Negative-Dimensional Fields

Alien Mathematicians

Introduction to Negative-Dimensional Fields

Negative-Dimensional Fields are hypothetical mathematical structures defined with the following properties:

- A negative-dimensional field, denoted F_{-d}, has a negative dimension $-d$ over a base field K, such that:

$$\dim_K F_{-d} = -d, \quad d > 0.$$

- Adding structure to F_{-d} leads to a contraction or reduction of the dimension, as opposed to an expansion seen in classical fields.

Definition of Negative-Dimensional Fields

Definition 1.1: Negative-Dimensional Field

- Let K be a base field.
- A negative-dimensional field F_{-d} is a commutative division ring with identity.
- The dimension of F_{-d} over K is given by:

$$\dim_K F_{-d} = -d.$$

Example of Negative-Dimensional Field

Example 1.1: Hypothetical Construction

- Suppose F_{-d} is a negative-dimensional extension of a base field K.
- Adding elements to F_{-d} leads to a reduction in the dimension, reflecting a collapse or contraction of the field structure.
- For example, each basis element reduces the effective dimension.

Inverse Automorphisms

Definition 2.1: Inverse Automorphism

- An inverse automorphism $\sigma : F_{-d} \to F_{-d}$ is a bijection that preserves the field operations:

$$\sigma(a+b) = \sigma(a) + \sigma(b), \quad \sigma(ab) = \sigma(a)\sigma(b).$$

- It reverses the dimension:

$$\dim_K \sigma(F_{-d}) = d.$$

Inverse Galois Groups

Definition 2.2: Inverse Galois Group

- The inverse Galois group of a negative-dimensional field extension F_{-d}/K is defined as:

$$\mathrm{Gal}(F_{-d}/K) = \{\sigma \in \mathrm{Aut}(F_{-d}) \mid \sigma(k) = k \text{ for all } k \in K\}.$$

- The group captures the contraction symmetries of F_{-d}.

Theorem 2.1: Inverse Automorphisms Form a Group

Theorem 2.1: The set of inverse automorphisms $\mathrm{Gal}(F_{-d}/K)$ forms a group under composition.

Proof.

- **Closure:** If $\sigma, \tau \in \mathrm{Gal}(F_{-d}/K)$, then $\sigma \circ \tau$ is an automorphism.
- **Identity:** The identity map is trivially an automorphism.
- **Inverses:** Each automorphism σ has an inverse σ^{-1}.

□

Definition of Negative-Dimensional Galois Extensions

Definition 3.1: Negative-Dimensional Galois Extension

- A negative-dimensional field extension F_{-d}/K is called a Galois extension if it is co-normal and co-separable.
- Co-normality means that irreducible polynomials collapse into fewer factors.
- Co-separability means that the roots of polynomials in F_{-d} are anti-distinct.

Theorem 4.1: Inverse Galois Correspondence

Theorem 4.1: There exists a one-to-one correspondence between supergroups of $\mathrm{Gal}(F_{-d}/K)$ and contracted field extensions.

Inverse Galois Problem

Problem 5.1: Given a finite group G, does there exist a negative-dimensional field extension F_{-d}/K such that:

$$\mathrm{Gal}(F_{-d}/K) \cong G?$$

Next Steps in the Infinite Series

Modular Expansion:

- Future slides will introduce additional properties and extensions of the negative-dimensional fields.
- Infinite recursive concepts, such as inverse cohomology, and connections to algebraic geometry.