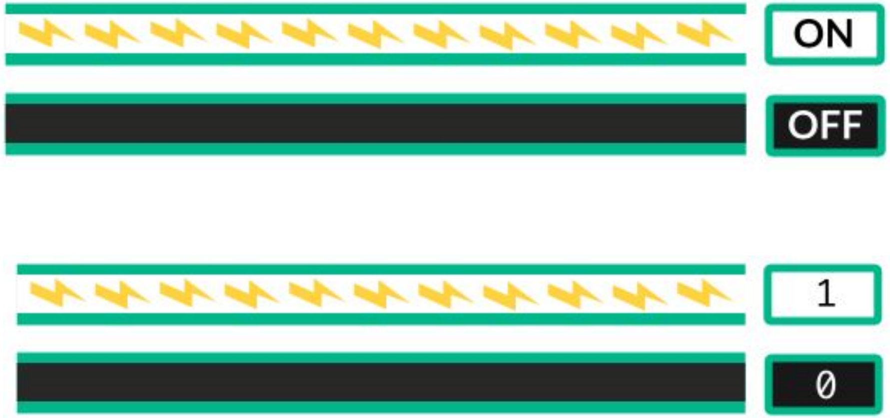
Number Systems

ACSL Contest #1

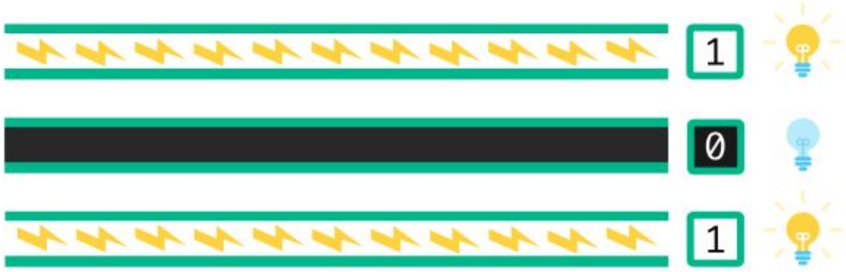
Intro to Computer Number Systems

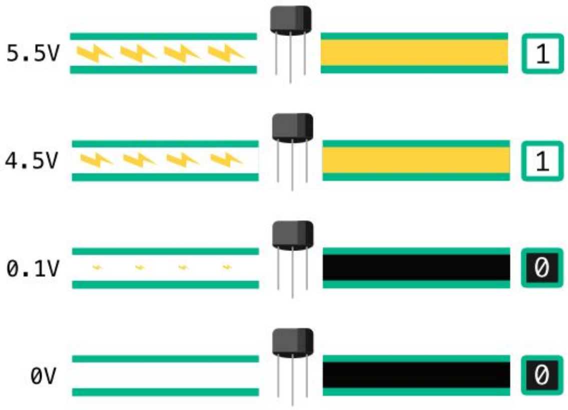
- All digital computers are electronic devices and ultimately can do one thing: detect whether an electrical signal is on or off.
- That basic information, called a *bit* (**bi**nary digi**t**), has two values: a 1 (or *true*) when the signal is on, and a 0 (of *false*) when the signal is off.



Intro to Computer Number Systems

- Larger values can be stored by a group of bits.
- Disclaimer: saying the wire is on or off is an abstraction that simplifies how computers actually work.
    - Transistor: a tiny physical device that acts like a digital switch in computers. When there's enough electricity flowing, it's on. Otherwise, it's off.





Number Systems

- A *number system* is the way we name and represent numbers.
    - The number system we use in our daily life is known as **decimal number system** or **base 10** because it is based on 10 different digits: 0, 1, 2, ..., 8, and 9.
- In CS, 3 additional number systems are commonly used: **binary** (base 2), **octal** (base 8), and **hexadecimal** (base 16).
    - Binary: how numbers are stored in a computer.
    - Octal: user-friendly representation.
        - Represents 3 binary bits.
    - Hexadecimal: user-friendly representation.
        - Represents 4 binary bits.

Number Systems

- The decimal number 53201, stored as in the computer as the binary number 10000001111000101100, is represented by the octal number 2017054, and the hex number 81E2C.
- In general, N bits have 2^N different values.

Number System	Base	Digits Used	Examples
Binary	2	0,1	10110_2, 10110011_2
Octal	8	0,1,2,3,4,5,6,7	75021_8, 231_8, 60012_8
Decimal	10	0,1,2,3,4,5,6,7,8,9	97425_{10} or simply 97425
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F	$54A2DD0F_{16}$

Decimal System

- The decimal value of a decimal number is simply sum of each digit multiplied by its place value:

$$12345 = 1 \times 10^4 + 2 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0 = 10000 + 2000 + 300 + 40 + 5 = 12345$$

$$3079 = 3 \times 10^3 + 0 \times 10^2 + 7 \times 10^1 + 9 \times 10^0 = 3000 + 70 + 9 = 3079$$

Binary System

- Works the same as the decimal system, the only difference is the value each place represents.

and so on

$$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$$
$$2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$$

$$\underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}$$
$$2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$$

$$(1 \times 16) + (0 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1)$$

$$16 + 0 + 4 + 0 + 1$$

$$= 21$$

(decimal system)

Octal System

- Works the same as the decimal system, the only difference is the value each place represents.

and so on

___	___	___	___	___
8^4	8^3	8^2	8^1	8^0

0	0	1	5	3
8^4	8^3	8^2	8^1	8^0

Hexadecimal System

- Base logic remains the same; however, A, B, C, D, E, and F function as digits that represent the decimal quantities 10, 11, 12, 13, 14, and 15, respectively.

and so on

___	___	___	___	___
16^4	16^3	16^2	16^1	16^0

0	0	1	5	C
16^4	16^3	16^2	16^1	16^0

Review: Converting to Decimal from Any System

- The decimal value of any number of any base is the sum of each digit multiplied by its place values:

$$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 4 + 0 + 1 = 13_{10}$$

$$175_8 = 1 \times 8^2 + 7 \times 8^1 + 5 \times 8^0 = 1 \times 64 + 7 \times 8 + 5 \times 1 = 64 + 56 + 5 = 125_{10}$$

$$A5E_{16} = 10 \times 16^2 + 5 \times 16^1 + 14 \times 16^0 = 10 \times 256 + 5 \times 16 + 14 \times 1 = 2560 + 80 + 14 = 2654_{10}$$

Converting from Decimal to Any System

- The algorithm to convert a number from any base to decimal requires finding how many times successive powers of the base go into the number, starting with the largest power of the base less than the starting number.

$3206_{10} = x_8$. Find x.

1) Identify largest power of eight that is less than or equal to 3206.

$$3206 = 6 \cdot 8^3 + 134$$

Start as far over to the left as possible and fill in

carry over remainder

repeat

repeat

6	2	0	6
8^3	8^2	8^1	8^0

2) Identify value that when multiplied by the next 8 power down, gets as close as possible to the remainder

$$134 = 2 \cdot 8^2 + 6$$

3) Repeat until all slots are filled and the remainder = 0

Converting From Octal & Hexadecimal —> Binary

- Octal —> Binary ~ replace each octal digit w/ its corresponding 3 binary bits:

$$375_8 = \;011 \;\; 111 \;\; 101_2 = 11111101_2$$

- Hex —> Binary ~ replace each hex digit w/ its corresponding 4 binary bits:

$$FD_{16} = \;1111 \;\; 1101_2 = 11111101_2$$

Converting From Binary —> Octal & Hexadecimal

- Binary —> Octal: group the bits by 3s starting at the right and convert each group.

$$10000001111000101100 = 10\ 000\ 001\ 111\ 000\ 101\ 100 = 2017054_8$$

- Binary —> Hexadecimal: group the bits by 4s starting at the right and convert each group.

$$10000001111000101100 = 1000\ 0001\ 1110\ 0010\ 1100 = 81E2C_{16}$$

ACSL: What you need to know...

- ACSL problems will focus on “converting between binary, octal, decimal, and hexadecimal, basic arithmetic of numbers in those bases, and occasionally, fractions in those bases.”

To be successful in this category, you must know the following facts cold:

1. The binary value of each octal digit 0, 1, ..., 7
2. The binary value of each hex digit 0, 1, ..., 9, A, B, C, D, E, F
3. The decimal value of each hex digit 0, 1, ..., F
4. Powers of 2, up to 4096
5. Powers of 8, up to 4096
6. Powers of 16, up to 65,536

4. (2^12)

5. (8^4)

6. (16^4)

ACSL: Useful chart for 1-4

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14

Sample Problem 1

Solve for x where $x_{16} = 3676_8$

- Option 1: Octal —> Decimal —> 16 = x
- Option 2: Octal —> Hexadecimal using the binary representation of the number

$$
\begin{aligned}
3676_8 &= 011\ 110\ 111\ 110_2 && \text{convert each octal digit into base 2} \\
&= 0111\ 1011\ 1110_2 && \text{group by 4 bits, from right-to-left} \\
&= 7\ \mathrm{B}\ \mathrm{E}_{16} && \text{convert each group of 4 bits into a hex digit}
\end{aligned}
$$

Sample Problem 2

Solve for x in the following hexadecimal equation: $x = \mathrm{F5AD}_{16} - \mathrm{69EB}_{16}$

- Option 1: Hex —> Decimal —> Subtract in decimal —> Hex
- Option 2: Perform arithmetic in base 16 (hint: work right-to-left, from least significant digits to the most)

The rightmost digit becomes 2, because D-B=2.

The next column is A-E. We need to *borrow* a one from the 5 column, and 1A-E=C

In the next column, 4-9=B, again, borrowing a 1 from the next column.

Finally, the leftmost column, E-6=8

Combining these results of each column, we get a final answer of $8BC2_{16}$.

Source Links

- http://www.categories.acsl.org/wiki/index.php?title=Computer_Number_Systems
- https://www.khanacademy.org/computing/computers-and-internet/xcae6f4a7ff015e7d:computers/xcae6f4a7ff015e7d:from-electricity-to-bits/a/from-electricity-to-bits?modal=1