Example of Mathematics in [∞-]-Dimensional UnicodeLang

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July 13, 2024

Abstract

This document illustrates a mathematical concept that can only be adequately documented in [∞-]-dimensional UnicodeLang. The chosen example is the representation of quantum states in an infinite-dimensional Hilbert space, which poses significant challenges for the current 2-dimensional Unicode system.

1 Introduction

Infinite-dimensional Hilbert spaces are fundamental in functional analysis and quantum mechanics. Representing these spaces and the operations within them requires more than the current 2-dimensional Unicode system can provide.

2 Infinite-Dimensional Hilbert Space

Definition: An infinite-dimensional Hilbert space $\mathcal{H}$ is a complete inner-product space:

$$\mathcal{H} = \left\{ \psi \mid \psi : \mathbb{N} \to \mathbb{C}, \sum_{n=1}^{\infty} |\psi(n)|^2 < \infty \right\}$$

with an inner product defined by:

$$\langle \psi, \phi \rangle = \sum_{n=1}^{\infty} \psi(n)\overline{\phi(n)}$$

3 Quantum State Representation

Consider a quantum state ψ in $\mathcal{H}$:

$$|\psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$$

where c_n are complex coefficients, and $|n\rangle$ are the basis states.

3.1 Current UnicodeLang Limitation

Representing $|\psi\rangle$ in a 2-dimensional Unicode system is cumbersome and loses the intuitive visualization of infinite dimensions.

3.2 [∞-]-Dimensional UnicodeLang Representation

An [∞-]-dimensional UnicodeLang can encode the infinite-dimensional state vector and its operations compactly and intuitively:

$$
\begin{array}{cccccc}
c_0 & c_1 & c_2 & \cdots & c_n & \cdots \\
|0\rangle & |1\rangle & |2\rangle & \cdots & |n\rangle & \cdots
\end{array}
$$

4 Conclusion

This example demonstrates the necessity of [∞-]-dimensional UnicodeLang for representing and working with infinite-dimensional mathematical concepts. Current 2-dimensional Unicode systems are insufficient for such tasks, highlighting the need for advanced representation systems.