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Homework 2

1. smallest $\rightarrow$ largest $\Theta(1) < \Theta(\lg n) < \Theta(n) < \Theta(n^k) < \Theta(k^n) < \Theta(n!) < \Theta(n^n)$

$1, 100, \lg^{(2)} n, \lg n, \lg^2 n, \sqrt{n}, n, n \lg n,$
$n^2, n^2 \lg n, n^3, 2^{(\lg n)}, 2^n, 2^{2^n}, 2^{2^{(n+1)}}, n!, n^n$

2. a. $\lg^2 n$ grows faster than $\lg n$
because $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\lg n}{\lg^2 n} = \lim_{n\to\infty} \frac{1}{\lg n} = 0$

therefore, $g(n) > f(n)$ as $n \to \infty$

b. $\lg(n) > \lg(\lg(n))$ as $n \to \infty$
$\Theta(n) > \Theta(\lg(n))$
because $n > \lg(n)$ as $n \to \infty$
therefore $\lg(n) > \lg(\lg(n))$

c. $f(n)$ and $g(n)$ grow at the same asymptotal rate

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{2^n}{2^{n+\epsilon}} = \lim_{n\to\infty} \frac{1}{2^{\epsilon}} \neq 0, \infty, -\infty$$

therefore, $f(n)$ and $g(n)$ must grow at the same rate since one doesn't dominate the other.

3. a.
$$\lim_{n\to\infty} \frac{3n^2+6n+1}{n^2} = \lim_{n\to\infty} \frac{6n+6}{2n} = \lim_{n\to\infty} \frac{6}{2} = 3$$

$0 < 3 < \infty$ therefore $f(n) = \Theta(n^2)$

b. $f(n) = O(n^2)$ because $f(n) = \Theta(n^2)$. $\Theta(n^2)$ implies that $O(n^2)$ and $\Omega(n^2)$ meaning both the upper and lower bounds are n^2.

4.
$$\lim_{n\to\infty} \frac{2^n}{2^{2^n}} = \lim_{n\to\infty} \frac{1}{2^{(2^n - n)}} = 0 \text{ therefore,}$$

$O(2^{2^n}) > O(2^n)$, so $f(n) = O(2^{2^n})$

5. a. False, Big-O only implies that there is an upper bound. if $f(n)$ or $g(n)$ is significantly larger than the other then $f(n) = O(g(n))$ would not necissarily imply $g(n) = O(f(n))$.
Ex: $f(n) = n^2$, $g(n) = n^3$
$f(n) = O(n^3)$, but $g(n) \neq O(n^2)$

b. True, Θ implies that both O and Ω are tightly bound to the function, therefore if $f(n) = \Theta(g(n))$, $g(n) = \Theta(f(n))$. As $n \to \infty$, $f(n)$ and $g(n)$ must grow at the same rate.

6. STRASSEN(A, B)

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let C be nxn matrix
if A.rows == 1
    C11 = A11 · B11
else
    Partition A, B, and C into n/2 x n/2 matrices
    S1 = B12 - B22
    S2 = A11 + A12
    S3 = A21 + A22
    S4 = B21 - B11
    S5 = A11 + A22
    S6 = B11 + B22
    S7 = A12 - A22
    S8 = B21 + B22
    S9 = A11 - A21
    S10 = B11 + B12
    P1 = STRASSEN(A11, S1)
    P2 = STRASSEN(S2, B22)
    P3 = STRASSEN(S3, B11)
    P4 = STRASSEN(A22, S4)
    P5 = STRASSEN(S5, S6)
    P6 = STRASSEN(S7, S8)
    P7 = STRASSEN(S9, S10)
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23. $C_{11} = P_5 + P_4 - P_2 + P_6$
24. $C_{12} = P_1 + P_2$
25. $C_{21} = P_3 + P_4$
26. $C_{22} = P_5 + P_1 - P_3 - P_7$
27. return C