A DYNAMICAL DEFORMATION APPROACH TO THE RIEMANN HYPOTHESIS

PU JUSTIN SCARFY YANG

ABSTRACT. We construct a deformation framework for understanding the emergence of the Riemann zeta function's critical line symmetry. Using the parameterized Dirichlet product family

$$L_t(s) := \prod_p \left(1 - \frac{1}{p^s}\right)^{-t}, \quad t \in [0,1],$$

we define and study the associated modulus-squared field $\mathcal{F}_t(s)$ and provide numerical, analytic, and variational evidence for the convergence of its modulus minima to $\Re(s) = 1/2$. We propose a formal attractor principle and outline a program for AI-assisted theorem verification. A newly observed "tortoise and hare" effect provides insight into the dynamical geometry of zero formation not previously captured in classical literature.

CONTENTS

1. Deformation Setup and Motivation

We study the family:

$$L_t(s) := \prod_p \left(1 - \frac{1}{p^s}\right)^{-t}$$

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and its logarithmic modulus square:

$$\mathcal{F}_t(s) := \log |L_t(s)|^2 = 2t \cdot \Re \left[\sum_p \sum_{k=1}^{\infty} \frac{1}{k} \cdot \frac{1}{p^{ks}} \right].$$

This field encodes interference structure which becomes increasingly focused as $t \to 1^-$. Our goal is to study its minima (modulus valleys) and show they converge toward the critical line $\Re(s) = \frac{1}{2}$.

2. The Functional Equation and $\Xi_t(s)$

We approximate a completed version:

$$\Xi_t(s) := \pi^{-s/2}\Gamma(s/2) \cdot L_t(s), \quad \text{such that} \quad \Xi_1(s) = \Xi(s).$$

We conjecture that symmetry:

$$\Xi_1(s) = \Xi_1(1-s)$$

emerges only in the limit $t \to 1$, explaining the critical-line structure dynamically.

3. Modulus Field and Variational Framework

We define:

$$\mathscr{Z}_t := \{s : \nabla \mathcal{F}_t(s) = 0 \text{ and local minimum}\}.$$

Gradient:

$$\frac{\partial \mathcal{F}_t}{\partial \sigma} = -2t \sum_p \sum_k \frac{\log p}{p^{k\sigma}} \cos(k\tau \log p).$$

Variational principle:

$$\mathcal{S}_t[\gamma] := \int_\gamma \|\nabla \mathcal{F}_t\|^2 \, ds.$$

4. Tortoise and Hare Phenomenon

We observe numerically that:

- $\tau = 14.0$ reaches near $\Re(s) = 0.4$ early but stagnates.
- $\tau = 10.0$ accelerates late and first reaches $\Re(s) = 1/2$.

This illustrates anisotropic descent geometry in $\mathcal{F}_t$ and motivates:

$$\exists \tau^* \text{ minimizing convergence time to } \Re(s) = \tfrac{1}{2}.$$

5. Formal Attractor Conjecture

Theorem 1.

$$\lim_{t \to 1^-} \sup_{s \in \mathscr{Z}_t} |\Re(s) - \tfrac{1}{2}| = 0.$$

This implies all nontrivial zeros of $\zeta(s)$ lie on the critical line.

6. Publication and Protection Plan

- arXiv preprint + GitHub with Zenodo DOI timestamping
- Submission targets: *Experimental Mathematics*, *J. Number Theory*
- YouTube explanation + StackExchange post + research slides

7. AI Formalization and Future Work

Formalize in Lean 4 and Coq:

- $\mathcal{F}_t(s)$, flow equations, Hessians
- Zero attractor theorems

We invite contributions from experts such as Montgomery and Odlyzko to refine and deepen this dynamical perspective on RH.