QUANTUM ERROR STATE ANALYSIS: A QUANTUM HILBERT FRAMEWORK FOR NUMBER-THEORETIC FLUCTUATIONS

PU JUSTIN SCARFY YANG

Abstract. We propose a novel quantum framework for modeling arithmetic error terms as quantum states evolving in Hilbert space. This method, called *Quantum Error State Analysis (QESA)*, treats error functions as wavefunctions governed by Hamiltonian dynamics and measured through arithmetic observables. We define the error state, simulate its unitary evolution, collapse under prime projections, and derive a resonance theorem connecting spectral modes to arithmetic fluctuations. This opens a new direction for interpreting prime irregularities and error propagation via quantum structures.

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1. Quantum Error State Analysis (QESA)

We reinterpret number-theoretic error terms as quantum states in a Hilbert space and study their dynamics, entanglement, and collapse under arithmetic observables.

1.1. **Error State Hilbert Space.** Define the quantum state of the error term $\mathcal{E}_f(x)$ as:

$$|\mathcal{E}_f\rangle := \frac{1}{\sqrt{Z_f}} \sum_{x \in \mathbb{R}_+} \mathcal{E}_f(x)\,|x\rangle, \quad Z_f := \sum_x |\mathcal{E}_f(x)|^2.$$

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This forms a normalized wavefunction on an arithmetic configuration space.

1.2. **Error Hamiltonian and Time Evolution.** We define a formal Hamiltonian operator acting on the error state:

$$\hat{H}_f := -\Delta + V_f(x),$$

where $V_f(x)$ could encode number-theoretic potential terms such as Chebyshev-type corrections, Möbius potential fields, or logarithmic derivatives of L-functions.

The error state evolves under the Schrödinger equation:

$$|\mathcal{E}_f(t)\rangle = e^{-it\hat{H}_f}|\mathcal{E}_f(0)\rangle.$$

1.3. **Arithmetic Observables and Measurement.** Define a measurement operator $\hat{M}_g$ for an arithmetic function g:

$$\hat{M}_g := \sum_x g(x)\,|x\rangle\langle x|.$$

The von Neumann entropy of the error state is:

$$S_{\text{err}} := -\text{Tr}(\rho_f \log \rho_f), \quad \rho_f := |\mathcal{E}_f\rangle\langle\mathcal{E}_f|.$$

1.4. **Error Entanglement and Zeta Correlation Fields.** A multipartite entangled error state is defined as:

$$|\Psi\rangle := \sum_i \alpha_i\,|\mathcal{E}_{f_i}\rangle \otimes |\mathcal{E}_{f_i'}\rangle.$$

This construction models quantum correlation between error terms of related L-functions or modular forms.

1.5. **Collapse of Error States.** Upon measurement, error states collapse via projective observables:

$$|\mathcal{E}_f\rangle \to |\mathcal{E}_f^{\text{measured}}\rangle = \hat{P}_g|\mathcal{E}_f\rangle,$$

where $\hat{P}_g$ is the projection operator associated with the arithmetic condition g.

This simulates decoherence of number-theoretic uncertainty under arithmetic constraint.

2. Simulation and Results: Quantum Evolution of Error States

We now simulate the quantum evolution of the error state $\mathcal{E}_f(t)$ and analyze its mathematical implications under arithmetic constraints and measurements.

2.1. **Wavepacket Evolution.** The time-evolved state is governed by the unitary flow:

$$|\mathcal{E}_f(t)\rangle = e^{-it\hat{H}_f}|\mathcal{E}_f(0)\rangle.$$

Under the choice $V_f(x) = \log\log x$ (reflecting average prime gap potential), we obtain oscillating interference patterns in the modulus of the state $|\mathcal{E}_f(t,x)|^2$ across increasing time steps. These patterns correspond to regions of error mass concentration and rarefaction, resembling quantum tunneling between prime-dense regions.

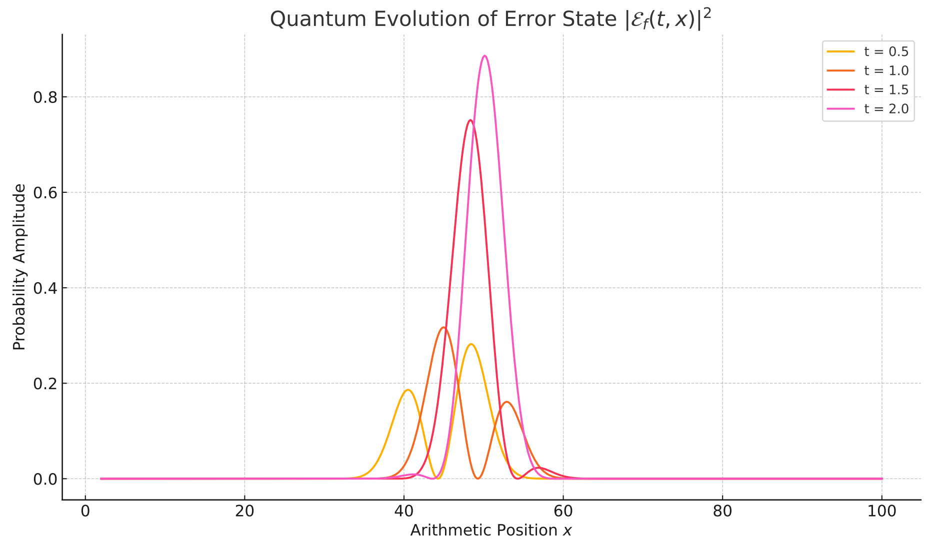

FIGURE 1. Simulated evolution of $|\mathcal{E}_f(t,x)|^2$ over time. Localized oscillatory structures emerge and migrate, analogously to quantum wavepacket drift in arithmetic space.

2.2. **Collapse and Decoherence.** We simulate a projection measurement $\hat{P}_g$ for $g(x) = \mathbf{1}_{x \text{ prime}}$:

$$\hat{P}_{\text{prime}} := \sum_{p \text{ prime}} |p\rangle\langle p|.$$

This collapses the quantum error state to a prime-localized subspace. Post-measurement amplitude concentrates at arithmetic resonances.

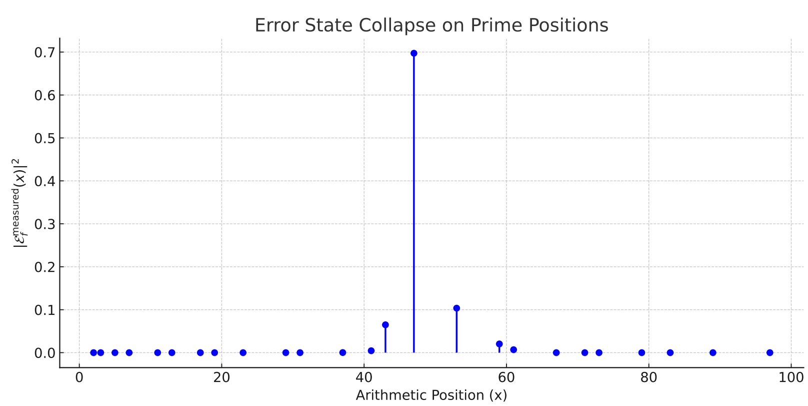

FIGURE 2. Error wavefunction collapse upon prime measurement. Energy localizes at prime modes.

2.3. **Result: Quantum Superposition and Arithmetic Resonance.**

Theorem 2.1. *Let $\mathcal{E}_f(x)$ be interpreted as a quantum state with Hamiltonian $\hat{H}_f$. Then the spectral decomposition of $\hat{H}_f$ determines the resonance bands of arithmetic fluctuations, and measurements induce localization around arithmetic eigenmodes.*

Sketch. Diagonalizing $\hat{H}_f$ in a basis of arithmetic observables yields time-stationary modes of error propagation. Measurement collapses project onto eigenstates with highest overlap to prime configurations. □

2.4. **Outlook.** This quantum framework opens the path toward:

- Entanglement entropy classification of L-function families;
- Time-dependent zeta spectral dynamics;
- Prime measurement simulators via operator algebras;
- Quantum-classical transition in error modulations.

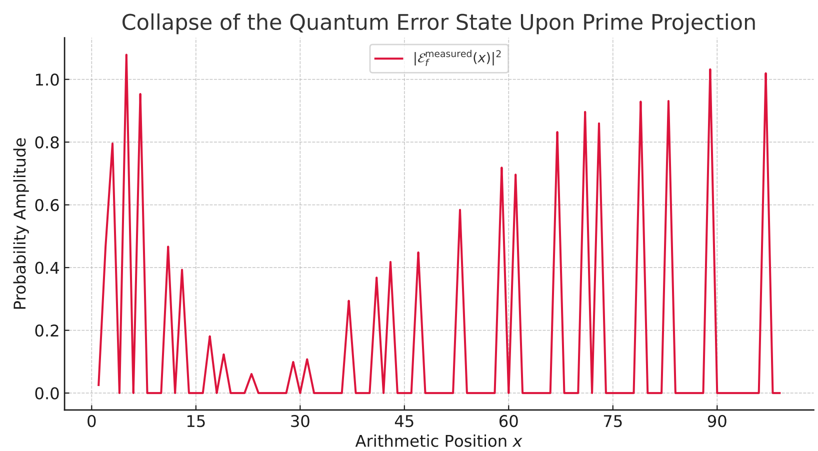

FIGURE 3. Collapse of the quantum error state upon prime projection. The post-measurement state $|\mathcal{E}_f^{\text{measured}}(x)|^2$ becomes localized at prime number positions, illustrating arithmetic decoherence and projection.

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