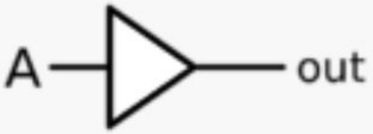
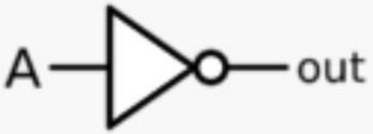
Digital Electronics

ACSL Contest #4

Overview

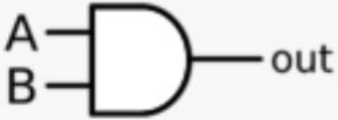
- A digital circuit is constructed from logic gates.
- Each logic gate performs a function of boolean logic based on its inputs, such as AND or OR.
- Each circuit can be represented as a Boolean Algebra expression
  - This topic is an extension of Boolean Algebra

Definitions

<table>
<tr><th>NAME</th><th>GRAPHICAL SYMBOL</th><th>ALGEBRAIC EXPRESSION</th><th>TRUTH TABLE</th></tr>
<tr><td>BUFFER</td><td>
</td><td>X = A</td><td><table><tr><th>INPUT</th><th>OUTPUT</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr></table></td></tr>
<tr><td>NOT</td><td>
</td><td>$X = \overline{A}$ or $\neg A$</td><td><table><tr><th>INPUT</th><th>OUTPUT</th></tr><tr><td>A</td><td>X</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table></td></tr>
</table>

Definitions

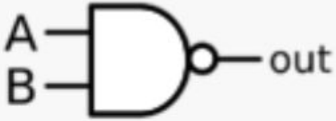
AND



X = AB or $A \cdot B$

INPUT		OUTPUT
A	**B**	**X**
0	0	0
0	1	0
1	0	0
1	1	1

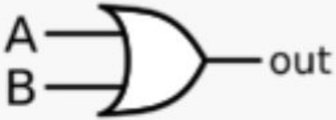
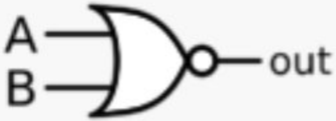
NAND



X = $\overline{AB}$ or $\overline{A \cdot B}$

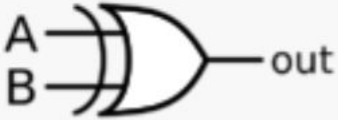
INPUT		OUTPUT
A	**B**	**X**
0	0	1
0	1	1
1	0	1
1	1	0

Definitions

<table>
<tr>
<td>OR</td>
<td>
</td>
<td>$X = A + B$</td>
<td>
<table>
<tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr>
<tr><th>A</th><th>B</th><th>X</th></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>1</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
</table>
</td>
</tr>
<tr>
<td>NOR</td>
<td>
</td>
<td>$X = \overline{A + B}$</td>
<td>
<table>
<tr><th colspan="2">INPUT</th><th>OUTPUT</th></tr>
<tr><th>A</th><th>B</th><th>X</th></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
<tr><td>0</td><td>1</td><td>0</td></tr>
<tr><td>1</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td><td>0</td></tr>
</table>
</td>
</tr>
</table>

Definitions

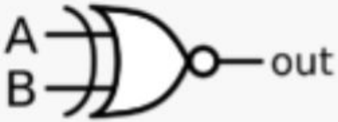
XOR



$X = A \oplus B$

INPUT		OUTPUT
A	**B**	**X**
0	0	0
0	1	1
1	0	1
1	1	0

XNOR



$X = \overline{A \oplus B}$ or $A \odot B$

INPUT		OUTPUT
A	**B**	**X**
0	0	1
0	1	0
1	0	0
1	1	1

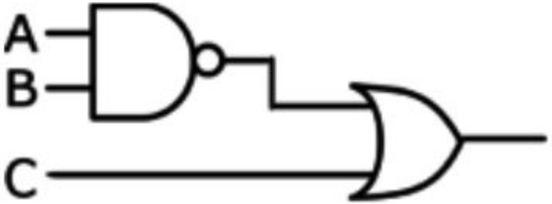
Online Tools

- The Logisim application is a wonderful tool for exploring this topic.
- Logisim is free to download and use; among its many features is support to automatically draw a circuit from a Boolean Algebra expression; to simulate the circuit with arbitrary inputs; and to complete a truth table for the circuit.
- From the application, you can import the circuit corresponding to the Sample Problem 1 from the file located at http://www.acsl.org/misc/wiki-digital-electronics-sample1.circ. There are many YouTube videos that show how to use the Logisim application; including a very nice 4 minute tutorial.

Logisim contains many additional advanced features that are beyond the scope of ACSL problems.

Sample Problem 1

Find all ordered triplets (A, B, C) which make the following circuit FALSE:



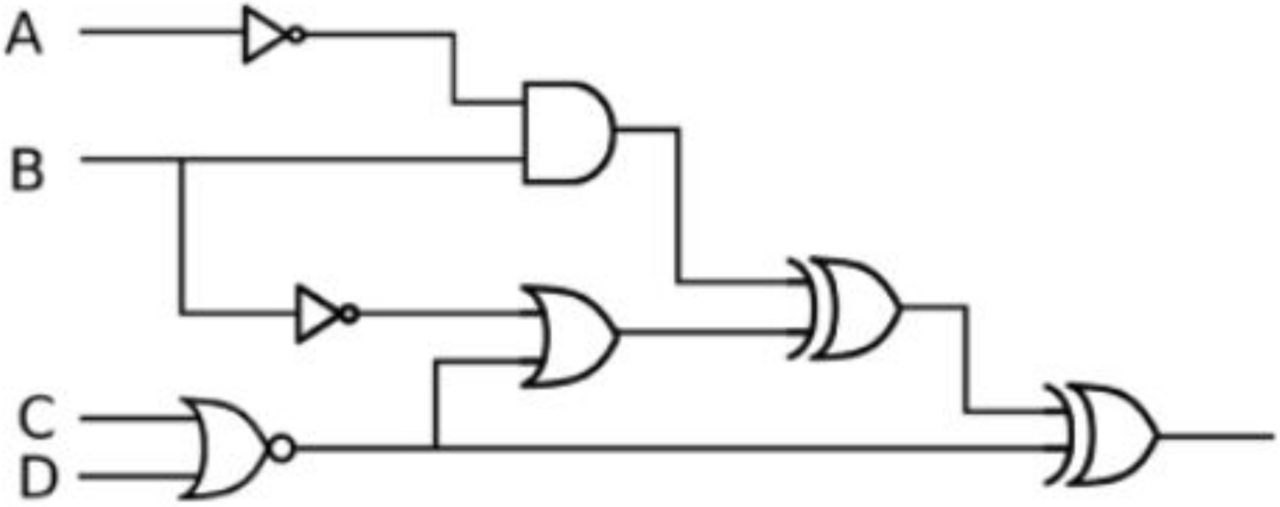
Solution:

One approach to solving this problem is to reason about that inputs and outputs are necessary at each gate. For the circuit to be FALSE, both inputs to the file OR gate must be false. Thus, input C must be FALSE, and the output of the NAND gate must also be false. The NAND gate is false only when both of its inputs are TRUE; thus, inputs A and B must both be TRUE. The final answer is (TRUE, TRUE, FALSE), or (1, 1, 0).

Another approach to solving this problem is to translate the circuit into a Boolean Algebra expression and simplify the expression using the laws of Boolean Algebra. This circuit translates to the Boolean expression $\overline{AB} + C$. To find when this is FALSE we can equivalently find when the $\overline{\overline{AB} + C}$ is TRUE. The expression becomes $\overline{\overline{AB}} \cdot \overline{C}$ after applying DeMorgan's Law. The double NOT over the AB expression cancels out, to become $AB\overline{C}$. The AND of 3 terms is TRUE when each term is TRUE, or A=1, B=1 and C=0.

Sample Problem 2: Question

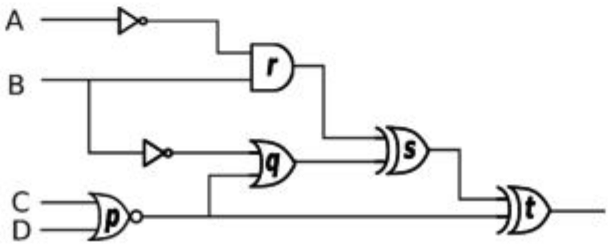
How many ordered 4-tuples (A, B, C, D) make the following circuit TRUE?



Sample Problem 2: Answer

Solution:

We'll use a truth table to solve this problem. The rows in the truth table will correspond to all possible inputs - 16 in this case, since there are 4 inputs. The output columns will be the output of each gate, other than the NOT gates. The diagram below labels each of the gates; it's useful to keep the column straight when working the truth table.

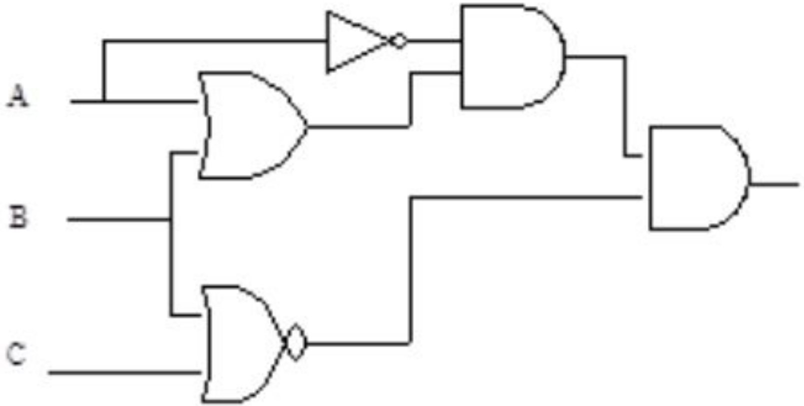


INPUT				OUTPUT				
A	B	C	D	p	q	r	s	t
				$\overline{C+D}$	$p+\overline{B}$	$\overline{A}B$	$r \oplus q$	$s \oplus p$
0	0	0	0	1	1	0	1	0
0	0	0	1	0	1	0	1	1
0	0	1	0	0	1	0	1	1
0	0	1	1	0	1	0	1	1
0	1	0	0	1	1	1	0	1
0	1	0	1	0	0	1	1	1
0	1	1	0	0	1	1	1	1
0	1	1	1	0	0	1	1	1
1	0	0	0	1	1	0	1	0
1	0	0	1	0	1	0	1	1
1	0	1	0	0	1	0	1	1
1	0	1	1	0	1	0	1	1
1	1	0	0	1	1	0	1	0
1	1	0	1	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0

From the truth table, there are 10 rows where the final output is TRUE.

Sample Problem 3

Simplify the Boolean expression that this circuit represents.



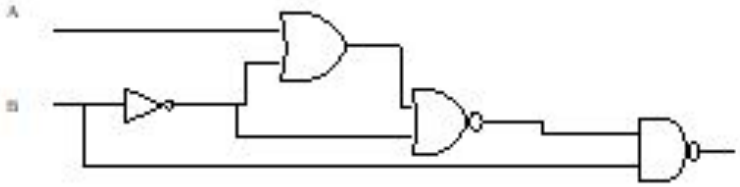
Solution:

The circuit translates as follows:

$$(\overline{A}(A+B))\overline{B+C} = (\overline{A}A + \overline{A}B)\overline{B}\,\overline{C} = (0+\overline{A}B)\overline{B}\,\overline{C} = \overline{A}B\overline{B}\,\overline{C} = 0$$

Past Contests Senior

3. Digital Electronics
Which ordered pairs make the circuit FALSE?



3. Digital Electronics The circuit translates to: $\overline{(\overline{(A+\overline{B})+\overline{B}})B}$

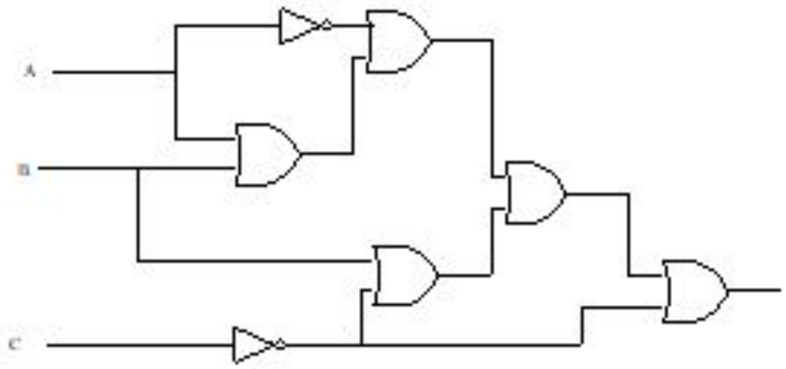
$\overline{(\overline{(A+\overline{B})+\overline{B}})B} = \overline{\overline{(A+\overline{B})+\overline{B}}}+\overline{B} = A+\overline{B}+\overline{B}+\overline{B} = A+\overline{B}$

This is only FALSE when both are FALSE. Only one ordered pair satisfies this condition: (0, 1)

3. (0, 1)

Past Contests Senior

4. Digital Electronics
Simplify the Boolean expression represented by the digital circuit.



4. Digital Electronics

The circuit translates to: $((\overline{A}+(A+B))+(B+\overline{C}))+\overline{C}$

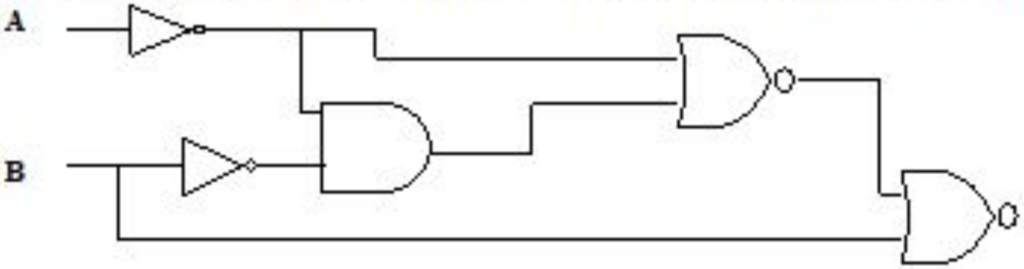
$((\overline{A}+(A+B))+(B+\overline{C}))+\overline{C} =$

$=1+B+\overline{C} \; = 1$

4. 1

Past Contests Intermediate

3. **Digital Electronics** Which ordered pairs make the circuit TRUE?



3. Digital Electronics

The circuit translates to: $\overline{\overline{\overline{A}+\overline{\overline{A}\,\overline{B}}}+B}$

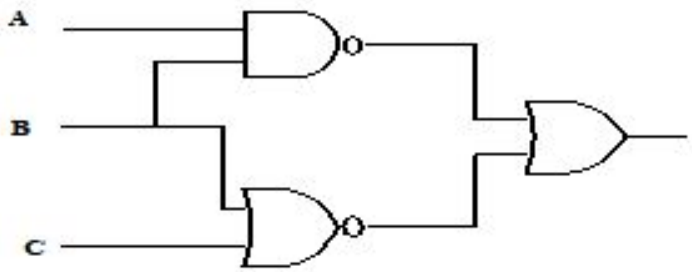
$\overline{\overline{\overline{A}+\overline{\overline{A}\,\overline{B}}}+B} = (\overline{\overline{\overline{A}+\overline{\overline{A}\,\overline{B}}}})\overline{B} = (\overline{A}+\overline{\overline{A}\,\overline{B}})\overline{B} = \overline{A}\,\overline{B}$

When both are FALSE, the expression is TRUE. (0, 0)

3. (0, 0)

Past Contests Intermediate

4. **Digital Electronics** How many ordered triples make the circuit FALSE?



4. **Digital Electronics**

The circuit translates to: $\overline{AB} + \overline{B + C}$

$\overline{AB} + \overline{B+C} = (\overline{A} + \overline{B}) + \overline{B}\,\overline{C} = \overline{A} + (\overline{B} + \overline{B}\,\overline{C}) = \overline{A} + \overline{B}(1 + \overline{C}) = \overline{A} + \overline{B}$

This is FALSE only when both are TRUE. (1, 1, *)

4. 2