Constructing Fields Larger than $\mathbb{C}$ Using Automorphic Forms, Motives, and L-functions

Mathematician

Lecture 0: Introduction and Course Overview

Outline

Motivation

- Why explore fields larger than $\mathbb{C}$?
- The role of automorphic forms, motives, and L-functions.
- Potential applications in number theory, geometry, and physics.

Course Structure

- Part 1: Foundations of Automorphic Forms.
- Part 2: Introduction to Motives and Their Constructions.
- Part 3: Leveraging L-functions for Field Construction.
- Part 4: Applications and Extensions.
- Indefinite Expansion: New developments and insights as the course progresses.

Preliminary Definitions

- Automorphic Forms: Functions that are invariant under a group action.
- Motives: Abstract objects in algebraic geometry that generalize the notion of algebraic cycles.
- L-functions: Complex functions that encode information about algebraic structures.

$$\text{Example: } L(s,\chi) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s}$$

Next Steps

- Setting up the necessary mathematical background.
- Overview of tools and techniques to be used.
- Discussion on the roadmap of the course.