

# DEMOGRAPHIC RESEARCH

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## ***DEMOGRAPHIC RESEARCH***

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*Research article*

## **(R)Markdown template for Demographic Research Journal**

**First Last**

**First Last**

This article is part of the Special Collection on “Data Visualization,” organized by Guest Editors Tim Riffe, Sebastian Klüsener, and Nikola Sander.

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## **(R)Markdown template for Demographic Research Journal**

**First Last**<sup>1</sup>

**First Last**<sup>2</sup>

### **Abstract**

#### **BACKGROUND**

What is the motivation for this submission? Why read it?

#### **OBJECTIVE**

What specific question(s) does this submission address?

#### **METHODS**

How does the submission reach its objective? What data? What methods?

#### **RESULTS**

What are the main findings?

#### **CONCLUSIONS**

What do the findings mean?

#### **CONTRIBUTION**

What new contribution does this submission make to the scientific literature?

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## 1. Introduction

Lorem ipsum dolor sit amet, id mel oratio lucilius eloquentiam, nam cu mazim erant aliquip. Nam et minim abhorreant, ferri minimum facilisis ad sit. In cum summo civibus appareat. Mei cu legimus accusata dissentiet. Qui illud gloriatur te. Probatus accommodare ut est, sed et atqui equidem dignissim.

Soluta legimus qui id, nam semper malorum ut. Est magna clita civibus id. Pro ne summo animal. Ne mucius partiendo sit, ne eos natum quodsi periculis.

Te usu movet nominavi, eu eum quod consul. Justo eligendi concludaturque no eam, aliquam fuisset convenire vis ne, purto vide instructor ex duo. An solet appetere sit, ea vis dolore aliquid scaevola, has omnes dolores eu. Eu per enim dolor civibus, no usu consul propriae. Vix id sale civibus definitiones, quis soluta an eum.

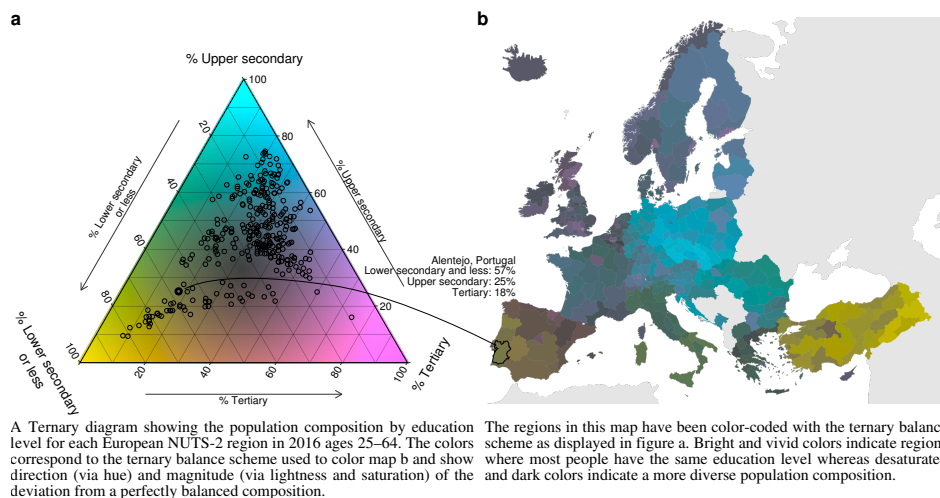
## 2. Figures and Tables

Variably referred to as de Finetti-, simplex-, or triangle plot, the ternary diagram is based upon a coordinate system that maps each point within an equilateral triangle to a unique three-part composition and as such has found use wherever the problem domain spans three parts of a whole. The diagram emerged during the 18th century as a means of illustrating relative mixtures of primary colors (Howarth 1996). It was subsequently adopted as the standard method to depict phase transitions in three-component alloys (Bancroft 1897), the genotype composition of a population (De Finetti 1926), soil composition (Davis and Hammond 1927), or the potential for flammability given different mixtures of three gases (Zabetakis 1965). In the social sciences, ternary diagrams depict population compositions along demographic characteristics, with an early example appearing in the USSR's first census report showing the distribution of workers across labor market segments in various regions (Kvitkin 1932).<sup>3</sup>

---

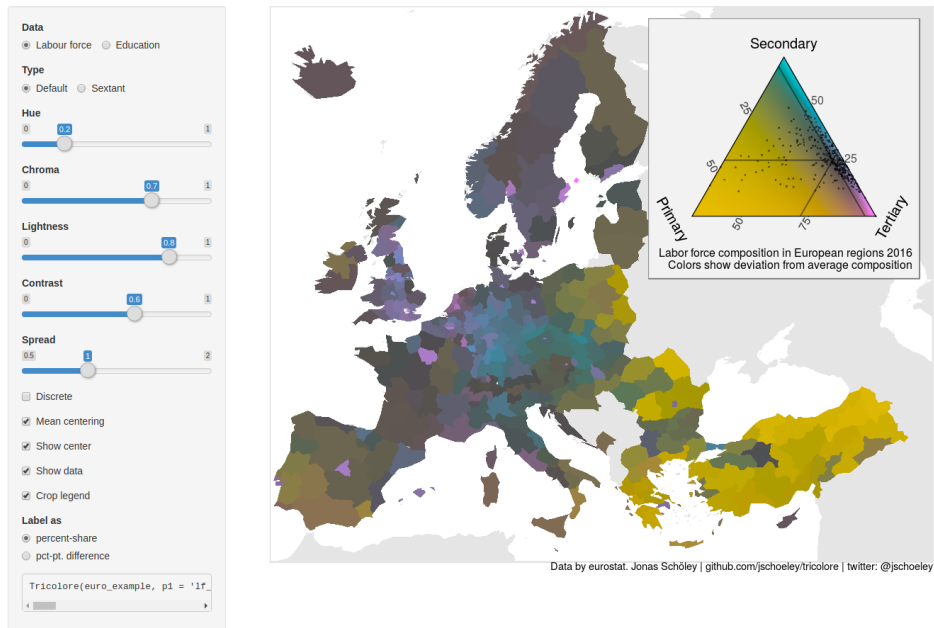
<sup>3</sup>A footnote.

**Figure 1: Demonstration of the ternary balance scheme showing the composition of educational attainment by region in Europe 2016. Data by Eurostat.**



**Figure 2:** The “tricolore” package for the statistical programming language R implements the centered ternary balance color scheme and provides a user interface for quickly testing different parametrizations.

Tricolore: A flexible color scale for ternary compositions



**Table 1:**      **A table with a caption that stretches across two lines. Nearly there...  
Done. Does it align nicely?**

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa

### 3. Equations

Given the observed death counts  $D_{jk}$  in age group  $j$  and stratum  $k$  and associated person-days of exposure to risk  $O_{jk}$  I fit the model

$$\begin{aligned} D_{jk} &\sim \text{Pois}(\lambda_{jk} O_{jk}) \\ \lambda_{jk} &= e^{\beta_{0k} + \beta_{1k} \log(x_{jk} + 1) + \beta_{2k} \log^2(x_{jk} + 1)}, \end{aligned} \quad (1)$$

where  $\lambda_{jk}$  are mortality rates by age group and stratum. For each stratum, a smooth hazard is recovered by evaluating  $\lambda_{jk}$  over a continuous range of ages  $x$ .

The stratum specific coefficients  $\beta_{0k}$ ,  $\beta_{1k}$ ,  $\beta_{2k}$  are sums of baseline coefficients  $\beta$ , prematurity effects  $\beta^{\text{Pm}}$ , prematurity-birth weight interactions  $\beta^{\text{Pm} \times \text{Bw}}$ , and prematurity-birth weight-Apgar interactions  $\beta^{\text{Pm} \times \text{Bw} \times \text{Ap}}$  resulting in the multilevel structure

$$\begin{pmatrix} \beta_{0k} \\ \beta_{1k} \\ \beta_{2k} \end{pmatrix} = \underbrace{\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}}_{\text{lvl 0 baseline coef.}} + \underbrace{\begin{pmatrix} \beta_{0,p[k]}^{\text{Pm}} \\ \beta_{1,p[k]}^{\text{Pm}} \\ \beta_{2,p[k]}^{\text{Pm}} \end{pmatrix}}_{\text{lvl 1 deviations by prematurity}} + \underbrace{\begin{pmatrix} \beta_{0,p[k],b[k]}^{\text{Pm} \times \text{Bw}} \\ \beta_{1,p[k],b[k]}^{\text{Pm} \times \text{Bw}} \\ \beta_{2,p[k],b[k]}^{\text{Pm} \times \text{Bw}} \end{pmatrix}}_{\text{lvl 2 deviations by birth weight given prematurity}} + \underbrace{\begin{pmatrix} \beta_{0,p[k],b[k],a[k]}^{\text{Pm} \times \text{Bw} \times \text{Ap}} \\ \beta_{1,p[k],b[k],a[k]}^{\text{Pm} \times \text{Bw} \times \text{Ap}} \\ \beta_{2,p[k],b[k],a[k]}^{\text{Pm} \times \text{Bw} \times \text{Ap}} \end{pmatrix}}_{\text{lvl 3 deviations by Apgar given prematurity and birth weight}},$$

where  $p[k]$ ,  $b[k]$  and  $a[k]$  denote the prematurity, birth weight and Apgar level associated with stratum  $k$ . Except for the baseline  $\beta$ 's each set of coefficients is assumed to be drawn from a multivariate-Normal distribution with zero mean and covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_{\beta_0}^2 & \sigma_{\beta_0\beta_1} & \sigma_{\beta_0\beta_2} \\ \sigma_{\beta_0\beta_1} & \sigma_{\beta_1}^2 & \sigma_{\beta_1\beta_2} \\ \sigma_{\beta_0\beta_2} & \sigma_{\beta_1\beta_2} & \sigma_{\beta_2}^2 \end{pmatrix},$$

with separate estimates for levels one to three.



## **4. Lists**

- Ringo
  - Drums
- George
  - Guitar
- Paul
  - Bass
  - Piano
- John
  - Guitar
  - Piano

## **5. Section**

### **5.1 Subsection**

#### **5.1.1 Subsubsection**

## **6. Acknowledgments**

I'd like to thank my husband whose caring love allowed me to fully concentrate on my work.

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## Appendix A

```

1  #' Get Stratified Life-table From Survival Data
2  #'
3  #' Aggregate individual level survival data into a stratified
4  #' life-tables with prespecified age-groups using the assumptions of
5  #' piecewise-constant hazards.
6  #'
7  #' @param df a data frame
8  #' @param x time at censoring or event
9  #' @param event event indicator, 0 if censoring, 1 if event
10 #' @param cuts vector of cuts for life-table age groups
11 #' @param ... strata
12 #'
13 #' @author Jonas Schöley
14 GetLifeTable <- function (df, x, event, cuts, ...) {
15
16   x_i = enquos(x); event_i = enquos(event); strata = enexprs(...)
17
18   lt <-
19     df %>%
20     # aggregate individual level event and censoring times
21     # into predefined age groups
22     mutate(
23       j =
24         .bincode(
25           x = !!x_i, breaks = cuts,
26           # [a, b)
27           right = FALSE, include.lowest = FALSE
28         ),
29       x_jk =
30         cuts[j]
31     ) %>%
32     group_by(!!!strata, j, x_jk) %>%
33     summarise(
34       # total observed deaths in interval
35       D_jk = sum(!event_i),
36       # total observed censorings in interval
37       C_jk = sum(!(!event_i)),
38       # average time spent in interval for
39       # those who leave during interval (by death or censoring)
40       a_jk = mean(!x_i - first(x_jk))
41     ) %>%
42     ungroup() %>%
43     # add predefined age intervals to table where no deaths or
44     # censorings occurred
45     complete(
46       nesting(x_jk = head(cuts, -1), j = 1:length(head(cuts, -1))),
47       nesting(!!!strata),
48       fill = list(D_jk = 0, C_jk = 0, a_jk = NA)

```

```

49   ) %>%
50   arrange(!!!strata, x_jk) %>%
51   mutate(
52     k = group_indices(., !!!strata)
53   ) %>%
54   group_by(k) %>%
55   # calculate life-table
56   mutate(
57     # width of the interval
58     n_jk =
59       c(diff(x_jk), last(cuts)-last(x_jk)),
60     # observed population alive at start of interval
61     N_jk =
62       head(cumsum(c(sum(C_jk+D_jk), -(C_jk+D_jk))), -1),
63     # total person-time of exposure over interval
64     O_jk =
65       (N_jk-D_jk-C_jk)*n_jk +
66       ifelse(is.na(a_jk), 0, a_jk*(D_jk+C_jk)),
67     # mortality rate over interval
68     m_jk =
69       ifelse(O_jk == 0, 0, D_jk/O_jk),
70     # probability of surviving interval
71     # assuming constant force of mortality
72     p_jk =
73       exp(-m_jk*n_jk),
74     # probability of death during the interval given survival
75     # to interval
76     q_jk =
77       1-p_jk,
78     # probability of survival until interval start
79     l_jk =
80       cumprod(c(1, head(p_jk, -1))),
81     # probability of death in interval
82     d_jk =
83       c(-diff(l_jk), last(l_jk)*last(q_jk))
84   ) %>%
85   # distribution of stratum-specific exposures by age
86   group_by(j) %>%
87   mutate(
88     pi_jk = O_jk/sum(O_jk)
89   ) %>%
90   ungroup() %>%
91   select(
92     k, j, !!!strata, x_jk, n_jk, N_jk, a_jk, O_jk, D_jk, C_jk,
93     m_jk, q_jk, l_jk, d_jk, pi_jk
94   )
95
96   return(lt)
97
98 }

```

## Appendix B

```
1 plot(cars)
```

**Figure B-1:**      **The cars data set**

