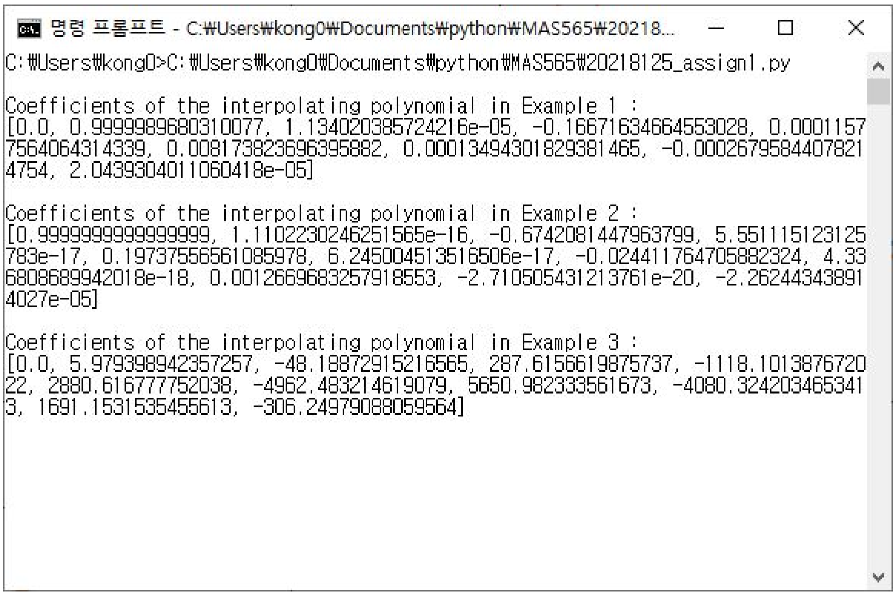
Computer Assignment

The program which does the required is submitted via KLMS along with this document. Given the two endpoints of the interval `a` and `b`, the step size `h`, and a function `f`, the program computes the interpolating polynomial using the divided difference scheme. The computed interpolating polynomials are printed out, as the following figure.

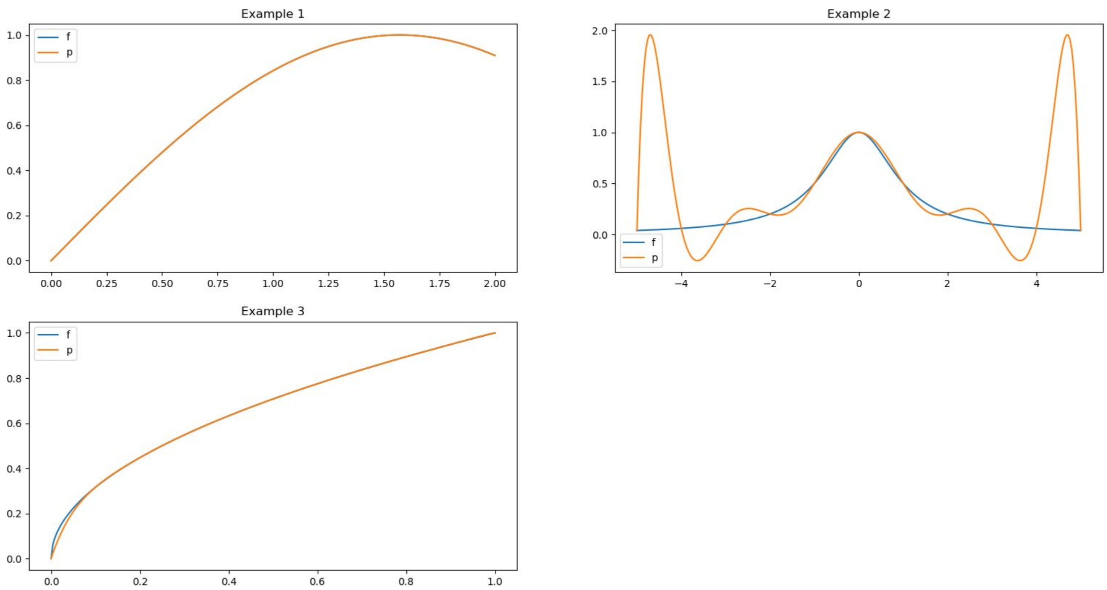


```
명령 프롬프트 - C:₩Users₩kong0₩Documents₩python₩MAS565₩20218...
C:₩Users₩kong0>C:₩Users₩kong0₩Documents₩python₩MAS565₩20218125_assign1.py

Coefficients of the interpolating polynomial in Example 1 :
[0.0, 0.9999989680310077, 1.134020385724216e-05, -0.16671634664553028, 0.0001157
7564064314339, 0.008173823696395882, 0.00013494301829381465, -0.0002679584407821
4754, 2.0439304011060418e-05]

Coefficients of the interpolating polynomial in Example 2 :
[0.9999999999999999, 1.1102230246251565e-16, -0.6742081447963799, 5.551115123125
783e-17, 0.19737556561085978, 6.245004513516506e-17, -0.024411764705882324, 4.33
6808689942018e-18, 0.0012669683257918553, -2.710505431213761e-20, -2.26244343891
4027e-05]

Coefficients of the interpolating polynomial in Example 3 :
[0.0, 5.979398942357257, -48.18872915216565, 287.6156619875737, -1118.101387672022, 2880.616777752038, -4962.483214619079, 5650.982333561673, -4080.324203465341
3, 1691.1531535455613, -306.24979088059564]
```

The results are the list of coefficients of the resulting polynomial, from the lowest degree term to the highest. For example the first result indicates that the interpolating polynomial is

$$p(x) = 0.0 + 1.000x + 1.134 \times 10^{-5}x^2 + \cdots + 2.044 \times 10^{-5}x^8.$$

The original function f and the interpolating polynomial p plotted together, for each example, is shown as in the following figure.



The first function $f(x) = \sin x$ looks well interpolated by the interpolating polynomial. Such a behavior is actually exactly as expected, because we have $\left\| f^{(n)}(x) \right\|_\infty \leq 1$ for any positive integer n, exactly the case discussed in Problem 2.4.

The second function, the Runge function $f(x) = \dfrac{1}{1+x^2}$, is badly interpolated, especially on the region near the end points of the given interval. It is indeed an example of a function where increasing the number of nodes does not increase the quality of the interpolation.

The third function $f(x) = \sqrt{x}$ is also not so well interpolated near $x = 0$. Indeed f has a vertical asymptote at $x = 0$, which is a trait a polynomial cannot have. As a result the error is even visible with our bare eyes.