Empirical prediction intervals
applied to short-term mortality forecasts and excess deaths

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joint work with

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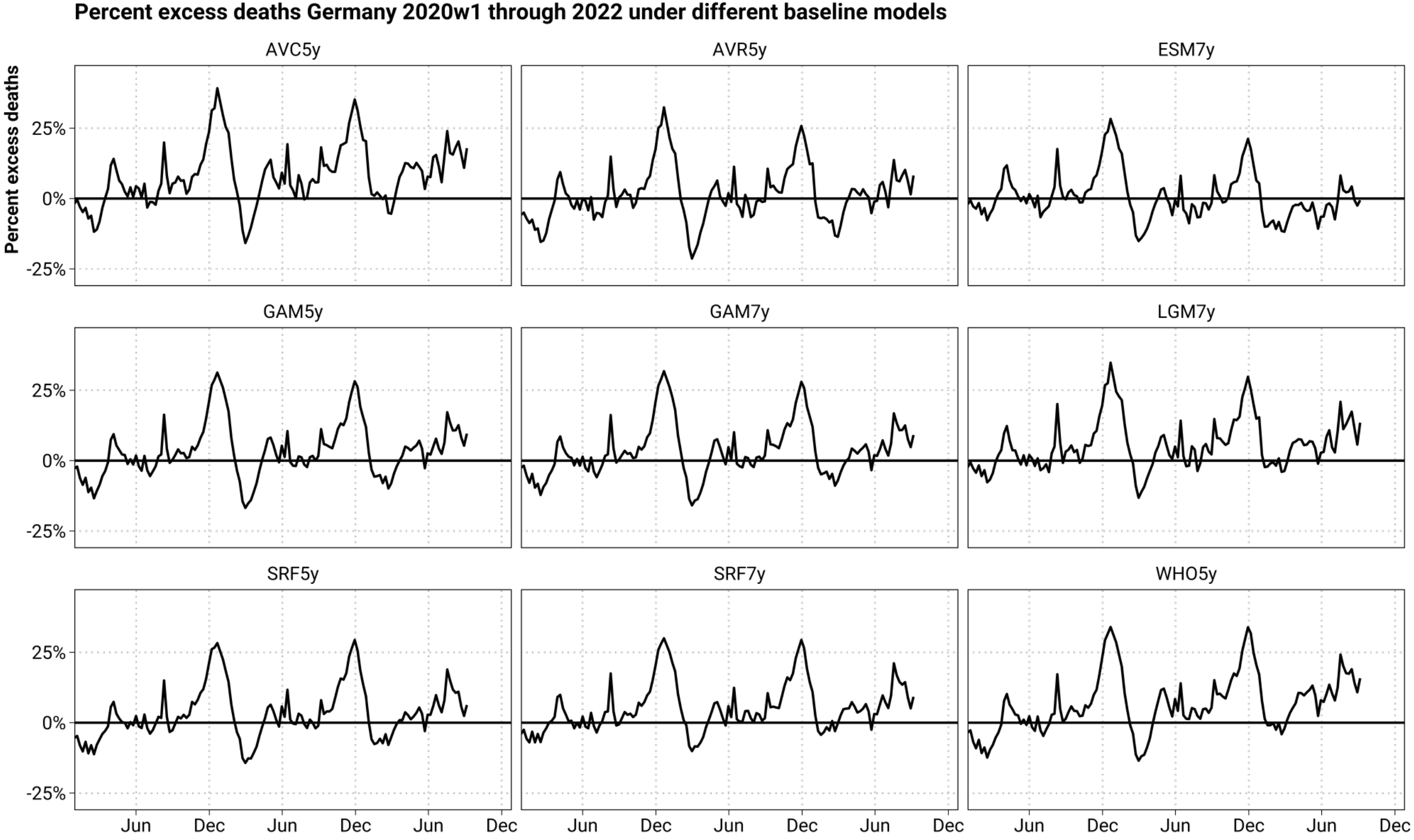
FOR DEMOGRAPHIC RESEARCH

Excess deaths as a forecasting challenge



How many deaths in a week had C19 not occurred?

Excess deaths as a forecasting challenge

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Excess = Observed - Expected

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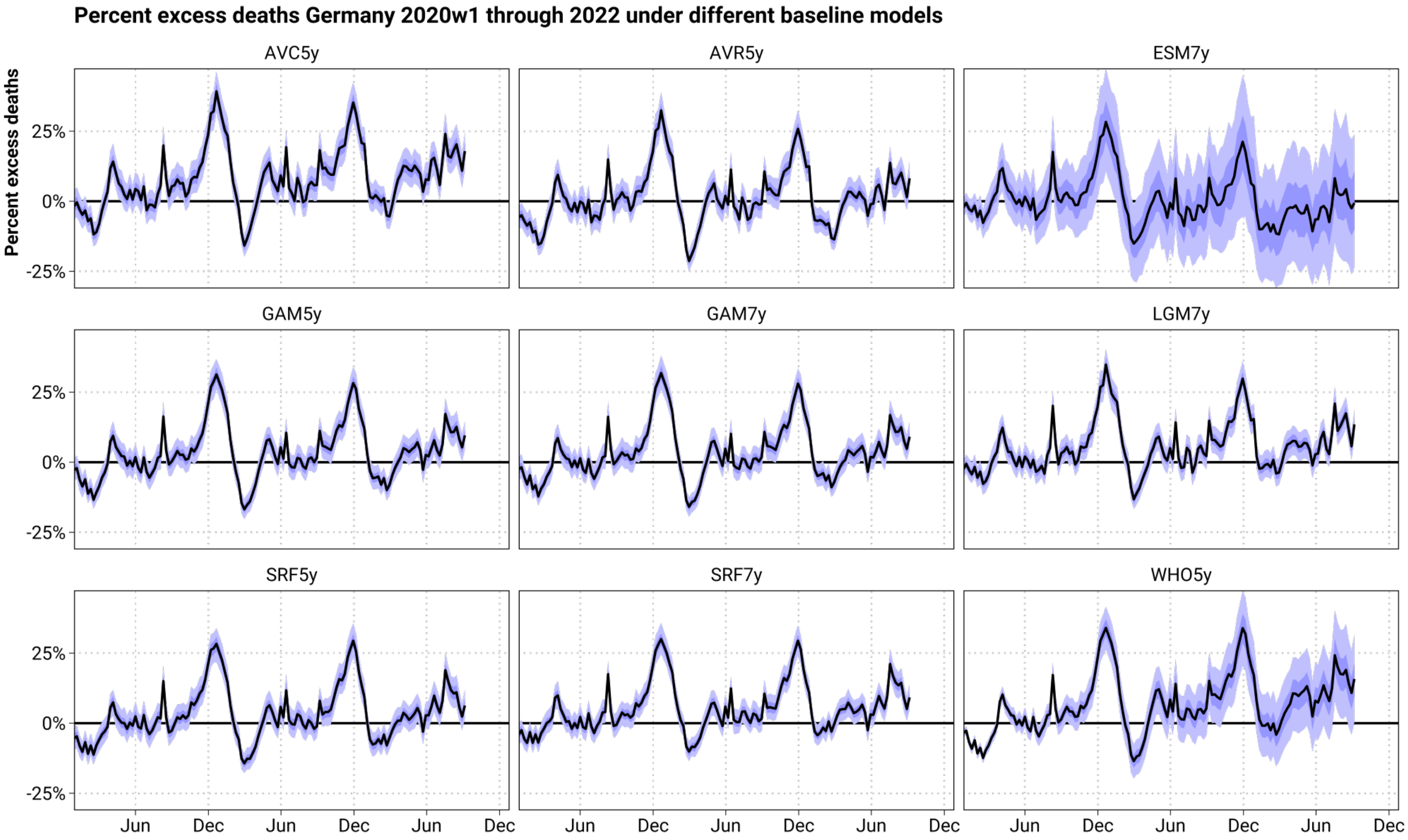

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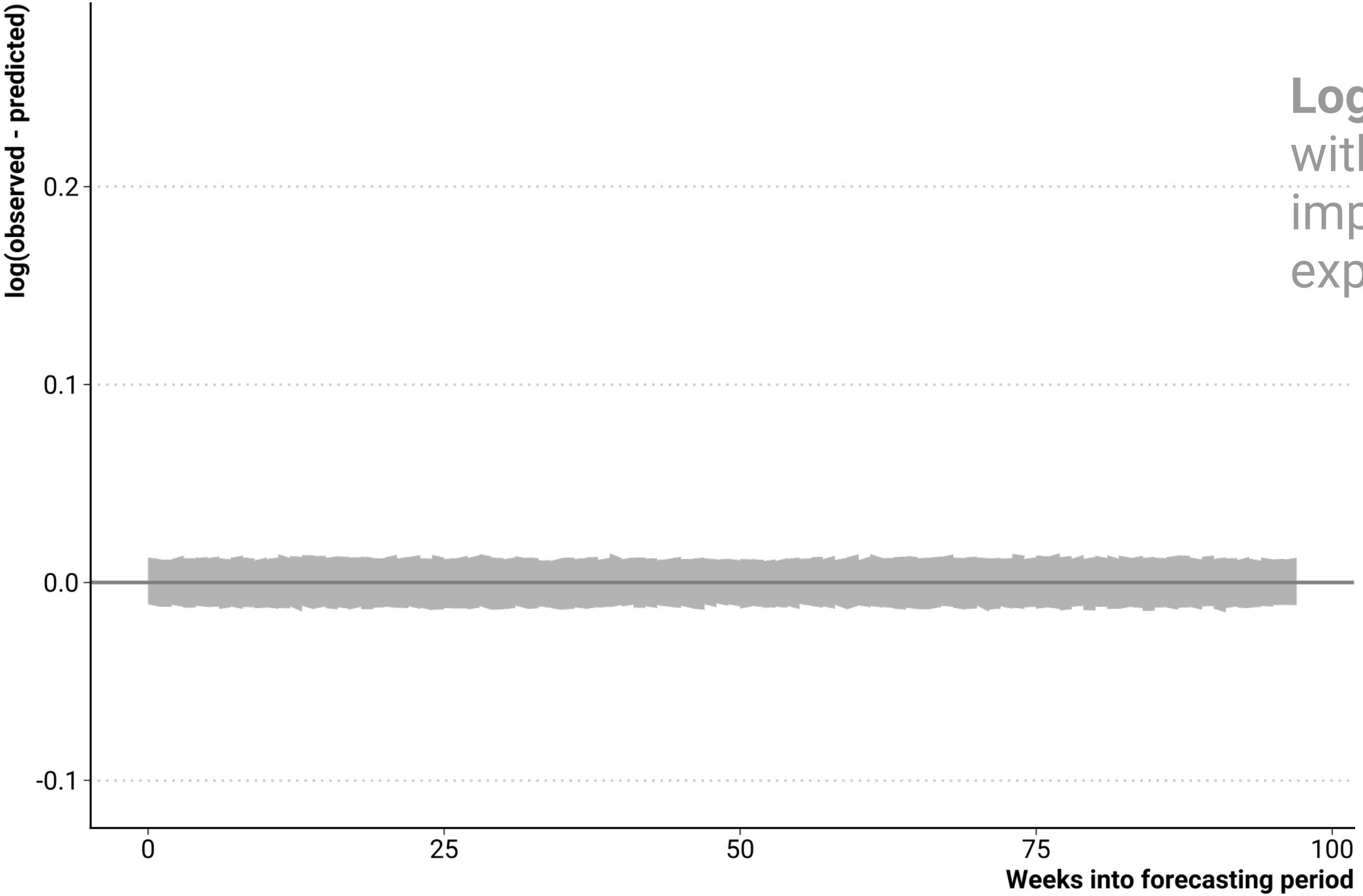
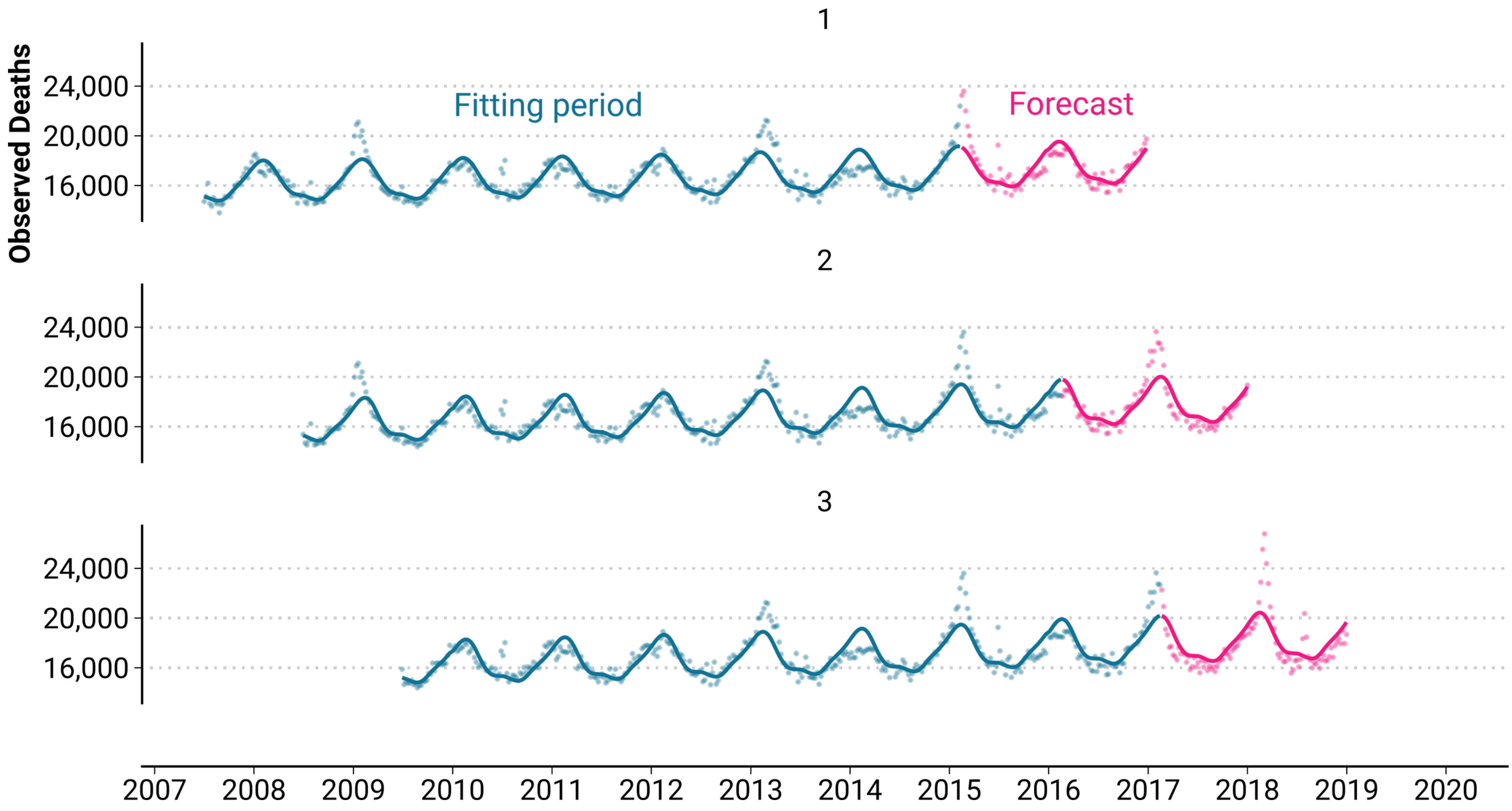


Log prediction error region with **90% nominal coverage** implied by Poisson-GAM expected deaths model

The logic of out-of-sample validation



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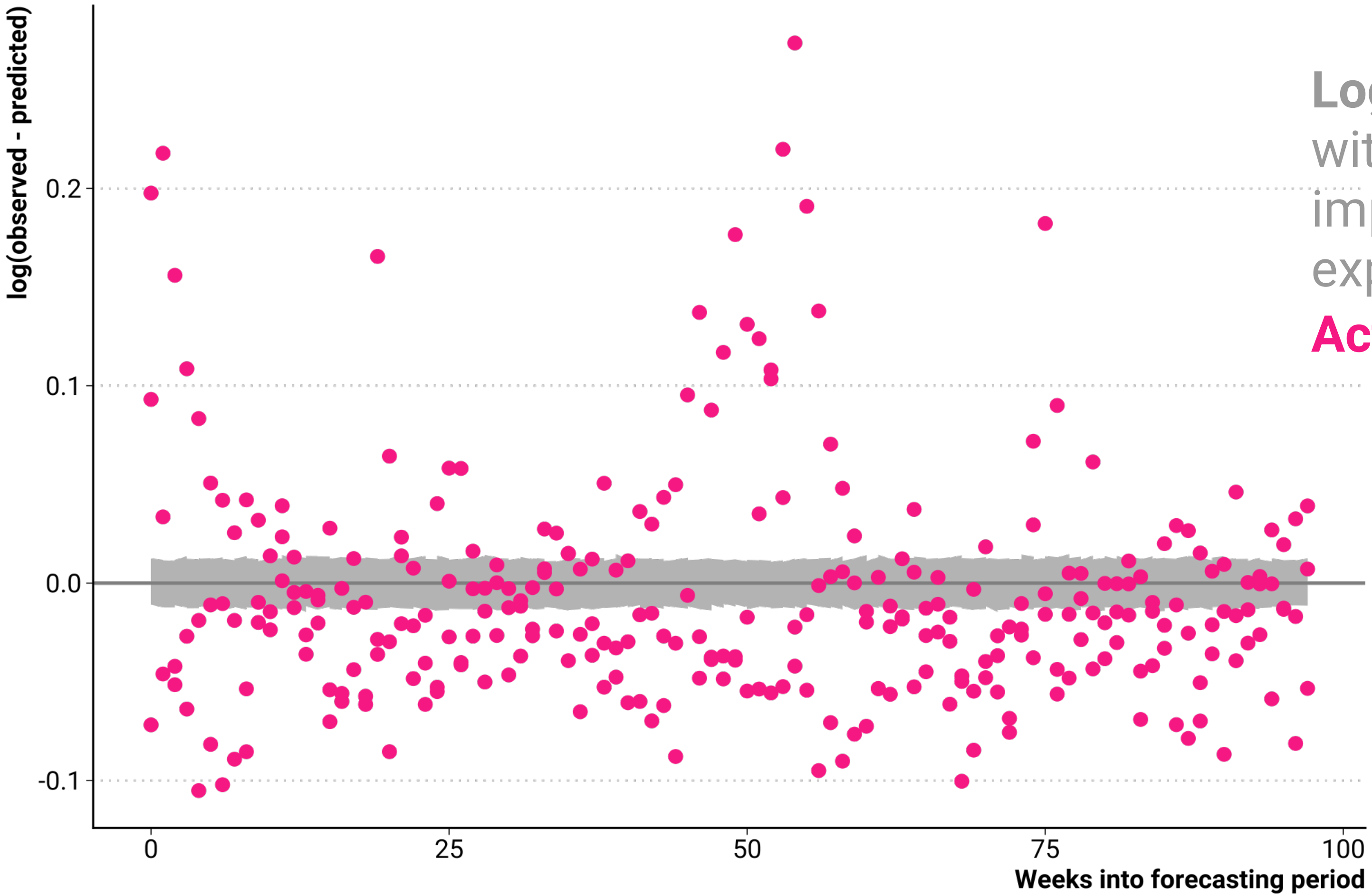
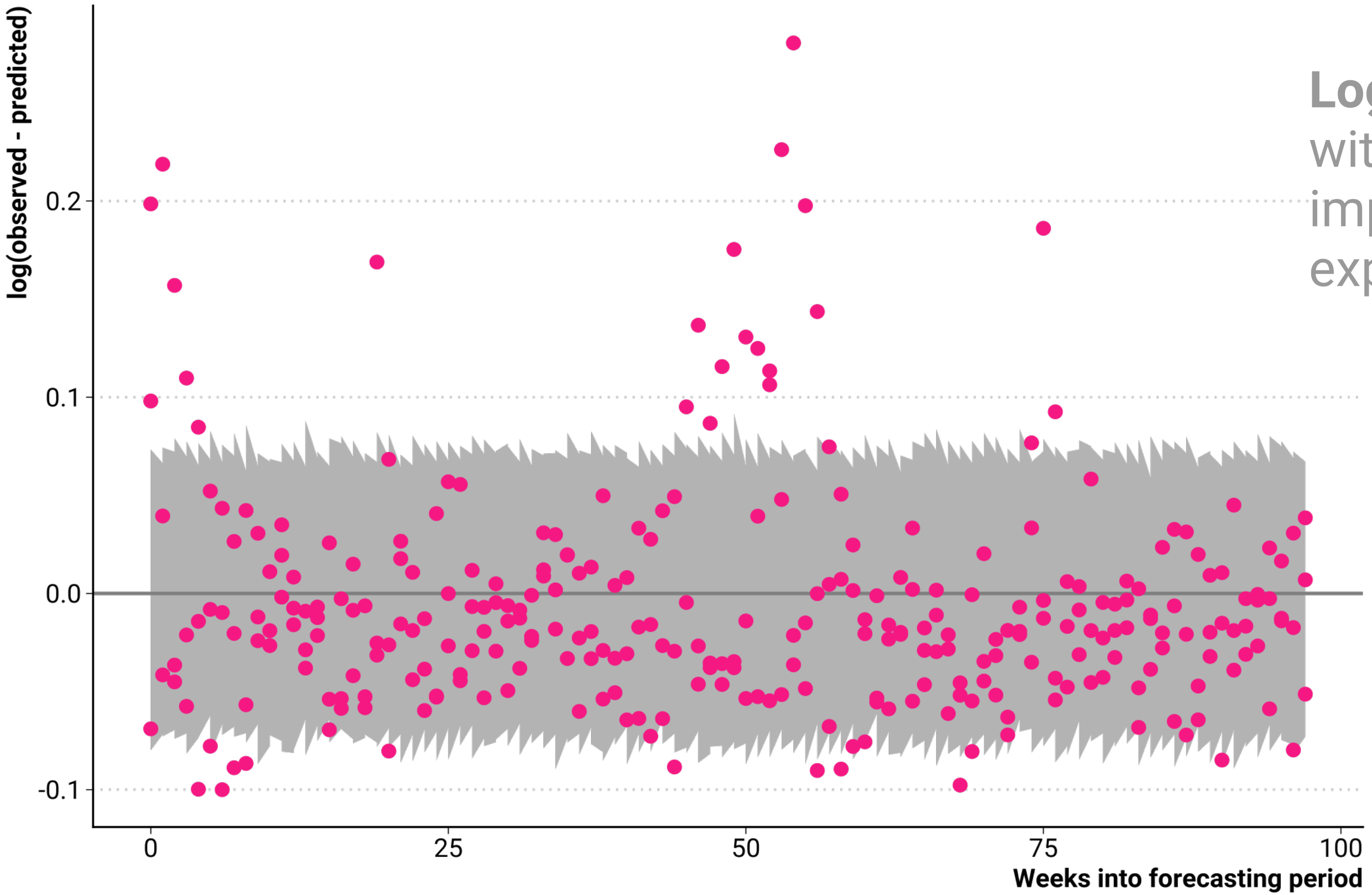


Log prediction error region with **90% nominal coverage** implied by Poisson-GAM expected deaths model

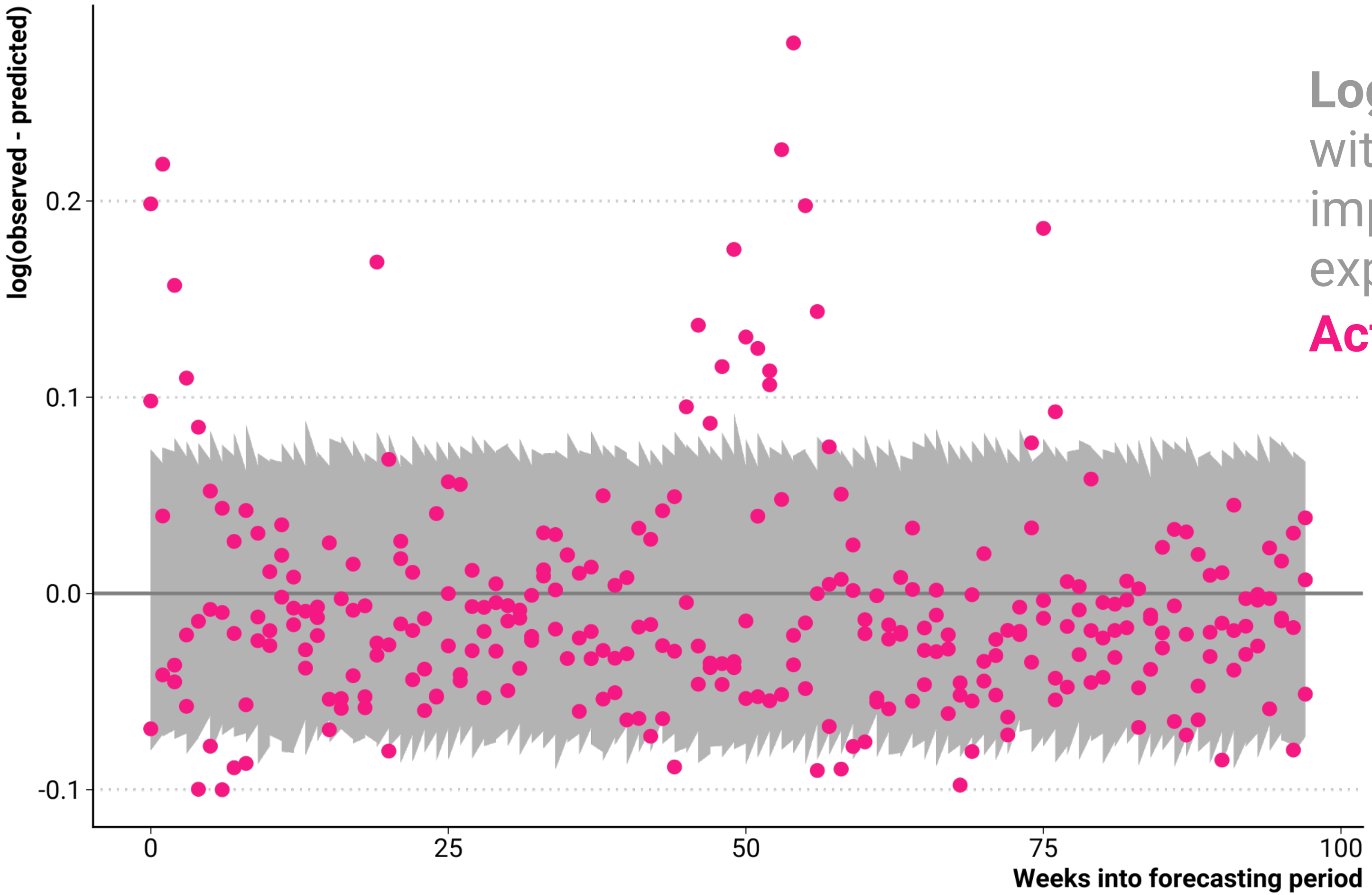
Actual coverage ~22%

Excess deaths as a forecasting challenge



Log prediction error region with **90% nominal coverage** implied by NegBin-GAM expected deaths model

Excess deaths as a forecasting challenge



Log prediction error region with **90% nominal coverage** implied by NegBin-GAM expected deaths model

Actual coverage ~87%

The logic of out-of-sample validation

Cross-validation

The prediction error of your model is
estimated from out of sample validation on known data.

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Empirical prediction intervals

The distribution of prediction errors of your model is estimated from out of sample validation on known data.

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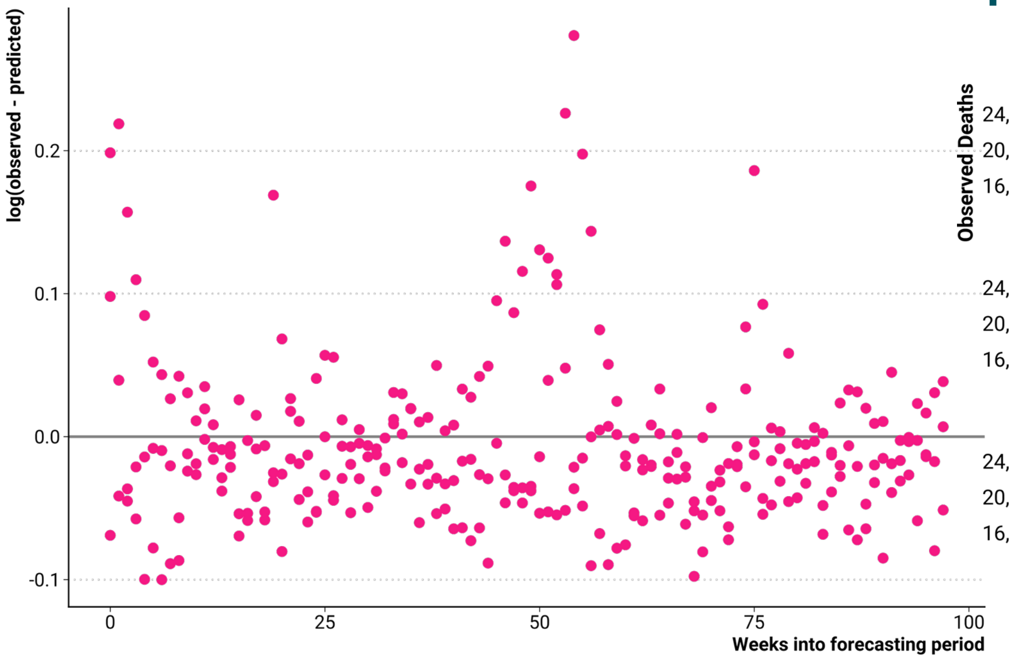
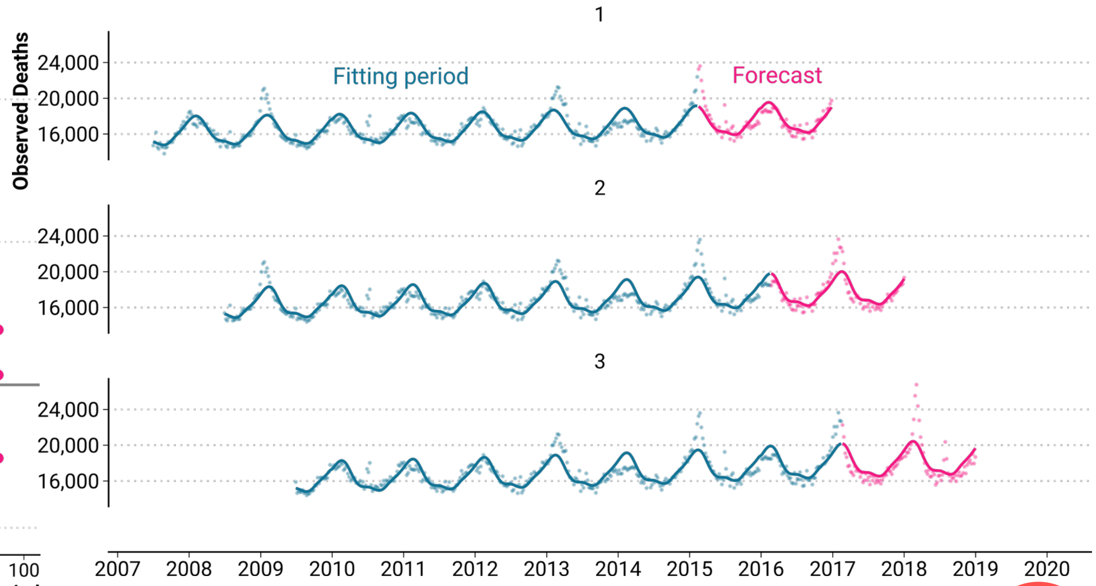




The logic of out-of-sample validation

Cross-validation
The prediction error of your model is
estimated from out of sample validation on known data.

Empirical prediction intervals
The distribution of prediction errors of your model is
estimated from out of sample validation on known data.

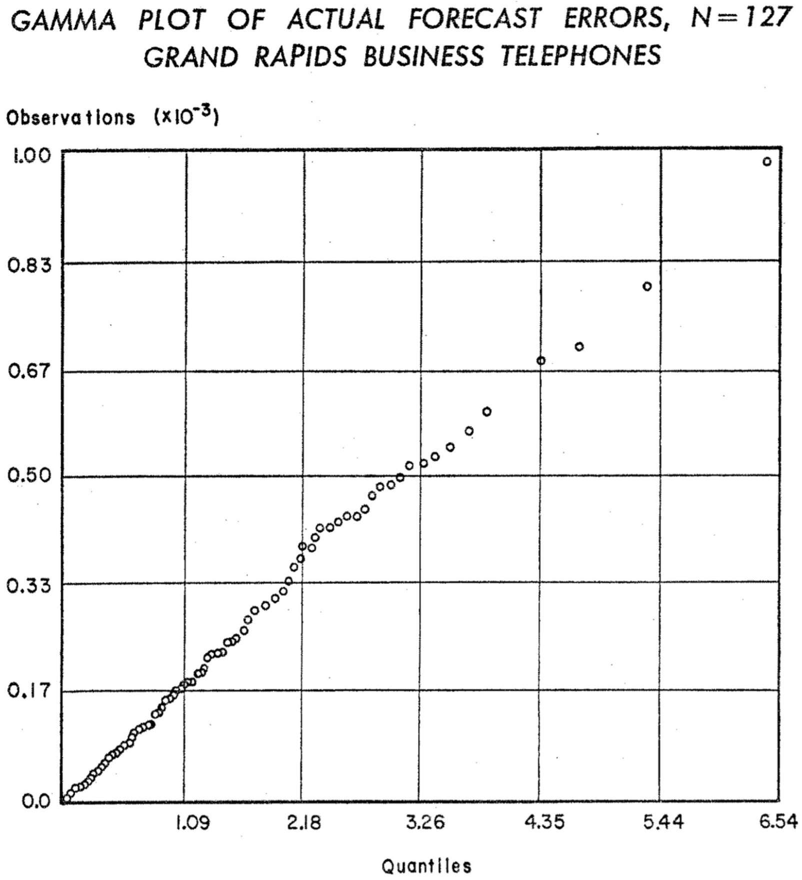
You're model is only as precise as has been demonstrated on a comparable forecasting challenge.

Some history

1971 Williams & Goodman
A Simple Method for the Construction of Empirical Confidence Limits for Economic Forecasts
Journal of the American Statistical Association
doi.org/10.2307/2284223

$$y_t = \gamma_1 t + \gamma_2 t^2 + \sum_{i=1}^{12} \beta_i m_{it} + e_t$$

A simple method for the construction of empirical confidence intervals for time series forecasts is described. The procedure is to go through the series making a forecast from each point in time. The comparison of these forecasts with the known actual observations will yield an empirical distribution of forecasting errors. This distribution can then be used to set confidence intervals for subsequent forecasts. The technique appears to be particularly useful when the mechanism generating the series cannot be fully identified from the available data or when limits based on more standard considerations are difficult to obtain.



Some history

1983 Stoto
The Accuracy of Population Projections
Journal of the American Statistical Association
doi.org/10.1080/01621459.1983.10477916

$$P_T = P_0 \exp \int_0^T r(t)dt \ .$$

The average growth rate over the projection period is

$$\bar{r} = \frac{1}{T} \int_0^T r(t)dt \ ,$$

Population projections are key elements of many planning and policy studies but are inherently inaccurate. This study of past population projection errors provides a means for constructing confidence intervals for future projections. We first define a statistic to measure pro-

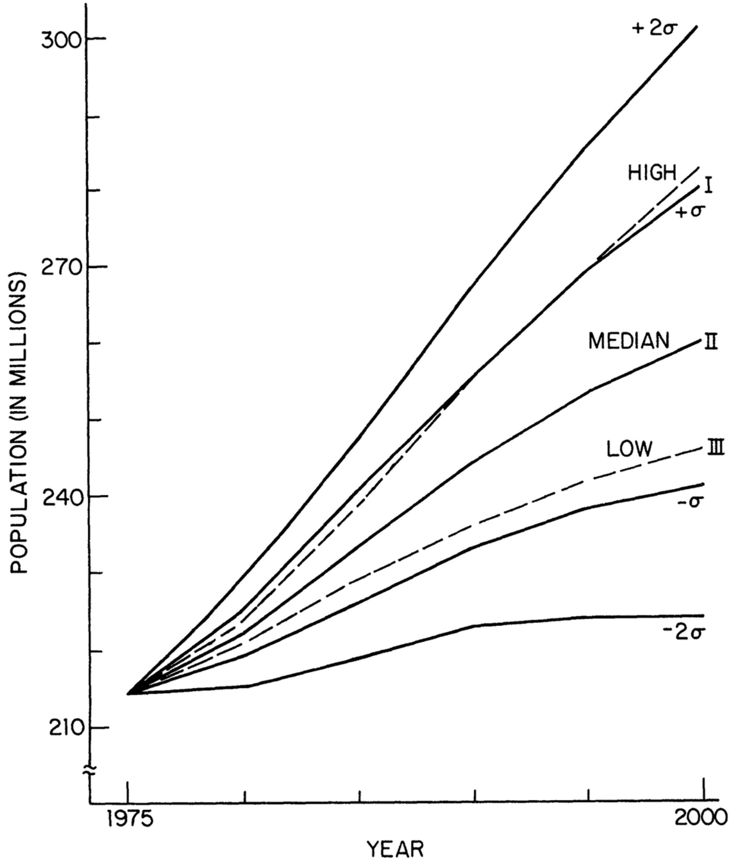


Figure 4. U.S. CENSUS PROJECTIONS (HIGH, MEDIAN, LOW) AND σ = .3 CONFIDENCE INTERVALS

Some history

1986 Cohen
Population forecasts and confidence intervals for Sweden: a comparison of model-based and empirical approaches
Demography
doi.org/10.2307/2061412

1987 Smith
Tests of Forecast Accuracy and Bias for County Population Projections
Journal of the American Statistical Association
doi.org/10.1080/01621459.1987.10478528

1987 Mandrakis & Hibon
Confidence Intervals. An Empirical Investigation of the Series in the M-Competition
International Journal of Forecasting
doi.org/10.1016/0169-2070(87)90045-8

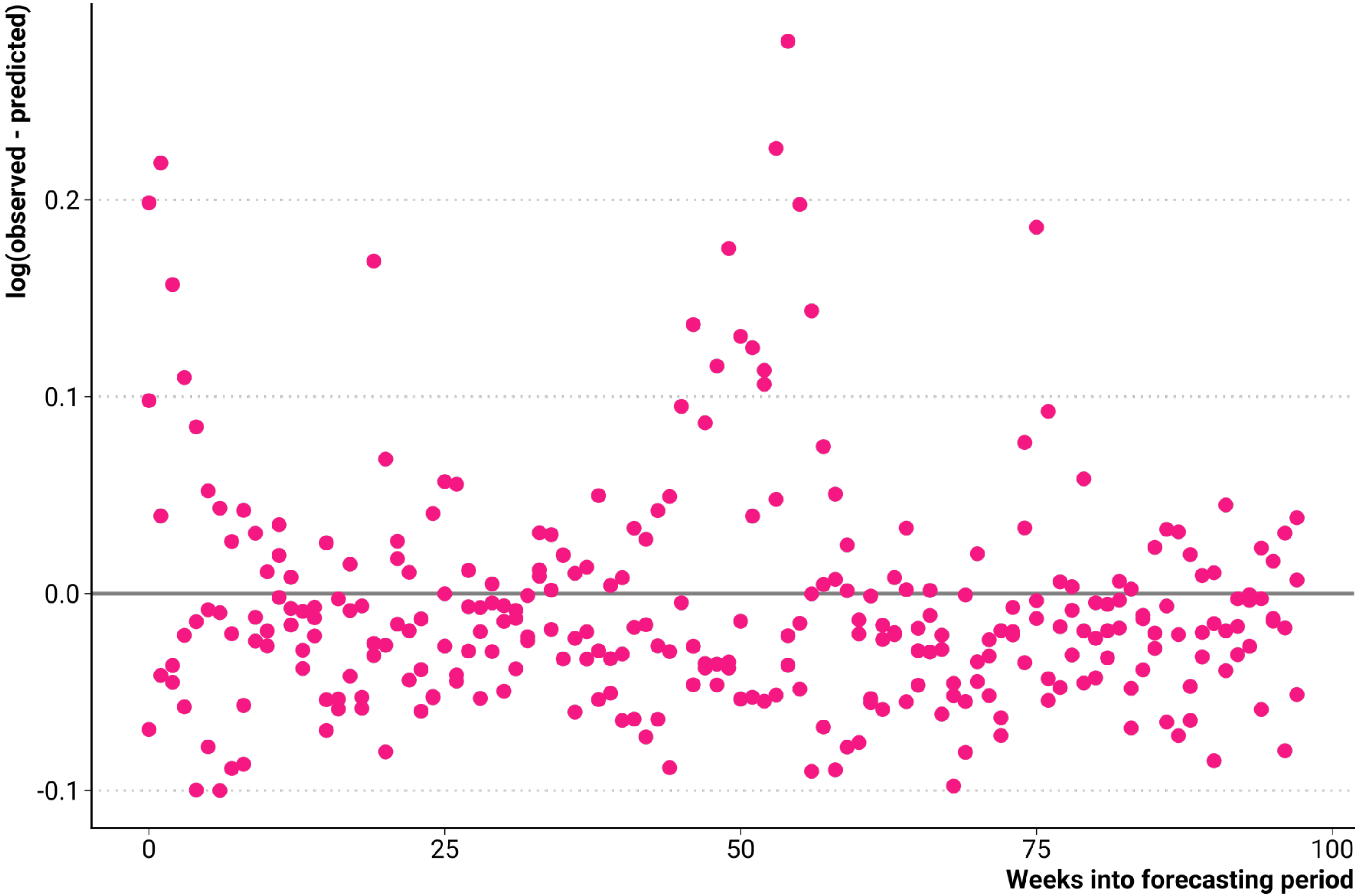
1988 Smith
Stability over time in the distribution of population forecast errors
Demography
doi.org/10.2307/2061544

Some history

2008 Shafer & Vovk
A Tutorial on Conformal Prediction
Journal of Machine Learning Research
doi.org/10.5555/1390681.1390693

"Conformal prediction uses past experience to determine precise levels of confidence in new predictions."

Modeling the distribution of empirical errors



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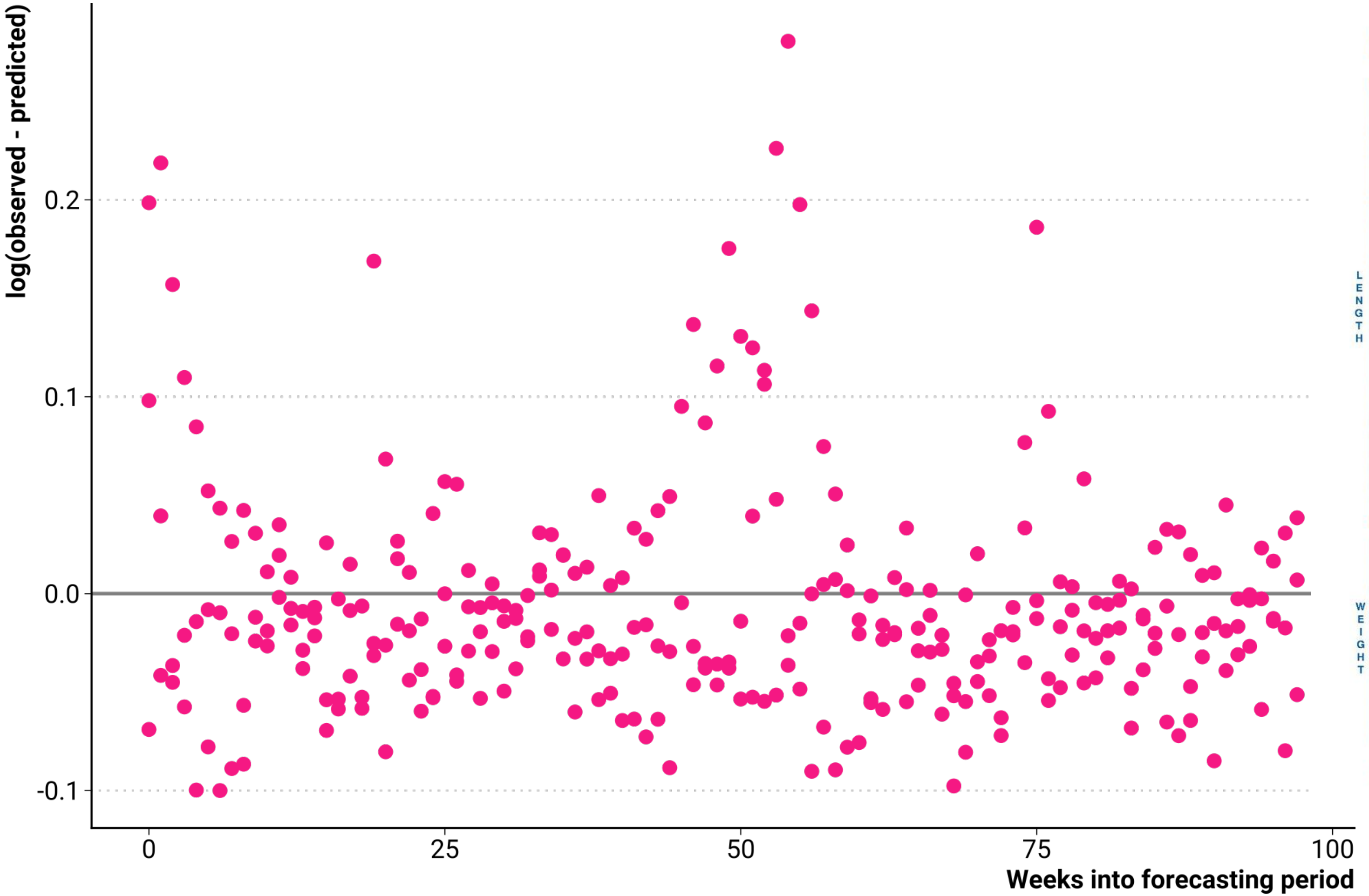

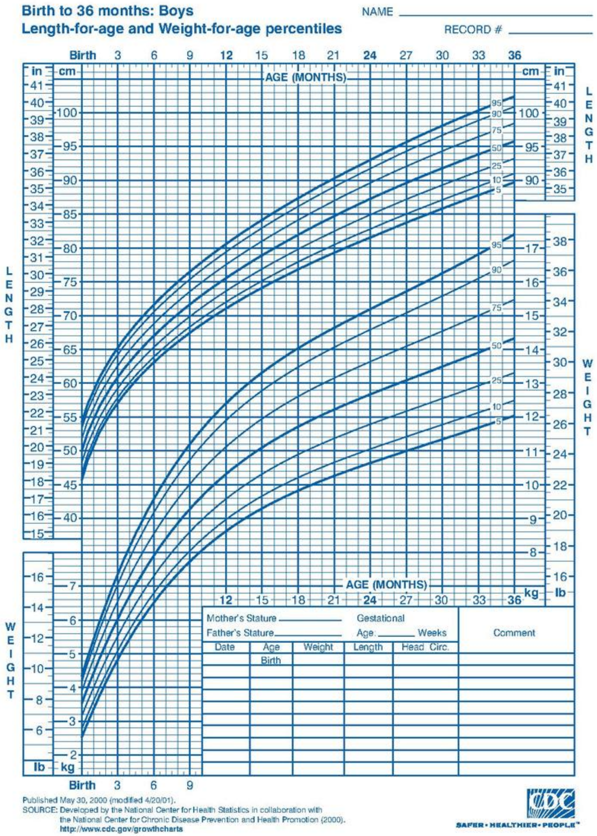

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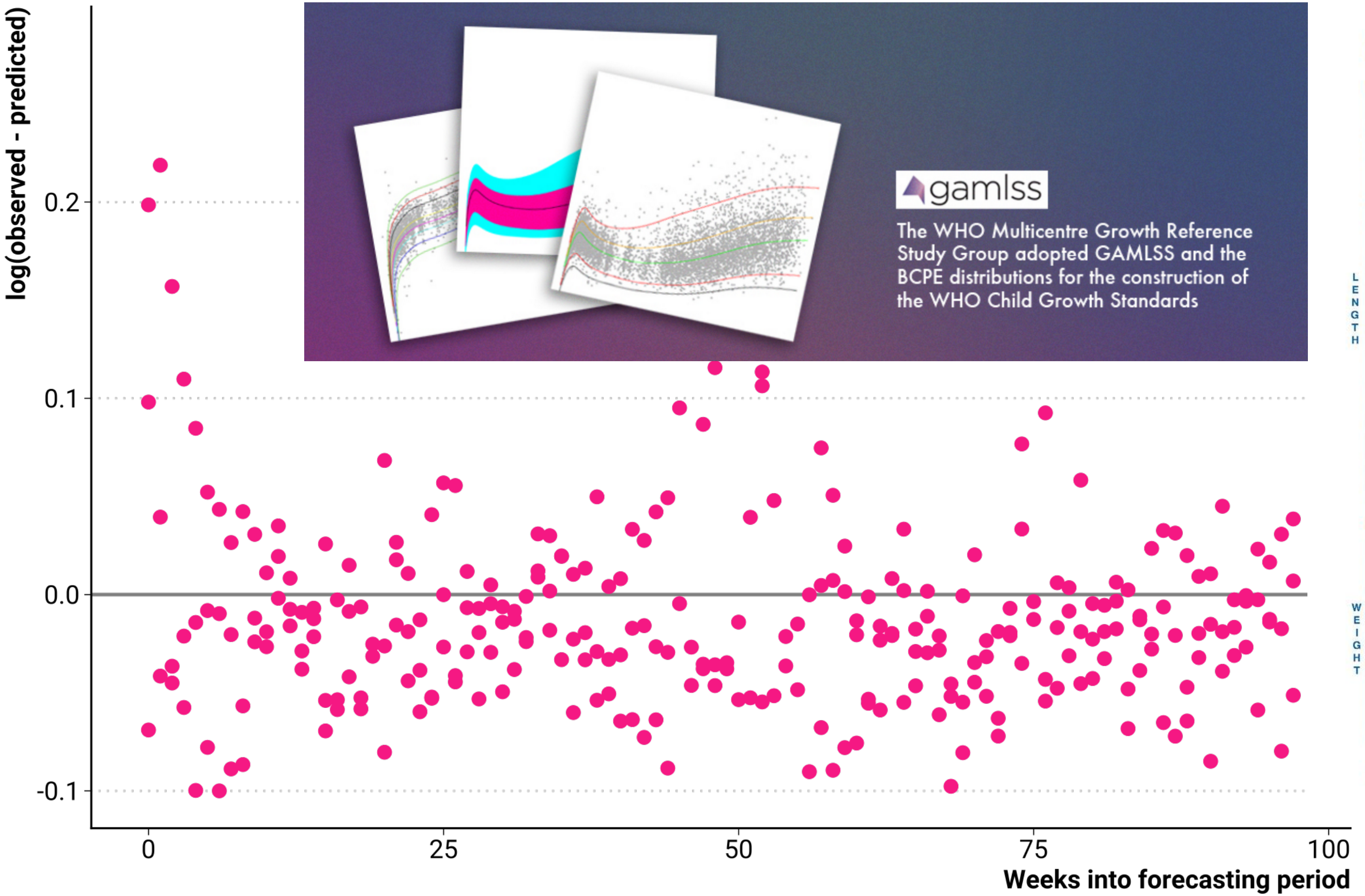
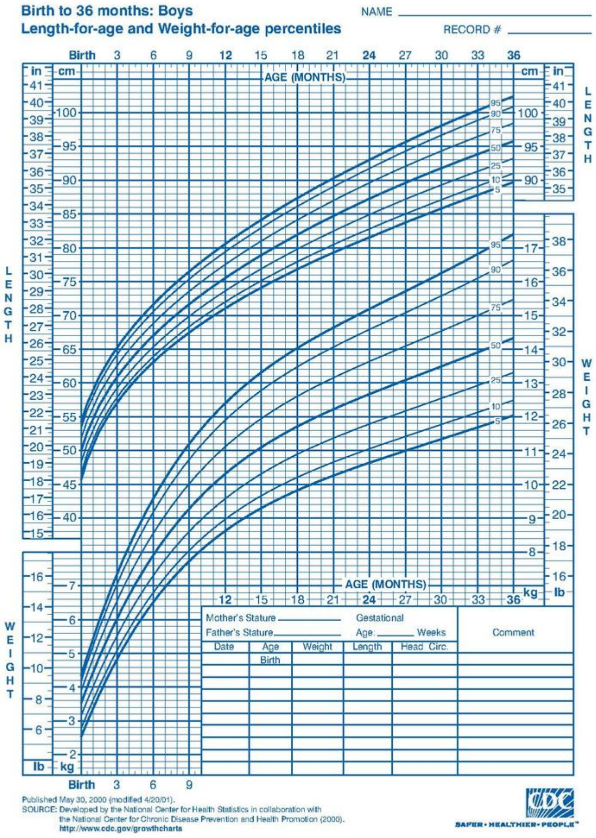
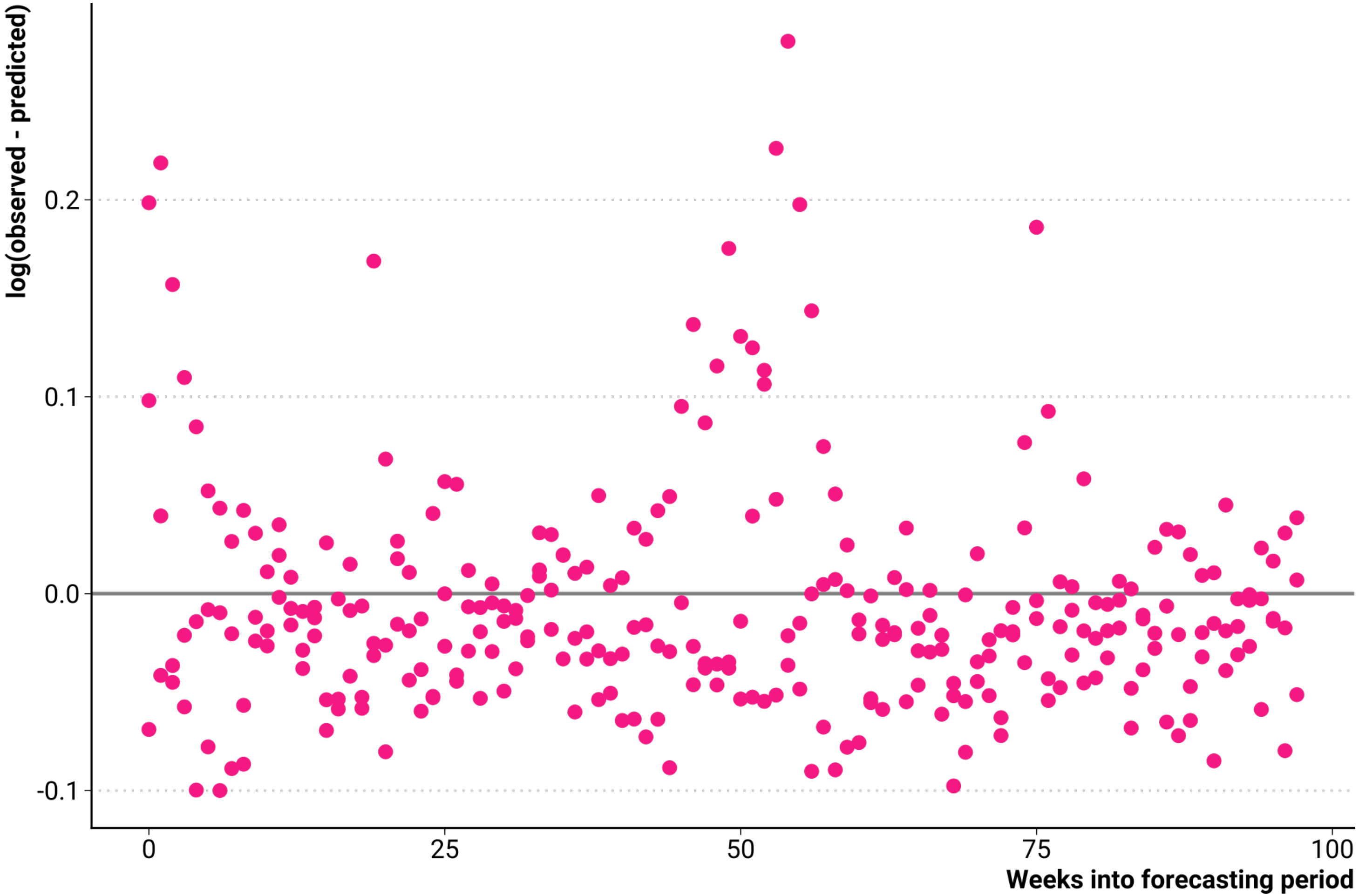




Modeling the distribution of empirical errors



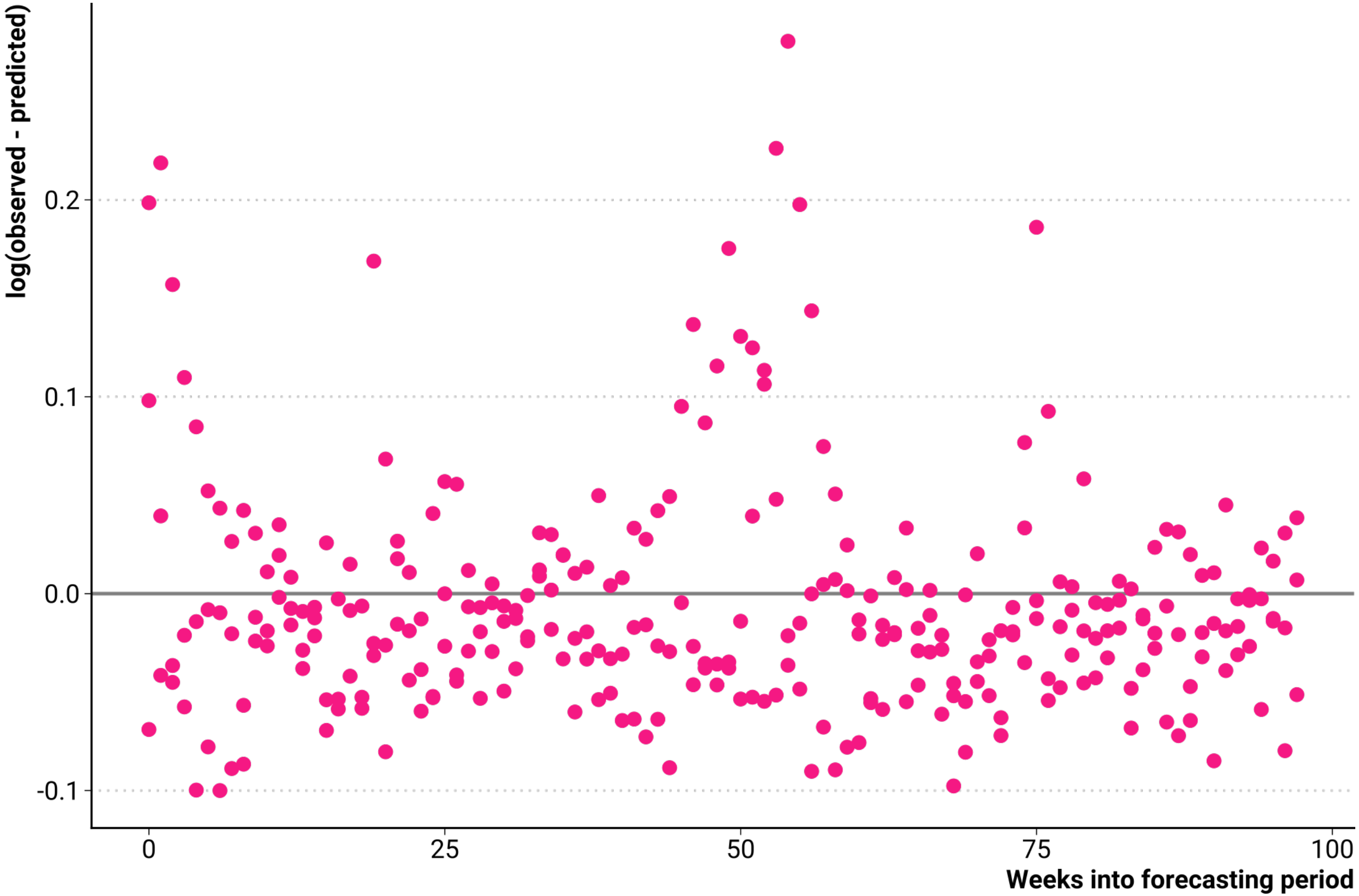
log (observed/predicted) ~ Skewed-Normal(μ_t, σ_t, v_t)

$\mu_t = b_{\mu 0}$

$\sigma_t = \exp(b_{\sigma 0} + b_{\sigma t} t + s_{\sigma}(t))$

$v_t = b_{v0} + s_v(t)$

Modeling the distribution of empirical errors



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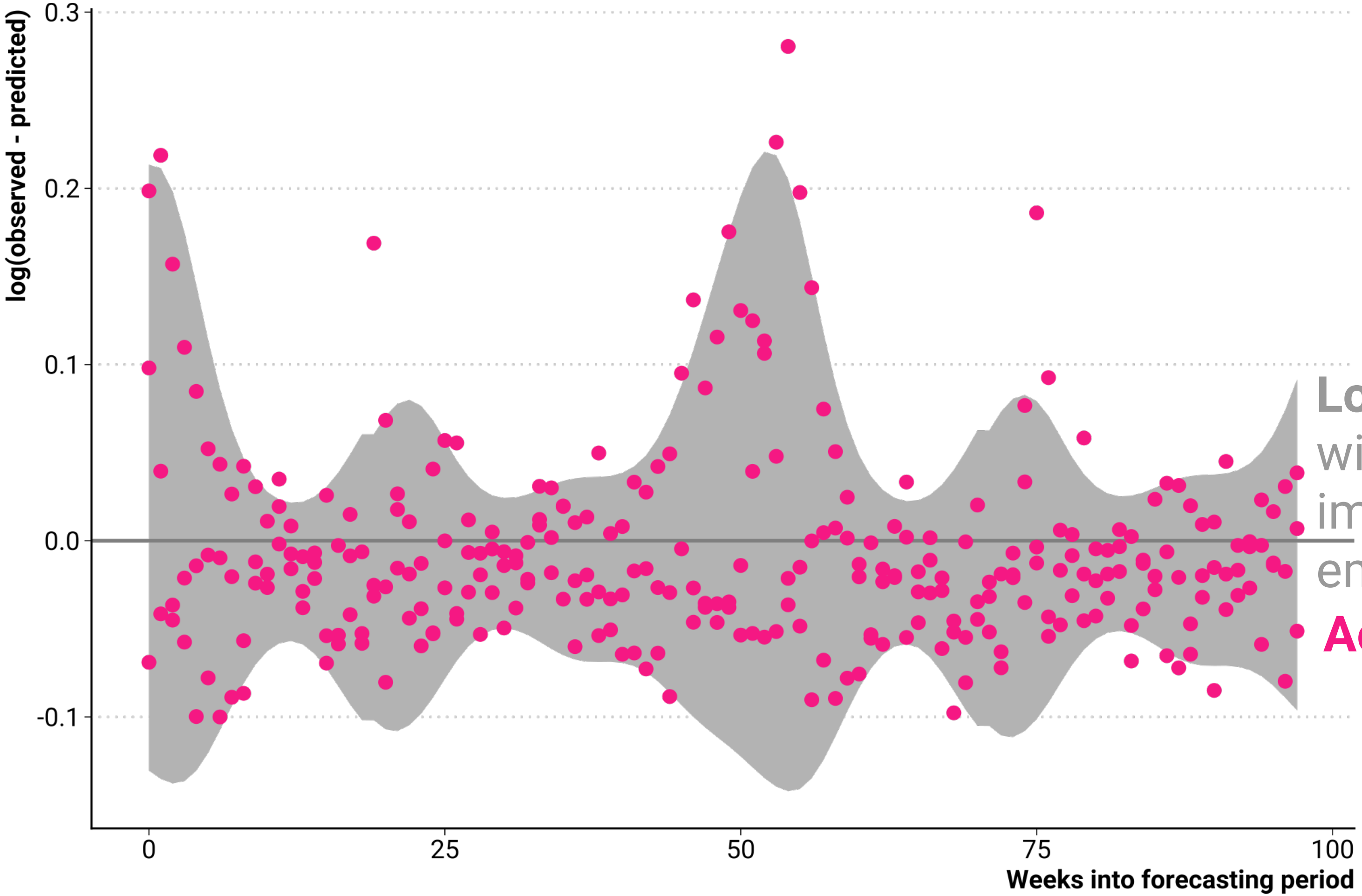


log (observed/predicted) ~ Skewed-Normal(μ_t, σ_t, ν_t)

$\mu_t = b_{\mu 0}$

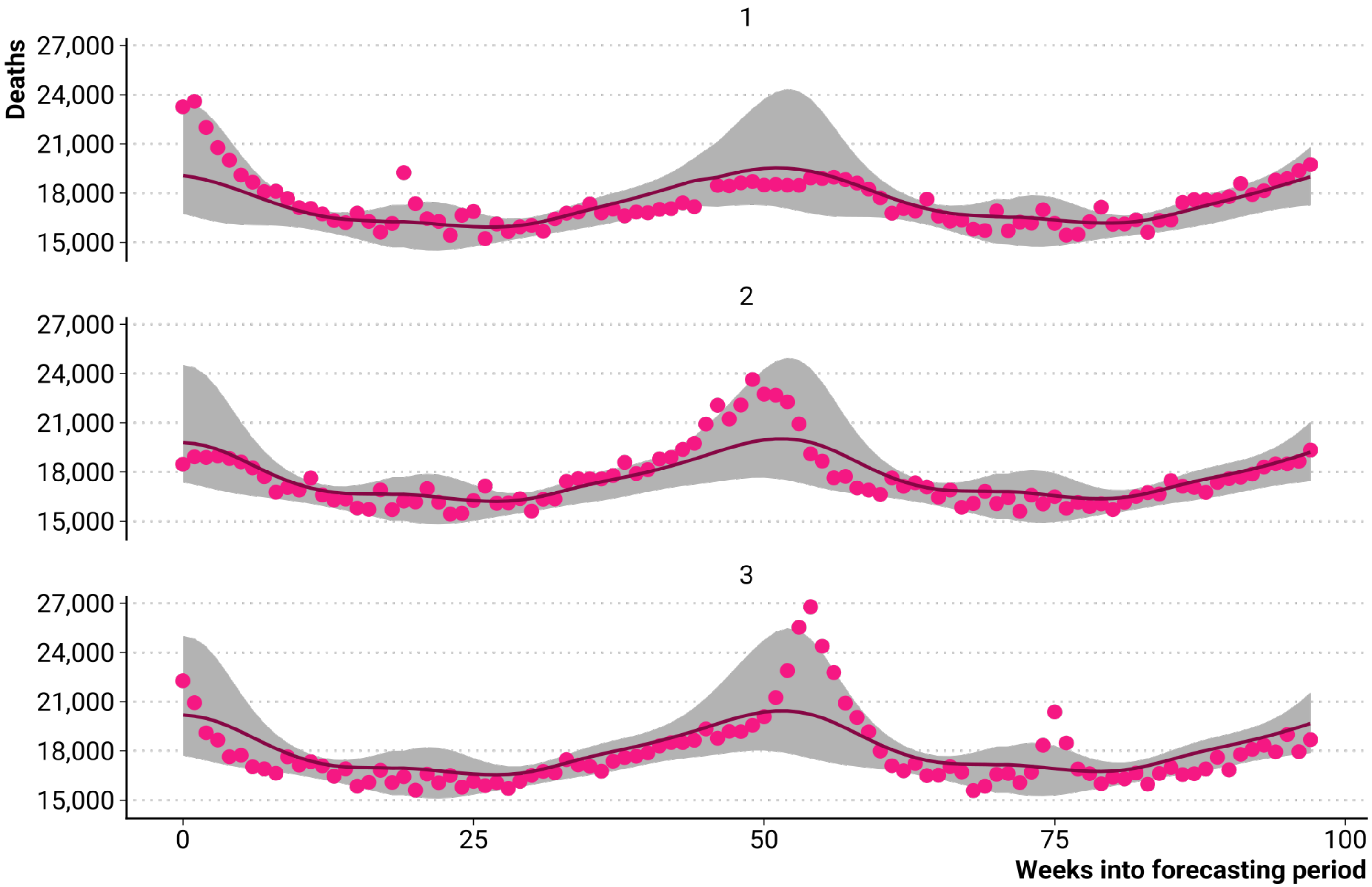
$\sigma_t = \exp(b_{\sigma 0} + b_{\sigma t} t + s_\sigma(t))$

$\nu_t = b_{\nu 0} + s_\nu(t)$

Log prediction error region with **90% nominal coverage** implied by Skewed-Normal empirical prediction intervals

Actual coverage ~92%

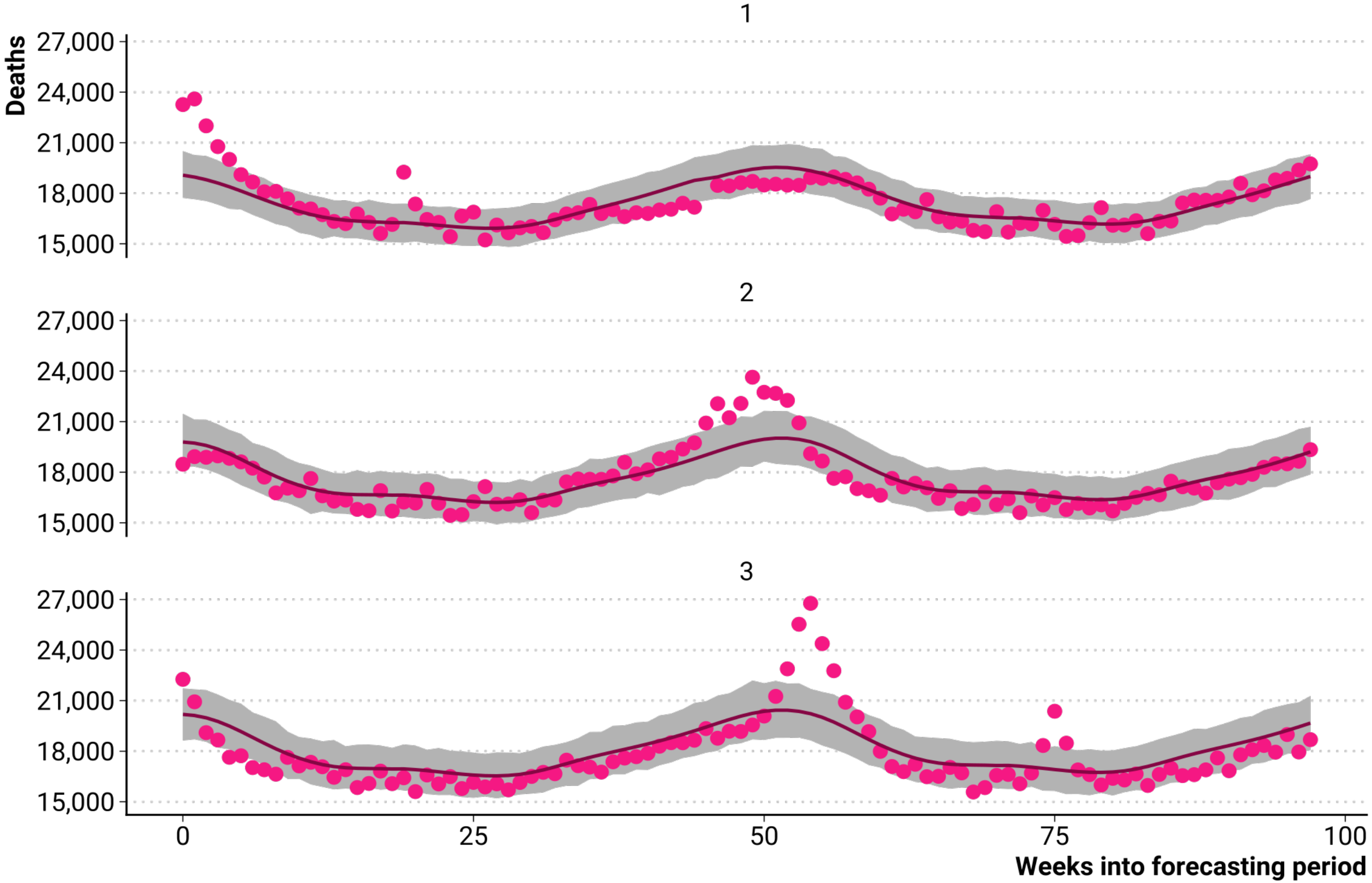
Modeling the distribution of empirical errors



Prediction interval with **90% nominal coverage** via Skewed-Normal empirical error model

Actual coverage ~92%

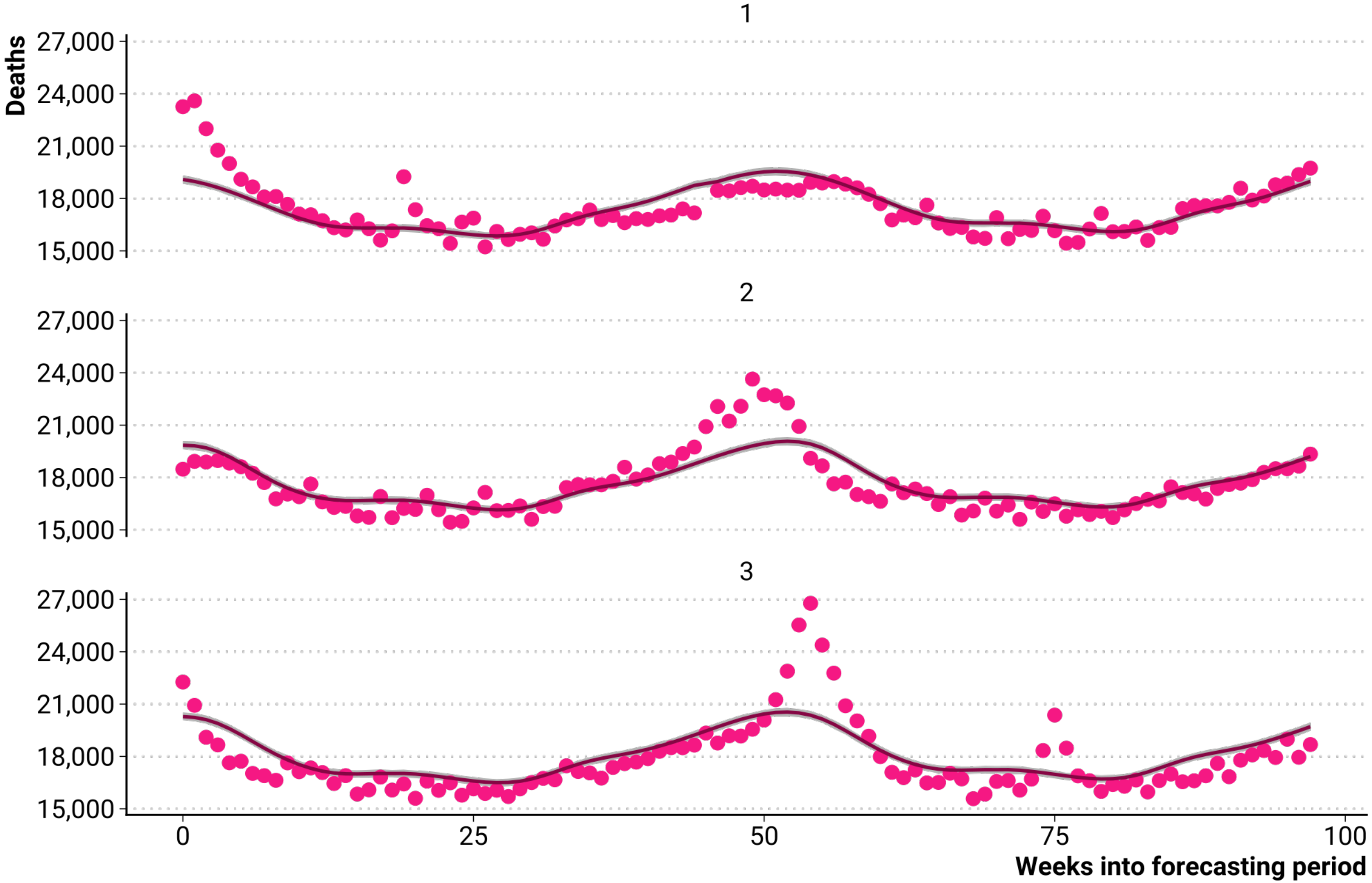
Modeling the distribution of empirical errors



Prediction interval with **90% nominal coverage** via Negative-Bin. GAM model

Actual coverage ~87%

Modeling the distribution of empirical errors



Prediction interval with **90% nominal coverage** via Poisson GAM model

Actual coverage ~22%

Evaluating the prediction interval

6.2 Interval Score

Interval forecasts form a crucial special case of quantile prediction. We consider the classical case of the central $(1-\alpha) \times 100\%$ prediction interval, with lower and upper endpoints that are the predictive quantiles at level $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$. We denote a scoring rule for the associated interval forecast by $S_\alpha(l, u; x)$, where l and u represent for the quoted $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ quantiles. Thus, if the forecaster quotes the $(1-\alpha) \times 100\%$ central prediction interval $[l, u]$ and x materializes, then his or her score will be $S_\alpha(l, u; x)$. Putting $\alpha_1 = \frac{\alpha}{2}$, $\alpha_2 = 1-\frac{\alpha}{2}$, $s_1(x) = s_2(x) = 2\frac{x}{\alpha}$, and $h(x) = -2\frac{x}{\alpha}$ in (42), and reversing the sign of the scoring rule, yields the negatively oriented *interval score*,

$$S_\alpha^{\text{int}}(l, u; x) = (u - l) + \frac{2}{\alpha}(l - x)\mathbb{1}\{x < l\} + \frac{2}{\alpha}(x - u)\mathbb{1}\{x > u\}. \quad (43)$$

Gneiting & Raftery (2007). Strictly Proper Scoring Rules. 10.1198/016214506000001437

Evaluating the prediction interval

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Prediction interval with **90% nominal coverage**

Coverage NB ~87%
Interval Score NB ~5979

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Evaluating the prediction interval

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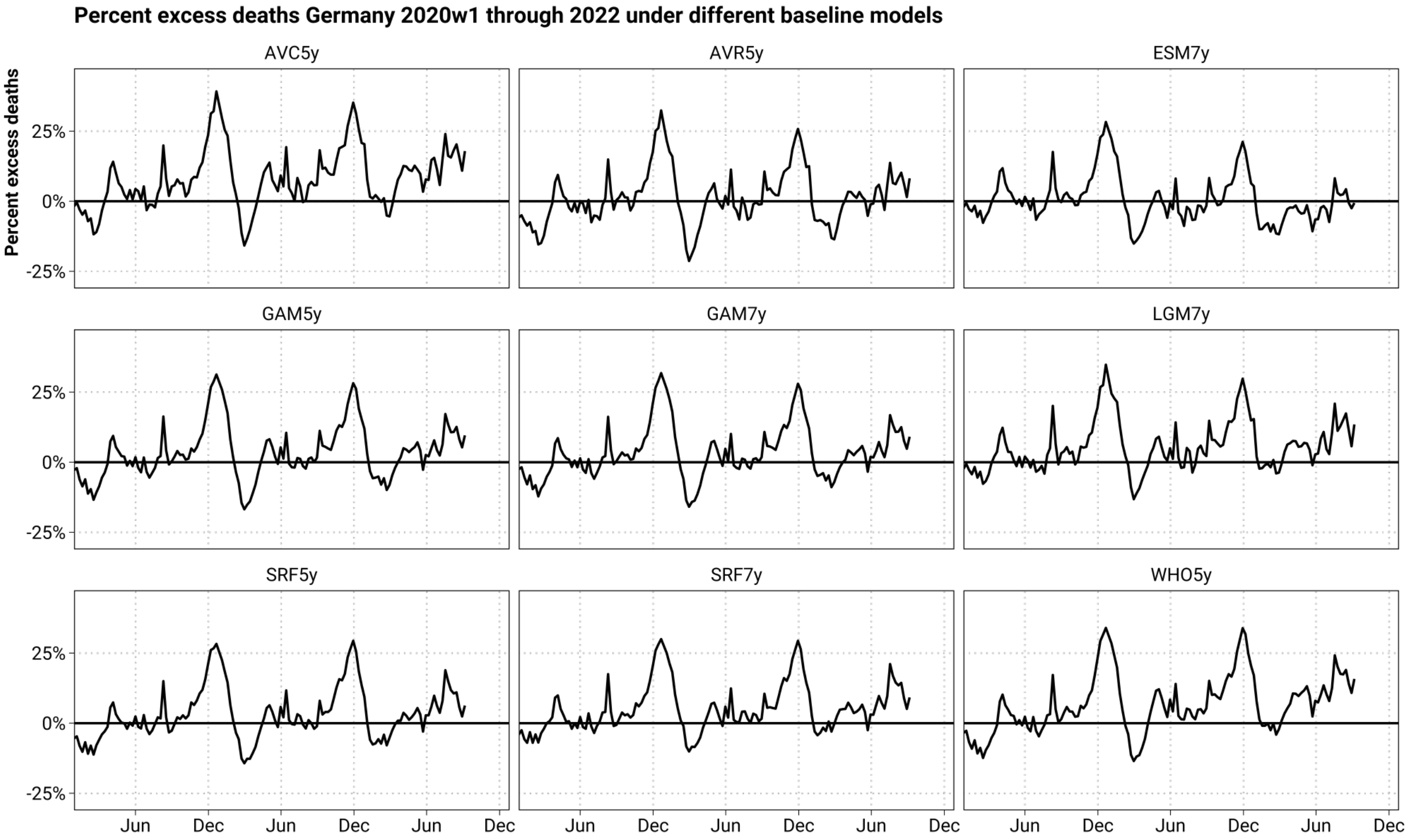
Prediction interval with **90% nominal coverage**

Coverage NB ~87%
Interval Score NB ~5979

Coverage SNO ~92%
Interval Score SNO ~4815

Gneiting & Raftery (2007). Strictly Proper Scoring Rules. 10.1198/016214506000001437

Excess deaths as a forecasting challenge



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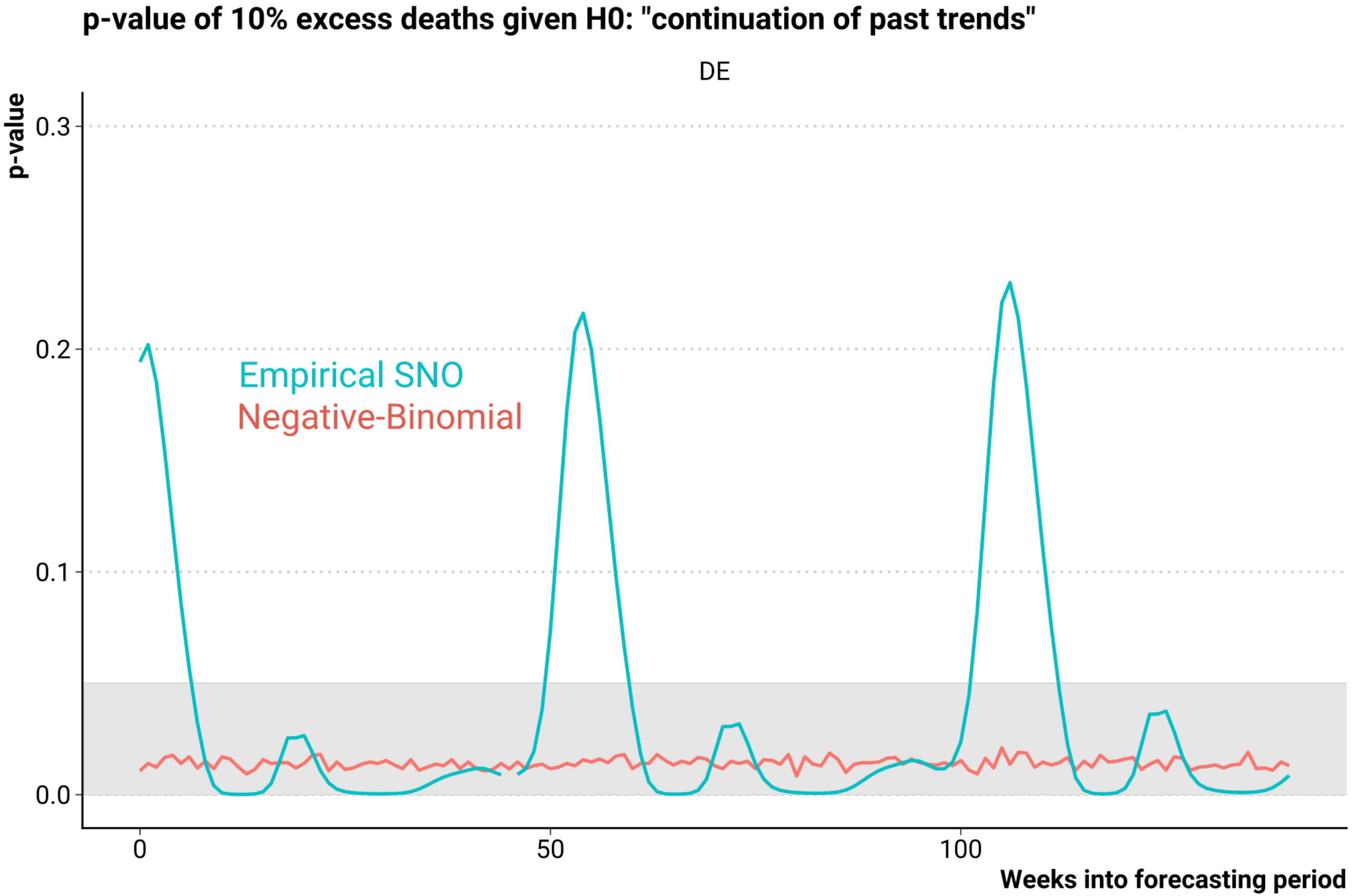

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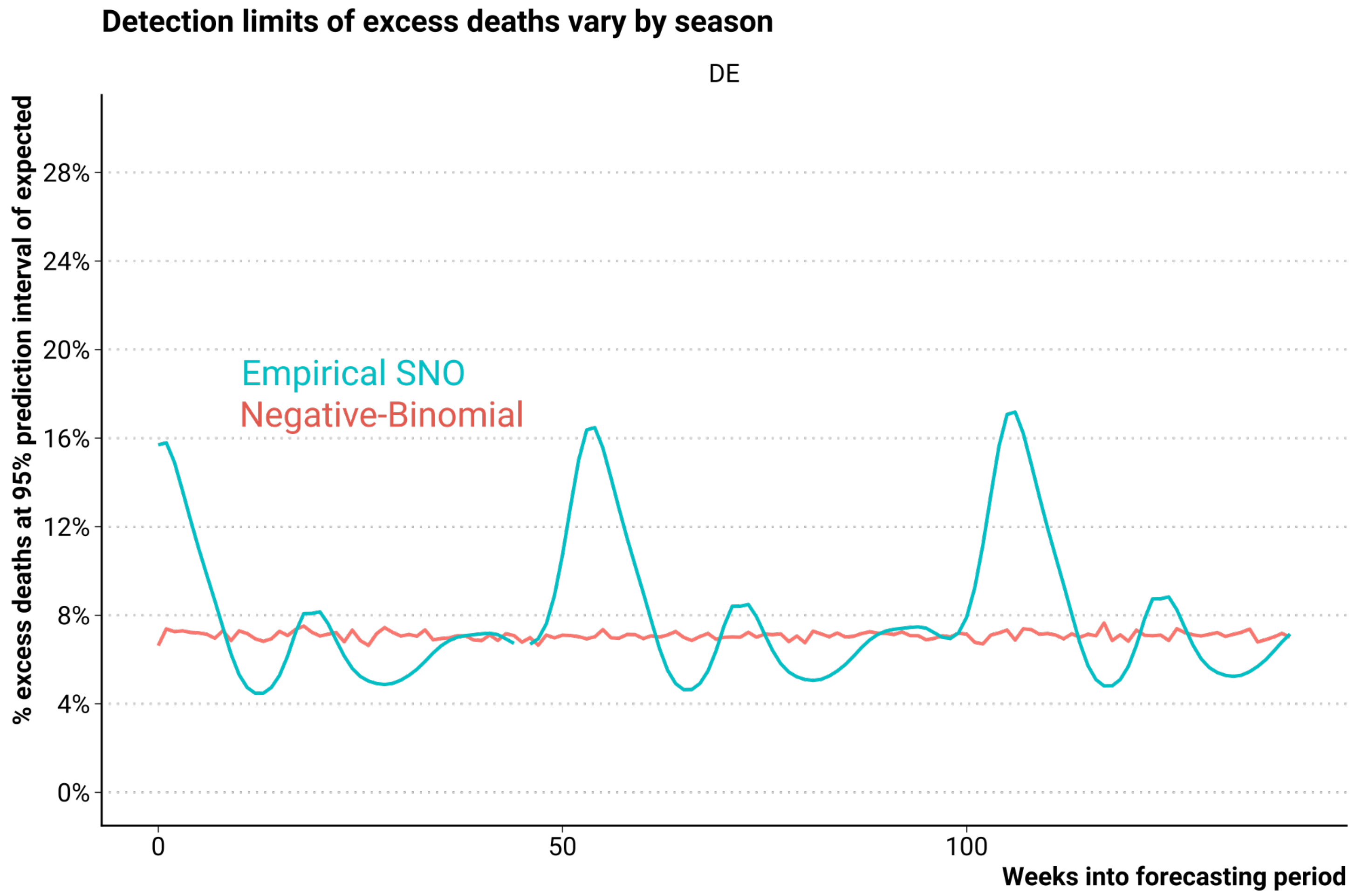
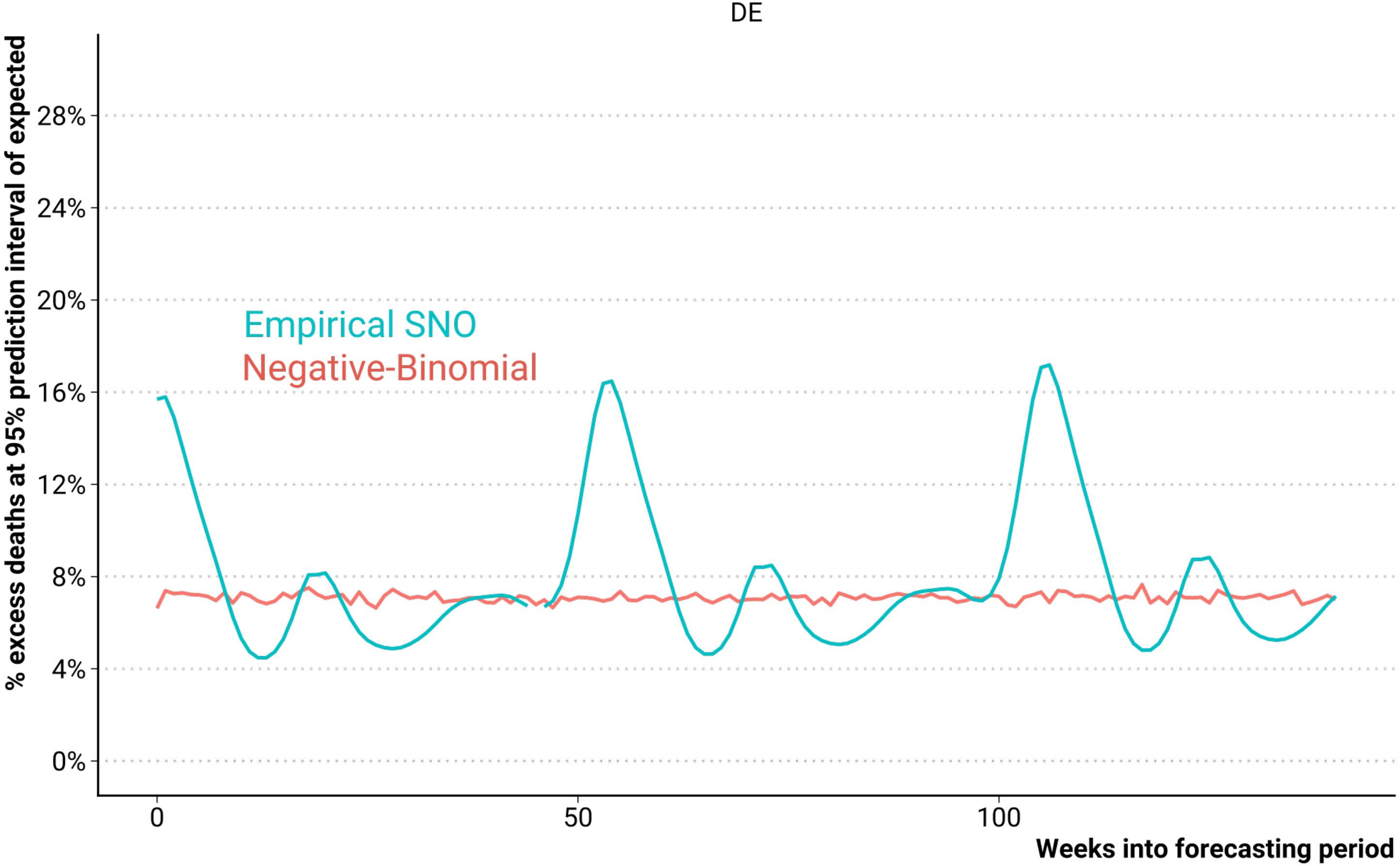


Excess deaths as a forecasting challenge

Detection limits of excess deaths vary by season



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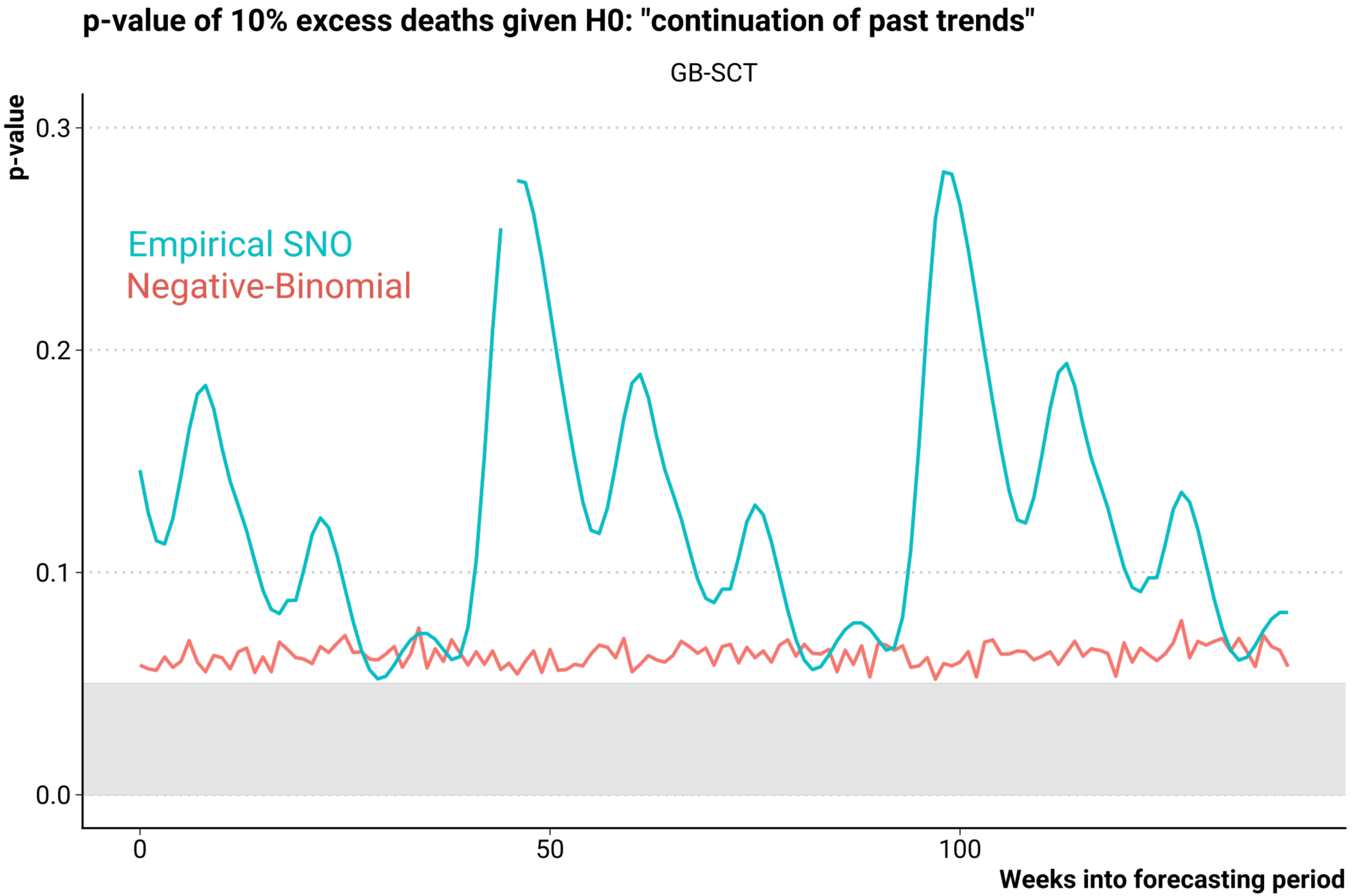

Reproducible analysis

github.com/jschoeley

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