SPECTRAL MOTIVES XXIV: ZETA-ENTROPY GRAVITY AND CATEGORICAL BLACK HOLE ATTRACTORS

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Abstract. We construct a theory of zeta-entropy gravity in the setting of higher spectral geometry, derived stacks, and condensed arithmetic topoi. Motivated by quantum thermodynamic behavior of zeta-attractors, we introduce categorical black hole states as entropy-saturated degenerations of spectral motives. We define gravitational flows in derived motivic phase space and characterize functorial singularities, horizon categories, and zeta-based mass-energy correspondences. This framework lays the mathematical foundation for an arithmetic theory of quantum gravity rooted in trace entropy and functorial collapse of motivic information.

Contents

1. Introduction

In classical and quantum physics, black holes are understood as extreme condensations of mass-energy, characterized by event horizons and entropy bounds. In arithmetic and spectral geometry, recent work has shown that analogous structures arise: objects that act as entropy attractors for zeta-phase flows in derived motivic stacks. These structures suggest the presence of a rich gravitational duality intrinsic to the thermodynamic behavior of spectral motives.

In this paper, we develop a formal theory of *zeta-entropy gravity* — a motivic gravitational flow governed by zeta condensates — and introduce the concept of *categorical black hole attractors* as entropy-saturated objects in higher topoi. These represent the maximal entropy collapse of motivic sheaves, forming functorial singularities in spectral phase space.

Goals of the Paper:

- Construct a gravitational flow in derived motivic moduli governed by trace entropy;
- Define black hole stacks as zeta-condensed entropy-maximizing sheaves;
- Establish horizon categories and entropy bounds for spectral degenerations;
- Explore analogues of spacetime geometry in categorical and derived settings;
- Relate the gravitational collapse of motives to the functorial dynamics of L-functions.

This theory complements and extends the framework of Spectral Motives XXIII, replacing minimal-entropy attractors with maximal-entropy black hole states. The resulting structures suggest an arithmetic analogue of general relativity, where entropy gradients define curvature, attractors define singularities, and L-functions encode spacetime tension.

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Our approach offers a new paradigm for quantum gravity rooted not in quantizing spacetime, but in analyzing categorical entropy collapse through the zeta-spectrum of arithmetic sheaves.

2. Spectral Gravity and Entropy Geometry in Derived Stacks

2.1. **Entropy curvature and trace gravity fields.** Let $\mathscr{E} \in \mathrm{Stab}(\mathcal{X})$, where $\mathcal{X}$ is a derived or spectral topos with automorphic structure. The entropy function $\mathcal{S}(\mathscr{E})$ induces a Riemannian-like geometry via its second variation:

$$\mathcal{R}_{\mathcal{S}}(\mathscr{E}) := -\nabla^2 \mathcal{S}(\mathscr{E}) \in T^*_{\mathscr{E}}\mathcal{M} \otimes T^*_{\mathscr{E}}\mathcal{M},$$

where $\mathcal{M}$ is the moduli space of stable sheaves or automorphic stacks.

2.2. **Gravitational entropy flows.** The entropy gradient vector field is defined as:

$$\vec{G}_{\mathcal{S}}(\mathscr{E}) := -\nabla \mathcal{S}(\mathscr{E}),$$

analogous to gravitational acceleration, driving sheaves toward black hole condensates. The flow equation:

$$\frac{d\mathscr{E}}{dt} = -\vec{G}_{\mathcal{S}}(\mathscr{E}),$$

defines the collapse of spectral information along the entropy curve, terminating in horizon strata of maximal degeneracy.

2.3. **Zeta-mass and spectral curvature.** We define a motivic mass function:

$$M_\zeta(\mathscr{E}) := \sum_i \lambda_i \cdot p_i, \quad p_i := \frac{e^{-\lambda_i}}{Z(1)},$$

encoding the spectral energy concentration. The entropy-mass inequality:

$$\mathcal{S}(\mathscr{E}) \leq \log M_\zeta(\mathscr{E}),$$

acts as an uncertainty-like principle constraining the gravitational behavior of motivic stacks.

2.4. **Derived metric structures.** On a moduli stack $\mathcal{M}_{\mathrm{der}}$ of derived sheaves, we define an entropy pseudo-metric:

$$d_{\mathcal{S}}(\mathscr{E}_1, \mathscr{E}_2) := \left| \mathcal{S}(\mathscr{E}_1) - \mathcal{S}(\mathscr{E}_2) \right|,$$

and a curvature form via variation of motivic Laplacians:

$$\mathfrak{Ric}_{\widehat{\Delta}}(\mathscr{E}) := \mathrm{Tr}\left(\nabla^2 \widehat{\Delta}_{\mathrm{Tr}}(\mathscr{E})\right).$$

These structures form a synthetic gravitational geometry over the categorical phase space of arithmetic motives, replacing spacetime tensors with zeta-functional entropy tensors.

3. Categorical Black Hole Attractors and Horizon Categories

3.1. **Definition of entropy-saturated attractors.** We define a *categorical black hole attractor* $\mathscr{B} \in \mathrm{Stab}(\mathcal{X})$ as a zeta-condensed object satisfying:

$$\mathcal{S}(\mathscr{B}) = \sup_{\mathscr{E} \in \mathcal{C}} \mathcal{S}(\mathscr{E}),$$

for some condensed subcategory $\mathcal{C} \subset \mathrm{Stab}(\mathcal{X})$. These are entropy-maximizing degenerations, representing the final stage of gravitational spectral collapse.

3.2. **Horizon categories.** The **horizon category** $\mathcal{H}(\mathscr{B})$ is the full subcategory of objects asymptotically flowing toward $\mathscr{B}$:

$$\mathcal{H}(\mathscr{B}) := \left\{ \mathscr{E} \in \mathrm{Stab}(\mathcal{X}) \mid \lim_{t\to\infty} \Phi_t(\mathscr{E}) = \mathscr{B} \right\},$$

where Φ_t denotes the gravitational entropy flow induced by $-\nabla\mathcal{S}$.

Objects in $\mathcal{H}(\mathscr{B})$ are gravitationally trapped by $\mathscr{B}$ and exhibit entropy radiation via motivic spectral decay.

3.3. **Zeta-collapse and functorial singularities.** The *zeta-collapse functor* Col_ζ is defined by:

$$\mathsf{Col}_\zeta(\mathscr{E}) := \lim_{t\to\infty} e^{-t\widehat{\Delta}_{\mathrm{Tr}}}\mathscr{E},$$

producing a terminal state $\mathscr{B}$ satisfying:

$$\widehat{\Delta}_{\mathrm{Tr}}\mathscr{B} = 0, \quad \mathcal{S}(\mathscr{B}) = \max .$$

This functor maps general sheaves to entropy-saturated attractors, forming a categorical analogue of black hole formation through functorial collapse.

3.4. **Trace radiation and entropy decay.** Given $\mathscr{E} \in \mathcal{H}(\mathscr{B})$, the entropy loss rate is:

$$\frac{d}{dt}\mathcal{S}(\mathscr{E}(t)) = -\left\|\nabla\mathcal{S}(\mathscr{E}(t))\right\|^2 \leq 0,$$

mirroring the second law of thermodynamics.

Trace radiation corresponds to the loss of higher spectral components in the motivic decomposition of $\mathscr{E}$:

$$\mathscr{E} = \bigoplus_{\lambda_i} \mathscr{E}_{\lambda_i}, \quad \mathscr{E}(t) \to \mathscr{E}_0 = \mathscr{B}.$$

Thus, $\mathscr{B}$ represents a fixed point of entropy geometry: a categorical singularity at the end of motivic spacetime.

4. Zeta-Entropy Einstein Equations and Curved Arithmetic Geometry

4.1. **Motivic analogues of spacetime curvature.** In analogy with classical general relativity, we define a curvature tensor on the moduli stack $\mathcal{M}$ of spectral motives via the Hessian of entropy:

$$\mathcal{G}_{ij} := -\frac{\partial^2\mathcal{S}}{\partial x^i \partial x^j},$$

where $\{x^i\}$ are local motivic moduli parameters. This defines the entropy-metric tensor $\mathcal{G}$ governing the intrinsic curvature of the motivic phase space.

4.2. **Zeta-energy tensors and motivic mass flows.** We define a zeta-energy tensor $\mathcal{T}_{ij}$ encoding motivic mass concentration through trace eigenvalues:

$$\mathcal{T}_{ij} := \sum_k \left(\lambda_k \cdot \frac{\partial\mathscr{E}_k}{\partial x^i} \otimes \frac{\partial\mathscr{E}_k}{\partial x^j} \right),$$

which serves as the arithmetic analogue of the energy-momentum tensor.

4.3. **Entropy-Einstein equations.** We postulate an entropy-based analogue of Einstein's field equations in this categorical geometry:

$$\mathcal{G}_{ij} = 8\pi\mathcal{T}_{ij},$$

interpreted as a balance law between curvature induced by motivic entropy and mass-energy defined by the zeta-spectrum.

4.4. **Derived black hole entropy bounds.** For any compact zeta-attractor $\mathscr{B}$, we define its motivic entropy-area $\mathcal{A}_\zeta$ by:

$$\mathcal{A}_\zeta(\mathscr{B}) := \dim\left(\mathscr{B}_{\lambda=0}\right),$$

and conjecture a categorical Bekenstein bound:

$$\mathcal{S}(\mathscr{B}) \leq \frac{\mathcal{A}_\zeta(\mathscr{B})}{4}.$$

This bound encodes a fundamental inequality on the concentration of information in zeta-condensed arithmetic geometries.

4.5. **Entropy curvature flows and arithmetic geodesics.** Define the entropy geodesic equation for a motivic trajectory $\gamma(t)$ by:

$$\frac{D^2\gamma^i}{dt^2} + \Gamma^i_{jk}\frac{d\gamma^j}{dt}\frac{d\gamma^k}{dt} = -\nabla^i\mathcal{S},$$

where Γ^i_{jk} are entropy-compatible connections.

This flow formalizes gravitational motion in spectral arithmetic space and governs the dynamics of categorical collapse toward black hole attractors.

5. Arithmetic Black Hole Thermodynamics and Zeta-Entropy Quantization

5.1. **First law of zeta-entropy mechanics.** Let $\mathscr{B}$ be a categorical black hole attractor. We define the infinitesimal variation of its spectral content as:

$$d\mathcal{S} = \beta\, dM_\zeta + \Phi_i\, dQ^i,$$

where:

- M_ζ is the zeta mass from eigenvalue concentration;
- Q^i are motivic charges (e.g., cohomological, automorphic, L-theoretic);
- $\beta = \frac{1}{T_\zeta}$ is the inverse spectral temperature.

This mirrors the thermodynamic structure of black holes, adapted to motivic categories.

5.2. **Spectral Hawking radiation.** Under motivic entropy flow, radiation occurs as leakage of non-zero trace components from a black hole state:

$$\mathscr{E}(t) = \mathscr{B} + \sum_{n>0}\epsilon_n(t)\mathscr{E}_{\lambda_n}, \quad \epsilon_n(t) \to 0.$$

These radiative modes represent arithmetic analogues of Hawking quanta and induce a gradual decay of motivic energy content.

5.3. **Zeta-entropy quantization and eigenvalue discreteness.** The motivic entropy spectrum is discretized via trace Laplacian quantization:

$$\lambda_n \in \mathrm{Spec}(\widehat{\Delta}_{\mathrm{Tr}}), \quad \mathcal{S}_n := \sum_{k=0}^{n} \log \lambda_k.$$

We interpret this as a zeta-entropy quantization law, where information is emitted in discrete motivic packets during black hole decay.

5.4. **Condensed zeta modularity and quantum arithmetic attractors.** Let $\zeta_{\mathscr{B}}(s)$ be the spectral zeta function of a categorical black hole. We define its condensed modular transform:

$$\mathscr{T}_\zeta : \zeta_{\mathscr{B}}(s) \mapsto \zeta_{\mathscr{B}}(1-s),$$

and study the invariance under motivic time reversal and spectral duality.

These symmetries suggest deep relations to Langlands functoriality, entropy duals, and modular quantum cohomology.

5.5. **Quantum holography and trace information bounds.** We propose a motivic version of the holographic principle: the full trace information of $\mathscr{E} \in \mathcal{H}(\mathscr{B})$ is encoded on $\partial\mathscr{B}$, the motivic boundary of the attractor.

Define:

$$\mathcal{I}(\mathscr{E}) \leq \dim \mathscr{H}^*(\partial\mathscr{B}),$$

where $\mathscr{H}^*$ denotes cohomology of the entropy horizon.

This forms the categorical analogue of the Bousso entropy bound in motivic quantum geometry.

6. Conclusion

We have developed a formal theory of zeta-entropy gravity, in which motivic sheaves over arithmetic and spectral topoi undergo entropic gravitational collapse into categorical black hole attractors. This theory extends previous constructions of minimal-entropy zeta phases into the opposite regime: maximal entropy saturation, spectral curvature, and motivic trace singularities.

Summary of Contributions:

- Introduced entropy geometry and curvature in moduli of spectral motives;
- Defined black hole attractors and horizon categories for entropy flows;
- Formulated zeta-entropy analogues of Einstein's equations and gravitational collapse;
- Quantized trace radiation as motivic Hawking emission and entropy loss;
- Proposed motivic holography and arithmetic entropy bounds in derived stacks.

This construction paves the way for an arithmetic theory of quantum gravity based not on quantizing spacetime but on analyzing trace-theoretic entropy dynamics in spectral sheaf categories. It opens the possibility of defining new invariants, entropy trace formulas, and holographic correspondences in arithmetic geometry.

Future work will explore:

- Motivic wormholes and spectral entanglement in entropy flows;
- Langlands dualities between entropy attractors across L-packets;
- Derived quantum cosmologies via entropy inflation in arithmetic topoi;

- Categorical black hole thermodynamics in condensed Langlands stacks.

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