

## Homework #8

## 1.a. FLOYD\_WARSHALL(W)

1.  $n = W.\text{rows}$
2.  $D = W$
3. for  $k=1$  to  $n$  do
4.     for  $i=1$  to  $n$  do
5.         for  $j=1$  to  $n$  do
6.              $d_{ij} = \min(d_{ij}, d_{ik} + d_{kj})$
7. return  $D$

Since  $d_{ik}$  and  $d_{kj}$  are never overwritten in each iteration, you can get away with using the same matrix for the whole algorithm. After the outer loop terminates  $k=n$  and  $D$  will be computed the same way to make  $D^{(n)}$ .

## b. FLOYD\_WARSHALL(W)

1.  $n = W.\text{rows}$
2.  $D = W$
3.  $P = \text{new } n \times n \text{ matrix}$
4. for  $i=1$  to  $n$  do
5.     for  $j=1$  to  $n$  do
6.         if ( $i=j$ ) or ( $d_{ij} = \infty$ ) then
7.              $p_{ij} = \text{NIL}$
8.         else
9.              $p_{ij} = i$
10. for  $k=1$  to  $n$  do
11.     for  $i=1$  to  $n$  do
12.         for  $j=1$  to  $n$  do
13.             if  $d_{ij} > d_{ik} + d_{kj}$  then
14.                  $d_{ij} = d_{ik} + d_{kj}$
15.                  $p_{ij} = p_{kj}$
16. return  $P$

2. No, it would only imply that any NP problem can be solved in polynomial time, not necessarily  $O(n^5)$ . By definition any NP problem can reduce to NP-complete in polynomial time but that transformation could be  $> O(n^5)$ .

3. a.  $\text{SORT} = \{A \mid A \text{ is an array and it is in sorted order}\}$   
b.  $\text{MIN} = \{A, x \mid A \text{ is an array of numbers with a minimum of } x\}$   
c.  $\text{LONGEST\_PATH} = \{G, B \mid G \text{ is some graph with no path } \geq B\}$

4. if  $|S'| \leq k$  then  
    found = false  
    for  $c$  in  $C$  do  
        for  $x$  in  $c$  do  
            for  $s$  in  $S'$  do  
                if  $s = x$  then  
                    found = true  
            if found = true then  
                found = false  
    else  
        return false  
    return true  
return false

Since a solution to the decision problem can be found in polynomial time, the hitting set problem is in NP

5. If  $\pi$  traverses each vertex only once  
if  $\pi$  contains real edges search  
if  $K = \pi.\text{length}$   
return true  
return false

6.  $\text{HAM-PATH} = \{G \mid G \text{ is a graph}$   
and there is a simple path that  
contains every vertex in  $G\}$

7.  $A(G, \pi)$   
if  $\pi.\text{length} = G.V - 1$   
if  $\pi$  contains no repeat vertices  
return true  
return false

8.  $\text{HAM-CYCLE}(G, \pi)$   
if  $\pi[1] = \pi[\pi.\text{length}]$   
return  $\neg \text{HAM-PATH}(G, \pi[1 \dots \pi.\text{length}-1])$   
return false + 1

• Hamilton cycle is a path with one  
extra condition

9.  $\text{HAM-PATH}(G, \pi)$   
if  $\pi.\text{length} = G.V - 1$   
return longest-path( $G, \pi, \pi.\text{length}$ )  
return false