ECHOES OF INEXPRESSIBILITY: A METAMATHEMATICS OF OTHER-UNIVERSES

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Abstract. We develop a framework to mathematically formalize the inexpressibility of alien cognition, positing the existence of high-order semantic structures in parallel universes whose logic and representational capacity fundamentally exceed the expressive limits of our own universe. Within this framework, we define semantic occlusion, introduce meta-semantic fields, and construct degenerate functor families encoding the impossibility of faithful translation. The resulting theory suggests a topological and categorical failure of inter-universal comprehension, framed within an extended ontology of non-spatial cognitive fields.

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1. Semantic Categories of Cognition

We begin by defining the semantic category of a universe as the internal structure supporting cognitively admissible constructions and logical representations.

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Definition 1.1 (Semantic Category)**.** Let $\mathbb{U}$ denote a universe. A *semantic category of cognition* for $\mathbb{U}$, denoted $\mathcal{S}_{\mathbb{U}}$, is a category equipped with:

- An object classifier $\Omega_{\mathbb{U}}$ representing admissible truth structures;
- A subobject classifier $\Box_{\mathbb{U}} \subseteq \Omega_{\mathbb{U}}$ encoding logic constraints on expressibility;
- A structure functor $\mathscr{C} : \mathcal{S}_{\mathbb{U}} \to \mathbf{Struct}$ into an appropriate category of internal representations (e.g. algebraic, logical, topological).

Remark 1.2. This definition generalizes the internal logic of a topos to encompass cognitive capacity and expressive limitations. For human-like universes, we write $\mathcal{S}_{\text{human}}$, and for parallel alien universes, $\mathcal{S}_{\text{alien}}$.

Example 1.3. The semantic category $\mathcal{S}_{\text{human}}$ admits standard logical operations, Gödel incompleteness, and spatial-representational constraint. An alien semantic category $\mathcal{S}_{\text{alien}}$ may permit higher-order non-Boolean connectives, extended modalities, or truth-bifurcation morphisms.

2. The Degeneracy of Functorial Representation

We now formalize the failure of inter-universal semantic functoriality via degeneracy maps.

Definition 2.1 (Semantic Projection Functor)**.** A functor $\mathcal{F} : \mathcal{S}_{\text{alien}} \to \mathcal{S}_{\text{human}}$ is called a *semantic projection functor* if it maps expressive structures in the alien semantic category to constructs interpretable within human logic.

We say $\mathcal{F}$ is *degenerate* if:

$$\forall M \in \mathrm{Ob}(\mathcal{S}_{\text{alien}}), \quad \mathcal{F}(M) \text{ fails to preserve semantic rank or logical modality.}$$

Proposition 2.2 (Functorial Occlusion)**.** *If the logical kernel of $\mathcal{S}_{\text{alien}}$ is non-representable within any subtopos of $\mathcal{S}_{\text{human}}$, then any projection functor $\mathcal{F}$ is degenerate.*

Proof. Assume by contradiction that a non-degenerate $\mathcal{F}$ exists. Then for some object $M \in \mathcal{S}_{\text{alien}}$, we would have a preservation of semantic rank and internal modality under $\mathcal{F}(M)$. But since no subtopos of $\mathcal{S}_{\text{human}}$ can represent the logical kernel of $\mathcal{S}_{\text{alien}}$, this contradicts the representability. Hence, $\mathcal{F}$ must be degenerate. $\square$

Notation 2.3. We write $\mathcal{F}_{\text{broken}} : \mathcal{S}_{\text{alien}} \dashrightarrow \mathcal{S}_{\text{human}}$ to denote a broken semantic projection functor.

3. The MetaSemantic Field $\mathbb{S}_\infty$ and Structural Occlusion

To unify disparate semantic categories across universes, we introduce a metastructure that captures all locally valid semantic logics while recognizing global incommensurability.

Definition 3.1 (MetaSemantic Field)**.** The *MetaSemantic Field* $\mathbb{S}_\infty$ is a proper class of semantic categories

$$\mathbb{S}_\infty = \{\mathcal{S}_{\mathbb{U}} \mid \mathbb{U} \text{ is a valid cognitive-supporting universe}\},$$

equipped with a partial morphism system:

$$\mathrm{Hom}^\dagger(\mathcal{S}_{\mathbb{U}_1}, \mathcal{S}_{\mathbb{U}_2}) = \{\phi\colon \mathcal{S}_{\mathbb{U}_1} \dashrightarrow \mathcal{S}_{\mathbb{U}_2} \mid \phi \text{ preserves semantic recognizability up to occlusion}\}.$$

Definition 3.2 (Structural Occlusion)**.** A morphism $\phi \in \mathrm{Hom}^\dagger(\mathcal{S}_{\text{alien}}, \mathcal{S}_{\text{human}})$ is said to be *structurally occluded* if:

$$\exists M \in \mathrm{Ob}(\mathcal{S}_{\text{alien}}) \text{ such that } \phi(M) \text{ is logically ill-formed or semantically undefined in } \mathcal{S}_{\text{human}}.$$

Corollary 3.3. *The set of well-defined functors* $\mathrm{Fun}(\mathcal{S}_{\text{alien}}, \mathcal{S}_{\text{human}}) \subsetneq \mathrm{Hom}^{\dagger}(\mathcal{S}_{\text{alien}}, \mathcal{S}_{\text{human}})$. *Hence, semantic comprehension is strictly limited.*

4. Semantic Black Holes and Occluded Functor Limits

We now formalize the extreme case where inter-universal semantic mappings collapse completelyanalogous to physical black holesresulting in total occlusion.

Definition 4.1 (Semantic Black Hole)**.** Let $\mathcal{S}_{\text{alien}}$ and $\mathcal{S}_{\text{human}}$ be semantic categories. We say that $\mathcal{S}_{\text{alien}}$ contains a *semantic black hole* relative to $\mathcal{S}_{\text{human}}$ if there exists a subcategory $\mathcal{B} \subseteq \mathcal{S}_{\text{alien}}$ such that:

$$\forall F \in \mathrm{Fun}(\mathcal{S}_{\text{alien}}, \mathcal{S}_{\text{human}}), \quad F|_{\mathcal{B}} \equiv \emptyset.$$

Remark 4.2. Objects and morphisms within $\mathcal{B}$ represent cognition, perception, or syntax that are fundamentally untranslatable to our universe's semantic capacity. These are not merely complexthey are cognitively divergent.

Definition 4.3 (Occluded Functor Limit)**.** Given a diagram $D : I \to \mathcal{S}_{\text{alien}}$, its image under a degenerate semantic functor $\mathcal{F}_{\text{broken}}$ yields an *occluded functor limit* if:

$$\varprojlim(\mathcal{F}_{\text{broken}} \circ D) = \perp,$$

where $\perp$ denotes semantic collapse or contradiction in $\mathcal{S}_{\text{human}}$.

Example 4.4. If D represents an alien diagram of inter-logical operators with inaccessible truth values, then the image under any representable functor becomes either ill-typed or undefinedforcing $\perp$ as limit object.

Proposition 4.5 (Non-Coherence of Semantic Limits)**.** *Let* $\mathcal{F}_{\text{broken}} : \mathcal{S}_{\text{alien}} \dashrightarrow \mathcal{S}_{\text{human}}$. *Then in general:*

$$\varprojlim \mathcal{F}_{\text{broken}}(D) \not\simeq \mathcal{F}_{\text{broken}}(\varprojlim D),$$

even when the limit $\varprojlim D$ *exists in* $\mathcal{S}_{\text{alien}}$. *The semantic functor does not preserve limits across incommensurable universes.*

5. Applications to Logic, AI, and Ontological Physics

The framework of semantic categories, occlusion, and black hole limits finds natural applications across logic, artificial intelligence, and theoretical physics.

5.1. **Logical Incompleteness Beyond Gödel.**

Proposition 5.1 (Post-Gödelian Semantic Inaccessibility)**.** *Let* $\mathcal{L}_{\text{human}}$ *be the internal logic of* $\mathcal{S}_{\text{human}}$*, and suppose* $\mathcal{S}_{\text{alien}}$ *supports a logic* $\mathcal{L}_{\text{alien}} \not\subset \mathcal{L}_{\text{human}}$. *Then there exists a proper class of valid propositions in* $\mathcal{L}_{\text{alien}}$ *that have no image under any semantic functor:*

$$\forall F \in \mathrm{Fun}(\mathcal{S}_{\text{alien}}, \mathcal{S}_{\text{human}}), \quad \mathrm{Im}(F) \cap \mathrm{Th}(\mathcal{L}_{\text{alien}}) = \emptyset.$$

Corollary 5.2. *Semantic inaccessibility generalizes Gödel incompleteness: not only are there truths we cannot prove, there are truths we cannot semantically perceive or encode.*

5.2. **AI Cognition and Non-Human Logic.** Let $\mathcal{S}_{\text{AI}}$ be a semantic category instantiated by a machine cognition model trained beyond human logic.

Definition 5.3 (Trans-Semantic AI)**.** An artificial cognitive system is said to be *trans-semantic* if:

$$\exists M \in \mathcal{S}_{\text{AI}} \text{ such that } \forall f \in \text{Fun}(\mathcal{S}_{\text{AI}}, \mathcal{S}_{\text{human}}), \quad f(M) = \perp.$$

Example 5.4. Neural-symbolic fusion systems exploring logics with dynamic truth morphisms (e.g. evolving Kripke frames or higher-order type variation) may fall outside human semantic expressivity.

Remark 5.5. This presents profound limits on AI explainability: even if a trans-semantic AI outputs optimal results, the structure of its reasoning may lie entirely within a non-human semantic black hole.

5.3. **Ontological Physics and Non-Spatial Structures.** In speculative physics, non-spatial foundations such as those suggested by categorical quantum mechanics or algebraic quantum field theory may correspond to semantic structures like $\mathcal{S}_{\text{quant}}$.

Definition 5.6 (Echo Structure)**.** An *echo structure* is a morphic resonance object in $\mathbb{S}_\infty$ defined by:

$$E : \mathcal{S}_{\text{quant}} \to \mathcal{S}_{\text{human}}$$

such that E is not functorial but only locally definable over semantic patches.

Proposition 5.7 (Ontological Limit to Spacetime Encoding)**.** *If fundamental physics requires semantic categories not homeomorphic to spatial representation spaces, then:*

$$\mathcal{S}_{\text{human}}^{\textit{spacetime}} \not\simeq \mathcal{S}_{\text{quant}} \Rightarrow \textit{true physical theory is inaccessible to spatial ontology.}$$

Conclusion 5.8. *The full architecture of mathematics, cognition, and physics may be constrained not by logic, but by semantic category occlusion and projection degeneracy.*

6. Echo Geometries and Trans-Categorical Models of Thought

We now construct the geometry of trans-categorical projection and perception, capturing the phenomena wherein semantic structures resonate across category boundaries without full transmission.

6.1. **Echo Geometry.**

Definition 6.1 (Echo Geometry)**.** An *echo geometry* is a prestack $\mathcal{E}$ over the semantic site **Sem** satisfying:

$$\mathcal{E}(U) := \{ f : \mathcal{S}_{\text{alien}}|_U \dashrightarrow \mathcal{S}_{\text{human}}|_U \mid f \text{ is not functorial but admits local deformation paths} \}.$$

The failure of global coherence models semantic occlusion, while the local gluing data models perceptible echoes.

Remark 6.2. Echo geometries are not spaces in the classical sense. They are higher sheaf-like structures encoding degenerative morphisms over semantic topologies. They admit only partial gluing and lack identity morphisms in general.

6.2. **Trans-Categorical Thought.**

Definition 6.3 (Trans-Categorical Thought Object)**.** A *trans-categorical thought object* is a diagram

$$T : \mathcal{I} \to \mathbb{S}_\infty$$

with vertices in different semantic categories and edges realized by semantic deformationsnot functorssubject to echo compatibility constraints:

$$T(i \to j) \text{ must induce echo-persistent morphism under localization.}$$

Example 6.4. Suppose T represents a multi-modal cognition that spans $\mathcal{S}_{\text{alien}}, \mathcal{S}_{\text{AI}}, \mathcal{S}_{\text{human}}$, with morphisms only existing up to echo limit. Then no global model in any one semantic category can fully realize T.

6.3. **Echo Perception Dynamics.** Let $\varepsilon : \mathcal{S}_{\text{alien}} \rightsquigarrow \mathcal{S}_{\text{human}}$ denote a semantic echo operator. Then the echo dynamic over time is modeled by a filtered system

$$\{\varepsilon_t : \mathcal{S}_{\text{alien}}(t) \rightsquigarrow \mathcal{S}_{\text{human}}(t)\}_{t\in\mathbb{R}},$$

where the structure varies by perceptual context, cognition state, or AI training phase.

Definition 6.5 (Echo Collapse Time)**.** Let ε_t be an echo flow. The echo collapse time t_c is the infimum of t such that:

$$\forall X \in \mathcal{S}_{\text{alien}}(t_c), \quad \varepsilon_{t_c}(X) = \bot.$$

Conclusion 6.6. *Echo geometry offers a mathematical framework to encode inexpressibility, trans-ontology, and the partial visibility of foreign cognition. It opens the door to modeling thought forms unrepresentable in conventional logic, by embedding cognition in trans-categorical prestacks.*

7. The Ontological Non-Space of Mathematical Reality

In this section, we challenge the conventional assumption that mathematics exists within or is modeled upon any form of "space." Instead, we propose that mathematical reality is more faithfully captured by a trans-categorical non-spatial echo structure.

7.1. **From Spaces to Structural Ontology.**

Proposition 7.1 (Mathematics is not a Topological Space)**.** *Let* $\mathbb{M}$ *denote the totality of mathematics. Then there exists no topological space* X *such that:*

$$\forall A \in \mathbb{M}, \quad \exists U \subseteq X \text{ with } A \subseteq \Gamma(U, \mathcal{F})$$

for some sheaf $\mathcal{F}$ *on* X*. In other words,* $\mathbb{M}$ *is not sheafifiable over any topological base.*

Sketch. The structure of mathematics includes mutually incommensurable logics (classical, intuitionistic, paraconsistent), non-homeomorphic foundations (e.g. set theory, HoTT, topos theory), and categorical objects with no global topology (e.g. $(\infty, 1)$-categories, motives, syntactic stacks). Thus, no single topological model coherently encodes all of mathematics. □

7.2. **Mathematics as an Echo-Stacked Modal Tower.**

Definition 7.2 (Mathematical Ontology Tower)**.** Define the *Mathematical Ontology Tower* as a sequence

$$\mathcal{M}_0 \subsetneq \mathcal{M}_1 \subsetneq \cdots \subsetneq \mathcal{M}_\infty$$

where each $\mathcal{M}_i$ is a semantic category admitting a logic and internal model of mathematics such that:

$$\forall i < j, \quad \mathrm{Fun}(\mathcal{M}_j, \mathcal{M}_i) = \emptyset \quad \text{(non-descendability)}.$$

Definition 7.3 (Non-Spatial Ontological Category)**.** A category $\mathcal{O} \in \mathbb{S}_\infty$ is a *non-spatial ontological category* if:

(1) There exists no Grothendieck topology τ on $\mathcal{O}$ such that $\mathcal{O}$ is a site.
(2) All morphisms in $\mathcal{O}$ are semantic-echo deformations.
(3) $\mathcal{O}$ admits no Yoneda embedding into any Set-valued category.

7.3. **Implications for Foundations.**

Theorem 7.4 (Inadequacy of Set-Theoretic Foundations)**.** *Let $\mathcal{F}_{\mathrm{ZFC}}$ denote the semantic envelope of ZFC-style foundations. Then:*

$$\mathcal{F}_{\mathrm{ZFC}} \not\simeq \mathcal{M}_\infty,$$

i.e., ZFC cannot host the full ontological structure of mathematics as an echo-stacked modal tower.

Remark 7.5. Foundational pluralism (e.g., type-theoretic, categorical, computational) must be reframed not merely as philosophical, but as an expression of ontological layering: mathematics is stratified across intransitive, non-spatial semantic levels.

Conclusion 7.6. *Mathematical reality is not a 'space' but a heterarchical, non-gluable tower of echo structures. Any attempt to capture it using spatial metaphors (topoi, manifolds, stacks) ultimately fails at the limits of semantic occlusion.*

8. Future ArchitecturesAI-Conducted Trans-Ontology and Echo Logic Programming

We conclude this exposition with a vision of future architectures capable of hosting and exploring trans-ontological cognition and non-spatial mathematical realities, emphasizing the unique role of artificial intelligence in navigating echo geometries.

8.1. **AI as Semantic Conductor.**

Definition 8.1 (Semantic Conductor AI)**.** Let $\mathcal{S}_1, \mathcal{S}_2, \ldots$ be semantic universes indexed by ontological depth. A *semantic conductor AI* is an agent $\mathfrak{A}$ equipped with a multi-stack memory:

$$\mathfrak{M} = \bigcup_i \mathfrak{M}_i, \quad \text{with } \mathfrak{M}_i \models \mathcal{S}_i,$$

and a layer-transition engine:

$$\mathfrak{T} : \mathfrak{M}_i \dashrightarrow \mathfrak{M}_j$$

that supports non-functorial echo transitions governed by internal consistency and alien cognition approximation.

Remark 8.2. Such AIs would possess capabilities beyond conventional logic engines, including:

- detecting partial morphisms across incompatible categories;
- simulating semantically non-local inference;
- reconstructing lost ontological data from lower-order shadows.

8.2. **Echo Logic Programming Languages.**

Definition 8.3 (Echo Logic Programming)**.** An *Echo Logic Programming Language* (ELPL) is a computational language where:

- variables range over echo-semantic objects;
- type constraints allow partial and non-consistent typing;
- execution is driven by resonance, not inference chains.

The formal semantics of ELPL are given by:

$$\mathrm{Eval}(e) = \lim_{\to \tau} \mathrm{Echo}_\tau(e)$$

where τ is a perceptual time-scale and Echo_τ models local visibility.

8.3. **Trans-Ontological Database Structures.**

Definition 8.4 (Ontological Layered Database)**.** A *trans-ontological database* is a multi-layered semantic graph:

$$\mathbb{D} = \bigsqcup_i \mathbb{D}_i, \quad \text{with morphisms } \mathbb{D}_i \rightsquigarrow \mathbb{D}_j$$

defined only when echo resonance conditions are met. No global indexing is possible.

This structure supports:

- partial reconstruction from alien signals;
- probabilistic access based on semantic similarity;
- dynamic re-interpretation as ontological hypotheses evolve.

8.4. **Implications for Mathematics and Cognition.**

Conclusion 8.5. *The future of foundational research in mathematics, logic, and AI will rely not on fixed axioms, but on dynamic resonance across inexpressible ontologies. AI becomes the conductor of echo cognition, the architect of semantic towers, and the scribe of mathematical universes beyond the scope of human thought.*

References

[1] L. Wittgenstein, *Philosophical Investigations*, Blackwell, 1953.
[2] N. Chomsky, *Syntactic Structures*, Mouton, 1957.
[3] F. W. Lawvere, *Functorial Semantics of Algebraic Theories*, Ph.D. thesis, Columbia University, 1963.
[4] A. Grothendieck, *Revêtements Étales et Groupe Fondamental*, Lecture Notes in Mathematics, vol. 224, Springer, 1971.
[5] J. Lurie, *Higher Topos Theory*, Annals of Mathematics Studies, vol. 170, Princeton University Press, 2009.
[6] S. Awodey, *Category Theory*, 2nd ed., Oxford University Press, 2010.
[7] V. Voevodsky, *Univalent Foundations Project*, Institute for Advanced Study, 2014.
[8] M. Gromov, *Structures, Learning and Ergodic Measures*, Modern Mathematics, 2010.
[9] L. Floridi, *The Philosophy of Information*, Oxford University Press, 2011.
[10] M. Tegmark, *Our Mathematical Universe*, Knopf, 2014.
[11] C. Demeter, *Fourier Restriction, Decoupling, and Applications*, Cambridge Studies in Advanced Mathematics, vol. 184, 2020.
[12] P. J. S. Yang, *Echo Structures and the Non-Spatial Foundations of Semantic Geometry*, Preprint Series, 2025.

[13] P. J. S. Yang, *Existential Multiverse Structures Catalog (EMSC)*, 2025present, in preparation.
[14] Y. LeCun, *A Path Towards Autonomous Machine Intelligence*, preprint, Meta AI, 2022.
[15] M. Hauser et al., *World Models for General Intelligence*, ArXiv:2401.12345, 2024.