U.S. Crime Analysis

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In this script, I will analyze the U.S. Crime dataset included in the data folder of this repository. I will use a variety of exploratory and modeling techniques to do so, including, but not limited to:

- Outlier Detection
- Linear Regression
- Principal Component Analysis (PCA) + Lin. Reg.
- Regression Trees
- Random Forest
- Variable Selection

Let's start by taking a look at our data:

```
## # A tibble: 6 x 16
##       M    So    Ed   Po1   Po2    LF   M.F   Pop    NW    U1    U2 Wealth
Ineq
##   <dbl> <int> <dbl> <dbl> <dbl> <dbl> <dbl> <int> <dbl> <dbl> <dbl>  <int>
<dbl>
## 1  15.1     1   9.1   5.8   5.6 0.51   95      33  30.1 0.108   4.1   3940
26.1
## 2  14.3     0  11.3  10.3   9.5 0.583 101.     13  10.2 0.096   3.6   5570
19.4
## 3  14.2     1   8.9   4.5   4.4 0.533  96.9    18  21.9 0.094   3.3   3180
25
## 4  13.6     0  12.1  14.9  14.1 0.577  99.4   157   8    0.102   3.9   6730
16.7
## 5  14.1     0  12.1  10.9  10.1 0.591  98.5    18   3    0.091   2      5780
17.4
## 6  12.1     0  11    11.8  11.5 0.547  96.4    25   4.4 0.084   2.9   6890
12.6
## # ... with 3 more variables: Prob <dbl>, Time <dbl>, Crime <int>
```

```
##        M               So                Ed              Po1
##  Min.   :11.90   Min.   :0.0000   Min.   : 8.70   Min.   : 4.50
##  1st Qu.:13.00   1st Qu.:0.0000   1st Qu.: 9.75   1st Qu.: 6.25
##  Median :13.60   Median :0.0000   Median :10.80   Median : 7.80
##  Mean   :13.86   Mean   :0.3404   Mean   :10.56   Mean   : 8.50
##  3rd Qu.:14.60   3rd Qu.:1.0000   3rd Qu.:11.45   3rd Qu.:10.45
##  Max.   :17.70   Max.   :1.0000   Max.   :12.20   Max.   :16.60
##       Po2              LF               M.F              Pop
##  Min.   : 4.100   Min.   :0.4800   Min.   : 93.40   Min.   :  3.00
##  1st Qu.: 5.850   1st Qu.:0.5305   1st Qu.: 96.45   1st Qu.: 10.00
```

```
##  Median : 7.300   Median :0.5600   Median : 97.70   Median : 25.00
##  Mean   : 8.023   Mean   :0.5612   Mean   : 98.30   Mean   : 36.62
##  3rd Qu.: 9.700   3rd Qu.:0.5930   3rd Qu.: 99.20   3rd Qu.: 41.50
##  Max.   :15.700   Max.   :0.6410   Max.   :107.10   Max.   :168.00
##        NW              U1                U2             Wealth
##  Min.   : 0.20   Min.   :0.07000   Min.   :2.000   Min.   :2880
##  1st Qu.: 2.40   1st Qu.:0.08050   1st Qu.:2.750   1st Qu.:4595
##  Median : 7.60   Median :0.09200   Median :3.400   Median :5370
##  Mean   :10.11   Mean   :0.09547   Mean   :3.398   Mean   :5254
##  3rd Qu.:13.25   3rd Qu.:0.10400   3rd Qu.:3.850   3rd Qu.:5915
##  Max.   :42.30   Max.   :0.14200   Max.   :5.800   Max.   :6890
##       Ineq            Prob              Time           Crime
##  Min.   :12.60   Min.   :0.00690   Min.   :12.20   Min.   : 342.0
##  1st Qu.:16.55   1st Qu.:0.03270   1st Qu.:21.60   1st Qu.: 658.5
##  Median :17.60   Median :0.04210   Median :25.80   Median : 831.0
##  Mean   :19.40   Mean   :0.04709   Mean   :26.60   Mean   : 905.1
##  3rd Qu.:22.75   3rd Qu.:0.05445   3rd Qu.:30.45   3rd Qu.:1057.5
##  Max.   :27.60   Max.   :0.11980   Max.   :44.00   Max.   :1993.0
```

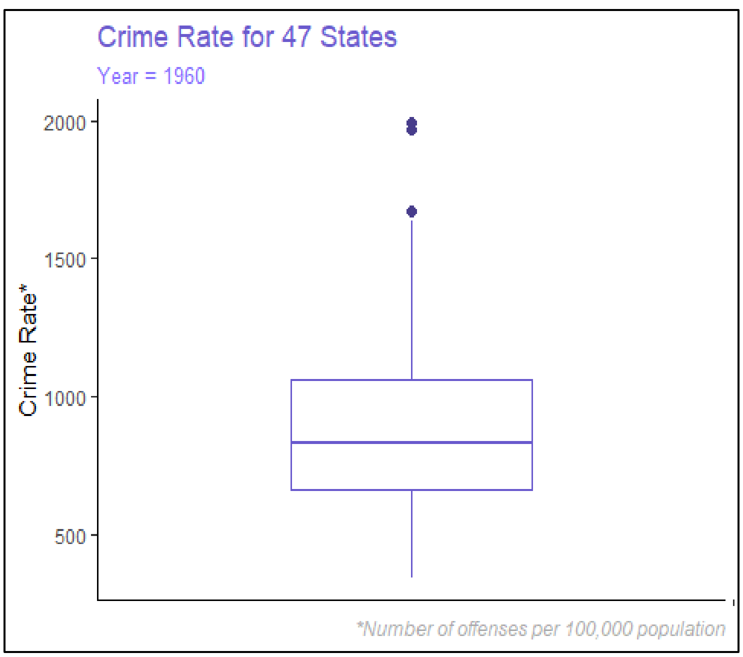
Below is a list of the variables in data along with their associated descriptions. We want to predict the last column, Crime, based on the other predictor variables.

Variable	Description
M	percentage of males aged 14-24 in total state population
So	indicator variable for a southern state
Ed	mean years of schooling of the population aged 25 years or over
Po1	per capita expenditure on police protection in 1960
Po2	per capita expenditure on police protection in 1959
LF	labour force participation rate of civilian urban males in the age-group 14-24
M.F	number of males per 100 females Pop state population in 1960 in hundred thousands
NW	percentage of nonwhites in the population

U1	unemployment rate of urban males 14-24
U2	unemployment rate of urban males 35-39
Wealth	wealth: median value of transferable assets or family income
Ineq	income inequality: percentage of families earning below half the median income
Prob	probability of imprisonment: ratio of number of commitments to number of offenses
Time	average time in months served by offenders in state prisons before their first release
Crime	crime rate: number of offenses per 100,000 population in 1960

Outliers

In this section, we'll first test to see whether there are any outliers in the last column (number of crimes per 100,000 people). To do so, we'll use the grubbs.test function in the outliers package in R.



Now that we have a clear idea of what the outliers could look like, let's use the grubbs.test to better understand these outliers.

```
##
##  Grubbs test for one outlier
##
## data:  data_grubbs$Crime
## G = 2.81287, U = 0.82426, p-value = 0.07887
## alternative hypothesis: highest value 1993 is an outlier

## [1] "highest value 1993 is an outlier"

## [1] 0.07887486
```

Since the p-value is above .05 (.079, to be exact), we cannot say with confidence that there is an outlier in the set. Let's move on to some modelling.

Linear Regression

In the first section, we'll use a simple linear regression model to predict on our crime data. Start by scaling the data so it's standardardized. Avoid column 2 because it's an indicator (factor) for southern states.

```
##
## Call:
## lm(formula = Crime ~ ., data = data_scaled)
##
## Coefficients:
## (Intercept)            M           Ed          Po1          Po2
LF
##    0.003348     0.285399     0.544723     1.481515    -0.791075    -0.0693
61
##         M.F          Pop           NW           U1           U2         Weal
th
##    0.132622    -0.072154     0.111784    -0.271628     0.366412     0.2399
19
##        Ineq         Prob         Time           So
##    0.729010    -0.285431    -0.063748    -0.009834

##
## Call:
## lm(formula = Crime ~ ., data = data_scaled)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.02321 -0.25361 -0.01731  0.29214  1.32554
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.003348   0.152841   0.022  0.98267
## M            0.285399   0.135547   2.106  0.04344 *
## Ed           0.544723   0.179589   3.033  0.00486 **
```

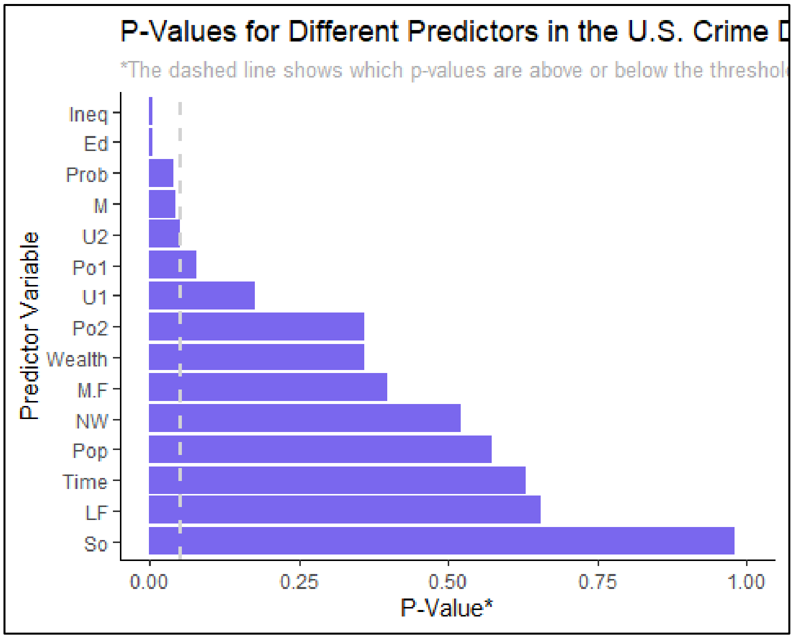
```
## Po1          1.481515   0.815350   1.817  0.07889 .
## Po2         -0.791075   0.849313  -0.931  0.35883
## LF          -0.069361   0.153568  -0.452  0.65465
## M.F          0.132622   0.155075   0.855  0.39900
## Pop         -0.072154   0.126938  -0.568  0.57385
## NW           0.111784   0.172308   0.649  0.52128
## U1          -0.271628   0.196261  -1.384  0.17624
## U2           0.366412   0.179791   2.038  0.05016 .
## Wealth       0.239919   0.258630   0.928  0.36075
## Ineq         0.729010   0.234330   3.111  0.00398 **
## Prob        -0.285431   0.133588  -2.137  0.04063 *
## Time        -0.063748   0.131294  -0.486  0.63071
## So          -0.009834   0.384616  -0.026  0.97977
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5405 on 31 degrees of freedom
## Multiple R-squared:  0.8031, Adjusted R-squared:  0.7078
## F-statistic: 8.429 on 15 and 31 DF,  p-value: 0.0000003539
```

The first two items we should look at are the R-squared value (80.3%) and the Adjusted R-squared value (71.8%). The next item we should look at is the overal p-value, which is p-value: 0.0000003539. This means that our crime data can be explained by the amalgam of predictor variables we provided.

```
##  (Intercept)            M           Ed          Po1          Po2
LF
##  0.003347768  0.285399152  0.544722603  1.481514913 -0.791074563 -0.069361
443
##          M.F          Pop           NW           U1           U2       Wea
lth
##  0.132622452 -0.072154040  0.111784244 -0.271627975  0.366411688  0.239918
991
##         Ineq         Prob         Time           So
##  0.729009903 -0.285430950 -0.063748223 -0.009834067
```

As an example of what this means, if M (percentage of males aged 14-24 in total state population) increases by 1, we would expect crime to increase by 87.83 crimes per 100,000 population.

But which of the variables mattered and which did not? Well, to start, let’s look at the individual p-values for each variable we have. We’ll do this by pulling out the p-values and coefficient names.



Let's confirm what we see above, especially since U2 seems on the border. Filter our data frame to only show p-values less than .05 and then order our results.

Now that we have the coefficients that matter, let's re-run the linear model only on the coefficients we see above

```
##
## Call:
## lm(formula = Crime ~ ., data = data_signif)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.3780 -0.6568 -0.1441  0.3563  2.4827
##
## Coefficients:
##                          Estimate               Std. Error t value Pr(>|t|)
## (Intercept)  0.0000000000000000604  0.1310586847078026862   0.000  1.00000
## M            0.1168920934073441331  0.1735000214758070369   0.674  0.50417
## Ed           0.4298365505224954752  0.2080133416220263098   2.066  0.04499
*
## Ineq         0.2772215506837938381  0.2349037199509093621   1.180  0.24458
## Prob        -0.4310280070497272686  0.1505129457305220686  -2.864  0.00651
**
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8985 on 42 degrees of freedom
## Multiple R-squared:  0.2629, Adjusted R-squared:  0.1927
## F-statistic: 3.745 on 4 and 42 DF,  p-value: 0.01077
```

We see from this model that our R-squared value dropped significantly. Very strange, and not what I expected. Let's try bringing in the two variables that were just above the .05 threshold (U@ and Po1).

```
##
## Call:
## lm(formula = Crime ~ ., data = data_updated)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.3658 -0.1913 -0.0181  0.3614  1.3014
##
## Coefficients:
##                            Estimate             Std. Error t value
## (Intercept) -0.0000000000000001558  0.0790947003719782299   0.000
## M            0.2589438160798205324  0.1059969678013346767   2.443
## Ed           0.4632372705133553925  0.1255970724844087949   3.688
## Ineq         0.7046465259134735426  0.1501861745051313868   4.692
## Prob        -0.2273487874016537624  0.0938579544450350439  -2.422
## Po1          0.9315281453651754751  0.1080581397067851696   8.621
##                    Pr(>|t|)
## (Intercept)        1.000000
## M                  0.018964 *
## Ed                 0.000656 ***
## Ineq       0.0000300483391 ***
## Prob               0.019930 *
## Po1        0.0000000000947 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5422 on 41 degrees of freedom
## Multiple R-squared:  0.7379, Adjusted R-squared:  0.706
## F-statistic: 23.09 on 5 and 41 DF,  p-value: 0.00000000005926
```

Great! Our R-squared value went up to 76.6% and our adjusted R-squared went up to 73.1%. In particular, we notice that our R-squared value has gone down (it was originally 80.3%); however, our adjusted R-squared value has gone up (it was originally 70.1%). This makes sense because adjusted R-squared factors in the number of variables, and should get closer to 1 the less we overfit our model.

Now, let's predict on a fake dataset given by the original prompt to see what we get.

```
##              (Intercept)                        M                       Ed
## -0.00000000000000155812  0.25894381607982053239 4  0.46323727051335539250 9
##                     Ineq                     Prob                      Po1
##  0.70464652591347354260 2 -0.22734878740165376243 0  0.93152814536517547505 7
```

```
##        1
## 33.59023
```

The linear regression model estimates that the Crime rate based on this data will be 35.79. This translates to about 36 crimes per 100,000 population. This seems extremely off from our original dataset, so we'll instead resort to our original, unscaled data which included every variable.

Now let's take a look at the confidence interval for this new data.

```
##        fit      lwr      upr
## 1 33.59023 24.58019 42.60027
```

The same linear regression model estimates 798 crimes per 100,000 population. It also estimates that the lower limit for our data is -886.6 and the upper limit is 2482.3. This result makes more sense based on the fact that the median of our original crime is **831**.

Principal Component Analysis

In this section, I'll apply Principal Component Analysis, a method for trimming down the number of variables in our dataset and for variable selection. This *should* help with model-building later on.

```
## # A tibble: 6 x 15
##       M    So    Ed   Po1   Po2    LF   M.F   Pop    NW    U1    U2 Wealth
Ineq
##   <dbl> <int> <dbl> <dbl> <dbl> <dbl> <dbl> <int> <dbl> <dbl> <dbl>  <int>
<dbl>
## 1  15.1     1   9.1   5.8   5.6 0.51   95      33  30.1 0.108   4.1   3940
26.1
## 2  14.3     0  11.3  10.3   9.5 0.583 101.     13  10.2 0.096   3.6   5570
19.4
## 3  14.2     1   8.9   4.5   4.4 0.533  96.9    18  21.9 0.094   3.3   3180
25
## 4  13.6     0  12.1  14.9  14.1 0.577  99.4   157   8   0.102   3.9   6730
16.7
## 5  14.1     0  12.1  10.9  10.1 0.591  98.5    18   3   0.091   2     5780
17.4
## 6  12.1     0  11    11.8  11.5 0.547  96.4    25   4.4 0.084   2.9   6890
12.6
## # ... with 2 more variables: Prob <dbl>, Time <dbl>
```

Run our first Principle Component Analysis using the prcomp() function which uses Singular value decomposition (SVD), which examines the covariances / correlations between individual observations.

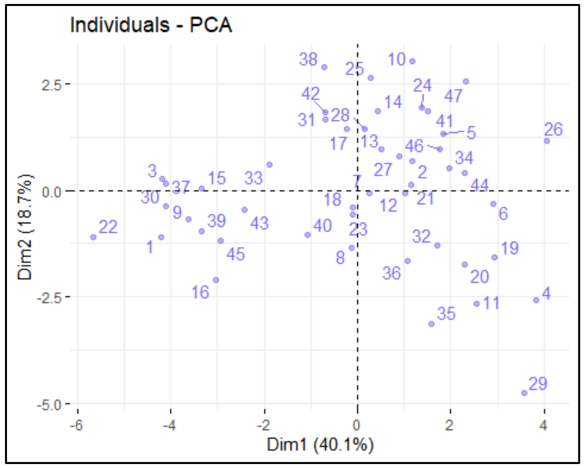
```
## Standard deviations (1, .., p=15):
##  [1] 2.45335539 1.67387187 1.41596057 1.07805742 0.97892746 0.74377006
##  [7] 0.56729065 0.55443780 0.48492813 0.44708045 0.41914843 0.35803646
## [13] 0.26332811 0.24180109 0.06792764
##
```

```
## Rotation (n x k) = (15 x 15):
##                PC1         PC2          PC3         PC4         PC5
## M      -0.30371194  0.06280357  0.1724199946 -0.02035537 -0.35832737
## So     -0.33088129 -0.15837219  0.0155433104  0.29247181 -0.12061130
## Ed      0.33962148  0.21461152  0.0677396249  0.07974375 -0.02442839
## Po1     0.30863412 -0.26981761  0.0506458161  0.33325059 -0.23527680
## Po2     0.31099285 -0.26396300  0.0530651173  0.35192809 -0.20473383
## LF      0.17617757  0.31943042  0.2715301768 -0.14326529 -0.39407588
## M.F     0.11638221  0.39434428 -0.2031621598  0.01048029 -0.57877443
## Pop     0.11307836 -0.46723456  0.0770210971 -0.03210513 -0.08317034
## NW     -0.29358647 -0.22801119  0.0788156621  0.23925971 -0.36079387
## U1      0.04050137  0.00807439 -0.6590290980 -0.18279096 -0.13136873
## U2      0.01812228 -0.27971336 -0.5785006293 -0.06889312 -0.13499487
## Wealth  0.37970331 -0.07718862  0.0100647664  0.11781752  0.01167683
## Ineq   -0.36579778 -0.02752240 -0.0002944563 -0.08066612 -0.21672823
## Prob   -0.25888661  0.15831708 -0.1176726436  0.49303389  0.16562829
## Time   -0.02062867 -0.38014836  0.2235664632 -0.54059002 -0.14764767
##                 PC6         PC7         PC8         PC9        PC10
PC11
## M      -0.449132706 -0.15707378 -0.55367691  0.15474793 -0.01443093  0.394
46657
## So     -0.100500743  0.19649727  0.22734157 -0.65599872  0.06141452  0.233
97868
## Ed     -0.008571367 -0.23943629 -0.14644678 -0.44326978  0.51887452 -0.118
21954
## Po1    -0.095776709  0.08011735  0.04613156  0.19425472 -0.14320978 -0.130
42001
## Po2    -0.119524780  0.09518288  0.03168720  0.19512072 -0.05929780 -0.138
85912
## LF      0.504234275 -0.15931612  0.25513777  0.14393498  0.03077073  0.385
32827
## M.F    -0.074501901  0.15548197 -0.05507254 -0.24378252 -0.35323357 -0.280
29732
## Pop     0.547098563  0.09046187 -0.59078221 -0.20244830 -0.03970718  0.058
49643
## NW      0.051219538 -0.31154195  0.20432828  0.18984178  0.49201966 -0.206
95666
## U1      0.017385981 -0.17354115 -0.20206312  0.02069349  0.22765278 -0.178
57891
## U2      0.048155286 -0.07526787  0.24369650  0.05576010 -0.04750100  0.470
21842
## Wealth -0.154683104 -0.14859424  0.08630649 -0.23196695 -0.11219383  0.319
55631
## Ineq    0.272027031  0.37483032  0.07184018 -0.02494384 -0.01390576 -0.182
78697
## Prob    0.283535996 -0.56159383 -0.08598908 -0.05306898 -0.42530006 -0.089
78385
## Time   -0.148203050 -0.44199877  0.19507812 -0.23551363 -0.29264326 -0.263
63121
##               PC12        PC13        PC14          PC15
```

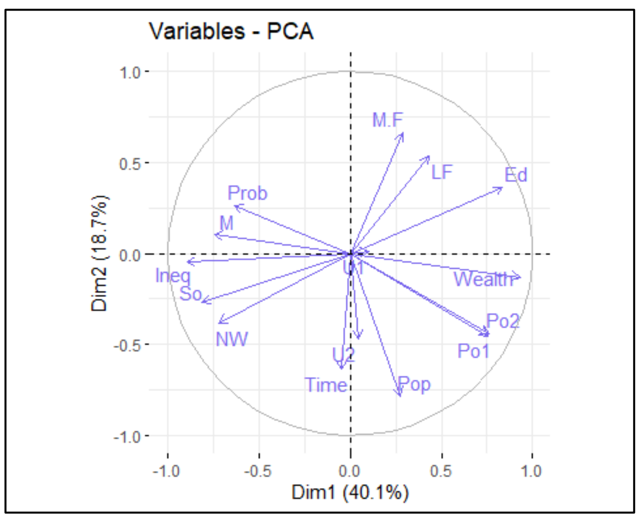
```
## M       0.16580189 -0.05142365  0.04901705  0.0051398012
## So     -0.05753357 -0.29368483 -0.29364512  0.0084369230
## Ed      0.47786536  0.19441949  0.03964277 -0.0280052040
## Po1     0.22611207 -0.18592255 -0.09490151 -0.6894155129
## Po2     0.19088461 -0.13454940 -0.08259642  0.7200270100
## LF      0.02705134 -0.27742957 -0.15385625  0.0336823193
## M.F    -0.23925913  0.31624667 -0.04125321  0.0097922075
## Pop    -0.18350385  0.12651689 -0.05326383  0.0001496323
## NW     -0.36671707  0.22901695  0.13227774 -0.0370783671
## U1     -0.09314897 -0.59039450 -0.02335942  0.0111359325
## U2      0.28440496  0.43292853 -0.03985736  0.0073618948
## Wealth -0.32172821 -0.14077972  0.70031840 -0.0025685109
## Ineq    0.43762828 -0.12181090  0.59279037  0.0177570357
## Prob    0.15567100 -0.03547596  0.04761011  0.0293376260
## Time    0.13536989 -0.05738113 -0.04488401  0.0376754405

## Importance of components:
##                           PC1    PC2    PC3     PC4     PC5     PC6     PC
7
## Standard deviation     2.4534 1.6739 1.4160 1.07806 0.97893 0.74377 0.5672
9
## Proportion of Variance 0.4013 0.1868 0.1337 0.07748 0.06389 0.03688 0.0214
5
## Cumulative Proportion  0.4013 0.5880 0.7217 0.79920 0.86308 0.89996 0.9214
2
##                            PC8     PC9    PC10    PC11    PC12    PC13   P
C14
## Standard deviation     0.55444 0.48493 0.44708 0.41915 0.35804 0.26333 0.2
418
## Proportion of Variance 0.02049 0.01568 0.01333 0.01171 0.00855 0.00462 0.0
039
## Cumulative Proportion  0.94191 0.95759 0.97091 0.98263 0.99117 0.99579 0.9
997
##                           PC15
## Standard deviation     0.06793
## Proportion of Variance 0.00031
## Cumulative Proportion  1.00000
```

We can see that, from our Cumulative Proportion dimension, the first six principle components explain about 90% of the variation in the data.



Above is a graph of the individual observations. Individual states with a similar profile of factors will be grouped. Next, we'll create a graph of the variables (columns in our dataset).
- Positively correlated variables point to the same side of the plot.
- Negatively correlated variables point to opposite sides of the plot.

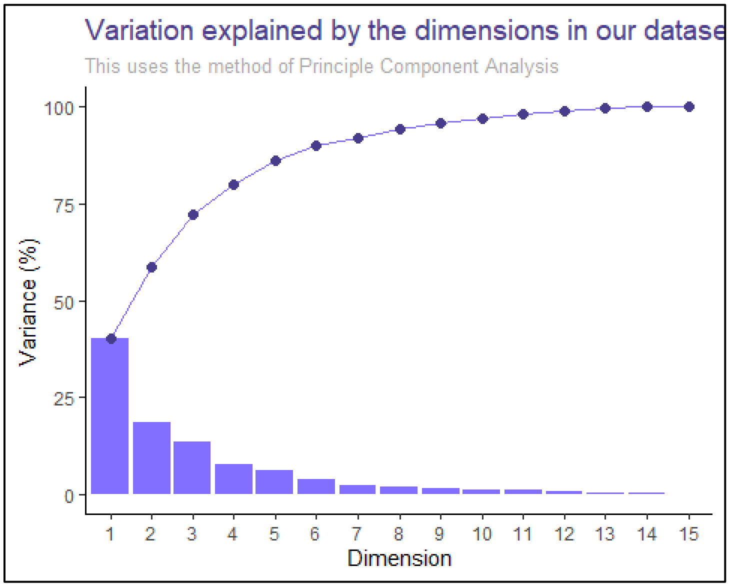


What are our eigenvalues?

```
##         eigenvalue variance.percent cumulative.variance.percent
## Dim.1   6.018952657       40.1263510                    40.12635
## Dim.2   2.801847026       18.6789802                    58.80533
## Dim.3   2.004944334       13.3662956                    72.17163
## Dim.4   1.162207801        7.7480520                    79.91968
## Dim.5   0.958298972        6.3886598                    86.30834
## Dim.6   0.553193900        3.6879593                    89.99630
## Dim.7   0.321818687        2.1454579                    92.14176
## Dim.8   0.307401270        2.0493418                    94.19110
## Dim.9   0.235155292        1.5677019                    95.75880
## Dim.10  0.199880931        1.3325395                    97.09134
## Dim.11  0.175685403        1.1712360                    98.26258
## Dim.12  0.128190107        0.8546007                    99.11718
```

```
## Dim.13 0.069341691        0.4622779                     99.57945
## Dim.14 0.058467765        0.3897851                     99.96924
## Dim.15 0.004614165        0.0307611                    100.00000
```

Note that the variance.percent values noted here align with our screeplot above. This also now tells us that 90% of the variance in our data is explained by just 6 dimensions. Let's graph the cumulative variance percentages so it's easier to visualize.



Just like earlier, we'll bring our crime data (our dependent variable) back in and see how our predictions are using the principle components.

```
##
## Call:
## lm(formula = Crime ~ ., data = pca_vals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -377.15 -172.23   25.81  132.10  480.38
##
## Coefficients:
##             Estimate Std. Error t value             Pr(>|t|)
## (Intercept)   905.09      35.35  25.604 < 0.0000000000000002 ***
## PC1            65.22      14.56   4.478          0.000061443 ***
## PC2           -70.08      21.35  -3.283              0.00214 **
## PC3            25.19      25.23   0.998              0.32409
## PC4            69.45      33.14   2.095              0.04252 *
## PC5          -229.04      36.50  -6.275          0.000000194 ***
## PC6           -60.21      48.04  -1.253              0.21734
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 242.3 on 40 degrees of freedom
```

```
## Multiple R-squared:  0.6586, Adjusted R-squared:  0.6074
## F-statistic: 12.86 on 6 and 40 DF,  p-value: 0.00000004869
```

Notice that our adjusted R-squared value is 60.7%. This is significantly lower than what we saw in the previous homework (somewhere around 75-80%), but we've significantly reduced the number of variables we're using.

With that in hand, we'll transform our data back, since PCA performs a linear transformation to our dataset.

```
## [1] "Great work. Let's keep on going."
```

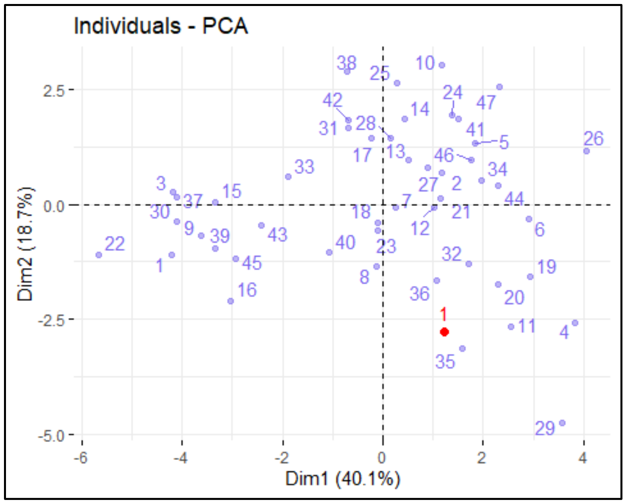
```
##        V1
##  Min.   : 273.7
##  1st Qu.: 705.3
##  Median : 884.0
##  Mean   : 905.1
##  3rd Qu.:1102.2
##  Max.   :1874.5
```

The estimates represent what the PCA model expects our crime data to be based on the six principle components we identified. These are not exact, because our PCA model really only explains about 90% of the variation within our dataset.

As we did earlier, now, let's predict on the dataset we've been provided. First we'll store all the new data we've been provided into a data frame.

```
##        PC1       PC2      PC3       PC4       PC5      PC6         PC7
PC8
## 1 1.224044 -2.767641 0.533605 -1.146837 -1.206098 2.333343 -0.1535916 -1.3
91625
##        PC9       PC10       PC11      PC12      PC13      PC14     PC15
## 1 1.460274 -0.4525158 -0.3466498 1.663782 -1.811307 -2.174071 1.288675
```

So where does this new "state" fit? We'll use the graph we saw earlier and add this new point, manipulated utilizing our PCA, as a red dot on the graph.



Now we'll predict what the crime rate would now be by leveraging the new data that's had PCA applied.

```
##        1
## 1248.427
```

Thus we predict that the crime rate in the new state would be 1248 crimes per 100,000 population. This seems within the range of reason for our data, especially given that the mean of our crime data is **905.0851064** and the range goes from **342** to **1993**.

Decision Trees

In this section, I'll find the best model available using: - Regression Trees - Random Forest

Regression Tree

Recursive partitioning is a fundamental tool in data mining. It helps us explore the stucture of a set of data, while developing easy to visualize decision rules for predicting a categorical (classification tree) or continuous (regression tree) outcome. Our formula below will be in the format outcome ~ ., which allows us to predict on all of our variables.

We'll grow our tree using the rpart function from the rpart package to create our regression tree.

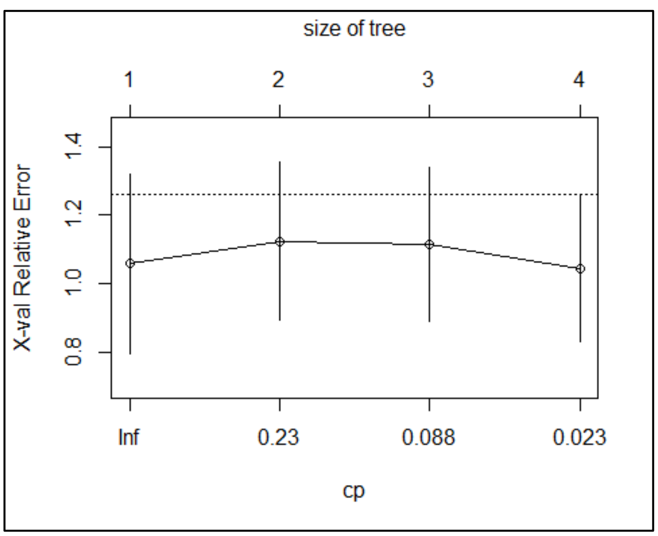
```
## Call:
## rpart(formula = Crime ~ ., data = data_scaled_fix, method = "anova")
##   n= 47
##
##           CP nsplit rel error   xerror      xstd
## 1 0.36296293      0 1.0000000 1.059023 0.2612971
## 2 0.14814320      1 0.6370371 1.123717 0.2301792
## 3 0.05173165      2 0.4888939 1.113252 0.2247664
## 4 0.01000000      3 0.4371622 1.045636 0.2140906
```

```
##
## Variable importance
##    Po1    Po2 Wealth   Ineq   Prob      M     NW    Pop   Time     Ed
LF
##     18     17     11     11     10     10      9      5      4      4
1
##
## Node number 1: 47 observations,    complexity param=0.3629629
##   mean=-1.039358e-16, MSE=0.9787234
##   left son=2 (23 obs) right son=3 (24 obs)
##   Primary splits:
##       Po1    < -0.2860126 to the left,  improve=0.3629629, (0 missing)
##       Po2    < -0.2944798 to the left,  improve=0.3629629, (0 missing)
##       Prob   < -0.2305884 to the right, improve=0.3217700, (0 missing)
##       NW     < -0.2395015 to the left,  improve=0.2356621, (0 missing)
##       Wealth < 1.022034   to the left,  improve=0.2002403, (0 missing)
##   Surrogate splits:
##       Po2    < -0.2944798 to the left,  agree=1.000, adj=1.000, (0 split)
##       Wealth < 0.07894027 to the left,  agree=0.830, adj=0.652, (0 split)
##       Prob   < -0.1536433 to the right, agree=0.809, adj=0.609, (0 split)
##       M      < -0.4833422 to the right, agree=0.745, adj=0.478, (0 split)
##       Ineq   < -0.5639655 to the right, agree=0.745, adj=0.478, (0 split)
##
## Node number 2: 23 observations,    complexity param=0.05173165
##   mean=-0.6088395, MSE=0.2264937
##   left son=4 (12 obs) right son=5 (11 obs)
##   Primary splits:
##       Pop < -0.3708059 to the left,  improve=0.4568043, (0 missing)
##       M   < 0.5112762  to the left,  improve=0.3931567, (0 missing)
##       NW  < -0.4583118 to the left,  improve=0.3184074, (0 missing)
##       Po1 < -0.9253348 to the left,  improve=0.2310098, (0 missing)
##       U1  < -0.1368969 to the right, improve=0.2119062, (0 missing)
##   Surrogate splits:
##       NW   < -0.4583118 to the left,  agree=0.826, adj=0.636, (0 split)
##       M    < 0.5112762  to the left,  agree=0.783, adj=0.545, (0 split)
##       Time < -0.6063828 to the left,  agree=0.783, adj=0.545, (0 split)
##       Ed   < 0.2558061  to the right, agree=0.739, adj=0.455, (0 split)
##       Po1  < -0.9589833 to the left,  agree=0.739, adj=0.455, (0 split)
##
## Node number 3: 24 observations,    complexity param=0.1481432
##   mean=0.5834712, MSE=1.003931
##   left son=6 (10 obs) right son=7 (14 obs)
##   Primary splits:
##       NW   < -0.2395015 to the left,  improve=0.2828293, (0 missing)
##       M    < -0.6424812 to the left,  improve=0.2714159, (0 missing)
##       Time < -0.6628885 to the left,  improve=0.2060170, (0 missing)
##       M.F  < 0.3047006  to the left,  improve=0.1703438, (0 missing)
##       Po2  < 0.6174944  to the left,  improve=0.1659433, (0 missing)
##   Surrogate splits:
##       Ed   < 0.792143   to the right, agree=0.750, adj=0.4, (0 split)
```

```
##       Ineq < -0.7895516 to the left,  agree=0.750, adj=0.4, (0 split)
##       Time < -0.6628885 to the left,  agree=0.750, adj=0.4, (0 split)
##       Pop  < -0.1738065 to the left,  agree=0.708, adj=0.3, (0 split)
##       LF   < 0.6757556  to the right, agree=0.667, adj=0.2, (0 split)
##
## Node number 4: 12 observations
##   mean=-0.9168028, MSE=0.135826
##
## Node number 5: 11 observations
##   mean=-0.2728796, MSE=0.1090716
##
## Node number 6: 10 observations
##   mean=-0.04701877, MSE=0.3727469
##
## Node number 7: 14 observations
##   mean=1.033821, MSE=0.9680209
```

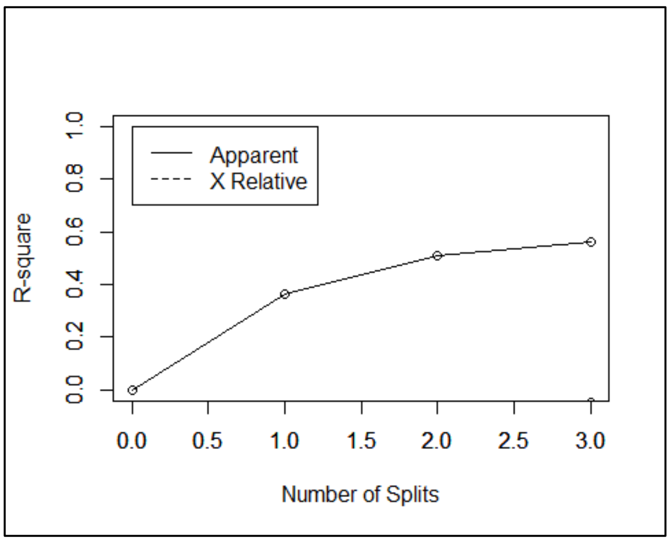
Immediately, we see that our four most important variables in this tree model are:
1. Po1 (per capita expenditure on police protection in 1960)
2. Po2 (per capita expenditure on police protection in 1959)
3. Wealth (median value of transferable assets or family income)
4. Ineq (percentage of families earning below half the median income)

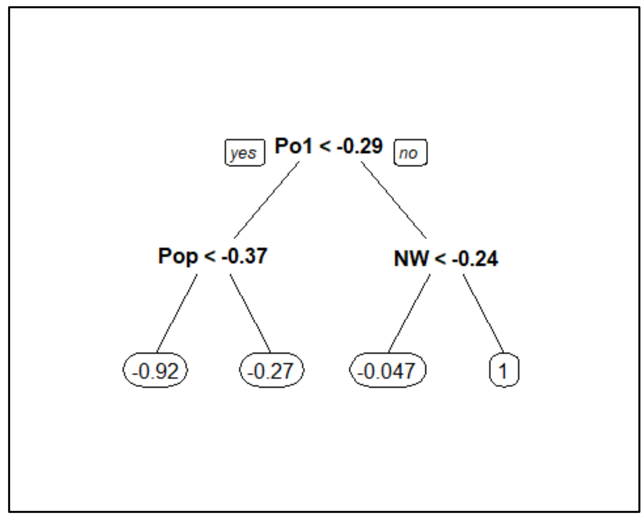
```
##           CP nsplit rel error   xerror      xstd
## 1 0.36296293      0 1.0000000 1.059023 0.2612971
## 2 0.14814320      1 0.6370371 1.123717 0.2301792
## 3 0.05173165      2 0.4888939 1.113252 0.2247664
## 4 0.01000000      3 0.4371622 1.045636 0.2140906
```



Notice from the graph of the relative errors, that our best split occurs when cp = 4, which is where the lowest cross valiadation error occurs. This means that the optimal number of splits for our tree is 4. This value occurs at cp = **0.01**.

```
##
## Regression tree:
## rpart(formula = Crime ~ ., data = data_scaled_fix, method = "anova")
##
## Variables actually used in tree construction:
## [1] NW  Po1 Pop
##
## Root node error: 46/47 = 0.97872
##
## n= 47
##
##          CP nsplit rel error xerror    xstd
## 1 0.362963      0   1.00000 1.0590 0.26130
## 2 0.148143      1   0.63704 1.1237 0.23018
## 3 0.051732      2   0.48889 1.1133 0.22477
## 4 0.010000      3   0.43716 1.0456 0.21409
```

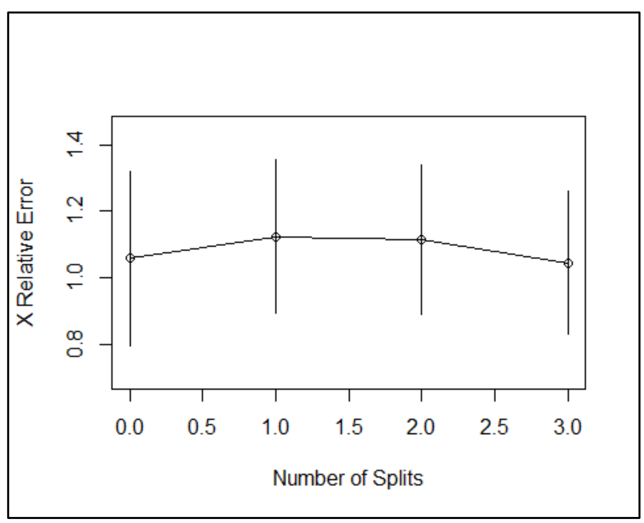




From the visualization of the regression tree above, we can see that Po1 is the most significant variable (which aligns with the output of our rpart() function), and then Pop and NW are the next most important. After that, not many of the variables are that important for our purposes.

Random Forest

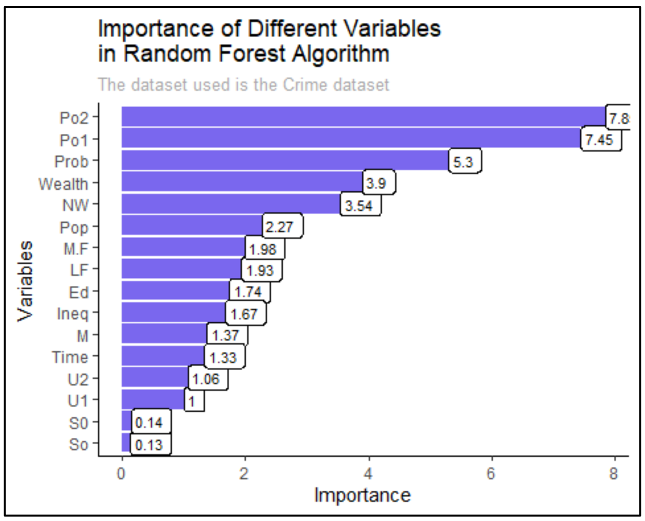
Now we'll compare our results to those we get from running a Random Forest model. The Random Forest model will make 500 decision trees and "vote" on the nodes that work best (i.e. provide the highest accuracy).



```
##
## Call:
##  randomForest(formula = Crime ~ ., data = data_scaled_fix)
##                Type of random forest: regression
##                      Number of trees: 500
## No. of variables tried at each split: 5
##
##           Mean of squared residuals: 0.570131
##                     % Var explained: 41.75
```

Here we note that the mean of the squared residuals is **0.570131**. The Random Forest algorithm we ran explains about 56.7% of the variation in our data, which is not great.

Which variables were the most important according to the Random Forest algorithm?



It's interesting to note here that the three most important variables for the Random Forest algorithm are similar to the three we found in the Regression Tree earlier:
1. Po1 (per capita expenditure on police protection in 1960)
2. Po2 (per capita expenditure on police protection in 1959)
3. Wealth (median value of transferable assets or family income)

Variable Selection

In this section, I'll attempt multiple methods for narrowing down our dataset by selecting the most important variables. As I narrow down the dataset, I'll compare different linear regression models to see if our accuracy improves.

Stepwise Regression

First we'll find the full linear regression model and then run Stepwise Regression, in which we'll slowly pick apart the full model by slicing out negligible coefficients.

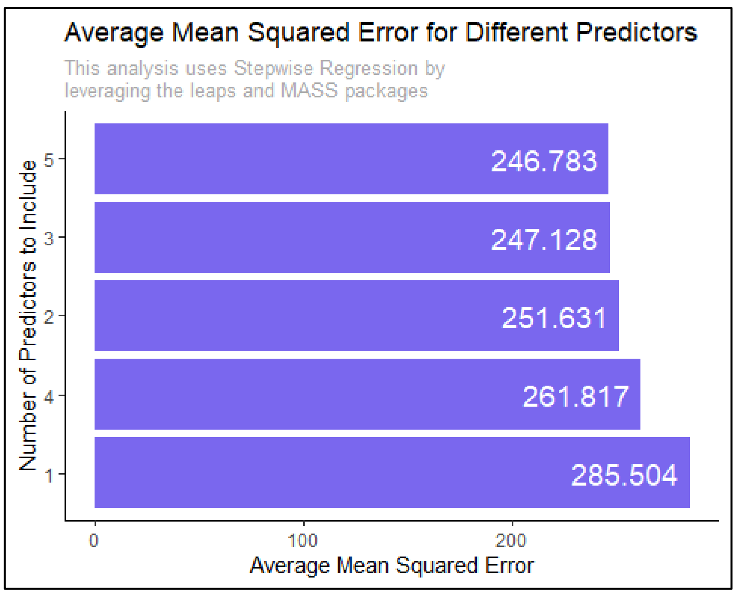
```
##
## Call:
## lm(formula = Crime ~ M + Ed + Po1 + M.F + U1 + U2 + Ineq + Prob,
##     data = data_scaled)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.14981 -0.28718  0.00784  0.31581  1.24960
##
## Coefficients:
##                           Estimate             Std. Error t value     Pr(>
|t|)
## (Intercept) -0.0000000000000003591  0.0737492532532140616   0.000      1.0
0000
## M            0.3032430653453815350  0.1088472638915121971   2.786      0.0
0828
## Ed           0.5209921876198559954  0.1525872689769799673   3.414      0.0
0153
## Po1          0.7887902674851503537  0.1192860516465484993   6.613 0.000000
0826
## M.F          0.1702060389736743118  0.1036268867503082197   1.642      0.1
0874
## U1          -0.2837258784713196924  0.1556584746932818675  -1.823      0.0
7622
## U2           0.4090916262204149501  0.1582795086237918925   2.585      0.0
1371
## Ineq         0.6326935328246856560  0.1439940625623334636   4.394 0.000086
3344
## Prob        -0.2231607928097278648  0.0876319928046151164  -2.547      0.0
1505
##
## (Intercept)
## M           **
## Ed          **
## Po1         ***
## M.F
## U1          .
## U2          *
## Ineq        ***
## Prob        *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5056 on 38 degrees of freedom
## Multiple R-squared:  0.7888, Adjusted R-squared:  0.7444
## F-statistic: 17.74 on 8 and 38 DF,  p-value: 0.0000000001159
```

This puts our R-squared value around 74% and p-value close to 0

Next we'll use the caret package – specifically the leaps and the MASS packages to fit our linear regression model using stepwise selection (leapSeq). Set up repeated 10-fold cross-validation, which will let us estimate the RMSE (average prediction error) for each of the 5 models specified by nvmax.

```
##   nvmax     RMSE  Rsquared      MAE   RMSESD RsquaredSD    MAESD
## 1     1 285.5037 0.5797686 231.0552 75.93507  0.3766473 64.65062
## 2     2 251.6307 0.5885311 191.6186 90.14342  0.3475418 60.22092
## 3     3 247.1283 0.5489734 197.2120 99.55289  0.3557170 77.35470
## 4     4 261.8166 0.5968752 206.0392 83.13210  0.3131785 69.53834
## 5     5 246.7830 0.6003050 199.6644 81.81135  0.2985455 60.59434

## We can see that an nvmax of 5 is the best model with an RMSE of 246.783
```



Our final model and coefficients are:

```
## Subset selection object
## 15 Variables  (and intercept)
##        Forced in Forced out
## M          FALSE      FALSE
## So         FALSE      FALSE
## Ed         FALSE      FALSE
## Po1        FALSE      FALSE
## Po2        FALSE      FALSE
## LF         FALSE      FALSE
## M.F        FALSE      FALSE
## Pop        FALSE      FALSE
## NW         FALSE      FALSE
## U1         FALSE      FALSE
## U2         FALSE      FALSE
## Wealth     FALSE      FALSE
## Ineq       FALSE      FALSE
```

```
## Prob        FALSE      FALSE
## Time        FALSE      FALSE
## 1 subsets of each size up to 5
## Selection Algorithm: 'sequential replacement'
##          M   So  Ed  Po1 Po2 LF  M.F Pop NW  U1  U2  Wealth Ineq Prob Time
## 1  ( 1 ) " " " " " " "*" " " " " " " " " " " " " " " " "    " "  " "  " "
## 2  ( 1 ) " " " " " " "*" " " " " " " " " " " " " " " " "    "*"  " "  " "
## 3  ( 1 ) " " " " "*" "*" " " " " " " " " " " " " " " " "    "*"  " "  " "
## 4  ( 1 ) "*" "*" "*" "*" " " " " " " " " " " " " " " " "    " "  " "  " "
## 5  ( 1 ) "*" " " "*" "*" " " " " " " " " " " " " " " " "    "*"  "*"  " "

## (Intercept)          Ed         Po1        Ineq
##  -3275.4088    157.8695    124.3143     75.0575
```

An asterisk indicates that a given variable is going to be included in our final model. For example, since we got the lowest RMSE of **246.783036** value with **5** predictors, we are going to include:

1. Ed (mean years of schooling of the population aged 25 years or over)

2. Po1 (per capita expenditure on police protection in 1960)

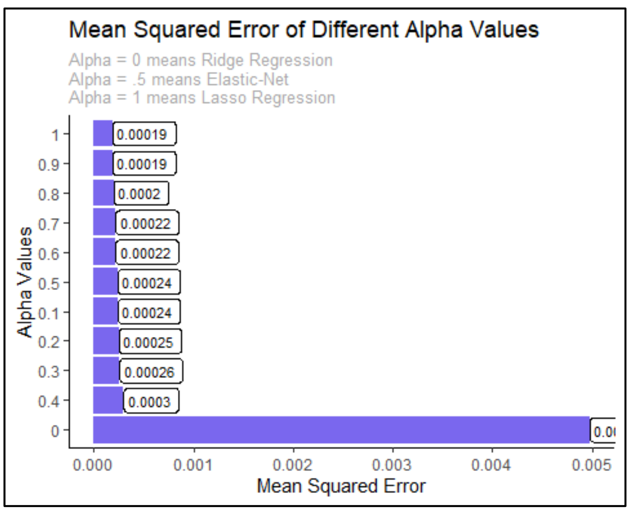
3. Ineq (income inequality: percentage of families earning below half the median income)

Thus our final model becomes:

```
## Crime = -3275.409+ (Ed x 157.8695) + (Po1 x 124.3143) + (Ineq x 75.0575)
```

Lasso and Elastic-Net Regression

After doing some research, I found that both variable selection methods use the same glmnet() functions. The only difference is an input parameter, denoted 'alpha', which differs between the two. So we'll just look at them simultaneously, along with a number of other variations of alpha values.



```
## From above, we can see that an alpha value of 0.9 produces our lowest Mean Squared Error of 0.00019.
```

Find our Model

Now that we know which alpha value gives us the lowest Mean Squared Error, we'll pull that model out and treat that as our linear regression model.

Find the alpha value's lambda.1se, which is the largest value of lambda such that error is within 1 standard error of the minimum.

```
## [1] 0.01398437
```

Run our prediction on our model. Note that s probably refers to the size of the penalty we are setting.

```
##                1
##  [1,] 0.98059186
##  [2,] 0.01001711
##  [3,] 0.98059186
##  [4,] 0.01001711
##  [5,] 0.01001711
##  [6,] 0.01001711
##  [7,] 0.98059186
##  [8,] 0.98059186
##  [9,] 0.98059186
## [10,] 0.01001711
## [11,] 0.01001711
## [12,] 0.01001711
## [13,] 0.01001711
## [14,] 0.01001711
## [15,] 0.98059186
## [16,] 0.98059186
## [17,] 0.01001711
## [18,] 0.98059186
## [19,] 0.01001711
## [20,] 0.01001711
## [21,] 0.01001711
## [22,] 0.98059186
## [23,] 0.01001711
## [24,] 0.01001711
## [25,] 0.01001711
## [26,] 0.01001711
## [27,] 0.01001711
## [28,] 0.01001711
## [29,] 0.01001711
## [30,] 0.98059186
## [31,] 0.01001711
## [32,] 0.01001711
## [33,] 0.98059186
```

```
## [34,] 0.01001711
## [35,] 0.01001711
## [36,] 0.01001711
## [37,] 0.98059186
## [38,] 0.01001711
## [39,] 0.98059186
## [40,] 0.98059186
## [41,] 0.01001711
## [42,] 0.01001711
## [43,] 0.98059186
## [44,] 0.01001711
## [45,] 0.98059186
## [46,] 0.01001711
## [47,] 0.01001711
```

Get our mean squared error

```
## [1] 0.0001944134
```

Now that we have our glmnet conducted, we'll figure out which coefficients we need so we can build out our linear regression model.

```
## Warning in model.matrix.default(mt, mf, contrasts): the response appeared
on the
## right-hand side and was dropped

## Warning in model.matrix.default(mt, mf, contrasts): problem with term 1 in
## model.matrix: no columns are assigned

##
## Call:
## lm(formula = Crime ~ ., data = new_crime)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -563.09 -246.59  -74.09  152.41 1087.91
##
## Coefficients:
##             Estimate Std. Error t value            Pr(>|t|)
## (Intercept)   905.09      56.42   16.04 <0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 386.8 on 46 degrees of freedom
```

We can see that our Adjusted R-squared value is 66%. Not all of the coefficients seem to have low p-values, and are thus not significant, so I would suggest taking more of a manual approach to sifting through the data and selecting variables.

Thanks for reading!