

Name: Joshua SchmidtDate: 9/16/2019*I pledge my honor that I have abided by the Stevens Honor System.* - Joshua Schmidt

Point values are assigned for each question.

Points earned:        / 100, =        %

1. Find an upper bound for  $f(n) = n^4 + 10n^2 + 5$ . Write your answer here:  $O(n^4)$  (4 points)

Prove your answer by giving values for the constants  $c$  and  $n_0$ . Choose the smallest integral value possible for  $c$ . (4 points)

$$f(n) = n^4 + 10n^2 + 5 < 2n^4, n_0 = 4, c = 2$$

2. Find an asymptotically tight bound for  $f(n) = 3n^3 - 2n$ . Write your answer here:  $\Theta(n^3)$  (4 points)

Prove your answer by giving values for the constants  $c_1$ ,  $c_2$ , and  $n_0$ . Choose the tightest integral values possible for  $c_1$  and  $c_2$ . (6 points)

$$f(n) = 3n^3 - 2n < 3n^3, n_0 = 1$$

$$f(n) = 3n^3 - 2n > 2n^3, n_0 = 1$$

$$c_1 = 2, c_2 = 3, n_0 = 1$$

3. Is  $3n - 4 \in \Omega(n^2)$ ? Circle your answer: yes / **no**. (2 points)

If yes, prove your answer by giving values for the constants  $c$  and  $n_0$ . Choose the smallest integral value possible for  $c$ . If no, derive a contradiction. (4 points)

$$3n - 4 < n^2, n_0 = 1 \rightarrow -1 < 1$$

The function has a value lower than Omega, which should not be possible.

4. Write the following asymptotic efficiency classes in **increasing** order of magnitude.

$O(n^2), O(2^n), O(1), O(n \lg n), O(n), O(n!), O(n^3), O(\lg n), O(n^n), O(n^2 \lg n)$  (2 points each)

$O(1)$ ,  $O(\lg n)$ ,  $O(n)$ ,  $O(n \lg n)$ ,  $O(n^2)$ ,  $O(n^2 \lg n)$ ,  $O(n^3)$ ,  $O(2^n)$ ,  $O(n!)$ ,  $O(n^n)$ ,

5. Determine the largest size  $n$  of a problem that can be solved in time  $t$ , assuming that the algorithm takes  $f(n)$  milliseconds.  $n$  must be an integer. (2 points each)

a.  $f(n) = n, t = 1 \text{ second}$  1000

b.  $f(n) = n \lg n, t = 1 \text{ hour}$  204100

c.  $f(n) = n^2, t = 1 \text{ hour}$  360000

d.  $f(n) = n^3$ ,  $t = 1$  day      864000

e.  $f(n) = n!$ ,  $t = 1$  minute      8

6. Suppose we are comparing two sorting algorithms and that for all inputs of size  $n$  the first algorithm runs in  $4n^3$  seconds, while the second algorithm runs in  $64n \lg n$  seconds. For which integral values of  $n$  does the first algorithm beat the second algorithm?  $n_0 = 7$  (4 points)

Explain how you got your answer or paste code that solves the problem (2 point):

I tried different values for  $n$  until I got one where the second algorithm was less than the first. This occurred at 7.

7. Give the complexity of the following methods. Choose the most appropriate notation from among  $O$ ,  $\Theta$ , and  $\Omega$ . (8 points each)

```
int function1(int n) {
    int count = 0;
    for (int i = n / 2; i <= n; i++) {
        for (int j = 1; j <= n; j *= 2) {
            count++;
        }
    }
    return count;
}
```

Answer:  $\Theta(n \lg n)$

```
int function2(int n) {
    int count = 0;
    for (int i = 1; i * i * i <= n; i++) {
        count++;
    }
    return count;
}
```

Answer:  $\Theta(n^{1/3})$

```
int function3(int n) {
    int count = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            for (int k = 1; k <= n; k++) {
                count++;
            }
        }
    }
    return count;
}
```

Answer:  $\Theta(n^3)$

```

int function4(int n) {
    int count = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            count++;
            break;
        }
    }
    return count;
}

```

Answer:  $\Theta(n)$

```

int function5(int n) {
    int count = 0;
    for (int i = 1; i <= n; i++) {
        count++;
    }
    for (int j = 1; j <= n; j++) {
        count++;
    }
    return count;
}

```

Answer:  $\Theta(n)$