Homework 2 (Computational)

Due on Tuesday, September 25, at 05:20 pm

Solve the following 5 problems to get full credit for this homework assignment. Please follow the instructions for homework assignments. I reserve the right to deduct points if you do not follow these rules.

Preliminaries

1 Download and install Matlab

You can get a free copy through `https://accessuh.uh.edu` (software download).

Read the instructions carefully.

Important: Use your UH email address to create your MathWorks account.

If you are not familiar with Matlab you should look for free online tutorials.

Matlab has an amazing documentation/help: `https://www.mathworks.com/help/matlab`.

You can also use the `doc` command in your Matlab command window to open the documentation.

To learn more about a Matlab command type `doc cmd` in your command window.

Example: `doc inv` ↵ (other option: `help inv` ↵).

2 Download and install CVX (website: `http://cvxr.com/cvx`)

Follow the instructions on the webpage/in the documentation.

Read the instructions carefully.

After installation: Go to the examples folder and run the quickstart (assuming your in the CVX folder):

`cd examples` ↵

`quickstart` ↵

3 Download the examples I have created and run them.

The code relevant for this class can be found here:

`https://github.com/andreasmang/optik`

If you are not familiar with `git`, I strongly encourage you to get familiar.

Assignments

Try to understand the code I provide (I do not expect you to understand what the solvers do at this point; we will learn about this later in class).

1. Run `ex01_lsq.m` located in the `examples` folder. This code computes the solution of the unconstrained least squares problem

   $$\min_x \|Ax - b\|_2$$

   using CVX. Nothing needs to be submitted for this part of the homework.

2. Run `ex02_lsq.m` and `ex03_lsq.m` located in the `examples` folder. Nothing needs to be submitted for this part of the homework.

   (a) The solvers are located in the `kernels` folder. In `ex02_lsq.m` we solve an unconstrained least squares problem using Matlab and CVX routines. In `ex03_lsq.m` we solve the constrained least squares problem given by

   $$\min_x \|Ax - b\|_2 \quad \text{subject to} \quad l \preceq x \preceq u.$$

   Try to understand the code.

   (b) Modify the problem sizes in `ex02_lsq.m` (i.e., n and m) and explore the behavior.

3. Create your own script by extending `ex01_lsq.m` to solve the constrained least squares problem given by

   $$\min_x \|Ax - b\|_2 \quad \text{subject to} \quad x \succeq 0.$$

   Compare your result to the unconstrained case (i.e., plot the solution for both problems in one graph/figure). Submit your script as described in the homework instructions.

4. One of the many traps in optimization is working with an erroneous derivative. The following test provides a simple way of checking the implementation of a derivative. To this end, let f be a multivariate function $f : \mathbf{R}^n \to \mathbf{R}$ and let $v \in \mathbf{R}^n$ be an arbitrary vector in the Taylor expansion

   $$f(x + hv) = f(x) + h\nabla f(x)^\mathsf{T} v + \mathcal{O}(h^2).$$

   The vector g is the derivative of f if and only if the difference

   $$|f(x + hv) - f(x) - hg^\mathsf{T} v|$$

   is essentially quadratic in h. Implement a derivative check for the least squares problem

   $$\min_x \frac{1}{2}\|Ax - b\|_2^2.$$

   Write a general purpose function called `checkDerivative` that takes a function handle and the vector x as input. A common choice for the steps size is `h = logspace(-1,-10,10)`. Notice that you should use random vectors to make sure that the derivative check generalizes. The script `ex04_lsq.m` illustrates some ideas that you might find helpful. For the derivative check, also plot the trend of the error (first and second order) versus the step size h. Submit a script that runs this derivative check, an implementation of the objective function and its derivative, as well as your implementation of the derivative check as described in the homework instructions.

5. Implement a derivative check for the regularized least squares problem

   $$\min_x \frac{1}{2}\|Ax - b\|_2^2 + \frac{\beta}{2}\|x\|_2^2,$$

   with regularization parameter $\beta > 0$. Submit a script to run the derivative check, and your implementation of the objective function and its derivate as described in the homework instructions.