MATH312 **Spring 2020**

Final Exam-written part

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Note:

1. Sign and include this page as the cover page. Submit it along with your write-up for the exam problems.
2. Scan all the pages (cover page and your write-up) and convert into a single PDF file for submission.
3. This is an open book exam in which academic integrity is honored. While you can consult textbooks, notes and software, use of online tutorial services is strictly prohibited and will result in a zero grade on this exam. Moreover, details before the final numerical answer is expected for full credits.
4. **MAKE SURE TO WRITE DOWN CORRESPONDING NULL AND ALTERNATIVE HYPOTHESES WHENEVER A HYPOTHESIS TESTING TASK IS INVOLVED.**

Item	Points	Score
Prob 1	50	
Prob 2	35	
Total	80	

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1.a. $H_0: \mu_d \geq 0$

$H_1: \mu_d < 0$

$\bar{x}_d = -6.78$

$s_d = \sqrt{55.8818} = 7.475$

$\alpha = 0.05, \; v = 10 - 1 = 9$

- $t_{0.05,9} = -1.833$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{-6.78 - 0}{7.475/\sqrt{10}} = -2.868$$

$-2.868 < -1.833$, so we reject the null hypothesis and accept H_1.

- Based on the t-test, there is sufficient evidence to suggest that the mean SvO_2 levels significantly increases 6-hr after admission.

1.b.

d	rank	signed-rank
2.4	1	1
-2.5	2	-2
-3.5	3	-3
5.5	4	4
-5.8	5	-5
-7.1	6	-6
-12.2	7	-7
-13.2	8	-8
-13.7	9	-9
-17.7	10	-10

$N = 10$

$$\bar{r} = \frac{1-2-3+4-5-6-7-8-9-10}{10} = -4.5$$

$H_0: E(\bar{r}) = 0 \quad H_1: E(\bar{r}) < 0 \quad z_{0.05} = -1.64$

$$N < 20 \rightarrow z^* = \frac{-4.5 - 0.5 - 0}{\sqrt{(11)\cdot(21)/(60)}} = -2.55 < -1.64$$

So Null hypothesis is rejected and we conclude: mean SvO_2 increases

u is random variable from distribution

1.c. $H_0: p(u > 0) = \pi = 0.5$
$H_1: \pi < 0.5$

In the sample:
2 are > 0 and 8 are < 0
$b(2;10,1/2) = 0.044 + 0.01 + 0.001 = 0.055$
$0.055 \approx 0.05$, so we reject the null hypothesis and accept the alternative that the mean SVO_2 levels must be increased because the mean of differences is very likely to be < 0

1.d. $F^* = \frac{S_1^2}{S_2^2} = \frac{121.6454}{47.1288} = 2.5811$

$F_{0.05,9,9} = 3.179$ and $F_{0.95,9,9} = \frac{1}{3.179} = 0.315$

$3.179 > 2.5811 > 0.315$ so we do not reject the equal variance assumption

- We can use t-test:

$H_0: \mu_x = \mu_y$ or $\mu_d = 0$ $\leftarrow \mu_d = \mu_x - \mu_y$
$H_1: \mu_x < \mu_y$ or $\mu_d < 0$

$$S_p^2 = \frac{(10-1)(121.6454) + (10-1)(47.1288)}{10 + 10 - 2} = 84.3871$$

$$t = \frac{\bar{Y}_1 - \bar{Y}_2 - 0}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} = \frac{52.41 - 59.19}{\sqrt{\frac{84.3871}{10} + \frac{84.3871}{10}}} = -1.65$$

$-t_{0.05,18} = -1.734$ $\quad -1.65 > -1.734$

so we do not reject the null hypothesis and conclude that $\mu_x = \mu_y$.

- This result is clearly different from parts a-c most likely because this test assumed they were independent, which is not the case

2.a.

X	Y_1	Y_2	Y_3
10	6	9.14	8.26
8	5	8.14	4.04
13	7.5	8.74	18.94
9	5.5	8.77	5.9
11	6.5	9.26	11.18
14	8	8.1	23.9
6	4	6.13	1.58
4	3	3.1	0.4
12	7	9.13	14.72
7	4.5	7.26	2.62
5	3.5	4.74	0.86
mean: 9	6.5	7.5	8.4

	$X-\mu_x$	$Y_1-\mu_{Y_1}$	$Y_2-\mu_{Y_2}$	$Y_3-\mu_{Y_3}$
	1	0.5	1.64	-0.14
	-1	-0.5	0.64	-4.36
	4	2	1.24	10.54
	0	0	1.27	-2.5
	2	1	1.76	2.78
	5	2.5	0.60	15.5
	-3	-1.5	-1.37	-6.82
	-5	-2.5	-4.4	-8
	3	1.5	1.63	6.32
	-2	-1	-0.24	-5.78
	-4	-2	-2.76	-7.54
S_{xy}: Sum$((x-\mu_x)(y-\mu_y))$		55	55	250.58
sum $(X-\mu_x)^2$	110	27.5	41.28	625
sqrt ^	10.49	5.24	6.42	25
$\sqrt{S_{xx} S_{yy}}$		55	67.38	262.22
Pearson (r)		1	0.816	0.956

$\left(\frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}\right)$

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rank X	rank Y_1	rank Y_2	rank Y_3
5	5	2	5
7	7	6	7
2	2	5	2
6	6	4	6
4	4	1	4
1	1	7	1
9	9	9	9
11	11	11	11
3	3	3	3
8	8	8	8
10	10	10	10

	$(\text{rank Dif } Y_1)^2$	$(\text{rank Dif } Y_2)^2$	$(\text{rank Dif } Y_3)^2$
	0	9	0
	0	1	0
	0	9	0
	0	4	0
	0	9	0
	0	36	0
	0	0	0
	0	0	0
	0	0	0
	0	0	0
	0	0	0
Sum:	0	68	0
Spearman (r_s):	1	0.69	1

$$\left(1-\left(\frac{6\sum d_i^2}{n(n^2-1)}\right)\right)$$

2.b. Using my results from part a each of the 3 cases can be determined

Y_1: $r = 1$, $r_s = 1$, so $r = r_s$ (case 1)

Y_2: $r = 0.816$, $r_s = 0.69$, so $|r| > |r_s|$ (case 3)

Y_3: $r = 0.956$, $r_s = 1$, so $|r| < |r_s| = 1$ (case 2)

1. $r = r_s$ when there is a perfect correlation $1 = 1$ or $-1 = -1$
   - This case is shown in the first scatterplot where you can see a perfect line through the data

2. $|r| < |r_s| = 1$ occurs when all of the points generally move upword, but not in a perfect linear pattern. The rank is equal to 1 or -1 because each point remains higher or lower than the one before respectively.
   - This case is shown in the third scatterplot where each point is higher than the last, but it is not a perfect line shape.

3. $|r| > |r_s|$ occurs when points do not go up or down from the one before. This makes the x rank and y rank differ and result in a low $|r_s|$ score. However, there is still some correlation, so $|r|$ is reasonable.
   - This case is shown in the second scatterplot because the points seem to rise, but then come down after a point.