Various Air Platform Models

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Abstract

When conducting a simulation of an air-platform, many questions arise: What assumptions can I make about my model?, what level of fidelity is appropriate for conducting the required simulation? Are there ways of utilizing mathematics to analytically find solutions to the problems being posed?, and more. In this brief, we outline some various air-platform models and discuss some of the advantages and disadvantages of reduced and increased fidelity models. State space representations are also shown which aid the aspiring control theorist to conduct guidance, navigation, and control based simulations of aircraft which are appropriate for their problem set.

2D Simple Motion

The equations of motion for a simple 2D Platform can be seen below:

$$\begin{aligned} \dot{x} =& V cos\theta \\ \dot{y} =& V sin\theta \\ \theta =& u_1 \end{aligned} \tag{1}$$

In this model the control is the heading of the agent at a given time t. The position is given by x(t) and y(t), solutions to the differential equations. And as a result, the agent may change his heading instantaneously at any time. This is unrealistic, but allows for closed form solutions. Utilization of 2D Models requires that the altitude changes or differences between aircraft be small relative to their range or flight path. Subsonic cruising at an altitude for a long duration or range is a good example of a case where the 2D Simple Motion Model could be utilized.

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2D Motion with Turning Rate Constraints

In order to consider cases when the relative range between aircraft is small, or the motion of the aircraft is small relative to the turns it can make, we need to consider a minimum turning radius. Below is a model which limits the turn radius:

$$\begin{aligned} \dot{x} =& V cos\theta \\ \dot{y} =& V sin\theta \\ \dot{\theta} =& u_1 \quad |u_1| \leq \ \omega_{max} \, \forall t \in [t_0, t_f] \end{aligned} \tag{2}$$

Now we have imposed a turning rate constraint, ω_{max}, which limits the rate by which the agent my rotate in two-dimensional space. While this seems like a simple addition to the equation of motion, the use of Pontryagin's Minimum is now required for finding candidate solutions to the differential equations since limits on the dynamics exist. By introducing this limit, numerical methods are most likely required.

3D Simple Motion

In cases where the difference in altitude is large enough, relative to the range of multiple aircraft, we need to consider the change in altitude. There are also many other reasons to consider altitude in an aircraft model; but, kinematically, the need for altitude comes from changes in altitude large enough to consider stack height or climb/descent:

$$\begin{aligned} \dot{x}(t) =& V \cos\theta(t) \cos\psi(t) \\ \dot{y}(t) =& V \cos\theta(t) \sin\psi(t) \\ \dot{h}(t) =& V \sin\theta(t) \\ \theta(t) =& u_1(t) \\ \psi(t) =& u_2(t) \end{aligned} \tag{3}$$

For the equations of motion, we utilize the height, h, in order to reduce confusion with the North, East, Down axis assigned to the body frame. Although we still utilize the x and y axis, generally confusion with z and h is common.

3D Simple Motion with Acceleration (Throttle)

In an effort to model acceleration or the ability for an aircraft to vary in speed, we need to introduce some dynamics on the velocity, V. It is important that the initial conditions of the aircraft lie within the speed limits $[V_{idle}, V_{max}]$ for model stability.

$$
\begin{aligned}
\dot{x}(t) =& V\cos\theta(t)\cos\psi(t) \\
\dot{y}(t) =& V\cos\theta(t)\sin\psi(t) \\
\dot{h}(t) =& V\sin\theta(t) \\
\dot{V}(t) =& u_1(t) \quad u_1 \in [a_{min}, a_{max}] \mid V \in [V_{idle}, V_{max}] \\
\theta(t) =& u_2(t) \quad \theta \in [\theta_{min}, \theta_{max}] \\
\psi(t) =& u_3(t)
\end{aligned} \tag{4}
$$

It is important to note that we now have three inputs to the aircraft model: Acceleration, Pitch angle, and Heading angle. For this model, the aircraft may change its heading and pitch angle instantaneously and as a result, this model is not good for dealing with climb and heading rate constraints.

3D Motion with Climb and Heading Rate Constraints and Throttle

We can augment the dynamics of the aircraft model to include the heading angle and climb angle. By doing so, we can now limit the rate by which the aircraft can change its direction. This would be useful if trying to consider a kinematic analysis at closer range, but the inclusion of gravity and mass still are not present in the model.

$$
\begin{aligned}
\dot{x}(t) =& V\cos\theta(t)\cos\psi(t) \\
\dot{y}(t) =& V\cos\theta(t)\sin\psi(t) \\
\dot{h}(t) =& V\sin\theta(t) \\
\dot{V}(t) =& u_1(t) \quad u_1 \in [a_{min}, a_{max}] \mid V \in [V_{idle}, V_{max}] \\
\dot{\theta}(t) =& u_2(t) \quad u_2 \in [A_{dive}, A_{climb}] \mid \theta \in [\theta_{min}, \theta_{max}] \\
\dot{\psi}(t) =& u_3(t) \quad |u_3| \leq \xi_{max}
\end{aligned} \tag{5}
$$

New rates have been defined such as those dealing with climb and heading rate constraint, A and ξ respectively.

3D Point Mass Model of Aircraft

In an effort to add forces to the model, we want to consider the addition of Thrust, T; Drag, D; and gravity, g. Using look-up tables this model provides states such as altitude, airspeed, pitch angle, and angle of attack in order to obtain values of thrust and drag.

$$
\begin{aligned}
\dot{x}(t) =& V\cos\theta(t)\cos\psi(t) \\
\dot{y}(t) =& V\cos\theta(t)\sin\psi(t) \\
\dot{h}(t) =& V\sin\theta(t) \\
\dot{V}(t) =& \frac{1}{m}(\eta(t)T(t)\cos\alpha(t) - D(t)) - g\sin\theta(t) \\
\dot{\theta}(t) =& \frac{1}{mV}\left((\eta(t)T(t)\sin\alpha(t) + L(t)) - gm\cos\theta(t)\right) \\
\dot{\psi}(t)(t) =& \frac{\sin\sigma(t)}{mV\cos\theta(t)}\left(\eta(t)T(t)\sin\alpha(t) + L(t)\right) \\
\dot{\alpha}(t) =& u_1 \\
\dot{\sigma}(t) =& u_2 \\
\eta(t) =& u_3
\end{aligned}
\tag{6}
$$

Using this model, a more realistic model of an aircraft can be implemented, but at this point, analytic solutions to optimal controls problems are no longer possible. The use of computer programs is required in order to analyze the motion tracks of the aircraft. Furthermore, the tables which determine Lift and Drag are platform specific, and as a result, the analysis gained only applies to a specific aircraft.

3D Point Mass Model of Missile

Similar to the point mass model of the aircraft, the point mass model of a missile utilizes lookup tables in order to achieve the motion tracks of the missile. What is different about this model, is that a time delayed acceleration is applied to produce changes of velocity. This model has two inputs which are acceleration in the pitch and acceleration in the heading directions.

$$
\begin{aligned}
\dot{x}(t) =& V\cos\theta(t)\cos\psi(t) \\
\dot{y}(t) =& V\cos\theta(t)\sin\psi(t) \\
\dot{h}(t) =& V\sin\theta(t) \\
\dot{V}(t) =& \frac{1}{m}(T(t) - D(t)) - g\sin\theta(t) \\
\dot{\theta}(t) =& \frac{1}{V}\left(a_\theta(t) - g\cos\theta(t)\right) \\
\dot{\psi}(t)(t) =& \frac{a_\psi(t)}{V\cos\theta} \\
\dot{a_\theta}(t) =& \frac{u_1 - a_\theta}{\tau} \\
\dot{a_\psi}(t) =& \frac{u_2 - a_\psi}{\tau}
\end{aligned}
\tag{7}
$$