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Homework 1

Minimum

Linear Algebra

1. $3 \cdot 5 + 5 \cdot 1 = 23$

2. $\begin{bmatrix} 4 \cdot 3 + 3 \cdot 5 \\ 1 \cdot 3 + 2 \cdot 5 \end{bmatrix} = \begin{bmatrix} 27 \\ 13 \end{bmatrix}$

3. yes, $X^{-1} = \frac{1}{8-3} \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2/5 & -3/5 \\ -1/5 & 4/5 \end{bmatrix}$

4. $\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & -5 \\ 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ Rank $= 2$

Calculus

1. $y' = \frac{3}{2}x^2 - 3$

2. $2\cos(2)e^{-x} - 2\cos(2)xe^{-x} = (1-x)(2\cos(2)e^{-x})$

Prob and Stat

1. $\frac{2}{5}$

2. $\frac{2 \cdot (1 - 2/5)^2 + 3(-2/5)^2}{5} = \frac{6/5}{5} = 6/25$

3. $(0.5)^5 = 0.03125$

4. $h^3 \cdot t^2 = p \qquad h + t = 1$

$h^3 \cdot (1-h)^2 = p \qquad h^3 \cdot (1 - 2h + h^2) = p$

$h^5 - 2h^4 + h^3 = p$

$p' = 5h^4 - 8h^3 + 3h^2 = 0 = h^2(5h^2 - 8h + 3) = 0$

$h = \frac{8 \pm \sqrt{64 - 60}}{10} = \frac{8 \pm 2}{10} = 1, 3/5, 0$

$0^3 \cdot 1^2 = 0 \quad 1^3 \cdot 0^2 = 0 \quad (3/5)^3 \cdot (2/5)^2 = 0.03456$

h = heads t = tails $h = 3/5 \quad t = 2/5$

5. $(z = F, y = C) = 0.3$

$0.5 + 0.2 + 0.15 + 0.3 = \mathbf{0.7}$

Big-O Notation

1. Both are true because both functions are restricted by the logarithmic function, so they have the same assymptote behavior
2. $g(n) = O(f(n))$ because 3^n is significantly larger than n^3 and can act as an upper bound
3. $g(n) = O(f(n))$ because $\lim_{n \to \infty} \frac{2^n}{3^n} = 0$, which means that $f(n)$ is significantly larger

MEDIUM

Probability and Stats

1. a. True
   b. False
   c. False
   d. True
   e. True
2. $E_{xy}[(X - E_x[X])(Y - E_y[Y])]$
   $= E_{xy}[XY - XE_y[Y] - YE_x[X] + E_x[X]E_y[Y]]$
   $= E_{xy}[XY] - 2E_x[X]E_y[Y] + E_x[X]E_y[Y]$
   $= E_{xy}[XY] - E_x[X]E_y[Y]$
3. $\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Linear Algebra

1. $\begin{bmatrix} a & b \\ b & c \end{bmatrix}^{-1} = \frac{1}{ac - b^2} \begin{bmatrix} c & -b \\ -b & a \end{bmatrix}$ the values in the b positions do not change from eachother so it will stay symmetric or $A = A^T$
2. $A \cdot A^{-1} = I$ $A^{-1} = B$ $|A| \cdot |B| = |AB| = |I| = 1$
   $|B| = 1/|A| = |A^{-1}|$

3. $\sum_{i=1}^{n} x \geq \sqrt{\sum_{i=1}^{n} x^2}$

4. If the vector x can be represented by the base, then its transpose can be used to find x_i because of the properties of the dot product and that it is a scalar

Calculus

1. $\ln\left(\frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right) = \ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right) + \ln\left(e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right)$

$= -\ln(\sqrt{2\pi}) - \ln(\sigma) - ((x-\mu)^2/2\sigma^2)$

2. $\frac{-x^2+2x\mu-\mu^2}{2\sigma^2} \rightarrow \frac{2x-2\mu}{2\sigma^2}$