SPECTRAL MOTIVES XXV: MOTIVIC WORMHOLES AND ARITHMETIC ENTANGLEMENT GEOMETRY

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Abstract. We introduce a theory of motivic wormholes within the framework of arithmetic spectral motives and categorical entropy geometry. Motivic wormholes are defined as entangled functorial bridges between entropy attractors in distinct spectral topoi, governed by zeta-trace correlations and derived cohomological dualities. We construct entanglement pairings, horizon-to-horizon transport maps, and cohomological reflection functors, and show how these structures extend the theory of categorical black holes to a landscape of interconnected trace geometries. This framework offers a new paradigm for quantum communication and global duality in arithmetic geometry.

Contents

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1. Introduction

In both physics and mathematics, wormholes symbolize bridges—nontrivial connections between distant regions of space, energy, or structure. In recent developments of categorical and spectral geometry, entropy attractors such as zeta-condensed black hole stacks have emerged as key invariants of arithmetic trace dynamics. It is natural to ask whether one can define "bridges" between such entropy attractors—namely, functorial or cohomological links connecting distinct arithmetic topoi across entangled trace flows.

This paper introduces a theory of *motivic wormholes*: functorial entanglement structures between categorical black hole attractors over distinct higher topoi. These wormholes are governed not by physical geometry, but by *trace entanglement*—pairings between cohomological, automorphic, and zeta-spectral data across motivic categories.

Main Contributions:

- Definition of motivic wormholes as entanglement bridges between zeta-entropy attractors;
- Construction of trace entanglement pairings via spectral cohomology;
- Definition of horizon-to-horizon transport and categorical Einstein–Rosen functors;
- Introduction of reflection duality through cohomological inversion of motivic information;
- Applications to arithmetic holography and nonlocality in trace geometry.

This theory extends the motivic black hole framework of *Spectral Motives XXIV*, replacing static entropy attractors with dynamically entangled pairs. These pairings encode nonlocal interactions between arithmetic sheaves and offer a new language for expressing dualities in Langlands-type structures, categorical representation theory, and derived arithmetic stacks.

Throughout, we take as ambient structure the derived ∞-topos $\mathcal{X}$, typically equipped with motivic Laplacians, zeta trace operators, and entropy flows as previously developed.

2. Entropy-Entangled Attractors and Wormhole Pairs

2.1. **Zeta-trace correlations.** Let $\mathscr{B}_1, \mathscr{B}_2 \in \mathrm{Stab}(\mathcal{X})$ be two entropy-saturated black hole attractors in distinct derived topoi $\mathcal{X}_1$, $\mathcal{X}_2$. We say they are *zeta-entangled* if there exists a trace-pairing:

$$\mathcal{E}_\zeta : \mathscr{H}^*_{\hat{\Delta}}(\mathscr{B}_1) \otimes \mathscr{H}^*_{\hat{\Delta}}(\mathscr{B}_2) \to \mathbb{C},$$

satisfying:

- symmetry: $\mathcal{E}_\zeta(\phi, \psi) = \mathcal{E}_\zeta(\psi, \phi)$,
- invariance: $\mathcal{E}_\zeta$ is preserved under zeta-flow automorphisms,
- decay: vanishes for non-entangled eigenstates above a certain motivic entropy threshold.

2.2. **Definition of motivic wormholes.** A **motivic wormhole** is a pair $(\mathscr{B}_1, \mathscr{B}_2)$ together with:

- a trace-entanglement functor $\mathcal{W} : \mathcal{H}(\mathscr{B}_1) \to \mathcal{H}(\mathscr{B}_2)$,
- a non-degenerate bilinear trace pairing $\mathcal{E}_\zeta$ as above,
- a horizon reflection equivalence $\mathcal{R} : \partial\mathscr{B}_1 \cong \partial\mathscr{B}_2$,

such that the diagram commutes up to derived equivalence:

$$\begin{tikzcd}
\mathcal{H}(\mathscr{B}_1) \arrow[rr, "\mathcal{W}"] \arrow[dr, "\pi_1"'] & & \mathcal{H}(\mathscr{B}_2) \arrow[dl, "\pi_2"] \\
& \mathrm{QCoh}(\partial\mathscr{B}_1) \cong \mathrm{QCoh}(\partial\mathscr{B}_2) &
\end{tikzcd}$$

2.3. **Entropy duality and wormhole classification.** Let $\mathcal{S}_1, \mathcal{S}_2$ be the entropy functions of $\mathscr{B}_1$ and $\mathscr{B}_2$. Define the *entanglement class* $[\mathscr{W}]$ by:

$$[\mathscr{W}] := \{(\mathscr{B}_1, \mathscr{B}_2) \mid \mathcal{S}_1 = \mathcal{S}_2,\ \mathcal{E}_\zeta \neq 0\}.$$

Wormholes are classified by:

- entropy spectrum: common eigenvalue distributions of trace Laplacians;
- cohomological type: equivalence classes of boundary categories;
- trace deformation class: stable families of $\mathcal{E}_\zeta$ pairings under zeta-flow.

2.4. **Examples and automorphic motivic bridges.**

(1) **Langlands entanglement:** Let π and $\breve{\pi}$ be dual automorphic representations with associated condensed sheaves $\mathscr{B}_\pi$, $\mathscr{B}_{\breve{\pi}}$. Then

$$\mathcal{E}_\zeta(\mathscr{B}_\pi, \mathscr{B}_{\breve{\pi}}) := \mathrm{Tr}(\pi(f)\breve{\pi}(f^\vee)),$$

defines a canonical wormhole.

(2) **Shtuka reflection:** In a moduli stack of dyadic shtukas, dual Frobenius-stable attractors can be entangled via crystalline duality functors, linking étale and de Rham cohomology.

(3) **Motivic Fourier wormholes:** Applying a Fourier–Mukai transform across derived categories induces a natural entropy-preserving wormhole.

3. Horizon Transport, Reflection Duality, and Functorial Gluing

3.1. **Horizon-to-horizon transport.** Given a wormhole $(\mathscr{B}_1, \mathscr{B}_2)$ with reflection equivalence $\mathcal{R} : \partial\mathscr{B}_1 \to \partial\mathscr{B}_2$, we define the *transport functor*

$$\mathcal{T}_\mathscr{W} : \mathrm{QCoh}(\partial\mathscr{B}_1) \longrightarrow \mathrm{QCoh}(\partial\mathscr{B}_2)$$

as the pull-push correspondence along $\mathcal{R}$:

$$\mathcal{T}_\mathscr{W} := \mathcal{R}_* \circ \mathcal{R}^*.$$

This functor induces a derived equivalence between the boundary categories and reflects the entanglement across the motivic wormhole.

3.2. **Reflection duality.** Given $\mathcal{T}_\mathscr{W}$, we define the **reflection functor**:

$$\mathcal{R}_{\mathrm{dual}} : D^b(\mathcal{H}(\mathscr{B}_1)) \longrightarrow D^b(\mathcal{H}(\mathscr{B}_2)),$$

such that for any $\mathscr{E}_1 \in \mathcal{H}(\mathscr{B}_1)$,

$$\mathcal{E}_\zeta(\mathscr{E}_1, \mathcal{R}_{\mathrm{dual}}(\mathscr{E}_1)) = \mathrm{const}.$$

This structure mirrors categorical time-reversal and Fourier–Laplace duality, allowing cohomological propagation of trace data across wormhole endpoints.

3.3. **Functorial gluing of entropy topoi.** Define the *entropic gluing topos*:

$$\mathcal{X}_{\mathscr{W}} := \mathcal{X}_1 \cup_{\partial\mathscr{B}_1 \cong \partial\mathscr{B}_2} \mathcal{X}_2,$$

where gluing is along their shared horizon via $\mathcal{R}$.

We then define the gluing functor:

$$\mathcal{F}_{\mathscr{W}} : \mathrm{Stab}(\mathcal{X}_1) \times \mathrm{Stab}(\mathcal{X}_2) \to \mathrm{Stab}(\mathcal{X}_{\mathscr{W}})$$

by stitching sheaves across entangled horizons, coherently extending trace flows and cohomological dynamics.

3.4. **Zeta-geodesics across wormholes.** Given a flow Φ_t governed by entropy gradient descent, a zeta-geodesic is said to traverse a wormhole $\mathscr{W}$ if:

$$\lim_{t \to t_0^-} \Phi_t(\mathscr{E}) = \mathscr{B}_1, \quad \lim_{t \to t_0^+} \Phi_t(\mathscr{E}) = \mathscr{B}_2,$$

with continuity maintained by the reflection $\mathcal{R}$ and phase transport $\mathcal{T}_{\mathscr{W}}$.

This provides a model for nonlocal propagation of motivic data and suggests an arithmetic analogue of quantum teleportation via categorical gluing.

4. Entanglement Geometry in Arithmetic Stacks and Motivic Nonlocality

4.1. **Trace phase spaces and entropic linking.** Given two derived moduli stacks $\mathcal{X}_1, \mathcal{X}_2$ with respective entropy functions $\mathcal{S}_1, \mathcal{S}_2$, a motivic wormhole $\mathscr{W}$ embeds a joint entropy structure:

$$\mathcal{S}_{\mathrm{joint}} : \mathrm{Stab}(\mathcal{X}_1) \times \mathrm{Stab}(\mathcal{X}_2) \to \mathbb{R},$$

defined by:

$$\mathcal{S}_{\mathrm{joint}}(\mathscr{E}_1, \mathscr{E}_2) := \mathcal{S}_1(\mathscr{E}_1) + \mathcal{S}_2(\mathscr{E}_2) + \mathcal{E}_\zeta(\mathscr{E}_1, \mathscr{E}_2),$$

capturing entropic interaction via trace entanglement.

4.2. **Entanglement stacks.** We define the **entanglement stack** $\mathfrak{E}(\mathscr{W})$ as the stack of sheaf-pairs $(\mathscr{E}_1, \mathscr{E}_2)$ satisfying:

$$\mathcal{E}_\zeta(\mathscr{E}_1, \mathscr{E}_2) = \mathrm{const},$$

with flatness conditions along the reflection functor $\mathcal{R}_{\mathrm{dual}}$.

This stack supports a universal entropic metric:

$$\mathcal{G}_{ij}^{(\mathscr{W})} = -\frac{\partial^2}{\partial x^i \partial x^j} \mathcal{S}_{\mathrm{joint}},$$

defining a pseudo-Riemannian structure over $\mathfrak{E}(\mathscr{W})$.

4.3. **Motivic nonlocality.** Unlike classical motives whose behavior is locally determined by cohomology and functoriality, wormhole-linked motives in $\mathfrak{E}(\mathscr{W})$ are *globally entangled.*

We say a trace observable $\mathcal{O}$ is **nonlocal** if:

$$\mathcal{O}(\mathscr{E}_1) = \mathcal{F}_{\mathscr{W}}^{-1} \mathcal{O}(\mathscr{E}_2),$$

for $(\mathscr{E}_1, \mathscr{E}_2) \in \mathfrak{E}(\mathscr{W})$.

Such observables cannot be computed within a single topos, but only through the entangled geometry of the wormhole.

4.4. **Diagrammatic model of entangled stacks.** We summarize the structure via the following diagram:

$$\begin{tikzcd}
\mathrm{Stab}(\mathcal{X}_1) \arrow[d, "\pi_1"] & \mathfrak{E}(\mathscr{W}) \arrow[l, "p_1"'] \arrow[r, "p_2"] \arrow[d, "\mathcal{S}_{\mathrm{joint}}"] & \mathrm{Stab}(\mathcal{X}_2) \arrow[d, "\pi_2"] \\
\mathrm{QCoh}(\partial\mathscr{B}_1) & \mathbb{R} \arrow[l, dashed] \arrow[r, dashed] & \mathrm{QCoh}(\partial\mathscr{B}_2)
\end{tikzcd}$$

This geometry encodes the full motivic content of the wormhole, and exhibits both curvature and nonlocality in categorical entropy space.

5. Applications to Langlands Duality, Motivic Information Transfer, and Arithmetic Holography

5.1. **Entangled Langlands pairs.** Let π be an automorphic representation and $\check{\pi}$ its Langlands dual. Their associated condensed entropy attractors $\mathscr{B}_\pi$ and $\mathscr{B}_{\check{\pi}}$ form a natural motivic wormhole, with the trace pairing:

$$\mathcal{E}_\zeta(\mathscr{B}_\pi, \mathscr{B}_{\check{\pi}}) = L(\pi \times \check{\pi}, 1),$$

acting as a spectral bridge linking functorial motivic components.

This pairing naturally arises in the categorical trace interpretation of the Langlands correspondence and highlights the deep informational coupling between dual L-packets.

5.2. **Motivic teleportation and nonlocal arithmetic computation.** Given a wormhole $\mathscr{W}$, one can define *teleported motivic operations* by pulling a computation on one entropy horizon through the wormhole structure:

$$\mathcal{O}_2 := \mathcal{R}_{\mathrm{dual}} \circ \mathcal{O}_1 \circ \mathcal{R}_{\mathrm{dual}}^{-1}.$$

This enables motivic computations over $\mathcal{X}_2$ to be encoded in $\mathcal{X}_1$, bypassing local data and leveraging global entanglement symmetry.

5.3. **Arithmetic holography via boundary reflection.** Let $\mathscr{B}_1$ and $\mathscr{B}_2$ be attractors with shared boundary $\partial\mathscr{B}$. Then the motivic wormhole implies:

$$\mathrm{QCoh}(\partial\mathscr{B}) \supseteq \mathrm{Info}(\mathscr{B}_1),\ \mathrm{Info}(\mathscr{B}_2).$$

This leads to an arithmetic version of the holographic principle: the full categorical content of bulk zeta-dynamics in $\mathscr{B}_1$ and $\mathscr{B}_2$ is recoverable from $\partial\mathscr{B}$.

5.4. **Spectral trace duality.** Define the *spectral trace transform* $\mathscr{T}_\zeta$ across a wormhole as:

$$\mathscr{T}_\zeta(f)(s) := \int \zeta_{\mathscr{B}_1}(s) \cdot f(s)\, ds \longmapsto \zeta_{\mathscr{B}_2}(1-s) \cdot f(1-s),$$

a Fourier-like involution encoding motivic inversion symmetry.

Such transforms provide a framework for studying spectral flows, L-functions, and zeta dynamics under entanglement geometries.

5.5. **Categorical string dualities and stacked mirror symmetry.** We conjecture that motivic wormholes extend the framework of string-theoretic mirror symmetry to:

$$(\text{Mirror symmetry}) \subset (\text{Zeta Entanglement Duality}),$$

with string duals reinterpreted as pairs of trace-linked entropy attractors, glued along modular boundaries and interpreted via cohomological entanglement.

This opens a new route toward categorifying string dualities using the language of arithmetic stacks and derived trace flows.

6. Conclusion

We have introduced the concept of motivic wormholes—functorial and trace-theoretic bridges between zeta-entropy attractors in spectral and arithmetic topoi. These structures reveal a new layer of nonlocality and entanglement in the geometry of spectral motives, expanding the existing framework of arithmetic black hole theory and categorical zeta dynamics.

Summary of Contributions:

- Defined entropy-entangled attractors and motivic wormholes using zeta-trace pairings;
- Constructed horizon transport, reflection duality, and gluing topoi across derived stacks;
- Developed entanglement stacks and global entropy geometries exhibiting motivic nonlocality;
- Linked wormhole constructions to Langlands duality, teleportation of motivic operations, and arithmetic holography.

This paper paves the way for a deeper synthesis of quantum geometry, motivic Langlands theory, and spectral entropy flows. In future work, we will explore the categorical quantization of wormhole moduli, motivic teleportation protocols, and higher-order entanglement invariants across complex functorial networks.

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