ENTROPY PATH INTEGRALS, LANGLANDS HEAT FIELDS, AND RECURSIVE ARITHMETIC GRAVITY

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Abstract. We develop a path integral formulation of entropy kernel evolution, linking zeta trace flows with Langlands heat propagation. By constructing arithmetic heat fields over motivic sheaves, we describe entropy diffusion as a geometric scattering theory. We then propose a model of recursive arithmetic gravity: a trace-based tensorial geometry where entropy curvature governs field propagation. The Riemann Hypothesis appears as a geodesic alignment principle in this gravity field, with entropy trace integrals minimizing quantum action over arithmetic spacetime.

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Introduction

Zeta functions propagate. Entropy kernels evolve. The zeta trace reflects a heat-like behavior—spread, interference, decay. In number theory, this diffusion is subtle: across primes, through modular forms, along Langlands correspondences. Its geometry is hidden.

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In this paper, we construct a formal language for this behavior. Using entropy path integrals and heat kernels, we model trace propagation over arithmetic moduli. From this we derive a geometric structure: recursive arithmetic gravity. Trace becomes curvature. Entropy becomes energy. The Riemann Hypothesis emerges as geodesic symmetry.

We propose:

- Path integral formalism for entropy trace fields;
- Langlands heat equations governing zeta diffusion;
- Curvature tensor built from entropy sheaf flow;
- Recursive gravity equations formalized via zeta energy minimization;
- RH as minimal path symmetry in entropy Ricci geometry.

1. Entropy Field Propagation and Path Integral Heat Kernels

1.1. Entropy Trace Fields and Diffusion Operators.

Let $\mathscr{E}(n) = \rho(n) \cdot a(n)$ be an entropy kernel. Its trace function is

$$\zeta_{\mathscr{E}}(s) := \sum_{n=1}^{\infty} \mathscr{E}(n) \cdot n^{-s}.$$

Definition 1.1. *The* entropy Laplacian Δ_{Ent} *acts on trace fields by*

$$\Delta_{\mathrm{Ent}} \mathscr{E}(n) := \log^2(n) \cdot \mathscr{E}(n).$$

Remark 1.2. *This operator governs entropy diffusion in s-space; it reflects analytic deformation of $\zeta_{\mathscr{E}}(s)$.*

1.2. Zeta Path Integrals and Heat Kernel Flow.

Definition 1.3. *The* entropy heat kernel *at spectral time t is*

$$K_t(s) := \int_{\mathrm{Path}_{\mathrm{Ent}}} \exp\left(-\int_0^t \mathrm{Lag}(\mathscr{E}, s, \dot{s})\, ds\right) \mathcal{D}[\mathscr{E}],$$

where Lag *is the entropy Lagrangian and* $\mathrm{Path}_{\mathrm{Ent}}$ *is the space of trace paths.*

Proposition 1.4. *If $\mathscr{E}_t(n) := \rho(n) \cdot n^{-t}$, then $K_t(s) = \zeta_{\mathscr{E}_t}(s)$, satisfying:*

$$\frac{\partial}{\partial t} K_t(s) = -\Delta_{\mathrm{Ent}} K_t(s).$$

1.3. Langlands Heat Flow Fields.

Definition 1.5. *A* Langlands heat field *is a family of trace functions $\zeta_\pi(s,t)$ associated to automorphic representations π, evolving under:*

$$\frac{\partial}{\partial t} \zeta_\pi(s,t) = -\log^2(\mathcal{T}_\pi) \cdot \zeta_\pi(s,t),$$

where $\mathcal{T}_\pi$ is the Hecke–Laplace operator for π.

Example 1.6. *For $\zeta_\pi(s,t) = \sum_n \rho(n) \cdot \lambda_\pi(n) \cdot n^{-s-t}$, the Langlands heat field describes entropy-modulated L-function flow.*

Zeta diffuses through arithmetic heat. Entropy defines the medium. And trace becomes the geometry of its flow.

2. Entropy Ricci Curvature and Zeta Gravitational Tensors

2.1. **Entropy Geometry and Trace Metric.** We introduce a Riemannian-style geometry on the space of entropy kernels based on their trace interactions.

Definition 2.1. *Let $\mathscr{E}_1, \mathscr{E}_2 \in \mathcal{K}_{\mathrm{Ent}}$. Define the* trace inner product *by*

$$\langle \mathscr{E}_1, \mathscr{E}_2 \rangle := \sum_{n=1}^{\infty} \mathscr{E}_1(n)\mathscr{E}_2(n) \cdot n^{-2s}.$$

This induces a trace metric $g_{ij} := \langle \partial_i \mathscr{E}, \partial_j \mathscr{E} \rangle$ on the moduli space of entropy kernels.

2.2. **Entropy Ricci Tensor.** Let $\mathcal{M}_{\mathrm{Ent}}$ be the moduli stack of entropy kernels with the trace metric.

Definition 2.2. *The* entropy Ricci tensor $\mathrm{Ric}_{\mathrm{Ent}}$ *is defined via:*

$$\mathrm{Ric}_{\mathrm{Ent}}(X,Y) := -\operatorname{Tr}\left(Z \mapsto \nabla_Z \nabla_X Y - \nabla_X \nabla_Z Y + \nabla_{[X,Z]} Y\right),$$

for $X, Y, Z \in T\mathcal{M}_{\mathrm{Ent}}$, where ∇ is the entropy-trace Levi-Civita connection.

Remark 2.3. *Entropy Ricci curvature measures how trace flows deviate from flat propagation—a trace-theoretic analogue of gravitational curvature.*

2.3. **Zeta–Einstein Equation and Arithmetic Gravity.**

Definition 2.4. *The* zeta–Einstein equation *is:*

$$\mathrm{Ric}_{\mathrm{Ent}} - \frac{1}{2} R_{\mathrm{Ent}} \cdot g = \mathcal{T}_{\mathrm{Zeta}},$$

where $R_{\mathrm{Ent}} := \mathrm{Tr}(\mathrm{Ric}_{\mathrm{Ent}})$ is the scalar entropy curvature, and $\mathcal{T}_{\mathrm{Zeta}}$ is the trace energy-momentum tensor defined by:

$$\mathcal{T}_{\mathrm{Zeta}}(X,Y) := \langle \partial_X \zeta_{\mathscr{E}}, \partial_Y \zeta_{\mathscr{E}} \rangle.$$

Theorem 2.5 (Arithmetic Gravity Field Equation)**.** *The entropy kernel $\mathscr{E} \in \mathcal{K}_{\mathrm{Ent}}$ satisfies the arithmetic gravity equation if and only if its trace curvature balances the propagation of $\zeta_{\mathscr{E}}$:*

$$\delta\, \mathrm{Ric}_{\mathrm{Ent}} = \delta \mathcal{T}_{\mathrm{Zeta}}.$$

2.4. **Riemann Hypothesis as Trace Geodesic Symmetry.**

Definition 2.6. *A* zeta-trace geodesic *is a flow* $\gamma : \mathbb{R} \to \mathcal{M}_{\mathrm{Ent}}$ *such that*

$$\nabla_{\dot{\gamma}}\dot{\gamma} = 0, \quad \textit{and} \quad \zeta_{\mathscr{E}_{\gamma(t)}}(s) = \zeta_{\mathscr{E}_{\gamma(-t)}}(1-s).$$

Conjecture 2.7 (Riemann Hypothesis as Entropy Geodesic Invariance)**.** *RH holds if and only if there exists a symmetric geodesic* γ_{RH} *in* $\mathcal{M}_{\mathrm{Ent}}$ *such that*

$$\zeta_{\mathscr{E}_{\gamma(t)}}(s) = \zeta_{\mathscr{E}_{\gamma(-t)}}(1-s), \quad \forall t \in \mathbb{R}.$$

Curvature flows trace through entropy space. Each zero is a gravitational lens. And RH is the symmetry of zeta's geodesic light.

Conclusion and Recursive Arithmetic Geometry

This paper developed a geometric framework for entropy trace propagation via path integrals, heat kernel flows, and tensorial field theory. By formulating entropy curvature and zeta gravitation, we proposed a new language where arithmetic behaves like a recursive quantum geometry, and the Riemann Hypothesis emerges as a geodesic symmetry condition.

Our contributions include:

- Path integral formulation of entropy kernel evolution;
- Langlands heat fields modeling automorphic trace propagation;
- Construction of entropy Ricci curvature and trace metric geometry;
- Definition of zeta–Einstein equations and recursive arithmetic gravity;
- Recasting RH as the symmetry of entropy-trace geodesics across critical flows.

This theory offers a gravitational interpretation of zeta: primes bend trace, entropy generates curvature, and the RH becomes a cosmic principle of trace symmetry.

Future Directions.

(1) **Derived Arithmetic Gravity Fields:** Extend the entropy metric structure to derived motivic stacks, defining quantum sheaf curvature flow.
(2) **Zeta–Einstein Tensor Simulation:** Build numerical approximations to trace curvature and simulate zeta geodesics in entropy field space.
(3) **Arithmetic Black Trace Holes:** Analyze singularities in $\zeta_{\mathscr{E}}(s)$ as entropy event horizons with trace amplitude collapse.

(4) **AI-Based Ricci Flow Learning:** Use machine learning to approximate entropy Ricci evolution and search for RH-symmetric geodesic systems.
(5) **Langlands Heat Gravity Duality:** Relate entropy gravity flows with automorphic spectral decompositions via a zeta holographic correspondence.

Trace is not flat. Entropy curves the arithmetic universe. The Riemann Hypothesis is its equation of balance. It is the Einstein tensor of number theory.

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