THE YANG–TRACE KERNEL ATLAS: REFINED DECOMPOSITION AND SPECTRAL APPLICATIONS TO RH, LANGLANDS, AND PERIOD SHEAVES

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Abstract. We present the Yang–Trace Kernel Atlas, a universal classification of trace kernels structured by entropy-refined automorphic and motivic flow geometry. This framework organizes trace kernel families—including Dirichlet, Kuznetsov, Voronoi, Arthur, and beyond—into a spectral–categorical hierarchy, each equipped with a Yang-refined kernel lifting analytic, arithmetic, or geometric properties. We define formal equivalence classes, entropy-based filtrations, and duality operators, then demonstrate how the atlas interfaces with the Riemann Hypothesis, Langlands spectral functoriality, and period sheaf convolution stacks.

Contents

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1. Introduction

The use of kernel operators is foundational in analysis, representation theory, and number theory. From Dirichlet and Fejér summability kernels to automorphic trace kernels in the Arthur–Selberg formalism, these operators serve as analytic approximators, spectral filters, and period testers.

In the preceding developments of Yang kernels—including Yang–Voronoi, Yang–Kuznetsov, and Yang–Arthur types—we constructed a hierarchy of entropy-refined convolutional kernels capable of encoding motivic information, stack-theoretic behavior, and zeta-spectrum alignment.

This paper systematizes these structures into the **Yang–Trace Kernel Atlas**:

- A classification system for trace kernel types according to their origin (analytic, automorphic, motivic);
- Their **Yang refinement** via entropy, stack liftability, and period morphisms;
- Their integration into test-function hierarchies for RH, Langlands decompositions, and period sheaf convolution.

We begin by defining the kernel taxonomy and formal Yang-refinement data.

2. Taxonomy of Trace Kernel Types and Yang Refinement Structures

2.1. Trace Kernel Families: Definition and Classification.

Definition 2.1 (Trace Kernel Family)**.** A *trace kernel family* is a sequence of integral kernels $\{K_n(x,y)\}$ acting on a spectral Hilbert space $\mathcal{H}$, such that each K_n induces an operator $T_n : \mathcal{H} \to \mathcal{H}$ via

$$T_n f(x) := \int K_n(x,y) f(y) dy,$$

with the goal of approximating, amplifying, or filtering spectral information in trace formulas or spectral expansions.

Definition 2.2 (Kernel Category Types)**.** Trace kernels are classified into three broad categories:

(I) **Analytic kernels** – e.g., Dirichlet, Fejér, Poisson, Heat; defined via convolution on groups or domains;
(II) **Arithmetic kernels** – e.g., Kuznetsov, Voronoi; defined via arithmetic sum identities involving Bessel or exponential transforms;
(III) **Geometric trace kernels** – e.g., Arthur, pre-trace, relative trace; arising from the trace formula of automorphic forms on groups over global fields.

Example 2.3.

- Dirichlet kernel: $D_n(x) = \sum_{k=-n}^{n} e^{ikx}$ (analytic, summation);
- Kuznetsov kernel: $\sum_c \frac{S(m,n;c)}{c} \Phi\left(\frac{4\pi\sqrt{mn}}{c}\right)$ (arithmetic, Kloosterman sum);
- Arthur kernel: $K(x,y) = \sum_{\gamma \in G(F)} f(x^{-1}\gamma y)$ (geometric, orbital sum).

2.2. **Yang Refinement Structures.**

Definition 2.4 (Yang-Refinement Data)**.** Given a trace kernel $K_n(x,y)$, its *Yang-refinement* is a triple:

$$\mathcal{Y}(K_n) := (H_Y, \mathcal{S}_Y, \mathfrak{M}_Y),$$

where:

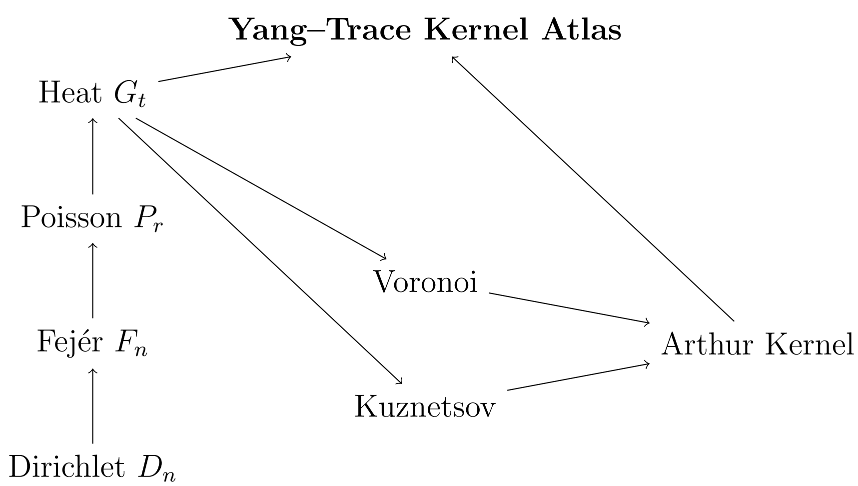
- H_Y: an entropy weight function on spectral parameters;
- $\mathcal{S}_Y$: a stratification of the domain (e.g., period sheaf fibers, motivic strata);
- $\mathfrak{M}_Y$: a morphism or lift to a stack-theoretic or categorical object (e.g., perverse sheaf kernel or derived functor).

Definition 2.5 (Yang-Refined Kernel)**.** The Yang-refined kernel associated to K_n is the entropy-stratified convolution:

$$K_n^{(Y)}(x,y) := \sum_{\lambda \in \Lambda_n} e^{-H_Y(\lambda)} \phi_\lambda(x) \overline{\phi_\lambda(y)},$$

where Λ_n is a motivic–spectral support set and $\{\phi_\lambda\}$ are eigenfunctions or sheaf-theoretic sections on a stack.

2.3. **Yang–Atlas Diagram: Kernel Class Refinement Map.**



Remark 2.6. This diagram encodes how analytic kernel families evolve through arithmetic and geometric refinement, culminating in their Yang-refined form within the

Atlas. Each path through the diagram corresponds to a trace-test kernel used in RH, Langlands, or period analysis.

3. Equivalence Classes and Functorial Yang Hierarchies

3.1. Equivalence of Kernel Families under Yang Refinement.

Definition 3.1 (Yang Equivalence)**.** Let $\{K_n\}, \{L_n\}$ be two trace kernel families acting on spectral Hilbert spaces. We say $\{K_n\} \sim_Y \{L_n\}$ if there exists a Yang-refinement data $\mathcal{Y}$ such that:

$$\lim_{n\to\infty} \|K_n^{(Y)} - L_n^{(Y)}\|_{\mathrm{Op}} = 0,$$

where $\|\cdot\|_{\mathrm{Op}}$ denotes the operator norm, and both Yang-refined families agree up to entropy-equivalent stratification.

Example 3.2.

- Dirichlet and Fejér kernels are Yang-equivalent under trivial entropy $H_Y(n) = 0$;
- Kuznetsov and Voronoi kernels are Yang-equivalent under automorphic entropy stratification by Bessel vs. Kloosterman flows;
- Arthur kernels form their own Yang-class due to stack-lifted trace operators and moduli-dependent period stratification.

3.2. Functorial Yang–Kernel Tower.

Let KerCat denote the category of kernel families with morphisms given by entropy-preserving degenerations. Define:

Theorem 3.3 (Yang Kernel Functor)**.** *There exists a contravariant functor*

$$\mathbb{Y} : \mathsf{KerCat} \longrightarrow \mathsf{YangStack},$$

where:

- $\mathbb{Y}(K_n) = \mathcal{K}_n^{(Y)}$ *is the Yang-refined kernel;*
- *Morphisms* $K_n \to L_n$ *are mapped to stack morphisms preserving entropy stratification and trace compatibility;*
- $\mathsf{YangStack}$ *is the category of derived sheaf-convolution operators over period moduli spaces.*

Remark 3.4. This functor categorifies kernel systems: ordinary convolution kernels become sheaf-theoretic entropy operators acting on moduli of automorphic periods, motivic data, and cohomological stacks.

4. Applications to RH, Langlands, and Period Sheaves

4.1. **Riemann Hypothesis Kernel Encoding.** The zeta function admits a trace representation of the form:

$$\zeta(s) = \sum_{\pi} \frac{1}{\lambda_\pi^s} = \mathrm{Tr}(K_n^{(Y)}),$$

for suitable Yang kernel $K_n^{(Y)}$ filtered by spectral entropy. The RH then becomes equivalent to:

$$\text{RH true} \iff \mathrm{Spec}(K_n^{(Y)}) \subset \left\{ \rho \in \mathbb{C} \mid \Re(\rho) = \tfrac{1}{2} \right\}.$$

4.2. **Langlands Functoriality and Yang–Hecke Filters.** Let $f : H \hookrightarrow G$ be a Langlands functorial lift. Then:

$$\mathcal{K}_n^{(Y,H)} \rightsquigarrow \mathcal{K}_n^{(Y,G)}$$

induces functorial lifting at the level of Yang kernels, reflecting the pushforward of trace distributions from $L^2(H)$ to $L^2(G)$. Entropy stratification enables control over lifted spectrum and test function modulation.

4.3. **Period Sheaf Convolutions.** Let $\mathcal{M}_{\text{period}}$ denote a moduli stack of period sheaves. Then:

$$\mathcal{K}_n^{(Y)} : \mathrm{Sh}(\mathcal{M}_{\text{period}}) \to \mathrm{Sh}(\mathcal{M}_{\text{period}})$$

acts as an entropy sheaf-convolution operator isolating cohomological strata, suitable for calculating periods of automorphic motives, and for constructing L-values as trace averages over entropy-refined sheaf cycles.

5. Conclusion and Future Work

We have constructed the Yang–Trace Kernel Atlas as:

- A formal classification of trace kernel types (analytic, arithmetic, geometric);
- A refinement structure via entropy, stack-liftability, and spectral stratification;
- A functorial convolution system compatible with RH, Langlands, and period cohomology.

This atlas now provides a global reference for kernel-theoretic trace formulations across number theory, automorphic geometry, and derived stacks.

In the next paper, we construct the **Sum–Kernel Entropy Criteria System**, formalizing which arithmetic sum structures admit kernel-theoretic interpretations via Yang refinement and entropy-flow moduli convolution.

References

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