Math 6366 Optimization: Homework 03

Due on Oct 11, 2018

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Problem 1

Consider the optimization problem

$$\begin{aligned} \underset{x\in\mathbb{R}^2}{\text{minimize}} \quad & f_0(x) \\ \text{subject to} \quad & 2x_1 + x_2 \geq 1 \\ & x_1 + 3x_2 \geq 1 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}$$

Make a sketch of the feasible set.

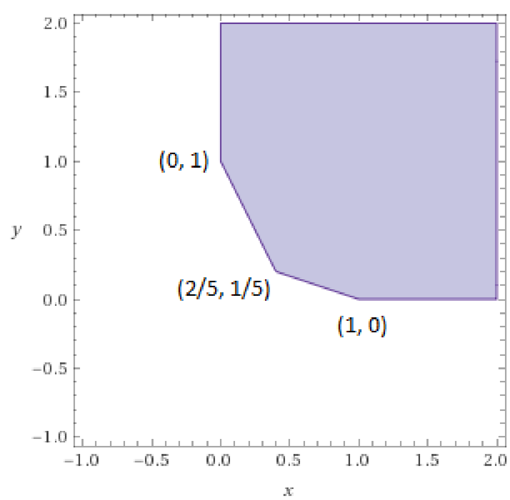

Figure 1:

For each of the following objective functions, give the optimal set and the optimal value.

a $f_0(x) = x_1 + x_2$.

The optimal point is $(2/5, 1/5)$ and the optimal value is $3/5$.

b $f_0(x) = \max\{x_1, x_2\}$.

The optimal point is $(1/3, 1/3)$ and the optimal value is $1/3$.

c $f_0(x) = x_1^2 + 9x_2^2$.

Problem 2

Verify that $x^* = \begin{bmatrix} 1 \\ 0.5 \\ -1 \end{bmatrix}$ is optimal for the optimization problem

$$\begin{aligned} \underset{x \in \mathbb{R}^3}{\text{minimize}} \quad & \frac{1}{2} x^\top A x + q^\top x + r \\ \text{subject to} \quad & -1 \leq x_i \leq 1, i = 1, 2, 3 \end{aligned}$$

where $A = \begin{bmatrix} 13 & 12 & -2 \\ 12 & 17 & 6 \\ -2 & 6 & 12 \end{bmatrix}$, $q = \begin{bmatrix} -22 \\ -14.5 \\ 30 \end{bmatrix}$ and $r = 1$.

This is a quadratic, convex objective, so the optimal value occurs when the gradient is zero.

$$\nabla f_0(x) = Ax + q^\top = 0$$

$$\nabla f_0(x^*) = \begin{bmatrix} 13 & 12 & -2 \\ 12 & 17 & 6 \\ -2 & 6 & 12 \end{bmatrix} * x^* + \begin{bmatrix} -22 \\ -14.5 \\ 30 \end{bmatrix}^\top = \begin{bmatrix} -1 \\ 0 \\ 19 \end{bmatrix}$$

So, at x^*, the gradient is not zero in all directions. But the objective value only gets better for increasing x_1 and decreasing x_3, which would put us outside of the constraints. The gradient for x_2 is zero, so the value of x_2 is optimal here.