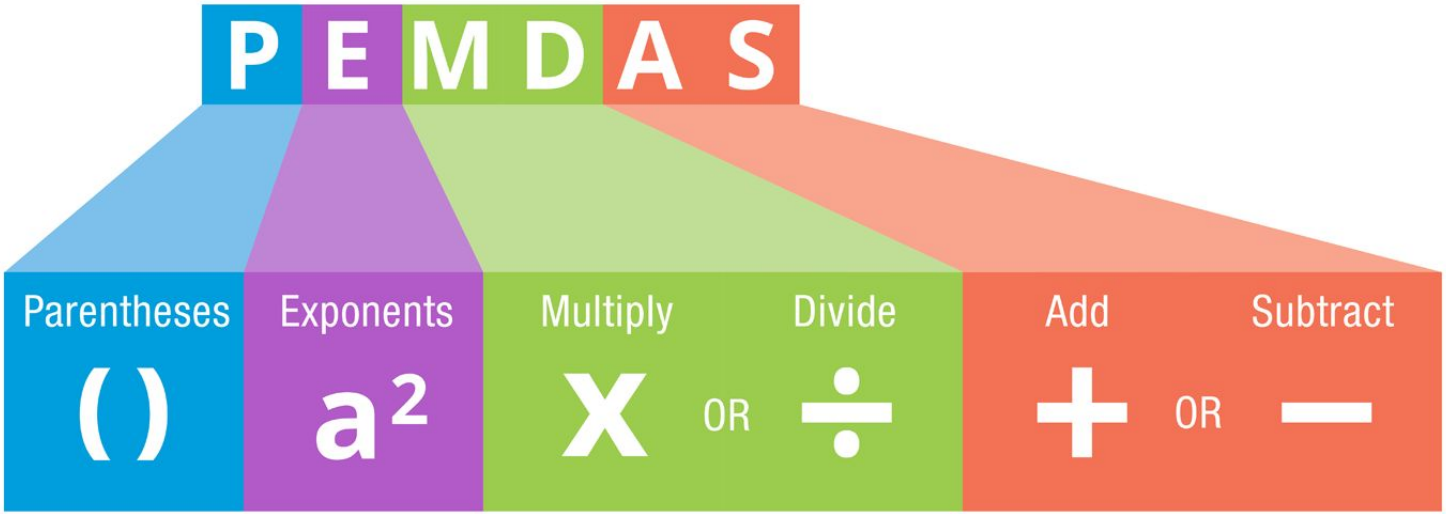
Prefix/Infix/Postfix Notation

ACSL Contest #2 Topics

Evaluating arithmetic

If asked to evaluate this expression, in what order would you do so?

$$5 + \frac{8}{3-1}$$



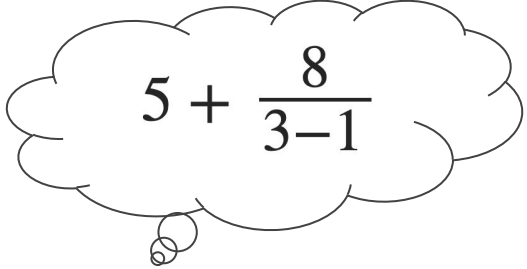
Definition of Infix

- Infix is a form of mathematics notation.
- “It is the notation commonly used in arithmetical and logical formulae and statements.”
- “It is characterized by the placement of operators between operands—‘infixed operators’—such as the plus sign in 2 + 2.”

How does Infix relate to programming?

- Think about the mental process you go through to use PEMDAS for evaluating an expression/equation: “The algorithm to evaluate an *infix* expression is complex, as it must address the order of precedence.”
- “Two alternative notations have been developed which lend themselves to simple computer algorithms for evaluating expressions.”
  - **Prefix and Postfix.**

Prefix and Postfix



- In ***prefix*** notation, each **operator** is placed **before** its **operands**.
  - The expression above would be `+ 5 / 8 - 3 1`.
- In ***postfix*** notation, each **operator** is placed **after** its **operand**.
  - The expression above is `5 8 3 1 - / +`.
- In *prefix* and *postfix* notations, there is no notion of order of precedence, nor are there any parentheses. The evaluation is the same regardless of the operators.

ACSL Information

- “Problems in this category ask you to convert between prefix, infix, and postfix, or to evaluate an expression in prefix or postfix.”
- All numbers are single digits.

General Formula for Infix -> Postfix/Prefix Conversion

Infix -> Postfix/Prefix:

1. Fully parenthesize the infix expression. It should now consist solely of “terms”: a binary operator sandwiched between two operands.
2. Write down the operands in the same order that they appear in the infix expression.
3. Look at each term in the infix expression in the order that one would evaluate them, i.e., inner-most parenthesis to outer-most and left to right among terms of the same depth.
4. For each term, write down the operand before (after) the operators.

Infix to Prefix

The following sequence of steps illustrates converting $X = \left(AB - \frac{C}{D}\right)^E$ from infix to prefix:

(X = (((A * B) - (C / D)) ↑ E))
X A B C D E
X `* A B` C D E
X `* A B` `/ C D` E
X `- * A B / C D` E
X `↑- * A B / C D E`
`= X ↑ - * A B / C D E`

Infix to Postfix

Infix to Postfix

The following sequence of steps illustrates converting $X = \left(AB - \frac{C}{D}\right)^E$ from infix to postfix:

(X = (((A * B) - (C / D)) ↑ E))
X A B C D E
X `A B *` C D E
X `A B *` `C D /` E
X `A B * C D / -` E
X `A B * C D / - E ↑`
`X A B * C D / - E ↑ =`

General Formula for Postfix/Prefix -> Infix Conversion

Postfix/Prefix -> Infix:

1. One way to convert from prefix (postfix) to infix is to make repeated scans through the expression.
2. Each scan, find an operator with two adjacent operators and replace it with a parenthesized infix expression.

ACSL's note: This is not the most efficient algorithm, but works well for a human. Here's what a more efficient algorithm looks like: https://www.geeksforgeeks.org/postfix-to-infix/

Prefix to Infix

Prefix to Infix

The following sequence of steps illustrates converting $(3 * 4 + \frac{8}{2})^{(7-5)}$ from its prefix representation to infix:

↑ + * 3 4 / 8 2 – 7 5
↑ + `(3*4)` / 8 2 – 7 5
↑ + `(3*4)` `(8/2)` – 7 5
↑ `(3*4)+(8/2)` - 7 5
↑ `((3*4)+(8/2))` `(7–5)`
`(((3*4)+(8/2))↑(7–5))`

Postfix to Infix

Postfix to Infix

The following sequence of steps illustrates converting $(3 * 4 + \frac{8}{2})^{(7-5)}$ from its postfix representation to infix:

3 4 * 8 2 / + 7 5 - ↑
(3∗4) 8 2 / + 7 5 - ↑
(3∗4) (8/2) + 7 5 - ↑
((3∗4)+(8/2)) 7 5 - ↑
((3∗4)+(8/2)) (7−5) ↑
(((3∗4)+(8/2))↑(7−5))

Past Contest Problems: Senior

1. **Prefix/Infix/Postfix**
   Convert the following infix expression to prefix:

$$\frac{C(A+B)}{A^2} - \frac{BC + A^2}{B+C}$$

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1. **Prefix/Infix/Postfix**

$\frac{C(A+B)}{A^2} - \frac{BC+A^2}{B+C}$ = (C*(A+B))/A² − (B*C + A²)/(B+C)

= (C*(+AB))/(↑A2) − ((*BC) + (↑A2))/(+BC)

= ((*C+AB)/(↑A2)) − ((+*BC↑A2)/(+BC))

= ((/*C+AB↑A2) − (/+*BC↑A2+BC))

Past Contest Problems: Senior

2. **Prefix/Infix/Postfix**

Given a@b = min{a, b} Evaluate the following postfix expression.
(Note: all numbers are single digits)

3 2 2 ↑ @ 1 1 @ + 2 @ 2 2 ↑ @ 2 + 3 1 − @

2. Prefix/Infix/Postfix

3 2 2 ↑ @ 1 1 @ + 2 @ 2 2 ↑ @ 2 + 3 1 − @

= 3 (2 2 ↑) @ (1 1 @) + 2 @ (2 2 ↑) @ 2 + (3 1 −) @

= (3 4 @) 1 + 2 @ 4 @ 2 + 2 @ = (3 1 +) 2 @ 4 @ 2 + 2 @

= (4 2 @) 4 @ 2 + 2 @ = (2 4 @) 2 + 2 @ = (2 2 +) 2 @ = 4 2 @ = 2

Past Contest Problems: Intermediate

1. **Prefix/Infix/Postfix**

   Convert this infix expression into a postfix expression.

a * b + c * (a + b) / a – a / b / c

1. **Prefix/Infix/Postfix**

a * b + c * (a + b) / a – a / b / c = (a * b) + c * (a + b) / a – (a / b) / c
= (a b *) + (c * (a b +))/ a – ((a b /) / c)
= (a b *) + ((c a b + *) / a) – (a b / c /)
= ((a b *) + (c a b + * a /)) – (a b / c /)
= ((a b * c a b + * a / +) – (a b / c /))
= a b * c a b + * a / + a b / c / –

Past Contest Problems: Intermediate

– – –

2. Prefix/Infix/Postfix

Evaluate the following prefix expression:
(Note: all numbers are single digits)

– * 4 ↑ – / 8 2 3 2 / / 8 4 2

2. Prefix/Infix/Postfix

– * 4 ↑ – / 8 2 3 2 / / 8 4 2 = – * 4 ↑ – (/ 8 2) 3 2 / (/ 8 4) 2
= – * 4 ↑(– 4 3) 2 (/ 2 2)
= – * 4 (↑ 1 2) 1
= – (* 4 1) 1
= – 4 1
= 3

Source Links

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https://www.acsl.org/get-started/study-materials

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