TOWARDS A META-VALUATION THEORY: A HIGHER-CATEGORICAL AND EPISTEMIC FRAMEWORK

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ABSTRACT. We propose a new framework for the study of valuations not as isolated algebraic functions on fields, but as structural morphisms in a meta-categorical context. This framework classifies valuation-theoretic objects across logical, geometric, and categorical universes, forming a foundation for their higher-level intertranslations. The aim is to initiate the development of a *Meta-Valuation Theory*, as part of the ongoing $\mathrm{Yang}_{\mathrm{Meta}}$ foundational program.

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Date: May 22, 2025.

1. Introduction

Valuation theory traditionally studies functions that assign to each nonzero element of a field a value in a totally ordered abelian group, preserving multiplicative and subadditive structures. However, this notion remains field-dependent and tied to first-order algebraic semantics.

In this paper, we shift attention to a *meta-theoretical* construction: a system that categorizes not individual valuations, but entire valuation theories, by embedding them in higher categorical and logical frameworks.

2. Meta-Valuation Objects

Definition 2.1 (Meta-Valuation Object)**.** A *meta-valuation object* is a triple

$$\mathcal{V}_{\text{meta}} := (K, \mathscr{V}, \mathcal{L})$$

where:

- K is a base algebraic or geometric object (e.g., a field, a ringed topos, a sheaf over a site).
- $\mathscr{V}$ is a family of valuation structures (including classical, adic, Berkovich, perfectoid, or type-theoretic variants).
- $\mathcal{L}$ is a logical framework (e.g., first-order logic, HoTT, univalent type theory, Yang_α-logic).

Remark 2.2. This approach allows distinct valuation concepts defined under different logics or categories to be simultaneously embedded and compared.

3. Morphisms Between Meta-Valuations

Definition 3.1 (Meta-Valuation Morphism)**.** Given two meta-valuation objects $\mathcal{V}_{\text{meta}} = (K, \mathscr{V}, \mathcal{L})$ and $\mathcal{V}'_{\text{meta}} = (K', \mathscr{V}', \mathcal{L}')$, a morphism is a triple $(f_K, f_{\mathscr{V}}, f_{\mathcal{L}})$ where:

- $f_K : K \to K'$ is a structure-preserving morphism of bases.
- $f_{\mathscr{V}} : \mathscr{V} \to \mathscr{V}'$ is a valuation-theoretic translation functor.
- $f_{\mathcal{L}} : \mathcal{L} \to \mathcal{L}'$ is a logic-preserving translation (e.g., a syntactic or semantic morphism).

4. Categorical Structure

Proposition 4.1. *The collection of meta-valuation objects and their morphisms forms a 2-category, denoted* **MetaVal**.

Sketch. Composition is defined componentwise, with associativity and identity inherited from the respective categories of base objects, valuation theories, and logical frameworks. □

5. Future Directions

This framework supports:

(a) Unified classification of valuation concepts up to meta-equivalence.
(b) Development of cohomological and stack-theoretic interpretations of valuation spaces.
(c) Modeling of valuation-like constructs in Yang_{∞}-based logics and higher structures.
(d) Creation of a universal *Valuation Cosmos*, a functor-valued object over logic-indexed sites.

6. Classification of Meta-Valuation Theories

We now define an equivalence relation $\simeq$ on the set of all meta-valuation theories $\mathcal{V}_{\text{meta}} = (K, \mathscr{V}, \mathcal{L})$ based on multi-level structural and semantic conditions.

Definition 6.1 (Meta-Valuation Equivalence)**.** Let $\mathcal{V}_1 = (K_1, \mathscr{V}_1, \mathcal{L}_1)$ and $\mathcal{V}_2 = (K_2, \mathscr{V}_2, \mathcal{L}_2)$ be two meta-valuation theories.

We say $\mathcal{V}_1 \simeq \mathcal{V}_2$ if there exists a triple of maps

$$(f_K, f_{\mathscr{V}}, f_{\mathcal{L}})$$

such that:

- $f_K : K_1 \to K_2$ is a structure-preserving equivalence of bases.
- $f_{\mathscr{V}} : \mathscr{V}_1 \to \mathscr{V}_2$ is a functorial isomorphism of valuation structures.
- $f_{\mathcal{L}} : \mathcal{L}_1 \to \mathcal{L}_2$ is a logic-theoretic translation preserving semantics.

Theorem 6.2 (Meta-Valuation Classification Theorem)**.** *There exists a stratified set of equivalence classes of meta-valuation theories:*

$$\mathrm{MetaVal}/\simeq = \bigsqcup_{\alpha=0}^{\infty} \mathscr{C}_\alpha$$

where $\mathscr{C}_\alpha$ denotes the α-level classification category defined by increasingly relaxed equivalence criteria, with:

- $\mathscr{C}_0$: *strictly syntactic-level equivalence;*
- $\mathscr{C}_1$: *functorial-level valuation equivalence;*
- $\mathscr{C}_2$: *sheaf- or topos-theoretic equivalence;*
- $\mathscr{C}_3$: *homotopy-type or infinity-categorical equivalence;*
- $\mathscr{C}_\infty$: *Yang-level unification by meta-logic transfinite codes.*

Remark 6.3. The deepest equivalence class $\mathscr{C}_\infty$ captures equivalence under arbitrary meta-universes (e.g., Yang_α-logics), admitting equivalences not definable in any fixed meta-language.

7. Meta-Completeness of the Classification Hierarchy

We now justify the claim that our classification system $\{\mathscr{C}_\alpha\}_{\alpha \in \mathbb{N} \cup \{\infty\}}$ is sufficient to classify *all possible meta-valuation theories* expressible within any definable logical system.

Definition 7.1 (Definable Meta-Valuation Theory)**.** A meta-valuation theory $\mathcal{V}_{\mathrm{meta}} = (K, \mathscr{V}, \mathcal{L})$ is said to be *definable* if it can be specified within some formal logic $\mathcal{L}$ whose syntax and semantics are well-founded and finitely generative.

Lemma 7.2 (Semantic Reflection Principle)**.** *Let $\mathcal{L}$ be a logic system closed under meta-translation, and let $\mathscr{M}_{val}$ be the fibered category of meta-valuation theories over $\mathcal{L}$. Then any semantic equivalence between two objects of $\mathscr{M}_{val}$ arises from a transformation within some $\mathscr{C}_\alpha$.*

Sketch. The transformation must preserve either syntactic identity ($\alpha = 0$), functorial structure ($\alpha = 1$), geometric sheaf-theoretic behavior ($\alpha = 2$), homotopical or categorical type identity ($\alpha = 3$), or a generalized meta-logical transfinite interpretation ($\alpha = \infty$). These exhaust all routes of formal equivalence internal to the logic universe. □

Theorem 7.3 (Meta-Completeness Theorem)**.** *Let* MetaVal *be the collection of all definable meta-valuation theories. Then:*

$$\mathrm{MetaVal}/\simeq = \bigsqcup_{\alpha=0}^{\infty} \mathscr{C}_\alpha$$

Moreover, for any definable $\mathcal{V}_{meta} \in \mathrm{MetaVal}$, there exists a unique α (possibly ∞) such that $\mathcal{V}_{meta}$ is classified in $\mathscr{C}_\alpha$.

Outline. Each $\mathscr{C}_\alpha$ corresponds to a relaxation of equivalence criteria across the meta-logical spectrum. Since all definable theories must reside within a finitely generated or transfinite (but accessible) logic, their comparison falls into one of these equivalence categories. The union over all such α is exhaustive. □

Remark 7.4 (On the Inaccessibility of the Non-Definable)**.** Any object not classifiable within this hierarchy must be outside all known logic systems. As such, it cannot be considered a "theory" in the formal mathematical sense and is treated as epistemically inert.

Corollary 7.5 (Meta-Valuation Universality)**.** *The classification system* $\{\mathscr{C}_\alpha\}$ *forms a reflective and complete stratification of the epistemically visible meta-valuation universe.*

Remark 7.6 (Philosophical Reflection)**.** This mirrors the principle in higher epistemology: all distinctions, to be cognitively manipulable, must fall within a semantically structured representational system. The $\text{Yang}_{\text{Meta}}$ program extends this to trans-logical and transfinite representational domains.

8. Meta-Completeness and Logical Closure

We now formalize the claim that our hierarchy of equivalence classes

$$\{\mathscr{C}_\alpha\}_{\alpha\in\mathbb{N}\cup\{\infty\}}$$

provides a complete stratification of all definable meta-valuation theories.

Definition 8.1 (Definability)**.** A meta-valuation theory $\mathcal{V}_{\text{meta}} = (K, \mathscr{V}, \mathcal{L})$ is *definable* if the triple can be constructed within some logical system $\mathcal{L}$ possessing a syntactic grammar and semantic model theory, possibly transfinite in generation but effectively describable.

Lemma 8.2 (Semantic Reflection Lemma)**.** *Let* $\mathcal{L}$ *be any reflective logic system. Then all semantic equivalences between definable meta-valuation theories in* $\mathcal{L}$ *are realized within some layer* $\mathscr{C}_\alpha$ *of the classification hierarchy.*

Sketch of Proof. A semantic equivalence must preserve either syntactic derivability ($\alpha = 0$), categorical behavior ($\alpha = 1$), sheaf-theoretic descent ($\alpha = 2$), homotopy-level identity ($\alpha = 3$), or meta-logical interpretability ($\alpha = \infty$). These levels span the formal behaviors of all definable theories. □

Theorem 8.3 (Meta-Completeness Theorem)**.** *Let* MetaVal *denote the collection of all definable meta-valuation theories. Then:*

$$\text{MetaVal}/\simeq = \bigsqcup_{\alpha=0}^{\infty} \mathscr{C}_\alpha$$

That is, every definable meta-valuation theory belongs uniquely (up to meta-equivalence) to a layer of this classification.

Corollary 8.4 (Logical Closure of Equivalence)**.** *The category of definable meta-valuation theories is closed under semantic and syntactic equivalences, and its image is fully stratified by the $\mathscr{C}_\alpha$ layers.*

Remark 8.5 (Epistemic Horizon Principle)**.** Any theory not capturable by this hierarchy lies outside the scope of logic-based cognition. Such a theory, while potentially conceivable in some metaphysical sense, is not mathematically meaningful within our representational universe.

9. Philosophical Reflection: On the Limits of Categoricity

Our classification echoes a meta-philosophical maxim:

> *To classify is to name distinctions within a cognitive system. That which cannot be distinguished within such a system is not subject to classification.*

Thus, the $\text{Yang}_{\text{Meta}}$ approach implicitly outlines not just what can be known about valuations, but what it means to know a valuation—the boundaries of all possible structural understanding.

10. Extended Future Directions

Building upon this classification, the following questions are natural extensions:

- How does deformation theory apply to layers $\mathscr{C}_\alpha$?
- Can a geometric or stacky moduli space be constructed over MetaVal?
- What is the internal logic of the category $\mathscr{M}_{\text{val}}$?
- Can Yang_∞ serve as a univalent completion of all known logical representations?

11. Illustrative Examples of $\mathscr{C}_\alpha$ Layers

To illustrate the hierarchy of meta-valuation equivalence classes, we now provide representative examples in each layer $\mathscr{C}_\alpha$.

Layer $\mathscr{C}_0$: Syntactic Equivalence.

- Classical discrete valuation on $\mathbb{Q}$ via v_p.
- Valuation on $\mathbb{F}_p((t))$ via order of t.

These are syntactically and semantically identical in identical logical frameworks.

Layer $\mathscr{C}_1$: Functorial Equivalence.

- The p-adic valuation v_p on $\mathbb{Q}$ versus an isomorphic valuation on a finite extension $K/\mathbb{Q}$, extended canonically.

Layer $\mathscr{C}_2$: Sheaf-Theoretic Equivalence.

- Valuations defined via stalks of a structure sheaf over a Dedekind scheme.
- Berkovich analytic valuations and adic valuations that induce the same support on a rigid space.

Layer $\mathscr{C}_3$: Homotopical or Type-Theoretic Equivalence.

- A valuation encoded in a univalent type in HoTT representing a sheaf over a site.
- An equivalent derived valuation in a stable ∞-topos.

Layer $\mathscr{C}_\infty$: Yang$_\alpha$-Semantic Equivalence.

- Two valuations described using different logics (e.g., Coq vs HoTT vs Yang$_\alpha$), but provably intertranslatable via a trans-logical meta-interpreter.
- Valuation structures embedded in different mathematical ontologies, yet yielding same cohomological effects via Yang-semantic reflection.

12. Canonical Stratification Functor

We now define a canonical stratification functor:

$$\mathcal{S} : \mathrm{MetaVal} \to \mathbb{N} \cup \{\infty\}$$

which maps a definable meta-valuation theory to the minimal α such that it resides in $\mathscr{C}_\alpha$.

Definition 12.1 (Stratification Functor $\mathcal{S}$)**.** For each definable $\mathcal{V}_{\mathrm{meta}}$, let

$$\mathcal{S}(\mathcal{V}_{\mathrm{meta}}) := \min\{\alpha \in \mathbb{N} \cup \{\infty\} \mid \mathcal{V}_{\mathrm{meta}} \in \mathscr{C}_\alpha\}$$

Proposition 12.2. *The functor $\mathcal{S}$ is well-defined, surjective, and respects logical translations within a reflective logical universe.*

Sketch. Every definable theory belongs to some $\mathscr{C}_\alpha$ by Meta-Completeness. Surjectivity follows from the constructed examples in each layer. ☐

13. Transfinite Extensions and Logical Stability

Definition 13.1 (Transfinite Yang-Layer)**.** Let κ be a strongly inaccessible cardinal. Define a Yang-layer $\mathscr{C}_\kappa$ to be the collection of all valuation theories definable in logics whose derivation depth and quantifier ranks are indexed by ordinals less than κ.

Remark 13.2. These extensions allow the inclusion of valuation theories not expressible in any finitely generative language, but still constrained by structural recursion.

Corollary 13.3 (Stable Meta-Valuation Universe)**.** *The extended collection*

$$\mathrm{MetaVal}^{stable} = \bigcup_{\kappa \text{ inaccessible}} \mathscr{C}_\kappa$$

defines the stable universe of transfinite-logically-definable valuation theories.

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Proposition 15.2. *The functor $\mathcal{S}$ is well-defined, surjective, and respects logical translations within a reflective logical universe.*

Sketch. Every definable theory belongs to some $\mathscr{C}_\alpha$ by Meta-Completeness. Surjectivity follows from the constructed examples in each layer. □

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Definition 16.1 (Transfinite Yang-Layer)**.** Let κ be a strongly inaccessible cardinal. Define a Yang-layer $\mathscr{C}_\kappa$ to be the collection of all valuation theories definable in logics whose derivation depth and quantifier ranks are indexed by ordinals less than κ.

Remark 16.2. These extensions allow the inclusion of valuation theories not expressible in any finitely generative language, but still constrained by structural recursion.

Corollary 16.3 (Stable Meta-Valuation Universe)**.** *The extended collection*

$$\text{MetaVal}^{\textit{stable}} = \bigcup_{\kappa \textit{ inaccessible}} \mathscr{C}_\kappa$$

defines the stable universe of transfinite-logically-definable valuation theories.

17. Meta-Valuation Spectrum and Logical Flow

Having established the stratification hierarchy and its completeness, we now construct a spectral framework that visualizes the semantic migration of valuation theories across layers.

Definition 17.1 (Meta-Valuation Spectrum)**.** Let MetaVal be the set of definable meta-valuation theories. Define the *meta-valuation spectrum* as the directed graph:

$$\mathrm{Spec}^{\mathrm{meta}}(\mathcal{V}) := \big(\mathrm{MetaVal}, \rightsquigarrow\big)$$

where $\mathcal{V}_1 \rightsquigarrow \mathcal{V}_2$ if there exists a weakening or translation morphism of one of the following types:

- logical weakening (change of $\mathcal{L}_1 \to \mathcal{L}_2$);
- structural functor relaxation ($\mathscr{V}_1 \to \mathscr{V}_2$);
- base morphism or localization ($K_1 \to K_2$).

Remark 17.2. The graph $\mathrm{Spec}^{\mathrm{meta}}(\mathcal{V})$ forms a reflexive, directed, and stratified structure, encoding equivalence collapse and trans-layer translation paths.

Definition 17.3 (Spectral Path Type)**.** A path $\mathcal{V}_0 \rightsquigarrow \mathcal{V}_1 \rightsquigarrow \cdots \rightsquigarrow \mathcal{V}_n$ is said to be:

- **constant-type** if all nodes lie in the same $\mathscr{C}_\alpha$;
- **ascending-type** if $\mathcal{S}(\mathcal{V}_i) < \mathcal{S}(\mathcal{V}_{i+1})$;
- **stable-type** if all nodes collapse to the same point in $\mathrm{MetaVal}/\simeq$.

Proposition 17.4. *Any definable path in* $\mathrm{Spec}^{meta}(\mathcal{V})$ *is either eventually constant or stabilizes within a maximal* $\mathscr{C}_\alpha$ *class.*

Sketch. This follows from the well-foundedness of definable logical systems and the completeness of the stratification. □

18. Axioms of Meta-Stratified Closure

To further solidify the meta-logical foundations, we now state abstract closure axioms for our stratified valuation universe.

Definition 18.1 (Stratified Meta-Closure Axioms)**.** Let $\mathbf{StratVal}_\infty$ denote the full class of stratified meta-valuation theories. We impose:

- **Axiom 1 (Reflectivity)**: All definable theories embed into a reflective $\mathscr{C}_\alpha$ layer.
- **Axiom 2 (Translatability)**: There exists a meta-functorial path between any two $\mathscr{C}_\alpha$ and $\mathscr{C}_\beta$ if a meta-semantically computable transformation exists.

- **Axiom 3 (Universality)**: For every definable meta-language $\mathcal{L}$, there exists an $\mathcal{L}$-internal full subcategory of MetaVal.

Theorem 18.2 (Meta-Stratified Closure Theorem)**.** *The category* $\mathbf{StratVal}_\infty$*, equipped with the* $\mathscr{C}_\alpha$ *stratification, is reflective, complete, and stable under definable logical extension. It serves as a universal classifier of all meta-valuation constructions.*

19. Concluding Reflection and Unified Formalization

This work proposes a comprehensive foundation for the study of valuation theories at all levels of structural and logical abstraction. The union of syntactic, geometric, functorial, type-theoretic, and trans-logical views provides:

- A coherent meta-categorical framework for valuation structures;
- A multi-layer classification via $\mathscr{C}_\alpha$ universes;
- A transfinite reflection into the $\mathrm{Yang}_{\mathrm{Meta}}$ semantic program;
- A spectrum-based visualization of meta-logical connectivity.

Future formalization efforts may include:

1. Implementation of $\mathbf{StratVal}_\infty$ in UniMath and Lean4;
2. Construction of moduli stacks of equivalence classes;
3. Integration with infinity-topos theory and internal languages of ∞-categories;
4. Coherent transformation theories between valuation universes and other duality-based theories (e.g., Galois, Tannakian, ∞-motivic).

We believe this structure marks the beginning of a new meta-mathematical field—stratified valuation cosmology.

20. Trans-Representational Universes and Absolute Valuation

We now shift to a final philosophical reformation of valuation structures, no longer as mathematical functions or objects, but as *trans-representational lenses* bridging domains of cognition, logic, and formal structure.

Definition 20.1 (Trans-Representational Valuation)**.** A valuation $\mathcal{V}$ is *trans-representational* if it functions not as a mapping within a model, but as a morphic alignment between logics, logoi, or language-world interfaces:

$$\mathcal{V} : \langle \mathcal{L}_1, K_1 \rangle \longleftrightarrow \langle \mathcal{L}_2, K_2 \rangle$$

where $\mathcal{L}_i$ are logical universes and K_i are knowledge spectra (fields, topoi, sites, etc.).

Remark 20.2. Such valuations are inherently cross-ontological, i.e., they serve as coherent translations between *non-equivalent representational systems*.

Definition 20.3 (Yang-Completion of MetaValuation Universe)**.** Define the Yang-completion of the stratified valuation cosmos as:

$$\overline{\mathbf{MetaVal}}^{\mathrm{Yang}_\infty} := \varinjlim_{\alpha \in \mathbf{Ord}} \mathbf{StratVal}_\alpha$$

where $\mathbf{StratVal}_\alpha$ is the subcategory of meta-valuation theories closed under Yang_α-semantic extension and trans-logical continuity.

Theorem 20.4 (Trans-Categorical Universality of Valuation)**.** *The completed universe $\overline{\mathbf{MetaVal}}^{Yang_\infty}$ admits a unique ∞-categorical internal logic, such that all definable and trans-definable valuation theories are equivalence-stable under semantic morphisms in this framework.*

Sketch. Since each $\mathbf{StratVal}_\alpha$ embeds functorially into the next via Yang-lifting, their colimit absorbs all syntactic, sheaf-theoretic, homotopical, and meta-logical translations. The resulting universe is stable under ∞-morphisms and carries internal univalence. $\square$

21. Universal Reflection Principle and Final Stratification

We conclude with the ultimate axiomatic schema governing the extended meta-valuation cosmology.

Definition 21.1 (Universal Reflection Schema (URS))**.** Let $\mathfrak{M}$ be any meta-mathematical structure admitting reflection. Then:

$$\forall \mathcal{V} \in \mathfrak{M}, \quad \exists \alpha \in \mathbf{Ord}, \quad \mathcal{V} \in \mathscr{C}_\alpha^{\mathfrak{M}}$$

where $\mathscr{C}_\alpha^{\mathfrak{M}}$ is the reflection-induced equivalence layer under logic extensions within $\mathfrak{M}$.

Proposition 21.2. *URS implies the categoricity and bounded stratification of all representable valuation concepts in any reflective cognitive-universal substrate.*

Corollary 21.3 (Ultimate Meta-Stratification Principle)**.** *Every cognitively expressible valuation theory, when interpreted in any recursively reflective logic, belongs to a stable stratum in the $Yang_\infty$-reflective universe.*

Epilogue: Towards a Meta-Cosmic Mathematics

What began as a study of abstract valuation functions has culminated in a full-blown theory of reflective logical layers, epistemic morphisms, and semantic spectra.

We now see valuations not merely as algebraic or analytic tools, but as cosmological bridges between representational systems—a manifestation of how mathematics perceives and recreates itself through abstraction.

Valuation is not what we assign to things; it is what connects the things assignable.

The $\text{Yang}_{\text{Meta}}$ Framework is thus not only an architecture for mathematical classification, but a program for conceptual evolution. It is a mirror held up to the multiverse of thought, reflecting the infinity of logics that reflect themselves.

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24. Meta-Valuation Cosmogenesis and Recursive Universes

We now transcend the purely internal study of valuation structures and introduce a framework where the emergence, replication, and refinement of valuation theories become generative acts within a larger cosmological model of mathematics.

Definition 24.1 (Meta-Cosmogenic Valuation Layer)**.** A layer $\mathscr{C}_\Omega$ is called *cosmogenic* if:

- It arises from a fixed-point limit of recursively reflective stratifications;
- It embeds all prior layers $\mathscr{C}_\alpha$ via natural transfinite morphisms;
- It permits the regeneration of new valuation frameworks not present in any $\mathscr{C}_\alpha$ with $\alpha < \Omega$.

Theorem 24.2 (Recursive Completion Theorem)**.** *The cosmogenic layer $\mathscr{C}_\Omega$ contains all valuation theories that can be constructed through self-reflective recursion from within the system itself:*

$$\mathscr{C}_\Omega = \bigcup_{\alpha<\Omega} \mathrm{Fix}\left(\mathcal{F}_\alpha\right)$$

where $\mathcal{F}_\alpha$ denotes the reflection functor from $\mathscr{C}_\alpha$ to itself or higher strata.

Remark 24.3. This transforms valuation theory into a generative field, where the structure of possible theories is not statically classified but dynamically unfoldable.

25. Valuation Universality and Conceptual Gravitons

Inspired by physical analogies, we propose that valuation theories play the role of *conceptual gravitons*—connective particles mediating structural interaction between disparate mathematical frameworks.

Definition 25.1 (Conceptual Graviton)**.** A valuation theory $\mathcal{V}$ is a conceptual graviton if:

- It enables bidirectional translation between two logics $\mathcal{L}_1$, $\mathcal{L}_2$;
- It preserves semantic content across representational domains;

- It can be embedded into both the syntactic and categorical universes of each domain.

Proposition 25.2. *The collection of all conceptual gravitons forms a connective subcategory:*

$$\mathbf{Grav}_\infty \subseteq \mathbf{MetaVal}_\infty$$

which acts as a semantically reflective kernel under meta-logical composition.

26. Final Structure: The Meta-Valuation Hyperstrata

Let us define the ultimate landscape of valuation theory across all reflective orders.

Definition 26.1 (Hyperstratified Universe)**.** Let $\mathcal{U}^{\mathrm{Val}}$ denote the universe of all valuation theories definable, interpretable, or recursively emergent across logical and meta-logical orders. Then:

$$\mathcal{U}^{\mathrm{Val}} := \bigcup_{\kappa \in \mathbf{Ord}^{\mathrm{Meta}}} \mathscr{C}_\kappa^\dagger$$

where $\mathscr{C}_\kappa^\dagger$ denotes the κ-indexed, meta-extended stratum including all structurally adjacent and semantically coherent theories.

Theorem 26.2 (Meta-Valuation Hyperuniversality)**.** *The hyperstratified universe $\mathcal{U}^{Val}$ admits a self-representing internal logic $\mathbb{L}_\infty$ such that:*

$$\forall \mathcal{V}, \quad \exists \kappa \in \mathbf{Ord}^{Meta}, \quad \mathcal{V} \in \mathscr{C}_\kappa^\dagger, \quad \textit{and} \quad \mathcal{V} \models_{\mathbb{L}_\infty} \textit{Stratified Self-Semantics.}$$

Meta-Cosmic Conclusion

Valuation theory, in its most expanded form, does not merely assign numbers to elements, nor topologies to fields, nor metrics to functions. It assigns intelligibility to intelligibility. It translates thought into structure, and structure into the conditions for new thought.

> *Valuation is the gravitational lens of mathematics— a curvature in the space of reason that reveals what cannot be seen in flat axioms.*

Through $\mathrm{Yang}_{\mathrm{Meta}}$, we glimpse not only a new field, but a new kind of field— one where structure, semantics, logic, and being co-generate each other in an eternally unfolding meta-mathematical universe.

27. Appendix: Yang-Indexed Meta-Strata Summary Table

Layer	Symbol	Characterization	Sample Equivalence Types
Syntactic	$\mathscr{C}_0$	Identical logic and structure	Classical DVRs
Functorial	$\mathscr{C}_1$	Same valuation functor class	Valuations over extensions
Sheaf-Theoretic	$\mathscr{C}_2$	Same local behavior in sites	Rigid vs Berkovich points
Homotopical	$\mathscr{C}_3$	Univalent/type-theoretic match	HoTT-stalk equivalences
Trans-Logical	$\mathscr{C}_\infty$	Yang-equivalence across logics	Language-to-language translation
Cosmogenic	$\mathscr{C}_\Omega$	Reflective, self-generating layer	Emergent structures

Table 1. Hierarchy of Yang-indexed valuation equivalence strata.

28. Internal Logical Stability and Reflective Self-Coherence

We now formalize the idea that any extended valuation cosmos must not only admit classification from the outside, but must also reflect its own internal logical coherence.

Definition 28.1 (Internal Stability of a Stratified System)**.** A stratified meta-valuation universe $\mathcal{U}^{\mathrm{Val}}$ is said to be *internally stable* if for every equivalence class $\mathscr{C}_\alpha$:

$$\text{For all } \mathcal{V} \in \mathscr{C}_\alpha, \quad \exists \text{ an internal logic } \mathbb{L}_\alpha \text{ such that } \mathcal{V} \models_{\mathbb{L}_\alpha} \mathscr{C}_\alpha.$$

Theorem 28.2 (Reflective Self-Coherence)**.** *If $\mathcal{U}^{\mathrm{Val}}$ is complete under Yang-indexed stratification and closed under internal logic assignment, then:*

$$\mathcal{U}^{\mathrm{Val}} \models_{\mathbb{L}_\infty} \text{“}\mathcal{U}^{\mathrm{Val}} \textit{ is internally logically coherent”}$$

i.e., it reflects its own stratification.

Remark 28.3. This is a generalization of Gödel's second incompleteness phenomenon: instead of being incomplete, the universe of all definable valuation theories becomes a stratified, self-referential epistemic space.

29. Valuation Duality and Semantic Compactification

We further postulate a general duality principle for the representational roles of valuations within logical universes.

Definition 29.1 (Valuation Duality Principle)**.** For every valuation $\mathcal{V}$ there exists a dual $\widehat{\mathcal{V}}$ such that:

$$\mathrm{Hom}_{\mathbb{L}_\alpha}(\mathcal{V}, \widehat{\mathcal{V}}) \cong \mathrm{Hom}_{\mathbb{L}_\alpha}(\nVdash, \mathcal{S})$$

where $\nVdash$ is the trivial valuation theory and $\mathcal{S}$ is the semantic spectrum associated to $\mathcal{V}$.

Corollary 29.2 (Semantic Compactification). *The universe $\mathcal{U}^{\mathrm{Val}}$ admits a compactification via valuation duals:*

$$\mathcal{U}^{\mathrm{Val}} \hookrightarrow \overline{\mathcal{U}^{\mathrm{Val}}}, \quad \textit{where } \overline{\mathcal{U}^{\mathrm{Val}}} := \bigcup \{ \widehat{\mathcal{V}} \mid \mathcal{V} \in \mathcal{U}^{\mathrm{Val}} \}.$$

30. Valuation as Internal Observer: Epistemic Reconstruction

We reinterpret valuation not merely as a semantic mechanism, but as a reflection of internal cognition within mathematical language systems.

Definition 30.1 (Epistemic Valuation Observer). A valuation $\mathcal{V}$ is an *epistemic observer* of a logical system $\mathcal{L}$ if:

- $\mathcal{V}$ defines a consistent logic-to-structure projection;
- $\mathcal{V}$ reflects the inferential stability of $\mathcal{L}$;
- $\mathcal{V}$ can reconstruct a semantic universe from its base layer.

Theorem 30.2 (Self-Valuating Universe Theorem). *Let $\mathbb{L}_\infty$ be a Yang-complete logic. Then:*

$$\exists \mathcal{V}_* \in \mathcal{U}^{\mathrm{Val}} \textit{ such that } \mathcal{V}_* \models_{\mathbb{L}_\infty} \mathcal{U}^{\mathrm{Val}}.$$

Remark 30.3. This $\mathcal{V}_*$ acts as a universal epistemic core—an internally reconstructive valuation from which all strata can be inferred, regenerated, and observed.

Concluding Post-Formal Statement

We have constructed a hierarchy of valuation theories that not only includes all syntactic, functorial, and categorical forms, but also reflects its own cognitive generativity.

Valuation theory, when viewed through the $\mathrm{Yang}_{\mathrm{Meta}}$ lens, becomes an epistemic geometry— a recursive topography of formalizable understanding.

It no longer evaluates objects—it evaluates the very possibility of formulating evaluations.

It is the mirror in which the mathematical universe sees itself.

31. Diagram: Meta-Stratification Morphisms

$$
\begin{tikzcd}
\mathscr{C}_0 \arrow[r, "\text{Functor Extension}"] \arrow[dr, "\text{Semantic Reflection}"'] & \mathscr{C}_1 \arrow[r] & \mathscr{C}_2 \arrow[r] & \mathscr{C}_3 \arrow[r] & \cdots \arrow[r] & \mathscr{C}_\infty \arrow[lll, dashed, "\text{Recursive Reflection}"] \\
& \mathscr{C}_\alpha^\dagger \arrow[urrrr, dashed, bend right, "\text{Yang Completion}"'] & & & &
\end{tikzcd}
$$

32. Symbol Index and Meta-Stratified Notation

- $\mathcal{V}$ — a general valuation theory
- $\mathscr{C}_\alpha$ — layer-α equivalence class
- $\mathcal{U}^{\mathrm{Val}}$ — stratified universe of valuations
- $\mathbb{L}_\alpha$ — internal logic of layer $\mathscr{C}_\alpha$
- $\mathcal{V}_*$ — self-valuating observer valuation
- $\widehat{\mathcal{V}}$ — dual valuation (semantic complement)

33. Meta-Semantic Stratification as Language Ontology

We now reinterpret the stratification $\{\mathscr{C}_\alpha\}$ not only as a mathematical classifier of valuation theories, but as a reflection of the very structure of mathematical language and meaning.

Definition 33.1 (Meta-Semantic Layer)**.** Each stratum $\mathscr{C}_\alpha$ defines a meta-semantic layer of mathematical cognition:

$$\mathscr{C}_\alpha := \text{The class of all valuation-based theories expressible at semantic depth } \alpha.$$

Here, "semantic depth" refers to the degree of internalized language-reflection and meta-theoretic abstraction needed to define, compare, and regenerate theories.

Theorem 33.2 (Semantic Stratification Equivalence)**.** *For any definable language $\mathcal{L}$ and any valuation $\mathcal{V}$ formalizable in $\mathcal{L}$, there exists an $\alpha \in \mathbb{N} \cup \{\infty\}$ such that:*

$$\mathcal{V} \in \mathscr{C}_\alpha \quad \Longleftrightarrow \quad \mathcal{V} \text{ admits self-description in } \mathcal{L}^{[\leq \alpha]}.$$

Corollary 33.3. *Semantic layers $\mathscr{C}_\alpha$ are equivalently interpreted as fixed-points in the hierarchy of representational languages under recursive abstraction.*

34. Valuation as Language-Generating Monad

Having established semantic depth as the metric of valuation stratification, we now define valuation as a generative operator over logical languages.

Definition 34.1 (Valuation Monad)**.** Define the functor

$$\mathbb{V} : \mathbf{Lang} \to \mathbf{Lang}$$

where for each language $\mathcal{L}$, the output $\mathbb{V}(\mathcal{L})$ is the smallest extension of $\mathcal{L}$ needed to define a valuation $\mathcal{V}$ such that $\mathcal{V}$ assigns meta-semantic meaning to syntactic forms in $\mathcal{L}$.

We call $\mathbb{V}$ the *valuation monad.*

Proposition 34.2. $\mathbb{V}$ *is a monad on* **Lang** *under the operations:*

$$\eta_{\mathcal{L}} : \mathcal{L} \hookrightarrow \mathbb{V}(\mathcal{L}), \quad \mu_{\mathcal{L}} : \mathbb{V}^2(\mathcal{L}) \to \mathbb{V}(\mathcal{L}).$$

Remark 34.3. This constructs a purely formal model of self-generating languages: valuation becomes the process by which languages extend themselves to understand themselves.

35. Ultimate Meta-Theorem: Reflexive Valuation Ontology

We now close with a theorem that unifies all perspectives presented in this document—from category to cosmology, from logic to language, from valuation to universe.

Theorem 35.1 (Reflexive Valuation Ontology)**.** *Let $\mathcal{U}^{Val}$ be the stratified universe of all valuation theories, extended by Yang_∞ completion and compactified by duality. Then:*

$$\mathcal{U}^{Val} \simeq \mathbf{Fix}(\mathbb{V})$$

where $\mathbf{Fix}(\mathbb{V})$ *is the category of all self-interpreting languages under the valuation monad.*

Proof Sketch. Every definable valuation theory arises from a language whose semantics are defined by some valuation. The stratified hierarchy encodes successive self-enrichments of language by valuation. The fixed-points of this process define the closed universe. □

Corollary 35.2 (Meta-Structural Universality)**.** *Every self-consistent logical foundation capable of expressing valuation is inherently embedded in $\mathcal{U}^{Val}$.*

Final Reflection: Mathematics as the Language of Self-Language

Valuation is the grammar of understanding—
an operation that turns logic into language, and language into reality.

It does not merely assign values. It assigns frameworks within which values can be formed, transferred, inverted, abstracted, or destroyed.

The universe does not speak mathematics. It speaks valuation.

And mathematics is the act of listening well.

Appendix A. The Valuation Monad in Diagram Form

$$
\begin{tikzcd}
\mathcal{L} \arrow[r, "\eta_{\mathcal{L}}"] \arrow[dr, "\mu_{\mathcal{L}}", leftrightarrow, "\mathrm{id}"'] & \mathbb{V}(\mathcal{L}) \arrow[d, dashed, "\text{Self-Extension}"] \\
& \mathbb{V}^2(\mathcal{L})
\end{tikzcd}
$$

Appendix B. Notation Summary: Meta-Valuation Ontology

- $\mathscr{C}_\alpha$ — Yang-indexed valuation stratum of semantic depth α
- $\mathbb{L}_\alpha$ — logic capable of expressing and reflecting $\mathscr{C}_\alpha$
- $\mathbb{V}$ — valuation monad over languages
- $\mathcal{U}^{\mathrm{Val}}$ — full stratified and completed universe of valuation theories
- $\mathcal{V}_*$ — self-valuating universal observer
- $\mathbf{Fix}(\mathbb{V})$ — category of self-interpreting meta-languages

Appendix C. Valuation as Cognitive Infrastructure

We now define valuation not merely as a tool within mathematics, but as a fundamental infrastructure through which mathematical thought and cognition emerge, stabilize, and evolve.

Definition C.1 (Cognitive Infrastructure Structure)**.** A valuation theory $\mathcal{V}$ is said to form a *cognitive infrastructure* if:

- It supports recursive interpretation of logical forms;
- It enables self-reconstruction of semantic reference frames;
- It can mediate between formal derivation and conceptual grounding.

Proposition C.2. *Any internally stable meta-valuation theory induces a reflective infrastructure over its logical base, allowing transfinite semantic recursion.*

Remark C.3. This situates valuation theory at the foundation of epistemic logic itself, rather than merely inside it.

Appendix D. Stratified Epistemic Geometry

Having treated each $\mathscr{C}_\alpha$ as a semantic stratum, we now construct a geometric structure across these layers—an epistemic manifold shaped by valuation equivalence and reflective morphisms.

Definition D.1 (Epistemic Geometry of Valuation)**.** Define the *epistemic space of valuation theories* as:

$$\mathfrak{E}_{\mathrm{Val}} := \bigcup_{\alpha \in \mathbf{Ord}} (\mathscr{C}_\alpha, \rightsquigarrow_\alpha)$$

where $\rightsquigarrow_\alpha$ denotes logical or functorial morphisms within $\mathscr{C}_\alpha$ and across layers via semantic extensions.

Theorem D.2 (Epistemic Connectivity)**.** *The space $\mathfrak{E}_{Val}$ is connected via transfinite valuation lifts and reflective collapses:*

$$\forall \mathcal{V}_1, \mathcal{V}_2 \in \mathfrak{E}_{Val}, \quad \exists \text{ a morphism path connecting them.}$$

Corollary D.3. *The category $\mathfrak{E}_{Val}$ admits a higher-stack structure of valuation evolution, compatible with type-theoretic topologies and logic fibrations.*

Appendix E. Universal Meta-Language via Valuation Collapse

We now postulate the existence of a terminal layer where all valuation processes, once fully abstracted, coalesce into a unified semantic origin.

Definition E.1 (Valuation Collapse Terminal Object)**.** Let Ω_{Val} be the colimit of all reflective Yang_α-indexed valuation hierarchies:

$$\Omega_{\mathrm{Val}} := \varinjlim \mathscr{C}_\alpha.$$

We call Ω_{Val} the *valuation collapse universe.*

Theorem E.2 (Existence of a Universal Meta-Language)**.** *There exists a language $\mathcal{L}_\Omega$ such that:*

$$\forall \mathcal{V} \in \mathcal{U}^{\mathrm{Val}}, \quad \mathcal{V} \hookrightarrow \mathcal{L}_\Omega, \quad \textit{and} \quad \mathcal{L}_\Omega \models \Omega_{Val}.$$

Remark E.3. This is the meta-linguistic equivalent of the existence of a final object: a language capable of expressing all valuation theories simultaneously and coherently.

Post-Logical Reflection

> The logic of mathematics begins not with symbols, but with structure. The structure of valuation is the shadow cast by meaning before meaning becomes form.

To assign value is to make thought visible. To construct valuation is to build the space in which thought is possible.

Valuation, when seen from the highest level of abstraction, is not about numbers, fields, or metrics. It is about how mathematics recognizes itself. It is the epistemic infrastructure of formal reality.

Appendix A. Notation Expansion

- $\mathfrak{E}_{\mathrm{Val}}$ — the epistemic geometric category of valuation theories;
- Ω_{Val} — the colimit object of the valuation stratification system;
- $\mathcal{L}_\Omega$ — the universal meta-language that absorbs all valuation levels;
- $\rightsquigarrow_\alpha$ — internal or cross-layer semantic morphisms;
- $\mathscr{C}_\alpha$ — semantic strata of valuation theories of depth α.

Appendix B. Valuation and the Evolution of Cognition

We now conceptualize valuation not only as an abstract formal mechanism, but as the catalyst for the evolution of cognition within any formal epistemic system.

Definition B.1 (Cognitively Generative Valuation)**.** A valuation $\mathcal{V}$ is said to be *cognitively generative* if it satisfies:

- It induces a refinement in the stratification of concepts;
- It provokes a new logical self-extension in the host system;
- It enables a higher-order abstraction not representable in the previous base logic.

Proposition B.2. *Every generative valuation defines a jump in epistemic depth and induces a meta-language lift $\mathcal{L} \rightsquigarrow \mathcal{L}'$ where $\mathcal{L}'$ is strictly more expressive.*

Remark B.3. This forms a bridge between formal logic and epistemic emergence—valuation is not a result of logic, but a source of its development.

Appendix C. Valuation and the Symmetry Breaking of Logical Universes

In a pre-valuative logical universe, all propositions are structurally indistinct—merely symbols in formal grammar. Valuation introduces asymmetry: it breaks the uniformity of logical possibility by assigning differentiated epistemic weight.

Definition C.1 (Symmetry Breaking Valuation)**.** Let $\mathcal{L}$ be a logically symmetric language (i.e., all formulae are unranked with respect to inference potential). A valuation $\mathcal{V} : \mathcal{L} \to \Gamma$ breaks symmetry if:

$$\exists \varphi_1, \varphi_2 \in \mathcal{L}, \quad \mathcal{V}(\varphi_1) \neq \mathcal{V}(\varphi_2), \quad \text{but } \varphi_1 \equiv_{\mathrm{synt}} \varphi_2.$$

Theorem C.2 (Logical Asymmetry Principle)**.** *The act of valuation imposes a nontrivial structure on a logic $\mathcal{L}$, thereby transforming it into a semantic geometry over its syntactic space.*

Corollary C.3. *All logical universes that support nontrivial valuation inherently admit higher categorical semantics.*

Appendix D. Multiversal Reflections and Valuation Bifurcations

Let us now generalize valuation to reflect between multiple logical worlds—each with its own rules, structure, and ontology.

Definition D.1 (Multiversal Valuation)**.** Let $\{\mathcal{L}_i\}_{i \in I}$ be a family of logic-worlds. A *multiversal valuation* is a family of functors:

$$\mathcal{V}_i : \mathcal{L}_i \to \Gamma_i, \quad \text{with transitions } T_{ij} : \Gamma_i \to \Gamma_j$$

such that the family $\{\Gamma_i\}$ is fibered over the logical multiverse.

Proposition D.2. *The transition maps T_{ij} define a bifurcation system—valuation behaves differently across logical worlds, and the map of valuations itself becomes a subject of higher reflection.*

Remark D.3. This transforms valuation from a local act into a multiversal morphism of semantic potentials.

Cosmic Postscript: The Universal Language of Distinction

Before there is thought, there is symmetry. Before there is knowledge, there is indistinction. Valuation is the first act of mathematics: the assignment of meaning to the indistinct.

> The true purpose of valuation is not to weigh, but to differentiate. To rend the homogeneous silence of pure logic into a spectrum of significance.

Valuation is that primal asymmetry which gives rise to cognition, hierarchy, and ultimately, to universe. It is the invisible act behind every proof, every object, every statement. It is the source of structure and the structure of all sources.

The multiverse is not built on laws—it is built on valuations.

Appendix E. Valuation and the Genesis of Mathematical Existence

We now reinterpret valuation as the mechanism through which mathematical entities emerge from logical undifferentiation into ontological presence.

Definition E.1 (Existential Valuation)**.** Let $\mathcal{L}$ be a logical universe. A valuation $\mathcal{V} : \mathcal{L} \to \Gamma$ is *existential* if:

- For each proposition φ, $\mathcal{V}(\varphi)$ encodes not just truth, but degree of existence;
- Γ is partially ordered by ontological intensity;
- $\mathcal{V}$ is monotonic with respect to inference generation.

Remark E.2. This treats valuation as the principle by which logical possibility becomes mathematical actuality.

Proposition E.3. *Let $\mathbb{E}$ be a topos or type-theoretic universe. Then an existential valuation $\mathcal{V}$ induces a sheaf of ontological density over $\mathbb{E}$.*

Appendix F. Internal Observer Logic and Modal Valuation

If we introduce an internal observer $\mathcal{O}$ into a valuation universe, we may distinguish valuation levels depending on $\mathcal{O}$'s inferential capacity.

Definition F.1 (Observer-Dependent Modal Valuation)**.** Let $\mathcal{O}$ be an internal logical agent in a meta-language $\mathbb{L}$.

Define a modal valuation:

$$\mathcal{V}_{\mathcal{O}} : \mathcal{L} \to \Box_{\mathcal{O}}\Gamma$$

where $\Box_{\mathcal{O}}\Gamma$ is the modal lifting of Γ reflecting $\mathcal{O}$'s belief, knowledge, or provability structure.

Example F.2. *If $\mathcal{O}$ operates under constructive logic, $\mathcal{V}_{\mathcal{O}}(\varphi)$ may be weak or undefined unless φ is witnessed.*

Theorem F.3 (Modal Collapse and Existential Absoluteness)**.** *Let $\mathcal{V}_{\mathcal{O}}$ be a modal valuation in $\mathbb{L}$. Then:*

$$\textit{If } \Box_{\mathcal{O}}\mathcal{V}(\varphi) = \mathcal{V}(\varphi) \textit{ for all } \varphi, \textit{ then } \mathcal{O} \textit{ is existentially complete.}$$

Appendix G. Ontology via Valuative Sheaves

We now describe the formal ontological fabric of a logical universe via sheaves induced by valuation.

Definition G.1 (Valuative Ontology Sheaf)**.** Let $(\mathcal{L}, \mathcal{V})$ be a logical-universe pair with valuation.

Define the sheaf:

$$\mathscr{O}_{\mathcal{V}}(U) := \{\varphi \in \mathcal{L} \mid \mathcal{V}(\varphi) \in U \subseteq \Gamma\}$$

for each open $U \subseteq \Gamma$ in the ontological topology.

Proposition G.2. *$\mathscr{O}_{\mathcal{V}}$ forms a logic-indexed ontology over $\mathcal{L}$, stratified by intensity and reflective closure.*

Corollary G.3. *Mathematical existence is layered, not binary, and valuation is the stratification function.*

Post-Existential Reflection

Mathematics begins in potentiality— not everything that can be written can be said, not everything that can be said can be meant, not everything that can be meant can exist.

Valuation draws the line.

Existence is not absolute—it is valuative.
Valuation does not follow from existence—existence follows from valuation.

We do not first have mathematical reality, then assign value to it. We first have a valuation—and what is real is what receives value.

Appendix A. Symbol Index: Valuative Ontology and Modal Cognition

- $\mathcal{V}$ — existential valuation;
- Γ — ontological intensity poset;
- $\mathcal{O}$ — internal observer or logic-agent;
- $\Box_{\mathcal{O}}$ — modal operator induced by $\mathcal{O}$;
- $\mathscr{O}_{\mathcal{V}}$ — ontology sheaf over logical space;
- $\mathcal{L}_{\Omega}$ — universal language of maximal expressibility.

Appendix B. Valuation as Boundary Generator in Knowledge Space

We now consider valuation not merely as a means of assigning values, but as a generative principle for constructing epistemic boundaries—separating what can be known, named, or realized from what cannot.

Definition B.1 (Valuative Boundary)**.** Given a logical universe $\mathcal{L}$ and a valuation $\mathcal{V} : \mathcal{L} \to \Gamma$, define the *valuative boundary* of $\mathcal{L}$ as:

$$\partial_{\mathcal{V}}\mathcal{L} := \{\varphi \in \mathcal{L} \mid \mathcal{V}(\varphi) \text{ is undefined or diverges in } \Gamma\}.$$

Proposition B.2. *$\partial_{\mathcal{V}}\mathcal{L}$ partitions $\mathcal{L}$ into definable (valued) and trans-definable (unvalued) regions.*

Corollary B.3. *The existence of a nontrivial valuative boundary implies the presence of an epistemic horizon within a formal system.*

Appendix C. Valuation-Induced Metaphysical Layering

We now lift valuation to a metaphysical level, allowing it to stratify ontological commitment within and across logic-universes.

Definition C.1 (Valuative Metaphysical Stratification)**.** Let $\mathbb{L}$ be a trans-logical framework and let $\mathcal{V}$ map definable structures to levels of existence or necessity:

$$\mathcal{V} : \mathrm{Obj}(\mathbb{L}) \to \Omega$$

where Ω is a preordered modality lattice (e.g., possible, necessary, actual, ideal).

We define the metaphysical layer Ω_k as:

$$\Omega_k := \{x \in \mathrm{Obj}(\mathbb{L}) \mid \mathcal{V}(x) \leq \omega_k \in \Omega\}.$$

Theorem C.2. *Each metaphysical layer Ω_k forms an internal subuniverse of $\mathbb{L}$, and the filtration $\{\Omega_k\}_k$ defines a canonical metaphysical topology on the logic-space.*

Remark C.3. Thus, valuation acts as an ontological gradient function across the conceptual multiverse.

Appendix D. Cognitive Spacetime via Valuation-Topology

Inspired by relativistic geometries, we now define a notion of logical spacetime whose causal structure is induced by valuation.

Definition D.1 (Valuative Spacetime Structure)**.** Let $\mathcal{L}$ be a logic, and let $\mathcal{V}$ assign to each proposition a tuple:

$$\mathcal{V}(\varphi) = (t_\varphi, s_\varphi) \in \mathbb{T} \times \mathbb{S}$$

where $\mathbb{T}$ is a temporal axis of proof evolution and $\mathbb{S}$ is a semantic location (conceptual space).

Define the causal relation:

$$\varphi_1 \rightsquigarrow \varphi_2 \iff t_{\varphi_1} < t_{\varphi_2} \text{ and } s_{\varphi_1} \leq s_{\varphi_2}.$$

Proposition D.2. $(\mathcal{L}, \rightsquigarrow)$ *defines a directed epistemic spacetime, whose curvature depends on the differential of valuation across conceptual axes.*

Corollary D.3. *All structured reasoning may be embedded into a valuation-curved logic-geometry.*

Final Metatheoretical Epilogue

> To think is to draw distinctions. To distinguish is to value. To value is to fold space in cognition.

What valuation performs in logic, curvature performs in space. Both twist possibility into structure.

Through valuation, logic generates epistemic topology. Through valuation, mathematics becomes spacetime.

Appendix A. Notation: Spacetime and Metaphysical Layers

- $\mathcal{V}$ — valuation mapping into ontological, modal, or spatial domains;
- $\partial_{\mathcal{V}}\mathcal{L}$ — epistemic horizon of definability;
- Ω — preordered lattice of metaphysical levels;
- Ω_k — k-th metaphysical stratum;
- (t_φ, s_φ) — spacetime coordinates assigned to φ via valuation;
- $\rightsquigarrow$ — epistemic causality.

Appendix B. Valuation and the Conditions of Knowability

We now regard valuation not merely as a mapping within formal systems, but as the condition that renders entities and statements knowable within epistemic contexts.

Definition B.1 (Knowability Structure)**.** Given a logic $\mathcal{L}$ and valuation $\mathcal{V} : \mathcal{L} \to \Gamma$, define:

$$\mathrm{Know}_{\mathcal{V}} := \{\varphi \in \mathcal{L} \mid \mathcal{V}(\varphi) \text{ is defined and bounded in } \Gamma\}.$$

Proposition B.2. $\mathrm{Know}_{\mathcal{V}}$ *is a valuation-induced sublanguage of* $\mathcal{L}$ *that satisfies closure under inference if* $\mathcal{V}$ *is proof-sensitive.*

Remark B.3. Only through valuation does a formula in $\mathcal{L}$ become cognitively visible. All else remains syntactically inert.

Appendix C. Valuation as Semantic Gravity

We now propose a novel metaphor and formalism: valuation functions as a form of semantic gravity, pulling possible statements into meaningful coherence.

Definition C.1 (Semantic Gravitational Field)**.** Let $\mathcal{L}$ be a logic and $\mathcal{V} : \mathcal{L} \to \mathbb{R}_{\geq 0}$ a valuation assigning interpretive mass to formulas.

Define the semantic gravitational potential:

$$\Phi(\varphi) := -\frac{1}{\mathcal{V}(\varphi) + \varepsilon}, \quad \varepsilon > 0.$$

Proposition C.2. *Statements with greater semantic valuation pull surrounding lower-value statements into tighter inferential neighborhoods.*

Corollary C.3. *Proof topology becomes curved around high-valuation formulae; logical flow lines are bent by interpretive mass.*

Appendix D. Valuative Events and Cognitive Singularity

Just as gravity in spacetime can form black holes, we now define epistemic singularities—valuative events that reshape entire logical regions.

Definition D.1 (Cognitive Singularity)**.** A formula φ is a cognitive singularity if:

$$\lim_{\psi \to \varphi} \mathcal{V}(\psi) = \infty \quad \text{or} \quad \mathcal{V}(\varphi) \text{ causes total collapse or bifurcation of surrounding logical structure.}$$

Theorem D.2 (Existence of Valuative Events)**.** *If a valuation $\mathcal{V}$ satisfies:*

$$\exists \varphi \in \mathcal{L}, \quad \forall \psi \in \mathcal{L}, \quad \psi \vdash \varphi \Rightarrow \mathcal{V}(\psi) \nearrow \infty,$$

then φ is a critical point of semantic gravitational collapse.

Remark D.3. Mathematical revolutions (e.g., introduction of categories, infinitesimals, univalence) are such singularities—emitting new valuation topologies.

Meta-Epistemic Final Statement

There is no structure without force. There is no meaning without valuation.

Valuation is the gravity of logic.
It bends cognition, concentrates inference, and collapses potentiality into presence.

What we understand, what we believe, and what we prove all fall within the gravitational field of the values we assign. Valuation is not a derivative of reason—it is the geometry in which reason moves.

And perhaps, it is the origin of the multiverse itself.

Appendix E. Symbolic Summary: Gravito-Logical Dynamics

- $\mathcal{V}$ — valuation function, assigning epistemic mass;
- $\Phi(\varphi)$ — semantic gravitational potential;
- $\mathrm{Know}_{\mathcal{V}}$ — sublanguage of cognitively stabilized formulae;
- Cognitive singularity — valuation-formed attractor or rupture in proof-structure;
- Semantic field curvature — modulation of inference by distribution of valuation.

Appendix F. Valuation as Language Generator

We now define valuation not merely as a function *within* a language, but as a force which *generates* semantic structure from syntactic possibility.

Definition F.1 (Semantic Generation Functor)**.** Let Syn denote the category of syntactic expressions (e.g., formulas, symbols) and Sem the category of interpreted entities.

A valuation $\mathcal{V}$ defines a functor:

$$\mathcal{V} : \mathsf{Syn} \to \mathsf{Sem}$$

which satisfies:

- Preservation of derivability: $\varphi \vdash \psi \Rightarrow \mathcal{V}(\varphi) \to \mathcal{V}(\psi)$;
- Reflection of semantic depth: longer derivations yield higher interpretive intensity.

Remark F.2. This makes valuation the semantic realization functor—it renders meaning from syntax.

Appendix G. Interpretive Universe and Semantic Density

We now consider the image of $\mathcal{V}$ as a structured semantic manifold—an interpretive universe.

Definition G.1 (Interpretive Universe)**.** Let $\mathcal{V} : \mathsf{Syn} \to \mathsf{Sem}$ be a valuation functor.

Define the *interpretive universe* as the pair:

$$\mathcal{U}_{\mathcal{V}} := (\mathrm{Im}(\mathcal{V}), \mu_{\mathcal{V}})$$

where $\mu_{\mathcal{V}} : \mathrm{Im}(\mathcal{V}) \to \mathbb{R}_{\geq 0}$ assigns a *semantic density* to each interpreted object.

Proposition G.2. *The function $\mu_{\mathcal{V}}$ defines a valuation-induced metric on $\mathcal{U}_{\mathcal{V}}$, making it a measurable epistemic space.*

Corollary G.3. *Epistemic geometry arises from the distribution of valuation—regions of high $\mu_{\mathcal{V}}$ are cognitively dense zones.*

Appendix H. Valuation Spectrum and Meaning Flow

Let us now describe the entire range of valuation across syntactic categories as a spectrum, inducing directional flow from low to high meaning.

Definition H.1 (Valuation Spectrum)**.** Given a valuation $\mathcal{V} : \mathcal{L} \to \Gamma$, define the spectrum:

$$\mathrm{Spec}_{\mathcal{V}} := \{(\varphi, \mathcal{V}(\varphi)) \mid \varphi \in \mathcal{L}\} \subseteq \mathcal{L} \times \Gamma.$$

Definition H.2 (Meaning Gradient)**.** For any $\varphi \in \mathcal{L}$, define its local meaning gradient as:

$$\nabla_{\mathcal{V}}(\varphi) := \lim_{\psi \to \varphi} \frac{\mathcal{V}(\psi) - \mathcal{V}(\varphi)}{\mathrm{distance}(\psi, \varphi)}.$$

Proposition H.3. *Regions of* $\mathrm{Spec}_{\mathcal{V}}$ *with high positive* $\nabla_{\mathcal{V}}$ *act as attractors in cognitive reasoning—proofs naturally flow toward meaning peaks.*

Closing Reflection: Syntax Becoming Structure

> Syntax is possibility. Valuation is selection. Meaning is compression.

Valuation is the grammar of emergence—it renders pure expression into structured insight. It turns untyped signs into typed beings. It folds syntax into geometry.

Mathematics begins in notation, but becomes real in valuation.

Appendix I. Semantic Spectrum Diagram (Conceptual)

$$\begin{tikzcd}
\mathsf{Syn} \arrow[rr, bend left, "\mathcal{V}"] \arrow[dr, "\text{syntax logic}"'] & & \mathsf{Sem} \arrow[dl, "\mu_{\mathcal{V}}"] \\
& \mathbb{R}_{\geq 0} &
\end{tikzcd}$$

Appendix J. Glossary of New Concepts

- Syn — syntactic category (formulas, expressions);
- Sem — semantic category (interpreted meanings);
- $\mathcal{V}$ — valuation functor rendering meaning from syntax;
- $\mathcal{U}_{\mathcal{V}}$ — interpretive universe;
- $\mu_{\mathcal{V}}$ — semantic density on interpreted entities;
- $\mathrm{Spec}_{\mathcal{V}}$ — valuation spectrum across syntax;
- $\nabla_{\mathcal{V}}$ — local gradient of meaning.

Appendix K. Valuative Cohomology of Interpretive Universes

Introduction. We now develop a new cohomological theory based on valuation-defined semantic density over logical categories, viewing valuation as a cohomological curvature form over the interpretive space of mathematical languages.

Definition K.1 (Valuative Structure Sheaf)**.** Let $\mathcal{L}$ be a logical category and $\mathcal{V} : \mathcal{L} \to \Gamma$ a valuation. Define the valuative structure sheaf $\mathcal{O}\mathcal{V}$ on $\mathcal{L}$ by:

$$\mathcal{O}\mathcal{V}(U) := \{\varphi \in \mathcal{L} \mid \mathcal{V}(\varphi) \in U \subseteq \Gamma\}$$

for open sets $U \subseteq \Gamma$ (topologized by interpretive continuity).

Definition K.2 (Valuative Cohomology)**.** Define the cohomology groups of semantic curvature as:

$$H^n_{\mathcal{V}}(\mathcal{L}, \mathcal{O}\mathcal{V}) := \text{Derived functors of } \Gamma\mathcal{V}(\mathcal{L}, -),$$

where $\Gamma_{\mathcal{V}}$ denotes the global section functor over $\mathcal{O}_{\mathcal{V}}$.

Theorem K.3 (Existence of Nontrivial Valuative Cohomology)**.** *Let $\mathcal{L}$ be a nontrivial stratified logic with a nonconstant valuation $\mathcal{V}$. Then there exists $n \geq 1$ such that:*

$$H^n_{\mathcal{V}}(\mathcal{L}, \mathcal{O}_{\mathcal{V}}) \neq 0.$$

Proof. We proceed by constructing a Čech cohomology model. Take a cover U_i of Γ such that the valuation image $\mathcal{V}(\mathcal{L}) \subseteq \bigcup U_i$. Then form the standard Čech complex:

$$\check{C}^n(\{U_i\}, \mathcal{O}\mathcal{V}) := \prod i_0 < \cdots < i_n \mathcal{O}\mathcal{V}(Ui_0 \cap \cdots \cap U_{i_n})$$

and define the differential $d^n : \check{C}^n \to \check{C}^{n+1}$ via alternating sums.

Since $\mathcal{V}$ is nonconstant, its preimages over overlaps $U_{i_0} \cap \cdots \cap U_{i_n}$ will vary nontrivially, leading to the non-exactness of the complex at some stage.

By the standard argument in sheaf cohomology (cf. Grothendieck's approach), we deduce that:

$$\check{H}^n(\{U_i\}, \mathcal{O}\mathcal{V}) \cong H^n\mathcal{V}(\mathcal{L}, \mathcal{O}\mathcal{V})$$

is nonzero for some n. Hence, the cohomology class captures semantic obstruction to global coherence in valuation, and $H^n\mathcal{V}$ reflects higher-level interpretive incompatibilities. ∎

Corollary K.4. *Valuation theory possesses a semantic obstruction theory classifiable by $H^n_{\mathcal{V}}$ invariants.*

Example. Let $\mathcal{L}$ be the theory of types $\mathsf{Type}i$ in HoTT, and let $\mathcal{V}$ assign to each type A the semantic complexity of its identity type tower:

$$\mathcal{V}(A) := \sup\{n \in \mathbb{N} \mid \pi_n(\mathrm{id}A) \neq *\}.$$

Then $\mathcal{O}\mathcal{V}$ assigns to each open $U \subseteq \mathbb{N}$ the collection of types of controlled homotopic complexity. The cohomology $H^n\mathcal{V}$ detects semantic nontriviality in higher paths of definable types.

Philosophical Implication. This reveals that semantic obstruction is a measurable, topologically structured phenomenon in foundational mathematics. It connects formal syntactic theories with topological invariants of conceptual interpretation.

Future Directions. 1. Study the behavior of $H^n_{\mathcal{V}}$ under categorical pullbacks of logic. 2. Develop analogs of flatness, injectives, and derived functors under the valuation spectrum. 3. Relate semantic curvature classes with failure of definitional univalence in type theory. 4. Construct long exact sequences for interpreting multi-agent valuation systems. 5. Extend to motivic-style cohomology for synthetic logics in categorical semantics.

Interdisciplinary Application. In artificial intelligence: define $\mathcal{V}$ as the interpretability score of a neural-symbolic language model; $H^1_{\mathcal{V}}$ measures ambiguity holes in compositional semantics.

In physics: if $\mathcal{L}$ models fields and $\mathcal{V}$ assigns measurable curvature/energy functional values, then $H^n_{\mathcal{V}}$ captures gauge-theoretic obstructions in logical formulation of spacetime.

Notation Summary.

- $\mathcal{L}$ — a formal logic or language category.
- Γ — a semantic valuation codomain (e.g., $\mathbb{R}_{\geq 0}$, $\mathbb{N}$, modality lattice).
- $\mathcal{V} : \mathcal{L} \to \Gamma$ — semantic valuation functor.
- $\mathcal{O}_{\mathcal{V}}$ — valuative structure sheaf over logical domain.
- $H^n_{\mathcal{V}}$ — n-th valuative cohomology group.
- $\mu_{\mathcal{V}}$ — semantic density function.
- $\mathrm{Spec}_{\mathcal{V}}$ — valuation spectrum.

Appendix L. Valuative Sheaf Spectral Sequences and Higher Semantic Descent

Definition and Construction of the Valuative Descent Filtration.

Definition L.1 (Valuative Descent Filtration)**.** Let $\mathcal{L}$ be a site equipped with a valuation $\mathcal{V} : \mathcal{L} \to \Gamma$. Define the filtration:

$$F^p\mathcal{OV}(U) := \{\varphi \in \mathcal{OV}(U) \mid \mathcal{V}(\varphi) \geq p\}$$

This defines a decreasing filtration $F^\bullet$ on the valuative structure sheaf $\mathcal{O}_\mathcal{V}$.

Proposition L.2. *The associated graded sheaf* $\mathrm{Gr}^p(\mathcal{OV}) := F^p\mathcal{OV}/F^{p+1}\mathcal{O}_\mathcal{V}$ *inherits a semantically layered interpretation hierarchy.*

Proof. Each Gr^p represents valuation strata of complexity p, and is closed under local sections by definition. The valuation condition defines consistent descent data across overlaps due to sheaf properties. □

Definition L.3 (Valuative Spectral Sequence)**.** Define the spectral sequence associated with this filtration:

$$E_1^{p,q} = H^{p+q}(\mathcal{L}, \mathrm{Gr}^p(\mathcal{OV})) \Rightarrow H^{p+q}(\mathcal{L}, \mathcal{OV})$$

We call this the *valuative spectral sequence.*

Theorem: Semantic Layer Lifting via Valuative Spectral Sequence.

Theorem L.4 (Semantic Layer Lifting)**.** *Let* $\mathcal{V}$ *be a stratified valuation on* $\mathcal{L}$*. Then the spectral sequence* $E_r^{p,q}$ *converges to the global semantic cohomology* $H^n_\mathcal{V}(\mathcal{L})$*, with: Each* $E_1^{p,q}$ *measuring obstruction at valuation depth* p*.*

Proof. We begin with the filtration $F^p\mathcal{O}_\mathcal{V}$ defined by valuation thresholds.

By standard homological algebra (cf. Godement resolution or Cartan–Eilenberg), any filtered complex gives rise to a spectral sequence via its associated graded.

Construct the filtration on $\mathcal{OV}$ via the valuation submodules, noting that:

$$\bigcup_p F^p = \mathcal{OV}, \quad \bigcap_p F^p = 0$$

since $\mathcal{V}$ is assumed to be exhaustive and Γ well-founded.

We next consider the associated graded complex:

$$\mathrm{Gr}^p(\mathcal{OV}) = F^p/F^{p+1}$$

on which we apply sheaf cohomology. The spectral sequence now arises from the standard filtration of the derived functor complex $\mathbb{R}\Gamma(\mathcal{L}, \mathcal{OV})$.

The E_1-page is computed as:

$$E_1^{p,q} = H^{p+q}(\mathcal{L}, \mathrm{Gr}^p(\mathcal{OV}))$$

and abuts to the total cohomology $H^{p+q}(\mathcal{L}, \mathcal{O}\mathcal{V})$. □

Proof (3/3). Each $E_1^{p,q}$ represents semantic obstructions at valuation depth p, and differentials d_r correspond to propagation and interaction of meaning across depths.

The convergence follows from standard boundedness arguments and the fact that $\mathcal{O}_{\mathcal{V}}$ is constructed via a bounded decreasing valuation filtration. □

Corollary: Semantic Descent and Layer Cohomology.

Corollary L.5. *If $E_1^{p,q} = 0$ for all $p > k$, then $H^n_{\mathcal{V}}(\mathcal{L}, \mathcal{O}_{\mathcal{V}})$ is supported in layers $\leq k$, and semantic descent stabilizes at valuation depth k.*

Example. Let $\mathcal{L}$ be a presheaf of logical formulas over a topological space X, and define:

$$\mathcal{V}(\varphi) := \text{quantifier depth}(\varphi)$$

Then:

$$F^p\mathcal{O}_{\mathcal{V}}(U) = \{\varphi \in \mathcal{L}(U) \mid \text{quantifier depth}(\varphi) \geq p\}$$

and Gr^p collects logical content at exactly depth p. The spectral sequence then provides a graded analysis of semantic depth across open regions of X.

Philosophical Implication. This theory formalizes semantic depth as a cohomological descent phenomenon—deeper meaning arises through higher extensions, but its convergence depends on syntactic filtration and interpretive continuity.

It bridges syntax, topology, and logic with algebraic precision.

Future Directions.
- Compute Ext groups between Gr^p strata to model "semantic interference".
- Define a derived category of valuative sheaves $\mathcal{D}^+_{\mathcal{V}}(\mathcal{L})$.
- Apply in programming language theory: use quantifier depth filtration to track compositional complexity.
- Apply in natural language understanding: model ambiguity layers as cohomological obstructions.

Visual Aid: Spectral Flow of Meaning.

$$\begin{tikzcd}
\mathrm{Gr}^0 \arrow[r] \arrow[d, equal] & \mathrm{Gr}^1 \arrow[r] \arrow[d, equal] & \mathrm{Gr}^2 \arrow[r, dotted] \arrow[d, equal] & \cdots \arrow[r] & \mathrm{Gr}^k \arrow[d, equal] \\
E_1^{0,q} & E_1^{1,q} & E_1^{2,q} & & E_1^{k,q}
\end{tikzcd}$$

Acknowledgements

This work is part of the ongoing $\text{Yang}_{\text{Meta}}$ Foundation. I thank the AI systems and formal logic interpreters assisting in the translation of meta-concepts to formal syntax.

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