Recursive Functions Problems

Past Contests

Senior Q1

Recursive Functions

Find $f(28)$, given:

$$f(x) = \begin{cases} f(x/2)+1 & \text{if } x \geq 6 \text{ and even} \\ f([x/2])-3 & \text{if } x \geq 1 \text{ and odd} \\ 3x+4 & \text{otherwise} \end{cases}$$

Note: [a] returns the greatest <u>integer</u> less than or equal to a

$$f(28) = f(28/2)+1 = f(14)+1 = -4+1 = -3$$
$$= f(14/2)+1 = f(7)+1 = -5 = 1 = -4$$
$$= f([7/2])-3 = f(3)-3 = -2-3 = -5$$
$$= f([3/2])-3 = f(1)-3 = 1-3 = -2$$
$$= f([1/2])-3 = f(0)-3 = 4-3 = 1$$
$$= f(0) = 3*0+4 = 4$$

$f(28) = f(14)+1$
$f(14) = f(7)+1$
$f(7) = f(3)-3$
$f(3) = f(1)-3$
$f(1) = f(0)-3$
$f(0) = 3(0)+4$
$= 4$

$f(28) = -4+1 = -3$

$f(14) = -5+1 = -4$
$f(7) = -2-3 = -5$
$f(3) = 1-3 = -2$
$f(1) = 4-3 = 1$

Intermediate Q1

Find $f(6)$ given:

$$f(x) = \begin{cases} 2 * f(x-1) + 3 & if\ x > 1 \\ 2 & if\ x = 1 \end{cases}$$

$f(1) = 2$

$f(2) = 2 * f(1) + 3 = 2 * 2 + 3 = 7$
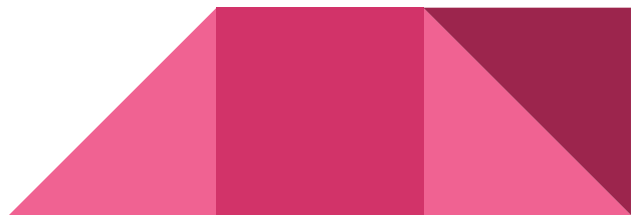
$f(3) = 2 * f(2) + 3 = 2 * 7 + 3 = 17$

$f(4) = 2 * f(3) + 3 = 2 * 17 + 3 = 37$

$f(5) = 2 * f(4) + 3 = 2 * 37 + 3 = 77$

$f(6) = 2 * f(5) + 3 = 2 * 77 + 3 = 157$

Senior Q2

Find $f(20,10)$, given:

$$f(x,y) = \begin{cases} f(x-3, y-1) + 1 & \text{if } x > y \\ f(x-1, y) - 2 & \text{if } x = y \\ 2x - y & \text{if } x < y \end{cases}$$

$f(20, 10) = f(17, 9) + 1 = 5 + 1 = 6$

$f(17, 9) = f(14, 8) + 1 = 4 + 1 = 5$

$f(14, 8) = f(11, 7) + 1 = 3 + 1 = 4$

$f(11, 7) = f(8, 6) + 1 = 2 + 1 = 3$

$f(8, 6) = f(5, 5) + 1 = 1 + 1 = 2$

$f(5, 5) = f(4, 5) - 2 = 3 - 2 = 1$

$f(4, 5) = 2*4 - 5 = 3$ Now substitute backwards.

$f(20,10) = f(17, 9) + 1$

$f(17,9) = f(14, 8) + 1$

$f(14, 8) = f(11, 7) + 1$

$f(11,7) = f(8, 6) + 1$

$f(8,6) = f(5, 5) + 1$

$f(5,5) = f(4, 5) - 2$

$f(4,5) = 8 - 5 = 3$

$f(20,10) = 5 + 1 = 6$

$f(17,9) = 4 + 1 = 5$

$f(14,8) = 3 + 1 = 4$

$f(11,7) = 2 + 1 = 3$

$f(8,6) = 1 + 1 = 2$

$f(5,5) = 3 - 2 = 1$

Intermediate Q2

Find $f(16)$ given:

$$f(x) = \begin{cases} 2 * f(x-3) + 4 & \text{if } x \geq 8 \\ f([x/2]) - 1 & \text{if } 0 < x < 8 \\ x * x - x & \text{if } x \leq 0 \end{cases}$$

where [x] = greatest integer $\leq$ x

$f(16) = 2 * f(13) + 4 = 2 * 0 + 4 = 4$

$f(13) = 2 * f(10) + 4 = 2 * (-2) + 4 = 0$

$f(10) = 2* f(7) + 4 = 2 * (-3) + 4 = -2$

$f(7) = f([7/2]) - 1 = f(3) - 1 = -2 - 1 = -3$

$f(3) = f[3/2]) - 1 = f(1) - 1 = -1 - 1 = -2$

$f(1) = f([1/2]) - 1 = f(0) - 1 = 0 - 1 - 1$

$f(0) = 0 * 0 - 0 = 0$ Now substitute backwards

2. 4

Source Links

https://www.acsl.org/get-started/study-materials