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Homework #9

1. a. $\{\phi : \phi$ has 2 different assignments that evaluate ϕ to true$\}$

b. We can reduce SAT into Double-SAT

- Enclose the entire expression in parenthesis and add an $\vee x$ to the end of ϕ
- This effectively creates another solution to the circuit when x is true
- call $A_{Double-SAT}(\phi)$ on the modified ϕ to see if another solution exists besides the created one

- This transformation can be done in polynomial time, therefore it is NP-Hard

2. a. $\{(G,K)$: G has a simple path of length k or greater$\}$

b. We can reduce HAM-PATH into Longest Path

- The Hamilton path is just the longest path problem where K is is equal to $|G.V| - 1$
- Call $A_{Longest\ Path}(G)$ if length of longest path is one less than total vertecies it is a hamiltonian path

3. a. $\{(G,H)$: H is isomorphic to a subgraph of G$\}$

b. We can reduce the Clique problem into the Subgraph-Isomorphism problem

- Call $A_{Subgraph-Isomorphism}(G,H)$ where H is a Complete graph
- Since, CLIQUE is NP-Hard, Subgraph-Isomorphism must also be NP-Hard.

4. a. $\{(G, K)$: G contains a set of K or more independent vertices$\}$.

b. The Clique problem can be reduced to the Independent set

- An independent set of vertices implies that there are no edges between any two vertices in the independent set
- By calling $A_{Independent\text{-}set}(G')$ where G' is a modified version of G with edges added between every vertex and the original edges removed (essentially an inverted version of G), we get returned an independent set that will be a complete subgraph from the original G.
- Since inverting the edges can be done in polynomial time, the Independent Set problem is NP-Hard.