§1.1 Linear Systems

In general,

$$a_{11} x_1 + \dots + a_{1n} x_n = b_1$$

$$\vdots$$

$$a_{n1} x_1 + \dots + a_{nn} x_n = b_n$$

n equation, n variables

Linear = (power of x_i = 1)

Trying to solve for $x_1 \dots x_n$.

Solved by <u>Row Operations</u>.

1. Add a multiple of one equation to another.

$$r_j' = a r_i + r_j$$

2. Multiply an eq'n by a constant.

$$r_i' = c r_i$$

3. Swap rows/equations.

Ex

$$
\begin{aligned}
2x - y + 2z &= 2 \\
-x - y + 3z &= 1 \\
3x \quad\quad - 2z &= 1
\end{aligned}
$$

$(1,1,1)$

Augmented matrix

$$
\left(\begin{array}{ccc:c}
2 & -1 & 2 & 2 \\
-1 & -1 & 3 & 1 \\
3 & 0 & -2 & 1
\end{array}\right)
$$

swap $r_1 r_2$

$r_1' = -r_1$

$$
\begin{pmatrix}
1 & 1 & -3 & -1 \\
\boxed{2} & -1 & 2 & 2 \\
\boxed{3} & \boxed{0} & -2 & 1
\end{pmatrix}
$$

$$\xrightarrow{r_2' = -2r_1 + r_2} \begin{pmatrix} 1 & 1 & -3 & -1 \\ 0 & -3 & 8 & 4 \\ \boxed{3} & 0 & -2 & 1 \end{pmatrix}$$

$$\boxed{r_3' = -3r_1 + r_3}$$

$$\begin{pmatrix} 1 & 1 & -3 & -1 \\ 0 & -3 & 8 & 4 \\ 0 & \boxed{-3} & 7 & 4 \end{pmatrix}$$

$$r_3' = -r_2 + r_3$$

$$\begin{pmatrix} 1 & 1 & -3 & -1 \\ 0 & -3 & 8 & 4 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\left.\begin{aligned} x + y - 3z &= -1 \\ -3y + 8z &= 4 \\ -z &= 0 \end{aligned}\right\}$$

"Back substitution"

$$\Rightarrow \boxed{z = 0}$$

$$-3y = 4$$

$$\Rightarrow \boxed{y = \frac{-4}{3}}$$

$$x + \left(\frac{-4}{3}\right) - 0 = -1$$

$$\boxed{x = \frac{1}{3}}$$

§ 1.2 Matrices

A matrix is an $n \times m$ array of numbers.

- n rows
- m columns

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix}$$

3×2

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \\ -5 & \pi \end{pmatrix}$$

Define $\boxed{M_{n\times m}(\mathbb{R})}$

to be the set of

$n\times m$ matrices w/

entries in $\mathbb{R}$.

(Recall $\mathbb{R}$ is the real numbers.)

Similarly define

$M_{n\times m}(\mathbb{C})$

to be set of $n\times m$

matrices w/

entries in $\mathbb{C}$

($\mathbb{C}$ is the complex numbers)