Rigorous Development of Yang$_?$ Geometry

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Abstract

This paper rigorously develops the concept of Yang$_?$ geometry for any value of "?", including integer, fractional, fractal, p-adic, and recursively defined dimensions. We provide examples and explore the relationships between different choices of "?".

1 Introduction to Yang$_\alpha$ Number Systems

The Yang$_\alpha$ number system is defined by a set of rules and properties that extend traditional number systems into higher-dimensional spaces.

1.1 Definition

For a given α, the Yang$_\alpha$ number system is characterized by a set of elements $\{y_i\}_{i\in\mathbb{Z}}$ where each y_i follows specific algebraic and topological rules.

1.2 Properties

- **Closure**: For any $y_i, y_j \in \{y_i\}$, $y_i + y_j, y_i \cdot y_j \in \{y_i\}$.
- **Associativity and Commutativity**: The operations are associative and commutative.
- **Identity Elements**: There exist identity elements 0 and 1 for addition and multiplication, respectively.

1.3 Dimensionality

The dimension $\dim(Y_\alpha)$ of a Yang$_\alpha$ space is defined such that it generalizes traditional integer dimensions to α.

2 Yang$_?$ Dimensional Geometries

2.1 General Framework for Yang$_?$ Dimensions

For any ?, the Yang$_?$ geometry is characterized by:

- A base space B and a fiber space F both defined in terms of Yang$_?$ elements.
- A projection map $\pi : E \to B$ where E is the total space, such that locally E looks like $B \times F$.

2.2 Mathematical Notation and Definitions

Let E be a Yang$_?$ geometric space. Define B as the base Yang$_?$ space with dimension $\dim_Y(B) = ?_B$. Define F as the fiber Yang$_?$ space with dimension $\dim_Y(F) = ?_F$.

The total dimension is given by:

$$\dim_Y(E) = ?_B + ?_F.$$

2.3 Types of Yang$_?$ Dimensions

- **Integer Dimensions:** For $? = n \in \mathbb{Z}$, we have $\dim_Y(E) = n$.
- **Fractional Dimensions:** For $? = q \in \mathbb{Q}$, we have $\dim_Y(E) = q$.
- **Fractal Dimensions:** For $? =$ fractal dimension, we define using the Hausdorff dimension $\dim_H$: $\dim_Y(E) = \dim_H(E)$.
- **p-adic Dimensions:** For $? = p$-adic dimension, where p is a prime, $\dim_Y(E) = p$.
- **Recursive and Hierarchical Structures:** For nested Yang systems, such as $? = Yang_\infty$, $\dim_{Y_n}(E) = \sum_{i=1}^{n} ?_i$.

3 Examples and Relationships Between Different Choices of "?"

3.1 Example 1: Integer and Fractional Dimensions

Consider $? = 3$ and $begin : math : text? = 2.5 end : math : text$:

- For $? = 3$, we have a 3-dimensional space.
- For $? = 2.5$, we have a space with a fractional dimension, which could correspond to a fractal structure with dimension 2.5.

Relationship: A 3-dimensional space can be embedded or projected into a fractional-dimensional space, showing how classical geometries relate to fractal geometries.

3.2 Example 2: p-adic and Integer Dimensions

Consider $? = p$ for a prime p and $? = n \in \mathbb{Z}$:

- For $? = 3$, we have a 3-dimensional space.
- For $? = p$, we have a space defined over the p-adic field $\mathbb{Q}_p$.

Relationship: p-adic spaces can be seen as analogs of integer-dimensional spaces but in a different number system, showing how classical geometries can extend into number theory contexts.

3.3 Example 3: Recursive Yang Systems

Consider $? = Yang_2$ and $? = Yang_\infty$:

- For $? = Yang_2$, we have a 2-level nested Yang system.
- For $? = Yang_\infty$, we have an infinitely nested system.

Relationship: Finite-level Yang systems can be extended to infinite-level systems, showing how hierarchies of dimensions can be built recursively.

4 Unified Framework and Applications

4.1 Generalization of Fibration Structures

We generalize the classical notion of fibrations to accommodate new dimensional contexts, ensuring they respect the properties of the base and fiber spaces.

4.2 Interactions and Properties of Generalized Fibrations

We investigate how these generalized fibrations interact and their topological, algebraic, and geometric properties.

4.3 Development of Theoretical Tools

We create new mathematical tools and techniques to work with these generalized fibrations, using advanced concepts from homotopy theory, category theory, and higher-dimensional algebra.

4.4 Potential Applications in Mathematics and Physics

We explore applications of these generalized fibrations in various areas, uncovering new relationships and insights.

5 Conclusion

This paper extends the concept of fibrations to negative, fractional, fractal, p-adic, Yang_α, and $Yang_{Yang_{\cdots Yang}}$ dimensions, providing rigorous definitions and exploring their implications.

References

[1] Virtual Dimensions in Algebraic Geometry.

[2] Fractal Geometry and Measure Theory.

[3] p-adic Geometry and Analytic Spaces.