Let me try to make my question more precise. I mean the B-K conjecture on special values of zeta. Where does K-theory (or motivic cohomology) come in here precisely?

* If you remember the proof of the BK conjecture on special values of zeta attached to Tate motives, K-theory comes in the guise of motivic cohomology , via the Quillen-Licht. conjecture (proved using the "other" BK conjecture). More precisely, K-theory of the ring of integers of a number field tensored by Z_p is iso. to motivic (or étale if p is odd) cohomology of twisted versions of Z_p. This p-adic "avatar" deals with the arithmetic interpretation of the special values, using cohomology to perform (co)descent on Iwasawa modules, and thus allowing the Main Conjecture to enter the game. Whereas the archimedean avatar is treated by classical complex analytic methods, which allow to express the special values in terms of polylog values. The combination of the two points of view is made possible by the existence of Beilinson's special element in K-theory (remember I compared it to Plato's "archetype"), which is sent by p-adic regulators onto the Deligne-Soulé elements (=special cyclotomic elements), and on the polylog. by the Borel-Beilinson regulator. Note that up to now, the existence of special elements is limited to abelian fields. Now that a non commutative MC is available, people are of course looking for non commutative special elements, using e.g. the (conjectural) Stark units, but this is still wishful thinking, as far as I know.
The same kind of road map should be applicable to the K-theory of elliptic curves, after the pioneering work of Soulé, but my knowledge of this domain is not precise enough. Of course, on the Iwasawa side, many MC are now available, but on the side of avatars,I think we still miss the essential objects and tools available in the cyclotomic theory, in spite of serious work by e.g. Kings et al. But I'm actually not an expert here.

Assuming the B-K conjecture (on L-values), what information does it provide on the K-theory (let us stick to the specific case of an elliptic curve) of E or on the motive cohomology of E?

* What kind of information precisely ? Sticking to the cyclotomic case, we have now analogues of the analytic class number formula for half the K-groups (depending on the parity of m in the index 2m -2, I never can remember what parity). The parity limitation is due to the limitation of the MC itself, which is roughly speaking restricted to totally real fields. One could think of extending this in the following sense : after all, the order of a group is an annihilator of this group, so one could ask for more elaborate annihilators involving special L-values and Galois action. For class groups (resp. K-groups), we have the Brumer-Stark (resp. Coates-Sinnott) conjectures. I can't cite all the work which has been done, and the many partial results which have been obtained since Iwasawa theory and the MC. But only the apperance of the Equivariant MC allowed to get complete proofs in certain cases. In the abelian situation,I dare say that the first proof of the CS conjecture is due to myself (JNTDN Bordeaux,17, 2005). I even showed annihilation results for the K-groups with "bad parity" indices (see my contribution to the Proc. of the Heidelberg Iw. Conf.). After the apparition on the non commutative EMC, many analogous non abelian results have been proved for the "good parity", mainly by Burns et al., and especially by Nickel (Proc. London Math. Soc., 106, 2013).
In the elliptic case, the same problems can be posed of course. But I know of no precise statement nor result. Buecause again, I'm not an expert here.