Excess mortality P-scores generally depend on the age structure of a population

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Abstract

The excess mortality P-score can be expressed as the weighted average of age-specific relative increases/decreases in the expected death rates wrt. the age distribution of deaths in a population under the expected death rates. This means that the P-score does not generally measure a pure effect of changing death rates. Two populations may experience the same relative change in age-specific death rates, yet show different P-scores. The only exception to the age-structure dependence of the P-score is when the age-specific relative changes in the expected death rates are constant over age.

The P-score is a widely used measure of excess death, defined as

$$P = D^O/D^E$$

where D^O denote the actual deaths in some population over some time interval and D^E denote the expected deaths under some counterfactual scenario. Often this counterfactual scenario relates to a situation where an alternative set of death rates m is acting on the population. We can express this situation as

$$P = \frac{\delta mT}{mT},$$

where T is the population-(exposure-time) under risk of death and m the corresponding expected death rate. The expected death rate is modified by factor δ in the actual scenario, yielding $D^O = \delta mT$, and $D^E = mT$.

To what degree does the P-score depend on the age composition of a population – in addition to the relative change in death rates? We call a measure age-dependent if it varies with the age composition of a population, even if the underlying rates remain the same.

In a population structured by integer age x we can re-write the P-score as

$$P = \frac{\sum_x \delta_x m_x T_x}{\sum_x m_x T_x} = \frac{\sum_x D_x^O}{\sum_x D_x^E} = \frac{D^O}{D^E},$$

where m_x, δ_x, and T_x are the age-specific expected death rates, the age-specific increase factors of expected mortality, and the age-specific population at risk.

It is now possible to analyze the age dependence of the P-score for different functions δ_x, the mortality rate effect.

Scenario 1: An age-constant proportional change in death rates. Given that $\delta_x = c$ actual death rates are elevated by a constant proportion in each age relative to the expected death rates, thus

$$P = \frac{\sum_x c m_x T_x}{\sum_x m_x T_x} = \frac{c \sum_x D_x^O}{\sum_x D_x^E} = c.$$

Given an age-constant proportional change in death rates, this proportional change is measured by the P-score irrespective of the age structure of the population – here, the P-score is not age-dependent.

We can prove that the P-score is independent of T_x if and only if $\delta_x = c$. For that, let's assume two different populations T_x^A and T_x^B. As the difference between the two population structures should have no bearing on the P-score we write

$$P = \frac{\sum_x \delta_x m_x T_x^A}{\sum_x m_x T_x^A} = \frac{\sum_x \delta_x m_x T_x^B}{\sum_x m_x T_x^B},$$

which can be rearranged to

$$\sum_x \delta_x m_x T_x^A \sum_x m_x T_x^B = \sum_x \delta_x m_x T_x^B \sum_x m_x T_x^A.$$

The equality is only true for $\delta_x = c$. If δ_x changes with age one can always choose two population structures T^A and T_B as to make the two sides unequal.

Scenario 2: An age-linear proportional change in death rates. For $\delta_x = a + bx$ we have

$$D^O = \sum_x (a + bx) m_x T_x = a \sum_x m_x T_x + b \sum_x x m_x T_x,$$

and thus

$$P = a + b \left(\frac{\sum_x x m_x T_x}{\sum_x m_x T_x} \right).$$

Note that the fraction above is the average age at death in the population under the expected death rates, which we can denote with $\bar{x}^E$, so

$$P = a + b\bar{x}^E.$$

Clearly, under the scenario of a linear proportional change in mortality across age, the P-score depends on the age structure of a population. If $b > 0$, i.e. older ages experience a higher proportional increase in death rates over expected, then the higher the average age at death in the population under the expected scenario, the higher the P-score. In other words, of two populations with equal δ_x and $b > 0$, the older population will have a higher P-score. The opposite holds for $b < 0$.

Scenario 3: An arbitrary proportional change in death rates. Assuming an arbitrary shape for δ_x over age, we can re-write P as the weighted average

$$P = \sum_x \frac{\delta_x m_x T_x}{\sum_x m_x T_x} = \sum_x \delta_x \pi_x,$$

with $\pi_x = \frac{m_x T_x}{\sum_x m_x T_x} = D_x^E / D^E$, i.e. the distribution of ages at death under the expected scenario.

So in the general case, the P-score depends on the age structure of the population, in addition to the age-pattern of changes in death rates relative to the expected rates.