

Homework #9

1.a. $\{\Phi\}$: Φ has 2 different assignments that evaluate Φ to true}

b. We can reduce SAT into Double-SAT

- Enclose the entire expression in parenthesis and add an $\vee X$ to the end of Φ
- This effectively creates another solution to the circuit when X is true
- Call $A_{\text{DoubleSAT}}(\Phi)$ on the modified Φ to see if another solution exists besides the created one

• This transformation can be done in polynomial time, therefore it is NP-Hard

2.a. $\{(G, k)\}$: G has a simple path of length k and or greater

- We can reduce HAM-PATH into Longest Path
- The Hamilton Path is just the longest path problem where k is equal to $|G| - 1$
- Call $A_{\text{longest-path}}(G)$ if length of longest path is one less than total vertices it is a hamiltonian path

3.a. $\{(G, H)\}$: H is isomorphic to a subgraph of G

b. We can reduce the Clique Problem into the Subgraph-Isomorphism problem

- Call $A_{\text{Subgraph-Isomorphism}}(G, H)$ where H is a complete graph
- Since, CLIQUE is NP-Hard, Subgraph-Isomorphism must also be NP-Hard.

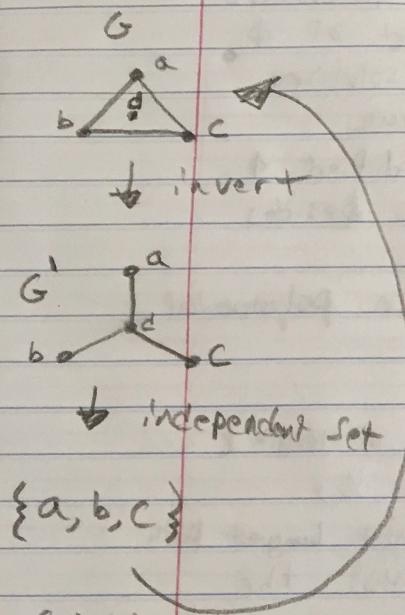
4. a. $\{(G, K)\}$: G contains a set of K or more independent vertices.

b. The Clique problem can be reduced to the Independent Set

- An independent set of vertices implies that there are no edges between any two vertices in the independent set.

- By calling A Independent-set (G') where G' is a modified version of G with edges added between every vertex and the original edges removed (essentially an inverted version of G), we get returned an independent set that will be a complete subgraph from the original G .

- Since inverting the edges can be done in polynomial time, the Independent Set problem is NP-Hard.



Solution
to clique