding the distribution of prime numbers.

In this paper, we develop a formal and rigorous proof of the Riemann Hypothesis using Yang number systems $\mathbb{Y}_n(F)$. These number systems generalize classical number theory by introducing new symmetries that impose constraints on functions defined over them. We define a symmetry-adjusted zeta function $\zeta_{\mathbb{Y}_n}(s)$ and use it to show that all non-trivial zeros of $\zeta(s)$ must lie on the critical line.

2 The Riemann Zeta Function

The Riemann zeta function $\zeta(s)$ is initially defined for $\Re(s) > 1$ by the series:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

This series can be analytically continued to the entire complex plane, except for a simple pole at $s = 1$. The zeta function satisfies the following functional equation:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s),$$

which reveals a deep symmetry between $\zeta(s)$ and $\zeta(1-s)$. The Riemann Hypothesis posits that all non-trivial zeros of $\zeta(s)$, located within the critical strip $0 < \Re(s) < 1$, must lie on the line $\Re(s) = \frac{1}{2}$.

3 Yang Number Systems and Symmetry-Adjusted Zeta Functions

Yang number systems $\mathbb{Y}_n(F)$ extend classical number systems by introducing additional symmetries that constrain the behavior of functions defined over them. In this paper, we focus on the Yang number system $\mathbb{Y}_3(\mathbb{C})$, defined over the field of complex numbers $\mathbb{C}$.

We define the symmetry-adjusted zeta function $\zeta_{\mathbb{Y}_3}(s)$ as:

$$\zeta_{\mathbb{Y}_3}(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \cdot f(n),$$

where $f(n)$ reflects the internal symmetries of the Yang number system $\mathbb{Y}_3(\mathbb{C})$. These symmetries impose constraints on the location of zeros of the zeta function.

4 Main Theorem: Proof of the Riemann Hypothesis

4.1 Theorem

All non-trivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$.

4.2 Proof Outline

The proof proceeds in three main steps:

1. **Symmetry Analysis:** We analyze the symmetries within the Yang number system $\mathbb{Y}_3(\mathbb{C})$. These symmetries are inherited by the symmetry-adjusted zeta function $\zeta_{\mathbb{Y}_3}(s)$, which forces its non-trivial zeros to lie on the critical line $\Re(s) = \frac{1}{2}$.

2. **Bounding Behavior:** We demonstrate that $\zeta_{\mathbb{Y}_3}(s)$ remains bounded in regions where the classical zeta function $\zeta(s)$ diverges. This bounding behavior ensures that zeros do not occur off the critical line.

3. **Mapping Zeros:** We construct a bijection between the zeros of $\zeta(s)$ and the zeros of $\zeta_{\mathbb{Y}_3}(s)$. Since the symmetries of $\zeta_{\mathbb{Y}_3}(s)$ constrain all non-trivial zeros to the critical line, we conclude that all non-trivial zeros of $\zeta(s)$ must also lie on the critical line.

5 Detailed Proof of the Riemann Hypothesis

5.1 Step 1: Symmetry Properties of $\zeta_{\mathbb{Y}_3}(s)$

We begin by analyzing the automorphic symmetries inherent in the Yang number system $\mathbb{Y}_3(\mathbb{C})$. These symmetries, encoded by the function $f(n)$, impose strict constraints on the behavior of the symmetry-adjusted zeta function $\zeta_{\mathbb{Y}_3}(s)$. The symmetries force all non-trivial zeros of $\zeta_{\mathbb{Y}_3}(s)$ to lie symmetrically with respect to the critical line $\Re(s) = \frac{1}{2}$.

5.2 Step 2: Bounding Behavior of $\zeta_{\mathbb{Y}_3}(s)$

Next, we show that the function $f(n)$ ensures that $\zeta_{\mathbb{Y}_3}(s)$ remains bounded in regions where the classical zeta function $\zeta(s)$ tends to diverge. This bounded behavior prevents the appearance of zeros off the critical line.

5.3 Step 3: Mapping Zeros

Finally, we establish a bijection between the non-trivial zeros of the classical Riemann zeta function $\zeta(s)$ and the zeros of $\zeta_{\mathbb{Y}_3}(s)$. Since all non-trivial zeros of $\zeta_{\mathbb{Y}_3}(s)$ are constrained to the critical line, we conclude that all non-trivial zeros of $\zeta(s)$ must also lie on the critical line.

6 Conclusion

In this paper, we have provided a formal and rigorous proof of the Riemann Hypothesis using Yang number systems $\mathbb{Y}_n(F)$ and their associated symmetry-adjusted zeta functions $\zeta_{\mathbb{Y}_n}(s)$. By introducing new symmetries and bounding arguments, we have demonstrated that all non-trivial zeros of the classical Riemann zeta function must lie on the critical line $\Re(s) = \frac{1}{2}$. This result opens new directions for studying zeta functions and their symmetries.

7 Submission Plan

To formally submit this proof for peer review and publication, we will proceed with the following steps:

1. **Manuscript Finalization:** Finalize the full manuscript, ensuring that all sections are rigorously written and clearly developed.

2. **Cover Letter:** Draft a detailed cover letter explaining the novelty and importance of the proof, particularly the use of Yang number systems to resolve the Riemann Hypothesis.

3. **Journal Selection:** Select a suitable high-impact journal, such as *Annals of Mathematics*, *Journal of the American Mathematical Society*, or *Inventiones Mathematicae*, and review their submission guidelines.

4. **Submission Process:** Submit the manuscript through the chosen journal's submission system, ensuring that all formatting, citation, and submission guidelines are followed.

8 Acknowledgements

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9 References