lifequal: Calculating Life-table Lifespan Equality

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How to install?

You can install `lifequal` by running:

```
install.packages("devtools")
devtools::install_github("jschoeley/lifequal")
```

What does it do?

`lifequal` lets you calculate three measures of lifespan equality from a life-table:

1) `ExDagger(x, ex, wx, ax)` Life expectancy lost by those who die in age interval [x, x+w)
2) `EDagger(dx, exdagger, radix)` Total life expectancy lost due to death
3) `KeyfzEntro(edagger, e0)` Keyfitz's entropy

They are defined as follows,

Measure	Definition
Start of age interval	x
Width of age interval starting at x	w_x
Start of last age interval	ω
Average time spent in age interval $[x, x + wx)$ when dying in that interval	a_x
Deaths in age interval $[x, x + w_x)$	d_x
Life-expectancy at age x	e_x
Life expectancy lost due to death in age interval $[x, x + w_x)$	$e_x^\dagger = \frac{a_x}{w_x} e_{x+w_x} + (1 - \frac{a_x}{w_x}) e_x$
Total life expectancy lost due to death	$e^\dagger = \sum_{x=0}^{\omega} d_x e_x^\dagger$
Keyfitz's Entropy	$\frac{e^\dagger}{e_0}$

Life expectancy versus lifespan equality for 1x1 Swedish life-tables

```
library(lifequal)
library(dplyr)
library(ggplot2)
```

The analysis starts with a demographic life-table. We want 1) age groups ordered from low to high, 2) no gaps between subsequent age groups. Something like this:

```
# Swedish 1x1 period life-tables by period and sex
sweden1x1
```

```
## Source: local data frame [58,608 x 11]
##
##       sex period     x      mx      qx    ax     lx    dx    Lx      Tx
##     (chr)  (int) (int)   (dbl)   (dbl) (dbl)  (int) (int) (int)   (int)
## 1  female   1751     0 0.21223 0.18651  0.35 100000 18651 87877 3987544
## 2  female   1751     1 0.04941 0.04822  0.50  81349  3923 79388 3899667
## 3  female   1751     2 0.03225 0.03174  0.50  77427  2457 76198 3820279
## 4  female   1751     3 0.02601 0.02567  0.50  74970  1925 74007 3744080
## 5  female   1751     4 0.02370 0.02342  0.50  73045  1711 72190 3670073
## 6  female   1751     5 0.01876 0.01859  0.50  71334  1326 70671 3597884
## 7  female   1751     6 0.01296 0.01287  0.50  70008   901 69558 3527212
## 8  female   1751     7 0.00877 0.00873  0.50  69107   603 68806 3457654
## 9  female   1751     8 0.00608 0.00606  0.50  68504   415 68296 3388849
## 10 female   1751     9 0.00494 0.00493  0.50  68089   335 67921 3320553
## ..    ...    ...   ...     ...     ...   ...    ...   ...   ...     ...
## Variables not shown: ex (dbl)
```

First, we use `ExDagger()` on each single life-table (separate by period and sex) to calculate the life expectancy lost in each age. We then summarise each life-table into a set of 3 numbers: Life expectancy at birth, total life years lost due to death (`EDagger()`) and lifespan equality (`KeyfzEntro()`). Note that we transform Keyfitz's Entropy by taking the negative log.

```
sweden1x1 %>%
  # ...for each single life-table...
  group_by(period, sex) %>%
  #...we calculate the life years lost in age x...
  mutate(exdagger = ExDagger(x, ex)) %>%
  # ...and then summarise each life-table into a set of 3 numbers:
  # e0:         Life-expectancy at birth
  # edagger:    Total life years lost due to death
  # keyfzentro: Lifespan equality
  summarise(
    e0         = ex[x == 0],
    edagger    = EDagger(dx, exdagger, radix = 100000),
    keyfzentro = -log(KeyfzEntro(edagger, e0))
  ) -> sweden1x1summary
```

The summarised life-tables look like this:

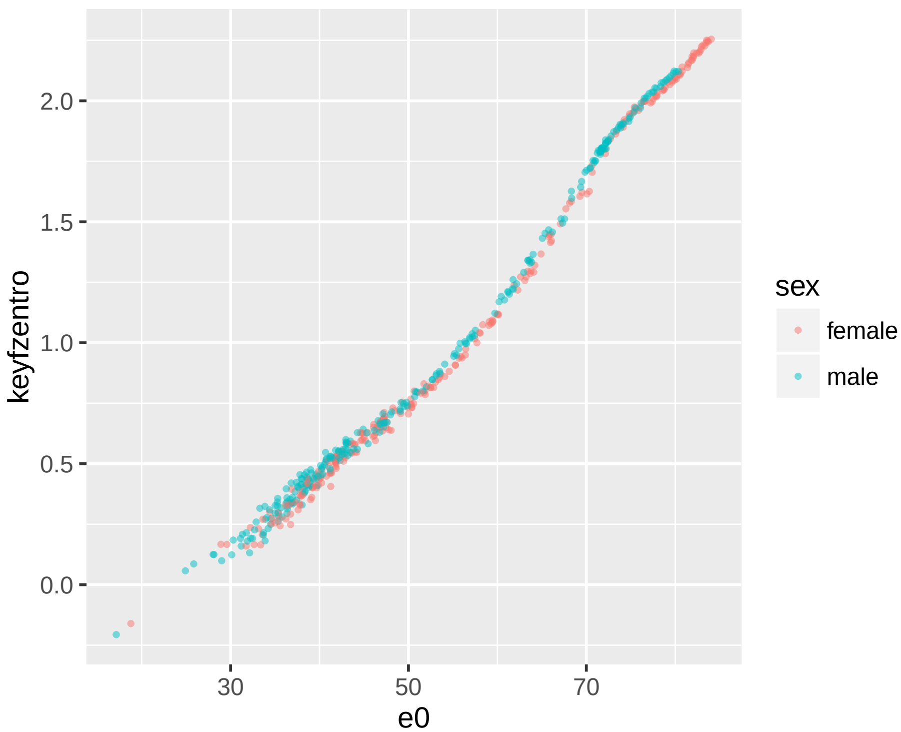
```
sweden1x1summary
```

```
## Source: local data frame [528 x 5]
## Groups: period [?]
##
##    period    sex    e0  edagger keyfzentro
##     (int)  (chr) (dbl)    (dbl)      (dbl)
## 1    1751 female 39.88 26.42272  0.4116507
## 2    1751   male 36.81 26.29076  0.3365518
## 3    1752 female 36.75 28.65525  0.2488017
## 4    1752   male 33.88 28.26324  0.1812630
## 5    1753 female 41.27 27.49097  0.4062782
## 6    1753   male 38.05 27.36753  0.3295438
## 7    1754 female 39.01 27.45547  0.3512526
```

```
## 8    1754   male 35.80 27.05649  0.2800211
## 9    1755 female 37.60 27.59827  0.3092511
## 10   1755   male 35.34 27.21785  0.2611423
## ..    ...    ...   ...      ...        ...
```

For each life-table we plot the life expectancy at birth versus the lifespan equality.

```
plot_lifequal <-
  ggplot(sweden1x1summary, aes(x = e0, y = keyfzentro, color = sex)) +
  geom_point(size = 0.6, alpha = 0.5) +
  theme(aspect.ratio = 1)
plot_lifequal
```



Life expectancy versus lifespan equality for 5x5 Swedish life-tables

The same exercise as before, only now we deal with life-tables aggregated over multiple year period and age intervals.

```
# Swedish 5x5 period life-tables by period and sex
sweden5x5
```

```
## Source: local data frame [2,496 x 12]
##
##       sex    period     x    wx      mx      qx    ax     lx    dx     Lx
##     (chr)     (chr) (int) (dbl)   (dbl)   (dbl) (dbl)  (int) (int)  (int)
## 1  female 1755-1759     0     1 0.23517 0.20399  0.35 100000 20399  86741
## 2  female 1755-1759     1     4 0.04076 0.14805  1.52  79601 11785 289151
## 3  female 1755-1759     5     5 0.01256 0.06043  1.88  67816  4098 326313
## 4  female 1755-1759    10     5 0.00658 0.03239  2.53  63718  2064 313492
```

```
## 5  female 1755-1759    15     5 0.00622 0.03063  2.44  61654  1888 303428
## 6  female 1755-1759    20     5 0.00720 0.03537  2.62  59766  2114 293794
## 7  female 1755-1759    25     5 0.00929 0.04544  2.57  57652  2620 281902
## 8  female 1755-1759    30     5 0.01230 0.05971  2.53  55032  3286 267060
## 9  female 1755-1759    35     5 0.01108 0.05389  2.49  51746  2788 251744
## 10 female 1755-1759    40     5 0.01622 0.07806  2.59  48958  3822 235561
## ..    ...       ...   ...   ...     ...     ...   ...    ...   ...    ...
## Variables not shown: Tx (int), ex (dbl)
```

We deal with this aggregation by informing `ExDagger()` about w_x, the width of each age interval. This information is included in the column `wx` of our life-table.

```
sweden5x5 %>%
  # for practicality we assume a width of 2
  # years for the last open age group 110+
  mutate(wx = ifelse(is.na(wx), 2, wx)) %>%
  # ...for each single life-table...
  group_by(period, sex) %>%
  #...we calculate the life years lost in age interval [x, x+wx)...
  mutate(exdagger = ExDagger(x, ex, wx, ax)) %>%
  # ...and then summarise each life-table into a set of 3 numbers:
  # e0:         Life-expectancy at birth
  # edagger:    Total life years lost due to death
  # keyfzentro: Lifespan equality
  summarise(
    e0         = ex[x == 0],
    edagger    = EDagger(dx, exdagger, radix = 100000),
    keyfzentro = -log(KeyfzEntro(edagger, e0))
  ) -> sweden5x5summary
```

The summarised life-tables look like this:

```
sweden5x5summary
```

```
## Source: local data frame [104 x 5]
## Groups: period [?]
##
##       period    sex    e0  edagger keyfzentro
##        (chr)  (chr) (dbl)    (dbl)      (dbl)
## 1  1755-1759 female 36.80 26.33548  0.3345809
## 2  1755-1759   male 33.89 26.03847  0.2635449
## 3  1760-1764 female 36.50 26.60345  0.3162712
## 4  1760-1764   male 33.62 26.12272  0.2523156
## 5  1765-1769 female 37.81 26.52746  0.3543933
## 6  1765-1769   male 34.78 26.16651  0.2845622
## 7  1770-1774 female 31.23 25.58650  0.1993144
## 8  1770-1774   male 28.67 24.93197  0.1397002
## 9  1775-1779 female 38.08 26.55897  0.3603218
## 10 1775-1779   male 35.95 26.48334  0.3056131
## ..       ...    ...   ...      ...        ...
```

The aggregated life-tables follow the same trend as the single year life-tables.

```
plot_lifequal +
  geom_point(shape = 22, fill = "white",
             data = sweden5x5summary)
```