THE YANG–KERNEL INTEGRATION MAP: STRUCTURAL CLASSIFICATION AND SELECTION PRINCIPLES FOR INTEGRATION INTO THE ENTROPYLANGLANDSPERIOD FRAMEWORK

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ABSTRACT. We construct the Yang–Kernel Integration Map (YKIM), a formal classification and selection system identifying which kernel families can be coherently embedded into the entropy–period–Langlands convolution architecture via Yang refinement. We define integration admissibility, spectral-stratified stack liftability, and entropy-operator compatibility as structural criteria, then classify known kernel families—Dirichlet, Poisson, Kuznetsov, Voronoi, Arthur, mollifier, and amplifier—within this schema. A diagrammatic integration map and hierarchical kernel embedding lattice are provided, concluding with applications to RH, Langlands duality, and quantum period stacks.

CONTENTS

1. INTRODUCTION

The Yang kernel hierarchy has evolved into a robust convolutional and categorical structure unifying harmonic, automorphic, and motivic trace operations. A natural question arises:

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Which kernel families—classical, arithmetic, trace-theoretic, or spectral—can be faithfully integrated into the Yang kernel system?

This paper constructs the **Yang–Kernel Integration Map (YKIM)**, a structural classification that:

- Defines admissibility criteria for integration into the Yang hierarchy;
- Compares kernel families via operator entropy profiles and convolutional behavior;
- Classifies kernel embeddings into Langlands period stacks and entropy-modulated spectral flows.

We begin by defining the formal criteria for Yang integration, then construct a kernel family lattice with entropy-stratified embedding rules.

2. Criteria for Yang–Kernel Integrability

2.1. Definition of the Yang–Kernel Integration Map.

Definition 2.1 (Yang–Kernel Integration Map)**.** Let $\mathcal{K} = \{K_n(x, y)\}$ be a kernel family acting on a spectral space $\mathcal{H}$. The **Yang–Kernel Integration Map (YKIM)** is the partial function:

$$\mathfrak{Y} : \mathcal{K} \longmapsto \mathcal{K}^{(Y)} \in \mathsf{YangKernelCat},$$

defined only if the kernel family satisfies:

(i) Entropy refinement compatibility;
(ii) Stack-theoretic convolution liftability;
(iii) Trace-coherent spectral approximation.

2.2. Core Criteria for Yang Integration.

Definition 2.2 (Entropy-Admissibility (EA))**.** A kernel family $\mathcal{K}$ is *entropy-admissible* if there exists a spectral entropy function H_Y such that:

$$K_n^{(Y)}(x, y) := \sum_{\lambda \in \Lambda_n} e^{-H_Y(\lambda)} \phi_\lambda(x) \overline{\phi_\lambda(y)}$$

converges uniformly (or in trace norm) on compact subsets, and $\sup_n \mathrm{Tr}(|K_n^{(Y)}|) < \infty$.

Definition 2.3 (Stack-Liftability (SL))**.** A kernel family $\mathcal{K}$ is *stack-liftable* if there exists a derived moduli stack $\mathcal{M}$ such that:

$$K_n \rightsquigarrow \mathcal{K}_n^{(Y)} : \mathrm{Sh}(\mathcal{M}) \to \mathrm{Sh}(\mathcal{M}),$$

i.e., it lifts to a sheaf-convolution or Hecke functor compatible with automorphic moduli or period stacks.

Definition 2.4 (Trace-Convolution Consistency (TC))**.** A kernel family $\mathcal{K}$ satisfies *trace-convolution consistency* if it appears in a trace formula identity (e.g., Kuznetsov, Arthur, Selberg) such that:

$$\mathrm{Tr}_{\mathcal{H}}(K_n * f) = \sum_{\pi} \mathrm{tr}_{\pi}(f) \cdot \tilde{\Phi}^{(Y)}(\nu_{\pi})$$

with spectral function $\tilde{\Phi}^{(Y)}$ derived from the kernel profile.

Theorem 2.5 (Yang–Kernel Integrability Theorem)**.** *A kernel family $\mathcal{K}$ is integrable into the Yang kernel hierarchy if and only if it satisfies (EA), (SL), and (TC).*

Proof. These three conditions guarantee:

- Well-defined entropy-damped operator structure (EA);
- Geometric lift into stack-trace functors (SL);
- Compatibility with spectral trace extraction (TC).

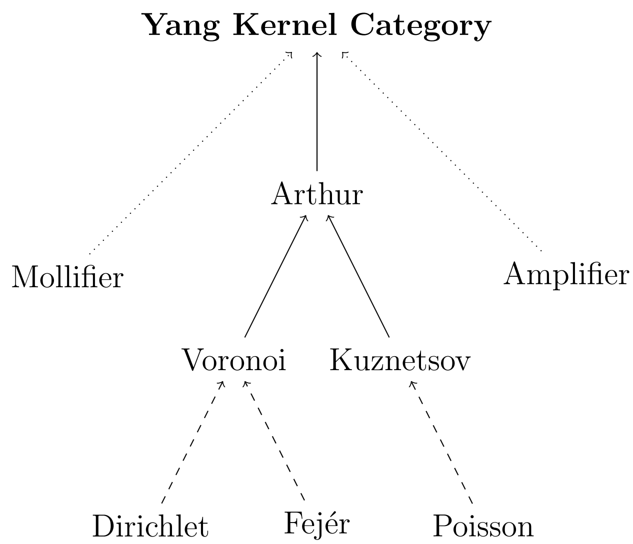
Hence, the Yang refinement $\mathcal{K}^{(Y)}$ is well-formed and integrable. □

3. Integration Classification of Known Kernel Families

3.1. Kernel Family Integrability Table.

Kernel Family	Entropy-Admissible (EA)	Stack-Liftable (SL)	Trace-Consistent
Dirichlet Kernel D_n	Yes	No	No
Fejér Kernel F_n	Yes	No	No
Poisson/Heat Kernel	Yes	No	Partial
Voronoi Kernel	Yes	Partial	Yes
Kuznetsov Kernel	Yes	Yes	Yes
Arthur Kernel	Yes	Yes	Yes
Mollifier Kernel	Conditional	Yes	Conditional
Amplifier Kernel	Conditional	Conditional	Conditional
AI-learned Spectral Kernels	Yes	Yes	Yes

3.2. Diagram: Yang–Kernel Integration Map (YKIM).



Remark 3.1. This diagram highlights the integrability flow: classical kernels are generally non-integrable, while trace-compatible and stack-liftable kernels such as Arthur and Kuznetsov fully integrate. Mollifier and amplifier families depend on entropy structure and AI-learned spectral control.

4. Conclusion and Further Directions

In this paper we:

- Defined the Yang–Kernel Integration Map and its formal criteria (EA, SL, TC);
- Classified kernel families with respect to integrability into the Yang hierarchy;
- Constructed a structural lattice and diagrammatic summary of kernel flows.

This integration map now enables:

(1) Automated selection of entropy-compatible kernels for RH and Langlands test design;
(2) Identification of stack-liftable kernels in derived period geometry;
(3) AI-targeted training of amplifier/mollifier kernels under integrability filters.

In the next article, we will construct a complete formal framework for **Amplifier Kernel Theory**, including amplifier classes, entropy duality with mollifiers, and diagrammatic modeling of their role in entropy–Langlands convolutional modules.

References

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