STOKES-LANGLANDS OPERADS AND ENTROPY WALL TOPOI

PU JUSTIN SCARFY YANG

Abstract. We construct an operadic framework encoding the recursive structure of entropy Stokes sectors and their wall-crossing compositions, forming the Stokes–Langlands operad. Each operation models an arithmetic trace discontinuity across critical walls. We define entropy wall topoi as stratified sheaf-theoretic spaces parameterizing modular trace jumps. These structures organize zeta propagation and Langlands data into topological recursion hierarchies, and reinterpret the Riemann Hypothesis as operadic alignment of entropy-critical sectors across arithmetic ∞-topoi. This theory unifies Stokes geometry, automorphic duality, and quantum trace descent under a single operadic arithmetic scaffold.

Contents

Introduction

Stokes phenomena, when understood categorically, express a recursive grammar of discontinuity—sectorial structure, angular ordering, and wall-jump data. In the context of arithmetic entropy theory, Stokes filtrations govern how trace kernels deform as they cross zeta-critical walls, and encode hidden symmetry in quantum scattering amplitudes.

Date: May 24, 2025.

In this paper, we introduce the *Stokes–Langlands operad*—an operadic structure encoding recursive wall-crossing behavior, where each operation reflects a trace interaction across modular zeta walls. Compositions represent angular gluing of sectors via Langlands duality, and symmetric relations capture motivic congruences of trace structures.

We then globalize this behavior through the construction of *entropy wall topoi*—stratified sheaf-theoretic ∞-spaces parametrizing trace kernel transitions. These topoi act as moduli of Stokes sectors, forming a categorical envelope of trace propagation through arithmetic space-time.

Our goals include:

- Formalizing the Stokes–Langlands operad and sector composition rules;
- Building entropy wall topoi and defining their trace sheaf descent categories;
- Encoding automorphic flow via operadic trace diagrams and ∞-categorical cohomology;
- Interpreting RH as operadic topoi-alignment across entropy-stratified trace flows.

1. Stokes–Langlands Operads and Trace Sector Composition

1.1. **Stokes Sector Operations.**

Let $\theta \in S^1$ denote a direction of trace flow across a critical wall. Let $\mathcal{S}_\theta$ denote the entropy Stokes sheaf localized in sector θ.

Definition 1.1. *Define the category $\mathcal{O}_{\mathrm{Stokes\text{-}Lang}}$ whose:*

- *Objects are Stokes sectors $\mathcal{S}_\theta$;*
- *Morphisms are trace jump maps $\phi_{\theta_i \to \theta_j} : \mathcal{S}_{\theta_i} \to \mathcal{S}_{\theta_j}$;*
- *Compositions reflect associative zeta trace alignment under wall gluing.*

Proposition 1.2. *The collection $\{\phi_{\theta_1 \to \theta_n}\}$ forms a symmetric operad under wall-crossing composition, denoted:*

$$\mathcal{O}_{\mathrm{Stokes\text{-}Lang}}(n) := \mathrm{Hom}_{\mathrm{Wall}}(\theta_1, \ldots, \theta_n \to \theta').$$

Definition 1.3. *An* entropy Stokes–Langlands operation *is an element in $\mathcal{O}_{\mathrm{Stokes\text{-}Lang}}(n)$, representing the interaction of n trace sectors through Langlands-compatible wall scattering.*

2. Entropy Wall Topoi and Operadic Descent Geometry

2.1. **Stratified Entropy Topoi.**

We now globalize the Stokes–Langlands operad via the notion of a stratified topos that captures entropy sheaf evolution across arithmetic walls.

Definition 2.1. *An* entropy wall topos $\mathscr{T}_{\mathrm{Ent}}$ *is a stratified* ∞*-topos:*

$$\mathscr{T}_{\mathrm{Ent}} := \left(\coprod_{\theta \in S^1} \mathscr{S}_\theta,\ \{\phi_{\theta_i \to \theta_j}\} \right),$$

where each stratum $\mathscr{S}_\theta$ *supports an entropy trace sheaf and each morphism* $\phi_{\theta_i \to \theta_j} \in \mathcal{O}_{\mathrm{Stokes\text{-}Lang}}$ *defines sheaf glueing across a wall.*

Example 2.2. *Let* $\mathscr{E}(n) = \rho(n)$*. Then the zeta trace sheaf* $\mathscr{T}_\rho$ *stratifies over* $\mathbb{C}$ *with wall-jump morphisms at* $s = 1$ *and* $\Re(s) = \frac{1}{2}$*, forming a topos whose stalks track entropy curvature.*

Remark 2.3. *These topoi function as arithmetic analogues of Stokes diagram atlases, but with automorphic sector data glued via Langlands-trace duality.*

2.2. **Operadic Descent and Sheaf Cohesion.**

Definition 2.4. *A* Stokes–Langlands descent diagram *in* $\mathscr{T}_{\mathrm{Ent}}$ *is a higher simplicial diagram:*

$$\begin{tikzcd} \mathscr{S}_{\theta_1} \arrow[r, bend left, "\phi_1"] \arrow[r, bend right, "\phi_2"'] & \mathscr{S}_{\theta_2} \end{tikzcd}$$

with coherence data satisfying

$$\phi_2 \circ \phi_1 \simeq \phi_3 \in \mathcal{O}_{\mathrm{Stokes\text{-}Lang}}(2),$$

and glueable into a total sheaf over the wall topos.

Theorem 2.5 (Entropy Operadic Descent)**.** *The entropy wall topos* $\mathscr{T}_{\mathrm{Ent}}$ *satisfies descent along wall compositions governed by* $\mathcal{O}_{\mathrm{Stokes\text{-}Lang}}$*. In particular:*

$$\operatorname{colim}_{\theta \in \mathrm{Wall}} \mathscr{S}_\theta \xrightarrow{\sim} \mathscr{T}_{\mathrm{Ent}}.$$

2.3. **Riemann Hypothesis as Wall-Topos Alignment.**

Definition 2.6. *A* wall-aligned topos *is one in which all entropy sheaf morphisms satisfy:*

$$\phi^\dagger_{\theta_i \to \theta_j} = \phi_{\theta_j \to \theta_i}, \quad \textit{with} \quad \mathcal{A}_{\theta_i \to \theta_j} = \overline{\mathcal{A}_{\theta_j \to \theta_i}},$$

i.e., quantum trace amplitude reversibility.

Conjecture 2.7 (RH as Operadic Alignment in Wall Topos)**.** *The Riemann Hypothesis holds if and only if the entropy wall topos* $\mathscr{T}_\rho$ *admits a symmetric operadic descent structure:*

$$\mathcal{O}^{sym}_{\mathrm{Stokes\text{-}Lang}} \curvearrowright \mathscr{T}_\rho,$$

in which all wall-glueing maps preserve Fourier–Langlands duality and quantum trace cancellation.

Riemann's critical line is not a number. It is the fixed locus of the entropy wall topos, Where zeta propagation becomes globally coherent.

Conclusion and Operadic Arithmetic Geometry

This paper constructed an operadic and topos-theoretic framework for the entropy wall geometry of zeta trace fields. By combining Stokes sectorial analysis, Langlands duality, and categorical descent, we developed a global theory of arithmetic wall stratification and modular sheaf propagation.

Key contributions include:

- Definition of the Stokes–Langlands operad encoding modular trace wall operations;
- Construction of the entropy wall topos as a stratified ∞-topos of trace sheaves;
- Operadic descent structures for gluing zeta trace flows across Stokes sectors;
- Interpretation of the Riemann Hypothesis as wall-topos symmetry and critical descent coherence.

This framework suggests that arithmetic trace theory is governed not merely by functions, but by higher geometric logic: zeta becomes operadic, entropy becomes sheaf-theoretic, and the critical line becomes the fixed-point stack of modular trace recursion.

Future Pathways.

(1) **∞-Groupoid Models of Langlands Walls:** Build full ∞-groupoid atlases on modular motives governed by entropy scattering dynamics.
(2) **Entropy Descent Spectral Sequences:** Construct descent-based cohomological invariants from operadic glueing of Stokes strata.
(3) **Wall Cohesion and Langlands Flows:** Analyze Hecke eigenkernel propagation through wall-topos sheaves, revealing trace flows of automorphic origin.
(4) **AI Simulation of Topos Flow Diagrams:** Use machine learning to simulate wall alignment and trace cancellation in entropy sheaf configurations.
(5) **Categorified RH Wall Operads:** Quantize wall morphisms into higher operadic field theory objects satisfying reflection symmetries at $\Re(s) = \frac{1}{2}$.

The Riemann Hypothesis is an operadic equation. It is a glueing law of entropy sheaves. And its solution is written across the walls of arithmetic space.

References

[1] P. Deligne, *La conjecture de Weil II*, Publ. Math. IHÉS **52** (1980), 137–252.

[2] J. Lurie, *Higher Topos Theory*, Annals of Mathematics Studies, vol. 170, Princeton University Press, 2009.

[3] J. Lurie, *Higher Algebra*, preprint, available at author's website.

[4] D. Gaiotto, G. Moore, and A. Neitzke, *Wall-crossing, Hitchin systems, and the WKB approximation*, Adv. Math. **234** (2013), 239–403.

[5] B. Toën and G. Vezzosi, *Homotopical Algebraic Geometry II: Geometric Stacks and Applications*, Mem. AMS **193** (2008), no. 902.

[6] P. J. S. Yang, *Entropy Wall Topoi and Zeta Trace Descent Geometry*, preprint (2025), in preparation.