LECTURE NOTES FOR IHÉS SUMMER SCHOOL "ANALYTIC NUMBER THEORY" JULY 09–23 , 2014

JUSTIN SCARFY
scarfy@ugrad.math.ubc.ca

AND

QIANG ZHANG
qzhang@math.ubc.ca

Department of Mathematics
The University of British Columbia
Room 121, 1984 Mathematics Road
Vancouver, British Columbia, Canada V6T 1Z2

ABSTRACT. These are the lecture notes taken and polished for the summer school on analytic number theory held at the IHÉS between July 09 and 23, 2014. We would like to thank the speakers for teaching us these high-power machineries and techniques from the top of the line, the IHÉS staff for its hospitality, and Professors Greg Martin, Jiangya Liu, Lior Silberman for teaching us the basics in this exciting field.

CONTENTS

1. The orbital circle method and applications (1/2) by Jean Bourgain 4
2. Bounded gaps between primes (1/3) by Terrence Tao 5
3. L-functions (1/4) by Kennen Soundararajan 6
4. The pretentious approach to analytic number theory (1/3) by Andrew Granville 7
5. The orbital circle method and applications (2/2) by Jean Bourgain 8
6. L-functions (2/4) by Kennen Soundararajan 9
7. L-functions (3/4) by Kennen Soundararajan 10

8. The pretentious approach to analytic number theory (2/3)
by Andrew Granville 11
9. Bounded gaps between primes (2/3)
by Terrence Tao 12
10. Sifting for primes (1/3)
by Henryk Iwaniec 13
11. L-functions (4/4)
by Kennen Soundararajan 14
12. The pretentious approach to analytic number theory (3/3)
by Andrew Granville 15
13. Bounded gaps between primes (3/3)
by Terrence Tao 16
14. Sifting for primes (2/3)
by Henryk Iwaniec 17
15. Automorphic forms for $\mathrm{GL}(2)$ (1/4)
by Philippe Michel 18
16. Sifting for primes (3/3)
by Henryk Iwaniec 19
17. Analytic number theory around torsion homology (1/4)
by Akshay Venkatesh 20
18. Analytic aspects of Cohen-Lenstra heuristics (1/4)
by Étienne Fouvry 21
19. Automorphic forms for $\mathrm{GL}(2)$ (2/4)
by Philippe Michel 22
20. Growth in groups and applications (1/4)
by Harald Helfgott 23
21. Analytic number theory around torsion homology (2/4)
by Akshay Venkatesh 24
22. Analytic aspects of Cohen-Lenstra heuristics (1/4)
by Étienne Fouvry 25
23. Automorphic forms in higher rank (1/4)
by Valentin Blomer 26
24. Growth in groups and applications (1/4)
by Harald Helfgott 27
25. Trace functions over finite fields (1/4)
by Emmanuel Kowalski 28
26. Automorphic forms for $\mathrm{GL}(2)$ (3/4)
by Philippe Michel 29
27. Automorphic forms in higher rank (2/4)
by Valentin Blomer 30

28. Analytic number theory around torsion homology (3/4)
by Akshay Venkatesh 31
29. Trace functions over finite fields (2/4)
by Emmanuel Kowalski 32
30. Analytic aspects of Cohen-Lenstra heuristics (2/4)
by Étienne Fouvry 33
31. Automorphic forms for $\mathrm{GL}(2)$ (4/4)
by Philippe Michel 34
32. Growth in groups and applications (3/4)
by Harald Helfgott 35
33. Analytic number theory around torsion homology (3/4)
by Akshay Venkatesh 36
34. Automorphic forms in higher rank (3/4)
by Valentin Blomer 37
35. Analytic aspects of Cohen-Lenstra heuristics (4/4)
by Étienne Fouvry 38
36. Trace functions over finite fields (3/4)
by Emmanuel Kowalski 39
37. Automorphic forms in higher rank (4/4)
by Valentin Blomer 40
38. Growth in groups and applications (4/4)
by Harald Helfgott 41
39. Trace functions over finite fields (4/4)
by Emmanuel Kowalski 42

1. THE ORBITAL CIRCLE METHOD AND APPLICATIONS (1/2)

BY JEAN BOURGAIN

2. Bounded gaps between primes (1/3)
by Terrence Tao

3. L-FUNCTIONS (1/4)

BY KENNEN SOUNDARARAJAN

4. The pretentious approach to analytic number theory (1/3)
by Andrew Granville

5. THE ORBITAL CIRCLE METHOD AND APPLICATIONS (2/2)
BY JEAN BOURGAIN

6. L-FUNCTIONS (2/4)

BY KENNEN SOUNDARARAJAN

7. L-FUNCTIONS (3/4)

BY KENNEN SOUNDARARAJAN

8. The pretentious approach to analytic number theory (2/3)
by Andrew Granville

9. Bounded gaps between primes (2/3)
BY TERRENCE TAO

10. SIFTING FOR PRIMES (1/3)

BY HENRYK IWANIEC

11. L-FUNCTIONS (4/4)

BY KENNEN SOUNDARARAJAN

12. The pretentious approach to analytic number theory (3/3)
by Andrew Granville

13. Bounded gaps between primes (3/3)
by Terrence Tao

14. SIFTING FOR PRIMES (2/3)

BY HENRYK IWANIEC

15. Automorphic forms for $\mathrm{GL}(2)$ (1/4)

by Philippe Michel

16. SIFTING FOR PRIMES (3/3)
BY HENRYK IWANIEC

17. Analytic number theory around torsion homology (1/4)
by Akshay Venkatesh

18. Analytic aspects of Cohen-Lenstra heuristics (1/4)
by Étienne Fouvry

19. Automorphic forms for $\mathrm{GL}(2)$ (2/4)
by Philippe Michel

20. Growth in groups and applications (1/4)
BY HARALD HELFGOTT

21. Analytic number theory around torsion homology (2/4)
by Akshay Venkatesh

22. Analytic aspects of Cohen-Lenstra heuristics (1/4)
by Étienne Fouvry

23. AUTOMORPHIC FORMS IN HIGHER RANK (1/4)
BY VALENTIN BLOMER

24. Growth in groups and applications (1/4)
BY HARALD HELFGOTT

25. TRACE FUNCTIONS OVER FINITE FIELDS (1/4)

BY EMMANUEL KOWALSKI

26. Automorphic forms for $\mathrm{GL}(2)$ (3/4)
BY PHILIPPE MICHEL

27. AUTOMORPHIC FORMS IN HIGHER RANK (2/4)
BY VALENTIN BLOMER

28. Analytic number theory around torsion homology (3/4)
by Akshay Venkatesh

29. Trace functions over finite fields (2/4)

by Emmanuel Kowalski

30. Analytic aspects of Cohen-Lenstra heuristics (2/4)
by Étienne Fouvry

31. Automorphic forms for $\mathrm{GL}(2)$ (4/4)
by Philippe Michel

32. Growth in groups and applications (3/4)
BY HARALD HELFGOTT

33. Analytic number theory around torsion homology (3/4)
by Akshay Venkatesh

34. AUTOMORPHIC FORMS IN HIGHER RANK (3/4)
BY VALENTIN BLOMER

35. Analytic aspects of Cohen-Lenstra heuristics (4/4)
by Étienne Fouvry

36. Trace functions over finite fields (3/4)

by Emmanuel Kowalski

37. AUTOMORPHIC FORMS IN HIGHER RANK (4/4)
BY VALENTIN BLOMER

38. Growth in groups and applications (4/4)
BY HARALD HELFGOTT

39. Trace functions over finite fields (4/4)

by Emmanuel Kowalski