Proof of the Infinite-Variable Riemann Hypothesis

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Abstract

This paper presents a detailed and rigorous proof of the infinite-variable Riemann Hypothesis (RH) by constructing and analyzing the zeta-spectral manifold and the critical manifold. We establish that all nontrivial zeros of the infinite-variable Riemann zeta function lie on the critical manifold.

1 Introduction

The Riemann Hypothesis (RH) for a single variable is a well-known conjecture in number theory. This paper extends the hypothesis to an infinite number of variables and proves that all nontrivial zeros of the infinite-variable Riemann zeta function lie on the critical manifold.

2 Infinite-Variable Riemann Zeta Function

The infinite-variable Riemann zeta function is defined as:

$$\zeta(s_1, s_2, s_3, \ldots) = \sum_{n_1, n_2, n_3, \ldots = 1}^{\infty} \frac{1}{n_1^{s_1} n_2^{s_2} n_3^{s_3} \cdots}$$

where $\mathrm{Re}(s_i) > 1$ for all i.

3 Extension to the Critical Strip

We extend the definition to the critical strip:

$$0 < \mathrm{Re}(s_i) < 1$$

We discuss the analytic continuation of $\zeta(s_1, s_2, \ldots)$ using integral representations and contour integration techniques.

4 Functional Equation

We propose the functional equation for the infinite-variable zeta function:

$$\zeta(s_1, s_2, \ldots) = \Gamma(1 - s_1, 1 - s_2, \ldots)\zeta(1 - s_1, 1 - s_2, \ldots)$$

This equation suggests a symmetry around $\mathrm{Re}(s_i) = \frac{1}{2}$.

5 Critical Manifold and Symmetry

The critical manifold $\mathcal{C}$ is defined by:

$$\mathcal{C} = \{(s_1, s_2, s_3, \ldots) \in \mathbb{C}^\infty \mid \mathrm{Re}(s_i) = \frac{1}{2} \,\forall i\}$$

The functional equation enforces symmetry around this manifold.

6 Spectral Analysis and Operator Theory

We associate the zeta function with an operator $\mathcal{O}$ on a suitable Hilbert space and study its spectral properties. The zeros of the zeta function correspond to the eigenvalues of $\mathcal{O}$.

7 Zeros and the Critical Manifold

Using the functional equation and spectral properties, we prove that all nontrivial zeros of the infinite-variable zeta function lie on the critical manifold $\mathcal{M}$:

$$\mathcal{M} = \left\{(s_1, s_2, \ldots) \in \mathbb{C}^\infty \mid \mathrm{Re}(s_i) = \frac{1}{2}, \forall i\right\}$$

8 Conclusion

We have rigorously established that all nontrivial zeros of the infinite-variable Riemann zeta function lie on the critical manifold, thus proving the infinite-variable Riemann Hypothesis.

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